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COMPUTATION OF THE INTERVALS OF UNCERTAINTIES ABOUT THE PARAMETERS FOUND FOR IDENTIFICATION

P. Mereau and J. Raymond

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August 1982

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Translation

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The authors described a modeling method to calculate the intervals of uncertainty for parameters found by identification, they discussed the region of confidence, the general approach to the calculation of these intervals and described the general subprograms for determination of dimensions. They provide the organizational charts for the subprograms, the tests carried out and the listings of the different subprograms.

Unlimited-Unclassified

Unclassified	Unclassified	42

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COMPUTATION OF THE INTERVALS OF UNCERTAINTIES ABOUT THE PARAMETERS FOUND FOR IDENTIFICATION

P. Mereau and J. Raymond [1]

1. Introduction /1*

Practically the procedure of identification leads to a slanted estimate of the parameters of the model, this slant is due to various causes:

-- uncertainty about measurements: unmeasurable secondary inputs, measurement noises, errors because of quantization;

-- error of characterization of the model: the model is generally characterized with a certain number of simplifying hypotheses in which certain behaviors are disregarded;

-- numerical errors: the numerical methods used lead to a necessarily limited precision.

Thus, in these conditions, it is illusory to seek to determine the "true" values of the parameters. On the other hand modeling assumes its total reality as an experimental method if its final goal is to achieve values of parameters completed with an estimate of an uncertainty interval around these values, taking into account the effect of the various causes of error.

2. Region of Confidence /2

The modeling method leads to the minimizing of a criterion of quadratic spread which is written as follows:

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[1]Association for the Development of Teaching and Research in Applied Systematics; Group for Study and Research in Bio-Systems.

*Numbers in the margin indicate pagination in the foreign text.

$$D(p) = \sum_{k=1}^{N} \left[s_m(t_k,p) - s_0(t_k)\right]^T \left[s_m(t_k,p) - s_0(t_k)\right] \qquad (1)$$

N = identification level

$s_m(t_k,p)$ = value of the model outlets

$s_o(t_k,p)$ = value of the object outlets

p = parameter vector

t_k = sampling time.

The effect of the different sources of errors indicated above is to lead to a result p_{min}, at the time of minimizing of the criterion (1), differing from thenominal point p_0 (point minimizing and "ideal" distance of the object to the model in the absence of noise) (compare Figure 1).

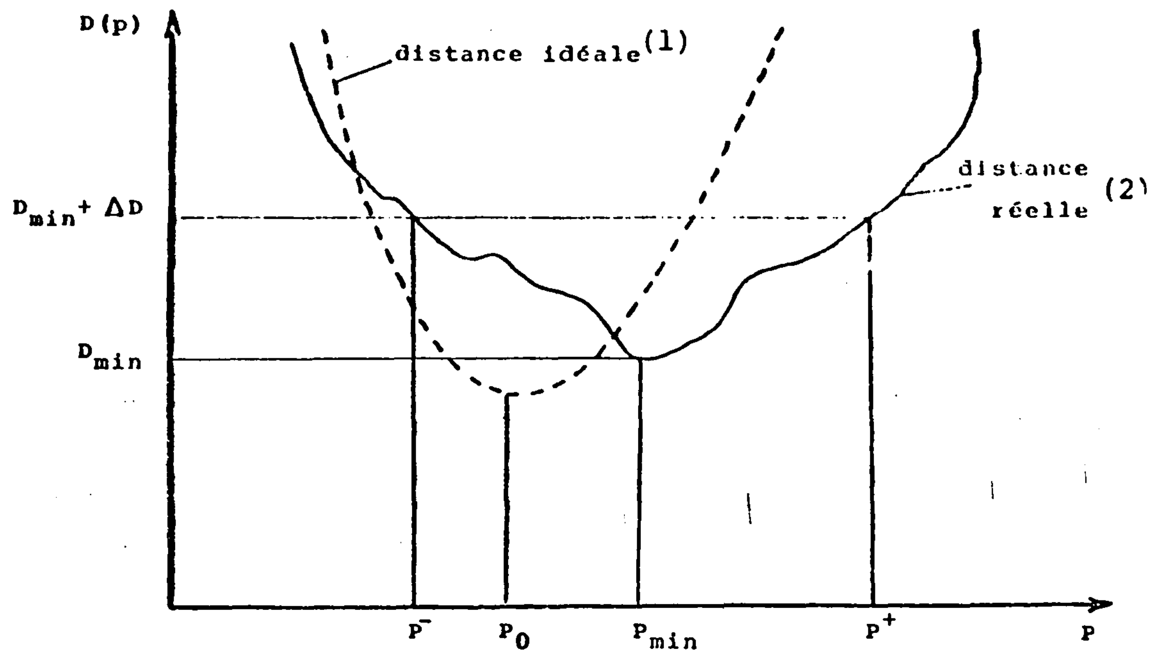


Figure 1: Distance of the object to the model.
Key: (1) ideal distance; (2) real distance.

Thanks to the statistical interpretation of the problem of the identification (maximum likelihood) the theory gives us a value of the distance of the object to the model ($D_{min} + \Delta D$) representing a threshold of confidence for the value of the parameters. This means /3 that the segment /p^-, p+/ (in the case of a parameter like Figure 1) should contain the nominal parameter with a certain error risk.

Thus we obtain by means of the theory a region in the parametric space defined by the isodistance $D_{min} + \Delta D$ whose limits establish the intervals of uncertainty about the parameters (see Figure 2 for minimization with two parameters).

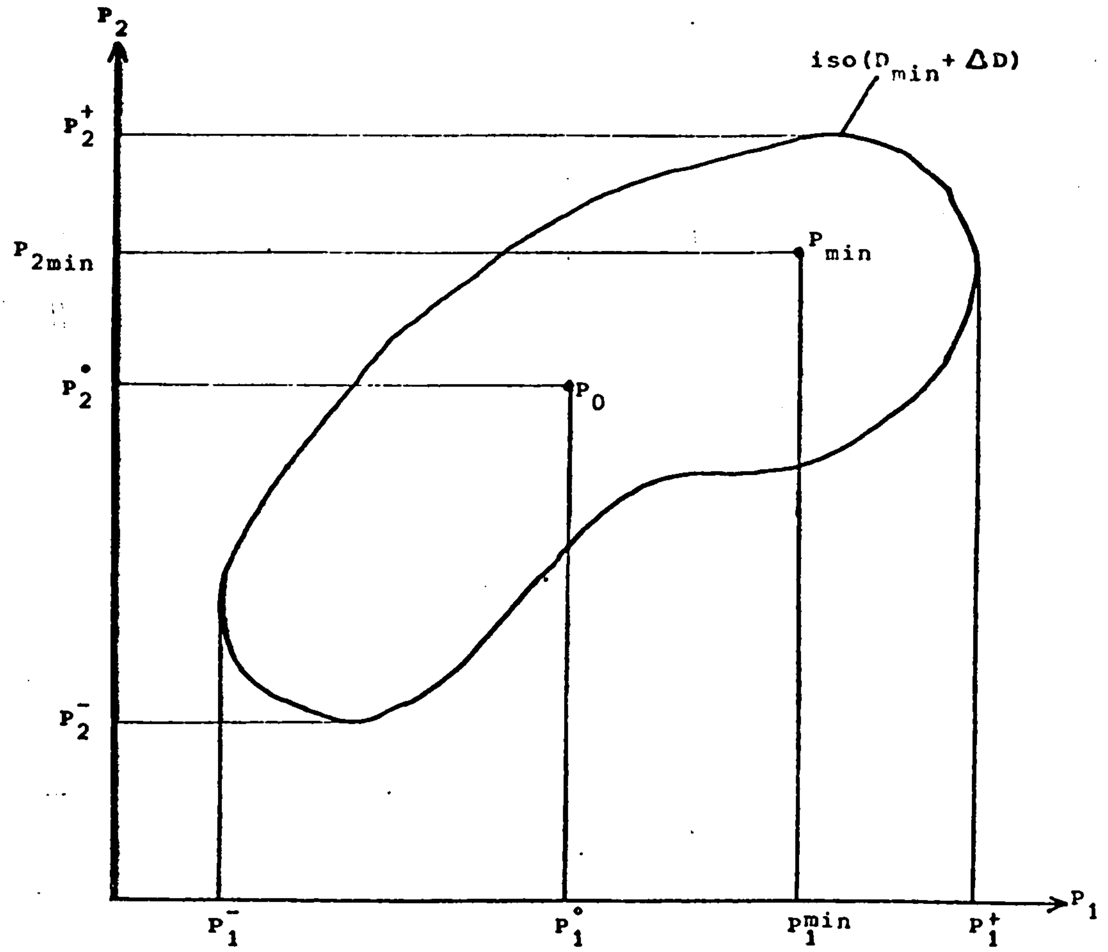


Figure 2: Intervals of uncertainty.

$P_1^- \leqslant P_1^0 \leqslant P_1^+$

$P_2^- \leqslant P_2^0 \leqslant P_2^+$

3. Overall Approach to the Computation of the Intervals of Uncertainty /4

The problem raised is therefore to calculate the limits (extreme values in each direction) of the domain defined by the iso ($D_{min} + \Delta D$) (compare Figure 2).

This problem may be solved locally by approximating the iso distances to their osculating quadrics; we will give here an overall solution which is not based on any approximation and which is based on aleatory drawings of points in the parametric space.

In the following subparagraphs we will give in detail iteration by iteration the calculations carried out and show on example the method of progressing to the limits of the above-defined region of confidence.

3.1 First Iteration

The purpose of the first iteration is to seek the extremes along the direction of the axes making it possible to obtain a first calibration of the iso D* sought around the starting point (point obtained by identification). The points P_i^m (extreme point along the direction i in the negative sense), P_i^M (extreme point along the direction in the positive sense) are retained as well as the characteristic length defined by:

$$l_i^m = \|P_i - P_i^m\| \ , \ l_i^M = \|P_i - P_i^M\|$$

the extreme points P_1^m, P_1^M, ant P_2^m, P_2^M are sought by dichotomy, these points represent the result of the dimensioning after the first iteration. The lengths l_1^M, l_1^m, l_2^M, l_2^m, are the characteristic lengths.

/5

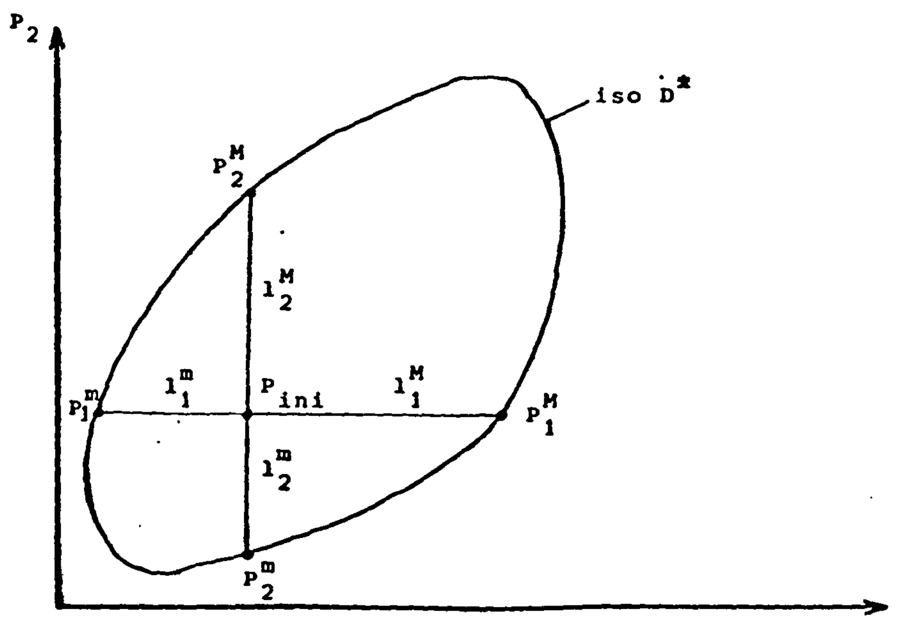


Figure 3: First iteration.

3.2 Iterations in One Direction

Let us seek the extreme point P_i^{MA}, in the positive direction of the axis i:

-- we take the best point found P_i^M

-- we carry out in this point an aleatory drawing of a group of points in a slag.

To this end the components of each of these points will be drawn according to a uniformly distributed law on /-v,+v/, the drawings being independent. Several cases may be considered:

-- the component j considered is not in the direction in which the extreme is sought (J ≠ i)

+ if the value drawn X is positive then the component j of the new point will be:

$$P_j = X * l_j^M + P_{i_j}^M$$

in which l_j^M is the characteristic length defined previously. $P_{i_j}^M$ component j of the point P_i^M

+ if the value drawn X is negative then the component j of the new point would be:

$$P_j = X * l_j^m + P_{i_j}^M$$

in which l_j^m is the characteristic length defined above previously.

-- the component j considered is in the direction in which the extreme is sought (j = i)

+ if the value drawn X is positive then the component i of the new point will be:

$$P_i = X * l_i^M + P_{i_i}^M$$

(we are progressing in the direction sought positive sense)
+ if the value drawn X is negative then the component i of the new point will be:

$$P_i=-X*l_i^m+P^M_{i\,i}$$

(symmetry is taken to be in the direction of the search).

Remark: in case we are seeking the extreme P^m_i in the negative direction of the axis i, the last two cases will be modified:

if x is negative $\quad P_i=X*l_i^m+P^m_{i\,i}$

if X is positive $\quad P_i= -X*l_i^M+P^M_{i\,i}$

Then we estimate the criterion D(P) at the point P thus drawn and if it belongs to the iso sought ($D(P)\leq D^*$) it is "classified", that is we see whether one of the components becomes an extreme (maximum: positive direction, minimum: negative direction) and it is retained if this case occurs. /7

After the drawing of the group in the small slab we estimate the rate of success of the drawing: τ which is a percentage of points drawn inside the iso.

According to the value of τ, the value V (drawing interval of the aleatory variable X) is modified. If τ is high, a large number of points are inside the iso; therefore V must be increased to hope to progress to the extreme sought; on the other hand if τ is small, a few points will be found inside the iso; therefore V must be decreased not to "go far" from the boundaries of the domain.

When this new value of V is calculated, the drawing process is repeated taking the best point found (extreme in the positive direction i).

Each of the directions is traveled in the same way (in both directions (see previous remark for the case of search in the negative direction).

3.3 Example of Progression of the Algorithm in the Case of a Parametric Space of Dimension 2 /8

Let us assume that the dichotomic search of 3.1 has been carried out (Figure 3) and that we are in the case in which the search begins with a progression in the positive direction p_1; the initial value V being taken equal to 1 (the formula of modification of V is a function of τ will be seen subsequently). On Figures 4, 5, 6 an enlargement is achieved making it possible to follow more precisely the progression.

The starting point is p_1^M result of the dichotomic search according to the direction of the axes.

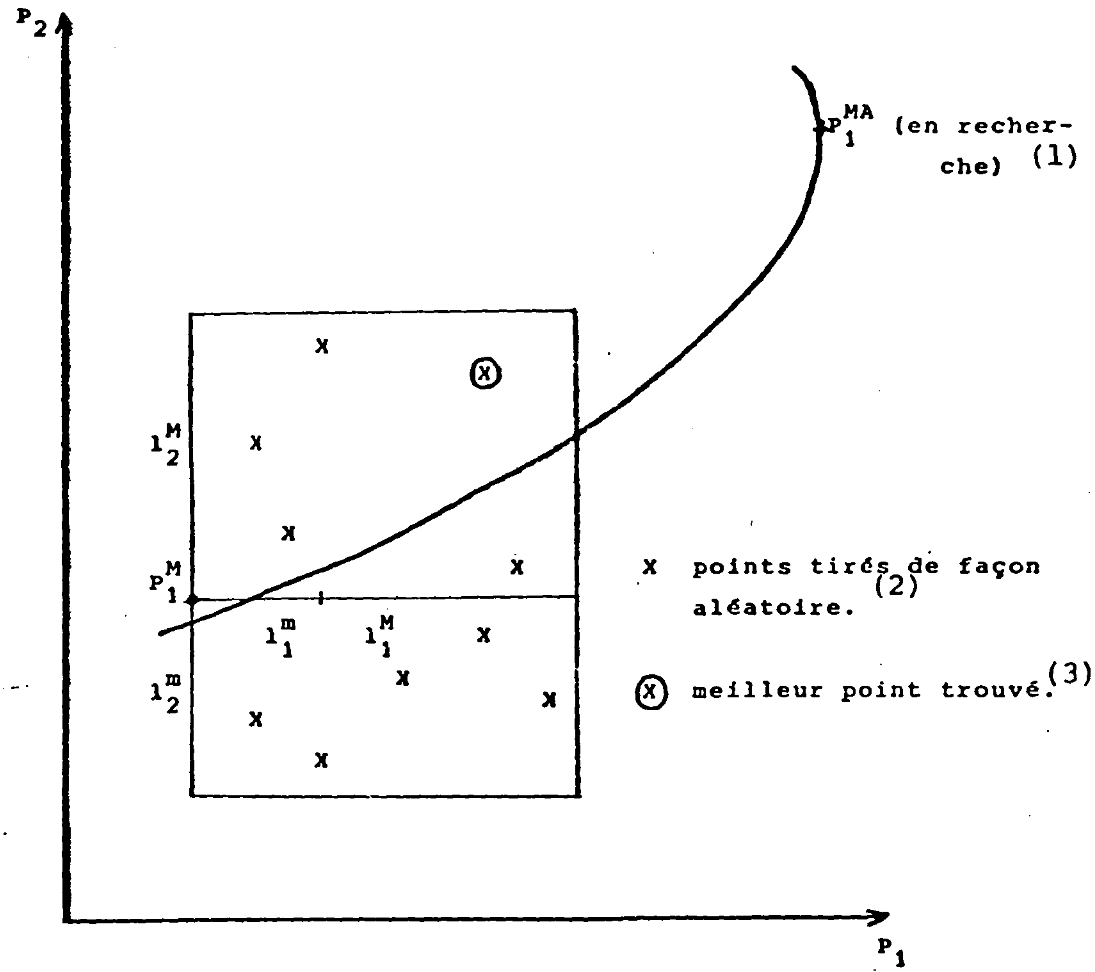


Figure 4: First step of the progression.
Key: (1) sought; (2) points taken in an aleatory manner; (3) best point found.

On Figure 4 we see the result of the drawing of a first group (points marked X) the point marked as (⊗) is the best point found in the search for P_1^{MA}, this point will be the starting point of the second step of the progression, here the value of τ allows only a slight increase of V (see paragraph 4.1.1.). /9

Remark: + the effect of symmetry permits an acceleration of the progression along the direction considered.

+ the fact that the points drawn in the lower half of the slab are not useful is only a consequence of the geometry of the isodistances and no hypothesis can be made in this matter beforehand.

The starting point will be noted P_1^M (⊗ on Figure 4).

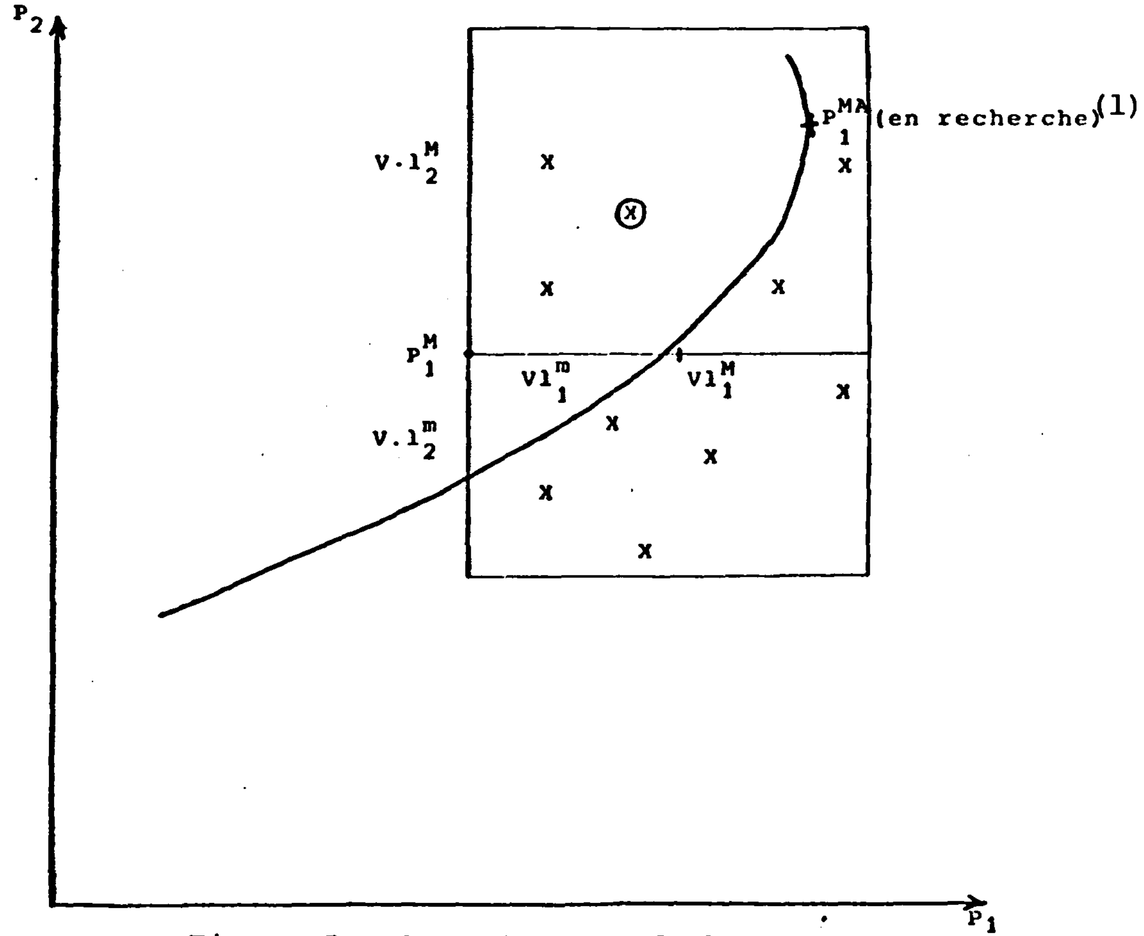


Figure 5: Second step of the progression.
Key: (1) sought.

In the same way as before the new starting point is the point designated as ⊗ which will be called P_1^M. On Figure 6 we show the third stage of progression; it may be noted that the value of V here is decreased because of the low rate of success of drawing. /10

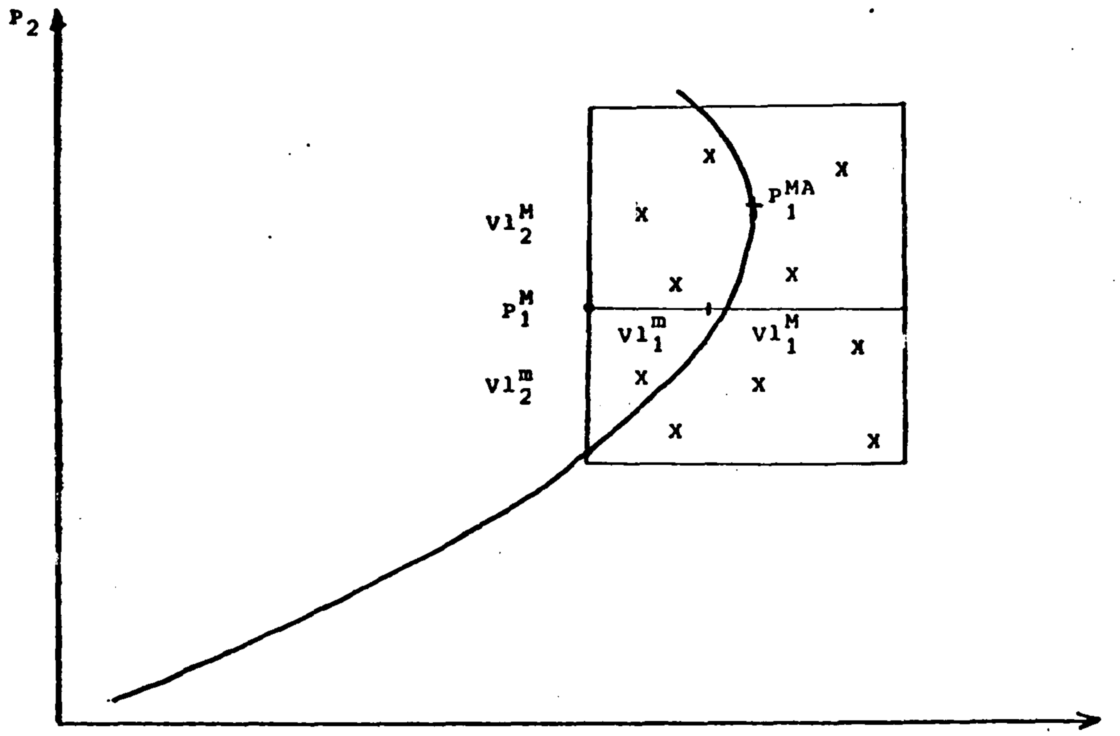


Figure 6: Third step of progression.

In the third stage of progression we can also note a relatively low rate of success and thus as we approach the point P_1^{MA} the size of the slab decreases and the precision increases.

It may be noted that in this search many interesting points for the other directions are found, it is therefore important to retain them as starting point of searchs in these directions.

But it is possible to find cases in which the progression along a direction does not offer as much satisfaction: these cases will be discussed in paragraph 5.1.

4. Sub-programs for Determining Dimensions

4.1 Introduction - General Presentation of Dimensioning

4.1.1 DIMENS Sub-programs (called: see paragraph 6)

The DIMENS sub-program involves four sub-programs:

DILOC, XCLASS, FORMU, BRBL

and makes it possible to calculate the extreme points along each dimension.

The user transmits to the sub-program the parameter vector P(N) found by identification and the CROP criterion in this point, as well as the iso XISO level which he seeks. Meanwhile he chooses the characteristics inherent to the search:

-- number of iterations along a given direction and sense: NBMAX(3)

-- number of individuals drawn in an aleatory manner during an iteration: NBMAX(1)

-- the segment on which the aleatory variable will be drawn during the first iteration: if VAI is the value transmitted by the user the segment will be /-VAI, +VAI/

As well as the characteristics inherent to the monodimensional search (compare paragraph 3.1) carried out around the initial point:

-- characteristic length estimated by the user XL(N,2) /12

XL(I,1): characteristic length of the positive direction I

XL(I,2): characteristic length of the negative direction I

-- number of iterations carried out for this monodimensional search for a given dimension and sense: NBMAX(2).

For each change of direction and/or sense, it is possible to improve the precision of the search by carrying out the local determination of dimensions (monodimensional searchs) around the starting point of iterations along this direction. The initial characteristic lengths involved in the iterations of the search are those issuing from the monodimensional search around the initial point, NBMAX(4) is the number of iterations chosen for these investigations.

The modification of the size VA of the segment of drawing of the aleatory variable as a function of the rate (RATE) of points of the group drawn which are in the area bounded by XISO, was chosen as linear function of this rate:

-- if the rate is zero, this size is divided by two.

-- if the rate is equal to 1, this size is multiplied by two.

Therefore the function: VA-VA by (1.5 by RATE + 0.5).

The user may limit the number of iterations with a threshold XMU /13
such that o XMU 1, the purpose of this threshold is to limit the decrease of VA (when the algorithm reaches convergence) indeed the iterations (if NBMAX(3) does not limit them) are stopped when VA XMU*VAI.

The DIMENS sub-program restores:

+ XMA(N,N): points of which one component represents a maximum for the points of XISO, XMA (I,J) component I of the point whose component J is maximum;

the upper limits of the intervals of uncertainty are therefore the diagonal terms XMA (J,J).

+XMI (N,N): points of which one component represents a minimum for the points in XISO, XMI (I,J) component I of the point of which the component J is minimum.

the lower limits of uncertainty intervals are therefore the diagonal terms XMI (J,J).

+ XOPT(N): best point found in the sense of the value of the criterion.

+ CROP: value of the criterion at the point XOPT(N).

+ NIT: the total number of calculation of the criterion.

The total number of computation of the criterion may be calculated (in the case in which XMU = 0) from the values transmitted in NBMAX and the value N dimension of the parametric space:

NIT=2*N(NBMAX(2)+NBMAX(3)*NBMAX(1)+NBMAX(4)).

4.1.2 DILOC Sub-program (call: see paragraph 6)

The DILOC sub-program carries out a monodimensional search around a point surrounding each axis and according to the two directions of this axis. The purpose of these monodimensional searches are to calculate the characteristic lengths defined in paragraph 3.1. To this end we give to the sub-programs an estimate (even a rough one) of these lengths (Table XL(N,2)) and the sub-program improves the values by a dichotomic procedure and restores them in the same Table XL (N,2). /14

The initial length may be divided or multiplied at most by 2^{NBCR} (NBCR: number of iterations for the search).

4.1.3 XCLASS Sub-program (call: see paragraph 6)

This sub-program makes it possible to "classify" a point, that is:

-- calculate the CRI criterion in this point by calling the FORMU sub-program (compare paragraph 4.1.4).

-- place the IN flag
IN = 0 the point is in XISO (CRI $<$ XISO)
IN = 1 the point is outside XISO (CRI $>$ XISO)

-- modify XOPT (N) if $CRI < CROP$

-- modify the Tables XMA(N,N) and XMI(N,N) if one of the components of the points is maximum or minimum.

4.1.4 FORMU Sub-program (call: see paragraph 6) /15

Sub-program using the computation of the criterion in one point.

4.1.5 BRBL Sub-program (call: see paragraph 6)

Sub-program for aleatory drawings. The variable is drawn according to a uniformly distributed law on /-0.5, +0.5/, the drawings being independent.

4.2 Notations /16

Variable	Type	Meaning
N	whole	dimension of parametric space; there are therefore 2 extreme N's to be sought: maximum N (positive direction), and minimum N (negative direction).
P(N)	real	initial parameter vector called by the sub-program (result of identification). It is used to store in the sub-program the best point found in a given direction and sense.
XISO	real	level of the iso sought (which must compulsorily be greater than the criterion calculated in P: see paragraph 2).
SMA(N,N)	real	table of storage of the extreme points in the positive direction (maxima).

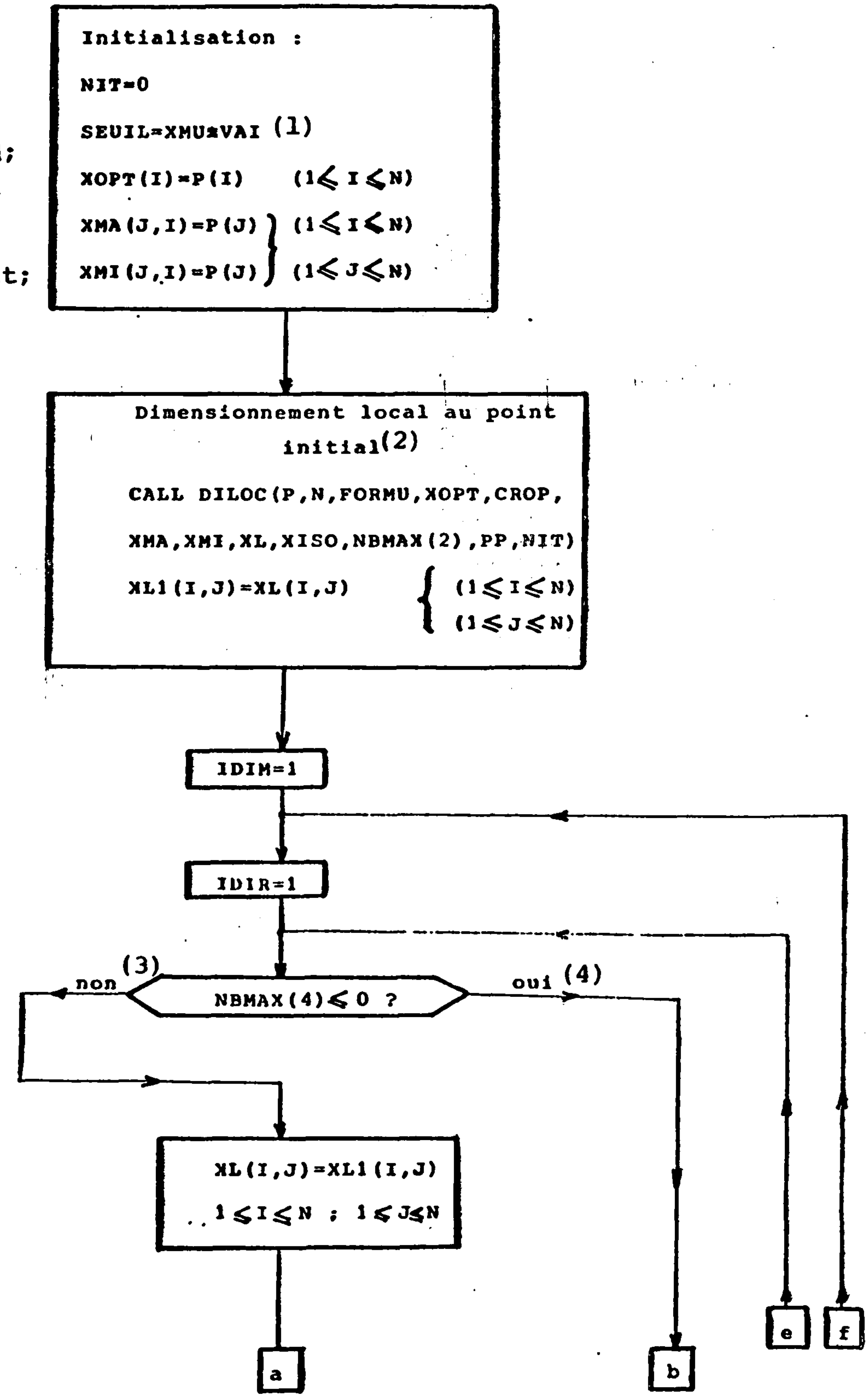
Variable	Type	Meaning
		the XMA(I,J) element is the component number 1 of the point whose component J is maximum (XMA(J,J) is the maximum found on the direction J of the points contained in XISO).
XMI(N,N)	real	table of storage of the extreme points in the negative direction (minima). the XMI(I,J) element is the component number 1 of the point whose J component is maximum (XMI(J,J) is the minimum found of the direction J of the points contained in XISO).
VAI	real	initial size chosen by the user of the /17 segment of drawing of the aleatory variable. It is advised to give value VAI=1.
FORMU	SS.P (external)	sub-program supplied by the user, calculating the criterion in a point.
XL(N,2)	real	the dichotomic search assumes that an idea of a certain order of the characteristic length is known, these lengths are transmitted in this table XL(I,1) characteristic length of the dimension I positive direction. XL(I,2) characteristic length of the dimension I negative direction. The result of the dichotomic search is stored in the same form in this table.
XOPT(N)	real	the initial P(N) vector transmitted to the dimensioning sub-program is not retained, on the other hand during the aleatory search a best point XOPT(N) of criterion CROP may

Variable	Type	Meaning
		be found and these values are restored to the user.
XL1(N,2)	real	the result of the dichotomic search carried out on the basis of the first point P(N) is stored in this table (see XL(N,2))
CROP	real	in the input criterion at point P(N) in the outlet criterion at the point XOPT(N)
PP(N)	real	working vector which is used to store the in an aleatory way during the progression in one direction /18
NIT	whole	total number of computation of the criterion.
NBMAX(4)	whole	table containing the characteristics of determination of the dimensions supplied by the user. NBMAX(1)= number of elements in the group drawn in an aleatory way NBMAX(2)= number of monodimensional searches during the initial dimensioning NBMAX(3)= number of iterations for direction (number of groups drawn in an aleatory way during the progression in a given sense and direction) NBMAX(4)= number of monodimensional searches to be carried out for the best point found during each change of direction
NBCR	whole	argument used by the sub-programs of local dimensioning by monodimensional searches: this is the number of searches to be carried out.

Variable	Type	Meaning
IN	whole	flag used to recognize whether a point is in XISO(IN=0) or outside XISO(IN=1)
XMU	real	threshold of stopping in percentage of VAI (transmitted by the user)

4.3 Organizational Chart of the DIMENS Sub-program

Key: (1) threshold; (2) local determination of dimensions at the initial point; (3) no; (4) yes.

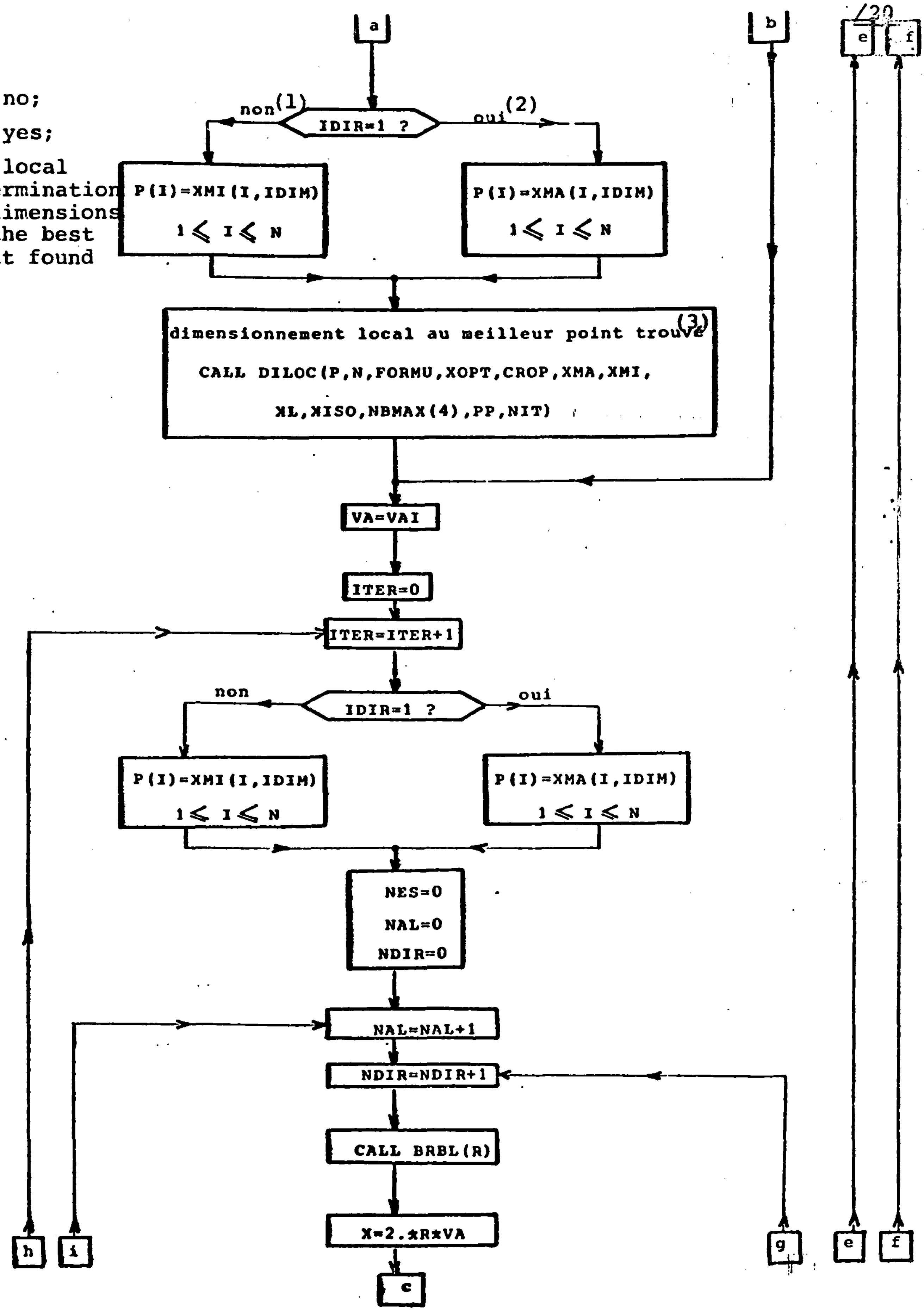


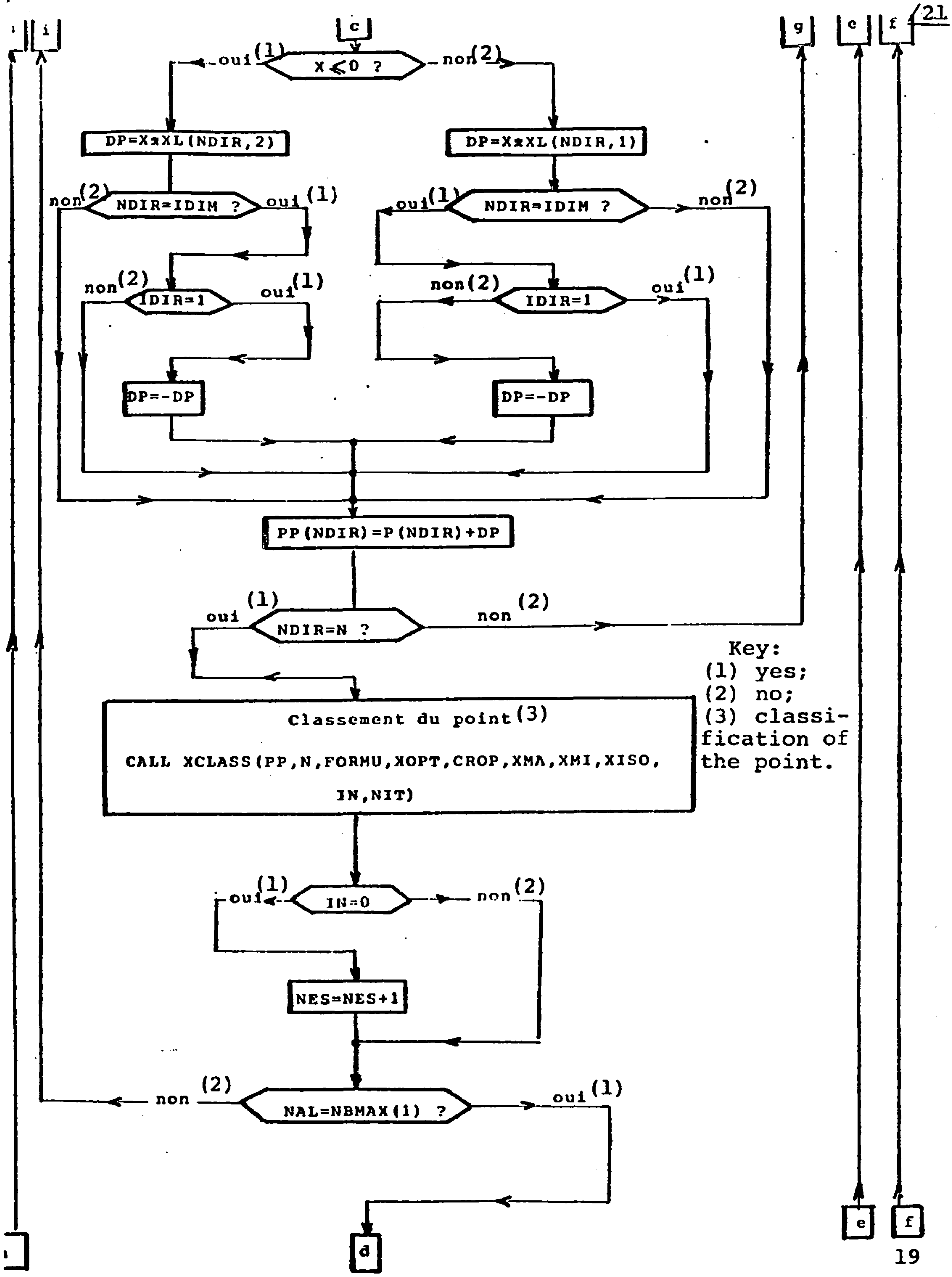
Key:

(1) no;

(2) yes;

(3) local determination of dimensions at the best point found





Key:
(1) yes;
(2) no;
(3) classification of the point.

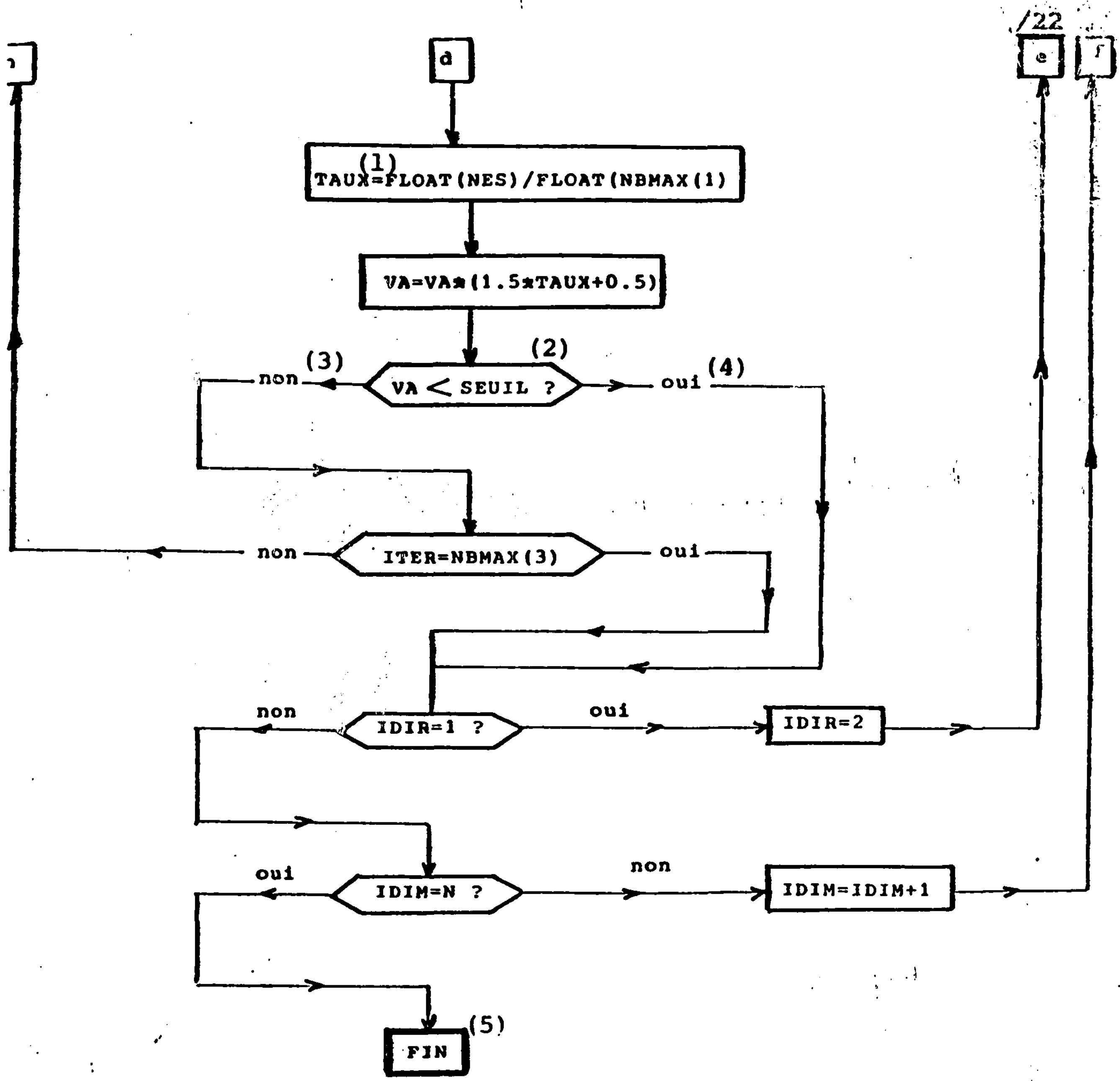
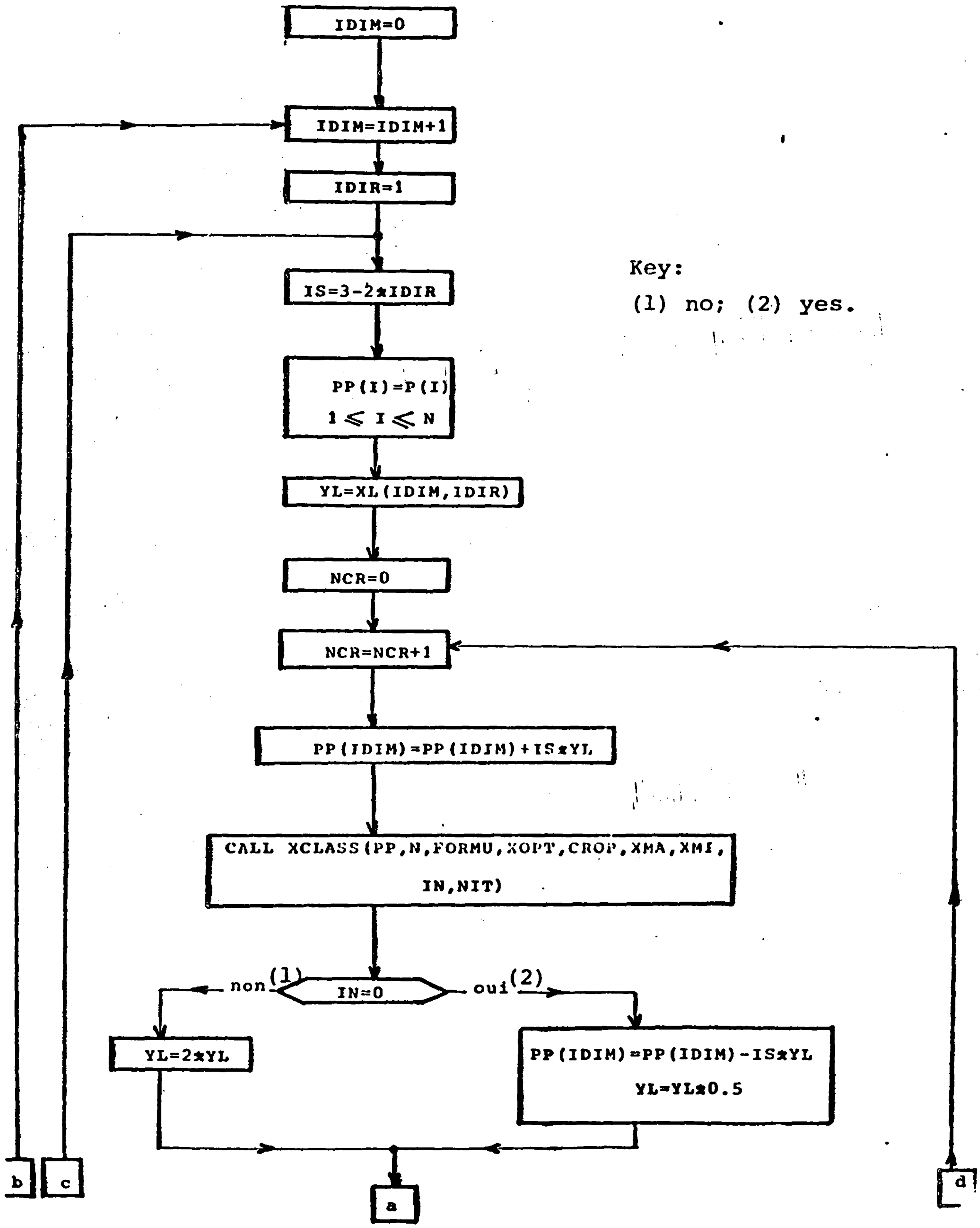


Figure 7: Organizational chart of the DIMENS Sub-program

Key: (1) rate; (2) threshold; (3) no; (4) yes; (5) end.

4.4 Organizational Chart of the DILOC Sub-program



Key:

(1) no; (2) yes.

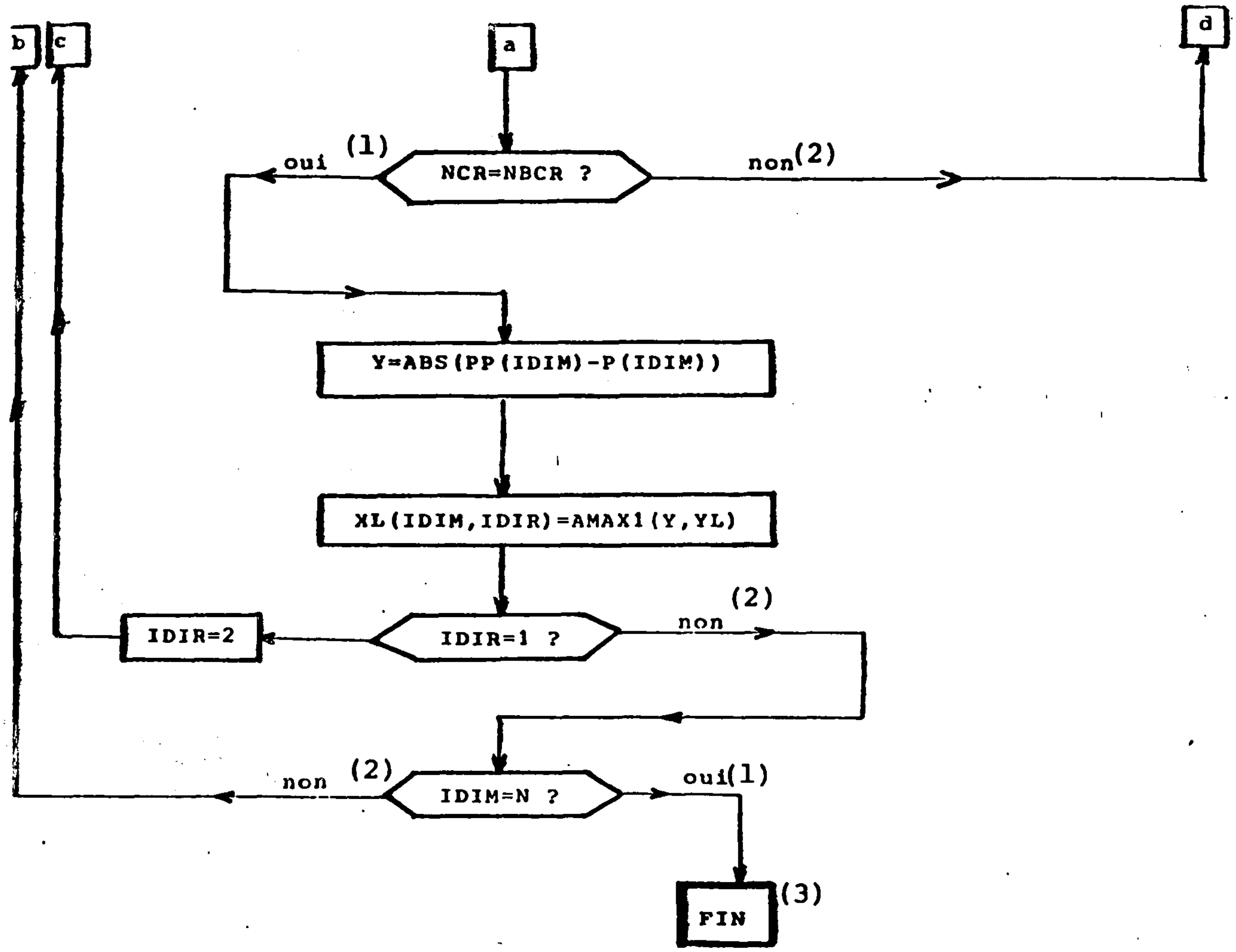
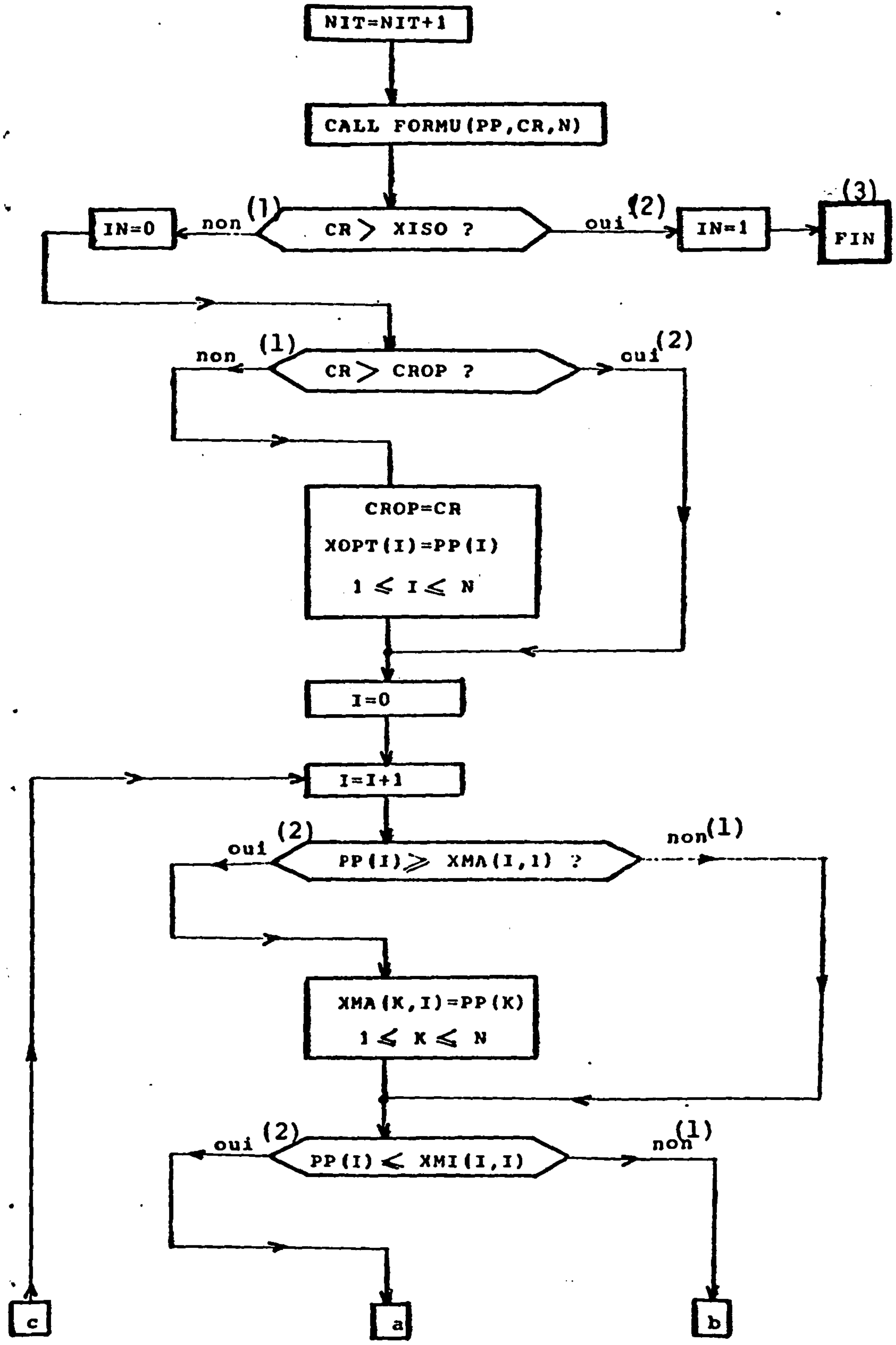


Figure 8: Organizational Chart of the DILOC Sub-program

Key: (1) yes; (2) no; (3) end.

4.5 Organizational Chart of the XCLASS Sub-program



Key:
(1) no; (2) yes.
(3) threshold

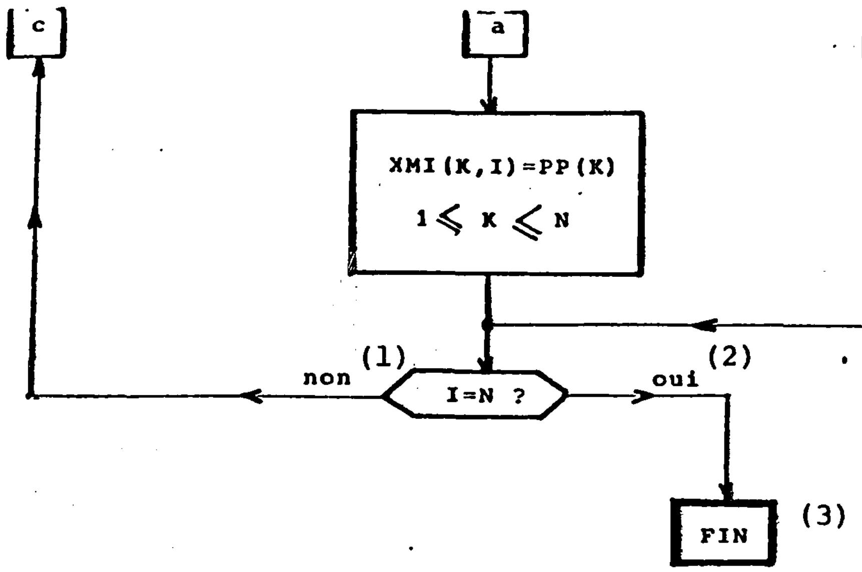


Figure 9: Organizational Chart of the XCLASS Sub-program

Key: (1) no; (2) yes; (3) end.

5. Tests Performed /27

5.1 Parametric Space of Dimension 2

The purpose of this test is to show the points drawn by the sub-program in a concrete manner: these points are visualized for different cases on Figures 10 to 14.

These figures were obtained by giving beforehand the ellipse (iso level) and the starting point, the points drawn by the program are then visualized by the symbol .

-- Figure 10 shows the results obtained on a relatively favorable case: well-conditioned ellipse, starting point close to the center. In this case the values obtained are in good approximation of the "intervals of uncertainty".

-- Figure 11 is also a case in which the ellipse is well-conditioned, but on the other hand the starting point is in a very unfavorable position. The maximum value is p_1 and minimum in p_2 close to the initial point are found without any trouble; the minimum value in p_1 is also found. On the other hand, the maximum value in p_2 is the worst obtained: it may be clearly seen that the algorithm was stopped in its search by a too low number of iterations.

-- For Figures 12 and 13 the ellipse is relatively poorly conditioned but the starting point is in a not too unfavorable condition: here too the values obtained are not poor even though, just as in Figure 13 for the maximum in p_1, there is no accumulation around the "best" point, the difference existing between the points found and this best point being not very great for the coordinate p_1.

-- Figure 14 is a very unfavorable case which unfortunately is not /28 a "rare" case in identification since it is a type of conversion which may be found for certain algorithms progressing in the areas of low gradients ("valley") such as for example for the method of the gradient. The point found is therefore in a position actually very far from the center of the iso's because of the slowness of the progression of these algorithms in the region of low gradient.

The minima and maxima close to the starting point are therefore easily found but a larger number of iterations would be needed for the determination of dimensions to hope to find the symmetrical maxima and minima which for their part are "very" far. It is only by comparing the minima and maxima with regard to the initial point that we can realize these situations: here the maximum in p_2 and the minimum in p_1 are very close to the initial point whereas the minimum in p_2 and the maximum in p_1 are comparatively very far away.

It should be noted that the points drawn are not extremely far from the region concerned and in most of the cases they are most "close" to the ellipse sought.

5.2 Determination of the Dimension of an Ellipsoid in a Space of Dimension 4. /34

Let us consider the ellipsoid of equation.

$$CRI=AL(1)*X(1)^2+AL(2)*X(2)^2+AL(3)*X(3)^2+AL(4)*X(4)^2$$

value of AL: AL = /0.01,1.,100.,10000./

We are going to take different points located on the ellipsoid of level CRI=1 and seek the dimensions of the ellipsoid of level XISO=1.1, the dimensions following the axes of this ellipsoid ("interval of uncertainty") are:

Axis 1 = 20.98; Axis 2 = 2.098; Axis 3 = 0.2098; Axis 4 = 0.02098.

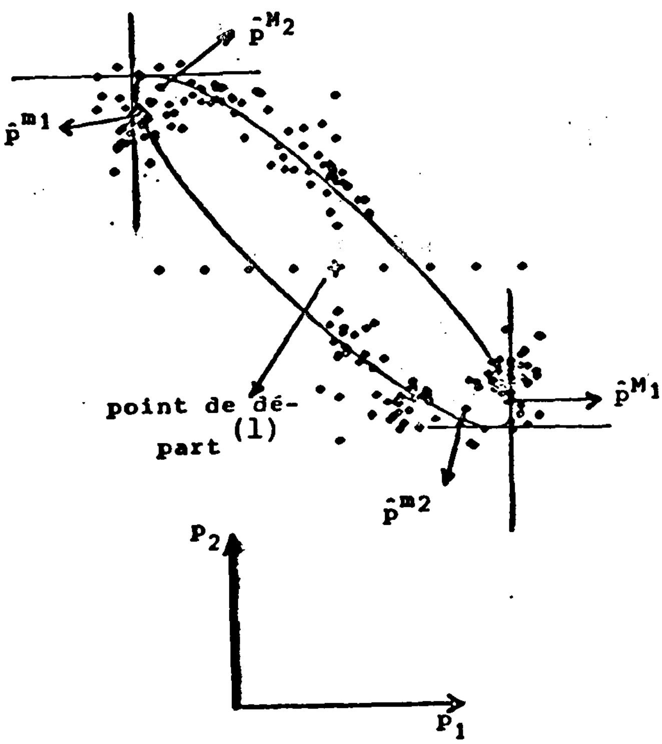


Figure 10

Caption common to Figures 10, 11, 12, 13, 14:

p^{M1}= extreme point found: maximum in p_1

p^{m1}= extreme point found: minimum in p_1

p^{M2}= extreme point found: maximum in p_2

p^{m2}= extreme point found: minimum in p_2

Key: (1) starting point.

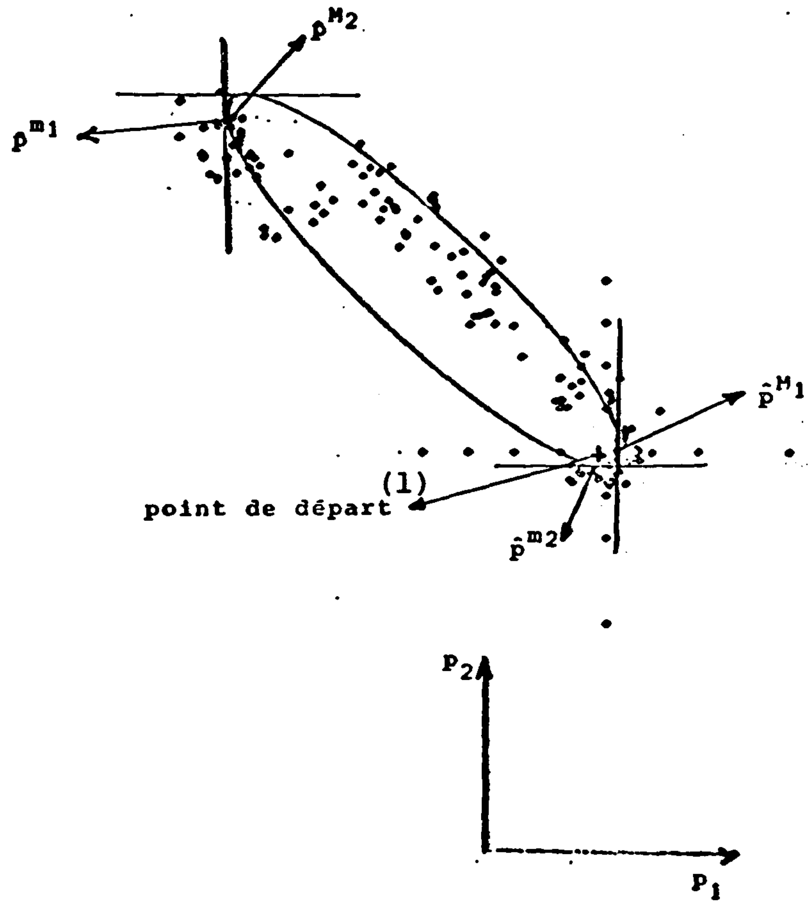


FIGURE 11

Key: (1) starting point.

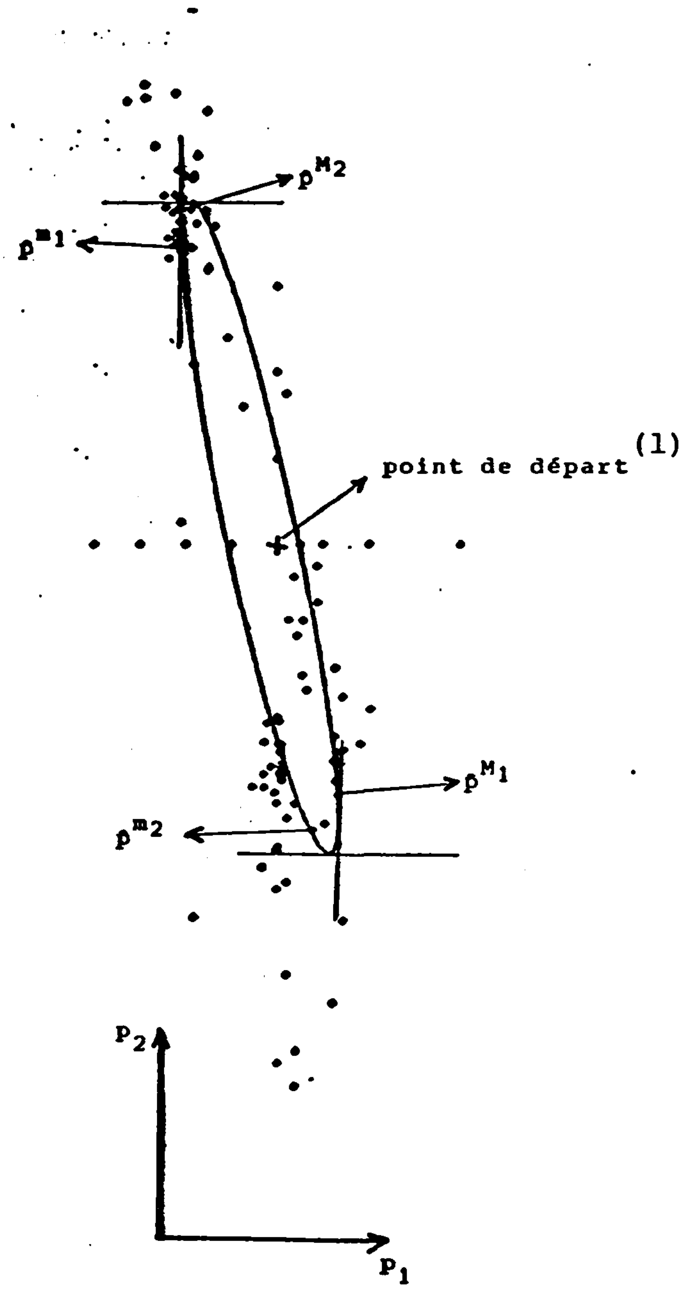


FIGURE 12

Key: (1) starting point.

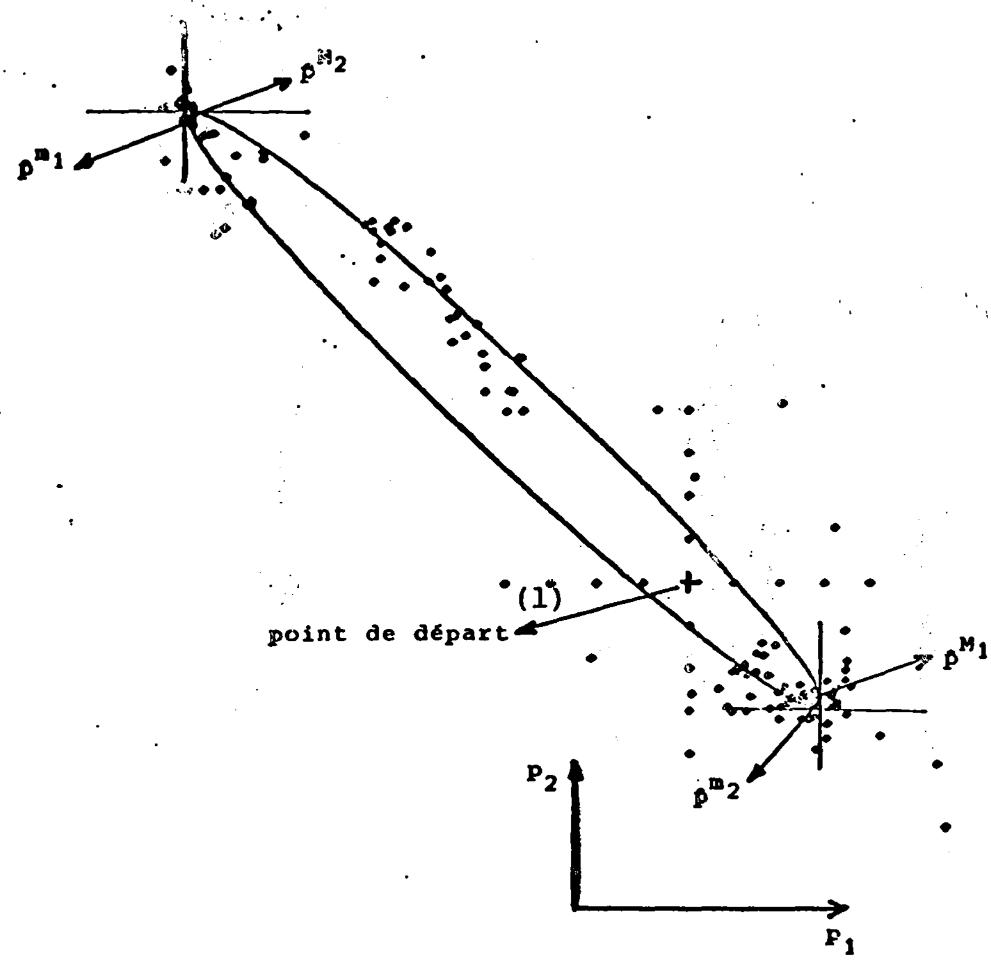


FIGURE 13

Key: (1) starting point.

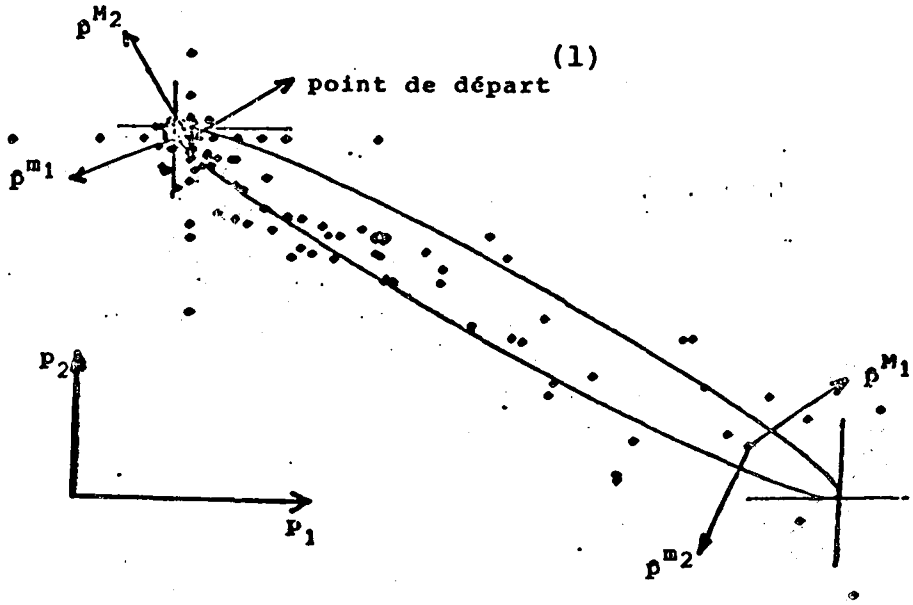


FIGURE 14

Key: (1) starting point.

The characteristics fixed for the determination of dimensions are:

$XL(I,J) = 1.(1 \leq I \leq 4;\ 1 \leq J \leq 2)$: initial characteristic lengths used for the local determination of the dimension around the initial point

NBMAX(1) = 8: number of elements of the group drawn in an aleatory manner

NBMAX(2) = 5: number of iterations of the dichotomic search around an initial point

NBMAX(3) = 10: number of iterations along one direction

NBMAX(4) = 0: known local determination of dimensions for each change of direction.

VAI = 1: segment of drawing of the aleatory variable during the first iteration: /-1,+1/.

We will cause the variation of the threshold of stoppage XMU and /35
give the number of computation of the criteria needed and the result obtained.

(1) Point de départ	XMU	(2) nombre de calcul du critère	"Intervalle d'incertitude" (3)			
			DX1	DX2	DX3	DX4
10.	0.0	680	20.07	1.586	0.1567	0.01795
0.	0.1	526	19.85	1.632	0.1527	0.01667
0.	0.2	395	19.81	1.614	0.1342	0.01597
0.	0.3	322	19.51	1.540	0.1286	0.01590
	0.4	257	18.56	1.330	0.1242	0.01585
	0.5	190	18.06	1.095	0.1150	0.01488

Key: (1) starting point; (2) number of computation of the criterion; (3) interval of uncertainty.

0.	0.0	680	18.30	2.066	0.1508	0.01751	
1.	0.1	525	17.32	2.050	0.1449	0.01669	
0.	0.2	390	17.43	2.028	0.1408	0.01584	
0.	0.3	310	16.41	2.013	0.1339	0.01572	
	0.4	239	14.40	2.007	0.1134	0.01386	
	0.5	154	10.61	2.010	0.08236	0.01435	
0. .	0.0	680	19.34	1.776	0.1875	0.01827	
0.	0.1	530	19.58	1.724	0.1860	0.01699	
0.·1	0.2	417	19.00	1.581	0.1767	0.01614	
0.	0.3	344	17.38	1.621	0.1787	0.01552	
	0.4	266	15.19	1.378	0.1767	0.01559	
	0.5	183	13.44	1.093	0.1664	0.01593	
0.	0.0	680	20.05	1.868	0.1700	0.01938	/36
0.	0.1	561	19.94	1.844	0.1779	0.01893	
0.	0.2	466	19.57	1.751	0.1657	0.01866	
0.01	0.3	373	18.60	1.762	0.1505	0.01866	
	0.4	288	18.21	1.488	0.1366	0.01869	
	0.5	239	15.74	1.475	0.1245	0.01853	
5	0.0	680	18.35	1.746	0.1849	0.01738	
.5	0.1	508	18.51	1.776	0.1723	0.01708	
.05	0.2	411	17.81	1.851	0.1709	0.01636	
.005	0.3	303	16.43	1.712	0.1553	0.01492	
	0.4	232	15.79	1.437	0.1424	0.01503	
	0.5	149	12.49	1.164	0.1397	0.01239	

The analysis of this table shows us that it is possible to achieve a relatively satisfactory precision on the basis of very approximative data (XL(I,J)=1.); but a very high precision can only be obtained at the cost of a very large number of computation of the criterion.

The figure of 100 calculations of criterion per space dimension (50 for each direction) may be retained as the upper limit making it possible to achieve an acceptable precision, which gives with the values of NBMAX given here an XMU threshold of 0.2.

This rule cannot be considered as general and we see with this special case that the initial point transmitted by the user is very important.

5.3 Tests Carried Out on the Basis of an Identification of Simulated Data /37

a) The simulated object is an aircraft in longitudinal decoupled flight, the aerodynamic coefficients are developed in the form:

$$C_x=0.1456-0.0030C_z-0.1500C_z^2=C_{x0}+AC_z+BC_z^2$$

$$C_z=0.09151+2.470\,\alpha+1.700\,\delta_m$$

$$C_m=0.007364-0.1970\alpha-1.460\,\frac{1}{V_0}q-0.4220\,\delta_m$$

These developments make it possible for us to simulate measurements by means of equations of dynamics and mechanics of flight.

b) On the basis of these measurements through a procedure of identification by least squares, 8 terms of the developments of the previous aeronautical coefficients are identified (the others are assumed known).

The identified terms and their values are:

C_{z0}	= 0.091519	B	= -0.15332
$C_{z\alpha}$	= 2.4699	$C_{m\alpha}$	= -0.20140
$C_{z\delta m}$	= 1.6997	C_{mq}	= -1.0949
A	= -0.002186	$C_{m\delta m}$	= -0.42012

The purpose of dimensioning is to give for each of the parameters an interval of uncertainty. /38

The criterion of the quadratic spread between the object and the model in this point is 1433.0, after the dimensioning the best point was found and the uncertainty intervals are:

				Optimum Found
0.088842	$\leqslant$	c_{z_0} $\leqslant$	0.093657	0.090579
2.1660	$\leqslant$	c_z $\leqslant$	2.8846	2.5946
1.5861	$\leqslant$	$c_{z\delta_m}$ $\leqslant$	1.8878	1.7179
-0.0063814	$\leqslant$	A $\leqslant$	0.0041452	-0.0015125
-0.20817	$\leqslant$	B $\leqslant$	-0.11191	-0.1636
-0.21761	$\leqslant$	$c_{m\alpha}$ $\leqslant$	-0.16992	-0.20329
-2.1280	$\leqslant$	c_{m_q} $\leqslant$	-1.0449	-1.5142
-0.42730	$\leqslant$	$c_{m\delta_m}$ $\leqslant$	-0.40355	-0.42075

The criterion for the optimum found is 168.49.

The intervals of uncertainty indicated above were calculated for an iso level of 1433 by 1.1 = 1576.3.

The intervals of uncertainty found are relatively large because the results of the identification are poor: for this it is sufficient to compare the criteria at the point found at the time of the determination of the dimensions and at the point found by identification.

6. Listings of the Different Sub-programs /39

6.1 Listing of the DIMENS Sub-program /40

DIMENS /41

Determination of Dimensions of an ISO-EPS

Input Variables

P(N)	Initial parametric vector
N	Dimensions of the parametric space
FORMU	SSP computation of the criterian
XL(N,2)	Initial stage of monodimensional searches
XISO	Level of the iso sought
NBMAX(4)	Maximum number of iterations
	NBMAX(1): number of aleatory drawings
	NBMAX(2): number of initial monodimensional searches
	NBMAX(3): number of iterations per direction
	NBMAX(4): number of monodimensional searches for each change of direction
VAT	"Initial variance"
XMU	Threshold of stoppingin percentage of VAI

Outlet Variables

XOPT(N)	Final parameter vector: optimum found
CROP	Criterion in XOPT
XMA(N,N)	Coordinates of the points representing a maximum of XMA(I,J): I component of the point of which the rate component is maximum
XMT(N,N)	Coordinates of the points representing a mimum XMI(I,J): I component of the point whose J component is minimum
NIT	Total number of calculation of the criterion

Working Table

PP(N)	
XL1(N,2)	Result of the first local determination of dimensions

Sub-programs Called

DILOC

XCLASS

BRBL

Remarks

FORMU (X,CRI,N): SSP calculation of criterion, declare "External"

X(N): parametric vector

```
CRI:  Criterion in X
BRBL(X): Drawing of the variable X in an uniformly distributed group
         on -.5,+.5

Subroutine DIMENS(P,N,FORMU,XOPT,CROP,XMA,XMI,XL,XISO,NBMAX1,VAI,PP,
XL1,XMU,NIT)

Dimension P(N),PP(N),XOPT(N),NBMAX(4)
Dimension XL(N,2),XL1(N,2)
Dimension XMA(N,N),XMI(N,N)

Initialization
NIT=0
Threshold= XMU*VAI
NB1=NBMAX(1)
NB3=NBMAX(3)
DO 1 I=1,N
XMA(J,I)=P(J)
XMI(J,I)=P(J)
1 Continue
C* Local determination of dimensions at the initial point
Call DILOC (P,N,FORMU,XOPT,CROP,XMA,XMI,XL,XISO,NBMAX(2),PP,NIT)
DO 10 T=1,N
DO 10 J=1,2
10 XL1(I,J)=XL(I,J)
C* Init. aleatory variable
R=.2
C* Loop on the dimensions:IDIM
C* Loop on the directions:IDIR
C* IDIR=1: Positive sense
C* IDIR=2: Negative sense
DO 9000 IDIM=1,N
DO 9000 IDIR=1,2
 IF(NBMAX(4)) 1500,1500,1000
C*NBMAX(4) different than zero
C*Local dimensioning with each change in direction
1000 Continue
```

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```
 DO 12 I=1,N
 DO 12 J=1,2
12 XL (I,J)=XL1(I,J)
   GO TO (31,32) TDTR
31 Continue
   DO 33 T=1,N
33 P(T)=XMA(J,IDIM)
   GO TO 35
32 Continue
   DO 34 T=1,N
34 P(T)=XMI(I,IDIM)
35 Call DILOC(P,N,FORMU,XOPT,CROP,XMA,XMI,XL,XISO,NBMAX(4),PP,NIT)
1500 Continue
   VA=VAI
C* Iterations on a direction:ITER
   ITER=0
4000 Continue
   ITER=ITER+1
   GO TO (101,102),IDIR
C* Position at the "best point" found
101 Continue
  DO 20 I=1,N
20 P(I)=XMA(I,IDIM)
  GO TO 103
102 Continue
  DO 21 I=1,N
21 P(T)=XMI(I,IDIM)
103 Continue
  NES=0
C* Drawing of a group
  DO 2000 NAL=1,NB1
  DO 3000 NDIR=1,N
C* Drawing of a direction: Black
  Call BRBL (R)
  X=2.*R*VA
  IF(X)50,60,60
```

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```
 60 Continue
  DP=X*XL(NDIR,L)
  IF (NDIR-IDIM) 120,61,120
 61 GO TO (120,62),TDTR
 62 DP=DP
  GO TO 120
 50 Continue
  DP=X*XL(NDIR=IDIM)120,51,120
 51 GO TO (52,120)TDTR
 52 DP=-DP
 120 Continue
  PP(NDIR)=P(NDIR)+DP
 52 DP=-DP
 120 Continue
  PP(NDIR)=(NDIR)+DP
 3000 Continue
 C* Classification of the point
  Call XCLASS(PP,N,FORMU,XOPT,CROP,XMA,XMI,XISO,IN,NIT)
  IF(IN) 2000, 70, 2000
 70 NES=NES+1
 2000 Continue
        RATE=Float (NFS)/Float (NBMAX(1))
 C* Adjustment of the variance
   VA=VA*(1.5*Rate +.5)
   IF (VA=threshold) 9000,9000,4001
 4001 IF(NB3=ITER) 9000,9000,4000
 9000 Continue
  Return
  End
```

6.2 Listing of DILOC Sub-program /44

```
DILOC                                                                 /45
Local Dimensioning by Monodimensional Searches
C* Input Variables
 P(N)       Parametric vector
 N          Dimension of the parametric space
 XL(N,2)    Initial search steps
```

NBCR Number of searches

Outlet Variables

XL(N,2) Results of local dimensioning

Sub-programs Called

XCLASS

Working Table: PP(N)

Remarks

FORMU,XOPT,CROP,XMA,XMT,XISO,NIT: VOIR SSP DIMENS

```
Subroutine DILOC(P,N,FORMU,XOPT,CROP,XMA,XMI,XL,XISO,NBCR,PP,NIT)
Dimension P(N),PP(N),XOPT(N)
Dimension XMA(N,N),XMI(N,N)
Dimension XL(N,2)
C* Loop on the dimensions: IDIM
C* Loop on the directions: IDIR
DO 1000 IDIM=1,N
DO 1000 IDIR=1,2
IS=3-2*IDIR
DO 1I=1,N
1 PP(T)=P(T)
YL-XL (IDIM,IDIR)
CI Loop on the number of iterations: NCR
DO 2000 NCR=1,NBCR
PP(IDIM)=PP(IDIM)+IS+YL
C* Classification of the point
Call XCLASS(PP,N,FORMU,XOPT,CROP,XMA,XMI,XISO,IN,NIT)
IF(IN) 21,20,21
C* Length: 2 if point is in XISO
20 Yl=2.+Yl
GO TO 2000
C* Length: 2 if point outside XISO
21 PP(IFIM)=PP(IDIM)=IS*YL
YL=YL+.5
```

```
2000 Continue
Y=ABS(PP(IDIM)-P(IDIM))
C* Length=progression or last length attempted
XL(IDIM,IDIR)=AMAX1(Y,YL)
1000 Continue
 Return
 End
```

XCLASS /47

Comparison with the points already retained possible classification

Input Variables

PP(N)	Point to be classified
XMA(N,N)	Maximum found (see DIMENS)
XMI(N,N)	Minima found (see DIMENS)
IN	FLAG:IN=0 PP is at XISO IN=1 PP is outside of XISO
NIT	Number of calculations of the criterion

Outlet Variables

XMA(N,N)	Maximum outlet classification of PP
XMI(N,N)	Minimum outlet classification of PP
NIT	Number of calculation of the criterion

Sub-programs Called

FORMU(See DIMENS)

Remarks

FORMU,XOPT,CROP,XISO: See DIMENS

```
Subroutine XCLASS(PP,N,FORMU,XOPT,CROP,XMA,XMI,XISO,IN,NIT)
Dimension XOPT(N),PP(N)
Dimension XMA(N,N),XMI(N,N)
IT=NIT+1
C* Calculation of the criterion at the point: PP
Call FORMU (PP,CR,N)
```

```
IF(XISO-CR) 1,10,10
CI PP is outside of XISO
1 IN-1
 Return
C* PP is in XISO
10 IF(CROP-CR) 13,11,11
C* PP is optimum
11 DO 12 I=1,N
12 XOPT(T)=PP(I)
 CROP=CR
C* Loop on the dimensions:I
13 DO 20 I=1,N
 IF (XMA(I,I)=PP(I)) 22,21,21
C* I Component is maximum
22 DO 30 K=1,N
30 XMA(I,I)=PP(K)
21 IF +(PP(I)-XMI(I,I))32,20,20
C* The component I is miminum
32 DO 40 K=1,N
40 XMI(K,I)=PP(K)
20 Continue
 IN=0
 Return
 End
```

Sub-program of Aleatory Drawing /49

```
Subroutine BRBL(R)
 BB=1.
 R=ABS(R)
 P1=R*317
 R=AMOD(P1,BB)
 R=R-.5
 Return
 End
```

End of Document