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# DEPARTMENT OF MATHEMATICS 

TEXAS A\＆M UNIVERSITY

## PROCEEDINGS OF THE <br> NASA WORKSHOP ON IMAGE ANALYSIS

Texas A\&M University College Station, Texas April 28-30, 1982

Original photography may be purchased from EROS Data Conter Sloux Falls, SD 57198

## Prepared for

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"Studies in Mathematical Pattern Recognition and Image Analysis"

# Introduction 

## by

K. S. Fu

Purdue University

The NASA Workshop on Image Analysis held on April 28-30, 1982 at Texas A\&M University, College Station, Texas, provided an opportunity for experts in the areas of pattern recognition, image processing, and remote sensing to assess past progress and to project future development in the area of image analysis with respect to remote sensing applications.

A block diagram of the general image analysis system is given in Figure 1. The preprocessing stage usually refers to filtering, enhancement, and/or coding of raw imagery data. The segmentation stage involves the determination of various regions of importance in the image. Features such as shape and texture measurement are then extracted from each region; a classification technique is often employed to recognize these regions. Once each region has been recognized and the relations among these regions have been identified, a complete description and possibly the interpretation of the image can be obtained through a structural analysis. A priori knowledge (the so-called "world model") of the images under study plays an important role in the design of each stage.

The program of the three-day workshop was devoted to the three major topics of image analysis: segmentation, shape and texture analysis, and structural analysis. A survey paper and two or three special papers were presented on each topic. Formal presentations were followed by panel discussions which assessed past progress and identified future research problems in each topic area.


Figure 1. Block diagram of image analysis system

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# NASA WORKSHOP ON IMAGE ANALYSIS 

Texas A\&M University
April 28-30, 1982
Room 510, Rudder Tower

## Wednesday, April 28:

| $8: 15-8: 30$ | Coffee and donuts |
| :--- | :--- |
| $8: 30-9: 30$ | Introduction |
|  | R. B. MacDonald, NASA/Johnson Space Center |
|  | Opening Remarks |
|  | L. F. Guseman, Jr. , Texas A\&M University |

9:30-10:00 Overview: Image Analysis
K. S. Fu, Purdue University

10:00-10:30 Coffee Break
10:30-11:30 Image Segmentation: A Survey
Robert M. Haralick, Iirginia Polytechnic and State University
11:30-12:30 Cooperative Processes in Image Segmentation Larry Davis, University of Maryland
12:30-2:00 Lunch

2:00-3:00 Dual Problems in Image Segmentation Jack Bryant, Texas A\&M University Susan Jenson, EROS Data Center

3:00 - 3:30 Coffee Break
3:30 - 5:00 Panel Discussion--Image Segmentation
Moderator: Azriel Rosenfeld
Panelists: R. K. Aggarwal
Jack Bryant
Larry Davis
Robert M. Haralick

Thursday, April 29:
8:15 - 8:30 Coffee and donuts
8:30 - 9:30 Shape and Texture Azriel Rosenfeld, University of Maryland

NASA WORKSHOP ON IMAGE ANALYSIS, cont'd.

| 9:30 | - 10:30 | Shape Identification Using 3-D Features Carolyn M. Bjorklund, Lockheed Palo Alto |
| :---: | :---: | :---: |
| 10:30 | - 11:00 | Coffee Break |
| 11:00 | - 12:00 | Automatic Photointerpretation Via Textural Feature Extraction <br> Julius F. Tou, University of Florida |
| 12:00 | - 1:30 | Lunch |
| 1:30 | - 2:30 | Target Screening <br> R. K. Aggarwal, Honeywell |
| 2:30 | - 3:00 | Coffee Break |
| 3:00 | - 4:00 | Panel Discussion--Shape and Texture <br> Moderator: Robert M. Haralick <br> Panelists: Carolyn M. Bjorklund Azriel Rosenfeld Julius Tou |
| Friday, April 30: |  |  |
| 8:15 | - 8:30 | Coffee and Donuts |
| 8:30 | - 9:30 | Structure Analysis Techniques for Remote Sensing Linda Shapiro, Virginia Polytechnic and State University |
| 9:30 | - 10:30 | Determining 3-D Motion and Structure from Image Sequences Thomas S. Huang, University of Illinois |
| 10:30 | - 11:00 | Coffee Break |
| 11:00 | - 12:30 | Panel Discussion--Image Structure Analysis <br> Moderator: Thomas S. Huang <br> Panelists: K. S. Fu <br> Linda Shapiro |

NASA WORKSHOP ON IMAGE ANALYSIS
April 28-30, 1982

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## Image Segmentation Survey

Robet M. Haralick
Virginia Polytechnic Institute and State University Dept. of Electrical Engineering Dept. of Computer Science Blacksburg, VA 24061

Image segmentation can be accomplished by a variety of techniques which in this survey we classify as:
Single linkage schemesHybrid linkage schemes
Centroid linkage schemes
Histogram Mode Seekirg Schemes
Spatial Clusterning schemes
Split and Merge Schemes

## Single Linkage Image Segnentation

Single linkage image segmentation schemes regard each pixel as a node in a graph. Neighboring pixels whose properties are similar enough are joined by an arc. The image segments are maximal sets of pixels all belonging to the same connected component. Single linkage image segmentation schemes are attractive for their simplicity. They do, however, have a problem with chaining, because it takes only one arc leaking from one region to a neighboring one to cause the regions to merge.

The simplest single linkage scheme defines similar enough by pixel difference. Two neighboring pixels are similar enough if the absolute value of the difference between their gray tone intensity value is small enough. For pixels having vector values, the ohyious generalization is to use a vector norm of the pixel difference vector. Instead of using a Euclidean distance, Asano and Yokoya (1981) suggest that two pixels be joined together if this absolute value of their difference is small enough compared to the average absolute value of the center pixel minus neighbor pixel for each of the neighhorhoods the pixels belong to. Haralick and Dinstein (1975), however, do report some success using the simpler Euclidean distance on LANDSAT data. The ease with which unwanted region chaining can occur with this technique limits its potential on complex or noisy data.

Hybrid single linkage techniques are more powerful than the simple single linkage technique. The hybrid techniques seek to assign a property vector to each pixel where the property vector depends on the $K x K$ neighborhood of the pixel. Pixels which are
similar, are similar because their neighborhoods in some special sense are similar. Similarity is thus established as a function of neighboring pixel values and this makes the technique better behaved on noisy data.

One hybrid single linkage scheme relies on an edge operator to establish whether two pixels are joined with an arc. Here an edge operator is applied to the image labeling each pixel as edge or non-edge. Neighboring pixels, neither of which are edges, are joined by an arc. The initial segments sre the connected components of the non-edge labeled pixels. The edge pixels can either be left assigned edges and be considered as background or they can be assigned to the spatially nearest region having a label.

The quality of this technique is highly dependent on the edge operator used. Simple operators such as the Roberts and Sobel operator may provide too much region linkage, for a region cannot be declared as a segment unless it is completely surroundrd by edge pixels. Haralick (1982) reports some success with thiv technique using the zero-crossing of second directional derivative edge operator.

Another hylorid technique first used by Levine and Leemet (1976) is based on the Jarvis and Patrick (1973) shared nearest neighbor idea. Using any kind of reasonable notion for similarity, each pixel examines its $K x K$ neighborhood and makes a list of the N pixels in the neighborhood most similar to it. Call this list the similar neighbor list, where we understand neighbor to be any pixel in the $K x K$ neighborhood. An arc joins any pair of
immediately neighboring pixels if there are enough pixels common to their shared neighbor lists; that is, if the number of shared neighbors is high enough.

To make the shared neighbor technique work well each pixel can be associated with a property vector consisting of its own gray tone intensity and a suitable average of the gray tone intensity of pixels in its KxK neighborhood. For example, we can have ( $x, a$ ) and ( $y, b$ ) denote the property vectors for two pixels where in the first pixel, $x$ is its gray tone intensity value and a is the average gray tone intensity value in its neighborhood. Likewise, for the second pixel, $y$ is its gray tone intensity value and $b$ is the average gray tone intensity value in its neighborhood. Similarity can be established by computing

$$
S=w_{1}(x-y)^{2}+w_{2}(x-b)^{2}+w_{3}(y-a)^{2}
$$

where $w_{1}, w_{2}$ and $w_{3}$ are non-negative weights. The pixels are called similar enough for small enough values of $s$.

## Begion Growing $L$ Centroid Li, age

In contrast to single linkage, in centroid linkage pairs of neighboring pixels are not compared for similarity. Rather, a pixel's value is compared to the centroid of an already existing but not necessarily completed segment. If the values are close enough, then the pixel is added to the segment and the segment's centroid is updated. If no neighboring region has a centroid close enough, nen a new segment is established having the given pixel's value as its first member. Such a region growing
technique was first suggested by Brice and Finnema (1970). Instead of using the absolute value of the difference as the measure of dis-similarity, Gupta, Kettig, Landgrebe, and Wintz (1973) suggest using the more appropriate t-test.

Simple single pass approaches which scan the image in a left right top down manner are, of course, unable to make the left and right sides of a $V$-shaped region belong to the same segment. To be more effective, the single pass must be followed by some kind of connected components algorithm in which pairs of neighboring regions having centroids which are close enough are put into the same segment.

One minor problem with centroid linkage schemes is their inherent dependence on the order in which pixels are examined. A left right top down scan does not yield the same initial regions as a right left bottom up scan or for that matter a column major scan. Usually, however, differences caused by scan order are minor.

## Histogram Mode Seeking

Histogram mode seeking is a measurement space clustering process in which the clusters in measurement space are mapped back to the image domain where the maximal connected components of the clusters constitute the image segments. For images which are single band images, calculation of this histogram in an array is direct. The measurement space clustering can be accomplished by determining the valleys in this histogram and declaring the clusters to be the interval of values between valleys. A pixel
whose value is in the $i^{\text {th }}$ interval is labeled with index $i$ and the segment it belongs to is a connected component of all pixels whose label is i.

For multiband images such as LANDSAT, determining the histogram in a multi-dimensional array is not feasible. For example, in a six band image where each band has intensitites between 0 and 99, the array would have to have $100^{6}=10^{12}$ locations. A large image might be 10,000 pixels per row by 10,000 rows. This only constitutes $10^{8}$ pixels, a sample too small to estimate probabilities in a space of $10^{12}$ values were it not for some constraints of reality: (1) there is typically a high correlation between the band to band pixel values and (2) there is a large amount of spatial redundancy in image data. Both these factors create a situation in which the $10^{8}$ pixels can be expected to contain only between $10^{4}$ and $10^{5}$ distinct 6-tuples. Based on this fact, the counting required for the histogram is easily done by hashing the 6-tuple into an array.

Clustering using the multidimensional histogram is more difficult than univariate histogram clustering. Goldberg and Shlien (1977, 1978) threshold the multidimensional histogram to select all N -tuples situated on the most prominent modes. Then they perform a measurement space connected components on these $N$ tuples to collect together all the $N$-tuples in the top of the most prominent modes. These measurement space connected sets form the cluster cores. The clusters are defined as the set of all N tuples closest to each cluster core. A variation on this idea is discussed by Matsumoto, Naka, and Yamamoto (1981)

An alternate possibility is to locate the highest mode and region grew around it in the multi-dimensional measurement space. The region growing includes all successive neighboring $N$-tuples whose probability is no higher than the $N$-tuple from which it is growing. This procedure identifies the most prominent mode and its associated mountain as the first cluster core. Then the same procedure is repeated on the remaining $N$-tuples until all multidimensional peaks and their associated cores have been accounted for. The clusters are defined as the set of all N-tuples closest to each core.

Rather than accomplish the clustering in the fuls measurement space, it is possible to work in multiple lower order projection spaces and than reflect these clusters back to the full measurement space. Suppose, for example, that the clustering is done on a four band image. If the clustering done in bands 1 and 2 yield clusters $c_{1}, c_{2}, c_{3}$ and the clustering done in bands 3 and 4 yield clusters $c_{4}$ and $c_{5}$ than each possible 4 -tuple from a pixel can be given a cluster label from the set $\left(c_{1}, c_{4}\right),\left(c_{1}, c_{5}\right)$, $\left.\left.i c_{2}, c_{4}\right),\left(c_{2}, c_{5}\right),\left(c_{3}, c_{4}\right),\left(c_{3}, c_{5}\right)\right\}$. A 4-tuple $\left(x_{1}, x_{2}, x_{3}, x_{4}\right)$ gets the cluster label $\left(c_{2}, c_{4}\right)$ if $\left(x_{1}, x_{2}\right)$ is in cluster $c_{2}$ and ( $x_{3}, x_{4}$ ) is in cluster $c_{4}$.

## Spatial Clustering

It is possible to determine the image segments without periorming an independent clustering in measurement space. Such techniques are called spatial clustering. In essence spatial clustering schemes combine the histogram mode seeking tecinique
with the region growing/centroid linkage technique. Haralick and Kelly (1969) suggested it be done by locating, in turn, all the peaks in measurement space. Then determine all pixel locations having a measurement on the peak. Beginning with a pixel corresponding to the highest peak not. yet processed, simultaneously perform a spatial and measurement space region growing in the following manner. Initially, each segment is the pixel from which we begin. Consider for possible inclusion into this segment the neighbors of this pixel (in general, the neighbors of the pixel we are growing from) if the neighbor's $N$ tuple value is close enough in measurement space to the pixel's value and if its probability is not larger than the probability of the pixel's value we are growing from.

## Split and Merge

The split method for segmentation begins with the entire image as the initial segment. Then it successively splits each of its current segments into quarters if the secment is not homogeneous enough. Homogeneity can be easily established by determining if the difference between the largest and smallest gray tone intensities is small enough. Algorithms of this type were first suggested by Roberston (1973) and Klinger (1973)

Because segments are successively divided into quarters the boundaries produced by the split technique tend to be squareish and slightly artificial. Sometimes adjacent quarters cuming from adjacent split segments need to be joined rather than remain separate. Horowitz and Pavlidis (1976) suggest a split and merge strategy to take care of this problem Chen and Pavlidis (1980)
suggested using statistical tests for uniformity rather than examination of the difference between largest and smallest gray tone intensities.

The data structures required to do a split and merge on images larger than $512 \times 512$ are extremely large.' Execution of the algorithm on virtual mememory computers results in so much paging that the dominant activity is paging rather than segmentation. Browning and Tanimoto (1982) give a description of a split and merge scheme where the split and merge is first accomplished on mutually exclusive subimage blocks and the resulting segments are then merged between adjacent blocks to take care of tne artificial block boundaries.

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## Cooperative processes in image segmentation

Larry S. Davis

This talk will survey recent research into the role of cooperative, or relaxation, processes in image segmentation. Cooperative processes [1] can be employed at several levels of the segmentation process as a preprocessing enhancement step $[2,3,4]$, during supervised or unsupervised pixel classification $[4,5]$ and, finally, for the interpretatin of image segments based on segment properties and relalions [6].

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6. L. Kitchen, "Scene analysis by region based constraint propagation," in preparation.
[^0]
# Cooperative Processes in Image Analysis 

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## Cooperative Processes

Goal: Assign symbolic and numerical labels to picture parts

- ASSIGN SYMBOLIC LAND-USE CATEGORIES TO PIXELS
- ASSIGN NUMERIC STEREO DISPARITY LABELS TO PIXELS

Constraint: Images are large, so labeling process must be fast

- SEQUENTIAL: TOO SLOW
- PARALLEL: TOO ERROR PRONE
Solution: Cooperative processes
- assess each part independently (parallel)
- compare assessments of "relevant" parts (parallel)
- COMPARISONS MUST BE LOCAL
- entire process is iterative


## Organization

A) Initial, independent part assessment
b) adJustments of assessments based on relationships between parts
C) iteration of step (b)
2. Application to motion detection Assume

> 1) IMAGE INTENSITY IS A CONTINUOUS, DIFFERENTIABLE FUNCTION $f(x, y, T)$
2) The intensity corresponding to any given scene POINT DOES INOT CHANGE OVER TIME
3) Both the motion (u,v) and the time interval, $\tau$, between frames is small enough that a Maclauren SERIES EXPANSION IS A GOOD LOCAL APPROXIMATION to the pictuke function.

APproximate $F(x+u, y+v, T+\tau)$ by a SERIES EXPANSION AbOUT ( $x, y, y$ ) (which cali be regarded as ( $0,0,0$ )).
$F(X+ن, Y+V, T+\tau)=F(X, Y, T)+F_{X} \cdot u+F_{Y} \cdot V+F_{T} \cdot \tau$
thilgher ORDER TERMS

Areitrarily set $\tau=1$ anl note that (2) implies $F(x+u, \gamma+V, T+\tau)=$ $F(X, y, T)$

$$
\left.-F_{T}=F_{X} U+F_{Y} V \quad \begin{array}{l}
\text { ORIGINAL PAGE IS } \\
\text { OF POOR QUALITY }
\end{array}\right\}
$$

NOTE:

1) If $\mathrm{F}_{\mathrm{X}}, \mathrm{F}_{\mathrm{Y}} \approx \dot{\sim}$, then motion information cannot be accurately determined
2) IF $F_{X}=J$, THEN $-F_{T}=F_{Y} \vee$ SO that $v$ is determined but U IS UNKNOWN
3) Let g and $\ell$ denote the gradient and level directions at a pixel.

$$
\begin{aligned}
& G=T A N^{-1} F_{Y} / F_{X} \\
& \ell \perp G, F_{\ell}=O
\end{aligned}
$$

Then $F_{T}=F_{G} D_{G} / D_{T}$ so component of velocity in the gradient direction is known, but not in the level direction.

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2. Rough estimates by Pseudo-is

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## Combining Motion Constraints

1. Assume that u,v are constant over small regions of the image and obtain a local least square solution for $u, v$.
C. Cafforio aind F. Rocca, "Tracking moving objects in T.V. Images," Signal Proc., 1, 1979, 133-140.
J. Limb and J. Murphy, "Estimating the velocity of moving images in T.V. signals," CGIP, 4, 1975, 3il-327.
2. Assume that u,v vary smoothly over the image and use relaxation techniques to compute "optimal" values at u,v. b.K.Y. Horn and b'. Schunck, "Jetermining optical flow," ARtificial Iintelligence, $1 \%=1901$, 185-203.


Supervised Relaxation

- Statistical models for various textures
$M_{1} \quad M_{2} \quad \ldots \quad M_{n}$

| $P_{1}$ | $P_{2}$ | $P_{3}$ |
| :--- | :--- | :--- |
| $P_{4}$ | $X$ | $P_{5}$ |
| $P_{6}$ | $P_{7}$ | $P_{8}$ |

1) $\operatorname{Prob}\left[x\right.$ interior to $\left.M_{1}\right]=\prod_{i} \operatorname{prob}\left[p_{j} \in M_{1}\right] \cdot \operatorname{prob}\left[x \in M_{1}\right]$
2) $\operatorname{Probi}\left[X \in M_{1}\right.$ and on a vertical edge separating
$M_{j}$ from $\left.M_{1}\right]=$

$$
\prod_{j=1,4,6} \operatorname{prob}\left[p_{j} \in M_{j}\right] \cdot \prod_{j \neq 1,4,6} \operatorname{prob}\left[p_{j} \in M_{1}\right] \cdot \operatorname{prob}\left[X \in M_{i}\right]
$$

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Label Set $=\left\{m_{1}, m_{2}, m_{3}, \ldots, m_{n}\right.$

$$
v_{12}, v_{23}, \ldots, v_{n-1, n},
$$

$$
H_{12}, H_{23}, \ldots, H_{n-1, n^{3}}
$$

## Iterative procedure:

For each pixel, X :

1) compute the most probable label for $X$
2) Smooth $X$ "appropriately"

EX: 1) X interior - average with all neighbors
2)

| $P_{1}$ | $P_{2}$ | $P_{3}$ |
| :--- | :--- | :--- |
| $M_{1}$ | $M_{1}$ | $M_{4}$ |
| $P_{4}$ | $X$ | $P_{5}$ |
| $M_{1}$ | $M_{1}$ | $M_{4}$ |
| $P_{6}$ | $P_{7}$ | $P_{8}$ |
| $M_{1}$ | $M_{1}$ | $M_{4}$ |

average X with
$\left(P_{1}, P_{2}, P_{4}, P_{6}, P_{7}\right)$

## ORIGINAL PACE IS OF PCNOR QUALITY


（a）
（b）


Figurc 3．Noisy squares（a）and initial labelling（b）．

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## ORIGINAL PAGE IS' OF POOR QUALITY



- Figum 13 b


## ORIGINAL PAGE IS


(a)

(b)

Figure 32. 15th iteration smoothing (a) and labelling (b). $3 \times 3$ simple smoothing.

## ORIGINAL PAGE IS OF POOR QUALITY.


(a)

(b)

Figure 33. 15th iteration smoothing (a) and labelling (b) $-E^{5}$.

## ORIGINAL PPAE IS OF POOR QUALITY


(a)

(b)

Figure 31. 15th iteration smoothing (a) and labelling (b) MITES $3 \times 3$.

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(a)

Figure 19 (a) Concrete in Grating
(b) Grating left and Bricks right


## ORIGINAL POOR QUALITY




UMITES
E/G 7 th Iteration Labelling

## ORIGINAL PAEE IS OF POOR QUALITY

## MuLTI-TEMPOPAL IMAGE SEGMENTATION

```
Objects - pixels
LabelS - region class names
```

Relaxation rules can be heuristically derived from the following ObSERVATION:

Fields are, in ge enal, much bigger than a single pixel and dO NOT CHANGE LABELS DURiNG A SINGLE GROWING SEASON.

$$
\begin{aligned}
P_{\lambda}^{K}(I, J, T)- & \text { Probability that the pixel at spatial location } \\
& (I, J, T) \text { and time } T \text { is in class } \lambda \text { after K itera- } \\
& \text { } \operatorname{tiONS} \text { OF the relaxation rule }
\end{aligned}
$$

RELAXATION RULES

Rule schema

$$
\begin{aligned}
P_{\lambda}^{K+1}(I, J, T)= & \frac{1}{N} P_{\lambda}^{K}(I, J, T) \underset{T^{\prime} \neq T}{\times} P_{\lambda}^{K}\left(I, J, T^{\prime}\right) \\
& * F\left(\left\{P_{\lambda}^{K},\left(I^{\prime}, J^{\prime}, T\right): \quad \lambda^{\prime} \in L,\left(I^{\prime}, J^{\prime}\right)\right.\right. \text { A NEIGHBOR } \\
& O F(I, J)\})
\end{aligned}
$$

SPECIFIC RULES

$$
F_{1}=\operatorname{mAX}_{I=0}^{7} P_{\lambda}^{K}\left(X_{I}, T\right)
$$

| $x_{7}$ | $x_{0}$ | $x_{1}$ |
| :---: | :---: | :---: |
| $x_{6}$ | $(1, J)$ | $x_{2}$ |
| $x_{5}$ | $x_{4}$ | $x_{3}$ |

$$
\begin{aligned}
& F_{2}=\max _{\substack{\operatorname{MAX}}}^{\prod_{I=0}^{2}} P_{\lambda}^{K}\left(X_{I}+J, T\right) \\
& F_{3}=\sum_{J=0,2,4,6}^{\sum} P_{\lambda}^{K}\left(X_{J}, T\right)
\end{aligned}
$$

## Scene Analysis－Tanks world

## TANKSWORLD

Ground
SKy
Smoke
Tank
Tree
Labels

Tree fragments
Ulnary constraints
Binary constraints
Existential constraints


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Examples
Binary constraints (arcs)Tank cannot surround smokeIf (tank, smoke) $\in L_{I} \times L_{J}$ and if $N_{I}$ surrounds $N_{J}$, thendelete (tank, smoke) from $L_{I} \times L_{J}$.
Existential constraints
A tree fragment must be adjacent to a tree fragmentIf tree fragment $\in L_{1}$ and if for no adjacent $L_{J}$ is(tree fragment, tree fragment) $\in \mathrm{L}_{\mathrm{I}} \times \mathrm{L}_{\jmath}$, then eliminatetree fragment from $L_{I}$.

## ORIGINAL PAGE IS OF POOR QUALITY

cooperative processes in thace awlysis

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#### Abstract

Abartact. This paper containg on overviev of the organization of cooperative (or zelazation) proceses for low-level viaion. Tvo exemples (one involving pirel claseification and the other motion disparity eatimation) are used to llustrate the various steps in applying a relaration algorith to inage analysis protleas.


Note: This paper also appears in the Proceedings of IFAC 82.

## 1. ISSUES

Many inage analyais probleas can be regarded an probleas of aseigning a label to each element in a set of picture parts (pisels or regions). For eximple, pixela can be assigned sybbollic land use category labela based on their apectral signaturas, or numer 1cal sotion diaparity labels based on local comparieons anor: it consecutive frases in a tine-varyinc inage. Both of these probleas will be used as eramples 'throughout this paper.

The large number of pixele in a digital inage deands that ruch labelling processes be very fast. One obvious solution to this proble is to make the labelling process highly parallel. Hovever, in a parallel process, each picture part would be analyzed independently of all other picture parts. Thus, parsilel processes fail to make use of contextuai information (which is often available), and ake man labelling errors.

In order to overcane this probles, one can assess the labelling possibilities for every part independently, and then compare each part's asaesseats to those of other, related parti, in order to correct inconaistencies. Since both the asseament and the coaparieon can be done independently for every part. each atage of the process is parallel. On the other hand, context is now being used at the coaparison stage, vhen related parta are able to comanicate and 'cooperate'. To heep the computational cost lou, the comparisons should be local; they ahould involve only parts that are directiy related (e.s., neighboring pisele). This localness can be compenseted for by iterating the comparison proceas, in order to allow information to propagate.

These considerations lead maturally to the deasg of a 'cooperative' approach to labelling picture parts which allows context to be used in the labelling proceas while atill pernitting fast parallel implementation and low computat lonal cost. Such processes are called 'relamation' processes, because of their resemblance to certais iterdt ive procesaes used in mumerical analyais. Very senerally, a relaration process is organised as followe:
(a) A list of possible labels is independently eelected for each part, based on it intrinaic characteristics. A meamure of confidence can al wo be associated vith each possible label.
(b) The posaibilities (and confidences) for each part are compared with thooe for related parte, beed on a model for the relationships betveen the possible labels of picture parta. Labele are deleted or modif ied or confidences are adjuated, co reduce inconsistencies.
(c) Step (b) can be iterated as many times as required.

This approach is very general: We have not apecified how to formiate label relationship sodels, choose poasibilities, egrimite confidences, or adjuat then; mor have ve discussed when the process should be iterated. and If so, how many times.

These issues are discussed is soee detail in [1], and, in general, are a function of the proble at hand. For a general discussion, the reader should coasult [1]. Ve vill. Inatead, consider two apecific probleas in detail - pixel claseification and notion
disparity eatiantion. The firat involves a syabollic label set and the aecond a numeric oae. The first involves conf idence adjuatement and the eecond, additionally, Involves modification of aumeric labale. Sectic an 2,3, and 4 will conalder atepa $a, b$, and $c$ above for these two probles.

## 2. 1MITIAL inatl ASSICNODT

The firat atep of a ralanation procese involvea asaigning initial labals to each picture part. If the given labal set is oyybollic, then thia is ordinarily doae uaing techaiques free otatiatical patzern recogaicion. Were, measurenento are computed for each picture part and, baeed on a priori sodele of the clase conditional denaitica of these meacureacate a probability that each label io the correct one for a pleture part can be coaputed. Theae probabilities eerve as the measurea of coaf idence referred to in Section 1. Some examples of aybollic label sets, $L$, and asociated picture part meaourceats, $\mathrm{K}_{\text {, }}$ ara:
(1) In a "epring-loaded" teaplate matching proble [2] $L$ would be $t$ he eet of eubteplate mases and M the crose-correlation of the aubteplate ot a particular inage poaition. See [3] for detalle.
(2) In dot cluater detection, $L=$ (interior point, dge point, noise point) and $M$ are meapur ments of local dot density. See (4) for detaile.
(3) In supervised pizel classification, 1 1s the aet of given clase names, and M ordinarily containe opectral and, posaibly, local textural features.

Pursuing the pixel clasaification in more detail. Figura la-t ahows ais fages of a Lacie teat site (1473). These ais inages are from three time acquisitions and two bands ( 2 and 3). Ground truth is avallable for this eite, and $1 t$ can be used to computa the class conditional probability denaitias at each time for the i-vactor of meaguresente at each tise. Dy modeling theas denaities as mornal, we can adopt the folloving aimple procedure for deadopt the follouing aiaple procedure for pixel at each time:
(1) Compute the folloving diatance meeoure of a pirel, m, frome clase, d:

$$
d_{\lambda}(x)=\log \left|\Sigma_{\lambda}\right|+(x-y)^{T} \Sigma_{\lambda}^{-1}(x-y)
$$

Here, $y_{\lambda}$ is the mean vector for clams $\lambda$ and $I_{\lambda}$ is ita covariance matrix.
(2) Next, coapute the probability that pixel a belongs to class $\lambda$ by:

$$
P_{\lambda}(x)=\left[1 / d_{\lambda}(x)\right] / \sum_{\lambda}\left[1 / d_{\lambda},(x)\right]
$$

If one vere forced to decide on a fined label for a pixel, x, then one would choose the for which $p$ (x) is maximal. As mentioned earlier, such an independent classification containa may erroris. Figura 2a-l illuatrates this. In the next section wo deacribe how aimple relanation procese can improve on these resulte.

In many apflicatioas, the matural labal oet Is a numeric property value rather than aymbllic, and the label set at each picture part is often initially represented by the mont current eatimate of the mat ilhaly property value at that picture part and a meabure of confidence often related to the varlance of local property values. For emaple, in relazation algoritime for grey level iage enhanceaent, the initial property value at each pinel is the pixal'e grey level. and the meagure of confidence, for plecewise conatant imagea, would be the gray ievel variance in sone neighborhood of the picture.

A mure conplicated example is motion dieparity eatimation. Mere, the initial property value is a motion vector which can be coeputed as follovs. At each pirel, one can compute a linear coastraint on the $z$ and $y$ component ( $u, v$ ) of that pisel's motion beed on the equation:

$$
\begin{equation*}
-I_{t}=I_{z} u+1_{y} v \tag{1}
\end{equation*}
$$

Here, 1 , is the teaporal inge intenaity change Ind I and I represent the opatial image intensity gradient. We assume here that grey leval is an invariant of motion. Assuaning that the inage motion is locally a translation, then one could coabine the equations at two adjacent points to obtain a unique velocity vector at each point, or, eore unique velocity vector ally, one can compute the "pseendo-intereection" of the 1 inear constrainte in a neighborhood of each pixel (5). In this case, the arror of the fit can be used as a measure of confidence at each pixel, although we will ignore the confidences in what follows.

As an exampla, Figure 3 coota ine two frames in a tima varying inage, and Figure 4 containg the initial velocity vector computed by the paeudo-interaection techaique.

## 3. DLaxution

After initial labellinge (and confidencea) are computed, one begita a sequence of iterat fons where the labels and confidences at each picture part are modified based on the diatribution of labela and coofidences on related picture parte and agee nodel of how labellings affect one another. Thus, the relamation algorith is besed on:
a) aodel for the neighborhood of a picture part, and
b) aodel for the interactions between i, bellinge of adjacent picture parts.

The neig hborhood solel for a relazation precess opec *A:.e which paire of picture parts directiy comunicate with one another in the relamation process, and detacaiaes the topelogy of the graph on vhich the relazation process operstes. This graph has Individual picture parts as sedes. Its arce conaect those pairs of parte that congunicate vith one another. The nelighborhoed model is ugually desigaed to eatablish connections only between 'yearby' parta.

A neighborhuod sodel 10 opecified by a oet of anighbor zelat leas F - $\left(\mathrm{r}_{1} ; \mathrm{r}_{2}, \ldots, \mathrm{r}_{\mathrm{p}}\right)$. Each rf is a biany relation difined over the applepriate est of picture parta. For example. If the picture parta are pirela, then the neighborhood eodel aight opecify that a pirel Io connected to erary pirel ite $3 \times 3$ naighberhood. In this case, there are ot ill several possibilities for the relations contained in the eet r . Por example, F aight be the set idirectly above, directiy below, etc.) which would diatiaguish betveen pairs of pointe that are horisontally adjaceat, vertically odjacent, etc.., or it could be the singletea relat ion "In the $3 x 3$ meighborhood". In the Latter cace, the connectioas between paira of pinels would not be recoverable froe the graph on which the relasation process vill operate. The choice of $\mathrm{r} v i 11$, in general, be deterained by the isotropy of the universe of labela. For eample, if ve are deaigning - relaxation procese for edge reinforcement then the relative positions of pixela are crucial aince edgea generally 'line up'. while if we are deaigning a relazation procase to enhance an iage'e grey levels, then the posit ional information may not be required.

When the picture parts are regions rather than'pixels, than connections inght be fored between edjacent regions only. In some situations, it aight be mecessary to diat iaguiah between regions that ara above, belou, inaide, surrounding, etc.

The interaction model defises how a picture part changes its label. ig besed on the labellinge of ita aeighbors. An interaction model is composed of two parte:
(1) a knowlede representation for the relationships between labels, and
(2) acchanise, or procedure, for applyiss the lmouledge in (1) to change, or update, labellinge.

For diacrate sybbllic labellings the ainplest hnowledge representation is a eet of the paire of labels that can afmultaneoualy be aseociated vith pairs of neighboring picture parta. It can be represented by a binary ralation defined ovar the universe of labele $D$. Intuitively. (d, d') f

If a pair of neighbora can afaltanoously be labelled with dand $f^{\prime}$. In genaral, there is a binary ralation asuociated vith aech neighbor relation.

The wost obvious updating eechanif for diserate, aymbilic labelling is a label ilecarding procese, which looks at paire of picture parte at a time. 4 ishel, $d_{\text {, can be }}$ deleted fro the labelliag of o pleture part if, for soe neighboring picture part, that neighbor does not conta in abel, $\mathrm{d}^{\prime}$. In ite laballige vith $\left(\mathrm{d}, \mathrm{d}^{\circ}\right)$ (R.

The binary relation moowledge reprenentation can be generalized to oybollic labellings with confidences for ach oyphollic label by opecifying a real-valued compatiblity function, $C$, whoee domis io DV. As befors, in seneral, a copatibility function io defined for each pictura relation in the oet F . A variety of applications have uoed compatibility functions whose range is ( $-1,1$ ). Intuitivaly, if $C\left(d, d^{\prime}\right)=-1$, than and $d^{\prime}$ are animially incompatibla, and the otrong presence of $d^{\prime}$ at one pleture part (1.e.. $d^{\prime}$ has a high likalihood at that part) ohould depreas tive likelihood of at a neighboring picture part. If $C\left(d, d^{\prime}\right)=1$, then $d$ and $d^{\prime}$ are maximally compatible, and the atroag presence of $d^{\prime}$ at a picture part should incresse the likelihood of $d$ at a neighboring picture part. Finally, if $C\left(d, d^{\prime}\right)=0$, then the presence of d' at a picture part ahould have no effect on the likelihood of at a neighborIng part. Internediate values of $\mathbf{C}$ ohould have interaediate effects.

Several mechanifas have been auggeated for applying this knowledge reprasentation to updating labellinga. For example, Rosenfeld et al. [6] euggested the formala

$$
\begin{equation*}
P_{1}^{\prime}(d)=P_{1}(d)\left(1+Q_{1}(d)\right) / M \tag{2}
\end{equation*}
$$

vhere

$$
\begin{equation*}
Q_{i}(d)=\sum_{j} \mathbf{a}_{1 j}{\underset{d^{\prime}}{ }} C\left(d, d^{\prime}\right) p_{j}\left(d^{\prime}\right) \tag{3}
\end{equation*}
$$

and $N$ is a normalising factor which guaranteea that $p_{i}(d)-1$. The $\mathrm{m}_{\text {a }}$ valuas can be used to give hif her weight to iloen naighbors at part 1 than others. Q (d) seasures the overall support of the nelghborhood of part 1 for oupport of the neighborhood of part i for label di it takas on valuas in the range The abive operation is applied in parallel at every part and for every label. The $p$ ' values then raplace the $p$ values, and the operation can be iterated.

Very often, hovever, a general updating rule Ithe (2) is Inappropriate because it feile to take advantage of knovledge about the apeciif problea at hand. For example, in the crop claseification probles eeveral plausible relaration rules can be derived from heuriatic arguaents besed on the following geaeral obeervation:

Fields are, in general, wuch bigger than a single pisel and do not change class during - ungle groving seasen.

If ve let pili,j,t) denote the probability that the pifel at opatial lecation ( $1, j$ ) and tien is in clase $\bar{\lambda}$ after $i$ iteratione of the relaration rule, then the follouting rule achens cas be used to derive various epecific rulea:

$$
\begin{aligned}
& \text { - } f\left(\int_{\lambda^{\prime}}^{l}\left(1^{*}, j^{*}, t\right): \lambda^{*}\left(A,\left(1^{*}, j^{\prime}\right) a\right.\right. \\
& \text { neighber of ( } 1, j \text { ) )? }
\end{aligned}
$$

Mere, $M$ is a mornalization factor $\left(-I_{i} P_{\lambda}^{m+1}\right.$
$(1, j, t))$ and $A$ is the eet of possible clasees. The second tern raflecta the faet that the clase of a pisel does not change over time, while the third tara mat repreaent the apatial dependence of $(1, j)$ befing in clase $\lambda$ on the ifkalihood that ite seighbora are in various claseas. We can identify at least three plausible funct fong for i:

$$
\text { 1) } f_{2}=\min _{1=0}^{7} p_{\lambda}\left(x_{2}, t\right)
$$

Here, $x_{1}$ is an 0 -neighbor of ( $1, k$ ) (aee
ifgure belou) and this rule can be interpreted as saying: If ( $j, k$ ) is in clase $\lambda$, then aince fielda are large, at least one apatial meighbor of ( $\mathrm{g}, \mathrm{k}$ ) mat be in clase $\lambda$.

| $x_{7}$ | $x_{0}$ | $x_{1}$ |
| :--- | :--- | :--- |
| $x_{6}$ |  | $x_{2}$ |
| $x_{5}$ | $x_{4}$ | $x_{3}$ |

$$
\text { 2) } f_{2}=\operatorname{mix}_{j=0}^{7} \prod_{j=0}^{2} p_{\lambda}^{k}\left(x_{i+j}, t\right) \text {, aubacripta } \bmod \text { B. }
$$

If fialde are roughly fectangular, then any polat vill have at laast three conaecutive neigbbors in the aeme ifald. Notice that oimply chooeing

$$
f_{2}^{\prime}=\prod_{j=0}^{7} p_{\lambda}^{k}\left(x_{j}, z\right)
$$

would at firat iteration lover the probabilitiea of corract label at border pirala, and propagete these lou probabilites into the center of the $110^{\circ}{ }^{\circ}$ at subsequent iterations.

$$
\text { 3) } f_{3}=\sum_{j=0}^{6_{j} 2} p_{2}\left(x_{j}, t\right)
$$

One potential advantage of the aus over the product is ita insenaitivity to one or two erroneously low probabilities. Uaing fy aleo
givea more equal veight to the tepporal and opetial information in the relanation rule. In the neat section ve vill conskder the offect of applying chis updating rule to the inages in Figure 1.

Mest, conaider the motipn eatimation probie. At each pisel we have $\mathrm{v}^{\text {i }}$, the eatimate of the velocity after the ith iteration of relaration. Now, we vill sasume that, locally, the pattors of inage motion vectore can be vell deacribed by a rigid inage plane not fon consiat ing of a traaslation and rotation (over large regiana this aight be a poor asounption aince tha inage notien is the projection of a 3-D rigid motion).

Conoider a $3 \times 3$ neighborhood of velecities

| $v_{1}$ | $v_{2}$ | $v_{3}$ |
| :--- | :--- | :--- |
| $v_{3}$ | $v_{0}$ | $v_{4}$ |
| $v_{7}$ | $v_{6}$ | $v_{3}$ |

whare $\boldsymbol{v}_{1}$ is the velocity at $\mathrm{P}_{\mathbf{1}}$ *
Since any 2-D risid motion can be represented ae a tranalation plue a rotation about a fired poiat, If our asmuption of lecal motions being rigid inage piane motions is correct, then the 300 pateern of velocities

should be a rotation about the center point. In general, if point a is rotating about point $b$ with velocity $v_{a}$, then

$$
v_{a} \cdot d_{b}=0
$$

when $d_{\text {be }}$ is the vector froe b to a. Therefore, it is otraightforvard to compute a least squares ant inate of the angular valocity, $\mathrm{w}_{\mathrm{p}}$, of $\mathrm{P}_{0}$, in any $3 \times 3$ (or ma ) inage seighbor hood.
In order to computa a value for $\mathrm{v}_{\mathrm{k}+1}^{\mathrm{m}} \mathrm{w}$ proceed with the folloulag paralli operation: Each pixel io a meeber of the outer Fing of $83 \times 3$ neighborhoods (one centered at each of its neighbora). Choose the neighbor, 1 vich minimal least equare error and oet $v_{0}^{k+1}-v_{i}^{k}+v_{1}$.

## 4. ITERATION

In iaportant proble in the application of relaxation procesees is deteraining a terinction critaria for the itaratione. The two obvious ternination criteris are:

1) "convergence" is the sence that the change (in probability vectors, auearic labela, etc.) froe one iteration to the neat is quificientiy mali (see (6) Ior a diacusesion of ceavergesce o relanatios procesees).
2) exhmut ing allocated compucat ional resources - a.8., in motion detection, one eay ealy heve a ohert $t$ tae (c 1 * t) to iterate the relagat ion process.

Sonet fines one finds that white the firat sevaral iteratioas of a relamation procese $t$ and to ifprove upon the faitial labeling, sLloulag the iterations to proceed setualiy degrades the rasulta. In melh cases one vould vat to corniance after only a fow iterations.

For the miti-temporis, miti-apectral pizel classification, ve find apiriciliy, that the largeat increase in classification accuracy oceura in the firat iteratior, vith subeequeat itarations having iltile affeet on the rasulta. Figure 5 coataing the claseification mapt for each major clase in Figure 1 after 5 iterat ions of the ralamtion algorith dencribed is Section 2 using function f3. Troe thase figures, it is clear that the relamation algorith the aubstantially eahmeced the detection of the fielde and the ovarall claselfication otrategy.

Mext, consider the notion eat fation prob1ea. Here, the critical factor in deterbining the mubler of iterations of the relayation procese is tiae, aince frases are arriving every .03 oeconds. Figure 6 shows the motion vectora after $S$ iterations of relazation. Athough the vectora in Figure 6 appear eoother thria those of Figure 4 . one meeds seae quantitat ive maceure for coaparing the. One such seamure is to see how well the velocity vectors predict inage atructure (eay, for coded trancaisaion). If if(,$t), t-1,2$ is the iatenaity at pisel I at tiae t , then 1.6

$$
e_{i}(E)=\left|g(x, 1)-g\left(x+v_{n}, 2\right)\right|
$$

and, finally.

$$
z_{k}=\sum_{i}(X)
$$

If the ralaration is truly improving the velocity vectere, then ve vould expect $E$ velocity vectora, then we vould expect phent to be monotosicaliy nonincreasing with eampli'.

## 5. COMCLIDDIMC EDURKS

telaration processes have potential upeed advastages because they cas be faplenented is parallal (hardvere peraitting). They have been auccasafully applied to a vide varlety of labelliag probleas by a groving
nuaber of inveat igatera. In apite of these succasses, Iftile is as yet haown about the deaign and control of theae processes. Mowever, o number of preaising approsehes to their theoratical formulation are being purcued, and it is hoped that a deeper understanding of their anture vill soon be achieved.

The eupport of the llational seience Poundetion under Grant.WCS-79-23422 is gratefulis achooviedged, se is the help of Janet Salames in priparing this paper.

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Pigure 1. Three acquiatitiona of LNMDSAT for 2 lande



Figure 2. Initial labeling (left coluan is ground truth, right contains pixels classified into that ground truth clasa)


Figure 3. Two frames of a traffic sequence


Figure 4. Initial motion vectors

Figure 7. Normalized values of $E_{k}$,


Figure S. Labeling after 5th iteration uaing $\mathrm{i}_{3}$.


Figure 6. Motion vectors after 5 iterations of relaxation


ORIGINAL PAGE IS OF POOR QUALITY


#### Abstract

Summary. The obvious duality between edge finding and segmentation was exploited in [1]. This work has been refined, including a more general model for multi-image data, and many more tests of the clustering program AMOEBA. Possible directions for future work in edge following and clustering are suggested, using (now) the duality between eegmentation and classification made possible by regarding AMOEBA as a segmentation to classification mapping.


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[^1]
## SHAPE AND TEXTURE

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Regions in a segmented image are characterized, for purposes of description and recognition, by their geometrical properties (size, shape, etc.), as well as by properties that depend on their pixel values (lightness, color/spectral signature, texture). Such properties are also used to define or modify the segmentation process itself, as discussed in the session on segmentation.

The methods used to measure the geometrical properties of a region depend on the data structure used to represent the region. The simplest representation is a binary "overlay" array that has l's at region pixels and 0 's elsewhere. However, other types of representations are often used that are more compact, and that may make it easier to extract certain types oi geometrical information. One classical approach is to represent regions by border codes, defining the sequence of moves from neighbor to neighbor that must be made in order to circumnavigate the border; curves can also be represented by such move sequences ("chain codes"). Another standard way of representing regions is as unions of maximal "blocks" contained in them - egg., maximal "runs" of region points on each row of the image, or maximal upright squares contained in the regin; the set of run lengths on each row, or the set of centers
and radii of the squares (known as the "medial axis"), completely determines the region. The square centers tend to lie on a set of arcs or curves that constitute the "skeleton" of the region; if we specify each such arc by a chain code, and also specify a radius function along the arc, we have a representation of the region as a union of "generalized ribbons", which are 2D analogs of the "generalized cylinders (or cones)" often used to represent 3D objects.

There has been recent interest in the use of hierarchically structured representations that incorporate both coarse and fine information about a region or feature. A hierarchical maximal-block representation based on recursive subdivision into quadrants, where the blocks can be represented by the nodes of a degree-4 tree (a "quadtree"), is described in (Samet and Rosenfeld, 1980). A hierarchical border or curve representation based on recursive polygonal approximation, with the segments represented by the nodes of a "strip tree", is discussed in (Ballard, 1981); on a border or curve representation based on quadrant subdivision see (Shneier, 1981).

Classically, textural properties have been derived from the autocorrelation or Fourier power spectrum; for example, the coarser the texture (in a given direction), the slower its autocorrelation falls off in that direction from the origin (zero displacemerit) and the faster its power spectrum falls off in that direction from zero frequency. A related approach, studied extensively by Julesz and Haralick, characterizes textures by their second-order intensity statistics, i.e., by the
frequencies with which given pairs of gray levels occur at given relative displacements. It has long been realized, however, that first order statistics of various local property values (e.g., responses of operators sensitive to local features such as edges, lines, line ends, etc.) are at least equally effective in texture discrimination.

More recent work (Beck et al., 1982) suggests that local processes of linking between local features, giving rise to "texture elements" or "primitives", also play a significant role in the perception of texture differences. Texture discrimination based on second-order statistics of local features (e.g., occurrences of edge elements in given relative positions and orientations) has begun to be investigated (e.g., Davis et al.. 1979). Texture analysis based on explicit extraction of primitives has also been explored (e.g., Maleson et al., 1977); here statistics derived from properties of the primitives, or of pairs of adjacent primitives, are used as textural properties.

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# SHAPE AND TEXTURE 

## Ariel Rosenfeld

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Regions in : segmented image are characterized, for purposes of description and recognition, by their geometrical properties (size, shape, etc.), as well as by properties that depend on their pixel values (lightness, color/spectral signature, texture). Such properties are also used to define or modify the segmentation process itself, as discussed in the session on segmentation.

The methods used to measure the geometrical properties of a region depend on the data structure used to represent the region. The simplest representation is a binary "overlay" array that has l's at region pixels and 0 's elsewhere. However, other types of representations are often used that are more compact, and that may make it easier to extract certain types of geometrical information. One classical approach is to represent regions by border codes, defining the sequence of moves from neighbor to neighbor that must be made in order to circumnavigate the border; curves can also be represented by such move sequences ("chain codes"). Another standard way of representing regions is as unions of maximal "blocks" contained in them - egg., maximal "runs" of region points on each row of the image, or maximal upright squares contained in the redion; the set of run lengths on each row, or the set of centers
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## 1. Shape

a) Moments

In this section, we review moments, which are
a very useful class of shape properties.
The ( $i, j$ ) moment of $f$ is defined by

$$
m_{i j} \equiv \sum_{x} \sum_{y} x_{y}^{i} j_{f(x, y)}
$$

(in the continuous case, $\Sigma \Sigma$ becomes $\iint d x d y$ ). The first few moments of the picture

$$
\begin{array}{lll}
2 & 1 & 1 \\
3 & 1 & 0 \\
3 & 2 & 1
\end{array}
$$

are as follows, if we take the origin at the pixel in the lower left-hand corner of the picture:

| $i$ | $j$ | $m_{i j}$ |
| ---: | ---: | ---: |
| 0 | 0 | 14 |
| 1 | 0 | 8 |
| 0 | 1 | 12 |
| 2 | 0 | 12 |
| 1 | 1 | 7 |
| 0 | 2 | 20 |

Moments can be given a physical interpretation by regarding gray level as nass, i.e., regarding $f$ as composed of a set of point masses located at the points $(x, y)$. Thus $m_{00}$ is the total mass of $f$, and $m_{02}$ and $m_{20}$ are the moments of inertia of $f$ around the $x$ and $y$ axes, respectively. The moment of inertia of $f$ around the origin $m_{0} \equiv \sum \Sigma\left(x^{2}+y^{2}\right) f(x, y)=m_{20}{ }^{+m_{02}}$. It is easily verified that $m_{0}$ is invariant under rotation of $f$

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about the origin.
Moreover, if $f$ is rescaled, say by the factor $c$, it is not hard to see that $m_{0}$ is multiplied by $c^{4}$. Thus we can normalize $f$ with respect to magnification by rescaling it to give $m_{0}$ a specified value. Alternatively, a ratio of two moments that have the same value of $i+j$, e.g. $m_{01} / m_{10}$, is invariant under magnification.

If we substitute $-x$ for $x$ in the definition of $m_{i j}$, we obtain $\sum \Sigma(-x)^{i} y^{j} f(-x, y)=(-1)^{i} \sum \sum x^{i} y^{j} f(-x, y)$, so that if $f$ is symmetric about the $y$ axis (i.e., $f(-x, y)=f(x, y)$ for all $x, y)$, we have $m_{i j}=(-1)^{i_{m}}{ }_{i j}$. Thus if $i$ is odd, $m_{i j}$ must be zero. Similarly, if $f$ is symmetric about the $x$ axis and $j$ is odd, $m_{i j}=0$; and if $f$ is symmetric about the origin $(f(-x,-y)=$ $f(x, y)$ for all $x, y)$, and $i+j$ is odd, $m_{i j}=0$. Moments for which $i, j$, or $i+j$ is odd can thus be used as measures of asymmetry about the $y$ axis, $x$ axis, and origin, respectively.

If $f$ is a binary-valued picture, say with $S$ as its set of l's, the moments of $f$ provide useful information about the
spatial arrangement of the points of S . To compute moments from the binary array representation $X_{S}$ of S , we simply sum the $x^{i} y^{j}$ values for all $(x, y)$ in $S$. To compute them from the run length representation of s , we compute them for each run and sum the results; for example, the ( $i, j$ ) moment of the run whose endpoints are $\left(x^{\prime}, y\right)$ and $\left(x^{\prime \prime}, y\right)$ is $y^{j} \sum_{x=x}^{x \prime \prime} x^{i}$. Similarly, they can be computed from the quadtree representation of $S$ by computing them for each black leaf node, based on its position in the tree, and summing the results.

They
are not easy to compute from the MAT representation, since the blocks overlap. They can be computed from the crack or chain code representations of the borders of $S$ in much the same way that area is computed from these representations. As an example, for each horizontal crack $c_{k}$, let $S_{k}$ be the vertical rectangle of width 1 extending from the bottom or the picture to $c_{k}$; then $S$ is the union of the $s_{k}$ 's for which $c_{k}$ is an upper boundary of $S$, minus the union ef those for which $c_{k}$ is a lower boundary. The coordinates of $c_{k}$ determine the moments of $S_{k}$, just as in the case of runs; to compute the ( $\mathrm{i}, \mathrm{j}$ ) moment of S , we add the ( $\mathrm{i}, \mathrm{j}$ ) moments of all the upper-boundary $\mathbf{S}_{\mathbf{k}}$ 's, and subtract the sum of the ( $\mathrm{i}, \mathrm{j}$ ) moments of all lower-boundary $\mathrm{s}_{\mathrm{k}}$ 's.
b) The centroid; central moments

The centroid of $f$ is the point ( $\bar{x}, \bar{y}$ ) defined by

$$
\begin{aligned}
& \bar{x}=m_{10} / m_{00} \\
& \bar{y}=m_{01} / m_{00}
\end{aligned}
$$

Thus the centroid of the 3 by 3 picture shown earlier is (4/7,6/7). It is easily verified that if $f$ is shifted, its centroid shifts by the same amount. (Proof: If we shift $f$ by $(\alpha, \beta)$, the origin is now at $(-\alpha,-\beta)$, and the new coordinates of $(x, y)$ are $(x+\alpha, y+\beta)$. Hence $\sum \Sigma(x+\alpha) f(x, y) / \Sigma \Sigma f(x, y)=$ $m_{10}+\alpha=\bar{x}+\alpha$, and similarly for $\left.\bar{y}.\right)$ Thus if we take the origin at the centroid of $f$, we have normalized $f$ with respect to translation. Note that since the centroid does not have integer coordinates, if we take it at the origin we should redigitize f; alternatively, we can normalize $f$ by taking the origin at the integer-coordinate point closest to the centroid. (Analogous remarks apply in the case of normalizing with respect to magnification in subsection (a).)

When we take the origin at the centroid, moments computed with respect to this origin are called central moments, and will be denoted by $\bar{m}_{i j}$. Evidently $\bar{m}_{00}=m_{00}$, and it can be verified that $\bar{m}_{10}=\bar{m}_{01}=0$. (Proof: Take $(\alpha, \beta)=(-\bar{x},-\bar{y})$ in the preceding paragraph to obtain $\bar{m}_{10}=\Sigma \Sigma(x-\bar{x}) f(x, y)=m_{10} \bar{x}_{00}=0$, and similarly for $\mathrm{m}_{01}$.)

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c) The principal axis

The moment of inertia of $f$ about the line $(y-\beta) \cos \theta=(x-\alpha) \sin \theta$, which is the line through $(\alpha, \beta)$ with slope $\theta$, is

$$
\Sigma \Sigma[(x-\alpha) \sin \theta-(y-\beta) \cos \theta]^{2} f(x, y)
$$

We can find the $\alpha, \beta$, and $\theta$ for which this is a minimum by differentiating it with respect to $\alpha$ and $B$ and equating the results to zero; this yields

$$
\begin{array}{ll}
\Sigma \Sigma[(x-\alpha) \sin \theta-(y-\beta) \cos \theta] f(x, y)=0 & (f r o m \partial / \partial \alpha \text { or } \partial / \partial \beta) \\
\Sigma \Sigma[(x-\alpha) \cos \theta+(y-\beta) \sin \theta] f(x, y)=0 & (f r o m \partial / \partial \theta)
\end{array}
$$

Multiplying the first equation by $\sin \theta$, the $\operatorname{second}$ by $\cos \theta$, and adding gives $\Sigma \Sigma(x-\alpha) f(x, y)=0$, so that $\alpha=\Sigma \sum x f(x, y) / \Sigma \Sigma f(x, y)$ $=\bar{r}$. Similarly, multiplying the first equation by $\cos \theta$, the second by $\sin \theta$, and subtracting gives $\beta=\bar{y}$. Thus the minimuminertia line passes through the centroid of $f$. This line is called the principal axis of $f$.

To find the slope of the principal axis, take the origin at the centroid; then the moment of inertia of $f$ about the line $\mathrm{y}=\mathrm{x} \tan \theta$ is

$$
\Sigma \Sigma(x \sin \theta-y \cos \theta)^{2} f(x, y)=\bar{m}_{20} \sin ^{2} \theta-2 \bar{m}_{11} \sin \theta \cos \partial+\bar{m}_{02} \cos ^{2} \theta
$$ Differentiating this with respect to $\theta$ and equating to zero gives

$$
2 \bar{m}_{20} \sin \theta \cos \theta-2 \bar{m}_{11}\left(\cos ^{2} \theta-\sin ^{2} \theta\right)-2 \bar{m}_{02} \cos \theta \sin \theta=0
$$

or $\bar{m}_{20} \sin 2 \theta-\bar{m}_{11} \cos 2 \theta-\bar{m}_{02} \sin 2 \theta=0$ so that $\tan 2 \theta=2 \bar{m}_{11} /\left(\bar{m}_{20}-\bar{m}_{02}\right)$

Since $\tan 2 \theta=2 \tan \theta /\left(1-\tan ^{2} \theta\right)$, we can obtain $\tan \theta$ as a root of the quadratic equation

$$
\tan ^{2} \theta+\frac{m_{20}-m_{02}}{m_{11}} \tan \theta-1=0
$$

It is easily seen that the last equation above is equivalent to

$$
\left(m_{11} \tan \theta+m_{20}\right)^{2}-\left(m_{20}+m_{02}\right)\left(m_{11} \tan \theta+m_{20}\right)+\left(m_{20^{m}} m_{02}^{-m_{11}}\right)=0
$$

This implies that $m_{11} \tan \theta+m_{20}$ is an eigenvalue of the matrix

$$
\left(\begin{array}{ll}
\mathrm{m}_{20} & \mathrm{~m}_{11} \\
\mathrm{~m}_{11} & \mathrm{~m}_{02}
\end{array}\right.
$$

Show that the principal axis is in the direction of the eigenvector correponding to the larger eigenvalue of this matrix.

A standard method of normalizing $f$ with respect to rotation is to rotate it so that its principal axis has some standard orientation, say vertical. (Here again, this involves redigitization.) More generally, f can be normalized with respect to various types of geometrical distortions by transforming it so as to give standard values to various combinations of its moments.

The principal axis of $f$ can be regarded as a line that "best fits" $f$. More generally, one can find higher-order curves that "best fit" $f$ in various senses. For example, given a general quadratic curve

$$
q(x, y) \equiv a x^{2}+b x y+c y^{2}+u x+v y+w=0
$$

we can attempt to find the values of the coefficients $a, b$, c, $u, v, w$ such that

$$
\Sigma \Sigma(q(x, y))^{2} f(x, y)
$$

is a minimum. The curve having these coefficients would be a sort of "quadratic principal axis" for f. Given f's bestfitting quadratic curve $q_{0}$, one can attempt to perform "shape normalization" on $f$ by transforming coordinates so that $q_{0}$ becomes some standard type of curve--for example, if $q_{0}$ is an ellipse, we could transform to make it a circle.

## 2. Texture

This section discusses statistical picture properties, and in particular, properties that can be used to describe the "visual texture" of a picture, or better, of a statistically homogeneous region in a picture. We will not attempt to define conditions under which a region would be called uniformly textured. Such regions are often described as consisting of large numbers of small uniform patches, or "primitive elements," arranged according to "placement rules," where the patch shapes and positions are governed by random variables.

## a) Gray level statistics

The histogram $p_{f}(z)$ of a digital picture $f$ tells us how often each gray level occurs in $f$; it provides an es:imate of the gray level probability density in the ensembie of pictures of which $f$ is a sample. If there are $k$ possible gray levels, $z_{1}, \ldots, z_{k}, p_{f}$ is a.k-element vector. Statistics computed from $p_{f}$ give us general information about this gray level population. For example,

1) The mean gray level of $f, \mu_{f} \equiv \frac{1}{N} \sum z p_{f}(z)$, where $N \equiv \sum_{P_{f}}(z)$ is the number of points in $f$, is a measure of the overall lightness/darkness of $f$. The median gray level, i.e., the gray level $m_{f}$ such that (about) half the points of $f$ are lighter than $m_{f}$ and half are darker, is another such measure.
2) The gray level variance of $f, \sigma_{f}^{2} \equiv \frac{1}{N} \Sigma\left(z-\mu_{f}\right){ }^{2} p_{f}(z)$, and the standard deviation $\sigma_{f}$, are measures of the overall contrast of $f^{*}$; if they are small, the gray levels of $f$ are all dose to the mean, while if they are large, $f$ has a large range of gray levels. Another such measure is the interquartile range $r_{f}$, which is defined as follows: Let $m_{l f}$ be the gray level such that $1 / 4$ of the pixels of $f$ are lighter than $m_{l f}$ and $3 / 4$ are darker; let $m_{3 f}$ be defined analogously, with the *Note that if we define one-dimensional moments by $m_{i} \equiv \sum z_{\mathrm{i}_{f}}(z)$, we have $N=m_{0}$ and $\mu_{f}=m_{1} / m_{0}$ (so that $\mu_{f}$ is the centroid of $p_{f}$ ); moreover, if we define central moments $\bar{m}_{i} \equiv \sum\left(z-\mu_{f}\right) i_{f}(z)$ by taking $\mu_{f}$ as the origin, we have $\sigma_{f}^{2}=\bar{m}_{2} / \bar{m}_{0}$
$1 / 4$ and $3 / 4$ interchanged; then $r_{f} \equiv\left|m_{1 f}-m_{3 f}\right|$. Other percentiles can be used here in place of the quartiles $m_{1 f}$ and $m_{3 f}$.

Second-order gray level statistics
Statistics computed from the histogram $\mathbf{p}_{f}$ are of only limited value in describing $f$, since $p_{f}$ remains the same no matter how the points of $f$ are permuted--for example, $p_{f}$ is the same when $f$ is half black and half white, when $f$ is a checkerboard, or when $f$ consists of salt-and-pepper noise. More insight into the nature of $f$ is obtained by studying how often the possible pairs of gray levels occur in given relative positions.

Let $\underline{\delta} \equiv(\Delta x, \Delta y)$ be a displacement, and let $M_{\delta}$ be the $k-b y-k$ matrix whose ( $i, j$ ) element is the number of times that a point having gray level $\mathbf{z}_{i}$ occurs in position $\delta \underline{f}$ relative to a point having gray level $z_{j}, l \leq i, j \leq k$. For example, if $f$ is

1122
0221
0021
1001
and $\delta$ is $(1,0)$, then $M_{\delta}$ is
212
111
022
Note that the size of $M_{\delta}$ depends only on the number of gray levels, not on the size of $f$. Elements near the main diagonal of $M_{\delta}$ correspond to pairs of gray levels that are nearly equal, while elements far from the diagonal correspond to pairs that are very unequal.

Let $N_{\delta}$ be the number of point pairs in $f$ in relative position $\delta$; this is less than the total number of points in $f$, since if $(x, y)$ is near the border of $f,(x+\Delta x, y+\Delta y)$ may lie outside $f$. Then in the matrix $P_{\delta} \equiv M_{\delta} / N_{\delta}$ (i.e., if we divide each element of $M_{\delta}$ by $N_{\delta}$ ), the ( $\left.i, j\right)$ element is an estimate of the joint probability that a pair of points in relative position $\delta$ will have the pair of gray levels $\left(z_{i}, z_{j}\right)$. $P_{\delta}$ is called a gray level cooccurrence matrix for $f$.

The matrices $P_{\delta}$, for various $\delta$ 's, provide useful information about the spatial distribution of gray levels in $f$. For example, suppose that $f$ is composed of patches of approximately constant gray level of a certain size s. If the length of $\delta$ is small relative to $s$, then the high-valued entries in $P$ will be concentrated near its main diagonal, since a pair of points $\delta$ apart will of ten have nearly the same gray level. On the other hand, if $\underline{\delta}$ is long relative to $s$, the entries in $P$ will be more spread out. If $f$ consists of elongated streaks oriented in a given direction, the spread of values in $P_{\delta}$ will depend on both the length and slope of $\underline{\delta}$. If directionality is not important, we can use matrices $\bar{P} \quad$ that are averages of $P_{\delta}$ 's (or matrices $\bar{M}$ that are sums of $M_{\delta}$ 's) for sets of displacements of a given size in various directions. For example, if $f$ is the 4 -by-4 picture shown above, and we use the displacements $(1,0),(0,1),(-1,0)$, and $(0,-1)$, then the
combined matrix $\bar{M}$ is
$\begin{array}{lll}8 & 4 & 4 \\ 4 & 6 & 5 \\ 4 & 5 & 8\end{array}$
(Note that $\bar{M}$ is symmetric, since the set of directions used is symmetric.)

In principle, a large set of $P_{\delta}$ matrices is needed to completely specify the second-order gray level statistics of f. In practice, however, matrices corresponding to large
displacements are not necessary. As $\underline{\delta}$ becomes long, the pairs of gray levels separated by $\underline{\delta}$ become uncorrelated, and $P_{\delta}(i, j)$ approaches the probability that a pair of randomly chosen points of $f$ have gray levels $z_{i}$ and $z_{j}$. Thus for practical purposes we need not use $\delta^{\prime}$ s having lengths greater than the distance over which f's gray levels remain correlated, or greater than the size of the "patches" of which $f$ is composed. In fact, the most important $P_{\delta}$ 's are usually those for which $\delta$ has length 1. Historically, gray level transition probabilities $p\left(z_{j} \mid z_{i}\right)$ have been used to characterize textures; $p\left(z_{j} \mid z_{i}\right)$ is the probability that a point has level $z_{j}$ given that the preceding point (with respect to a scan of the picture) has level $z_{i}$. Note that the joint probability $p\left(z_{i}, z_{j}\right)$, which is equal to $p\left(z_{i}\right) p\left(z_{j} \mid z_{i}\right)$, is just the $(i, j)$ element of $P_{\delta}$ for $\delta=(1,0)$. Other investigators have characterized textures by fitting a time series model to the sequence of gry levels, and using the parameters of this model as texture descriptors; this approach will not be discussed here in detail.

Haralick has suggested a number of statistics that can be used to describe a given cooccurrence matrix $P_{\delta}$. Four of these are:

1) "Contrast", $\sum \sum(i-j){ }^{\mathbf{2}} \mathrm{P}_{\delta}(i, j)$; this is the moment of i $j$ inertia of $P_{\delta}$ about its main diagonal. Evidertly, it is low when the diagonal concentration of $P_{\delta}$ is high, and vice versa.
2) "Inverse difference moment", $\sum_{i} \sum_{j} P_{\delta}(i, j) /\left[l+(i-j)^{2}\right]$; this is high when the diagonal concentration is high.
3) "Angular second moment", $\sum_{i} \dot{\sum} \dot{j}_{\delta}^{2}(i, j)$; this is lowest when the $P_{\delta}(i, j)$ 's are all equal, and high when they are very unequal, so that in particular it tends to be high when the diagonal concentration is high.
4) "Entropy", $-\sum_{i} \sum_{j} P_{\delta}(i, j) \log F_{\delta}(i, j)$; this is highest when the $P_{\delta}(i, j)$ 's are all equal, and hence is low when the diagonal concentration is high.

It should be pointed out that the arrangements of values in the cooccurrence matrices depend not only on the coarseness or busyness of the given picture, but also on its lightness and contrast. For example, if we stretch the grayscale of a picture, the entries in the matrices will spread away from the diagonal, since the pairs of gray levels will be farther apart. Features (1-2) defined above will be especially sensitive to such changes (this is why feature (1) is called "contrast"), while features (3-4) will be less so. To avoid confusing the effects of the first and second order statistics of the picture, it is common practice to normalize its grayscale (e.g., by histogram flattening) before computing the matrices, so that the first order statistics have standard values.

We can define cooccurrence matrices that may be more sensitive to the spatial structure of the given texture by
using only selected pairs of points in constructing the matrices, rather than using all possible pairs having a given relative position. For example, suppose that we consider only point pairs $(Q, R)$ in which $Q$ is on an edge (e.g., is at a local maximum of the gradient magnitude), and $R$ is a given distance $\delta$ away from $Q$ in the gradient direction. In the matrix $P_{\delta}^{\prime}$ defined in this way, diagonal conce.ttration still corresponds to coarseness, since if $\delta$ is smal.l relative to the texture patch size, $R$ should be interior to the patch on the edge of which $Q$ lies. However, $P_{\delta}^{\prime}$ may be more sensitive to coarseness changes than the $P_{\delta}$ matrices were, since $P_{\delta}^{\prime}$ is not influenced by point pairs that are both interior to patches.

## c) Local property statistics

Another way of obtaining information about the spatial arramement of the gray levels in $f$ is to compute statistics of various local property values $f$ ' measured at the points of $\mathbf{f}$.

As an illustration of how local properties can be used for texture description, let $\underline{\delta} \equiv(\Delta x, \Delta y)$ be a displacement, let $f_{\delta}(x, y) \equiv f(x, y)-f(x+\Delta x, y+\Delta y)$, and let $p_{\delta}$ be the histogram of $f_{\delta}$. Suppose that $f$ is composed of patches of size s. If $\delta$ is short relative $t$, $s$, the high entries in $p_{\delta}$ will be concentrated near 0 , since pairs of points $\underline{\delta}$ apart will usually have small differences in value; but if $\underline{\delta}$ is large, the entries in $p_{\delta}$ will be more spread out. (Note, in fact, that $p_{\delta}(z)$ is the sum of the entries in the matrix $M_{\delta}$ along the line parallel to its main diagonal for which $i-j=2$.$) Thus the$ concentration of $p_{\delta}$ near 0 is a measure of the "coarseness" of $f$ relative to $\delta$, or equivalently, the spread of $p_{\delta}$ away from 0 is a measure of the "busyness" of $f$. Here again, these properties may depend on direction. Similar remarks apply if we use absolute rather than signed differences; this simply folds $p_{\delta}$ over on itself at the origin.

The gray level (absolute) difference histograms $p_{\delta}$ are not affected by shifting the grayscale (as cooccurrence matrices are), but they are affected by stretching it; thus they too should be used in conjunction with grayscale normalization. Various statistics can be used to describe $P_{\delta}$, including its mean ( $\frac{1}{N} \Sigma z p_{\delta}(z)$, if we use absolute differences), its second moment ( $\sum z^{2} p_{\delta}(z)$; this is proportional to the "contrast" statistic for the corresponding cooccurrence matrix), its entropy $\left(-\Sigma p_{\delta}(z) \log p_{\delta}(z)\right)$, and so on.

A wide variety of local properties $f$ ' can be used in place of $f_{\delta}$ for texture description. For example, we can use combinations of differences, such as the gradient (magnitude) or Laplacian; matches to local templates, such as spot, line, corner, or line end detectors; and so on. $f$ ' can be a predicate, e.g. 1 if an above-threshold difference is present and 0 otherwise; in this case, the histogram consists of only two values, and its mean tells us how many edges (or spots, lines, etc.) are present in $f$ per unit area. Another possibility is to count local gray level maxima and minima in $f$; evidently, both the number of edges and the number of extrema per unit area are measures of "busyness". More generally, we can count occurrences of arbitrary local patterns of values in $f$.

We can use second order as well as first order local property statistics as texture descriptore, by constructing
cooccurrence matrices of the values of $f$ ' in given relative positions. If desired, we can use only selected pairs of points in constructing the matrices, e.g., pairs of extrema or pairs of above-threshold edge points, and we can use dizplacements at each point that depend on $f$ ', e.g., displacements in the gradient direction, as at the end of subsection (a).

Rather than using a set of local properties, e.g., iifferences computed for a set of displacements, we can use a single property and measure it for pictures derived from the original one by a set of local operations. For example, suppose that we use a sequence of local min (or max) operations, and at each step, measure the average gray level; the rate at which this decreases (or increases) is a measure of the coarseness of the high-valued (low-valued) patches in f. For a binary-valued $f$, the analogous idea is to shrink (or expand) the l's in $f$ repeatedly, and at each step, count the number of l's. This approach, using generadized shrinking and expanding operations, has been extensively
used for texture analysis in microscopy.

## d) Autocorrelation and power spectrum

In the previous two subsections we saw how various statistics of the cooccurrence matrix, difference histogram, etc. for a given displacement $\underline{\delta} \equiv(\Delta x, \Delta y)$ provide useful information about a texture. Thus the set of values of a given statistic $\alpha$ as a function of $\underline{\delta}$ (in particular, for relatively short $\delta^{\prime} s$ ) can be used as a texture descriptor.

For example, let $f_{\delta}^{n} \equiv f(x, y) f(x+\Delta x, y+\Delta y)$, and let $\alpha$ be the mean of $f_{\delta}^{\prime \prime}$; then $\alpha$ as a function of $\delta$ is just the autocorrelation $R_{f}$, i.e., the expected value of the product of the gray levels of a pair of points $\underline{\delta}$ apart. By the Cauchy-Schwartz inequality, this takes on its maximum value for $\underline{\delta}=(0,0)$. (Proof: $\frac{\sum f(x, y) f(x+\Delta x, y+\Delta y)}{\left[\Sigma f(x, y)^{2} \Sigma f(x+\Delta x, y+\Delta y)^{2}\right]^{1 / 2}} \leq 1$; but the two factors in the denominator are the same, so that the denominator is equal to $\Sigma f(x, y)^{2}$, which is $R_{f}(0,0)$.) The rate at which $R_{f}$ falls off as $\delta$ moves aryy from $(0,0)$ is a measure of the coarseness of $f$; the iclloff is slower for a coarse texture, and faster for a busy one.

Similarly, let $f_{\delta}^{2}=[f(x, y)-f(x+\Delta x, y+\Delta y)]^{2}$, and let $\alpha \equiv v$ be its mean, i.e., the expected squared gray level difference at two points $\underline{\delta}$ apart; this descriptor is sometimes called the variogram of $f$. (Compare the use of the mean of the absolute difference histogran as a texture descriptor in subsection (c).) The rate at which its value rises as $\underline{\delta}$ moves
away from $(0,0)$ is a measure of the coarseness of $f$; the rise is slow for a coarse texture and fast for a busy one. Note that $v(\underline{\delta})=E\left\{[f(x, y)-f(x+\Delta x, y+\Delta y)]^{2}\right\}=E\left\{f^{2}(x, y)\right\}+$ $E\left\{f^{2}(x+\Delta x, y+\Delta y)\right\}-2 E\{f(x, y) f(x+\Delta x, y+\Delta y)\}$; here the first two terms are just $R_{f}(0,0)$ and the third is $-2 R_{f}(\underline{\delta})$, so that $v(\underline{\delta})=2\left(R_{f}(0,0)-R_{f}(\underline{\delta})\right)$. If $f$ is isotropic, the values of $R_{f}$ and $v$ depend only on the length of $\underline{\delta}$, not on its direction, so that they become functions of a single variable.

A texture can be modeled as a correlated random field, e.g., as an array of independent identically distributed random variables, to which a filtering operator has been applied. This model suggests that a texture can be described by its autocorrelation and by the probability density of the original random variables; the latter can be approximated by a histogram after a "whitening" operation has been applied to decorrelate the texture. If we use the gradient or Laplacian as an approximate whitening operation, the histogram is just a histogram of difference values, as in the preceding subsection

The rourier power spectrum $|\mathrm{F}|^{2}$ and the autocorrelation $R_{f}$ are Fourier transforms of each other. Thus $|F|^{2}$ can also be used as a texture descriptor. The rate at which $|F|^{2}$ falls off as the spatial fequency ( $u, v$ ) moves away from $(0,0)$ is again a measure of the coarseness of $f$; the falloff is faster for a coarse texture and slower for a
busy one, since fine detail gives rise to more power at high spatial frequencies. Samples of $|F|^{2}$ taken over rings centered at $(0,0)$, or over sectors emanating from $(0,0)$ (to detect directional biases), have often been used as texture features.

Other transforms of $f$ can also be used as a source of texture features. In practice, features based on $|F|^{2}$ (or $R_{f}$ ) seem to be somewhat less effective for texture discrimination than features based on second-order or local property statistics. At the same time, computation of $|F|^{2}$ is more costly than computation of a few statistical features (recall that they usually need only be computed for a few $\delta$ 's), unless we compute it optically.

## e) Region-based descriptions

The texture descriptors considered so far are derived from local or point pair properties. We conclude by briefly discussing texture description in terms of homogeneous patches or "primitive" regions. Several types of texture models are based on such decompositions into regions. For example, textures can be generated by using a random geometric process to tessellate the plane into cells, or to drop objects onto the plant, and then selecting gray levels (or gray level probability Gensities) for the cells or objects in accordance with some probability law.

If we can explicitly extract a reasonable set of primitives from $f$, we can describe the texture of $f$ using statistics of properties of these primitives--e.g., the mean or standard deviation of their average gray level, area, perimeter, orientation (of principal axis), eccentricity, etc. Secondorder statistics can also be used--i.e., we can construct matrices for pairs of values of the area (etc.) at pairs of neighboring primitives (perhaps in directions defined by each primitive's orientation. Of course, this approach depends on being able to extract a good set of primitives from $f$ at a reasonable computational cost. A related, but much simpler, idea is to extract maximal homogeneous blocks (e.g., runs of constant gray level in various directions) from $f$, and describe $f$ in terms of (first or second order) statistics of the block sizes (e.g., run lengths).
In general, the description of textures in terms of primitives may be hierarchical; the primitives may be composed of subprimitives, etc., or they may be arranged into groupings which in turn form larger groupings, etc. This makes it possible to define placement rules for the primitives in the form of stochastic grammars. Texture analysis can thus be carried out, in principle, by parsing with respect to a set of such grammars.

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SHAPE IDE'N'RIFICATION USIING 3-D FEATURES

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Image analysis continues to pose significant difficulties for automatic computer analysis due to the unpredictability of object signatures, variability of scene and illumination content and information lost using twodimensional imagery. Systems providing both intensity and range information permit geometric analysis of the scene to be performed concurrently with grey scale analysis. Laser range imagery provides this capability. Pixel values in a range image measure the distance to the nearest surface along the ray; thus, physical measurements of shapes can be extracted. Two applications will be described. In the first, planar surfaces are identified and extracted for matching in scenes containing buildings [l]. In the second, vehicles (e.g. trucks and tanks) are discriminated based on features extracted from the $3-D$ data.

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 \&N83 15770
# Automatic Photointerpretation via Texture and Morphology Analysis <br> Julius T. Thou <br> Center for Information Research University of Florida <br> <br> ORIGINAL PAGE IS <br> <br> ORIGINAL PAGE IS OF POOR QUALITY 

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## Abstract

This paper discusses computer-based techniques for automatic photointerpretation based upon information derived from texture and morphology analysis of images. By automatic photointerpretation, we mean the determination of semantic descriptions of the content of the images by computer. Such descriptions include a narrative report identifying the objects in the image and describing their characteristics and relationships. Our approaches consist of two major tasks: (1) Morphology analysis, and (2) textural analysis. Morphology and shape information enables us to make a preliminary identification of the objects or regions. In both tasks, a growing knowledge-base is generated from past experience and a prior information. Objects with distinctly different morphology and shape are recognized and these contents in an image are interpreted. To make a finer identification and more accurate interpretation, we make use of textural information.

To perform semantic analysis of morphology, we have developed an heirarchical structure of knowledge representation. The simplest elements in a morphology are "strokes", which are used to form "alphabets". The "alphabets" are the elements for generating "words", which are used to describe the function or property of an object or a region. The "words" are the elements for constructing "sentences", which are used for semantic description of the content of the image. We realize that in many cases morphology alone may not be sufficient to make positive identification and accurate interpretation. Photointerpretation based upon morphology is then augmented by textural information. $\qquad$ iditirfionalily bean

To perform textural analysis, we make use of the pixel-vector approach. Each pixel or cluster of pixels is represented by a property vector which characterizes the pixels belonging to an object or a region in an image. Pixels of similar properties are extracted by a correlation and clustering technique. Since an object or a region may contain several types of pixels, an object may be decomposed into several clusters of pixels with different types and properties. The features of the decomposed objects are used in automatic photointerpretation. When objects can be decomposed into similar clusters, we determine the textural rythm as positive identification of the object or the region in addition to morphology information. The knowledge-base is augmented with acquired information from image analysis.

Some experimental results of our knowledge-based photointerpretation system will be discussed.

FLIR TARGET SCREENING
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## Summary

The remote sensing applications typically involve a sensor that acquires data from the phyiscal world, a processor to process this data and most likely a controller that performs certain mission related functions. Remote sensing is characterized by lack of control over the sensing environment and the scene being sensed. Thus, in addition to the problems posed by the recognition task itself, there are special problems due to uncontrolled environmental factors, including noise and coherent clutter, and in some cases, uncooperativeness on the part of the objects that are to be recognized (countermeasures, camouflage).

One application of remote sensing has been in the area of FLIR target recognition. The sophistication of reconnaissance and strike systems is constantly increasing due to the high threat operational environment. Thus, advanced forward looking infrared sensors are integrated on high performance aircraft. The fast loading and high information rate of advanced sensors has made it imposssible for a human to perform the target search/deduction/recognition task accurately, consistently, and in real time. A lot of work has been done by university/industry teams towards the development of FLIR target screening technology.

A typical target screener would consist of segmentation, detection, and recognition stages. Segmentation step would typically involve the ability to locate the regions of interest. The detection stage is to separate out the clutter from potential targets and the recognition step is to label the type of the target. The process to date includes not only the simulation of this technology in various laboratories around the country but also the development of real time hardware. Some of the hardware boxes have been tested in real time in helicopters, etc.

In FLIR much work has been done on the segmentation and recognition of invidual targets, some of it using structural as well as statistical methods. One key problem that still remains is the ability of these techniques to work under varied conditions without extensive retraining of the algorithms. 'rhis deficiency is what leads us to the development of what are known as multi-scenario target screeners. There the key ability for the screener is to learn from what it sees through its sensor and adapt accordingly without operator intervention. The work being done by Honeywell in this direction will be ciscussed at the conference.

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# Structural Analysis Techniques for Remote Sensing 

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## SUMMARY

Structural analysis uses knowledge of the properties of an entity, its parts and their relationships, and the relationships in which it participates at a higher level to locate and recognize objects in a visual scene. For example, Bajcsy and Tavakoli [1] used spectral and shape properties of roads along with knowledge about required connections to other roads in a system for computer recognition of roads from satellite images. Tenenbaum, et. al. [7] set up geometric correspondence; between sensed images and symbolic reference maps to aid in monitoring or tracking predefined targets. We will discuss the basic techniques required for structural analysis.

One problem is the representation of structural knowledge. Production systems and relational descriptions are the two major classes of representation used so far. Production systems include picture grammars [2] that use production rules to parse pictures and expert systems [4] where knowledge about pictures can be stored and retrieved. Relational descriptions include trees, graphs, $n$-ara
relations, complex relational structures, semantic nets [4], and frames [3].

A second problen is the development of efficient algorithms for using the structural information to help analyze an image. In [5] we defined a general model of a structural description and compared several inexact matching algorithms for finding the corresponder.ce between models and images. A simple scheme called forward checking showed the most promise. Parallel hardware can also be used to speed up the matching process. In general, the more the problem can be constrained by knowledge, the faster the matching can be done.

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Linda G. Shapiro
STRUCTURAL KNOWLEDGE OF AN ENTITY includes

1. Knowledge of its properties
2. Knowledge of its parts and their relationships
3. Knowledge of the relationships in which it participates at a highen level

Example: ORIGINAL PAGE IS ROAD NETWORK IN AN URBAN SETTING


1. One property is its charcoal gray color
2. its parts are thin straight regions arranged th a grid
3. arranged along it are rows of hooses and shrubbery

THE REPRESENTATION PROBLEM

How can we represent structural knowledge in a way so that it can be used by a computer whose job it is to analyze scenes?
\{SYNTACTIC
descriptions

$$
\begin{aligned}
& s \rightarrow() \\
& s \rightarrow(s) \\
& s \rightarrow s s
\end{aligned}
$$

RELATIONAL DESCRIPTIONS


REPRESENTING KNCIELENCE ABC!T MKTUPE:

PICTURE GRAMMARS
Extension of ohrore structiure jommar: ane associnted normen to $2 D$ duta

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ARRAY GRAMMAR
SAMFIE PREDUCTIISN:
2)

2)


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## DERIVATION CF A

 RIGHT TRIANGLE:

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ANOTHER TRIANGLE GRAMMAR
(TRIANGLE ( $x$ y $z$ )
( $($ VERTEX $X)$ (VERTEX Y)
(VERTEX $Z$ ) (ELS $X Y$ )
(ELS $X$ Z) (ELS Y Z)
(NON COLL $X Y Z$ )
C( VERTICES
(LIST Yo))))

ORIGINAL PAGE IS
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HOW DC YOU PARSE A PICTURE:

METHOD 1: CREATE A. STRINg O DESCRIBING THE PICTURE AND PAKCEIT.

METHOD 2: LSE THE GRAMMAR TS CHINE THE SEARCH FER CERTAIN RELATIENSHIRE IN THE PICTURE

RELATIONAL STRUCTURES

picture

- ADJACENT
$\rightarrow$ INSIDE
$\rightarrow$ LEFT OF RELATIONS


THE MATCHING PROBLEM ORIGINAL PAGE IS
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Given a structural model of an entity,

1) determine if a given entity has the same structural characteristics as the model.
or 2) find an entity in a given image with the same structural characteristics as the model.
or 3) determine if an entire image satisfies the constraints specified by the model.

EXACT MATCHING
Structural Description of an Object

$$
\begin{aligned}
& D=(P, R) . \\
& P=\left\{P_{1}, \ldots, P_{m}\right\} \text { is a set }
\end{aligned}
$$

of PRIMITIVES.
$P_{i} \subseteq$ Attributes $\times$ Values, $1 \pm i \pm m$

$$
R=\left\{P R_{1}, \ldots, P R_{k}\right\} \text { is a set. }
$$

of named $N$-any relations over $P$

$$
P R_{k}=\left(N R_{k}, R_{k}\right) \quad 1 \leqslant k \leq K
$$

$N R_{A}$ is a NAME, for $R_{k}$ $R_{A} \subseteq P^{M_{A}}$ for some $M_{A}$ is an $M_{k}$-ary relation

EXACT PRIMITIVE MATCHING

Candidate Primitive $C_{j}$ matches Prototype Primitive $P_{i}$ if $P_{i} \subseteq C_{j}$

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R


Rah

Figure l 1 illustrates the composition of
Composition of a Relation with a Function

Let $R \leqslant P^{N}$ and $h: P \rightarrow Q$ $R \circ h=\left\{\left(q_{1}, \ldots, q_{N}\right) \in Q^{N} \mid\right.$ there exists. $\left(p_{1}, \ldots, p_{N}\right) \in R$ with $h\left(p_{i}\right)=q_{i}, 1 \leqslant i \leqslant i$.


5


Roh © S
Hon
Figure: 1.2 illustrates a relational homocerphite
$h$ from binary relation $R$ to binary relation 5 . $\qquad$
A RELATIOXAL HOMOMCRPHISM
from $R$ to $S$ is a mapping
$h: P \rightarrow Q$ satisfying. $R \circ h \leqslant S$

EXACT MATCH DEFINITION
Let $D_{p}=(P, R)$ be a prototype structural description

$$
\begin{aligned}
& P=\left\{P_{1}, \ldots, P_{n}\right\} \\
& R=\left\{\left(N R_{1}, R_{1}\right), \ldots,\left(N R_{k}, R_{k}\right)\right\}
\end{aligned}
$$

Let $D_{c}=(Q, S)$ be a candidate structural description

$$
\begin{aligned}
& Q=\left\{Q_{1}, \ldots, Q_{m}\right\} \\
& S=\left\{\left(N S_{1}, S_{1}\right), \ldots,\left(N S_{k}, S_{k}\right)\right\}
\end{aligned}
$$

$D_{c}$ MATCHES $D_{p}$ if there is a mapping $h: P \rightarrow Q$ satisfying

1) $h\left(P_{i}\right)=Q_{j} \Rightarrow P_{i} \leq Q_{j}$
2) $N R_{i}=N S_{j} \Rightarrow R_{i} \circ h \leq S_{j}$

INEXACT MATCHING
Weighted Prototype. Structural Description.

$$
\begin{aligned}
& D=\left(P, w_{p}, R, W_{R}\right) \\
& P=\left\{P_{1}, \ldots, P_{n}\right\} \\
& w_{P}: P \rightarrow[0,1] \text { satisfies } \sum_{i} w_{P}(i)=1 \\
& R=\left\{\left(N R_{1}, R_{1}\right), \ldots,\left(N R_{K}, R_{k}\right)\right\} \\
& W_{R}=\left\{w_{1}, \ldots, w_{k}\right\} \\
& w_{R}: R_{R} \rightarrow[0,1] \text { satisfies } \\
& \quad \sum_{\Omega \in R_{R}} w_{R}(\Omega)=1 \quad \text { for } 1 \leq R \leq K
\end{aligned}
$$

$W_{p}$ is the primitive weighting function $W_{k}, 1 \leq k \leq k$, are the $N$-tuple. weighting functions

E-homomorphisms
Let $R \leq p^{N}, w: R \rightarrow[0,1]$, $S \leq Q^{N}$ 。

An E-homomorphism from $R$ to $S$ with respect to $w$ is a mapping $\boldsymbol{h}: \boldsymbol{P} \rightarrow \boldsymbol{Q}$ satisfying

$$
\sum_{\substack{ \\\Omega \circ h \notin S}} w(\Omega) \leq \epsilon
$$

the ERROR of $h$

INEXACT MATCH DEFINITION
Let $D_{p}$ be a weighted prototype structural description

$$
\begin{aligned}
& D_{P}=\left(P, w_{p}, R_{P}, W R_{P}\right) \\
& P=\left\{P_{1}, \ldots, P_{m}\right\}, R_{P}=\left\{\left(N R_{1}, R_{1}\right), \ldots,\left(N R_{k}, R_{k}\right)\right\} \\
& \quad W R_{P}=\left\{w_{1}, \ldots, w_{k}\right\}
\end{aligned}
$$

Let $D_{c}$ be a candidate. structural description

$$
\begin{aligned}
& D_{c}=\left(C, R_{c}\right) \\
& C=\left\{C_{1}, \ldots, C_{m}\right\}, R_{c}=\left\{\left(N S_{1}, S_{1}\right), \ldots,\left(N_{\left.S_{k}, S_{k}\right)}\right)\right.
\end{aligned}
$$

$D_{c}$ inexactly matches $D_{p}$ with respect to
attribute value thresholds $T=\left\{t_{a} \mid a \in A\right\}$. missing parts threshold $t_{m}$, and relation thresholds $E=\left\{\epsilon_{i} \mid P R_{i} \in R_{p}\right.$ ? If there is a mapping
$h: P \rightarrow \subset \cup\{$ null $\}$ satisfying

1) $h\left(P_{i}\right)=C_{j} \Rightarrow C_{j}$ 'inexactly matches $P_{i}$ with respect to $T$.
2) $\sum_{\substack{p_{i} \in p \\ h\left(p_{i}\right)=\text { null }}} w_{p}\left(p_{i}\right) \leq t_{m}$
3) $N R_{i}=N S_{j} \Rightarrow h$ is an $\epsilon_{i}$-homomorphism wet $w_{i}$ from $R_{i}$ to $S_{j}$

FINDING E-homomorphisms
Tree search with
a) BACKTRACKING
b) FORWARD CHECKING
c) LOOKAHEAD BY ONE

BACKTRACKING
Instantiate pairs

$$
\left(P_{i}, Q_{j}\right)
$$

until the error of the partial mapping constructed so far is greater than $\epsilon$. Then BACKUP.

FORWARD CHECKING
Like backtracking, BUT...
Keep a Future Error Table. FEET. $Q_{1}, Q_{2}, Q_{b}, Q_{m} \quad$ Miner $P_{P_{1}}:$

Pi:
Pm,
F.E.T. $(i, j)=$ the error that mapping $P_{i}$ to $Q_{j}$ will cause based on the partial mapping instantiated so far. $\operatorname{minera}(i)=$ the smallest error in row $i$ of F.E.T.

PROCEDURE FOR UPDATING F.E.T. expressed for binary relation $R \in P^{2}$ and binary relation $S \leqslant Q^{2}$ (both symmetric)

Procedure UPDATE (F.E.T, $P_{c}, Q_{c}$, Master)
Sum_of_ Errors: $=0$
for each uninstantiated primitive $P_{i}$ de begin
for each $Q_{j}$ still O.k. for $P_{i} \underline{\text { do }}$ begin

$$
\operatorname{gif}_{\text {if }}\left(P_{c}, P_{i}\right) \in R \text { and }\left(Q_{c}, Q_{j}\right) \notin S
$$

then $F \cdot E . T \cdot(i, j)=F . E . T .(i, j)+w\left(P_{c}, P_{i}^{i}\right.$
end
$\operatorname{MINERA}(i):=$ the new minimum in now $i$
Sum_of_Errors: $=$ Sum-of-Errors + MINERP(i) If Sum-of-Errors + Pastern $>\epsilon$.
then fail return
end
return (Sum-of-Errors)
end

IN FORWARD CHECKING
Before instantiating some pair $\left(P_{c}, Q_{c}\right)$

1) check if
error of partial labeling so far + F.E.T. $\left(P_{c}, Q_{c}\right)$

+ Sum of MINERR(i) for all uninstantiated primitives $P_{i}$ is less than or equal to $\epsilon$

2) call UPDATE to update F.E.T. and return new future error sum (and possibly to FAIL)

LOOKAHEAD BY ONE
Like forward checking BUTT also take into account error that can come about due to pairs of yet uninstantiated primitives ( $P_{i}, P_{i^{\prime}}$ ).


Fighin 11.، illustrates the number of allliseconde of CPU time on an Imy 370/158 as a function of number of units for $p=.5, c=1$, add three different search sethods.

THE ORGANIZATION PROBLEM!
How can we organize a large database of structural models for fast retrieval of those models most appropriate for use in analyzing a given in age.

Approach

1) Define a relational distance measure that is a metric
$D(R, S)=0 \leftrightarrow R$ isomorphic to $S$
$D(R, S)=D(S, R)$

$$
D(R, S) \leq D(R, T)+D(T, S)
$$

for all relational models $R, S, T$.
a) Using the relational distance meausure, cluster the models into groups of similar models
3) Select for each cluster C a representative $R_{c}$.
example representative Best Model B satisfying

$$
\sum_{R \in C} D(R, B)=\min _{R^{\prime} \in C} \sum_{R^{\prime} \in C} D\left(R, R^{\prime}\right)
$$

4) Match unknown entities to the cluster representatives and only examine further those clusters whose. representatives took promising

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For best results, the clustering algorithm and representative selection aigonithm should satisfy

For each cluster $C$ and its representative $R_{C}$, there is a threshold $T_{c}$ so that

1) $R \in C \Rightarrow D\left(R, R_{C}\right) \leq T_{C}$
2) $R \notin C \Rightarrow D\left(R, R_{C}\right)>T_{C}$

WHAT ABOUT INEXACT MATCHIN A:

What if unknown entity $U$ is not identical to any mode! in the database, but is closest to some model $M$ in cluster $C$ with representative $R_{C}$ and threshold $T_{C}$.

$$
\begin{aligned}
& D\left(U, R_{C}\right) \leq D(U, M)+D\left(M, R_{C}\right) \\
& D\left(M, R_{C}\right) \leq T_{C}
\end{aligned}
$$

If $D(U, M) \leqslant \epsilon$, then

$$
D\left(U, R_{c}\right) \leq T_{c}+E .
$$

In practice, we can choose an $\epsilon$ and increase each threshold by $\in$ so that any entity within $\epsilon$ of a model will match the representative of its cluster

A remote sensing problem
Giver: $\angle A N D S A T$ image of $a$ mountainous area

Problem: Automatically identify ridges, valleys, rivers, streams direction of flow.

This problem is being soled at UPI with the use of STRUCTURAL KNOWLEDGE.

Given: an image already segmented into bright and dark regions

APRIORI KNOWLEDC.E

1) picture was taken in April .no foliage
2) sun illumination 15 from east to west

RDGE-boundary that is dark on the left and bright on the right

$\leftarrow$ SUN
valley - boundary that is bright on the left and dark on the right.

elevation model
start with known points of lowest elevation and grow the elevation outwards.

we grow along the vel!ey and outward from the valley

CLASSIPYING HORIZOUTAL SEAMCNTS

a higher than $b$
$\Rightarrow x=$ valle $y$
a lower than b

$$
\Rightarrow x=\text { ridje. }
$$

PEAK - junction of four boundary segments with more ridges than valleys and with the length of it :s longest rife greater than a threshold.

STRUCTURAL. KAIRIWLEEDC.E: ABOUT RIVER TUNCTICRIS


ANGLES

$$
\begin{array}{llllc}
\text { SirS } & \text { S1-S2 } & \text { S2-S3 } & \begin{array}{l}
\text { UP } \\
\text { STREAM }
\end{array} & \begin{array}{c}
\text { DOWN } \\
\text { STREAM }
\end{array} \\
=180 & <90 & >90 & \text { S1,S2 } & S, 3 \\
=180 & >90 & <90 & S 2,53 & S 1 \\
<180 & >90 & >90 & S 1, S 3 & 52
\end{array}
$$

T. S. Huang<br>Coordinated Science Laboratory University of Illinois at Urbana-Champaign 1101 West Springfield Avenue Urban, Illinois 61801

## Summary

The determination of $3-D$ motion and structure from image sequences has many applications, including interframe TV coding, target tracking, and robot trajectory planning using visual feedback: Past work in the area led to results which involved the iterative solution of nonlinear equations [r-3], and the questions of convergence and uniqueness were not resolved. In this talk, we shall present a new method [4] of determining 3-D motion and struccure from two image frames. This method requires eight point correspondences between the two frames, from which 3-D motion and structure parameters are determined by solving a set of eight linear equations and a singular value decomposition of a $3 \times 3$ matrix. We also show that the solution thus obtained is unique,

## References

1. S. Ullman, The Interpretation of Visual Motion, MIT Press, 1979.
2. J. W. Roach and J. X. Aggarwal, Determining the movement of objects from a sequence of images, IEEE Trans, on PAMI, Vol. 2, PP. 554-562; Nov. 1980.
3. T. S. Huang and R. Y. Tai, 3-D motion estimation from image-space shifts, Proc. IEEE International Conf. on ASSP, March 30-April 1, 1981; Atlanta, GA.
4. R. Y. Tai and T. S. Huang, Uniqueness and estimation of 3-D motion parameters of rigid bodies with curved surfaces, Report R-921, Oct. 30, 1981, Coordinated Science Laboratory, University of Illinois, Urban, Illinois 61801.
[^3]3-D Motion + structure from Image sequences

Roger y. Thai and
Thomas s. Itnang courdinated science Laboratory Univ. of Elise is, Urbanachampaign

$$
\text { Applications: }\left\{\begin{array}{l}
\text { Image coding (BW Compression) } \\
\text { Enhanament } \\
\text { segmentation }
\end{array}\right.
$$

Robotics: use visual fred back To plan collioin-troe trajectories

Notation and statement of Pwblem:


$$
\left[\begin{array}{l}
x^{\prime}  \tag{1.22}\\
y^{\prime} \\
z^{\prime}
\end{array}\right]=R\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]+\left[\begin{array}{c}
\Delta x \\
\Delta y \\
\Delta z
\end{array}\right]
$$

where $(\Delta x, \Delta y, \Delta z)$ is the amcu $t$ of translation, and $R$ is a rotation matrix [1.6]

$$
R=\left[\begin{array}{lll}
n_{1}^{2}+\left(1-n_{1}^{2}\right) \operatorname{cns} \theta & n_{1} n_{2}(1-\cos \theta)+n_{3} \sin \theta & n_{1} n_{3}(1-\cos \theta)-n_{2} \sin \theta  \tag{1.23}\\
n_{1} n_{2}\left(1-c c r_{2}\right)-n_{3} \sin \theta & n_{2}^{2}+\left(1-n_{2}^{2}\right) \cos \theta & n_{2} n_{3}(1-\cos \theta)+n_{1} \sin \theta \\
n_{1} n_{3}(1-c \operatorname{si})+n_{2} \sin \theta & n_{2} n_{3}(1-\cos \theta)-n_{1} \sin \theta & n_{3}^{2}+\left(1-n_{3}^{2}\right) \cos \theta
\end{array}\right]
$$

where $n_{1}, n_{2}$ and $n_{3}$ are the directional cosines of the axis of rotation

$$
\begin{equation*}
n_{1}^{2}+n_{2}^{2}+n_{3}^{2}=1 \tag{1.24}
\end{equation*}
$$

and $\partial$ is the amount of rotation between the two frames.
Assume the amount of rotation, $\theta$, is small. Then

$$
\mathbf{R}=\left[\begin{array}{ccc}
1 & \mathrm{n}_{3} \theta & -\mathrm{n}_{2} \theta  \tag{1.25}\\
-\mathrm{n}_{3} \theta & 1 & \mathrm{n}_{1} \theta \\
\mathrm{n}_{2} \theta & -\mathrm{n}_{1} \theta & 1
\end{array}\right]
$$

I
And we have, using Eqs. (1.22) and (1.15) and after some algebraic manipularsion,
Problem: Find $R$ and $\left[\begin{array}{c}\Delta x \\ \Delta y \\ \Delta t\end{array}\right]$ from the two images at $t_{1}$ and $t_{2}$.

Motion Estionation: Approach
step!


$$
\begin{aligned}
& t=t_{1} \\
& \left.t=t_{2}( \rangle t_{1}\right)
\end{aligned}
$$

Find point correspondences

$$
\begin{aligned}
& \left(x_{1}, y_{1}\right) \rightarrow\left(x_{1}^{\prime}, y_{1}^{\prime}\right) \\
& \left(x_{2}, y_{2}\right) \rightarrow\left(x_{2}^{\prime}, y_{2}^{\prime}\right)
\end{aligned}
$$

Step 2

$$
\left\{\begin{array}{l}
x^{\prime}=x^{\prime}(x, y, \text { motion paranecters }) \\
y^{\prime}=y^{\prime}(x, y, \text { motion paraenctors })
\end{array}\right.
$$

Non linear equations! solve for the motion parameters.

Examples of derivation of the nonlinear Equations:
S. Ullman, The Interpretation of vised motion, MエT press, 1979.
J. IV. Reach and J. K. Aggariva!, Determining the movement of objects from a sequence of images, IEEE Trans. on SAmE, Nov. 1980.
T.S.Hearg and R.Y. Tsai, Three-dimensional motion estimation from imoige-space shifts, Proc. IEEE Int'l cont. ASSP, march 30 -april!, 1981, Atlanta, Georgia.
$\rightarrow$ prazely.
Nonlinear equations - solve by iterative methods
$\qquad$
Let $F=1$.

$$
\left\{\begin{array}{l}
\Delta x=x^{\prime}-x \\
\Delta y=y^{\prime}-y
\end{array}\right.
$$

Let $\Delta z=1$.
For small rotation,

$$
\left\{\begin{array}{l}
\Delta x=\frac{\left(y \varphi_{3}-\left(1+x^{2}\right) \varphi_{2}+x y \varphi_{1}\right) z+\Delta x-x}{\left(\varphi_{2} x-\varphi_{1} y\right) z+z+1} \\
\Delta y=\frac{\left(-x \varphi_{3}+\left(1+y^{2}\right) \varphi_{1}-x y \varphi_{2}\right) z+\Delta y-Y}{\left(\varphi_{2} x-\varphi_{1} y\right) z+z+1}
\end{array}\right.
$$

where $\varphi_{1}=n_{1} \theta, \varphi_{2}=n_{2} \theta, \varphi_{3}=n_{3} \theta$.
Eliminations $z$,

$$
\begin{aligned}
& \frac{\Delta x-X-\Delta X}{\Delta X\left(\varphi_{2} X-\varphi_{1} Y+1\right)-\left[Y \varphi_{3}-\left(1+x^{2}\right) \varphi_{2}+x Y \varphi_{1}\right]} \\
& =\frac{\Delta y-Y-\Delta Y}{\Delta Y\left(\varphi_{2} X-\varphi_{1} Y+1\right)-\left[-X \varphi_{3}+\left(1+Y^{2}\right) y_{1}-X Y \varphi_{2}\right]}
\end{aligned}
$$

unknowns: $\varphi_{1}, \varphi_{2}, \varphi_{3}, \Delta x$, and $4 y$. need at least 5 displacement vector $s$.

End of Stroy? No!
uniqueness?
Given the 2 image frames, Can we determine the motion parameters uniquely (aside from a scale factor in translation)?

A better motion estimation procedure where we can be sure of our solution?
solving NL effs. iteratively $\left\{\begin{array}{l}\text { nonconvengence. }\end{array}\right.$ has problems - local min.

This paper presents answers $\%$ these 2 questions.

Motion estimation for a Rigid planar patch
Assume the object points we consider lie in a plane

$$
a x+b y+c z=1
$$

at time $t=t$,
Then :

$$
\left\{\begin{array}{l}
x^{\prime}=\frac{a_{1} x+a_{2} y+a_{3}}{a_{7} x+a_{8} y+1} \\
y^{\prime}=\frac{a_{4} x+a_{5} y+a_{6}}{a_{7} x+a_{8} y+1}
\end{array}\right.
$$

where

$$
A \triangleq\left[\begin{array}{lll}
a_{1} & a_{2} & a_{3} \\
a_{4} & a_{5} & a_{6} \\
a_{7} & a_{8} & 1
\end{array}\right]=k\left\{R+\left[\begin{array}{c}
\Delta x \\
\Delta y \\
\Delta z
\end{array}\right][a, b, c]\right\}
$$

Question 1: Given the 2 image frames, are $\left\{a_{1}, a_{2}, \cdots, a_{8}\right\}$ unique? How do we find them?
Question 2: Given $A$, are $R, T \triangleq\left[\begin{array}{l}\Delta x \\ \Delta y \\ \Delta j\end{array}\right]$, and $g \triangleq\left[\begin{array}{l}a \\ b \\ c\end{array}\right]$ unique?

How do we find them?

Theorem Given a ons-une mapping from $R^{2} \pitchfork R^{2}: \quad(x, y) \rightarrow\left(x ; y^{\prime}\right)$

The matrix $A \triangleq\left[\begin{array}{lll}a_{1} & a_{2} & a_{3} \\ a_{4} & a_{5} & a_{6} \\ a_{7} & a_{8} & 1\end{array}\right]$ is uniquely determined.

Theories
The matrix $A$ is naiguely determined by a minimum of 4 point cusresposidences where no 3 points lie on a st. line.

$$
\begin{aligned}
& A=k\left\{R+T g^{T}\right\} \\
& {\left[\begin{array}{lll}
a_{1} & a_{2} & a_{3} \\
a_{4} & a_{5} & a_{8} \\
a_{3} & a_{8} & 1
\end{array}\right]=R\left\{R+\left[\begin{array}{l}
\Delta x \\
a_{y} \\
\Delta z
\end{array}\right]\left[\begin{array}{lll}
a & b & c
\end{array}\right]\right\}}
\end{aligned}
$$

Singular valence decomposition (SVD) of $A$ :

$$
A: U \Omega V=U\left[\begin{array}{ccc}
\lambda_{1} & 0 & 0 \\
0 & \lambda_{2} & 0 \\
0 & 0 & \lambda_{3}
\end{array}\right] V^{\top}
$$

Theorem
case ; $\quad \lambda_{1}>\lambda_{2}>\lambda_{3}$
R.T.g can be expressed in terms

2 solutions for $R, T, g$.
case ii $\quad \lambda_{1}=\lambda_{2} \neq \lambda_{3}$
1 solution for R,T. $g$.
case iii $\quad \lambda_{1}=\lambda_{2}=\lambda_{3}$

$$
\left\{\begin{array}{l}
1 \text { solution for } R \\
T=0 . \\
g \text { curbitravy }
\end{array}\right.
$$

Note. scute factor $w$ in $T g=(T \omega)\left(g w^{* 1}\right)$ )
is exclude $d$.

Theorem
Given 3 frames, ie.,

$$
R^{2}(x, y) \rightarrow R^{2}\left(x^{\prime}, y^{\prime}\right) \rightarrow R^{2}\left(x^{\prime \prime}, y^{\prime \prime}\right),
$$

the motion parameters $R_{1} T$, and the geornstyy $g$ are uniquely determined.

Theorem en
The motion parameters and the geometrical parameters are uniquely determined from 3 frames of 4 points (no 3 of them lie on a st. line).

Note. the 3 frames have $\pi$ we distinct.

Estimating the three-dimensional motion parameters of a rigid body with curved surface from 2 image frame

Theorem 1. The motion parameters are unignely determined given 7 image point correspondences, if the 7 points do not lie on 2 planes on? "f which contains the origin nor on a cone containing the origin.

Theorem 2. Given 8 point correspondences, the motion parameters can be determined in 2 steps:
(i) soive a set of 8 linear egs. to get the 8 "essential parameters"

$$
E=\left[\begin{array}{lll}
e_{1} & e_{2} & e_{3} \\
e_{4} & e_{5} & e_{6} \\
e_{7} & e_{8} & 1
\end{array}\right]
$$

(ii) Determine $R$ and $T$ by singular value decomposition of $E$.

$$
\left[\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
z^{\prime}
\end{array}\right]=R\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right]+T
$$

Where

$$
\left\{\begin{array}{l}
x^{\prime}=\frac{\left(r_{1} x+r_{2} Y+r_{3}\right) z+\Delta x}{\left(r_{7} x+r_{8} \eta+r_{9}\right) z+\Delta z} \\
y^{\prime}=\frac{\left(r_{4} x+r_{5} y+r_{6}\right) z+\Delta y}{\left(r_{7} x+r_{8} Y+r_{9}\right) z+\Delta z} \\
z=\frac{\Delta x-\Delta z \cdot x^{\prime}}{x^{\prime}\left(r_{7} x+r_{8} Y+r_{9}\right)-\left(r_{1} x+r_{2} Y+r_{3}\right)} \\
z=\frac{\Delta y-\Delta z \cdot Y^{\prime}}{Y^{\prime}\left(r_{7} x+r_{8} Y+r_{9}\right)-\left(r_{4} x+r_{5} Y+r_{6}\right)}
\end{array}\right.
$$

$$
\left[\begin{array}{lll}
x^{\prime} & y^{\prime} & 1
\end{array}\right] E\left[\begin{array}{l}
x  \tag{1}\\
y \\
1
\end{array}\right]=0
$$

where

$$
\begin{aligned}
& E \leqslant\left[\begin{array}{lll}
e_{1} & e_{2} & e_{3} \\
e_{4} & e_{5} & e_{6} \\
e_{7} & e_{8} & 1
\end{array}\right]=G R \\
& G=\left[\begin{array}{ccc}
0 & \Delta z & -\Delta y \\
-\Delta z & 0 & \Delta x \\
\Delta y & -\Delta x & 0
\end{array}\right]
\end{aligned}
$$

(i) Determine $E$.

Given 8 pt. Corresp. : $\left(X_{i}, Y_{i}\right) \rightarrow\left(X_{i}^{\prime}, r_{i}^{\prime}\right)$

$$
i=1,2, \cdots, \delta^{\circ}
$$

From (1),

$$
\left[\begin{array}{cccccccc}
x_{1}^{\prime} x_{1} & x_{1}^{\prime} Y_{1} & x_{1} & Y_{1}^{\prime} x_{1} & y_{1}^{\prime} Y_{1} & y_{1}^{\prime} & x_{1} & Y_{1} \\
\vdots & & & & & & \\
x_{8}^{\prime} x_{8} & x_{8}^{\prime} y_{8} & x_{8} & y_{8}^{\prime} x_{8} & y_{\delta}^{\prime} y_{8} & r_{8}^{\prime} & x_{8} & y_{8}
\end{array}\right]\left[\begin{array}{l}
e_{1} \\
\vdots \\
e_{8}
\end{array}\right]=\left[\begin{array}{c}
-1 \\
\vdots \\
-1
\end{array}\right]
$$

(ii) $R+T$ from $E$.

$$
E=U \Lambda V^{\top} \quad(\operatorname{SV} D)
$$

Then

$$
R=U\left[\begin{array}{ccc}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 5
\end{array}\right] V^{\top}
$$

or $U\left[\begin{array}{rrr}0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 5\end{array}\right] V^{T}$
Where $s=\operatorname{det}(u) \cdot \operatorname{det}(v)=+1$ or -1 and

$$
T=\left[\begin{array}{l}
\left(-\phi_{1}^{\top} \phi_{1}+\phi_{2}^{\top} \phi_{2}+\phi_{3}^{\top} \phi_{3}\right)^{\frac{1}{2}} \\
\left(\phi_{1}^{\top} \phi_{1}-\phi_{2}^{\top} \phi_{2}+\phi_{3}^{\top} \phi_{3}\right)^{\frac{1}{2}} \\
\left(\phi_{1}^{\top} \phi_{1}+\phi_{2}^{\top} \phi_{2}-\phi_{3}^{\top} \phi_{3}\right)^{\frac{1}{2}}
\end{array}\right]
$$

where $\phi_{i}^{T}=i$ th row of $E$.
Note. only one of the 2 solutions for $R$ yields positive $z$.

# 8. Technicolor Graphic Services. Inc. 

RE: OAB5-39
May 10, 1982

Dr. Larry Guseman
Department of Mathematics
Texas A\&M University
College Station, Texas
77843

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Dear Dr. Guseman:
I would like to thank you for inviting me to the NASA Workshop on Image Analysis. Since I work in the area of technology transfer and applications of remote sensing, I viewed the conference from the perspective of how fundamental research influences applied research.

In the area of image segmentation, we now have many tools for clustering and classification. The extension to be made from cu: rent capabilities will perhaps make use of ancillary data for classification refinement via digitized soils data, terrain illumination correction, etc. With many layers in a data base, total registration accuracy is a weak point, as well as mechanism to quickly process mixtures of raster and polygonal formats.

Shape and texture analysis seems to me to present the most exciting possbilities - particularly given the finer resolution of Landsat D. Since we know a human analyst can identify features from shape and texture, perhaps images can be enhanced to iteratively improve the interpretability until ultimately some features can be machine-recognized. The role of color in this process has been relatively untapped which to me implies that the extentsion of algorithms from 2-D to $N-D$ is not at all trivial and deserves a great deal of study.

The area of structural knowledge relates to our needs in that we of ten model a desired variable (irrigability, exploration potential, grazing capacity) as a function of other variables. A structure within which we can analyze and weight the variables, and arrangements of the variables, would be beneficial.

As to where to look for the evolution of new image processing capabilities, I personally have been dissatisfied with statistical approaches. They seem to deal with images in measurement space only and make assumptions that are not true often enough. The best forward strides I have seen are practical and arise from a well-defined problem.

Again, I enjoyed meeting you, found the workshop stimulating, and am looking forward to visiting A\&M again.



Susan K. Jenson Senior Applications Scientist Geoscience Section

May 13, 1982
Dr. Larry Guseman
Dept. of Mathematics Texas A\&M University College Station, TX 77840

Dear Larry,
The following are some suggestions for future research areas under the NASA Fundamental Research Program in Pattern Recognition:

1. Application of AI methodology to develop "expert systems" for various interpretation tasks.
2. Automatic registration of multisensor data, and of images with maps.
3. Study of VLSI architecture requirements for remote sensor data processing and analysis.
4. Development of database management techniques applicable to remote sensor data.

We hope these ideas will be useful. We enjoyed the meeting and look forward to seeing you again soon.

Sincerely,
$\frac{\text { Uuly }}{\text { Larry \$. Davis }} \begin{aligned} & \text { Associate Professor }\end{aligned}$
Azrfel Rosenfeld
AR: job

University of Illinois at Urbana-Champaign ${ }^{152}$

1101 West Springfield Avenue COORDINATED SCIENCE LABORATORY Urbane, Illinois
(217) $333-2511$

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217-333-6912
May 21,1982
Dear puff. Ginseman:
Sorry fir the delay, beat here are copies of some of my view graphs. I hope you still have time \& include them in the workshop pwaedings.

As I stated during the workshop, I think we need basic research in the area of relating $2-d$ images to 3-d scenes.

Sincerely,
Sue ituang


[^0]:    Original photo er why we y be purchased
    from EROS Data boater
    Sioux Falls, SD 57198

[^1]:    * Jack Bryant, Department of Mathematics, Texas A\&M University, College Station, Texas 77843.
    + Susan Jenson, Applications Branch, EROS Data Center, Sioux Falls, South Dakota 57198.

[^2]:    A third problem consists of techniques for storage and retrieval of relational models. A knowledge database will typically consist of a large number of models and/or frames. Matching an unimown image against all of them is impossible. Thus schemes for organizing the database of models for fast retrieval of those models most appropriate to a given image are essential. In [6] we discuss some preliminary schemes for database organization. The problem of organizing structural knowledge and knowledge in general is an important topic of current research.

[^3]:    * To be presented at the NASA Workshop on Image Structures, April 28-30, 1982, Texas AסM University, College Station, Texas 77842.

