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# The Radar Cross Section of Dielectric Disks

D.M. LeVine

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National Aeronautics and  
Space Administration

**Goddard Space Flight Center**  
Greenbelt, Maryland 20771

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### ABSTRACT

A solution is presented for the backscatter (monostatic) radar cross section of dielectric disks of arbitrary shape, thickness and dielectric constant. The result is obtained by employing a Kirchoff-type approximation to obtain the fields inside the disk. The internal fields induce polarization and conduction currents from which the scattered fields and the radar cross section can be computed. The solution for the radar cross section obtained in this manner will be shown to agree with known results in the special cases of normal incidence, thin disks and perfect conductivity. It will also be shown that the solution can be written as a product of the reflection coefficient of an identically oriented slab times the physical optics solution for the backscatter cross section of a perfectly conducting disk of the same shape. This result follows directly from the Kirchoff-type approximation without additional assumptions.

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## THE RADAR CROSS SECTION OF DIELECTRIC DISKS

### I. Introduction

Although much research has been done on the scattering properties of perfectly conducting disks (Meixner and Andrejewski, 1950; Andrejewski, 1952; Ruck, et. al., 1970; Bowman, et. al., 1969; Hodge, 1980) little has been done on scattering from dielectric disks (Ruck, et. al., 1970), and most of the work which has been done applies to thin dielectric disks. For example, Schiffer and Thielheim (1979) developed an analytic solution for scattering from a thin dielectric disk by employing a Rayleigh approximation to find the equivalent sources induced inside the disk. The approximation is valid as long as the thickness is small compared to wavelength and also small compared to the cross section of the disk. Weil and Chu (1976a-b) developed a numerical solution for the thin dielectric disk also solving for the equivalent sources induced inside the disk but used an approach similar to the moment method to obtain the internal currents. They chose a set of basis functions for the radial and angular dependence of the currents but kept only the first order term in a power series expansion in thickness. Their solution applies to disks whose cross section is comparable to wavelength. Using a different approach, Neugebauer (1956, 1957) treated scattering from thin dielectric disks by using Babinet's principle after modifying it to apply to dielectric screens. He then used a physical optics approximation to treat the aperture and obtained results for normal incidence which were in agreement with experiment (Neugebauer, 1957; Ruck, et. al., 1970). Recently, dielectric disks of arbitrary thickness were treated by Le Vine, et. al (1982). This was done using by solving for the currents induced inside the disk by assuming that the fields inside the disk are the same as the fields inside an identically oriented slab of the same thickness and dielectric properties. This is a high frequency approximation reasonable when edge effects are not important, as for example in disks whose cross section is large compared to wavelength. This approximation does not require the disk to be thin compared to wavelength and therefore should include the thin

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dielectric disk as a special case. The scattering properties of dielectric disks whose physical cross section is large compared to wavelength, but whose thickness is not necessarily small compared to wavelength, are important in a variety of applications such as in remote sensing of vegetation as a model for leaves (Lang, 1981; Lang, Seker and Le Vine, 1982) or in remote sensing of clouds as models for ice crystals.

The purpose of this paper is to present a solution for the backscatter radar cross section of a dielectric disk using the solution of Le Vine, et. al. (1982) for the scattered fields. It will be shown that this solution reduces to the results obtained by Schiffer and Thielheim (1979) when the disk's thickness is very small compared to wavelength, and in fact is a generalization of their result which includes the case of vertically polarized incident radiation. It will also be shown that this solution reduces to the physical optics solution for perfectly conducting disks when the conductivity becomes large, and that the solution agrees with the results obtained by Neugebauer (1957) in the special case of normally incident radiation. Finally, it will be shown that the solution for the backscatter radar cross section can be written as a product of the physical optics solution for the backscatter cross section of a perfectly conducting disk of the same shape times the magnitude squared of the reflection coefficient of an identically oriented dielectric slab. This result, which has been suggested intuitively from special cases (Ruck, et. al., 1970) follows directly from the Kirchoff-type approximation for the internal fields without further assumptions and applies (formally) at all angles of incidence.

In the first section to follow the solution obtained by Le Vine, et. al. (1982) for the fields scattered from an arbitrarily oriented dielectric disk will be outlined and then used to obtain an expression for the backscatter cross section. Numerical examples will be presented in this section for the radar cross section of a circular disk. In the following section it will be shown that this result agrees with the special cases which have been treated in the past (i.e., thin disk, normal incidence and perfect conductivity). Finally, in the last section, it will be shown that the solution

can be written as a product of the radar cross section of a perfectly conducting disk times the magnitude squared of the reflection coefficient of an identically oriented dielectric slab, a form which is intuitively satisfying at high frequencies and is convenient to use.

## II. Radar Cross Section

Consider a plane wave incident on an arbitrarily oriented dielectric disk. The plane wave is assumed to have polarization (direction of the electric field)  $\hat{q}$  and to be propagating in the  $\hat{i}$  direction:

$$\vec{E}_{inc}(\vec{r}) = \hat{q} E_0 e^{jk_0 \hat{i} \cdot \vec{r}} \quad (1)$$

and the disk is assumed to have cross sectional shape  $S(\vec{r})$ , thickness  $T$  and to be comprised of material with relative dielectric constant  $\epsilon_r = \epsilon_r' + j \epsilon_r''$ .  $\epsilon_r$  is assumed to be constant throughout the disk and the shape of the disk  $S(\vec{r})$  is arbitrary. ( $S(\vec{r}) = 1$  on the disk and  $S(\vec{r}) = 0$  otherwise.) It is desired to determine the radar cross section of the disk,  $\sigma_{pq}$  in the case of backscatter (monostatic cross section). Following convention (Ruck et. al., 1970; Ishimaru, 1978)  $\sigma_{pq}$  is defined in terms of the scattered electric  $\vec{E}_{scat}(\vec{r})$  with polarization in the  $\hat{p}$  direction due to an incident wave  $\vec{E}_{inc}(\vec{r})$  with polarization  $\hat{q}$  as follows:

$$\sigma_{pq} \triangleq \lim_{R \rightarrow \infty} 4 \pi R^2 \left| \frac{\hat{p} \cdot \vec{E}_{scat}}{\hat{q} \cdot \vec{E}_{inc}} \right|^2 \quad (2)$$

Equation 2 will be evaluated here for the case of backscatter using a Kirchoff-type approximation to obtain the scattered fields.

The formal solution for the scattered fields can be obtained in terms of the fields inside the disk using standard procedures (e.g. Ruck, et. al., 1970; Le Vine, et. al. 1982). One obtains:

$$\vec{E}_{scat}(\vec{r}) = k_0^2 \int \int \int_{\text{volume}} (\epsilon_r - 1) \vec{E}(\vec{r}') \cdot \vec{G}(\vec{r}/\vec{r}') d\vec{r}' \quad (3)$$

where  $\vec{G}(\vec{r}/\vec{r}')$  is the dyadic Green's function for free space:  $\vec{G}(\vec{r}/\vec{r}') = (\vec{I} + \frac{1}{k_0^2} \nabla \nabla)$   $\frac{e^{jk_0 R}}{4 \pi R}$

where  $R = |\vec{r} - \vec{r}'|$  is the distance between source point  $\vec{r}'$  and observation point  $\vec{r}$ . Equation 3 is to be evaluated in the far field of the disk ( $R \rightarrow \infty$ ) in which case  $\vec{G}(\vec{r}, \vec{r}') \cong (\vec{I} - \hat{\hat{o}} \hat{\hat{o}}) \frac{e^{jk_0(R_0 - \hat{\hat{o}} \cdot \vec{r}')}}{4\pi R_0}$  where  $R_0$  is the distance from the center of the disk to the observer and  $\hat{\hat{o}}$  is a unit vector from the coordinate origin at the center of the disk toward the observer. To obtain a solution the fields inside the disk must be obtained. These aren't known in general and will be approximated here by replacing the disk with an identically oriented slab with the same dielectric properties and thickness and using the fields inside the slab for  $\vec{E}(\vec{r})$  in Equation 3. This approximation is similar to the Kirchoff approximation employed in physical optics to solve for scattering from perfectly conducting surfaces, and ought to be reasonable when edge effects aren't important such as in the case of disks whose cross section is much greater than a wavelength. This approximation has the advantage that it employs a canonical form (i.e., the fields inside the disk are a solution to Maxwell's equations) and yields the correct solution in the limit of a disk with infinite cross section. The solution for the fields inside the slab and the subsequent evaluation of Equation 3 are straightforward but messy. The details are described in Le Vine, et. al. (1982) where it is shown that the scattered field of polarization type  $\hat{p}$  due to an incident wave of polarization type  $\hat{q}$  can be written in terms of a dyadic scattering amplitude,  $\vec{f}(\hat{i}, \hat{o})$  as follows

$$\hat{p} \cdot \vec{E}_{\text{scat}}(\vec{r}, q) = [\hat{p} \cdot \vec{f}(\hat{i}, \hat{o}) \cdot \hat{q}] E_0 \frac{e^{jkR_0}}{R_0} \quad (4)$$

Substituting this result into Equation 2 one obtains:

$$\sigma_{pq} = 4\pi |\hat{p} \cdot \vec{f}(\hat{i}, \hat{o}) \cdot \hat{q}|^2 \quad (5)$$

The general expression for  $\vec{f}(\hat{i}, \hat{o})$  in the case of arbitrary polarization, angle of incident and orientation is complicated. However, the expression simplifies in the special case of backscatter



( $\hat{o} = -\hat{i}$ ), and simplifies further and if one restricts attention to the case in which the normal to the disk,  $\hat{n}$ , is in the plane of incidence. This is a completely general case which can always be obtained for a single disk with a simple rotation of coordinates. In this case, choosing  $\hat{p}$  and  $\hat{q}$  to be unit vectors in the direction of horizontal (h) or vertical (v) polarization defined with respect to the plane of incidence, one obtains:

$$\sigma_{hh} = \left\{ [\text{sinc}(\alpha\Omega_+) - r_h e^{j2\alpha\Gamma} \text{sinc}(\alpha\Omega_-)] e_h^- S_o \right\}^2 \bar{S}^2(v_r) \quad (6a)$$

$$\sigma_{vv} = \left\{ [\gamma_+ \text{sinc}(\alpha\Omega_+) - r_v e^{j2\alpha\Gamma} \gamma_- \text{sinc}(\alpha\Omega_-)] \frac{e_v^-}{\sqrt{\epsilon_r}} S_o \right\}^2 \bar{S}^2(v_r) \quad (6b)$$

$$\sigma_{hv} = \sigma_{vh} = 0 \quad (6c)$$

where  $\alpha = \frac{1}{2} k_o T$  and  $S_o = \frac{1}{\sqrt{4\pi}} T k_o^2 (\epsilon_r - 1)$  are constants and

$$\Omega_{\pm} = \cos(\theta) \pm \Gamma \quad (7a)$$

$$\gamma_{\pm} = \sin^2(\theta) \mp \cos(\theta) \Gamma \quad (7b)$$

$$\Gamma = \sqrt{\epsilon_r - \sin^2 \theta} \quad (7c)$$

$$e_{h,v}^- = \frac{t_{h,v} \exp(-j\alpha\Omega_-)}{1 - r_{h,v}^2 \exp(j4\alpha\Gamma)} \quad (7d)$$

$$r_{h,v} = \frac{\cos \theta - \mathcal{E}_{h,v} \Gamma}{\cos \theta + \mathcal{E}_{h,v} \Gamma} \quad (7e)$$

$$t_{h,v} = \frac{2 \cos \theta}{\cos \theta + \mathcal{E}_{h,v} \Gamma} \sqrt{\mathcal{E}_{h,v}} \quad (7f)$$

In the preceding  $\mathcal{E}_h = 1$  and  $\mathcal{E}_v = 1/\epsilon_r$ ; and  $\theta$  is the angle between the direction of propagation of the incident wave,  $\hat{i}$ , and the normal to the disk,  $\hat{n}$ ; the coefficients  $r_{h,v}$  and  $t_{h,v}$  are the reflection and transmission coefficients of a halfspace with the same dielectric constants as the disk; and  $e_{h,v}^-$  are the amplitudes of the waves inside the disk propagating in the negative

direction (measured with respect to  $\hat{n}$ ) and excited by a unit amplitude incident wave of horizontal or vertical polarization, respectively. [The exponent, 2, in Equations 6 means magnitude squared: The quantity times its complex conjugate. This short hand will be adopted throughout this text.]

Notice that with the geometry being considered here there is no depolarization ( $\sigma_{hv} = \sigma_{vh} = 0$ ). This is not true when the disk is tilted so that  $\hat{n}$  is not in the plane of incidence as is evident from the more general form of the solution for the scattered fields (Le Vine, et. al., 1982; Figure 5). However, this depolarization is a consequence of the definition of the polarization vectors and for a single disk can always be removed by a suitable change of coordinates and appropriate new definition of the polarization vectors. Also notice that both  $\sigma_{hh}$  and  $\sigma_{vv}$  are proportional to  $\bar{S}(\bar{\nu}_t)$ .  $\bar{S}(\bar{\nu}_t)$  is the Fourier transform of the cross sectional shape  $S(\bar{r})$  of the disk evaluated at frequencies  $\bar{\nu}_t$  which in the case of backscatter are:  $\bar{\nu}_t = \frac{k_o}{\pi} \hat{n} \times (\hat{i} \times \hat{n})$  (See Le Vine, et. al., 1982; Appendix C). For a disk with a circular cross section  $\bar{S}(\nu_t) = \pi a^2 \frac{J_1(2 k_o a \sin \theta)}{k_o a \sin \theta}$  where  $a$  is the radius of the disk and  $\theta$  is the angle between  $\hat{i}$  and the normal to the disk,  $\hat{n}$ . In comparison to  $\bar{S}(\nu_t)$ , the factors in brackets  $\{ \}$  in Equations 6 are generally slowly varying functions of  $\theta$ .

To illustrate the characteristics of the solution several examples of the radar cross section of a circular disk as a function of  $\theta$  have been computed. These are shown in Figures 1-4. Figures 1 and 2 show  $\sigma_{hh}(\theta)$  and  $\sigma_{vv}(\theta)$  respectively for a circular disk with radius  $a = 20$  cm, thickness  $T = 5$  cm, relative dielectric constant  $\epsilon_r = 11$  and with incident radiation at frequency  $f = 9$  GHz ( $ka \cong 36$ ). Notice the peak at  $\theta = 0$  (normal incidence) and the rapidly oscillating decay as a function of  $\theta$ , which are due primarily to  $\bar{S}(\nu_t)$ . The effects of the remaining factors in Equations 6 can be seen at large  $\theta$  by comparing  $\sigma_{hh}$  and  $\sigma_{vv}$ . Figures 1 and 2 are representative of the radar cross section of circular disks. Increasing  $ka$  causes the radar cross section to become more peaked at  $\theta = 0$  and to oscillate more rapidly as a

function of  $\theta$  and a decrease in  $ka$  results in a broadened peak and slower oscillations, but in general the shape is not changed. Changing the dielectric constant or adding loss effects the amplitude of the cross section but except in special cases the behavior as a function of  $\theta$  remains as indicated in Figures 1 and 2. An example of the radar cross section of a circular disk with lossy dielectric constant is shown in Figure 3.  $\sigma_{hh}(\theta)$  is shown for a circular disk with radius  $a = 10$  cm, thickness  $T = .5$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and at a frequency  $f = 9$  GHz ( $ka \cong 18$ ). Notice the broadened peak at normal incidence and the reduced number of oscillations in comparison with Figure 1 which are due to the smaller  $ka$ ; however, the overall shape of the curve is similar to those for the lossless dielectric disk in Figures 1 and 2. One of the special cases where dramatic changes in radar cross section can occur independent of shape, is when the disk's thickness becomes an odd multiple of half a wavelength. Figure 4 shows  $\sigma_{hh}(\theta)$  for such a case. The disk is the same as in Figure 3 except that the relative dielectric constant is  $\epsilon_r = 11$  (no loss). In this case  $T = \lambda/2$  where  $\lambda$  is the wavelength inside the disk. When  $T = \lambda/2$  the waves inside the disk add in phase at the surface of the disk and as a result the reflection coefficient is very small. This is clearly evident in  $\sigma_{hh}(\theta)$  and is especially noticeable at normal incidence where the peak is significantly reduced in amplitude. If the dielectric is lossy, then the amplitude of the waves is changed as they propagate through the disk and so this resonance is much less pronounced. (For example, see Figures 10-13 in Le Vine, et. al., 1982).

### III. Special Cases:

In this section it will be shown that the radar cross section (Equations 6) agrees with the special cases available in the literature. It will be shown that at normal incidence Equations 6 yield values confirmed by experiment; that in the limit of perfect conductivity Equations 6 reduce to the physical optics solution; and that for thin disks they are consistent with the solution obtained using a Rayleigh approximation.

#### A. Normal Incidence

At normal incidence  $\tilde{S}(\nu_r) = A$  for all shapes where  $A$  is the area of the disk. Also in this case  $\Gamma = \sqrt{\epsilon_r}$ ;  $\gamma_{\pm} = \pm\sqrt{\epsilon_r}$  and  $\Omega_{\pm} = 1 \pm \sqrt{\epsilon_r}$ . Substituting these results into Equations 6 and writing the functions  $\text{sinc}(\alpha\Omega_{\pm})$  in terms of exponentials one obtains the following results after some straightforward manipulations:

$$\sigma_{hh}(\theta) = \sigma_{vv}(\theta) = \frac{k_0^2 A^2}{\pi} |R_n|^2 \quad (8)$$

where  $R_n$  is the reflection coefficient of a slab of thickness  $T$  centered at the origin. In the notation adopted here

$$R_n = \frac{1}{2} [1 - e^{j4\alpha\sqrt{\epsilon_r}}] r_h e_h^- e^{-j\alpha(1 + \sqrt{\epsilon_r})} \quad (9)$$

For disks which are large compared to wavelength, theory and experiment (Ruck, et. al., 1970) indicate that at normal incidence the radar cross section is:

$$\sigma_{hh}(0) = \sigma_{vv}(0) = \sigma_{oo} |R_n|^2 \quad (10)$$

where  $\sigma_{oo} = (k_0 A)^2 / \pi$  is the radar cross section at normal incidence of a perfectly conducting disk of area  $A$ , and  $R_n$  is the reflection coefficient of an infinite slab of the same thickness and dielectric constant as the disk. Clearly this is in agreement with Equation 8.

Equation 10 can be obtained from the solution Neugebauer derived for the scattered fields

due to a normally incident plane wave (Neugebauer, 1957; Ruck, et. al., 1970). This result agrees with the experiments of Severna and von Baekmann (1951; and Ruck, et. al., 1970). Neugebauer obtained the result through an application of Babinet's principle modified to include dielectric screens and in reporting this work Neugebauer implies that the screen must be thin; however such a restriction is not required to obtain Equation 8.

### B. Perfect Conductivity

Consider the limit when  $\epsilon_r$  becomes very large and imaginary ( $\epsilon_r \rightarrow j\epsilon_r''$  and  $\epsilon_r'' \rightarrow \infty$ ).

In this case:

$$\Gamma \rightarrow \frac{1}{2} (1 + j) \sqrt{\epsilon_r''} \quad (11a)$$

$$\Omega_{\pm} \rightarrow \pm \Gamma \quad (11b)$$

$$\Omega_+ + \Omega_- \rightarrow 2 \cos(\theta) \quad (11c)$$

$$e_{h,v}^- \rightarrow e^{-j\alpha\Omega_-} t_{h,v} \quad (11d)$$

Using these limiting forms in Equations 6 and 7 one obtains:

$$\sigma_{hh}(\theta) \rightarrow \left[ \frac{t_h}{\alpha\Omega_+} S_0 \right]^2 \bar{S}^2(\nu_t) \quad (12a)$$

$$\sigma_{vv}(\theta) \rightarrow \left[ \frac{\gamma + t_v}{\alpha\Omega_+ \sqrt{\epsilon_r''}} S_0 \right]^2 \bar{S}^2(\nu_t) \quad (12b)$$

and since  $t_h \rightarrow 2 \cos(\theta)/\Gamma$  and  $\gamma + t_v \rightarrow 2 \cos(\theta)$  as  $\epsilon_r'' \rightarrow \infty$  one obtains the following result in the perfectly conducting limit:

$$\sigma_{hh}(\theta) = \sigma_{vv}(\theta) = \frac{k_0^2}{\pi} \cos^2(\theta) \bar{S}^2(\nu_t) \quad (13)$$

For a circular disk of radius, a, this becomes:

$$\sigma_{hh}(\theta) = \sigma_{vv}(\theta) = \frac{[k_0(\pi a)^2]^2}{\pi} \cos^2 \theta \left[ \frac{J_1(2k_0 a \sin \theta)}{k_0 a \sin \theta} \right]^2 \quad (14)$$

which is the result obtained using physical optics for the perfectly conducting disk (Ruck, et al., 1970). This result is rather to be expected because the physical optics solution and the solution obtained here follow from a Kirchoff-type approximation and because the solution employed here for the fields inside the slab is valid for arbitrary dielectrics, including very lossy slabs.

### C. Thin Dielectric Disks:

The case when  $k_0 T \sqrt{\epsilon_r} \ll 1$  has been treated by Schiffer and Thielheim (1979) for the case of an incident horizontally polarized wave. In their formulation Schiffer and Thielheim begin with Equation 3 and make a Rayleigh approximation for the fields inside the disk. That is, they find the fields inside using results from electrostatics for very thin plates. One can show that the fields inside an infinite slab reduce to this approximation when the slab thickness goes to zero; consequently, one would expect Equations 6 to reduce to the Schiffer and Thielheim result when  $T \rightarrow 0$ . In fact, this is the case.

In the limit as  $T$  goes to zero, one has:

$$\text{sinc}(\alpha\Omega_{\pm}) \rightarrow 1 \quad (15a)$$

$$e_{h,v}^- \rightarrow \frac{t_{h,v}^-}{1 - r_{h,v}^-} \quad (15b)$$

Hence Equations 6 reduce to

$$\sigma_{hh} \rightarrow \left[ (1 - r_h) e_h^- S_o \right]^2 \tilde{S}^2(\bar{\nu}_t) = \left[ S_o \right]^2 \tilde{S}^2(\bar{\nu}_t) \quad (16a)$$

$$\sigma_{vv} \rightarrow \left[ (\gamma_+ - r_v \gamma_-) \frac{e_v^-}{\sqrt{\epsilon_r}} S_o \right]^2 \tilde{S}^2(\bar{\nu}_t) = \left[ \frac{\sin^2 \theta - \epsilon_r \cos^2 \theta}{\epsilon_r} \right]^2 \sigma_{hh} \quad (16b)$$

and in the case of a disk with circular cross section, one obtains:

$$\sigma_{hh} = \frac{[k_0(\pi a^2)]^2}{\pi} [k_0 T(\epsilon_r - 1)]^2 \left[ \frac{J_1(2k_0 a \sin \theta)}{2k_0 a \sin \theta} \right]^2 \quad (17)$$

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which is the same as obtained by Schiffer and Thielheim. (Note: Schiffer and Thielheim's Equation 27 should be  $F = [ 2(1 - \sin\theta \sin\theta) - (\cos\Delta - \cos\delta)^2 ]^{1/2}$  ).

An example of the radar cross section of a thin disk is shown in Figures 5-6. Figure 5 shows  $\sigma_{hh}(\theta)$  and Figure 6 shows  $\sigma_{vv}(\theta)$  for a disk with radius  $a = 20$  cm, thickness  $T = .04$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and at a frequency  $f = 5$  GHz. This case is representative of leaves (e.g. soybean leaves) at microwave frequencies. Notice that near normal incidence ( $\theta = 0$ ) the curves for  $\sigma_{hh}(\theta)$  and  $\sigma_{vv}(\theta)$  are very similar, but differences in the radar cross sections for the two polarizations are apparent for larger values of  $\theta$ . Figures 5 and 6 have the same general characteristics evident in Figures 1 and 2 for thicker disks because  $\bar{S}(\bar{\nu}_0)$  is the dominant feature of both cases.

The equivalence in the thin disk limit of the solution derived here (Equations 6) with a solution obtained using the Rayleigh approximation is not a coincidence. In fact, one can show that the fields inside a slab (the approximation employed to obtain Equations 6) yields the Rayleigh approximation when the slab is thin. This suggests that for thin enough disks Equations 6 may apply at both high and low frequencies. For example, let  $k_0 a > 10$  be the criterion for the high frequency approximation to be valid, and let  $k_0 T < 1/10$  be the criterion for the Rayleigh approximation to apply to thin disks. Now, if one chooses the thickness so that  $k_0 T < 1/10$  when  $k_0 a = 10$ , then for lower frequencies the Rayleigh approximation applies and for higher frequencies the Kirchoff-type approximation applies. Since Equations 6 embody both approximations, they apply in this case at all frequencies. Clearly such a disk must be very thin compared to its cross section (e.g.  $a/T > 100$ ); however, this is not unrealistic for leaves of plants such as soybeans.

#### IV. General Form of the Radar Cross Section

Equations 6 can be written as a product of the radar cross section of a perfectly conducting disk and the reflection coefficient of an identically oriented slab as will be shown in this section.

The reflection coefficient,  $R_{h,v}$ , of a slab of thickness  $T$  and dielectric constant  $\epsilon_r$  for horizontally (h) or vertically (v) polarized incident plane waves is:

$$R_{h,v} = \frac{1}{2} \left[ e_{h,v}^+ e^{j\alpha\Omega+} + r_{h,v} e_{h,v}^- e^{j\alpha\Omega-} \right] (1 + \lambda_{h,v}) \sqrt{\frac{\lambda_h}{\lambda_{h,v}}} e^{-j2\alpha} \quad (18)$$

where

$$\lambda_h = \epsilon_r \lambda_v = \Gamma / \cos \theta \quad (19a)$$

$$e_{h,v}^+ = -r_{h,v} e^{j4\alpha\Gamma} e_{h,v}^- \quad (19b)$$

The  $e_{h,v}^\pm$  are the amplitudes of the waves inside the slab propagating in the positive and negative  $\hat{n}$ -direction (i.e. positive or negative with respect to the normal to the disk). This expression for  $R_{h,v}$  is obtained in a straight forward manner by matching boundary conditions at the slab interfaces and solving for the amplitude of the reflected plane wave (e.g. Born and Wolf, 1959). The form given above assumes that the origin is at the center of the slab. After some simple rearrangements one obtains:

$$|R_h|^2 = \frac{1}{4} \left[ [1 - e^{j4\alpha\Gamma}] e^{-j\Omega-} e_h^- \frac{\Omega-}{\cos \theta} \right]^2 \quad (20a)$$

$$|R_v|^2 = \frac{1}{4} \left[ [1 - e^{j4\alpha\Gamma}] e^{-j\Omega-} \frac{e_v^-}{\sqrt{\epsilon_r}} \left[ \frac{\epsilon_r \cos \theta - \Gamma}{\cos \theta} \right] \right]^2 \quad (20b)$$

It is not difficult to show that Equations 6 can be written in the following form:

$$\sigma_{hh} = \sigma_\infty \left[ [\text{sinc}(\alpha\Omega+) - r_h e^{j2\alpha\Gamma} \text{sinc}(\alpha\Omega-)] e_h^- \frac{\alpha(\epsilon_r - 1)}{\cos \theta} \right]^2 \quad (21a)$$

$$\sigma_{vv} = \sigma_\infty \left[ [\gamma^+ \text{sinc}(\alpha\Omega+) - r_v e^{j2\alpha\Gamma} \gamma^- \text{sinc}(\alpha\Omega-)] \frac{e_v^-}{\sqrt{\epsilon_r}} \frac{\alpha(\epsilon_r - 1)}{\cos \theta} \right]^2 \quad (21b)$$



where  $\sigma_{\infty}$  is the physical optics solution for the backscatter cross section of a perfectly conducting disk:  $\sigma_{\infty} = \frac{(k_0 \cos \theta)^2}{\pi} \bar{S}^2(\nu_r)$ . Now, comparing Equations 20 and 21 and using the following relationships

$$\Omega_+ \Omega_- = 1 - \epsilon_r \quad (22a)$$

$$\Omega_+ \gamma_- = \epsilon_r \cos \theta + \Gamma \quad (22b)$$

$$\Omega_- \gamma_+ = \epsilon_r \cos \theta - \Gamma \quad (22c)$$

$$\text{sinc}(\alpha\Omega_{\pm}) = \frac{e^{j\alpha\Omega_{\pm}} - e^{-j\alpha\Omega_{\pm}}}{2j\alpha\Omega_{\pm}} \quad (22d)$$

$$r_h = \frac{\Omega_-}{\Omega_+} \quad (22e)$$

$$r_v = \frac{\epsilon_r \cos \theta - \Gamma}{\epsilon_r \cos \theta + \Gamma} = \frac{\Omega_- \gamma_+}{\Omega_+ \gamma_-} \quad (22f)$$

one obtains the following results:

$$\sigma_{hh} = \sigma_{\infty} |R_h|^2 \quad (23a)$$

$$\sigma_{vv} = \sigma_{\infty} |R_v|^2 \quad (23b)$$

That is, the radar cross section for backscatter from a dielectric disk is the radar cross section of a perfectly conducting disk of the same shape (physical optics form) times the reflection coefficient of an identically oriented (identical  $\theta$ ) slab of the same thickness and dielectric properties.

As a check on this result, notice that Equations 23 include all of the special cases discussed above. For example, for a perfectly conducting disk  $R_h = R_v = -1$  and so  $\sigma_{hh} = \sigma_{vv} = \sigma_{\infty}$ ; and in the case of a thin disk ( $T \rightarrow 0$ ) one has

$$R_h \rightarrow j4\alpha\Gamma \frac{(\cos \theta + \Gamma)(\cos \theta - \Gamma)}{(\cos \theta + \Gamma)^2 - (\cos \theta - \Gamma)^2} \quad (24a)$$

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$$R_v \rightarrow -j4\alpha\Gamma \frac{(\epsilon_r \cos \theta + \Gamma)(\epsilon_r \cos \theta - \Gamma)}{(\epsilon_r \cos \theta + \Gamma)^2 - (\epsilon_r \cos \theta - \Gamma)^2} \quad (24b)$$

which, when substituted into Equations 23, results in Equation 16 and agrees with the Schiffer and Thielheim result for  $\sigma_{hh}$  (Equation 17).

Recalling that there is no depolarization when  $\hat{i}$  and  $\hat{n}$  are in the plane of incidence one can now write the backscatter radar cross section in the general form:

$$\sigma_{pq} = \sigma_{\infty} |R_p|^2 \delta_{pq} \quad (25)$$

where

$$\delta_{pq} = \begin{cases} 0 & p \neq q \\ 1 & p = q \end{cases} \quad (26)$$

and p and q indicate either horizontal polarization (h) or vertical polarization (v); that is  $p \in \{h,v\}$  and  $q \in \{h,v\}$ .

### Conclusion

A solution for the radar cross section of a dielectric disk has been obtained by using a Kirchoff-type approximation for the fields inside the disk. The result takes the simple form

$$\sigma_{pq} = \sigma_{\infty} |R_p|^2 \delta_{pq}$$

where  $\sigma_{\infty}$  is the physical optics form for the backscatter cross section of a perfectly conducting disk of the same shape and  $R_p$  is the reflection coefficient for an incident wave of polarization  $p \in \{h, v\}$  of an identically oriented slab of the same thickness and dielectric properties as the disk. This special form applies when the normal to the disk is in the plane of incidence. The solution agrees with special cases previously obtained for scattering from dielectric disks: It reduces to the physical optics solution for scattering from perfectly conducting disks when the disk is very lossy; it reduces to the form obtained by Schiffer and Thielheim for very thin disks; and at normal incidence, it has the form obtained by Neugebauer and verified by experiment. This solution applies for arbitrary dielectric constant, arbitrary thickness and shape and arbitrary angle of incidence; however, the solution is inherently a high frequency approximation (because of the Kirchoff-type approximation) and therefore should be applied with caution near edge-on incidence or when the cross section of the disk is not large compared with wavelength.

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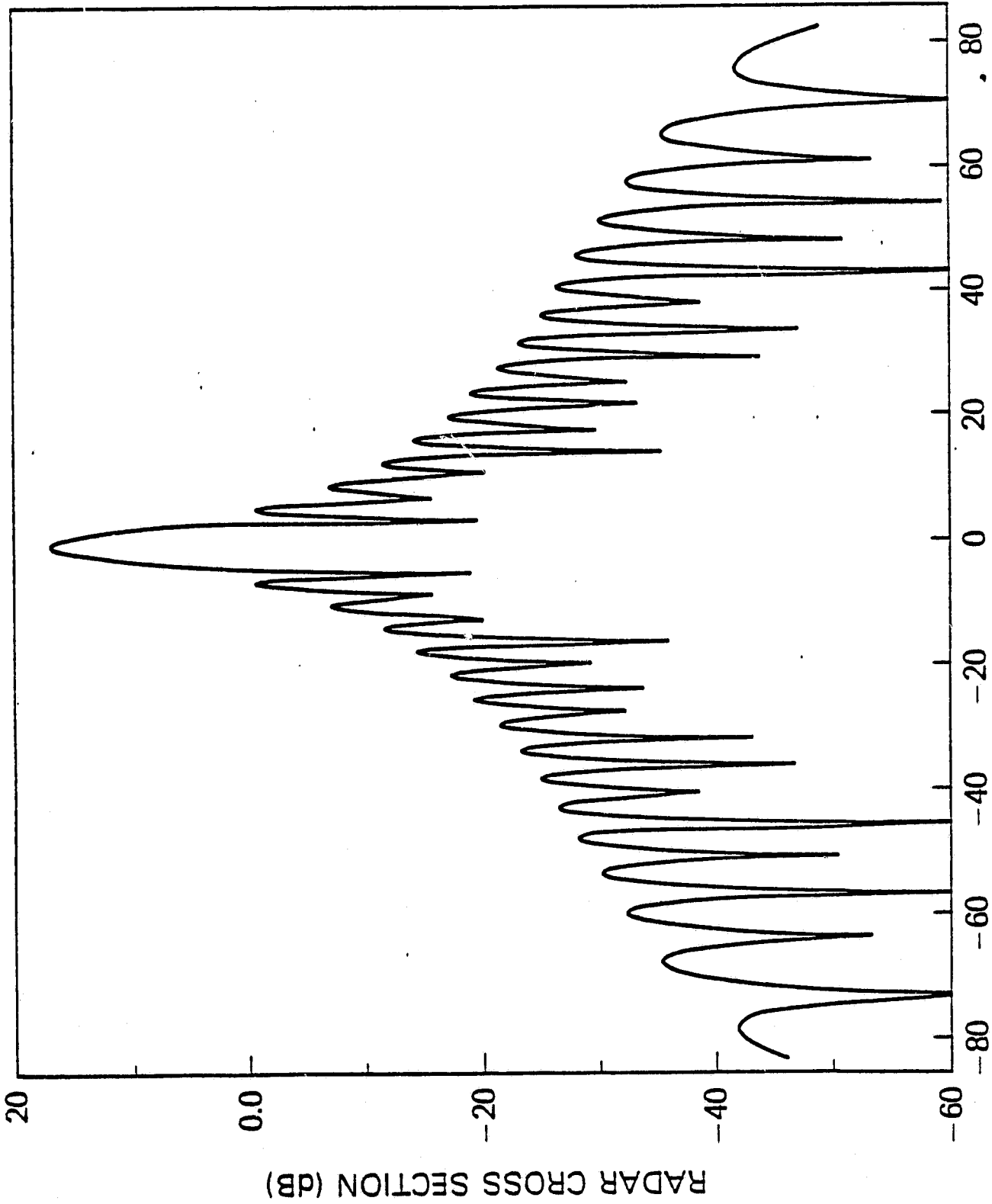
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Figure Captions

- Figure 1: Radar cross section  $\sigma_{hh}(\theta)$  for a circular disk with radius  $a = 20$  cm, thickness  $T = 5$  cm, relative dielectric constant  $\epsilon_r = 11$  and frequency  $f = 9$  GHz.
- Figure 2: Radar cross section  $\sigma_{vv}(\theta)$  for a circular disk with radius  $a = 20$  cm, thickness  $T = 5$  cm, relative dielectric constant  $\epsilon_r = 11$  and frequency  $f = 9$  GHz.
- Figure 3: Radar cross section  $\sigma_{hh}(\theta)$  for a circular disk with radius  $a = 10$  cm, thickness  $T = .5$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and frequency  $f = 9$  GHz.
- Figure 4: Radar cross section  $\sigma_{hh}(\theta)$  of a circular disk one half wave length thick: radius  $a = 10$  cm, thickness  $T = .5$  cm, relative dielectric constant  $\epsilon_r = 11$  and frequency  $f = 9$  GHz.
- Figure 5: Radar cross section  $\sigma_{hh}(\theta)$  of a thin circular disk. Disk radius  $a = 20$  cm, thickness  $T = .04$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and frequency  $f = 5$  GHz.
- Figure 6: Radar cross section  $\sigma_{vv}(\theta)$  of a thin circular disk. Disk radius  $a = 20$  cm, thickness  $T = .04$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and frequency  $f = 5$  GHz.

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Figure 1. Radar cross section  $\sigma_{rh}(\theta)$  for a circular disk with radius  $a = 20$  cm, thickness  $T = 5$  cm, relative dielectric constant  $\epsilon_r = 11$  and frequency  $f = 9$  GHz.

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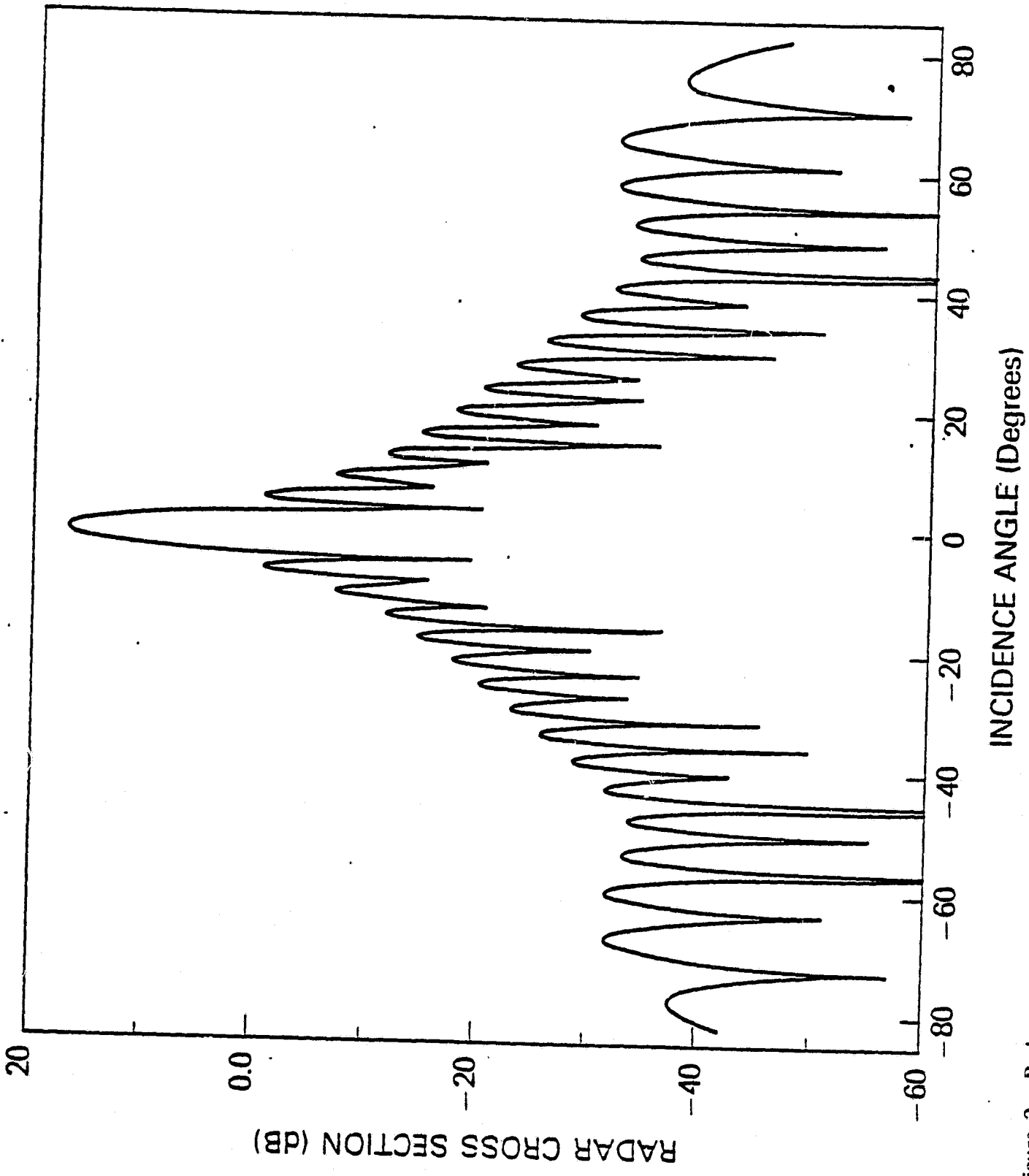
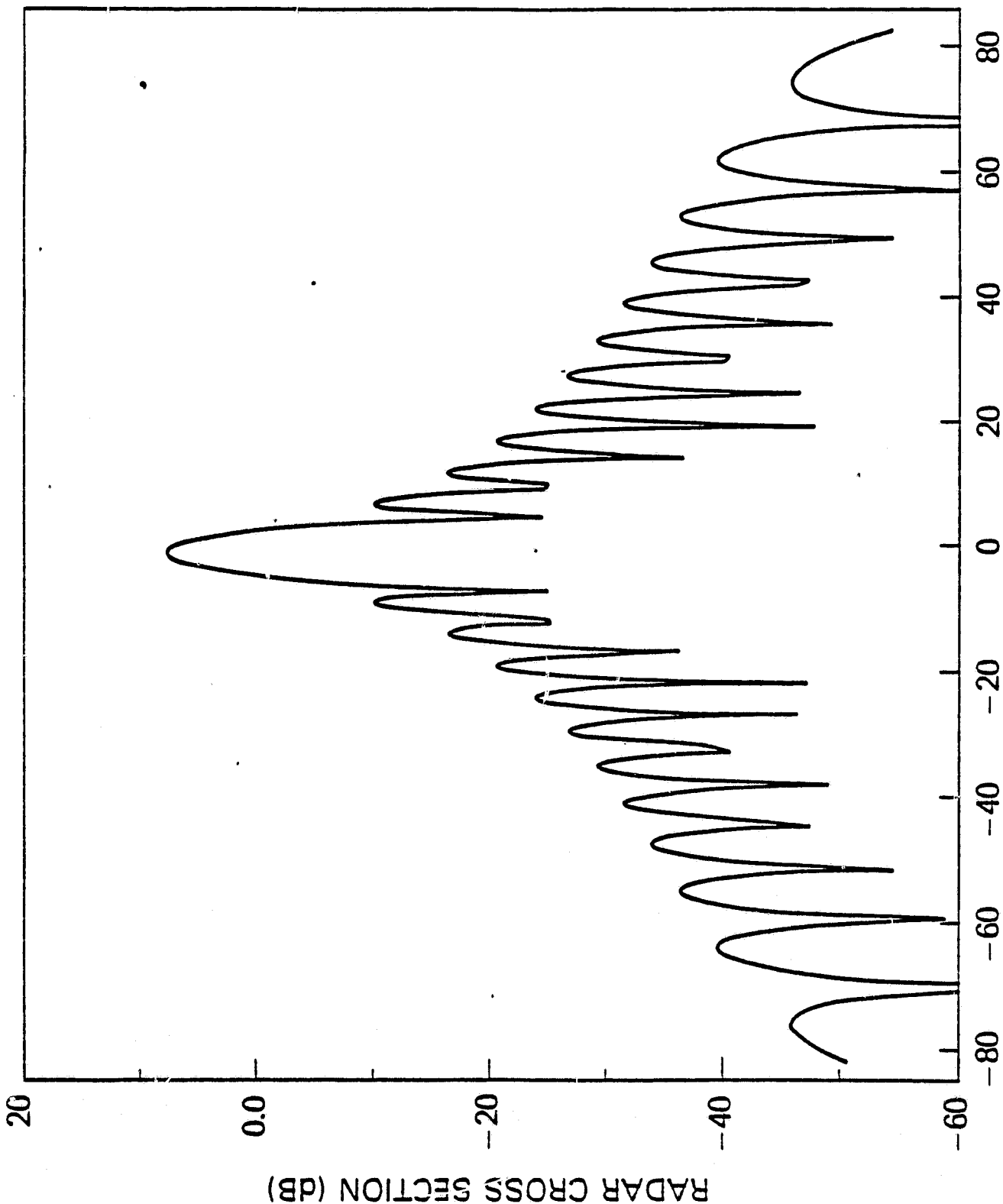


Figure 2. Radar cross section  $\sigma_{vv}(\theta)$  for a circular disk with radius  $a = 20$  cm, thickness  $T = 5$  cm, relative dielectric constant  $\epsilon_r = 11$  and frequency  $f = 9$  GHz.



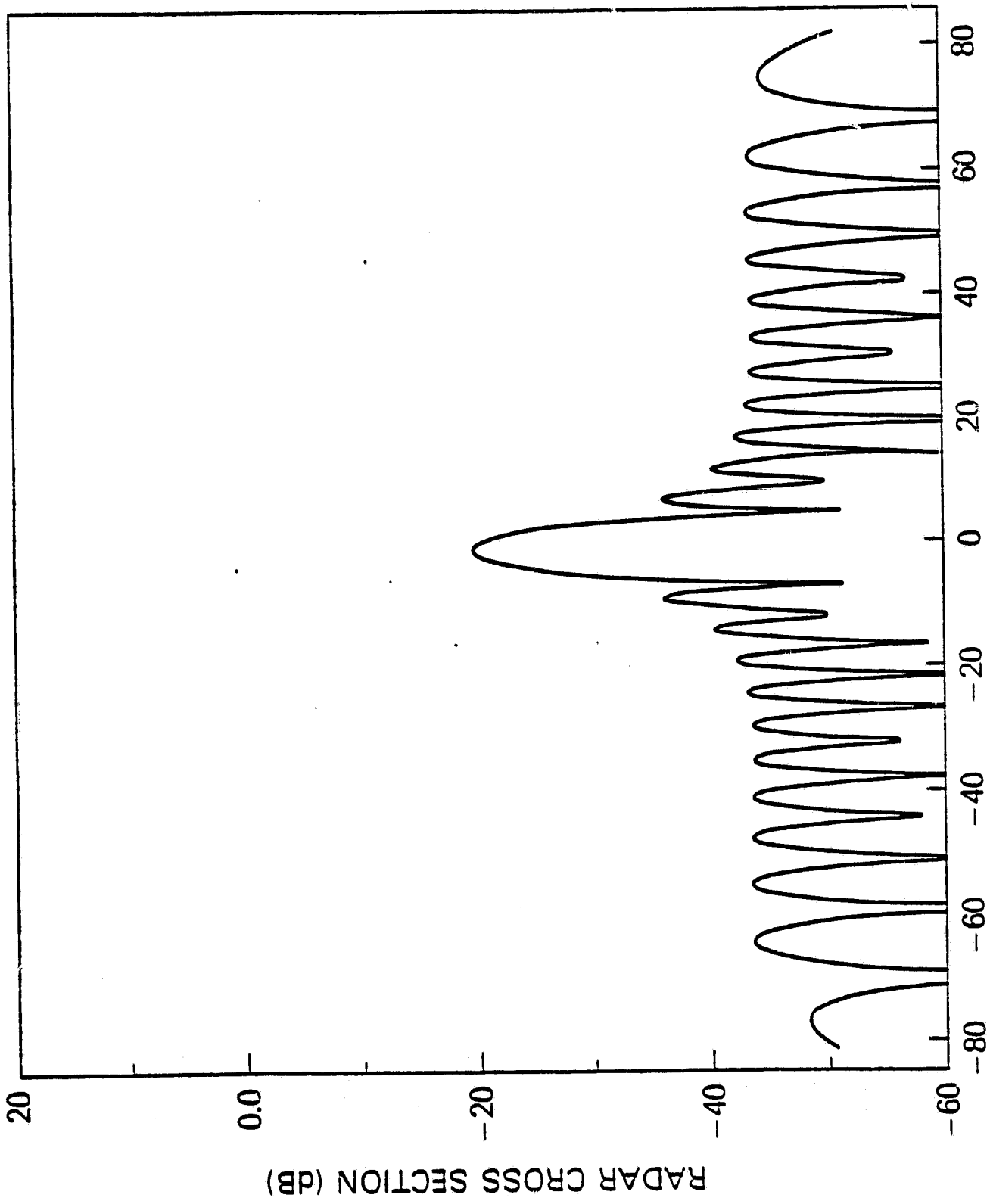
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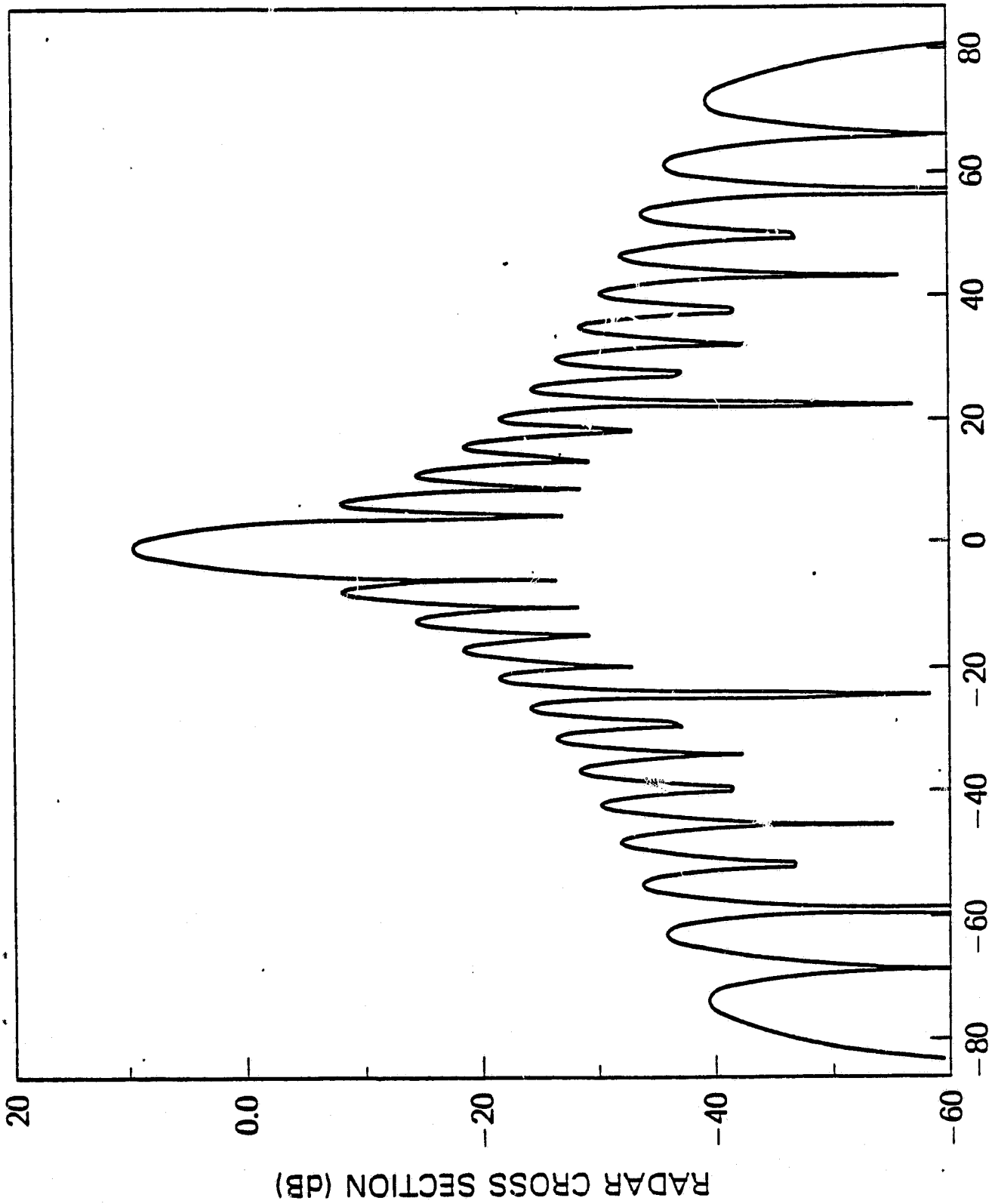
Figure 3. Radar cross section  $\sigma_{hh}(\theta)$  for a circular disk with radius  $a = 10$  cm, thickness  $T = .5$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and frequency  $f = 9$  GHz.

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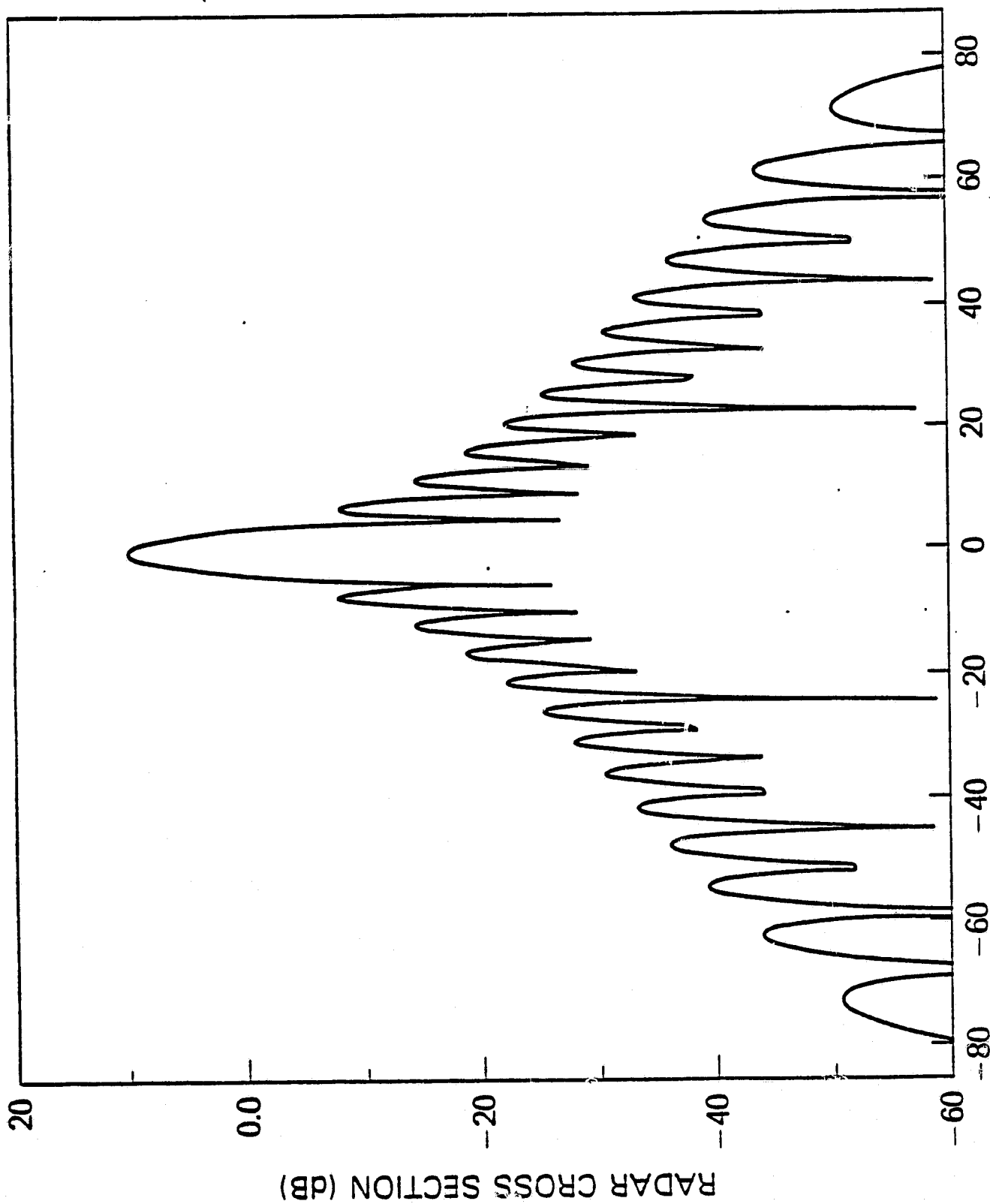
Figure 4. Radar cross section  $\sigma_{rh}(\theta)$  of a circular disk one half wavelength thick: radius  $a = 10$  cm, thickness  $T = .5$  cm, relative dielectric constant  $\epsilon_r = 11$  and frequency  $f = 9$  GHz.



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Figure 5. Radar cross section  $\sigma_{hh}(\theta)$  of a thin circular disk. Disk radius  $a = 20$  cm, thickness  $T = .04$  cm, relative dielectric constant  $\epsilon_r = 2.5 + j 11$  and frequency  $f = 5$  GHz.

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Figure 6. Radar cross section  $\sigma_w(\theta)$  of a thin circular disk. Disk radius  $a = 20$  cm, thickness  $T = .04$  cm, relative dielectric constant  $\epsilon_r = 25 + j 11$  and frequency  $f = 5$  GHz.