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Flow in a Torsionally Oscillating Filled Cylinder

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INTRODUCTION

Systems are being designed which will accommodate a large group of fluid science experiments. Experiments will require the capabilities for both isothermal and gradient freeze processing, interchangeability of samples for processing in the furnace, and high cooling rates (quench). Also thermal stability, heating rate, temperature range, temperature measurement accuracy, and sample temperature uniformity requirements must be specified. These requirements, and those for sample size, serve to place severe restrictions on the ability and means to accomplish another requirement – that of mixing multi-component samples in the low-gravity environment. A class of experiments will deal with solidification of mixtures of liquids which are mutually soluble only above certain temperatures and are immiscible liquid systems over certain temperature ranges (Fig. 1). These systems, in order to be in a well-homogenized state before solidification is initiated, must be heated to temperatures above this immiscible range and then subjected to some mixing process. This homogenization is not possible in an Earth-based laboratory due to rapid density segregation when the liquid system is cooled below the miscibility limit.

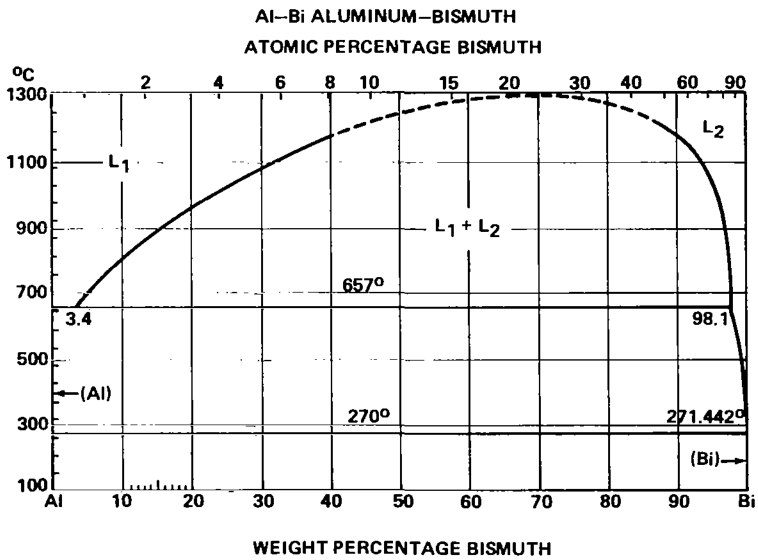


Figure 1. Al-Bi phase diagram.

SAMPLE CONFIGURATION AND PARAMETERS

Samples to be processed in typical low-gravity systems are expected to be cylindrical and to have diameters from 1.27 to 3.175 cm with lengths ranging from 2.54 to 25.4 cm. These samples will typically be a two-component metal system contained in a cylindrical can. The can will be fabricated from or lined with a material which is inert to all components of the sample. Figure 2 shows a sample configuration.

It has been proposed that torsional oscillations (typically 1 Hz frequency and ±45 deg amplitude) of the samples about their longitudinal axes (Fig. 3) be used to provide the required mixing. This technique has been suggested as one for which engineering obstacles could be overcome (design problems preclude most of the other possible techniques such as shaking, stirring, etc.). The problem is then to investigate this proposed technique and determine its efficacy in mixing liquid samples in a reasonable amount of time.

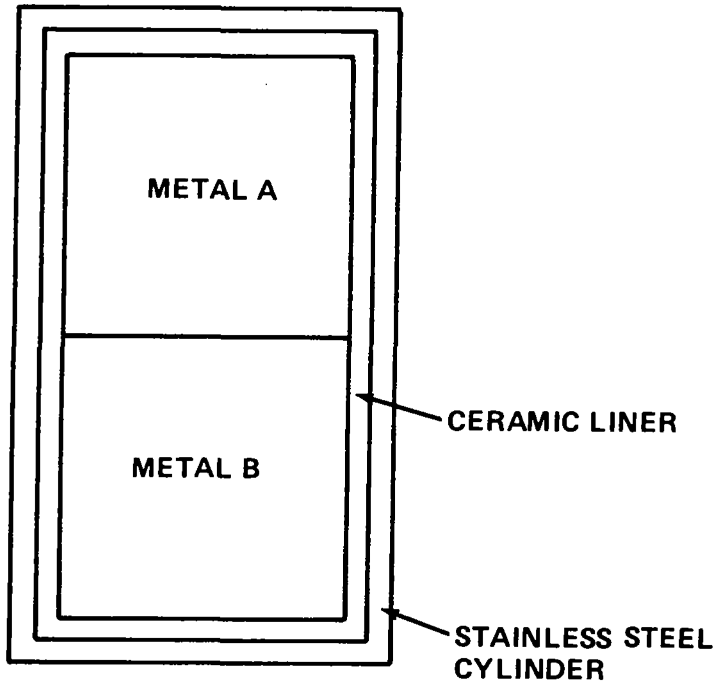


Figure 2. Typical sample configuration.

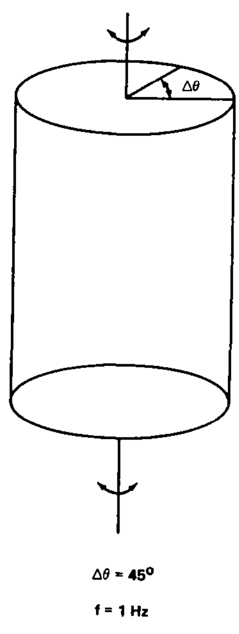
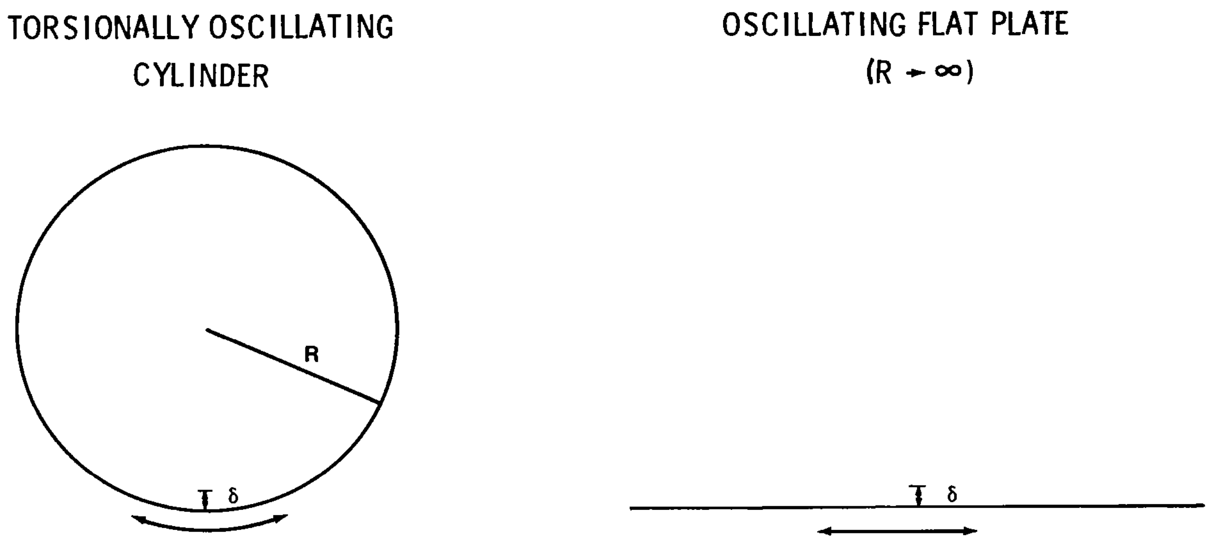


Figure 3. Torsional oscillations of cylinder.



δ = THICKNESS OF VISCOUS BOUNDARY LAYER

FOR δ<<R, THE R → ∞ MODEL APPLIES.

Figure 4. Plate-cylinder comparison.

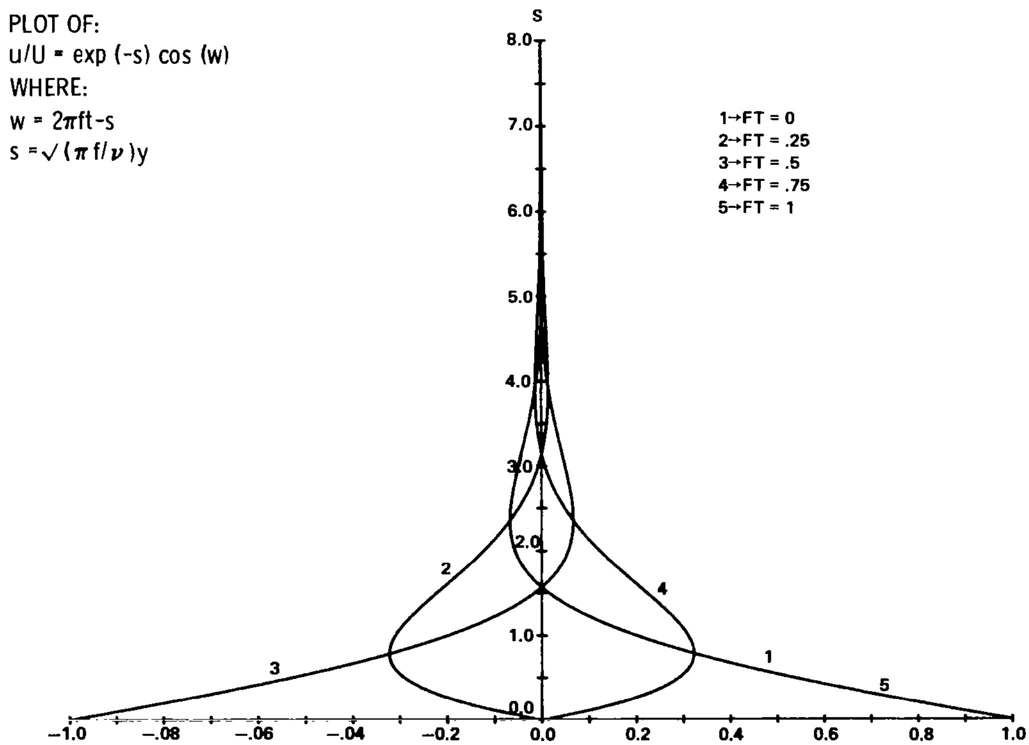


Figure 5. Velocity above oscillating plate.

Physics of the Proposed Mixing Process

When a closed cylinder, completely filled with fluid, is subjected to torsional oscillations of the type shown in Figure 3, the motion is communicated from the cylinder to the fluid through shear. As an aid to understanding the physical processes at work, the model may be broken into two parts. First, the side walls impart a motion to the fluid. If a very large cylinder is considered, then $R \rightarrow \infty$ (where R is the radius of curvature of the cylinder). That is, the oscillating cylinder is replaced with an oscillating plane (Fig. 4). If it is assumed that the model is described by the simplified Navier-Stokes equation

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

then according to Schlichting [1] the solution for the induced fluid velocity u(y,t) is

$$u(y,t) = u_o \, e^{-\sqrt{\frac{\omega}{2\nu}}\, y} \cos\left(\omega t - \sqrt{\frac{\omega}{2\nu}}\, y\right)$$

where:

u_o = the maximum velocity of the plate

ω = frequency of oscillation times 2π

y = vertical displacement above plate

ν = kinematic viscosity.

This solution is plotted (as nondimensional distance away from the plate against nondimensional velocity) in Figure 5 for various values of time. Figure 6 shows the same solution for a typical liquid metal (ν = 0.001 in CGS units) with velocity in centimeters. It is found that the motion of the fluid is essentially restricted to a layer adjacent to the plate with thickness $\delta \sim \sqrt{\nu/\omega}$. It is readily seen that δ is small compared to expected experimental cylinder radii. Therefore, there is reason to assume that the essential physics of side wall-fluid interaction has been captured by looking at the limiting case where $R \rightarrow \infty$ (flat plate). Further, it is noted that the only motion imposed on the fluid is parallel to the oscillating motion of the plate so that this will not contribute to mixing the fluid in the longitudinal direction. Also, the smallness of δ implies that the sidewall motion will not appreciably mix the fluid in the radial direction. It is concluded, then, that although shearing is present at the side wall, the motion induced in the fluid does not penetrate deeply into the fluid and does not contribute significantly to the mixing process.

The other mechanism for transmitting shearing forces into the fluid is due to the torsionally oscillating end walls of the cylindrical container. The flow generated by this machine is three-dimensional. Although this is a nonlinear process, it is helpful to examine separately the effects which determine the flow in the system in order to obtain understanding of the physics involved. First, the end walls of the cylinder carry along a thin layer of the fluid as they oscillate (as do the side walls discussed above). This is a circular motion which is time-dependent according to sin ωt. The tangential component of this

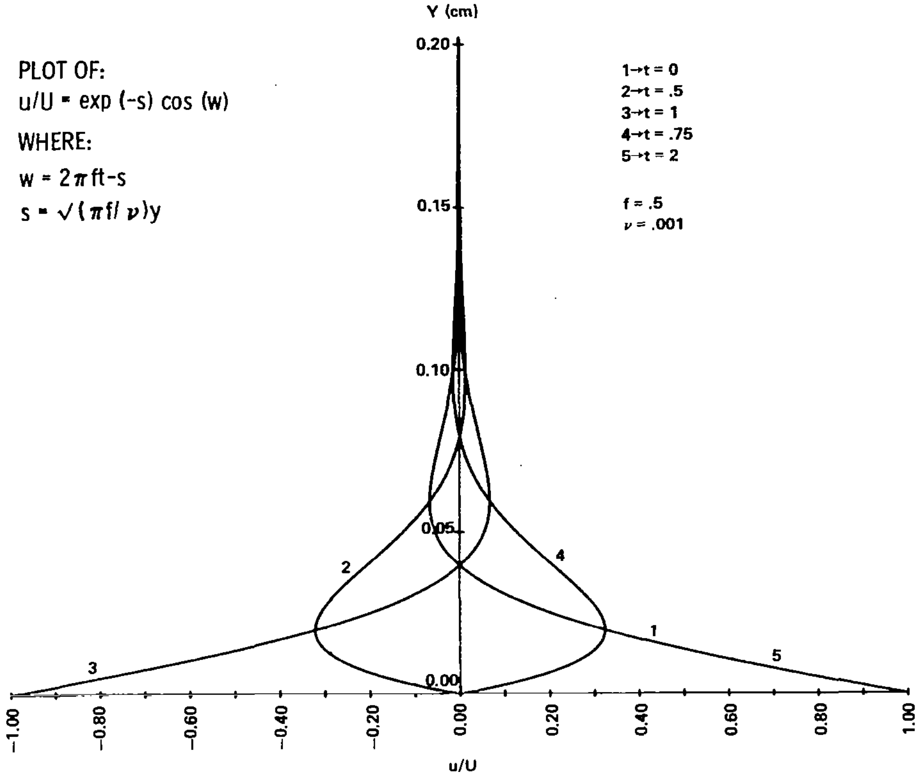


Figure 6. Velocity above oscillating plate.

motion varies directly as the radius from the axis of rotation and inversely as the distance away from the end wall. Since this motion due to shear at end walls is circular, there is a centrifugal force on any fluid element at the end wall which will be related to $\vec{\omega} \times (\bar{\omega} \times \bar{r})$ or $\omega^2 r$ which is always directed outward along a radius. That is, for all parts of the oscillatory cycle, a fluid parcel experiences an outward force. This leads to flow which is directed radially outward in a thin layer. Rosenblat [2] discusses flow due to a torsionally oscillating flat plate and shows that this radial flow is confined to a layer which has a thickness on the order of $1/\epsilon \sqrt{\nu/\lambda}$ where ϵ is the amplitude of oscillation, λ is the frequency of the oscillation, and ν is kinematic viscosity. For a typical liquid metal with $\nu = 0.001$ cm^2/sec and for $\lambda = 1$ Hz and $\epsilon = \pi/4$, it is found that the layer in which fluid is being transported radially outward has a characteristic thickness of about 0.04 cm. (For water, with λ and ϵ the same as above, $\delta \sim 0.13$ cm.) A plot of streamlines of flow about the axis of rotation of a torsionally oscillating plate is shown in Figure 8. (These results are from the analysis of Rosenblat and show the steady component of the flow.) In summary then, the fluid motion due to the motion of the end walls is complicated, but is due to three contributions. These are the radial motion of the fluid carried along by the end wall, the centrifugal effect which causes these rotating fluid elements to move radially outward, and an axial flow which is necessary for continuity [that is, the outward radial flow must be balanced by an axial inflow (Fig. 7)]. While the nonlinearity of the governing equations prohibits constructing a solution which is a superposition of these three effects, the picture that was sought could be established.

It remains, then, to bring together the side wall picture with the end wall picture to obtain a qualitative view of the fluid motion in the torsionally oscillating closed, completely filled cylinder. Figure 8 summarizes the various effects. As noted, the side walls contribute little to mixing. The end

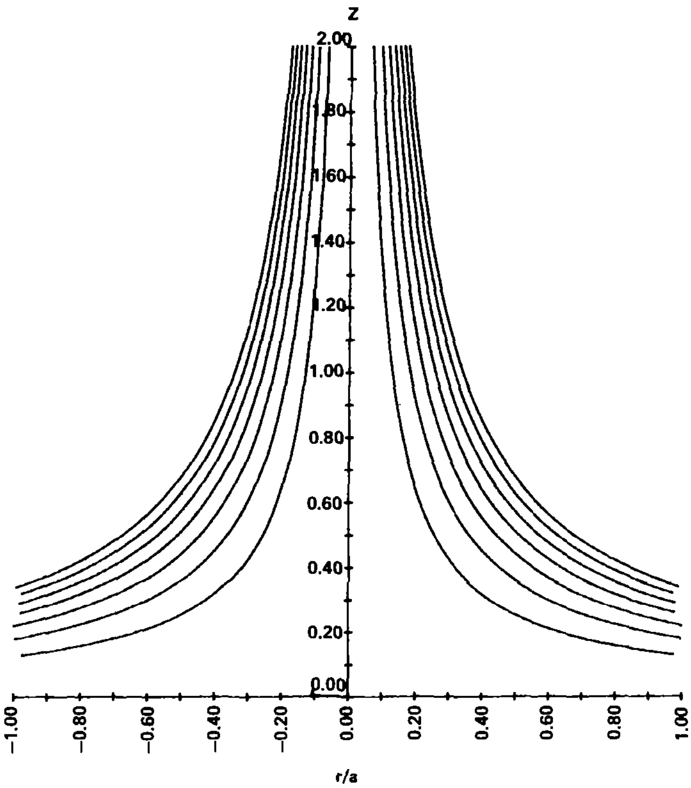


Figure 7. Flow streamlines above torsionally oscillating plate.

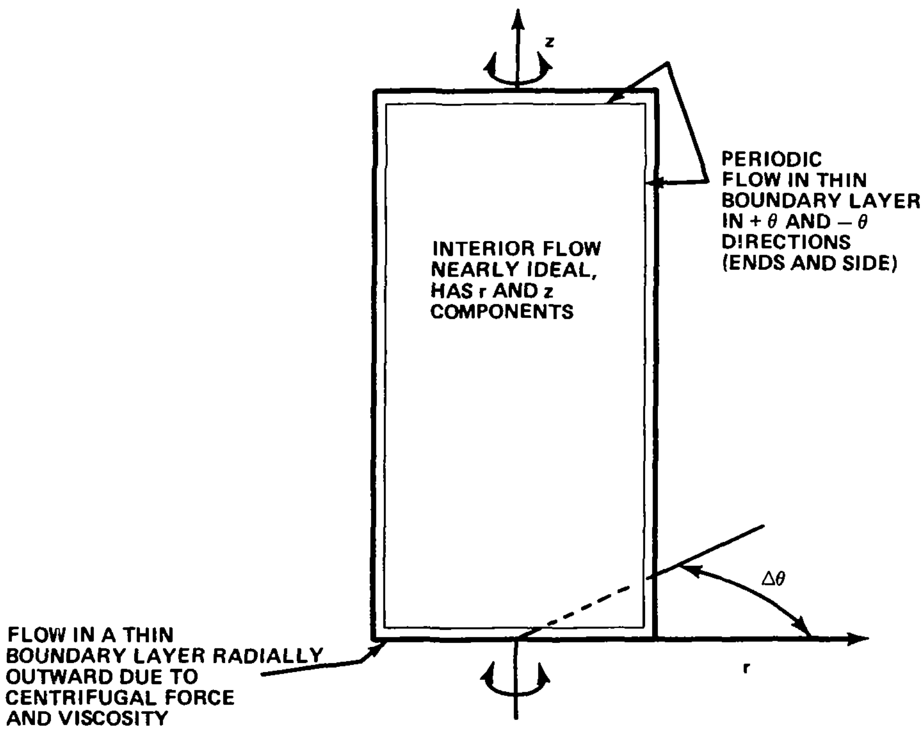


Figure 8. Summary of flow mechanisms in torsionally oscillating cylinder.

walls provide a flow which is always directed radially outward in addition to the oscillatory flow near the end walls. Continuity considerations, as noted above, require that fluid move toward the end walls (in an axial direction) to replace fluid being centrifugally pumped outward. The side walls serve to limit the outflow due to the centrifugally pumped fluid, i.e., they turn the flow so that it moves in a direction parallel to the side walls. This flow moves toward the center (axially) of the cylinder. Since these effects are occurring at both ends of the cylinder, the flow must turn at some point and come back to the end walls forming closed cells of flow of an undetermined size.

It has been shown that the viscous boundary layers in these systems are very thin both at the side walls and at the end walls. For the side walls, $\delta \approx 0.02$ cm and for the end walls, $\delta \approx 0.04$ cm for a typical liquid metal kinematic viscosity of 0.001 cm^2/sec. That is, viscous effects are important only in a very small volume of the liquid. This implies that we could obtain a description of the liquid in the interior region (most of the volume) in terms of an ideal fluid. That is, we could consider the flow as being due to a source-sink system at each end of the cylinder.

Here fluid moves in axially at the central section of the end wall so this region serves as a fluid sink. The fluid then moves out in a thin layer along the end wall and is expelled in an axial direction near the side wall, so the region where the side wall and end walls come together serves as a source of fluid. The flow due to a source-sink pair in two dimensions should give a qualitative picture of the flow in a section from the center out to the side wall. It must be noted that the source-sink model is highly idealized in that it uses assumptions of inviscid, irrotational flow. These assumptions lead to potential theory and yield solutions well-known in hydrodynamics and electrostatics, for example. This idealization yields information about the expected cell size. A source-sink pair has no net monopole term in the potential for the far field solution (i.e., for r greater than the characteristic source-sink dimension). The lowest surviving term will be a dipole term [3]. Therefore, we would expect that the flow velocity dies off rapidly (as $1/r^3$) as we move away from the source-sink region. That is, the flow should be concentrated primarily near the ends of the cylinder, in a region characterized by the length scale of the source-sink combination. This leads us to an estimation of the thickness of the convection cells which are generated by the end wall torsional oscillations and completes the qualitative description of flow in the cylinder (Fig. 9). A typical cell thickness then would be on the order of the radius of the cylinder. This cell thickness will be in reality a function of the fluid viscosity and density (i.e., the momentum transport) of the contained fluids. Higher viscosity could tend to expand the cell size while increasing density could tend to contract it. That is, the cell size would be expected to be a function of $\mu/\rho = \nu$, the kinematic viscosity.

The boundary layers discussed here are in the laminar flow regime. Levich [4] notes that the laminar to turbulent transition occurs for the boundary layer of a rotating disk for Reynolds numbers (defined by $Re = a^2\omega/\nu$, where a is disk radius) greater than 10^4 (theoretically 5×10^4). For a typical liquid metal system of the kind described here, $Re \approx 3 \times 10^3$.

Summary of Analysis

The flow in torsionally oscillating cylinders filled with a liquid metal system is expected to be laminar for cylinder diameters in the 1.27- to 3.18-cm range and for angular rates of π rad/sec. Further, viscous effects are expected to be strong only in boundary layers at each end and at the side walls of the cylinders. These layers are expected to be very thin compared to cylinder dimensions. Motion in the interior of the fluid can be treated as nonviscous. In this case, the centrifugal pumping effects (viscous effects, at the end walls would be replaced by a source-sink pair. This model shows that strongest flows will exist in toroidal cells at each cylinder end and of the order of the cylinder radius in thickness.

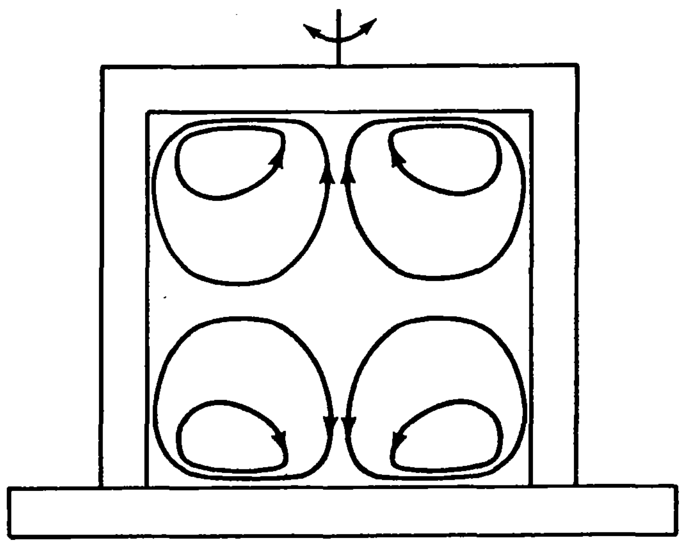


Figure 9. Flows in torsionally oscillating cylinder.

These analytical observations show that this process can be effective in mixing the liquids in the cylinder only if the length-to-diameter ratio of the cylinder is about one or less. For higher aspect ratios, the cells will not extend far enough into the cylinder to mix the fluid efficiently. Mixing could be improved by increasing the oscillation rate so that the laminar to turbulent transition occurs. This could be achieved by increasing the oscillation frequency by about a factor of 10.

Experimental Work

Although many insights into the physical processes at work in the torsionally oscillating cylinder of fluid may be obtained analytically by examining them separately, these cannot be completely quantitative (due to the nonlinearity of the process). It is necessary then, to examine this mixing process experimentally to confirm our analytical insights. It was noted earlier that the flight experiments of interest here will have diameters in the range of 1.27 to 3.18 cm and lengths ranging from 2.54 to 25.4 cm. Typical experimental fluids will be liquid metals which will have kinematic viscosities in the 10^{-3} cm^2/sec range.

The experimental approach to understanding the flow in these fluid systems requires that the following be accomplished:

1) Develop relevant model systems to replace the metal experimental fluids.

2) Develop means to trace motions of the fluid under test conditions so that details of the flow processes can be understood and an estimate of fluid velocities can be obtained.

3) Develop an optical system for illuminating and recording motions of tracer particles.

4) Develop means to observe the integrated effects of convection and diffusion on mixing in the fluid over longer periods of time so that total mixing times can be estimated.

5) Develop means for producing the required torsional oscillation which drives the mixing process.

Since we need to experimentally observe the details of fluid motions in the torsionally oscillating cylinders, we must use some model material rather than the actual experimental material due to the opacity of metals. We need some transparent material which can be marked in order to trace the fluid motions and to observe the mixing process. Water was the obvious choice of model material. Having chosen a material, the question of appropriate scaling laws arises. We need to use some scaling rule to preserve the essential physics of the mixing process in the model system. Since the primary driver of fluid motion leading to mixing was determined to be centrifugal pumping, or Eckman suction [5], at the endwalls, Eckman number scaling was used. Here, $E = \nu/\omega L^2$ is used as the parameter to be preserved in passing from experimental (metal) system to laboratory model (water) system. If we choose to keep the oscillation rate the same for the model system, then assuming a viscosity of 10^{-3} cm^2/sec for the metal system and 10^{-2} cm^2/sec for the model system, we obtain:

$$L_{model} = \sqrt{10}\, L_{experimental} \cdot$$

In order to preserve aspect ratio, L/D, which is also expected to be important in the mixing process, we must scale the diameter of the model system up by a factor of $\sqrt{10}$ also (Fig. 10).

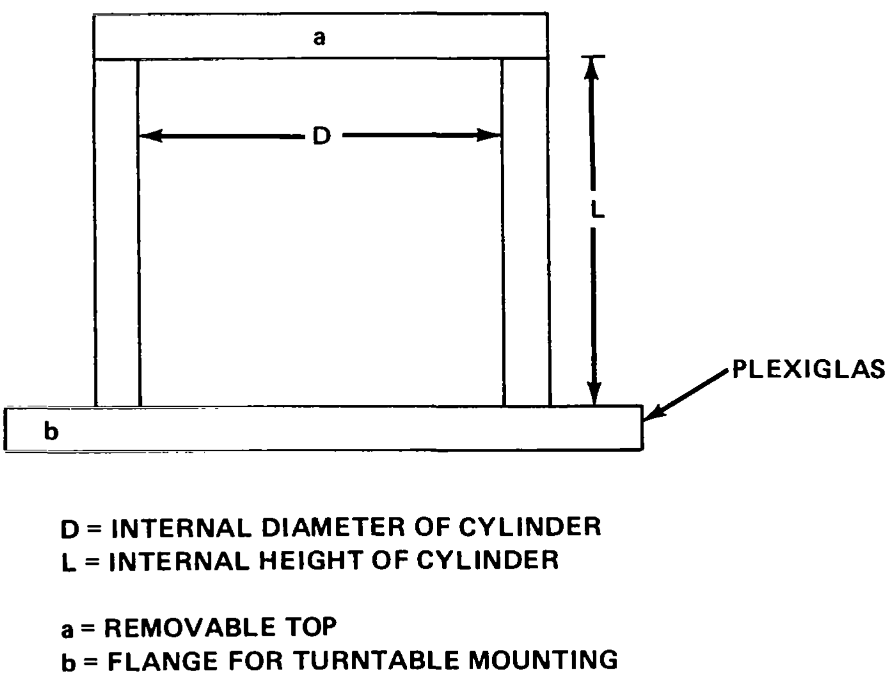


Figure 10. Sample cylinder – experimental model.

The Reynolds number ($Re = a^2 \omega/\nu$) for a typical liquid metal system is about 3×10^3. For the model system $Re \approx 8 \times 10^3$. This is still subcritical, so the model preserves the laminar flow characteristics expected in the liquid metal.

The second requirement can best be met by observing directly the motion of fluid parcels. This motion can be closely approximated by following the paths of near neutrally-buoyant tracer particles. We chose 20 μm latex spheres.

Applying Stokes Law for the sedimentation rate, V_s, of these spheres:

$$V_s = \frac{2a^2}{9\mu} g (\rho' - \rho)$$

where

a = sphere diameter = 10 μm

μ = viscosity of water = 0.01 stokes

ρ' = sphere density = 1.06 gm/cm^3

ρ = density of water = 1.00 gm/cm^3

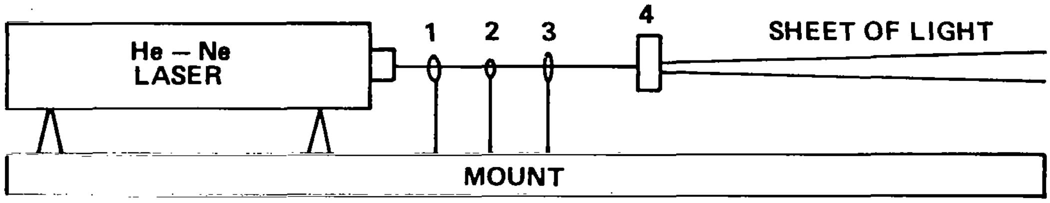
g = acceleration of gravity = 9.8×10^2 cm/sec^2 .

Then $V_s = 1.3 \times 10^{-3}$ cm/sec. This implies that these spheres will faithfully trace out the motion of the water as long as the observed velocities are large compared to 10^{-3} cm/sec. Since the observed velocities of the tracer particles in flow produced near the ends of the cylinders is on the order of 10^{-1} cm/sec, the latex spheres serve as useful flow markers in this experiment.

The illumination system developed for observation of motion of the marker particles is shown schematically in Figure 11. A pinhole is used to select the most intense central portion of a laser beam. The beam is then passed through a cylindrical lens so that it is focused in one-dimension. Past the focal point, the beam emerges as an expanding sheet of light. This sheet is passed through the experimental cylinder so that a thin section (less than 1-mm thick) containing the rotational axis is illuminated. Tracer particle motion in a plane can then be observed and photographed.

The integrated effects of convective and diffusive mixing are observed by injecting a red dye very carefully into the bottom of the sample cylinders. Visual observations of the mixing were recorded.

Torsional oscillations were produced by the system shown schematically in Figure 12. A turntable was used to drive a second turntable via a connecting rod with pivot points on each turntable. With this arrangement, the rotational motion of the first turntable was transformed to reciprocating motion of the second. The amplitude of oscillation of the second turntable was selectable through placement of the pivot points on each turntable.



1 – FOCUSING LENS
2 – PINHOLE
3 – COLLIMATING LENS
4 – CYLINDRICAL LENS

Figure 11. Illumination system.

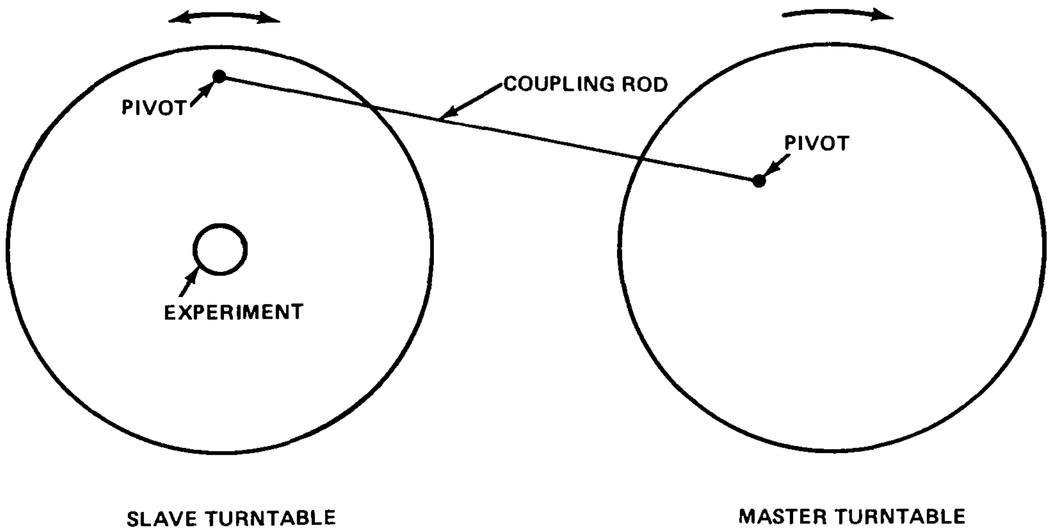


Figure 12. Turntable drive system.

The complete system for generating and observing flows is shown schematically in Figure 13.

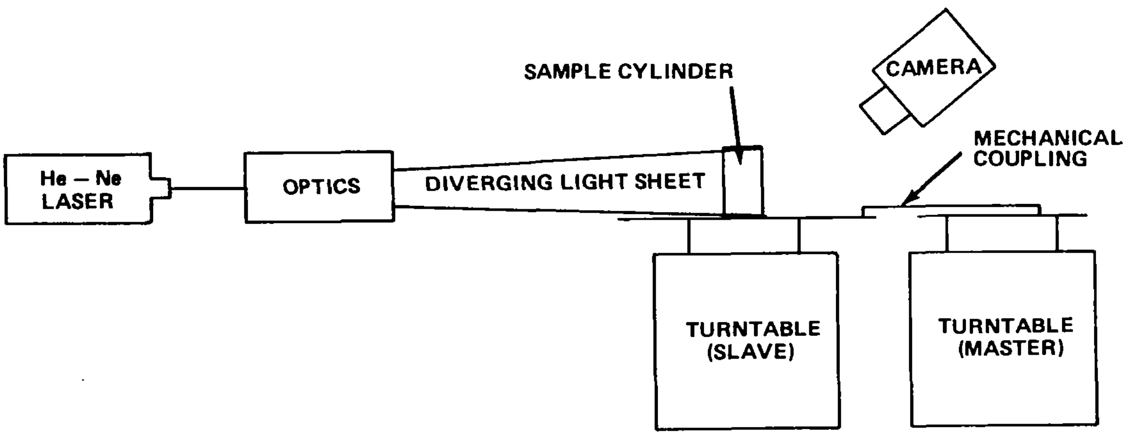


Figure 13. Experimental system.

Summary of Expected Fluid Behavior

For the experiment geometries, rotation rates, and material properties discussed, flows are expected to be laminar. The viscous effects are expected to be confined primarily to thin boundary layers along the cylinder sides and ends. Eckman pumping at the end walls will drive the flows. Convection cells of toroidal shape should develop at the cylinder ends due to the Eckman pumping and central return flow. The velocities due to end wall pumping should die off rapidly away from the ends (this should go as an inverse distance to the third power, for large r, since a distributed source and sink pair can be used to describe flow in the interior).

Experimental Observations

The laser illumination system was used in conjunction with latex sphere flow tracers for observing and recording flows in each of five torsionally oscillating cylinders having the geometries shown in the table below.

Cylinder Number	h (inches)	D (inches)
1	2	2
2	4	2
3	6	2
4	12	2
5	12	1

Recording was accomplished both by 35-mm still photographs and 16-mm movies. All cases observed showed very similar results. Flow cells quickly developed at the cylinder ends. These cells were toroidal with strong flow being observed only within a distance of less than a cylinder radius from the last end walls (Fig. 14). Varying the oscillation frequency up to the limit achievable by the coupled turntable system (about 1 Hz) did not appreciably change the structure of the flow cells. The velocities of the fluid in the cells did change, but the extent of penetration in the z direction change negligibly.

Integrated mixing effects were determined by observing the mixing of a red dye carefully placed at the bottom of the cylinders with a long needle attached to a syringe. These effects were as expected from the tracer particle studies. That is, strong mixing occurred in layers at the cylinder end wall, but the thicknesses of the well-mixed layers were less than a cylinder radius. Results were similar for all five cylinders. Duration of tests ranged from approximately 30 min to 3 hr.

Conclusions

Mixing of fluids in a closed, filled cylinder by torsional oscillations is a very inefficient process, except in a region lying within less than a cylinder radius from the end walls, when flow is in a laminar regime. This process can be recommended as a method of sample homogenization only if the cylinder length is no greater than the cylinder diameter.

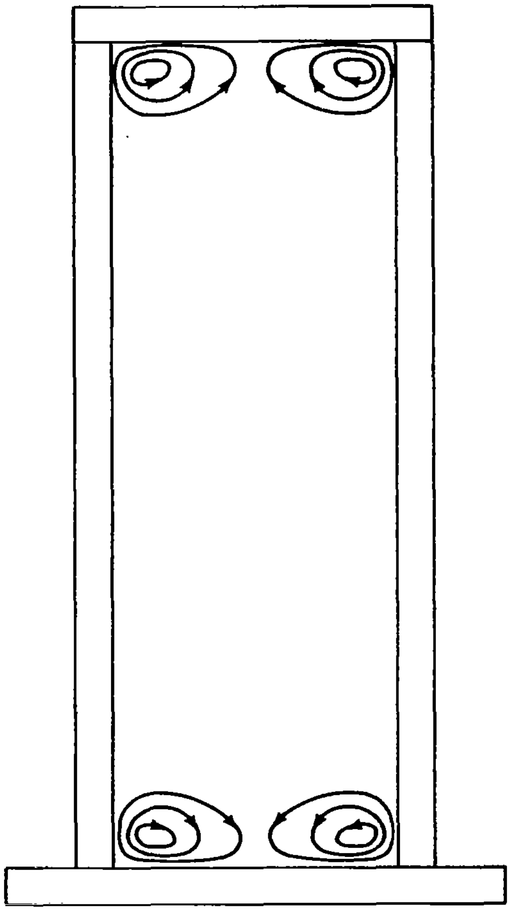


Figure 14. Flows in torsionally oscillating cylinder.

Methods for Enhancing the Mixing Process

Two techniques for enhancing the mixing process discussed here come to mind. First, increasing the oscillation frequency by a factor of 10 or more should cause the flow in the cylinder to become turbulent, thereby producing better mixing. This technique was not investigated because the higher oscillation frequencies were not accessible on the apparatus used. Second, baffles can be included at each end of the cylinder (Fig. 15). A series of tests were conducted using different baffle heights. It was observed that rapid mixing occurred in layers about 10 times the baffle height, extending out from the cylinder end walls in the z-direction. Mixing past this well-mixed layer was very slow. Use of baffles should allowing increasing the L/D ratio of the cylinders from about 1 to about 4 or 5 while still achieving rapid homogenization.

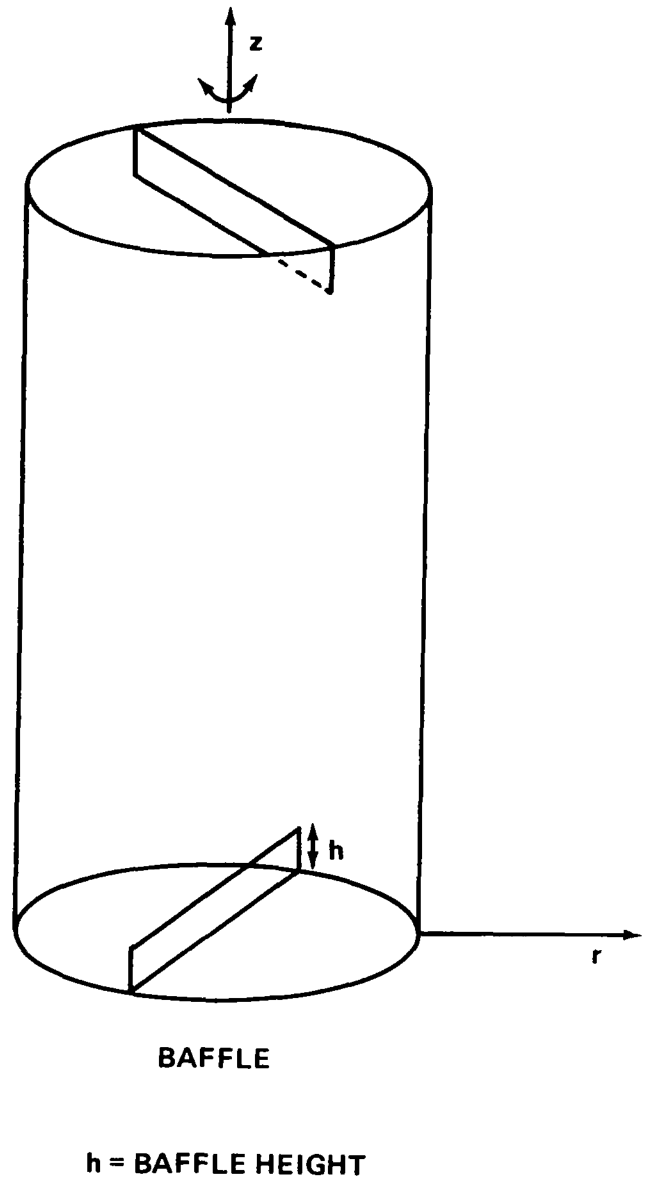


Figure 15. Baffle configuration.

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16. ABSTRACT

The flow of a liquid in a completely filled cylinder undergoing torsional oscillations about its longitudinal symmetry axis was studied analytically and experimentally. The objective of the studies was to determine the efficacy of the torsional oscillations in mixing the confined liquid. Flow was found to be confined primarily to toroidal cells at the ends of the cylinder. Cell thickness was about equal to the cylinder radius. The use of baffles at the end walls was shown to enhance the mixing process.

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