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S. S. Stahara, J. P. Elliott,
and J. R. Spreiter
Nielsen Engineering & Research, Inc.
Mountain View, California

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DEVELOPMENT OF A MULTIPLE-PARAMETER NONLINEAR PERTURBATION PROCEDURE FOR TRANSONIC TURBOMACHINERY FLOWS: PRELIMINARY APPLICATION TO DESIGN/OPTIMIZATION PROBLEMS

Stephen S. Stahara, James P. Elliott, and John R. Spreiter

Nielsen Engineering & Research, Inc.
Mountain View, CA

SUMMARY

An investigation was conducted to continue the development of perturbation procedures and associated computational codes for rapidly determining approximations to nonlinear flow solutions, with the purpose of establishing a method for minimizing computational requirements associated with parametric design studies of transonic flows in turbomachines. The results reported here concern the extension of the previously-developed successful method for single-parameter perturbations to simultaneous multiple-parameter perturbations, and the preliminary application of that multiple-parameter procedure in combination with an optimization method to blade design/optimization problems.

In order to provide as severe a test as possible of the method, attention is focused in particular on transonic flows which are highly supercritical. Flows past both isolated blades and compressor cascades, involving simultaneous changes in both flow and geometric parameters, are considered. Comparisons with the corresponding 'exact' nonlinear solutions display remarkable accuracy and range of validity, in direct correspondence with previous results for single-parameter perturbations. Initial applications of the perturbation method combined with an optimization procedure demonstrate the ability of the multiple-parameter method to work accurately in a design environment and establish its potential for reducing the computational work required in such applications by an order of magnitude.

1. INTRODUCTION

With the continuing success of advanced computational methods to determine solutions to increasingly complex fluid dynamic phenomena, it has become clearly apparent that in order to employ these methods in applications requiring routine high-frequency use, a means must be found to reduce the computational demands necessary in their straightforward application. While this need exists across a spectrum of aerodynamic users, it is particularly high in turbomachinery applications. There both the basic aerodynamic computation is time-consuming and the number of variable flow and geometry parameters are large, making any turbomachinery parametric or design study computationally expensive under the best of circumstances, and in many instances using more advanced codes, prohibitively so.

The ultimate objective underlying this study is to develop the means of reducing substantially the overall computational requirements necessary for turbomachinery design or parametric studies by minimizing the actual number of "expensive" numerical flow solutions required. That such procedures are achievable has been successfully demonstrated in the previous phase (ref. 1) of this study. In that work, a perturbation method was developed and tested on a large number of nonlinear flow problems involving single-parameter changes of a variety of flow and geometric parameters. Subcritical and supercritical flows past isolated blades and compressor cascades were considered, with particular emphasis placed on supercritical transonic flows which exhibited large surface shock movements over the parametric range studied. Comparisons of the perturbation predictions with the corresponding 'exact' nonlinear solutions indicated a remarkable accuracy and range of validity of the perturbation method.

The work reported here describes the continued extension and refinement of that perturbation technique, and has focused on the development of its capability for actual application to practical turbomachinery design problems requiring the highly repetitive use of computational codes to determine a large number of related flow solutions. Two primary tasks were involved. The first consisted of the extension of the method to treat simultaneous multiple-parameter perturbations. The second involved the combination of the perturbation method with an optimization procedure, and the preliminary application of that combination to blade design/optimization problems. The nature of the present work is both exploratory and developmental in that aspects of the procedure--such as its validity, range of accuracy, and computational economy for multiple-parameter perturbation problems and its workability in an optimization design environment--will be investigated, and a computational code for multiple-parameter perturbations will be developed.

2. ANALYSIS

2.1 Perturbation Concept and Previous Applications

The classical approach of performing a perturbation analysis--consisting of establishing and solving a series of linear perturbation equations--appears an an obvious choice for the current applications. However, results from the initial phase of this study (ref. 2) demonstrate that for applications to sensitive flows such as occur in transonic situations, the basic linear variation assumption fundamental to the technique is sufficiently restrictive that the permissable range of parameter variation is so small to be of little practical use. An interesting alternative to the linear perturbation equation approach has recently been successfully examined in which a correction technique is used that employs two or more nonlinear base solutions. For that method, the basic perturbation solution is determined simply by differencing two nonlinear base flow solutions removed from one another by some nominal change of a particular flow or geometrical quantity. A unit perturbation solution is then obtained by dividing that result by the change in the perturbed quantity. Related solutions are determined by multiplying the unit perturbation by the desired parameter change and adding that result to the base flow solution. This simple procedure, however, only works directly for continuous flows for which the perturbation change does not alter the solution domain. For those perturbations which change the flow domain, coordinate stretching is necessary to ensure proper definition of the unit perturbation solution. Similarly, for discontinuous flows, coordinate straining is necessary to account for movement of discontinuities due to the perturbation.

In a number of recent applications of the method (refs. 1-7), results have been obtained which demonstrate the accuracy, range of validity, and versatility of perturbation methods based on such ideas. The most extensive and systematic of these are provided in references 1 and 7, where results are reported for a variety of flow and geometry parameter perturbation case studies of nonlinear subsonic and transonic flows past both isolated blades and compressor cascades. In those results, particular emphasis was placed on strongly supercritical transonic flows which exhibit large surface shock movements over the parameter range studied. Comparisons of the perturbation results with the corresponding 'exact' nonlinear solution display remarkable accuracy across the spectrum of examples studied.

The basis of this accuracy lies in the use of coordinate straining. This provides the means in determining the unit perturbation to account properly for the displacement of discontinuities and maxima of high-gradient regions due to a parameter change. This in turn enables the perturbation method to maintain uncommon accuracy in regions of high gradients over large parametric ranges. In reference 1, a detailed examination was made of the effect of employing different classes of straining functions. Those results have illustrated deficiencies in certain classes of straining functions and have lead to the identification of a superior class of straining functions. These results are discussed in more detail in section 2.3.

2.2 Simultaneous Multiple-Parameter Perturbation Formulation

To provide the theoretical basis of the perturbation method as applied to simultaneous multiple-parameter perturbations of flows containing multiple shocks or high-gradient regions, consider the formulation of the procedure at the full potential equation level, since all of the results presented here are based on that level. Denote the operator L acting on the full velocity potential Φ as that which results in the two-dimensional full-potential equation for Φ, i.e.,

$$L[\Phi] = 0 \qquad (1)$$

If we now expand the potential in terms of zero- and higher-order components in order to account for the variation of M arbitrary geometrical or flow parameters q_j

$$\begin{aligned} \Phi &= \Phi_o + \sum_{j=1}^{M} \varepsilon_j \Phi_{1_j} + \ldots \\ q_j &= q_{o_j} + \Delta q_j \end{aligned} \qquad (2)$$

and then insert this into the governing equation (1), expand the result, order the equations into zero- and first-order components, and make the obvious choice of expansion parameters $\varepsilon_j = \Delta q_j$ we obtain the following governing equations for the zero- and M first-order components

$$\begin{aligned} L[\Phi_o] &= 0 \\ L_1[\Phi_{1_j}] + \frac{\partial}{\partial q_j} L[\Phi_o] &= 0 \end{aligned} \qquad (3)$$

Here L_1 is a linear operator whose coefficients depend on zero-order quantities and $\partial L[\Phi_o]/\partial q_j$ represents a 'forcing' term due to the $q_j{}^{th}$ perturbation. Actual forms of L_1 and the 'forcing' term are provided in reference 2 for a variety of flow and geometry parameter perturbations of a two-dimensional turbomachine, and in reference 4 for profile shape perturbations of an isolated airfoil. An important point regarding Equation (3) for the first-order perturbations Φ_{1_j} is that these equations represent a unit perturbation independent of the actual value of the perturbation quantity ε_j.

Appropriate account of the movement of a multiple number of discontinuities and maxima of high-gradient regions due to the perturbation is now accomplished by the introduction of strained coordinates (s,t) in the form

$$x = s + \sum_{j=1}^{M} \varepsilon_j x_1(s,t)$$
$$y = t + \sum_{j=1}^{M} \varepsilon_j y_1(s,t) \tag{4}$$

where

$$x_1(s,t) = \sum_{i=1}^{N} \delta x_i x_{1_i}(s,t)$$
$$y_1(s,t) = \sum_{i=1}^{N} \delta y_i y_{1_i}(s,t) \tag{5}$$

and $\varepsilon_j \delta x_i$, $\varepsilon_j \delta y_i$ represent individual displacements due to perturbation of the $q_j{}^{th}$ parameter of the N strained points, and $x_{1_i}(s,t)$, $y_{1_i}(s,t)$ are straining functions associated with each of the N strained points. Introducing the strained coordinate Equations (4) and (5) into the expansion formulation leaves the zero-order result in Equation (3) unchanged, but results in a change of the following form for the j^{th} perturbation

$$L_1[\Phi_{1_j}] + L_{2_j}[\Phi_o] + \frac{\partial}{\partial q_j} L[\Phi_o] = 0 \tag{6}$$

Here the operators are understood to be expressed in terms of the strained (s,t) coordinates, and the additional operator L_{2_j} arises specifically from displacement of the strained points. In references 3 and 4, specific expressions for L_{2_j} are provided for selected perturbations involving transonic small-disturbance and full-potential equation formulations. The essential point, however, with regard to perturbation Equation (6) expressed in strained coordinates is that it remains valid as before for a unit perturbation and independent of ε_j.

In employing the correction method, Equation (6) for the j^{th} unit perturbation is solved by taking the difference between two solutions obtained by the full nonlinear procedure after appropriately straining the coordinates. If we designate the solutions for some arbitrary dependent flow quantity Q as base Q_o and calibration Q_{c_j}, respectively, of the varied independent parameter q_j, we have for the predicted flow at some new parameter value q_j

$$Q(x,y) = Q_o(s,t) + \sum_{j=1}^{M} \varepsilon_j Q_{1_j}(s,t) + \ldots \tag{7}$$

where

$$Q_{1_j} = \frac{Q_{c_j}(\bar{x}_j,\bar{y}_j) - Q_o(s,t)}{\bar{\varepsilon}_j}$$

$$\left.\begin{aligned}
\bar{x}_j &= s + \sum_{i=1}^{N} \bar{\varepsilon}_j \delta x_i x_{1_i}(s,t) \\
\bar{y}_j &= t + \sum_{i=1}^{N} \bar{\varepsilon}_j \delta y_i y_{1_i}(s,t) \\
x &= s + \sum_{j=1}^{M} \frac{\varepsilon_j}{\bar{\varepsilon}_j}(\bar{x}_j - s) \\
y &= t + \sum_{j=1}^{M} \frac{\varepsilon_j}{\bar{\varepsilon}_j}(\bar{y}_j - t)
\end{aligned}\right\}$$

(Continued)

[Eq. (8) Continued]

$$\left.\begin{aligned} \bar{\varepsilon}_j &= q_{c_j} - q_{o_j} \\ \varepsilon_j &= q_j - q_{o_j} \end{aligned}\right\} \qquad (8)$$

In the following section, applications of the correction procedure are made to predict surface properties. Also provided is the particular form of the straining functions Equation (5) found to be most effective in those applications.

2.3 Application to Surface Properties

For the current applications, we have employed coordinate straining with the correction method to predict distributions of surface properties for simultaneous multiple-parameter perturbations of aerodynamic flows. In that instance where flow properties are required along some contour, the strained-coordinate solutions can be represented by

$$Q(x;\varepsilon) \sim Q_o(s) + \sum_{j=1}^{M} \varepsilon_j Q_{1_j}(s) + \ldots \qquad (9)$$

$$x \sim s + \sum_{j=1}^{M} \varepsilon_j x_1(s) + \ldots \qquad (10)$$

where x is the independent variable measuring distance along the contour or a convenient projection of that distance, s is the strained coordinate, and ε_j a small parameter representing the change in one of M flow or geometrical variables which we wish to vary simultaneously.

In order to determine the first-order corrections $Q_{1_j}(s)$, we require one base and M calibration solutions in which the calibration solutions are determined by varying each of the M arbitrary independent parameters q_j by some nominal amount from the base flow value while keeping the others fixed at their base values.

In this way, the first-order corrections $Q_{1_j}(s)$ can be determined as

$$Q_{1_j}(s) = \frac{Q_{c_j}(\bar{x}_j) - Q_o(s)}{\bar{\varepsilon}_j} \tag{11}$$

where Q_{c_j} is the calibration solution corresponding to changing the j^{th} parameter to a new value q_{c_j}, $\bar{x}_j$ is the strained coordinate pertaining to the Q_{c_j} calibration solution, and $\bar{\varepsilon}_j = q_{c_j} - q_{o_j}$ represents the change in the q_j parameter from its base flow value. If we now desire to keep invariant during the perturbation process a total of N points corresponding to discontuities or high-gradient maxima, we can represent the solution by

$$Q(x,\varepsilon_j) = Q_o(s) + \sum_{j=1}^{M} \varepsilon_j Q_{1_j}(s) \tag{12}$$

where $Q_{1_j}(s)$ is given above and

$$\bar{x}_j = s + \sum_{i=1}^{N} \bar{\varepsilon}_j \delta x_i x_{1_i}(s) \tag{13}$$

$$x = s + \sum_{i=1}^{N} \varepsilon_j \delta x_i x_{1_i}(s) \tag{14}$$

$$\bar{\varepsilon}_j = q_{c_j} - q_{o_j} \tag{15}$$

$$\varepsilon_j = q_j - q_{o_j} \tag{16}$$

$$\bar{\varepsilon}_j \delta x_i = (x_i^c - x_i^o)_j \tag{17}$$

$$\varepsilon_j \delta x_i = \frac{\varepsilon_j}{\bar{\varepsilon}_j}(x_i^c - x_i^o)_j \tag{18}$$

Here $\bar{\varepsilon}_j \delta x_i$ given in Equation (17) represents the displacement of the i^{th} invariant point in the j^{th} calibration solution from its base flow location due to the selected change $\bar{\varepsilon}_j$ in the q_j

parameter given by Equation (15), $\varepsilon_j \delta x_i$ given in Equation (18) represents the predicted displacement of the i^{th} invariant point from its base flow location due to the desired change ε_j in the q_j parameter given by Equation (16), and $x_{1_i}(s)$ is a unit-order straining function having the property that

$$x_{1_i}(x_k^o) = \begin{cases} 1 & k = i \\ 0 & k \neq i \end{cases} \qquad (19)$$

which assures alignment of the i^{th} invariant point between the base and calibration solutions.

In addition to the single condition Equation (19) on the straining function, it may be convenient or necessary to impose additional conditions at other locations along the contour. For example, it is usually necessary to hold invariant the end points along the contour, as well as to require that the straining vanish in a particular fashion in those locations. All of these conditions, however, do not serve to determine the straining uniquely. The nonuniqueness of the straining, nevertheless, can often be turned to advantage, either by selecting particularly simple classes of straining functions or by requiring the straining to satisfy further constraints convenient for a particular application.

The fact of nonuniqueness of straining function, however, raises a further question of the dependence of the final perturbation-predicted result on choice of straining function. An initial example of the effect of employing two different straining functions for a strongly supercritical flow was provided in reference 3, and in reference 1 a detailed examination was made of the dependence of perturbation results on several classes of different straining functions. Although it can be demonstrated (ref. 8) that the final perturbation-predicted result obtained when employing strained coordinates is formally independent of the particular straining function used--provided that the straining function moves the invariant points to the proper locations--the results of reference 1 demonstrate that, under certain conditions, particular classes of straining functions can induce spurious perturbation results. The underlying reason is that, while the perturbation-predicted results at and in the vicinity of invariant points are independent of the choice of straining function (provided invariant point locations are preserved), some classes of straining functions have the undesirable property of producing unwanted straining in certain regions removed from the invariant points. The correction for this deficiency, which was found in reference 1 and has proven effective in all case studies undertaken, is to employ linear piecewise-continuous straining functions. This both preserves the accuracy of the perturbation results in the vicinity of the invariant points, and introduces no excessive straining in regions removed from those locations.

For linear piecewise-continuous straining functions, the functional forms of the straining can be compactly written.

$$\bar{x}_j = s + \left\{ \frac{x^o_{i+1} - s}{x^o_{i+1} - x^o_i} \cdot (x^c_i - x^o_i)_j + \frac{s - x^o_i}{x^o_{i+1} - x^o_i} \cdot (x^c_{i+1} - x^o_{i+1})_j \right\} H(x^o_{i+1} - s) \cdot H(s - x^o_i) \tag{20}$$

where H denotes the Heaviside step function. As discussed above, it is usually necessary to hold invariant both of the end points along the contour in addition to the points corresponding to discontinuities or high-gradient maxima. Consequently, for the results reported here, the array of invariant points in the base and calibration solutions have been taken as

$$x^o_i = \{0, x^o_1, x^o_2, \ldots, x^o_n, 1\}$$
$$x^c_{i_j} = \{0, x^c_{1_j}, x^c_{2_j}, \ldots, x^c_{n_j}, 1\} \tag{21}$$

where the contour length has been normalized to unity and where n is the number of invariant points along the blade contour exclusive of the end points.

3. RESULTS

Because the ultimate utility of the perturbation methods being developed under this investigation is in engineering design or parametric analysis, the two primary objectives of the current study were to develop the simultaneous multiple-parameter capability of the method and then to examine the accuracy and range of validity of the multiple-parameter method in situations characteristic of that environment. Toward that end, we have tested the method in a series of problems involving simultaneous multiple-parameter variations of both flow and geometric quantities. As with the testing of the single-parameter method (ref. 1), emphasis was placed on transonic flows past both isolated blades and compressor cascades that are strongly supercritical and exhibit large surface shock movement over the parametric range studied. Additionally, we have coupled the multiple-parameter method with an optimization procedure to test the method's ability to perform in an actual design environment. These preliminary case studies of the combined perturbation/optimization method actually resulted in the most demanding tests of the perturbation method undertaken to date for observing its ability to work accurately under extreme interpolation/extrapolation conditions.

3.1 Simultaneous Multiple-Parameter Perturbations

In Figure 1, we present a comparison of results for the simultaneous perturbation of thickness ratio and oncoming Mach number of highly-supercritical flows past a series of isolated NACA four-digit (OOXX) blade profiles. The base flow chosen for these results is at $M_\infty = 0.820$ and $\tau = 0.120$, and is indicated on both plots shown in Figure 1 as the dashed line. Those results were obtained by solving the full-potential equation based on the finite-difference relaxation approximate-factorization method of reference 9. The body-fitted mesh employed had 75 points on both upper and lower surfaces. The calibration solution selected to account for Mach number changes is at $M_\infty = 0.800$ and $\tau = 0.120$, and is displayed as the dotted line in the plot on the left; while the calibration flow selected to account for thickness-ratio changes is at $M_\infty = 0.820$ and $\tau = 0.110$ and is displayed as the dotted line in the plot on the right. The open circles represent the perturbation-predicted solution for $M_\infty = 0.790$ and $\tau = 0.115$, which is a parameter extrapolation in M_∞ and interpolation in τ. Those results are meant to be compared with the 'exact' nonlinear results which is indicated as the solid line. We note that the indicated results for base, perturbation, and exact nonlinear solution in both plots in Figure 1 are the same; the reason for presenting two plots is to indicate clearly the separation between the base, the two calibration solutions, and the perturbation-predicted result. The straining employed is linear piecewise-continuous [see Eq. (20)], with leading and trailing edge and shock point held invariant. The shock point locations for the base and calibration flows for this example, as well as for all the multiple-parameter perturbation results presented here, were determined as the point where the pressure coefficient passed through critical with comprehensive gradient.

With regard to the results, we note that the comparison between the perturbation-predicted and the exact nonlinear result is, as in the case of single-parameter perturbation of these flows (see Figs. 4 to 6, ref. 1), extremely good, in particular in the region of the shock. The multiple-parameter perturbation result is able to accurately predict both shock location and the critical post-shock expansion behavior. Results for the region from the stagnation point to points just ahead of the shock are essentially identical to the exact nonlinear solution, as are results aft of the post-shock region. We note that the particular parameter values of $(M_\infty, \tau) = (0.790, 0.115)$ selected for the prediction solution represent reasonably substantial extrapolations and interpolations from the base and calibration values. Nevertheless, the perturbation method is able to treat simultaneous parameter variations over this range accurately.

Figure 2 presents analogous three-parameter perturbation results when angle-of-attack variations are included for the flows shown in Figure 1. Here, the base flow selected is at $\alpha = 0.2°$, $M_\infty = 0.800$, $\tau = 0.110$, and is indicated in all of the three plots provided as the dashed line. The calibration flow to account for angle-of-attack change is at $\alpha = 0.25°$ at the same (M_∞, τ) as the base flow, and is displayed as the dotted curve in the plot on the upper left. The corresponding calibration flow to account for Mach number change is at $M_\infty = 0.810$ at the same (τ, α) as the base flow, and is displayed in the upper right plot; while the calibration flow for thickness-ratio change is at $\tau = 0.115$ at the same (M_∞, α) as the base flow, and shown in the lower plot. The predicted result is for parameter values of $\alpha = 0.3°$, $M_\infty = 0.820$, $\tau = 0.100$ and again represents substantial extrapolations of all three parameters, as can be observed in Figure 2 from the relative differences between the base and calibration flows. The reason for selecting such small angles-of-attack for these flows was to preserve the shock wave on the lower surface, and thereby create a set of multiple-shock flows which were highly sensitive to parameter changes. The comparisons between perturbation and exact nonlinear results for this case is again extremely good, with the prediction of both the locations of the shocks on the upper and lower surface given very well, as well as the pressure distributions in the regions immediately ahead and behind those shocks. For these results, linear piecewise-continuous straining was employed with the invariant points being the lower surface trailing edge, lower surface shock, stagnation point, upper surface shock, and upper surface trailing edge.

The final multiple-parameter perturbation result is provided in Figure 3 for a four-parameter perturbation of strongly-supercritical full-potential flows past a cascade of blades having NACA four-digit profiles. The base flow is for an oncoming Mach number of $M_\infty = 0.780$, thickness-ratio $\tau = 0.110$, gap-to-chord ratio $t = 3.2$, and oncoming inflow angle $\alpha = 0.3°$, and is indicated on the four plots as the dashed line. Those results were obtained using the full-potential equation finite-area relaxation procedure of reference 10. The four calibration solutions to account for changes in the four varied parameters are provided in the four plots shown where the individual values of the calibration parameter varied are also indicated. Thus, the calibration solution for Mach number change is at $M_\infty = 0.790$ with (τ,t,α) at the base flow values, and is indicated as the dotted result in the plot at the upper left. Corresponding results for the other three parameters are shown in the remaining plots. The comparison of the predicted and exact nonlinear results are for parameter values of $M_\infty = 0.785$, $\tau = 0.115$, $t = 3.1$, $\alpha = 0.4°$. This particular set of flows was again selected because of the presence of multiple-shocks and high sensitivity to parameter change. As with the previous results for two- and three simultaneous parameter variations, we note that the perturbation predictions are once again remarkably accurate. The perturbation method is able both to track the location of the upper and lower surface shocks, as well as to predict the pressure characteristics in the pre-shock and post-shock regions.

3.2 Preliminary Application of Combined Multiple-Parameter Perturbation Method With Optimization Procedures For Blade Design Applications

The ultimate utility of the perturbation methods developed and evaluated here lies in their application to problems involving the high-frequency use of computational codes to determine a large number of related nonlinear flow solutions. In order to test the capability of the approximation method to work effectively in such practical applications, we have combined the method with the CONMIN optimization procedure (ref. 11) and have then made several preliminary case studies of the combination on isolated blade and compressor blade design/optimization problems. The objectives of these initial applications were to examine the feasibility and potential computational savings of the combined approximation/optimization procedure for some typical design problems, and to determine the accuracy of the perturbation-predicted optimization results.

The particular isolated blade design optimization problems selected for study involved the alteration of a baseline profile shape by adding to the baseline profile a set of shape functions according to the relation

$$Z(x) = Z_o(x) + \sum_{i=1}^{M} (DV_i - 1)\, F_i(x) \qquad (22)$$

where Z_o are the ordinates of the baseline profiles, F_i are the shape functions, and the coefficients DV_i are the design variables whose values are determined by the optimization program as a result of a search through design-variable solution space to achieve a desired design improvement. Here for convenience we have chosen the coefficients of F_i to be $(DV_i - 1)$ rather than DV_i. The general class of geometric shape functions employed here, and which have been found to be successful in previous applications involving optimization of supercritical airfoil sections (ref. 12), consists of exponential decay functions and sine functions. These are of the general form $(1 - x) \cdot x^p/e^{qx}$ and $\sin(\pi x^r)^n$, where the exponents p, q, r, and n are selected to provide a desired maximum at a particular chordwise location. The exponential functions are generally employed to provide adjustments near the leading edge, while the sine functions are used to provide maximum ordinate changes at particular chordwise stations. Illustrations of the chordwise variation of typical members of these shape functions are provided in Figure 4, and it can be seen that these functions smoothly concentrate ordinate thickness at selected locations.

For the initial application of the combined perturbation/optimization method, we have examined subcritical flow at $M_\infty = 0.10$ and $\alpha = 5°$ past a modified NACA 64A007 profile involving the nine profile shape functions

$$\begin{aligned} F_i &= 6(1 - x)x^{p_i}/e^{q_i x} & i &= 1,2 \\ F_i &= \sin(\pi x^{r_i})^2 & i &= 3,9 \end{aligned} \qquad (23)$$

where $(p_1,q_1) = (0.5,15)$, $(p_2,q_2) = (0.25,10)$, and $r_i = (0.37, 0.50, 0.66, 0.87, 1.16, 1.61, 2.41)$. The exponential functions achieve their maxima within 5% of chord, while those for the sine functions are at (15%, 25%, 35%, 45%, 55%, 65%, 75%) of chord.

A strategy which has proved convenient for performing optimization studies involving aerodynamic performance parameters (ref. 12) has been to recontour the profile shape so as to tailor the surface pressure distribution to conform to a desired distribution. This type of objective provides local control over the basic aerodynamic surface flow property of importance, and provides a means of attempting to achieve aft pressure gradients sufficiently weak to avoid separation. An important corollary advantage of using such an objective is that viscous separation can be minimized. This allows use of an inviscid aerodynamic flow solver in the optimization process rather than a much more computationally-expensive viscous solver, and assures that the optimization result thus obtained at the inviscid level is representative of the actual flow.

Consequently, for this initial case study the overall performance objective was, through modification of the surface contour, to tailor the pressure distribution along a portion of the upper surface so as to conform to a desired distribution. In particular, it was desired to minimize both the peaky behavior near the leading edge and the compressive gradient on the aft portion of the upper surface which existed at $M_\infty = 0.10$ and $\alpha = 5°$ on the NACA 64A007 baseline profile. This is illustrated schematically in Figure 5. The objective function was taken as the minimization of the mean squared error between the predicted and desired surface pressure distribution, i.e.,

$$OBJ = \sum_{k=1}^{K} [C_{P_{predicted}}(x_k) - C_{P_{desired}}(x_k)]^2 \qquad (24)$$

where K represents the number of chordwise locations x_k where desired and calculated surface pressures are compared.

Recall that in order to initiate the perturbation procedure in situations involving the simultaneous variation of M individual parameters from a baseline point, a matrix of M calibration solutions are required, each representing the solution change for a separate variation of each of the M parameters from its baseline value. Because optimum, or sometimes even typical, stepsizes for a particular optimization problem would not generally be known a priori, one of the primary goals of these initial studies was the demonstration that the perturbation method was capable of working effectively even under severe conditions imposed by a poorly-selected initial calibration solution matrix. This was accomplished by examining the sensitivity and accuracy of the perturbation predicted optimization results as a function of the initial design variable stepsizes of the calibration solution matrix.

The overall strategy, then, consisted of: (1) employing the perturbation method, based on some initial matrix of nonlinear aerodynamic solutions determined by an independent variation of each design variable, to provide all of the subsequent nonlinear aerodynamic solutions required by the optimization method for searches through design space, and (2) comparing the final perturbation-predicted optimization results for final design variable and objective function values with those determined by using the full nonlinear aerodynamic solver throughout. Figure 6 shows the results of such a sensitivity study, and indicates that even under extreme test case conditions caused by deliberately-selected poor choices of design variable stepsizes, the perturbation method performs exceptionally well, never breaking down or yielding spurious results. Indicated on the plots are the final optimized design variable values after five search cycles as predicted by the perturbation method (●) for four different choices of the initial stepsize for the design variables, i.e., δDV_i = (0.05, 0.02, 0.01, 0.001). Also shown are the corresponding final design variable values predicted when employing the nonlinear aerodynamic solver (ref. 13) throughout (○). As can be seen, for the extreme interpolation case $\delta DV_i = 0.05$, except for design variables 3 and 5, the perturbation prediction compares very favorably with the full nonlinear result. For $\delta DV_i = 0.02$, a more reasonable stepsize choice, the comparison of the perturbation result is quite good for all the design variables, while for $\delta DV_i = 0.01$, the perturbation prediction is essentially identical to the full nonlinear aerodynamic result. As a final illustration of the behavior of the perturbation method under extreme extrapolation conditions, the lower right-hand plot displays the perturbation predictions for $\delta DV_i = 0.001$. We note that for several of the design variables, the extrapolation range is of the order of 25 times the initial stepsize; yet, the perturbation predictions are quite reasonable and not spurious, which is remarkable and indicative of the robustness of the procedure.

Finally, we point out that <u>all</u> four of the perturbation-predicted results illustrated in Figure 6 are satisfactory in terms of the final objective function value obtained. These values are illustrated on the right of each of the plots. Provided for comparison are the initial (○ Initial) and final (○ Final) values obtained when using the nonlinear aerodynamic solver throughout. The value of the objective function evaluated at the final design variable point when using the perturbation method is indicated by the solid circle (●). However, the objective function value of real interest is the result indicated by the solid square (■) which represents that obtained by running the nonlinear aerodynamic solver at the perturbation-predicted final design point, and then using that solution to evaluate the objective function. This provides the overall ultimate check of the perturbation-predicted result. As can be seen from Figure 6, those results lie essentially on top of the final objective function result (○ Final) obtained when using the nonlinear aerodynamic solver throughout.

The computational savings attained for this application are shown in Figure 7. Here, a comparison of the computational work versus reduction in objective function per optimization cycle is provided when using the perturbation procedure (●) and when not using it but employing the nonlinear aerodynamic code (○) for each flow solution required by the optimizer. As can be seen, the computational time required for both the perturbation method and when using the full nonlinear aerodynamic solver throughout are the same for the first cycle, since both require a matrix of M + 1 (M=9 for this example) nonlinear aerodynamic solutions. After that, the perturbation-predicted results required essentially no computational time for cycles 2 through 5, and then a slight amount for the one additional call to the aerodynamic solver for the final check calculation (■). The reduction in the ratio of final to initial objective function is $OBJ/(OBJ)_i = 0.22$ and required approximately 20 CPU seconds on the CDC 7600. In comparison, the result when not employing the perturbation method required approximately 80 CPU seconds for the same reduction in objective function, indicating the perturbation method is able to save 75% of the computational work in this example.

Similar testing of the performance of the perturbation method for supercritical situations has also been carried out and has demonstrated a corresponding capability and potential computational savings. Because of the greater sensitivity of these shocked flows, two separate case studies were carried out in depth. Both of these involved recontouring of the upper surface of a NACA 0015 profile operating at the supercritical conditions of $M_\infty = 0.55$ and $\alpha = 6.7°$, and employed four design variables related to the shape functions.

$$F_i = \sin(\pi x^{q_i})^3 \quad i = 1,4 \tag{25}$$

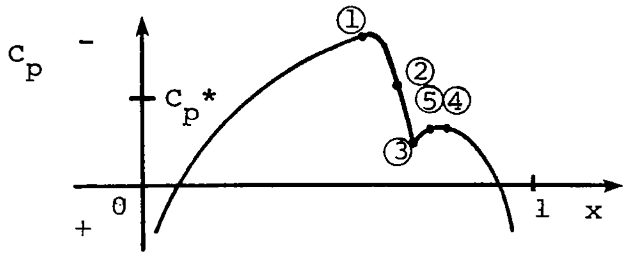
with $q_i = (0.301, 0.431, 0.576, 0.756)$. These functions have maxima at (10%, 20%, 30%, 40%) of chord. For the first of these supercritical studies, the objective function was chosen to be the drag coefficient squared, i.e.

$$OBJ = CD^2$$

For this problem, an evaluation of the accuracy of the perturbation-predicted optimization results as a function of initial calibration solution matrix was also made. The results of this study are provided in Figure 8, which displays the results of the perturbation-predicted final design variables (●) after eight search cycles for three different choices of the initial stepsize for the design variables, i.e., $\delta DV_i = (0.001, 0.002, 0.004)$. Also shown are the corresponding final design variable values predicted when not using the perturbation method but employing the nonlinear aerodynamic solver (ref. 13) throughout (○).

Because drag minimization, particularly at supercritical conditions, is an extremely sensitive optimization problem, this study provides one of the most severe tests of the perturbation procedure in a design optimization environment. This is so because the accurate prediction of drag at supercritical speeds depends almost entirely on the resolution of the flow behavior at and in the vicinity of the shock waves present on the surface of the profile. Hence, what is ultimately under evaluation in this example case study is the ability of the perturbation method to predict, under extreme extrapolation conditions and with simultaneous multiple-parameter perturbations, the location and strength of all surface shock waves and the flow behavior in the pre- and post-shock regions.

The most important results to emerge from the calculations involved in this case study were the discovery of a particular deficiency of the perturbation method in this regard, and the subsequent development of the means to improve the accuracy of the perturbation predictions in shock regions and other high gradient regions under extreme extrapolation conditions. The improvement in the basic procedure developed to meet these requirements consists of employing additional invariant points in those high gradient locations. For example, it was found that by characterizing a shock which has a post-shock expansion region, as sketched below,



with five invariant points--which correspond to: ① pre-shock minimum pressure, ② maximum gradient point, ③ post-shock maximum pressure, ④ post-shock minimum expansion pressure location, and ⑤ point of inflection between points ③ and ④ --rather than just the one point corresponding to the critical pressure location, which was standardly done in the past; that significantly improved perturbation results are obtained in the shock region for extreme solution extrapolations. This five invariant point characterization of the shock has been employed in determining the results of the two supercritical case studies reported here.

With regard to the drag minimization results indicated in Figure 8, we note that when selecting an initial design variable stepsize of $\delta DV_i = 0.001$, the agreement between the final perturbation-predicted design variable values and the exact nonlinear result is reasonable. The solution extrapolation indicated for design variables 2 and 3 are of the order of 5 times the initial design variable stepsize, but the perturbation method does not break down or provide spurious results for these strongly supercritical flows. We note that the optimization procedure working with the perturbation method is able to drive the perturbation-predicted drag (● P) to essentially zero. Although the final check of the perturbation-predicted design using the nonlinear solver (■ A) indicates a drag coefficient of 0.005, that represents nevertheless almost a factor of 3 in drag reduction from the baseline configuration.

When a somewhat more reasonable initial design variable stepsize of $\delta DV_i = 0.002$ is used, which reduces the solution extrapolations over that for $\delta DV_i = 0.001$, we note that the perturbation results closely approach those obtained when using the nonlinear aerodynamic solver throughout. The final drag value based on the perturbation design (■ A) is also improved, displaying over a factor of 5 in drag reduction. The final perturbation result obtained for an initial stepsize $\delta DV_i = 0.004$ and illustrated in the bottom plot displays almost exact agreement with the full nonlinear aerodynamic result. This particular choice of stepsize is the most reasonable of the three shown, since the design variable solution range to be searched by the optimizer requires moderate interpolation/ extrapolations from the calibration solution matrix. The final drag value (■ A) indicates over a factor of 5 drag reduction, and is very close to the final full nonlinear aerodynamic (⊙ Final) result.

The computational savings obtained for this application are indicated in Figure 9. Here a comparison of the computational work versus reduction in objective function per optimization search cycle is given for eight search cycles when using the approximation procedure (●) and for four cycles when using instead the full aerodynamic solver (○). We note for this example that when using the perturbation method, the optimization procedure requires approximately 40 CPU seconds for search cycles 2 through 8. This is in contrast to the essentially zero time needed for search cycles in the pressure distribution tailoring case study presented in Figure 7. The reason for the additional time in this minimization case study is that in order to evaluate the drag at the points in the design solution space required by the optimizer, additional geometry calculations are needed to determine the new surface slopes. Consequently, even though the perturbation procedure provides the means to determine the new pressure distributions at essentially no computational cost, because surface pressure integrations are required to evaluate the drag, calls to the geometry portion of the aerodynamic code are necessary to determine the new profile shapes. These geometry calculations cause the CPU time increase indicated. Finally, another 20 CPU seconds are needed for the final aerodynamic check solution (■), totaling 120 CPU seconds. That total represents just under a 60% computational savings of the 4 search cycle CPU time required when not employing the perturbation method.

The final supercritical case study presented in Figure 10 involved tailoring of the surface pressure distribution as done in the previous subcritical case, but this time specifically concentrating on alleviating the sharp gradient at the shock. It was specified in the following fashion. Using the pressure distribution from the optimized profile of the previous drag minimization case study, we returned to the original NACA 0015 baseline profile. Then, with that previous pressure distribution as the desired objective, by using the optimization method we attempted to reproduce the same optimized profile that resulted from the drag minimization problem. What resulted from these calculations was a demonstration that the perturbation method is capable in certain cases, of not only providing an enormous savings in computational cost but also an improved optimization result. In Figure 10, we provide the results of the sensitivity study of the perturbation method as a function of the initial design variable stepsize of the calibration solution matrix. In the plot on the left, the comparison is provided of the perturbation-predicted final design variables (●) based on an initial design variable stepsize $\delta DV_i = 0.001$ together with that obtained when using the full nonlinear aerodynamic solver throughout (△). Also shown is the 'optimum' full aerodynamic result (⊙) obtained from the drag minimization case study which is the result that is sought to be reproduced. What the results indicate is that the prediction obtained by not using the perturbation method is inferior to that obtained when employing it. Also, we note the large extrapolations involved in this perturbation result, i.e., about 4 times or 400% of the initial stepsize for design variables 1 and 2. These results are emphasized even more in the plot on the right which shows the corresponding perturbation results when using a more reasonable initial design variable stepsize of $\delta DV_i = 0.002$ to define the calibration solution matrix. In this instance, the perturbation prediction is essentially identical to the optimum result. We note that comparisons of the objective function reduction, shown on the scales to the right of the plots, display a reduction in objective function to almost zero for the perturbation result, while only an essentially 50% reduction for the full aerodynamic result.

This reduction in objective function is emphasized in Figure 11, which displays the comparison of computational work and objective function reduction per optimization cycle for the perturbation procedure and the full nonlinear aerodynamic result for this case study. Here, we see clearly that the perturbation method is able to drive the objective function to essentially zero, while the full aerodynamic procedure becomes fixed in a local minimum and is only able to reduce the objective function to 50% of its initial value. If we had carried the full aerodynamic result to eight optimization cycles, as was done with the perturbation result, the time savings would have been over an order of magnitude.

This result demonstrates that it is possible in certain instances for the perturbation method to provide not only an enormous savings in computational cost but also an improved optimization result. The latter is undoubtedly accomplished by the selection of a reasonable initial calibration solution matrix which permits the optimization method an enhanced rather than local view of the design solution space, thereby avoiding shallow local minimas in favor of a more global minima.

The final application, which has been carried out as far as possible in this phase, was directed toward laying the foundations of a practical turbomachinery blade design/optimization procedure coupled with the perturbation method. This preliminary version is based on TSONIC blade-to-blade solutions (ref. 14) with generalized circular-arc blade geometry routines BLADE (ref. 15) describing the blade profile, and employs the COPES/CONMIN optimization procedure (ref. 16).

The initial case study for the combined procedure involved, as a design objective, the minimization of the velocity diffusion on the blade suction surface, $q_{max,suction}$, i.e.

$$OBJ = \frac{q_{max,\ suction}}{q_{avg,\ exit}} \tag{27}$$

where $q_{avg,\ exit}$ is the average exit velocity in the freestream. Six design variables were employed and correspond to the following blade geometry parameters used to characterize NASA circular arc blade section profiles (ref. 15): blade outlet camber angle, transition location between fore and aft circular arc sections, maximum thickness location, inlet to outlet turning rate ratio, maximum thickness, and radius of the leading edge circle. During the optimization process, each of the design variables was constrained to remain within certain prescribed bounds in order to prevent a physically-unrealistic blade design from occurring. Furthermore, several active side constraints were additionally imposed both to insure design of a physically realistic blade and also to achieve certain desirable flow characteristics on the blade. The active side constraints employed were: (1) maintenance of nonzero local blade thickness, (2) maintenance of low velocity diffusion on the blade pressure surface, and (3) trailing edge closure. These constraints were enforced by employing bounding criteria on the following quantities according to:

$$0. < \frac{\text{thick}(x) - d_{tc}}{d_{t_c}} < 10. \qquad (28)$$

$$0. < \frac{q_{max,\ press}}{q_{min,\ press}} < 1.3 \qquad (29)$$

$$-1.0 < \frac{q_{ITE-2,suction} - q_{ITE-2,press}}{25.} < 1.0 \qquad (30)$$

Here, thick(x) is the local blade thickness, d_{tc} is the diameter of the trailing edge circle, $q_{max,press}$ is the maximum blade pressure surface velocity over the front half of the blade, $q_{min,press}$ is the minimum blade pressure surface velocity over the last two-thirds of the blade, and ($q_{ITE-2,suction}$, $q_{ITE-2,press}$) are the third last surface velocities on the grid near the trailing edge on the suction and pressure surface, respectively.

We have successfully completed a preliminary series of calculations of the new combined PERTURB/TSONIC/BLADE/COPES/CONMIN procedure in which the accuracy and sensitivity of the perturbation method was tested as a function of choice of the initial calibration solution matrix. The initial or base values of the design variables for the baseline blade profile, and the upper and lower bounds of the design variables that were specified for this test problem were:

Design Variable Number	Description	Lower Bound	Upper Bound	Initial Value
1	Outlet blade camber angle-KOCR	-15.0°	0.0°	-10.0°
2	Transition loc./chord-T	0.20	0.60	0.25
3	Max. thick. loc./chord-ZM	0.20	0.55	0.45
4	Inlet/outlet turning/chord-P	0.50	4.00	1.50
5	Max, thick./chord-TMX	0.03	0.10	0.05
6	L.E. rad./chord-THLE	0.003	0.012	0.005

The results of these calculations are summarized in Table 1. There we have provided comparisons of the final design variables and objective function predicted when employing full nonlinear TSONIC solutions throughout the optimization process with corresponding results when using the perturbation method. For the perturbation results, six different choices of the calibration solution matrix were made and are noted in the table. All the results represent converged solutions, with each calculation employing 10 optimization search cycles or less if no change in objective function should occur in three successive iterations.

Two separate results are presented for the case when the perturbation method is not employed and TSONIC solutions are used throughout the optimization search process. These were obtained on the Ames Research Center CDC-7600 and Lewis Research Center IBM 3033 employing the same TSONIC/BLADE/COPES/CONMIN code. The differences in the two final design results provides an indication of the sensitivity of this particular optimization problem with regard to choice of objective function, since the difference between these two results is due solely to the number of significant figures maintained in the respective calculations, i.e., 8 for the IBM 3033 and 14 for the CDC 7600. As is evident, there clearly is a sensitivity to search direction and final design result in this problem. We note that of the six design variables, the two which agree between the two results (ZM,TMX) have reached a limit boundary. The others have all trended in the same direction from the baseline value, but have reached somewhat different values at the final design point. We note that the objective functions have reached essentially the same level, indicating, as is often the case in such optimization problems, the existence of multiple local minimums.

The analogous results obtained employing the perturbation method with various choices of calibration solution matrix are exhibited as cases 1 to 6. We note that, with the exception of the design variable ZM which consistently moves to its upper bound regardless of the choice of calibration solution matrix, the final design variable predictions via the perturbation method basically exhibit a behavior quite similar to that obtained from the full nonlinear result when TSONIC solutions are used throughout. That is, the optimizer drives the design variables in generally the same direction from the baseline values as the full nonlinear result, but to somewhat different values. The exception to this is noted in cases 4 and 5, where the calibration solution matrices were selected such that TSONIC solution extrapolations rather than interpolations were involved for all or most of the design variables during the search procedure. In those two cases, we note further that the final objective function values obtained are somewhat inferior to those obtained in Cases 1, 2, 3, and 6, where solution interpolations were primarily involved. For those four cases, the final objective function result is almost identical to that obtained by the full nonlinear result.

The computational time needed to obtain the perturbation results in Cases 1 to 6 were 76-78 secs. of CDC 7600 CPU time per case. The benchmark full nonlinear CDC 7600 result shown in Table 1 required 644 secs. Thus, the perturbation method provides a savings of (644-78)/644 = 88% of the computational time for this example.

The primary conclusions to be drawn for this preliminary study are that the perturbation method can work effectively even for sensitively-defined optimization problems such as this and provide both meaningful final design results and large computational savings over not using the method. The choice of objective function such as was made for this case study, i.e., a point quantity located in a high-gradient region, requires that a reasonably good choice be made of the calibration solution matrix. This is so since if large solution extrapolations are required by the optimizer for minimization searches through design space, the final design result will most likely be less improved than would otherwise result if only modest solution interpolations/extrapolations were involved.

4. CONCLUSIONS AND RECOMMENDATIONS

An investigation was conducted to continue the development of perturbation procedures and associated computational codes for rapidly determining approximations to nonlinear flow solutions. The ultimate purpose is to establish a method for minimizing computational requirements associated with parametric design studies of transonic flows in turbomachines. The procedures being developed employ unit perturbations, determined from two or more nonlinear 'base' solutions which differ from one another by a nominal change in some geometry or flow parameter, to predict a family of related nonlinear solutions which can be either continuous or discontinuous. The results reported here relate to the extension of the previously-developed successful method for single-parameter perturbations: (1) to simultaneous multiple-parameter perturbations, and (2) to the preliminary application of the multiple-parameter procedure in combination with an optimization procedure and applied to blade design problems.

Calculations based on full-potential nonlinear solutions have been carried out to establish the accuracy and range of validity of the multiple-parameter capability. These involved flows past both isolated blades and compressor cascades involving simultaneous changes in both flow and geometric parameters, with attention focused on strongly supercritical situations involving large surface shock movements over the parameter ranges studied. Preliminary applications of the multiple-parameter perturbation method coupled with an optimization procedure were made for blade design problems in order to examine the capability of the method to produce accurate results in a typical design environment, and also to evaluate its potential for computational savings. Both subcritical and supercritical case studies were carried out involving multiple-design variables. Sensitivity studies were also performed to examine the accuracy dependence of the perturbation method on the choice of the initial calibration solution matrix.

Comparisons of the multiple-parameter perturbation results with the corresponding 'exact' nonlinear solutions display remarkable accuracy and indicate a large range of validity, in direct correspondence with previous results for single-parameter perturbations. The preliminary case studies of the multiple-parameter perturbation method combined with optimization procedures have clearly demonstrated the capability of the method to work accurately in a design environment where large solution extrapolations are often necessary, and have also established the methods' potential for reducing the computational work required in such applications by nearly an order of magnitude. The sensitivity studies indicate that for certain subcritical applications, the perturbation method is able to work quite accurately and effectively in spite of poor choices of the initial calibration solution matrix which require large extrapolations or interpolations. For supercritical flows, the initial calibration matrix choice is more important, but when employing multiple invariant point clustering about high-gradient locations (shocks, stagnation points, etc.), the perturbation method predictions can nevertheless maintain high accuracy in certain supercritical situations for extrapolations as large as 4 to 5 times the parameter separation between base and calibration solutions.

Based on these results, we conclude that perturbation methods formulated on these ideas are both accurate and clearly workable in design environments, and can provide the means for substantially reducing the computational work required in such applications. We suggest the development of the combined multiple-parameter perturbation procedure and optimization methods into a robust design tool for subcritical and supercritical turbomachinery blade design.

APPENDIX A

USER'S MANUAL FOR COMPUTER PROGRAM PERTURB

A.1 INTRODUCTION

The purpose of this appendix is to describe the operation of the computer code which was developed in conjunction with the theoretical work presented in this report, and to provide sufficient detail to permit convenient use and change of the program. The program computes and plots an arbitrary flow variable on a contour surface by employing the strained-coordinate perturbation method discussed in the main text.

A description of the general operating procedure of the program is given, together with complete description of both input and output. The program is written in FORTRAN IV and has been developed on the Ames Research Center CDC 7600 computer facility. Consequently, the plot package included in this version refers to system routines at that facility. In general, the plotting software must be supplied by the user according to the requirements of his operating system. This can be accomplished directly by replacing or modifying the subroutine DRVPLT. Typical program run times for flows involving approximately 150 surface mesh locations are 1 to 3 CPU seconds. The storage requirements are $105K_8$ for small core memory and no large core memory.

A.2 PROGRAM DESCRIPTION

The program calculates both continuous and discontinuous nonlinear perturbaton solutions which represent a multiple-parameter change in either geometry or flow conditions by employing a strained-coordinate procedure. The method utilizes unit perturbations, determined for each parameter from a previously-calculated common base solution and a calibration solution displaced from it by some reasonable change in the relevant parameter, to predict new nonlinear solutions over a range of parameter variation.

This current version of the procedure is configured to predict and plot an arbitrary flow variable (e.g., pressure coefficient) on the surface of a blade or airfoil, and can account for the motion of:

1. one or more critical points (shock points),

2. a stagnation point,

3. a maximum-suction-pressure point,

or simultaneously for any combination of these.

The program is also configured to compare the perturbation-predicted solutions with the corresponding 'exact' solutions obtained by employing the same 'expensive' computational procedure used to determine the base and calibration solutions.

The coordinate straining employed is piecewise linear with the end points and up to six interior points held invariant. At the option of the user, these additional interior points may be arbitrarily preselected, or chosen from among the minimum, maximum, and critical points automatically located by the program itself.

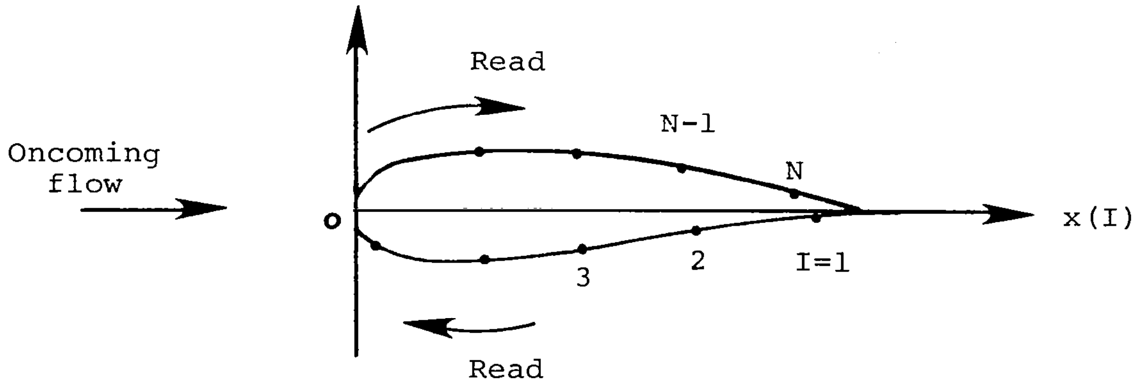
Critical or shock points are located on the basis of a user-supplied statement function defining the critical value of the dependent variable as a function of some single flow variable. The program default is with dependent variable y defined as pressure coefficient, with the independent variable being Mach number. In this case, the critical value is defined as

$$y_{crit} = C_p^* = \frac{2}{\gamma M_\infty^2} \left(\frac{2 + (\gamma - 1)M_\infty^2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma-1}} - 1 \qquad \text{(A-1)}$$

where γ is the ratio of specific heats. If instead of surface pressure coefficient, the surface velocity distribution were used, then the value of y_{crit} would be given by

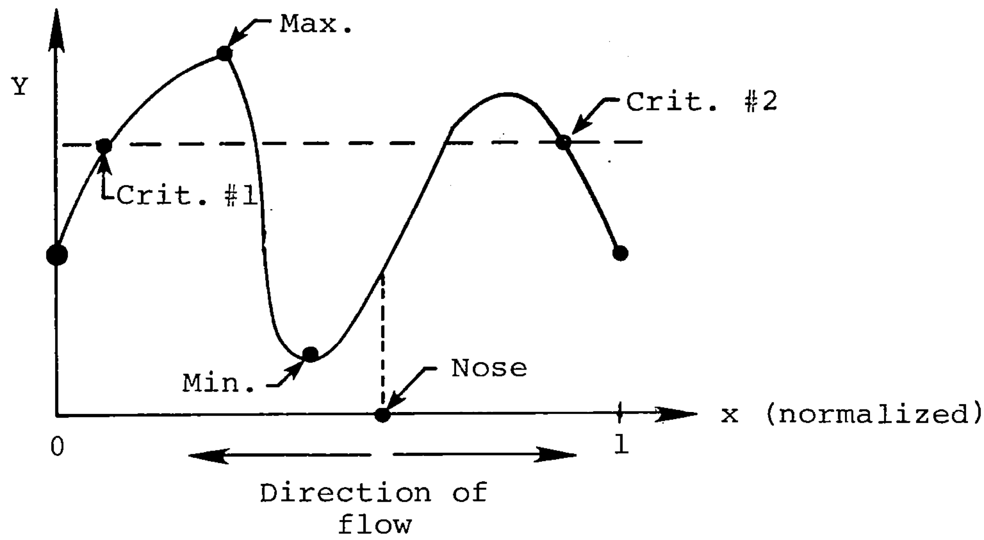
$$y_{crit} = \frac{V^*}{V_\infty} = \left(\frac{\gamma + 1}{2 + (\gamma - 1)M_\infty^2} \right)^{\frac{1}{\gamma-1}} \qquad \text{(A-2)}$$

Data for base, calibration, and comparison solutions (if available) are input as an array x(I) of coordinates a corresponding array y(I) giving the dependent variable at each coordinate location, where $1 \leq I \leq N$ and $N \leq 200$.



The leading edge is at x = 0; the data are read in beginning on the lower surface at the point farthest from the leading edge and proceeding clockwise around the surface as shown in the sketch. Data for the different solutions need not correspond to identical locations on the surface, except for the initial and final points, i.e., x(1) and x(N) must be the same for all cases. The program normalizes the x coordinates ($0 \leq x \leq 1$) such that x = 0 corresponds to I = 1 and x = 1 to I = $\bar{N}$.

The base and calibration solutions are searched for minimum, maximum, and critical points, e.g.,

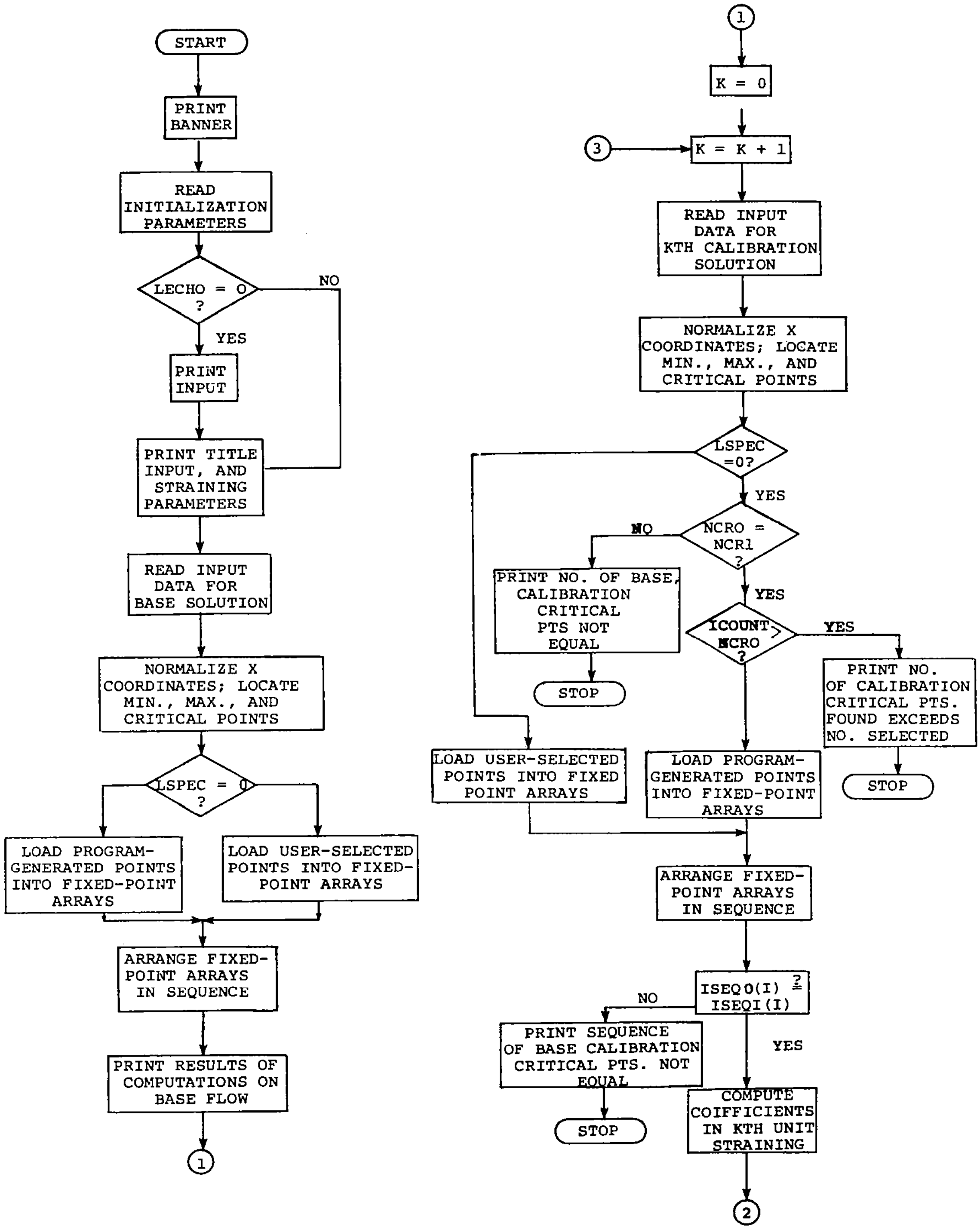


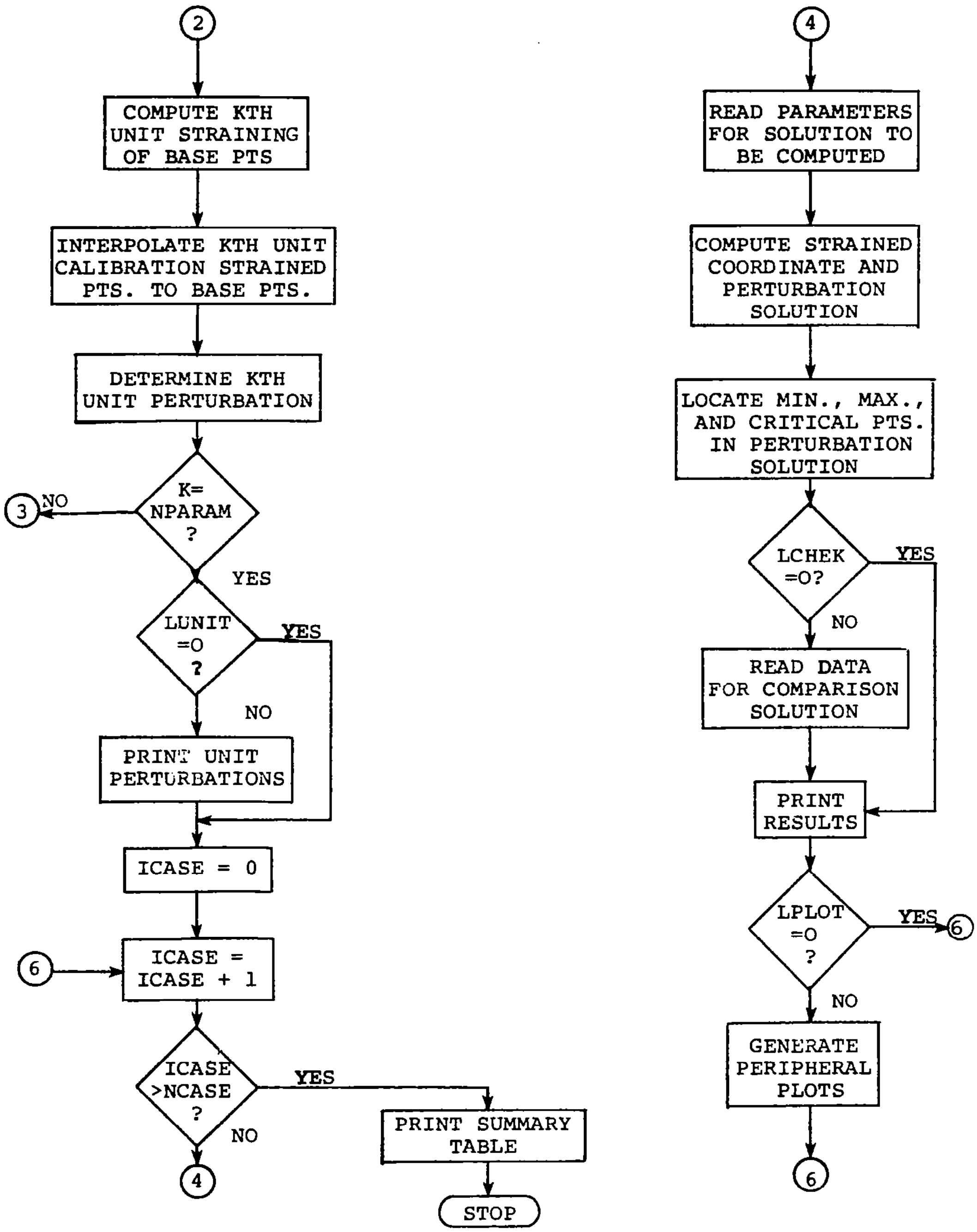
Note that the sign of dy/dx in physical coordinates is used in determining the critical points. For example, both critical points indicated on the above figure correspond to $dy/dx < 0$ in physical coordinates, since at point #1 the physical coordinate increases in the direction from right to left, whereas at point #2 it increases from left to right.

The points to be held invariant in straining are either selected from among those (1) located by the program or (2) individually specified by the user, after which the unit coordinate straining and unit perturbation are computed.

Data for the test cases is then read in and nonlinear perturbation solutions constructed from the unit perturbation.

A 3. PROGRAM FLOW CHART




2
COMPUTE KTH UNIT STRAINING OF BASE PTS
INTERPOLATE KTH UNIT CALIBRATION STRAINED PTS. TO BASE PTS.
DETERMINE KTH UNIT PERTURBATION
K= NPARAM ?
NO
3
YES
LUNIT =0 ?
YES
NO
PRINT UNIT PERTURBATIONS
ICASE = 0
6
ICASE = ICASE + 1
ICASE >NCASE ?
YES
NO
4
PRINT SUMMARY TABLE
STOP
4
READ PARAMETERS FOR SOLUTION TO BE COMPUTED
COMPUTE STRAINED COORDINATE AND PERTURBATION SOLUTION
LOCATE MIN., MAX., AND CRITICAL PTS. IN PERTURBATION SOLUTION
LCHEK =0?
YES
NO
READ DATA FOR COMPARISON SOLUTION
PRINT RESULTS
LPLOT =0 ?
YES
6
NO
GENERATE PERIPHERAL PLOTS
6

A.4 DICTIONARY OF INPUT VARIABLES

A	Scaling parameter in straining procedure. A = -x(1), where x(1) is location of first data point on lower surface (see PROGRAM DESCRIPTION).
B	Scaling parameter in straining procedure. B = x(N), where x(n) is location of last data point on upper surface (see PROGRAM DESCRIPTION).
LCHEK	Specifies whether or not perturbation solution is to be checked against an exact comparison solution. A printer plot is made in either case. LCHEK = 0 ... no comparison LCHEK = 1 ... comparison
LECHO	Controls whether or not input deck is printed. LECHO = 0 ... no print LECHO = 1 ... print
LOC0(I)	Array of length 6 of which NSELCT elements are read in. Specifies subscripts of those user-specified base flow points which are to be held invariant; operational only when LSPEC = 1.
LOC1(I)	Array of length 6 of which NSELCT elements are read in. Specifies subscripts of those user-specified points in the Kth calibration solution which are to be held invariant; operational only when LSPEC = 1.
LPLOT	Specifies whether or not an additional plot by a peripheral device is to be made. Software must be supplied by user in subroutine DRVPLT. LPLOT = 0 ... No peripheral plot LPLOT = 1 ... Peripheral plot
LSELCT(I)	Array of length 6 of which NSELCT elements are read in; operational only when LSPEC = 0, and specifies nature of points to be held invariant according to the code: 1 ... minimum point held invariant 2 ... maximum point held invariant 3 ... 1st critical point held invariant 4 ... 2nd critical point held invariant 5 ... 3rd critical point held invariant 6 ... 4th critical point held invariant Note that critical point ordering is determined from order of occurence starting at the lower surface at the point furthest from the leading edge and proceeding clock-wise around the surface (see PROGRAM DESCRIPTION).

Note that the code numbers can be assigned in any order, e.g.,

LSELCT(1) = 1		LSELCT(1) = 4
LSELCT(2) = 3	and	LSELCT(2) = 1
LSELCT(3) = 4		LSELCT(3) = 3

are equivalent, both corresponding to NSELCT = 3, with the minimum, and first and second critical points held invariant.

LSPEC — Controls how invariant points in straining are specified.

LSPEC = 0 ... invariant points selected from among those located by the program using the array LSELCT(1)

LSPEC = 1 ... invariant points preselected by user, using the arrays LOC0(I), LOC1(I)

LUNIT — Controls whether or not unit coordinate strainings and unit perturbation(s) are printed.

LUNIT = 0 ... no output
LUNIT = 1 ... output

M0,M1,M2 — Oncoming Mach numbers in base, calibration and computed perturbation solutions.

N — Number of locations for which data are input for base, calibration, and comparison solutions.

NCASE — Number of cases for which perturbation solutions are to be computed.

NPARAM — Number of parameters perturbed.

NSELCT — Number of points (in addition to end points) to be held invariant in straining; note: $1 \leq$ NSELCT ≤ 6.

PARNAM(K) — Array of 8-character strings which identify the parameters varied. NPARAM elements of the array are read in.

Q0(K) — Array of length 8 giving values of perturbation parameters in base solution. NPARAM elements of the array are read in.

Q1 — Value of Kth perturbation parameter in Kth calibration solution.

Q2(K) — Array of length 8 giving values of the perturbation parameters in the solution to be computed. NPARAM elements of the array are read in.

TITLE — Character string of length 80; identifies job and is printed as headline on first page of output. First nine characters are printed in upper-right-corner of banner page, and in upper-left corner of summary page.

VNAM — Character string of length 2 which symbolizes dependent variable, e.g., "CP" for pressure coefficient.

XBASE(I),XCALB(I),XCHEK(I)...

Arrays of surface coordinates in base, calibration, and comparison solutions.

YBASE(I),YCALB(I),YCHEK(I)...

Arrays of dependent variables in base, calibration, and comparison solutions.

A.5 PREPARATION OF INPUT DATA

A.5.1 Description of Input

Item 1 One card, identifies job-printed as headline on first page of output. First nine characters are printed in upper-right corner of banner page, and in upper-left corner of summary page.

Item 2 One card, containing the parameter, LECHO

Item 3 One card, containing the parameters N, NCASE, NPARAM, NSELCT, LSPEC, LUNIT, LCHEK, LPLOT.

Item 4 One card, containing the parameter array (LSELCT(I), I = 1, NSELCT). This item omitted if LSPEC = 1

Item 5 One card, containing the character string VNAM.

Item 6 One card, containing the character strings, PARNAM(K), K = 1, NPARAM

Item 7 One card, containing the scaling parameters A and B.

Item 8 One card, containing the parameter MO

Item 9 One card, containing the parameter array QO(K), K = 1, NPARAM

Item 10 One set of J cards, where J = 1 + INT(N/8), containing data for the base flow coordinates XBASE(I), I = 1, N

Item 11 One set of J cards, J as above, containing data for the dependent variable in the base solution, YBASE(I), I = 1, N

Item 12 One card, containing the parameter array LOC0(I), I = 1, NSELCT. This item omitted if LSPEC = 0

Item 13 One card, containing the parameters M1, Q1

Item 14 One set of J cards, J as above, containing data for the coordinates in the Kth calibration solution, XCAL(I), I = 1, N

Item 15 One set of J cards, J as above, containing data for the dependent variable in the Kth calibration solution, YCALB(I), I + 1, N

Item 16 One card, containing the parameter array LOC(I), I = 1, NSELCT. This item omitted if LSPEC = 0

Item 17 One card, containing the parameter MC

Item 18 One card, containing the parameter array Q2(k), K = 1 NPARAM

Item 19 One set of J cards, J as above, containing data for the coordinates in the comparison solution, XCHEK(I), I = 1, N. This item is required only when LCHEK = 1.

Item 20 One set of J cards, J as above, containing data for the dependent variables in the comparison solution, YCHEK(I), I = 1, N. This item is required only when LCHEK = 1.

NOTE: One set of items 13 through 16 is required for each of the NPARAM calibration solutions.

One set of items 17 through 20 is required for each of the NCASE solutions to be computed.

A.5.2 Format of Input Data

Item no. 1: 1 card (8A10)

Variable		Title						
Card column	10	20	30	40	50	60	70	80
Format Type		A						

Item no. 2: 1 card (I5)

Variable	LECHO	
Card column	5	
Format type	I	

Item no. 3: 1 card (16I5)

Variable	N	NCASE	NPARAM	NSELCT	LSPEC	LUNIT	LCHEK	LPLOT	
Card Column	5	10	15	20	25	30	35	40	
Format Type	I	I	I	I	I	I	I	I	

Item no. 4: (LSPEC=0): 1 card (16I5)

← LSELCT(NSELCT)

Variable	LSELCT(1)	LSELCT(2)	LSELCT(3)	------				
Card column	5	10	15	20	25	30	35	
Format Typr	I	I	I	I	I	I	I	

Item no. 5: 1 card (2A1)

Variable	VNAM	
Card column	2	
Format type	A	

Item no.6 1 card (10A8)

← PARNAM(NPARM)

Variable	PARNAM(1)	PARNAM(2)	PARNAM(3)	------				
Card column	8	16	24	32	40	48	56	
Format type	A	A	A	A	A	A	A	

Item no. 7. 1 card (8F10.6)

Variable	A	B	
Card column	10	20	
Format type	F	F	

Item no. 8. 1 card (8F10.6)

Varible	MO	
Card column	10	
Format type	F	

Item no. 9. 1 card (8F10.6) ↙Q0(NPARAM)

Variable	Q0(1)	Q0(2)	Q0(3)	-----				
Card column	10	20	30	40	50	60	70	
Format type	F	F	F	F	F	F	F	

Item no. 10. J cards, J=1+INT(N/8), 8 values per card (8F10.6)

Variable	XBASE(1)	XBASE(2)	XBASE(3	-----				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 11. J cards, J as above, 8 values per card (8F10.6)

Variable	YBASE(1)	YBASE(2)	Ybase(3	------				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 12. (LSPEC=1): 1 card (16I5) ↙LOC0(NSELCT)

Variable	LOC0(1)	LOC0(2)	LOC0(3	-----				
Card column	5	10	15	20	25	30	35	
Format type	I	I	I	I	I	I	I	

Item no. 13. 1 card (8F10.6)

Variable	M1	Q1	
Card column	10	20	
Format type	F	F	

Item no. 14. J cards, J as above, 8 values per card (F10.6)

Variable	XCALB(1)	XCALB(2)	XCALB(3)	-------				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 15. J cards, J as above, 8 values per card (8F10.6)

Variable	YCALB(1)	YCALB(2)	YCALB(3)	-------				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 16. (LSPEC=1): 1 card (16I5)

Variable	LOC1(1)	LOC1(2)	LOC1(3)	-------	← LOC1(NSELCT)			
Card column	5	10	15	20	25	30	35	
Format type	I	I	I	I	I	I	I	

Item no. 17. 1 card (8f10.6)

Variable	M2	
Card column	10	
Format type	F	

Item no. 18. 1 card (8F10.6)

Variable	Q2(1)	Q2(2)	Q2(3)	-------	← Q2(NPARAM)			
Card column	10	20	30	40	50	60	70	
Format type	F	F	F	F	F	F	F	

Item no. 19 (LCHEK=1): J cards , J=1+INT(N/8), 8 values per card (8F10.6)

Variable	XCHEK(1)	XCHEK(2)	XCHEK(3)	--------				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

Item no. 20. (LCHEK=1): J cards, J as above, 8 values per card (8F10.6)

Variable	YCHEK(1)	YCHEK(2)	YCHEK(3)	--------				
Card column	10	20	30	40	50	60	70	80
Format type	F	F	F	F	F	F	F	F

A.6 DESCRIPTION OF OUTPUT

The first output item consists of a banner page, and the card images of the input data, the latter only if LECHO = 1.

The second item is a page headed by the job title, listing;

1. the input parameters N, A, B and NPARAM relevant to the actual calculaton;

2. the straining option selected and the classifications of the straining points selected.

The third item is the results of the computations on the base solution. These include: the Mach number MO, values of the perturbation parameters of the base solution QO(k), K = 1, NPARAM, and the critical value of the dependent parameter, the locations of the minimum, maximum and critical points, and the locations of the invariant points. These results are then repeated for each of the NPARAM calibration solutions.

Results for unit straining of XBASE, and unit perturbations of the dependent variables are the fourth output item. This is done only if LUNIT = 1.

The fifth item (repeated for each case computed) summarizes the results of the perturbation calculation. The Mach number, the values of the perturbation paramters, and the critical values of the variable are printed first, followed by the locations of the minimum, maximum, and critical points in the perturbation solution and comparison solution (if any). Next, a legend is printed providing the maximum, minimum, and critical values of the dependent parameter and corresponding point symbols. Print symbols are also provided for the printer plot of the perturbation solutions P, comparison solution C (if any), and a common symbol ($) when there is agreement within printer plot accuracy between the two. Then follows a table listing XBASE, YBASE, XPERT (the strained coordinate), and YPERT (the computed value of the dependent variable). If LCHEK = 1, three additional columns list XCHEK (the computed solution) at the points given by XCHEK. This allows direct numerical comparison of YPERT with YCHEK, since the values of XPERT and XCHEK will not in general coincide. A printer plot is then provided of the perturbation result, together with the comparison solution (if any).

The final item is a table which summarizes the perturbation parameters for the base, calibration, and all predictive solutions.

A.7 ERROR MESSAGES

NUMBER OF CRITICAL POINTS IN
BASE AND CALIBRATION SOLUTIONS
ARE UNEQUAL - CALCULATION ENDED

This message will be printed if critical points are specified in straining (LSPEC = 0) and the number of critical points in base and calibration solutions are unequal. The remedy is to avoid use of critical points in straining, or to use base and calibration solutions having equal numbers of critical points.

NUMBER OF CRITICAL POINTS
SELECTED EXCEEDS NUMBER
ACTUALLY LOCATED - CALCULATION
ENDED

This message will be printed if more critical points are specified in straining (LSPEC = 0) than the number located by the program. The remedy is to specify a number of points less than or equal to the actual number.

ORDER OF SPECIFIED POINTS IN
BASE AND CALIBRATION SOLUTIONS
DOES NOT CORRESPOIND - CALCULATION ENDED

This message will be printed if the fixed points specified (LSPEC = 0) occur in a different sequence in the base and calibation solutions. The remedy is to use base and calibration solutions having the same qualitative features.

A.8 SAMPLE CASE

The sample case presented in this section provides some example results of perturbation calculations and comparisons with 'exact' nonlinear solutions for a multiple-shock flow for which partial results were provided in figure 3 of the main text. The calculation is for the simultaneous four-parameter $(M_\infty, \tau, t, \alpha)$ perturbation of strongly-supercritical full potential flows past a cascade of blades having NACA four-digit profiles. The base flow is for oncoming Mach number $M_\infty = 0.780$, thickness ratio $\tau = 0.110$, gap-to-chord ratio $t = 3.2$, and oncoming inflow angle α 0.3°. The calibration flows to account for perturbations in these parameters are at the following values of these parameters) (M_∞,τ,t,α) = (0.790,0.110,3.2,0.3), (0.780,0.120,3.2, 0.3), (0.780,0.110,3.0,0.3), (0.780,0.110,2.3,0.5). Perturbation results have been determined for 19 different solutions, which are summarized in the summary table after the print output. Results for several of those cases are presented here.

The input data is tabulated in figure A.1, with item numbers corresponding to those identified in Section A.5.1 and A.5.2. The first card, item 1, provides the title of the run. The next card, item 2, indicates that the input deck will not be printed (LECHO = 0). The next card, item 3, indicates that there are 191 points (N = 191) at which data will be input for the base, calibration, and comparison solutions; that there will be 19 cases (NCASE = 19) for which perturbation solutions are to be computed; that the number of parameters to be perturbed are 4 (NPARAM = 4); that there will be three invariant points (NSELCT = 3) in addition to the end points; that the invariant points will be located by the program (LSPEC = 0); that the information regarding the unit perturbation will be printed (LUNIT = 1); that there will be a comparison of the perturbation results with the exact solution (LCHEK = 1); and that there will be plots by a peripheral device of the output (LPLOT = 1). The next card, item 4, specifies that the three invariant points to be selected by the program are to be: the medium point (LSELECT(1) = 2), i.e., the stagnation point; the first critical point (LSELCT(2) = 3), i.e., the 1st shock point found when moving forward on the bottom surface from the leading edge; and the second critical point (LSELCT(3) = 4), i.e., the end shock point.

The next card, item 5, indicates that the dependent variable for print output will be symbolized by a 'CP' denoting the pressure coefficient. The next card, item 6, indicates that the parameters to be varied are "PARNAM(1) = MACH NO., PARNAM(L) = TAU, PARNAM(3) = PITCH, PARNAM(4) = ALPHA1. The next card, item 7, indicates that the coordinates of the data points to be read in will start at x = 1.0 on the lower surface and will end at x = 1.0 on the uppper surface, i.e., A = -1, B = 1. The next card, item 8, provides the oncoming Mach number of the base flow MO = 0.780. The next card, item 9, provides the base flow values of the parameters to be perturbed: Q0(1) = 0.780, Q0(2) = 0.110, Q0(3) = 3.2, Q0(4) = 0.30. The following 25 cards, item 10, provide the 191 base flow values of the surface coordinates XBASE(I), I = 1, N, while the next 5 cards, item 10, provide the 191 base flow values of the surface coordinates XBASE(I), I = 1, N, while the next 25 cards, item 11 provide the 191 base flow values of the dependent variable (pressure coefficient) YBASE(I), I = 1, N. The next card, item 13 provides the values of the oncoming Mach number M1 and the value of the 1st-perturbation parameter in the first calibration solution. Items 14 and 15, which correspond to the arrays of 191 points of coordinates XCALB(I), I = 1, N and dependent variable YCALB(I), I = 1, N, are provided as for the base flow in two sets of 25 cards each. Items 13, 14, and 15 are then repeated 3 more times corresponding to the total of NPARAM = 4 calibration solutions required. Items 17, 18, 19 and 20, of which there are 19 sets corresponding to the 19 cases to be studied, provide analogous information as items 8, 9, 10, and 11 of the base flow, but now refer to the 'exact' nonlinear results. Items 19 and 20, which correspond to the XCHEK(I), I = 1, N and YCHEK(I), I = 1, N arrays, respectively, have of course been previously compted at the indicated values of the perturbed parameters, and are included here for comparative purposes to enable assessment of the perturbation results.

Figure A.2 provides an abbreviated print output for sample case, while figure A.3 provides an abbreviated plot output of the results for the 19 cases, and display the base (----), calibration (····), perturbation (****), and 'exact" nonlinear (——) flow solutions. In these plots, the calibration solutions denoted "calibration no. 1 of 4", etc. correspond to those indicated in the summary table provided at the end of the print output for this case shown in Figure A.2.

```
TEST CASE - 4 PARAMETER PERTURBATION OF A SUPERCRITICAL CASCADE FLOW
    0
  191   19    4    3    0    1    1    1
    2    3    4
CP
MACH NO.   TAU    PITCH  ALPHA1
-1.0      1.0
    .780000
    .780000   .110000  3.200000   .300000
    .999495   .998713   .997404   .995569   .993212   .990334   .986938   .983028
    .978608   .973681   .968251   .962325   .955908   .949004   .941622   .933768
    .925451   .916680   .907466   .897818   .887747   .877264   .866380   .855106
    .843453   .831434   .819062   .806349   .793309   .779955   .766302   .752364
    .738155   .723692   .708989   .694061   .678926   .663598   .648095   .632431
    .616624   .600691   .584648   .568512   .552300   .536030   .519718   .503382
    .487040   .470708   .454404   .438147   .421952   .405838   .389822   .373922
    .358154   .342536   .327086   .311819   .296754   .281906   .267292   .252928
    .238830   .225014   .211494   .198286   .185405   .172863   .160676   .148856
    .137416   .126369   .115725   .105497   .095694   .086328   .077406   .068939
    .060934   .053397   .046336   .039757   .033665   .028066   .022965   .018360
    .014246   .010617   .007481   .004859   .002775   .001255   .000321   .000000
    .000306   .001225   .002732   .004802   .007411   .010536   .014156   .018261
    .022860   .027959   .033557   .039651   .046235   .053303   .060850   .068869
    .077353   .086293   .095682   .105510   .115766   .126440   .137520   .148995
    .160851   .173077   .185658   .198581   .211831   .225393   .239252   .253392
    .267798   .282454   .297342   .312446   .327750   .343235   .358886   .374684
    .390612   .406652   .422787   .438999   .455271   .471585   .487923   .504268
    .520603   .536909   .553170   .569369   .585489   .601512   .617422   .633202
    .648836   .664307   .679600   .694698   .709586   .724248   .738669   .752835
    .766729   .780338   .793648   .806645   .819315   .831645   .843623   .855237
    .866474   .877323   .887774   .897815   .907436   .916626   .925376   .933675
    .941515   .948886   .955781   .962194   .968118   .973549   .978480   .982907
    .986826   .990233   .993124   .995497   .997348   .998675   .999476
    .660763   .575142   .508550   .454292   .407733   .365898   .327433   .291440
    .257510   .225421   .195204   .167076   .141713   .120201   .103102   .089742
    .079017   .069975   .061487   .052188   .041329   .029199   .016319   .002961
   -.010696  -.024487  -.038230  -.051655  -.064464  -.076552  -.088017  -.098956
   -.109403  -.119377  -.128919  -.138069  -.146727  -.154608  -.161358  -.166584
   -.169807  -.170420  -.167580  -.160064  -.146078  -.122507  -.080024  -.403601
   -.858801  -.964078  -.972139  -.974261  -.974978  -.974565  -.973130  -.970687
   -.967228  -.962775  -.957354  -.950981  -.943667  -.935439  -.926325  -.916319
   -.905393  -.893513  -.880631  -.866686  -.851623  -.835382  -.817873  -.798979
   -.778626  -.756758  -.733216  -.707749  -.680122  -.650175  -.617610  -.581957
   -.542170  -.498376  -.452347  -.402844  -.346292  -.279184  -.198431  -.104356
    .000429   .118968   .264237   .447928   .664260   .885469  1.064121  1.149677
   1.119375   .987404   .799047   .601992   .426709   .282681   .162310   .055527
   -.040142  -.122747  -.192378  -.251825  -.303986  -.350902  -.394016  -.434147
   -.471923  -.508184  -.542111  -.572529  -.600062  -.625381  -.648825  -.670648
   -.690886  -.709565  -.726777  -.742591  -.757044  -.770186  -.782080  -.792762
   -.802242  -.810509  -.817508  -.823113  -.827167  -.829462  -.829670  -.827168
   -.820127  -.793036  -.563136  -.314794  -.291525  -.315153  -.323346  -.322847
   -.317122  -.308175  -.297215  -.285009  -.272067  -.258753  -.245291  -.231766
   -.218218  -.204700  -.191285  -.178085  -.165186  -.152520  -.139920  -.127253
   -.114431  -.101360  -.087898  -.073914  -.059507  -.044961  -.030523  -.016364
   -.002635   .010488   .022726   .033541   .042657   .050867   .059558   .069860
    .082752   .099296   .120145   .144679   .171777   .200771   .231320   .263287
    .296688   .331703   .368645   .408078   .450350   .494136   .527187
```

Figure A.1- Card input data for sample case

.790000	.790000						
.999495	.998713	.997404	.995569	.993212	.990334	.986938	.983028
.978608	.973681	.968251	.962325	.955908	.949004	.941622	.933768
.925451	.916680	.907466	.897818	.887747	.877264	.866380	.855106
.843453	.831434	.819062	.806349	.793309	.779955	.766302	.752364
.738155	.723692	.708989	.694061	.678926	.663598	.648095	.632431
.616624	.600691	.584648	.568512	.552300	.536030	.519718	.503382
.487040	.470708	.454404	.438147	.421952	.405838	.389822	.373922
.358154	.342536	.327086	.311819	.296754	.281906	.267292	.252928
.238830	.225014	.211494	.198286	.185405	.172863	.160676	.148856
.137416	.126369	.115725	.105497	.095694	.086328	.077406	.068939
.060934	.053397	.046336	.039757	.033665	.028066	.022965	.018360
.014246	.010617	.007481	.004859	.002775	.001255	.000321	.000000
.000306	.001225	.002732	.004802	.007411	.010536	.014156	.018261
.022860	.027959	.033557	.039651	.046235	.053303	.060850	.068869
.077353	.086293	.095682	.105510	.115766	.126440	.137520	.148995
.160851	.173077	.185658	.198581	.211831	.225393	.239252	.253392
.267798	.282454	.297342	.312446	.327750	.343235	.358886	.374684
.390612	.406652	.422787	.438999	.455271	.471585	.487923	.504268
.520603	.536909	.553170	.569369	.585489	.601512	.617422	.633202
.648836	.664307	.679600	.694698	.709586	.724248	.738669	.752835
.766729	.780338	.793648	.806645	.819315	.831645	.843623	.855237
.866474	.877323	.887774	.897815	.907436	.916626	.925376	.933675
.941515	.948886	.955781	.962194	.968118	.973549	.978480	.982907
.986826	.990233	.993124	.995497	.997348	.998675	.999476	
.625161	.636677	.547948	.490922	.441873	.397637	.357690	.320576
.285957	.253438	.222993	.194725	.169258	.147497	.130112	.117062
.106952	.098420	.090288	.081443	.071357	.060285	.048625	.036652
.024649	.012825	.001354	-.009465	-.019256	-.027841	-.035248	-.041464
-.046383	-.049859	-.051728	-.051769	-.049558	-.044424	-.035505	-.021635
-.000740	.033790	-.272131	-.797329	-.989708	-.996302	-.996581	-.996271
-.995378	-.993891	-.991799	-.989117	-.985865	-.982053	-.977659	-.972611
-.966842	-.960329	-.953063	-.945030	-.936219	-.926638	-.916295	-.905173
-.893227	-.880406	-.866638	-.851835	-.835915	-.818807	-.800429	-.780706
-.759623	-.737168	-.713171	-.687274	-.659135	-.628635	-.595586	-.559557
-.519680	-.475851	-.429425	-.379572	-.322975	-.255985	-.175553	-.082014
.022051	.139662	.283583	.465236	.678740	.896547	1.071722	1.154343
1.123192	.991649	.804492	.608914	.435017	.292151	.172715	.066673
-.028452	-.110695	-.180112	-.239458	-.291613	-.338613	-.381922	-.422426
-.461504	-.499119	-.533047	-.563342	-.591381	-.617542	-.642001	-.664798
-.686010	-.705736	-.724100	-.741189	-.757073	-.771837	-.785573	-.798348
-.810203	-.821169	-.831244	-.840398	-.848603	-.855854	-.862145	-.867460
-.871795	-.875184	-.877640	-.879096	-.879394	-.878209	-.869530	-.714721
-.329401	-.117701	-.151619	-.170257	-.180343	-.184574	-.184622	-.181605
-.176313	-.169347	-.161181	-.152229	-.142803	-.133002	-.122787	-.112126
-.101001	-.089373	-.077144	-.064229	-.050755	-.037019	-.023296	-.009799
.003304	.015860	.027572	.037833	.046295	.053816	.062008	.072070
.084631	.100396	.120360	.144252	.170602	.198812	.228312	.258942
.290546	.323158	.357105	.391082	.425405	.458203	.296419	
.780000	.120000						
.999496	.998714	.997407	.995575	.993220	.990346	.986955	.983051
.978636	.973715	.968293	.962374	.955963	.949067	.941691	.933844
.925533	.916769	.907562	.897920	.887856	.877380	.866502	.855233
.843587	.831573	.819206	.806497	.793460	.780109	.766457	.752520
.738311	.723846	.709141	.694210	.679070	.663736	.648225	.632552
.616735	.600791	.584735	.568584	.552357	.536070	.519740	.503384
.487021	.470667	.454340	.438058	.421838	.405697	.389653	.373724
.357927	.342279	.326797	.311499	.296401	.281521	.266874	.252478
.238347	.224498	.210945	.197705	.184792	.172219	.160001	.148150
.136681	.125605	.114935	.104681	.094854	.085465	.076523	.068038
.060016	.052468	.045400	.038819	.032731	.027141	.022055	.017479
.013421	.009888	.006887	.004422	.002497	.001117	.000283	.000000
.000268	.001087	.002453	.004364	.006817	.009807	.013330	.017381
.021952	.027035	.032624	.038714	.045300	.052375	.059934	.067969
.076471	.085433	.094844	.104695	.114977	.125678	.136786	.148290
.160176	.172433	.185045	.198000	.211282	.224876	.238768	[illegible]

Figure A.1- Continued

.267379	.282067	.296987	.312123	.327458	.342974	.358655	.374482
.390439	.406506	.422668	.438906	.455202	.471539	.487900	.504266
.520620	.536945	.553223	.569438	.585571	.601607	.617529	.633319
.648962	.664441	.679740	.694843	.709735	.724400	.738823	.752989
.766883	.780491	.793798	.806792	.819458	.831783	.843756	.855364
.866596	.877439	.887883	.897918	.907532	.916716	.925460	.933752
.941585	.948949	.955837	.962243	.968160	.973584	.978508	.982930
.986844	.990246	.993133	.995503	.997351	.998676	.999477	
.778187	.643254	.577572	.514959	.464661	.418858	.377585	.339040
.303090	.269002	.237151	.207670	.181000	.158080	.139916	.126059
.114999	.105725	.097068	.087718	.076980	.065137	.052698	.039960
.027159	.014504	.002216	-.009382	-.019920	-.029209	-.037262	-.044064
-.049508	-.053437	-.055667	-.055951	-.053831	-.048579	-.039253	-.024589
-.002415	.033819	-.241204	-.799151	-1.053651	-1.063071	-1.063441	-1.063136
-1.062176	-1.060557	-1.058273	-1.055341	-1.051783	-1.047609	-1.042793	-1.037260
-1.030937	-1.023798	-1.015828	-1.007010	-.997326	-.986775	-.975357	-.963040
-.949770	-.935492	-.920148	-.903678	-.886027	-.867129	-.846875	-.825114
-.801701	-.776498	-.749336	-.720000	-.688338	-.654386	-.617998	-.578270
-.533439	-.483303	-.429287	-.370502	-.304838	-.230921	-.146480	-.047241
.071713	.213735	.379255	.564986	.760419	.944831	1.086243	1.151976
1.129546	1.026020	.869853	.693094	.518423	.358587	.218632	.099338
-.001751	-.088718	-.165230	-.233282	-.294060	-.348440	-.397256	-.441646
-.483708	-.525064	-.563578	-.597755	-.628929	-.657892	-.684937	-.710125
-.733499	-.755206	-.775412	-.794224	-.811711	-.827947	-.843022	-.857015
-.869984	-.881972	-.892984	-.902988	-.911961	-.919897	-.926793	-.932637
-.937429	-.941207	-.943993	-.945726	-.946259	-.945310	-.938896	-.838800
-.411611	-.108785	-.148390	-.169889	-.181857	-.187388	-.188309	-.185828
-.180806	-.173899	-.165628	-.156444	-.146687	-.136477	-.125784	-.114585
-.102868	-.090601	-.077684	-.064024	-.049764	-.035231	-.020714	-.006418
.007479	.020791	.033204	.044119	.053225	.061398	.070178	.080759
.094133	.111067	.132206	.157348	.185172	.214778	.245656	.277888
.311239	.345517	.381103	.417216	.454758	.477775	.504653	
.780000	3.000000						
.999494	.998709	.997395	.995553	.993187	.990299	.986891	.982968
.978533	.973590	.968145	.962201	.955765	.948843	.941441	.933567
.925230	.916440	.907206	.897540	.887452	.876952	.866053	.854765
.843100	.831071	.818690	.805971	.792926	.779570	.765916	.751980
.737776	.723320	.708626	.693710	.678589	.663277	.647790	.632146
.616361	.600450	.584431	.568320	.552134	.535890	.519605	.503297
.486981	.470677	.454399	.438167	.421997	.405907	.389913	.374033
.358285	.342685	.327250	.311997	.296944	.282106	.267500	.253142
.239049	.225235	.211716	.198506	.185622	.173076	.160884	.149057
.137609	.126552	.115899	.105660	.095846	.086468	.077535	.069056
.061039	.053491	.046419	.039828	.033726	.028117	.023006	.018393
.014272	.010637	.007494	.004868	.002780	.001257	.000322	.000000
.000307	.001228	.002738	.004812	.007427	.010559	.014186	.018300
.022909	.028019	.033629	.039735	.046333	.053415	.060977	.069011
.077511	.086467	.095872	.105716	.115987	.126676	.137771	.149260
.161129	.173367	.185958	.198890	.212147	.225715	.239578	.253720
.268126	.282779	.297663	.312760	.328055	.343530	.359167	.374950
.390860	.406882	.422996	.439186	.455434	.471723	.488035	.504353
.520660	.536939	.553173	.569344	.585437	.601433	.617317	.633073
.648683	.664132	.679404	.694484	.709355	.724002	.738410	.752565
.766451	.780054	.793360	.806354	.819024	.831357	.843339	.854959
.866204	.877063	.887524	.897578	.907212	.916417	.925182	.933497
.941352	.948740	.955651	.962079	.968019	.973464	.978409	.982849
.986781	.990199	.993100	.995481	.997339	.998671	.999475	
.672452	.583881	.515448	.460229	.412948	.370572	.331613	.295165
.260771	.228218	.197536	.168979	.143232	.121423	.104205	.090833
.080141	.071169	.062745	.053481	.042646	.030556	.017735	.004462
-.009075	-.022705	-.036237	-.049388	-.061851	-.073514	-.084468	-.094789
-.104490	-.113567	-.122034	-.129895	-.136994	-.142975	-.147404	-.149788
-.149528	-.145860	-.137715	-.123451	-.100101	-.058454	-.403774	-.877544
-.979011	-.985637	-.987080	-.987308	-.986532	-.984870	-.982374	-.979017
-.974765	-.969621	-.963601	-.956710	-.948951	-.940342	-.930900	-.920613
-.909442	-.897339	-.884239	-.870059	-.854726	-.838176	-.820335	-.801124

Figure A.1- Continued

-.780518	-.758491	-.734847	-.709222	-.681345	-.651131	-.618334	-.582483
-.542556	-.498600	-.452362	-.402696	-.346011	-.278795	-.197933	-.103707
.001256	.119930	.265249	.448975	.665397	.886708	1.065269	1.150244
1.118913	.985640	.796132	.598308	.422638	.278441	.157926	.050969
-.044856	-.127566	-.197281	-.256824	-.309101	-.356156	-.399439	-.439783
-.477973	-.514751	-.548982	-.579556	-.607126	-.632579	-.656470	-.678856
-.699603	-.718693	-.736277	-.752499	-.767449	-.781197	-.793805	-.805309
-.815729	-.825080	-.833338	-.840434	-.846289	-.850815	-.853885	-.855303
-.854769	-.851647	-.840175	-.740515	-.426762	-.230610	-.267195	-.284058
-.290361	-.289899	-.284964	-.277072	-.267259	-.256245	-.244501	-.232272
-.219707	-.206939	-.194098	-.181343	-.168799	-.156420	-.144046	-.131552
-.118857	-.105875	-.092470	-.078517	-.064113	-.049551	-.035087	-.020898
-.007143	.005997	.018236	.029023	.038051	.046107	.054618	.064759
.077547	.094072	.114948	.139593	.166841	.195931	.226507	.258379
.291544	.326079	.362254	.400337	.440466	.479680	.499458	
.780000	.500000						
.999495	.998713	.997404	.995569	.993212	.990334	.986938	.983028
.978608	.973681	.968251	.962325	.955908	.949004	.941622	.933768
.925451	.916680	.907466	.897818	.887747	.877264	.866380	.855106
.843453	.831434	.819062	.806349	.793309	.779955	.766302	.752364
.738155	.723692	.708989	.694061	.678926	.663598	.648095	.632431
.616624	.600691	.584648	.568512	.552300	.536030	.519718	.503382
.487040	.470708	.454404	.438147	.421952	.405838	.389822	.373922
.358154	.342536	.327086	.311819	.296754	.281906	.267292	.252928
.238830	.225014	.211494	.198286	.185405	.172863	.160676	.148856
.137416	.126369	.115725	.105497	.095694	.086328	.077406	.068939
.060934	.053397	.046336	.039757	.033665	.028066	.022965	.018360
.014246	.010617	.007481	.004859	.002775	.001255	.000321	.000000
.000306	.001225	.002732	.004802	.007411	.010536	.014156	.018261
.022860	.027959	.033557	.039651	.046235	.053303	.060850	.068869
.077353	.086293	.095682	.105510	.115766	.126440	.137520	.148995
.160851	.173077	.185658	.198581	.211831	.225393	.239252	.253392
.267798	.282454	.297342	.312446	.327750	.343235	.358886	.374684
.390612	.406652	.422787	.438999	.455271	.471585	.487923	.504268
.520603	.536909	.553170	.569369	.585489	.601512	.617422	.633202
.648836	.664307	.679600	.694698	.709586	.724248	.738669	.752835
.766729	.780338	.793648	.806645	.819315	.831645	.843623	.855237
.866474	.877323	.887774	.897815	.907436	.916626	.925376	.933675
.941515	.948886	.955781	.962194	.968118	.973549	.978480	.982907
.986826	.990233	.993124	.995497	.997348	.998675	.999476	
.659353	.573792	.504775	.450760	.404477	.362946	.324633	.288795
.254899	.222842	.192768	.164747	.139580	.118098	.100840	.087427
.076676	.067604	.059072	.049710	.038771	.026541	.013537	.000031
-.013795	-.027783	-.041754	-.055442	-.068551	-.080977	-.092829	-.104214
-.115172	-.125737	-.135965	-.145916	-.155508	-.164478	-.172495	-.179185
-.184087	-.186606	-.185943	-.181005	-.170520	-.153313	-.128825	-.153113
-.577864	-.947484	-.949703	-.953889	-.955753	-.956074	-.955191	-.953169
-.950034	-.945830	-.940601	-.934372	-.927163	-.919007	-.909936	-.899950
-.889027	-.877137	-.864232	-.850244	-.835100	-.818717	-.800990	-.781818
-.761181	-.739091	-.715433	-.689911	-.662161	-.631824	-.598602	-.561885
-.521171	-.477603	-.432014	-.382322	-.325426	-.258011	-.177039	-.082772
.022283	.141093	.286239	.468878	.682778	.899860	1.072641	1.151190
1.113717	.975519	.782607	.582718	.406031	.261570	.141214	.034529
-.061031	-.143446	-.212778	-.271860	-.323642	-.370203	-.413026	-.452999
-.491391	-.528263	-.561422	-.590832	-.617893	-.643000	-.666471	-.688328
-.708478	-.726964	-.743941	-.759534	-.773825	-.786882	-.798764	-.809499
-.819092	-.827532	-.834774	-.840720	-.845258	-.848250	-.849481	-.848582
-.844747	-.832277	-.740646	-.437898	-.244913	-.282473	-.299178	-.304774
-.303452	-.297680	-.289019	-.278493	-.266799	-.254426	-.241684	-.228717
-.215607	-.202439	-.189306	-.176332	-.163616	-.151102	-.138627	-.126062
-.113324	-.100323	-.086919	-.072983	-.058612	-.044096	-.029682	-.015537
-.001814	.011308	.023549	.034378	.043518	.051758	.060481	.070823
.083734	.100208	.121124	.145915	.173205	.202435	.233136	.265239
.298921	.334230	.371701	.411706	.454665	.499081	.520980	
.775000							
.775000	.120000	3.000000	.500000				

Figure A.1- Continued

.999495	.998710	.997398	.995559	.993196	.990312	.986909	.982991
.978562	.973626	.968187	.962250	.955822	.948907	.941512	.933645
.925316	.916533	.907306	.897646	.887565	.877072	.866179	.854898
.843239	.831216	.818840	.806125	.793084	.779730	.766078	.752143
.737939	.723481	.708785	.693865	.678738	.663420	.647926	.632273
.616477	.600554	.584522	.568396	.552194	.535932	.519628	.503299
.486962	.470634	.454333	.438075	.421879	.405761	.389739	.373830
.358051	.342420	.326954	.311669	.296583	.281712	.267074	.252683
.238556	.224709	.211157	.197916	.185000	.172423	.160199	.148342
.136865	.125781	.115101	.104836	.094999	.085599	.076646	.068149
.060117	.052557	.045478	.038887	.032789	.027190	.022095	.017511
.013445	.009906	.006899	.004430	.002502	.001118	.000284	.000000
.000269	.001089	.002458	.004374	.006832	.009828	.013359	.017419
.021999	.027093	.032693	.038795	.045394	.052484	.060057	.068106
.076624	.085600	.095027	.104894	.115191	.125906	.137029	.148546
.160445	.172713	.185336	.198299	.211589	.225189	.239084	.253259
.267698	.282383	.297299	.312429	.327756	.343262	.358930	.374742
.390682	.406731	.422873	.439089	.455363	.471675	.488011	.504351
.520678	.536976	.553227	.569415	.585522	.601532	.617428	.633195
.648814	.664271	.679550	.694635	.709510	.724160	.738571	.752726
.766611	.780213	.793516	.806507	.819173	.831501	.843478	.855092
.866331	.877183	.887638	.897684	.907312	.916510	.925268	.933576
.941425	.948805	.955709	.962130	.968062	.973500	.978438	.982873
.986798	.990212	.993109	.995486	.997342	.998672	.999475	
.715525	.623070	.555465	.497218	.446870	.402235	.361863	.323841
.287908	.253822	.221641	.191749	.164876	.141936	.123510	.109349
.098232	.088874	.079998	.070272	.059061	.046658	.033509	.019908
.006148	-.007573	-.021107	-.034149	-.046297	-.057396	-.067547	-.076796
-.085068	-.092284	-.098398	-.103313	-.106713	-.108068	-.106738	-.101951
-.092684	-.077448	-.053362	-.010610	-.414414	-.948107	-1.047638	-1.052271
-1.052736	-1.052271	-1.050924	-1.048767	-1.045861	-1.042243	-1.037906	-1.032784
-1.026809	-1.019962	-1.012236	-1.003620	-.994096	-.983664	-.972325	-.960053
-.946805	-.932539	-.917210	-.900761	-.883128	-.864224	-.843912	-.822035
-.798479	-.773174	-.745954	-.716478	-.684354	-.649585	-.612367	-.571706
-.526192	-.476550	-.423026	-.363878	-.297691	-.223152	-.138029	-.038117
.081444	.223954	.389741	.575325	.769896	.952342	1.090345	1.151326
1.122932	1.013279	.851812	.670959	.493378	.331601	.190446	.070490
-.030901	-.117982	-.194510	-.262520	-.323211	-.377470	-.426154	-.470535
-.513169	-.554351	-.591894	-.625803	-.656994	-.685685	-.712217	-.736927
-.759926	-.781278	-.801110	-.819559	-.836732	-.852715	-.867573	-.881344
-.894055	-.905734	-.916389	-.926002	-.934560	-.942070	-.948530	-.953924
-.958244	-.961520	-.963754	-.964844	-.964537	-.961222	-.933540	-.610460
-.220865	-.151626	-.179805	-.195832	-.204080	-.206760	-.205402	-.201066
-.194513	-.186325	-.176975	-.166883	-.156364	-.145501	-.134240	-.122544
-.110388	-.097718	-.084426	-.070424	-.055847	-.041005	-.026183	-.011593
.002584	.016160	.028834	.040016	.049361	.057697	.066612	.077375
.090807	.107946	.129781	.155607	.184021	.214235	.246018	.279141
.313600	.349544	.386286	.424195	.463841	.502082	.514365	
.780000							
.780000	.115000	3.000000	.500000				
.999495	.998709	.997396	.995556	.993191	.990305	.986900	.982979
.978547	.973608	.968165	.962225	.955793	.948874	.941476	.933606
.925272	.916486	.907255	.897592	.887507	.877011	.866115	.854830
.843169	.831142	.818764	.806047	.793004	.779649	.765996	.752061
.737857	.723399	.708704	.693787	.678663	.663347	.647858	.632209
.616418	.600502	.584476	.568357	.552164	.535911	.519617	.503298
.486972	.470656	.454367	.438122	.421939	.405835	.389827	.373933
.358170	.342554	.327104	.311836	.296766	.281912	.267290	.252916
.238806	.224976	.211441	.198216	.185316	.172755	.160547	.148705
.137243	.126173	.115506	.105255	.095429	.086040	.077097	.068609
.060584	.053030	.045954	.039362	.033261	.027656	.022552	.017951
.013855	.010265	.007190	.004642	.002636	.001185	.000302	.000000
.000287	.001156	.002593	.004586	.007122	.010187	.013769	.017859
.022455	.027558	.033165	.039270	.045869	.052955	.060523	.068565
.077074	.086041	.095456	.105311	.115596	.126298	.137406	.148909
.160793	.173045	.185652	.198599	.211872	.225456	.239335	.253493

Figure A.1- Continued

.267915	.282584	.297484	.312597	.327907	.343397	.359050	.374847
.390772	.406807	.422935	.439138	.455399	.471699	.488023	.504352
.520669	.536958	.553200	.569379	.585479	.601482	.617372	.633133
.648748	.664201	.679476	.694558	.709431	.724080	.738490	.752644
.766530	.780132	.793437	.806430	.819098	.831428	.843408	.855024
.866266	.877122	.887580	.897630	.907262	.916463	.925225	.933536
.941388	.948772	.955679	.962104	.968040	.973481	.978423	.982861
.986789	.990205	.993104	.995484	.997340	.998671	.999475	
.700827	.610849	.540760	.484017	.435373	.391834	.351826	.314435
.279189	.245855	.214434	.185181	.158834	.136418	.118636	.104978
.094201	.085145	.076572	.067166	.056292	.044244	.031479	.018283
.004928	-.008395	-.021526	-.034166	-.045938	-.056694	-.066524	-.075469
-.083468	-.090448	-.096361	-.101110	-.104400	-.105718	-.104443	-.099832
-.090914	-.076274	-.053187	-.012386	-.401423	-.916541	-1.014031	-1.018527
-1.018993	-1.018565	-1.017292	-1.015242	-1.012474	-1.009027	-1.004893	-1.000006
-.994302	-.987764	-.980388	-.972166	-.963089	-.953160	-.942382	-.930729
-.918150	-.904595	-.890006	-.874326	-.857502	-.839474	-.820137	-.799347
-.776990	-.752989	-.727225	-.699469	-.669458	-.636926	-.601581	-.562768
-.519590	-.472765	-.422953	-.368337	-.306643	-.235628	-.152827	-.056288
.055611	.186640	.343244	.528224	.732213	.930643	1.084243	1.152652
1.119711	.996502	.819924	.629427	.451350	.297174	.166094	.053131
-.044882	-.129432	-.202386	-.266035	-.322394	-.372901	-.418740	-.461181
-.502119	-.541223	-.576673	-.608757	-.638152	-.665193	-.690397	-.713987
-.735954	-.756319	-.775226	-.792826	-.809216	-.824468	-.838636	-.851758
-.863864	-.874989	-.885143	-.894310	-.902473	-.909634	-.915786	-.920910
-.924997	-.928074	-.930140	-.931090	-.930691	-.928032	-.912610	-.546648
-.167509	-.159802	-.184662	-.198446	-.204887	-.206189	-.203851	-.198866
-.191925	-.183550	-.174160	-.164133	-.153755	-.143094	-.132081	-.120668
-.108824	-.096494	-.083570	-.069961	-.055793	-.041369	-.026965	-.012789
.000983	.014170	.026477	.037322	.046365	.054416	.063019	.073413
.086493	.103247	.124439	.149521	.177204	.206747	.237781	.270073
.303583	.338338	.374562	.412278	.451357	.486687	.496474	
.780000							
.780000	.120000	3.100000	.500000				
.999495	.998712	.997403	.995568	.993210	.990331	.986935	.983025
.978604	.973676	.968247	.962320	.955901	.948997	.941613	.933757
.925438	.916666	.907450	.897800	.887728	.877245	.866360	.855086
.843434	.831416	.819045	.806333	.793294	.779942	.766290	.752353
.738146	.723684	.708983	.694057	.678922	.663595	.648091	.632427
.616619	.600684	.584638	.568499	.552282	.536006	.519688	.503344
.486992	.470649	.454333	.438061	.421851	.405720	.389686	.373766
.357976	.342336	.326860	.311568	.296476	.281599	.266956	.252562
.238433	.224584	.211032	.197792	.184877	.172302	.160082	.148229
.136756	.125677	.115002	.104744	.094913	.085520	.076573	.068083
.060057	.052504	.045431	.038846	.032754	.027161	.022071	.017492
.013431	.009895	.006892	.004425	.002499	.001117	.000283	.000000
.000268	.001088	.002455	.004368	.006823	.009816	.013342	.017396
.021971	.027059	.032652	.038748	.045339	.052420	.059986	.068027
.076535	.085503	.094921	.104779	.115067	.125775	.136889	.148399
.160291	.172553	.185170	.198129	.211414	.225011	.238905	.253079
.267518	.282205	.297124	.312258	.327589	.343101	.358777	.374598
.390547	.406607	.422760	.438989	.455275	.471602	.487952	.504306
.520649	.536962	.553229	.569432	.585554	.601578	.617489	.633269
.648901	.664370	.679661	.694756	.709640	.724298	.738716	.752877
.766767	.780372	.793678	.806670	.819336	.831662	.843637	.855247
.866482	.877329	.887778	.897817	.907437	.916627	.925377	.933676
.941516	.948887	.955782	.962194	.968118	.973547	.978478	.982905
.986824	.990231	.993123	.995496	.997347	.998674	.999476	
.735231	.642357	.570206	.511220	.461138	.416331	.375339	.337064
.301120	.267169	.235260	.205633	.178999	.156436	.138661	.124649
.113249	.103878	.095443	.086371	.075751	.063919	.051533	.038912
.026213	.013632	.001444	-.010009	-.020386	-.029509	-.037369	-.043946
-.049145	-.052819	-.054787	-.054816	-.052471	-.047048	-.037620	-.022892
-.000402	.038267	-.348156	-.908725	-1.052134	-1.056972	-1.056912	-1.056213
-1.054885	-1.052919	-1.050303	-1.047052	-1.043186	-1.038714	-1.033609	-1.027791
-1.021187	-1.013767	-1.005518	-.996421	-.986453	-.975611	-.963891	-.951264

Figure A.1- Continued

-.937684	-.923106	-.907481	-.890752	-.872857	-.853707	-.833170	-.811084
-.787336	-.761852	-.734481	-.704914	-.672806	-.638053	-.600780	-.560151
-.514699	-.464950	-.411349	-.352299	-.286280	-.211972	-.127151	-.027632
.091396	.233180	.398011	.582396	.775553	.956501	1.093184	1.153325
1.125185	1.016414	.856124	.676477	.499970	.339082	.198623	.079197
-.021793	-.108556	-.184838	-.252662	-.313226	-.367414	-.416085	-.460625
-.503433	-.544571	-.582139	-.616233	-.647452	-.676123	-.702802	-.727756
-.751024	-.772652	-.792785	-.811574	-.829121	-.845497	-.860760	-.874953
-.888118	-.900297	-.911512	-.921755	-.931022	-.939328	-.946686	-.953101
-.958592	-.963227	-.967063	-.970094	-.972266	-.973487	-.973624	-.972436
-.953783	-.544528	-.120123	-.106248	-.130620	-.145161	-.153053	-.156106
-.155544	-.152266	-.146967	-.140231	-.132502	-.123987	-.114720	-.104720
-.094018	-.082617	-.070443	-.057413	-.043684	-.029611	-.015490	-.001511
.012137	.025217	.037417	.048192	.057280	.065462	.074030	.084126
.097121	.114220	.135718	.161004	.188854	.218509	.249622	.281961
.315461	.350089	.386020	.423017	.460380	.491526	.485025	
.780000							
.780000	.120000	3.000000	.400000				
.999495	.998710	.997398	.995559	.993196	.990312	.986909	.982991
.978562	.973626	.968187	.962250	.955822	.948907	.941512	.933645
.925316	.916533	.907306	.897646	.887565	.877072	.866179	.854898
.843239	.831216	.818840	.806125	.793084	.779730	.766078	.752143
.737939	.723481	.708785	.693865	.678738	.663420	.647926	.632273
.616477	.600554	.584522	.568396	.552194	.535932	.519628	.503299
.486962	.470634	.454333	.438075	.421879	.405761	.389739	.373830
.358051	.342420	.326954	.311669	.296583	.281712	.267074	.252683
.238556	.224709	.211157	.197916	.185000	.172423	.160199	.148342
.136865	.125781	.115101	.104836	.094999	.085599	.076646	.068149
.060117	.052557	.045478	.038887	.032789	.027190	.022095	.017511
.013445	.009906	.006899	.004430	.002502	.001118	.000284	.000000
.000269	.001089	.002458	.004374	.006832	.009828	.013359	.017419
.021999	.027093	.032693	.038795	.045394	.052484	.060057	.068106
.076624	.085600	.095027	.104894	.115191	.125906	.137029	.148546
.160445	.172713	.185336	.198299	.211589	.225189	.239084	.253259
.267698	.282383	.297299	.312429	.327756	.343262	.358930	.374742
.390682	.406731	.422873	.439089	.455363	.471675	.488011	.504351
.520678	.536976	.553227	.569415	.585522	.601532	.617428	.633195
.648814	.664271	.679550	.694635	.709510	.724160	.738571	.752726
.766611	.780213	.793516	.806507	.819173	.831501	.843478	.855092
.866331	.877183	.887638	.897684	.907312	.916510	.925268	.933576
.941425	.948805	.955709	.962130	.968062	.973500	.978438	.982873
.986798	.990212	.993109	.995486	.997342	.998672	.999475	
.657962	.680876	.580512	.523832	.474775	.427622	.385038	.346834
.310731	.276403	.244097	.214502	.187869	.165436	.147429	.133402
.122470	.113427	.105147	.096227	.085906	.074510	.062615	.050533
.038477	.026663	.015354	.004891	-.004359	-.012203	-.018621	-.023569
-.026928	-.028520	-.028128	-.025473	-.020052	-.011023	.002904	.024320
.062792	-.426025	-1.008072	-1.072871	-1.074088	-1.073538	-1.072448	-1.070836
-1.068721	-1.066065	-1.062837	-1.059039	-1.054683	-1.049770	-1.044269	-1.038097
-1.031174	-1.023471	-1.014973	-1.005662	-.995523	-.984554	-.972758	-.960102
-.946526	-.931959	-.916320	-.899530	-.881528	-.862259	-.841641	-.819551
-.795852	-.770387	-.742952	-.713336	-.681410	-.647145	-.610153	-.569662
-.524361	-.474318	-.420449	-.361446	-.295521	-.221326	-.136624	-.037193
.081839	.223787	.389020	.574157	.768549	.951307	1.090309	1.152922
1.127314	1.020678	.861946	.683339	.507472	.346936	.206627	.087206
-.013881	-.100803	-.177265	-.245274	-.306020	-.360382	-.409210	-.453743
-.496591	-.538075	-.575946	-.610206	-.641741	-.670787	-.697743	-.722927
-.746416	-.768272	-.788649	-.807699	-.825520	-.842175	-.857721	-.872205
-.885676	-.898183	-.909755	-.920387	-.930081	-.938854	-.946721	-.953690
-.959788	-.965091	-.969672	-.973542	-.976669	-.979003	-.980481	-.981048
-.979813	-.978397	-.529998	-.074422	-.090091	-.112678	-.126688	-.134556
-.137854	-.137704	-.134967	-.130348	-.124385	-.117346	-.109320	-.100377
-.090579	-.079945	-.068426	-.055972	-.042749	-.029103	-.015345	-.001706
.011614	.024411	.036365	.046863	.055580	.063350	.071654	.081662
.094415	.111091	.132345	.157364	.184883	.214492	.245418	.277320
.309953	.344221	.380022	.413320	.445721	.482948	.303599	

Figure A.1- Continued

.785000							
.785000	.110000	3.000000	.500000				
.999494	.998709	.997395	.995553	.993187	.990299	.986891	.982968
.978533	.973590	.968145	.962201	.955765	.948843	.941441	.933567
.925230	.916440	.907206	.897540	.887452	.876952	.866053	.854765
.843100	.831071	.818690	.805971	.792926	.779570	.765916	.751980
.737776	.723320	.708626	.693710	.678589	.663277	.647790	.632146
.616361	.600450	.584431	.568320	.552134	.535890	.519605	.503297
.486981	.470677	.454399	.438167	.421997	.405907	.389913	.374033
.358285	.342685	.327250	.311997	.296944	.282106	.267500	.253142
.239049	.225235	.211716	.198506	.185622	.173076	.160884	.149057
.137609	.126552	.115899	.105660	.095846	.086468	.077535	.069056
.061039	.053491	.046419	.039828	.033726	.028117	.023006	.018393
.014272	.010637	.007494	.004868	.002780	.001257	.000322	.000000
.000307	.001228	.002738	.004812	.007427	.010559	.014186	.018300
.022909	.028019	.033629	.039735	.046333	.053415	.060977	.069011
.077511	.086467	.095872	.105716	.115987	.126676	.137771	.149260
.161129	.173367	.185958	.198890	.212147	.225715	.239578	.253720
.268126	.282779	.297663	.312760	.328055	.343530	.359167	.374950
.390860	.406882	.422996	.439186	.455434	.471723	.488035	.504353
.520660	.536939	.553173	.569344	.585437	.601433	.617317	.633073
.648683	.664132	.679404	.694484	.709355	.724002	.738410	.752565
.766451	.780054	.793360	.806354	.819024	.831357	.843339	.854959
.866204	.877063	.887524	.897578	.907212	.916417	.925182	.933497
.941352	.948740	.955651	.962079	.968019	.973464	.978409	.982849
.986781	.990199	.993100	.995481	.997339	.998671	.999475	
.683828	.595218	.526126	.470329	.422695	.380080	.340960	.304398
.269961	.237390	.206713	.178160	.152432	.130566	.113322	.100126
.089705	.080955	.072679	.063584	.053035	.041328	.028930	.016120
.003144	-.009813	-.022577	-.034855	-.046298	-.056766	-.066335	-.075043
-.082845	-.089673	-.095473	-.100152	-.103431	-.104811	-.103685	-.099331
-.090820	-.076823	-.054844	-.016651	-.360073	-.854366	-.978642	-.984269
-.984822	-.984499	-.983355	-.981458	-.978867	-.975622	-.971718	-.967090
-.961679	-.955467	-.948456	-.940638	-.932009	-.922576	-.912349	-.901307
-.889400	-.876574	-.862760	-.847885	-.831892	-.814733	-.796334	-.776594
-.755441	-.732807	-.708512	-.682308	-.653998	-.623451	-.590278	-.553437
-.512248	-.468045	-.422015	-.372126	-.315111	-.247667	-.166760	-.072622
.032227	.150679	.295200	.476922	.689669	.905447	1.076827	1.153862
1.115557	.976821	.783784	.584150	.407933	.263961	.143962	.037494
-.057937	-.140271	-.209585	-.268725	-.320637	-.367399	-.410534	-.451206
-.490571	-.527676	-.561058	-.591267	-.618886	-.644389	-.668338	-.690824
-.711751	-.731133	-.749131	-.765890	-.781497	-.796012	-.809487	-.821960
-.833462	-.844029	-.853673	-.862376	-.870120	-.876903	-.882713	-.887525
-.891330	-.894146	-.895968	-.896680	-.896021	-.893007	-.868801	-.476915
-.130375	-.166810	-.187183	-.198348	-.203196	-.203557	-.200695	-.195457
-.188443	-.180120	-.170869	-.161039	-.150898	-.140509	-.129798	-.118707
-.107204	-.095235	-.082696	-.069493	-.055743	-.041744	-.027766	-.014010
-.000647	.012149	.024088	.034593	.043328	.051094	.059398	.069446
.082146	.098434	.119017	.143439	.170452	.199300	.229601	.261146
.293883	.327854	.363281	.400223	.438413	.473365	.480538	
.785000							
.785000	.115000	3.100000	.500000				
.999495	.998712	.997401	.995565	.993205	.990325	.986926	.983013
.978589	.973658	.968225	.962295	.955873	.948965	.941577	.933718
.925396	.916620	.907400	.897747	.887672	.877185	.866297	.855020
.843365	.831344	.818971	.806257	.793216	.779862	.766210	.752273
.738066	.723605	.708904	.693980	.678848	.663524	.648024	.632365
.616562	.600633	.584593	.568461	.552253	.535985	.519676	.503342
.487001	.470670	.454366	.438107	.421910	.405793	.389773	.373867
.358093	.342468	.327009	.311733	.296656	.281797	.267170	.252793
.238680	.224849	.211313	.198089	.185191	.172632	.160427	.148590
.137132	.126067	.115406	.105160	.095341	.085959	.077023	.068541
.060523	.052975	.045906	.039321	.033226	.027626	.022527	.017931
.013840	.010254	.007182	.004637	.002633	.001183	.000302	.000000
.000287	.001154	.002589	.004580	.007113	.010175	.013752	.017836
.022427	.027524	.033124	.039222	.045813	.052891	.060451	.068484

Figure A.1- Continued

.076984	.085942	.095348	.105195	.115470	.126164	.137264	.148759
.160637	.172883	.185484	.198426	.211695	.225276	.239153	.253311
.267733	.282403	.297306	.312423	.327739	.343235	.358895	.374701
.390636	.406682	.422822	.439037	.455311	.471626	.487964	.504308
.520640	.536944	.553202	.569396	.585511	.601529	.617434	.633208
.648836	.664301	.679588	.694681	.709563	.724220	.738636	.752797
.766688	.780293	.793600	.806594	.819262	.831591	.843568	.855181
.866419	.877269	.887721	.897764	.907388	.916581	.925334	.933637
.941480	.948854	.955753	.962169	.968096	.973529	.978463	.982893
.986815	.990224	.993118	.995493	.997345	.998674	.999476	
.708054	.628684	.555502	.497959	.448751	.404658	.364472	.326954
.291788	.258595	.227420	.198410	.172322	.150032	.132320	.118942
.108542	.099799	.091488	.082425	.072058	.060655	.048632	.036274
.023885	.011678	-.000174	-.011359	-.021480	-.030350	-.038005	-.044433
-.049518	-.053106	-.055039	-.055094	-.052834	-.047569	-.038404	-.024088
-.002259	.035163	-.345250	-.886977	-1.019274	-1.023550	-1.023440	-1.022733
-1.021432	-1.019524	-1.016994	-1.013857	-1.010133	-1.005832	-1.000927	-.995343
-.989007	-.981894	-.973992	-.965286	-.955761	-.945417	-.934255	-.922244
-.909331	-.895461	-.880575	-.864613	-.847523	-.829247	-.809680	-.788684
-.766142	-.741969	-.716034	-.688119	-.658002	-.625513	-.590209	-.551404
-.508252	-.461289	-.411374	-.356826	-.295257	-.224430	-.141900	-.045738
.065662	.196054	.351813	.535659	.738207	.935032	1.087172	1.154647
1.121995	.999766	.824454	.635189	.458138	.304762	.174308	.061846
-.035783	-.120048	-.192797	-.256299	-.312559	-.363011	-.408846	-.451467
-.492619	-.531731	-.567246	-.599429	-.628809	-.655920	-.681323	-.705146
-.727349	-.747981	-.767204	-.785161	-.801936	-.817586	-.832165	-.845717
-.858288	-.869924	-.880649	-.890454	-.899333	-.907299	-.914359	-.920516
-.925789	-.930241	-.933929	-.936844	-.938932	-.940104	-.940188	-.938894
-.934260	-.568149	-.134333	-.090919	-.115978	-.132691	-.142776	-.147634
-.148468	-.146268	-.141822	-.135787	-.128658	-.120671	-.111886	-.102351
-.092102	-.081139	-.069397	-.056823	-.043565	-.029941	-.016244	-.002707
.010486	.023167	.035029	.045455	.054087	.061782	.070159	.080451
.093374	.109714	.130386	.154983	.182132	.211150	.241598	.273262
.306012	.339921	.374955	.411358	.448035	.480512	.465808	
.785000							
.785000	.115000	3.000000	.400000				
.999495	.998709	.997396	.995556	.993191	.990305	.986900	.982979
.978547	.973608	.968165	.962225	.955793	.948874	.941476	.933606
.925272	.916486	.907255	.897592	.887507	.877011	.866115	.854830
.843169	.831142	.818764	.806047	.793004	.779649	.765996	.752061
.737857	.723399	.708704	.693787	.678663	.663347	.647858	.632209
.616418	.600502	.584476	.568357	.552164	.535911	.519617	.503298
.486972	.470656	.454367	.438122	.421939	.405835	.389827	.373933
.358170	.342554	.327104	.311836	.296766	.281912	.267290	.252916
.238806	.224976	.211441	.198216	.185316	.172755	.160547	.148705
.137243	.126173	.115506	.105255	.095429	.086040	.077097	.068609
.060584	.053030	.045954	.039362	.033261	.027656	.022552	.017951
.013855	.010265	.007190	.004642	.002636	.001185	.000302	.000000
.000287	.001156	.002593	.004586	.007122	.010187	.013769	.017859
.022455	.027558	.033165	.039270	.045869	.052955	.060523	.068565
.077074	.086041	.095456	.105311	.115596	.126298	.137406	.148909
.160793	.173045	.185652	.198599	.211872	.225456	.239335	.253493
.267915	.282584	.297484	.312597	.327907	.343397	.359050	.374847
.390772	.406807	.422935	.439138	.455399	.471699	.488023	.504352
.520669	.536958	.553200	.569379	.585479	.601482	.617372	.633133
.648748	.664201	.679476	.694558	.709431	.724080	.738490	.752644
.766530	.780132	.793437	.806430	.819098	.831428	.843408	.855024
.866266	.877122	.887580	.897630	.907262	.916463	.925225	.933536
.941388	.948772	.955679	.962104	.968040	.973481	.978423	.982861
.986789	.990205	.993104	.995484	.997340	.998671	.999475	
.662479	.679673	.569903	.511788	.460035	.416656	.376548	.337910
.301426	.268082	.237119	.208344	.182030	.159929	.142475	.128894
.118301	.109572	.101570	.092927	.082915	.071857	.060311	.048580
.036878	.025417	.014445	.004296	-.004672	-.012277	-.018507	-.023326
-.026622	-.028233	-.027969	-.025577	-.020594	-.012238	.000676	.020562
.056775	-.442134	-1.002884	-1.040202	-1.040447	-1.039858	-1.038750	-1.037151

Figure A.1- Continued

-1.035076	-1.032488	-1.029356	-1.025681	-1.021475	-1.016743	-1.011454	-1.005527
-.998884	-.991494	-.983347	-.974426	-.964716	-.954221	-.942944	-.930861
-.917924	-.904073	-.889231	-.873309	-.856225	-.837905	-.818266	-.797222
-.774708	-.750628	-.724749	-.696757	-.666477	-.633943	-.599074	-.561018
-.518083	-.470680	-.420476	-.366002	-.304548	-.233842	-.151417	-.055333
.056027	.186457	.342455	.526932	.730690	.929410	1.084056	1.154236
1.124336	1.004381	.830687	.642463	.466004	.312894	.182496	.069979
-.027756	-.112164	-.185085	-.248778	-.305237	-.355883	-.401888	-.444489
-.485647	-.525075	-.560875	-.593327	-.623090	-.650513	-.676152	-.700201
-.722639	-.743512	-.762983	-.781196	-.798231	-.814147	-.829001	-.842844
-.855724	-.867694	-.878781	-.888978	-.898283	-.906711	-.914273	-.920976
-.926847	-.931961	-.936389	-.940141	-.943185	-.945471	-.946937	-.947527
-.946741	-.940178	-.511288	-.080981	-.091545	-.112787	-.125803	-.133031
-.135962	-.135645	-.132878	-.128323	-.122491	-.115636	-.107837	-.099151
-.089638	-.079318	-.068139	-.056050	-.043210	-.029958	-.016600	-.003356
.009579	.022001	.033595	.043767	.052192	.059674	.067652	.077305
.089552	.105571	.126516	.151329	.178200	.206960	.236795	.267859
.300318	.333007	.365941	.400348	.433796	.480966	.312191	
.785000							
.785000	.120000	3.200000	.500000				
.999496	.998714	.997407	.995575	.993220	.990346	.986955	.983051
.978636	.973715	.968293	.962374	.955963	.949067	.941691	.933844
.925533	.916769	.907562	.897920	.887856	.877380	.866502	.855233
.843587	.831573	.819206	.806497	.793460	.780109	.766457	.752520
.738311	.723846	.709141	.694210	.679070	.663736	.648225	.632552
.616735	.600791	.584735	.568584	.552357	.536070	.519740	.503384
.487021	.470667	.454340	.438058	.421838	.405697	.389653	.373724
.357927	.342279	.326797	.311499	.296401	.281521	.266874	.252478
.238347	.224498	.210945	.197705	.184792	.172219	.160001	.148150
.136681	.125605	.114935	.104681	.094854	.085465	.076523	.068038
.060016	.052468	.045400	.038819	.032731	.027141	.022055	.017479
.013421	.009888	.006887	.004422	.002497	.001117	.000283	.000000
.000268	.001087	.002453	.004364	.006817	.009807	.013330	.017381
.021952	.027035	.032624	.038714	.045300	.052375	.059934	.067969
.076471	.085433	.094844	.104695	.114977	.125678	.136786	.148290
.160176	.172433	.185045	.198000	.211282	.224876	.238768	.252941
.267379	.282067	.296987	.312123	.327458	.342974	.358655	.374482
.390439	.406506	.422668	.438906	.455202	.471539	.487900	.504266
.520620	.536945	.553223	.569438	.585571	.601607	.617529	.633319
.648962	.664441	.679740	.694843	.709735	.724400	.738823	.752989
.766883	.780491	.793798	.806792	.819458	.831783	.843756	.855364
.866596	.877439	.887883	.897918	.907532	.916716	.925460	.933752
.941585	.948949	.955837	.962243	.968160	.973584	.978508	.982930
.986844	.990246	.993133	.995503	.997351	.998676	.999477	
.661722	.678267	.585987	.527478	.476984	.431536	.390440	.352390
.316752	.283125	.251496	.222221	.195997	.173576	.155526	.141933
.131485	.122800	.114686	.105950	.096014	.085154	.073817	.062308
.050930	.039923	.029502	.020011	.011871	.005315	.000371	-.002888
-.004276	-.003549	-.000424	.005487	.014865	.028888	.049898	.086166
-.343447	-.931343	-1.058572	-1.061377	-1.060836	-1.059817	-1.058293	-1.056303
-1.053852	-1.050898	-1.047403	-1.043362	-1.038780	-1.033653	-1.027946	-1.021575
-1.014464	-1.006578	-.997900	-.988404	-.978066	-.966879	-.954838	-.941911
-.928048	-.913204	-.897327	-.880361	-.862242	-.842884	-.822155	-.799897
-.775993	-.750364	-.722870	-.693239	-.661162	-.626443	-.589144	-.548570
-.503198	-.453370	-.399719	-.340790	-.274958	-.200899	-.116397	-.017290
.101185	.242234	.406111	.589317	.781097	.960601	1.096012	1.155336
1.127454	1.019540	.860392	.681922	.506475	.346476	.206724	.087845
-.012724	-.099152	-.175168	-.242787	-.303204	-.357297	-.405929	-.450604
-.493569	-.534662	-.572243	-.606392	-.637568	-.666271	-.693099	-.718234
-.741688	-.763529	-.783915	-.802992	-.820851	-.837552	-.853149	-.867692
-.881230	-.893814	-.905476	-.916214	-.926030	-.934945	-.942982	-.950155
-.956499	-.962090	-.967007	-.971273	-.974876	-.977793	-.980005	-.981514
-.982280	-.981084	-.969121	-.602843	-.131607	-.041453	-.066983	-.082755
-.092498	-.097775	-.099609	-.098812	-.096028	-.091637	-.085833	-.078766
-.070564	-.061299	-.050963	-.039546	-.027238	-.014397	-.001347	.011660
.024418	.036736	.048287	.058438	.066842	.074367	.082616	.092797

Figure A.1- Continued

.105560	.121717	.142429	.167102	.194234	.223200	.253537	.285364
.318030	.351580	.385872	.420524	.453360	.483402	.311886	
.785000							
.785000	.120000	3.100000	.400000				
.999495	.998712	.997403	.995568	.993210	.990331	.986935	.983025
.978604	.973676	.968247	.962320	.955901	.948997	.941613	.933757
.925438	.916666	.907450	.897800	.887728	.877245	.866360	.855086
.843434	.831416	.819045	.806333	.793294	.779942	.766290	.752353
.738146	.723684	.708983	.694057	.678922	.663595	.648091	.632427
.616619	.600684	.584638	.568499	.552282	.536006	.519688	.503344
.486992	.470649	.454333	.438061	.421851	.405720	.389686	.373766
.357976	.342336	.326860	.311568	.296476	.281599	.266956	.252562
.238433	.224584	.211032	.197792	.184877	.172302	.160082	.148229
.136756	.125677	.115002	.104744	.094913	.085520	.076573	.068083
.060057	.052504	.045431	.038846	.032754	.027161	.022071	.017492
.013431	.009895	.006892	.004425	.002499	.001117	.000283	.000000
.000268	.001088	.002455	.004368	.006823	.009816	.013342	.017396
.021971	.027059	.032652	.038748	.045339	.052420	.059986	.068027
.076535	.085503	.094921	.104779	.115067	.125775	.136889	.148399
.160291	.172553	.185170	.198129	.211414	.225011	.238905	.253079
.267518	.282205	.297124	.312258	.327589	.343101	.358777	.374598
.390547	.406607	.422760	.438989	.455275	.471602	.487952	.504306
.520649	.536962	.553229	.569432	.585554	.601578	.617489	.633269
.648901	.664370	.679661	.694756	.709640	.724298	.738716	.752877
.766767	.780372	.793678	.806670	.819336	.831662	.843637	.855247
.866482	.877329	.887778	.897817	.907437	.916627	.925377	.933676
.941516	.948887	.955782	.962194	.968118	.973547	.978478	.982905
.986824	.990231	.993123	.995496	.997347	.998674	.999476	
.704073	.639623	.589321	.533033	.485623	.441338	.400967	.362973
.327595	.293936	.262568	.233588	.207588	.185799	.168342	.154819
.144298	.135709	.127946	.119671	.110181	.099787	.089038	.078254
.067686	.057566	.048157	.039815	.032943	.027770	.024352	.022795
.023315	.026205	.031823	.040640	.053317	.070569	.090182	-.504042
-1.078807	-1.076694	-1.077608	-1.076717	-1.075380	-1.073596	-1.071393	-1.068798
-1.065803	-1.062357	-1.058415	-1.053968	-1.049017	-1.043556	-1.037548	-1.030906
-1.023551	-1.015448	-1.006580	-.996924	-.986462	-.975191	-.963111	-.950187
-.936358	-.921550	-.905683	-.888679	-.870478	-.851025	-.830238	-.807996
-.784159	-.758567	-.731016	-.701291	-.669279	-.634999	-.598113	-.557771
-.512524	-.462384	-.408468	-.349592	-.283857	-.209914	-.125532	-.026512
.091968	.233173	.397430	.581348	.774304	.955541	1.093194	1.154931
1.129584	1.023851	.866317	.688938	.514167	.354541	.214948	.096079
-.004585	-.091169	-.167361	-.235160	-.295754	-.350016	-.398797	-.443434
-.486398	-.527812	-.565676	-.600070	-.631613	-.660629	-.687686	-.713033
-.736706	-.758770	-.779383	-.798693	-.816788	-.833729	-.849572	-.864370
-.878178	-.891051	-.903023	-.914095	-.924269	-.933568	-.942014	-.949623
-.956435	-.962535	-.968009	-.972889	-.977172	-.980847	-.983909	-.986386
-.988299	-.989607	-.990021	-.980750	-.656028	-.164661	-.016430	-.043833
-.060634	-.071122	-.077066	-.079620	-.079624	-.077594	-.073809	-.068475
-.061767	-.053807	-.044617	-.034196	-.022756	-.010690	.001662	.014059
.026282	.038088	.049139	.058838	.066863	.074005	.081680	.091101
.103152	.119014	.139555	.163718	.190415	.218635	.247932	.278242
.309020	.340098	.370895	.400514	.422983	.422556	.279647	
.785000							
.785000	.120000	3.000000	.300000				
.999495	.998710	.997398	.995559	.993196	.990312	.986909	.982991
.978562	.973626	.968187	.962250	.955822	.948907	.941512	.933645
.925316	.916533	.907306	.897646	.887565	.877072	.866179	.854898
.843239	.831216	.818840	.806125	.793084	.779730	.766078	.752143
.737939	.723481	.708785	.693865	.678738	.663420	.647926	.632273
.616477	.600554	.584522	.568396	.552194	.535932	.519628	.503299
.486962	.470634	.454333	.438075	.421879	.405761	.389739	.373830
.358051	.342420	.326954	.311669	.296583	.281712	.267074	.252683
.238556	.224709	.211157	.197916	.185000	.172423	.160199	.148342
.136865	.125781	.115101	.104836	.094999	.085599	.076646	.068149
.060117	.052557	.045478	.038887	.032789	.027190	.022095	.017511
.013445	.009906	.006899	.004430	.002502	.001118	.000284	.000000

Figure A.1- Continued

.000269	.001089	.002458	.004374	.006832	.009828	.013359	.017419
.021999	.027093	.032693	.038795	.045394	.052484	.060057	.068106
.076624	.085600	.095027	.104894	.115191	.125906	.137029	.148546
.160445	.172713	.185336	.198299	.211589	.225189	.239084	.253259
.267698	.282383	.297299	.312429	.327756	.343262	.358930	.374742
.390682	.406731	.422873	.439089	.455363	.471675	.488011	.504351
.520678	.536976	.553227	.569415	.585522	.601532	.617428	.633195
.648814	.664271	.679550	.694635	.709510	.724160	.738571	.752726
.766611	.780213	.793516	.806507	.819173	.831501	.843478	.855092
.866331	.877183	.887638	.897684	.907312	.916510	.925268	.933576
.941425	.948805	.955709	.962130	.968062	.973500	.978438	.982873
.986798	.990212	.993109	.995486	.997342	.998672	.999475	
.692502	.665482	.605101	.550634	.500571	.456405	.415351	.377414
.341461	.307860	.275876	.246807	.221916	.200841	.183231	.169680
.159582	.151442	.144193	.136523	.127787	.118315	.108637	.099086
.089958	.081515	.074023	.067872	.063517	.061236	.061140	.063419
.068426	.076635	.088747	.106271	.134933	-.203590	-.834151	-1.091218
-1.094785	-1.093610	-1.092281	-1.090601	-1.088537	-1.086097	-1.083302	-1.080170
-1.076686	-1.072792	-1.068438	-1.063613	-1.058318	-1.052547	-1.046261	-1.039373
-1.031800	-1.023507	-1.014472	-1.004673	-.994088	-.982710	-.970539	-.957549
-.943688	-.928894	-.913088	-.896178	-.878069	-.858668	-.837876	-.815596
-.791755	-.766243	-.738805	-.709089	-.676952	-.642553	-.605960	-.566461
-.521557	-.470943	-.416747	-.357941	-.292318	-.218507	-.134265	-.035357
.083094	.224417	.389017	.573606	.767688	.950597	1.090412	1.154435
1.131605	1.028040	.872108	.695813	.521724	.362487	.223077	.104242
.003507	-.083211	-.159561	-.227524	-.288277	-.342692	-.391608	-.436211
-.479184	-.520923	-.559063	-.593589	-.625432	-.654814	-.682126	-.707683
-.731560	-.753831	-.774660	-.794194	-.812520	-.829698	-.845788	-.860848
-.874936	-.888109	-.900399	-.911808	-.922340	-.932020	-.940874	-.948926
-.956222	-.962855	-.968917	-.974445	-.979439	-.983893	-.987815	-.991249
-.994243	-.996790	-.998848	-1.000433	-.998749	-.843553	-.322219	.017305
-.014054	-.032675	-.044746	-.052267	-.056387	-.057797	-.056901	-.054003
-.049360	-.043162	-.035485	-.026364	-.016044	-.004953	.006544	.018193
.029754	.040952	.051435	.060604	.068111	.074707	.081809	.090667
.101827	.116652	.136982	.161606	.187816	.214920	.243500	.272636
.302397	.331981	.360940	.386762	.404828	.394883	.130523	
.790000							
.790000	.105000	3.000000	.500000				
.999494	.998708	.997393	.995551	.993183	.990293	.986883	.982957
.978519	.973573	.968124	.962177	.955738	.948812	.941406	.933529
.925189	.916396	.907159	.897489	.887397	.876894	.865992	.854700
.843033	.831001	.818618	.805896	.792850	.779492	.765838	.751901
.737698	.723242	.708549	.693635	.678516	.663207	.647725	.632085
.616305	.600400	.584387	.568283	.552105	.535870	.519594	.503295
.486991	.470697	.454431	.438211	.422054	.405976	.389996	.374131
.358397	.342811	.327392	.312154	.297116	.282294	.267704	.253362
.239283	.225485	.211982	.198788	.185918	.173388	.161209	.149397
.137962	.126919	.116278	.106052	.096250	.086882	.077959	.069490
.061481	.053940	.046872	.040284	.034182	.028572	.023457	.018836
.014696	.011020	.007814	.005108	.002936	.001335	.000344	.000000
.000329	.001307	.002894	.005053	.007747	.010943	.014609	.018743
.023359	.028472	.034084	.040190	.046785	.053863	.061418	.069444
.077934	.086881	.096275	.106107	.116366	.127043	.138124	.149599
.161455	.173678	.186255	.199172	.212414	.225966	.239813	.253940
.268331	.282968	.297836	.312919	.328199	.343658	.359281	.375049
.390946	.406954	.423055	.439232	.455468	.471745	.488047	.504355
.520652	.536922	.553147	.569310	.585395	.601385	.617264	.633014
.648620	.664065	.679334	.694411	.709280	.723926	.738333	.752488
.766374	.779978	.793284	.806281	.818953	.831288	.843272	.854895
.866143	.877005	.887470	.897527	.907164	.916373	.925141	.933459
.941318	.948709	.955623	.962055	.967998	.973447	.978395	.982838
.986772	.990192	.993096	.995478	.997337	.998670	.999475	
.667033	.578248	.510788	.455919	.409411	.367688	.329507	.293778
.260195	.228405	.198506	.170683	.145611	.124324	.107647	.094899
.084752	.076268	.068299	.059533	.049298	.037902	.025844	.013399
.000768	-.011868	-.024317	-.036290	-.047471	-.057732	-.067133	-.075714

Figure A.1- Continued

-.083445	-.090268	-.096128	-.100942	-.104445	-.106158	-.105485	-.101728
-.093999	-.081065	-.060753	-.026463	-.284347	-.752934	-.941137	-.949447
-.950180	-.950026	-.949061	-.947359	-.944980	-.941968	-.938319	-.933975
-.928876	-.923008	-.916375	-.908973	-.900798	-.891860	-.882171	-.871715
-.860454	-.848343	-.835317	-.821292	-.806192	-.789948	-.772485	-.753732
-.733694	-.712393	-.689665	-.665156	-.638574	-.609711	-.578083	-.543163
-.504222	-.462480	-.420231	-.375307	-.323205	-.259396	-.179919	-.087037
.011713	.116783	.246092	.421255	.641659	.876168	1.067841	1.154931
1.110278	.953740	.742873	.535015	.363593	.232617	.124434	.023671
-.070104	-.150552	-.216137	-.270583	-.317893	-.360914	-.401503	-.440596
-.478516	-.513694	-.545041	-.573290	-.599136	-.623238	-.645994	-.667397
-.687295	-.705710	-.722809	-.738730	-.753547	-.767314	-.780085	-.791899
-.802789	-.812788	-.821908	-.830129	-.837430	-.843802	-.849229	-.853683
-.857146	-.859629	-.861115	-.861467	-.860358	-.854050	-.764934	-.394621
-.133943	-.169288	-.188216	-.198260	-.202266	-.202007	-.198712	-.193196
-.186032	-.177665	-.168454	-.158729	-.148743	-.138552	-.128076	-.117251
-.106039	-.094391	-.082203	-.069375	-.056016	-.042423	-.028858	-.015508
-.002536	.009878	.021450	.031622	.040077	.047586	.055580	.065219
.077513	.093360	.113360	.137144	.163492	.191655	.221213	.252014
.283952	.317116	.351698	.387767	.425190	.459269	.467165	
.790000							
.790000	.110000	3.100000	.500000				
.999495	.998711	.997400	.995562	.993201	.990318	.986918	.983002
.978575	.973641	.968205	.962271	.955845	.948934	.941542	.933680
.925354	.916575	.907352	.897696	.887617	.877127	.866236	.854956
.843298	.831274	.818898	.806182	.793140	.779785	.766131	.752194
.737987	.723527	.708828	.693905	.678776	.663455	.647959	.632304
.616506	.600582	.584550	.568424	.552224	.535965	.519665	.503341
.487010	.470690	.454398	.438152	.421968	.405864	.389858	.373966
.358207	.342597	.327153	.311892	.296832	.281988	.267378	.253016
.238920	.225105	.211586	.198377	.185494	.172951	.160762	.148939
.137495	.126444	.115796	.105564	.095757	.086385	.077459	.068987
.060977	.053435	.046370	.039786	.033690	.028087	.022981	.018373
.014257	.010625	.007486	.004862	.002777	.001255	.000321	.000000
.000307	.001226	.002734	.004806	.007418	.010546	.014168	.018277
.022881	.027984	.033587	.039686	.046276	.053350	.060903	.068929
.077419	.086367	.095763	.105597	.115860	.126541	.137627	.149108
.160971	.173202	.185788	.198715	.211968	.225533	.239394	.253535
.267941	.282596	.297483	.312585	.327885	.343366	.359011	.374802
.390723	.406755	.422881	.439084	.455345	.471648	.487975	.504309
.520632	.536926	.553175	.569362	.585470	.601481	.617380	.633149
.648772	.664234	.679518	.694607	.709488	.724144	.738559	.752720
.766610	.780217	.793525	.806520	.819190	.831521	.843501	.855117
.866358	.877211	.887666	.897713	.907339	.916536	.925292	.933598
.941445	.948823	.955725	.962144	.968075	.973512	.978449	.982882
.986806	.990218	.993114	.995490	.997344	.998673	.999476	
.707432	.612060	.542263	.484845	.436521	.393388	.354028	.317341
.282951	.250492	.220011	.191700	.166248	.144614	.127576	.114562
.104289	.095738	.087765	.079090	.069059	.057964	.046296	.034344
.022341	.010490	-.001000	-.011818	-.021607	-.030195	-.037598	-.043804
-.048713	-.052183	-.054057	-.054124	-.051973	-.046943	-.038181	-.024501
-.003677	.031913	-.335856	-.858215	-.985689	-.989718	-.989627	-.988962
-.987729	-.985911	-.983496	-.980498	-.976937	-.972826	-.968140	-.962806
-.956755	-.949962	-.942418	-.934110	-.925028	-.915177	-.904561	-.893157
-.880914	-.867772	-.853661	-.838504	-.822244	-.804831	-.786193	-.766233
-.744885	-.722078	-.697625	-.671265	-.642793	-.612085	-.578863	-.542215
-.501134	-.456724	-.410554	-.360701	-.303768	-.236462	-.155791	-.062019
.042356	.160253	.304057	.484737	.696033	.910094	1.079858	1.155850
1.117873	.980227	.788544	.590153	.414890	.271618	.152183	.046196
-.048865	-.130952	-.200108	-.259138	-.310975	-.357695	-.400833	-.441708
-.481331	-.518497	-.551996	-.582202	-.609755	-.635434	-.659623	-.682338
-.703490	-.723142	-.741470	-.758601	-.774600	-.789516	-.803405	-.816316
-.828294	-.839386	-.849616	-.858973	-.867451	-.875057	-.881798	-.887673
-.892700	-.896936	-.900436	-.903189	-.905137	-.906189	-.906205	-.904864
-.885986	-.474721	-.089455	-.111546	-.131374	-.143271	-.149558	-.151691
-.150672	-.147253	-.142036	-.135536	-.128156	-.120073	-.111307	-.101871

Figure A.1- Continued

-.091786	-.081044	-.069575	-.057307	-.044381	-.031116	-.017796	-.004627
.008211	.020534	.032041	.042155	.050553	.058039	.066078	.075838
.088254	.104227	.124406	.148368	.174849	.203124	.232805	.263673
.295635	.328646	.362894	.398078	.433502	.461726	.455493	
.790000							
.790000	.110000	3.000000	.400000				
.999494	.998709	.997395	.995553	.993187	.990299	.986891	.982968
.978533	.973590	.968145	.962201	.955765	.948843	.941441	.933567
.925230	.916440	.907206	.897540	.887452	.876952	.866053	.854765
.843100	.831071	.818690	.805971	.792926	.779570	.765916	.751980
.737776	.723320	.708626	.693710	.678589	.663277	.647790	.632146
.616361	.600450	.584431	.568320	.552134	.535890	.519605	.503297
.486981	.470677	.454399	.438167	.421997	.405907	.389913	.374033
.358285	.342685	.327250	.311997	.296944	.282106	.267500	.253142
.239049	.225235	.211716	.198506	.185622	.173076	.160884	.149057
.137609	.126552	.115899	.105660	.095846	.086468	.077535	.069056
.061039	.053491	.046419	.039828	.033726	.028117	.023006	.018393
.014272	.010637	.007494	.004868	.002780	.001257	.000322	.000000
.000307	.001228	.002738	.004812	.007427	.010559	.014186	.018300
.022909	.028019	.033629	.039735	.046333	.053415	.060977	.069011
.077511	.086467	.095872	.105716	.115987	.126676	.137771	.149260
.161129	.173367	.185958	.198890	.212147	.225715	.239578	.253720
.268126	.282779	.297663	.312760	.328055	.343530	.359167	.374950
.390860	.406882	.422996	.439186	.455434	.471723	.488035	.504353
.520660	.536939	.553173	.569344	.585437	.601433	.617317	.633073
.648683	.664132	.679404	.694484	.709355	.724002	.738410	.752565
.766451	.780054	.793360	.806354	.819024	.831357	.843339	.854959
.866204	.877063	.887524	.897578	.907212	.916417	.925182	.933497
.941352	.948740	.955651	.962079	.968019	.973464	.978409	.982849
.986781	.990199	.993100	.995481	.997339	.998671	.999475	
.636410	.640716	.556503	.499191	.450753	.405588	.365236	.328020
.293174	.260239	.229559	.201334	.175962	.154478	.137313	.124093
.113807	.105382	.097654	.089290	.079585	.068861	.057661	.046281
.034932	.023819	.013185	.003352	-.005331	-.012689	-.018716	-.023377
-.026568	-.028133	-.027895	-.025620	-.020866	-.012892	-.000590	.018256
.052384	-.436488	-.976836	-1.006209	-1.006338	-1.005756	-1.004676	-1.003128
-1.001128	-.998638	-.995626	-.992097	-.988062	-.983529	-.978469	-.972803
-.966457	-.959404	-.951631	-.943123	-.933867	-.923863	-.913119	-.901615
-.889311	-.876158	-.862089	-.847012	-.830837	-.813476	-.794827	-.774803
-.753395	-.730614	-.706301	-.680092	-.651635	-.620822	-.587472	-.551138
-.510749	-.466343	-.419807	-.369962	-.313146	-.245958	-.165366	-.071639
.032671	.150480	.294335	.475497	.687959	.904000	1.076475	1.155433
1.120449	.985224	.795220	.597865	.423131	.280020	.160532	.054435
-.040730	-.122933	-.192250	-.251481	-.303538	-.350476	-.393799	-.434637
-.474230	-.511682	-.545442	-.576011	-.604004	-.629931	-.654337	-.677287
-.698687	-.718589	-.737163	-.754539	-.770786	-.785958	-.800118	-.813321
-.825614	-.837047	-.847643	-.857395	-.866297	-.874361	-.881598	-.888012
-.893632	-.898528	-.902770	-.906365	-.909277	-.911455	-.912834	-.913358
-.912668	-.896386	-.463761	-.064894	-.096097	-.114896	-.126421	-.132768
-.135201	-.134626	-.131755	-.127199	-.121436	-.114705	-.107073	-.098592
-.089316	-.079262	-.068381	-.056619	-.044129	-.031242	-.018255	-.005382
.007187	.019251	.030502	.040362	.048504	.055709	.063378	.072708
.084531	.100112	.120329	.144255	.170658	.198787	.228158	.258570
.290044	.322412	.356300	.388801	.421657	.446481	.275560	
.790000							
.790000	.115000	3.100000	.400000				
.999495	.998712	.997401	.995565	.993205	.990325	.986926	.983013
.978589	.973658	.968225	.962295	.955873	.948965	.941577	.933718
.925396	.916620	.907400	.897747	.887672	.877185	.866297	.855020
.843365	.831344	.818971	.806257	.793216	.779862	.766210	.752273
.738066	.723605	.708904	.693980	.678848	.663524	.648024	.632365
.616562	.600633	.584593	.568461	.552253	.535985	.519676	.503342
.487001	.470670	.454366	.438107	.421910	.405793	.389773	.373867
.358093	.342468	.327009	.311733	.296656	.281797	.267170	.252793
.238680	.224849	.211313	.198089	.185191	.172632	.160427	.148590
.137132	.126067	.115406	.105160	.095341	.085959	.077023	.068541

Figure A.1- Continued

.060523	.052975	.045906	.039321	.033226	.027626	.022527	.017931
.013840	.010254	.007182	.004637	.002633	.001183	.000302	.000000
.000287	.001154	.002589	.004580	.007113	.010175	.013752	.017836
.022427	.027524	.033124	.039222	.045813	.052891	.060451	.068484
.076984	.085942	.095348	.105195	.115470	.126164	.137264	.148759
.160637	.172883	.185484	.198426	.211695	.225276	.239153	.253311
.267733	.282403	.297306	.312423	.327739	.343235	.358895	.374701
.390636	.406682	.422822	.439037	.455311	.471626	.487964	.504308
.520640	.536944	.553202	.569396	.585511	.601529	.617434	.633208
.648836	.664301	.679588	.694681	.709563	.724220	.738636	.752797
.766688	.780293	.793600	.806594	.819262	.831591	.843568	.855181
.866419	.877269	.887721	.897764	.907388	.916581	.925334	.933637
.941480	.948854	.955753	.962169	.968096	.973529	.978463	.982893
.986815	.990224	.993118	.995493	.997345	.998674	.999476	
.712384	.639555	.578073	.502795	.454998	.422331	.387583	.348771
.313145	.280187	.249769	.221487	.195587	.173973	.157043	.144001
.133591	.124989	.117111	.108633	.098836	.088030	.076747	.065284
.053842	.042612	.031804	.021713	.012660	.004780	-.001979	-.007642
-.012137	-.015300	-.016836	-.016126	-.011659	.000498	.033867	-.267419
-.815832	-1.034558	-1.043369	-1.043158	-1.041805	-1.040002	-1.037807	-1.035245
-1.032307	-1.028943	-1.025108	-1.020795	-1.016006	-1.010735	-1.004947	-.998557
-.991487	-.983704	-.975190	-.965926	-.955895	-.945098	-.933537	-.921187
-.907998	-.893907	-.878838	-.862702	-.845417	-.826909	-.807096	-.785891
-.763229	-.739018	-.713026	-.684935	-.654566	-.621943	-.586977	-.549148
-.506443	-.458854	-.408561	-.354181	-.292877	-.222380	-.140247	-.044561
.066282	.196054	.351188	.534508	.736799	.933883	1.087036	1.156239
1.126641	1.007695	.835295	.648332	.472924	.320639	.190890	.078896
-.018431	-.102531	-.175222	-.238742	-.295074	-.345635	-.391603	-.434323
-.475630	-.515034	-.550862	-.583365	-.613093	-.640565	-.666354	-.690575
-.713187	-.734263	-.753971	-.772445	-.789753	-.805951	-.821100	-.835257
-.848478	-.860819	-.872310	-.882948	-.892732	-.901683	-.909821	-.917163
-.923747	-.929658	-.934979	-.939740	-.943937	-.947559	-.950602	-.953093
-.955057	-.956461	-.957238	-.951185	-.687577	-.207195	-.011918	-.040172
-.057404	-.068232	-.074462	-.077274	-.077526	-.075747	-.072223	-.067169
-.060762	-.053122	-.044273	-.034220	-.023168	-.011497	.000460	.012463
.024297	.035728	.046425	.055798	.063523	.070377	.077745	.086811
.098666	.114234	.133997	.157153	.183159	.211056	.239480	.268132
.295755	.329189	.371941	.413708	.440443	.429280	.394885	
.790000							
.790000	.115000	3.000000	.300000				
.999495	.998709	.997396	.995556	.993191	.990305	.986900	.982979
.978547	.973608	.968165	.962225	.955793	.948874	.941476	.933606
.925272	.916486	.907255	.897592	.887507	.877011	.866115	.854830
.843169	.831142	.818764	.806047	.793004	.779649	.765996	.752061
.737857	.723399	.708704	.693787	.678663	.663347	.647858	.632209
.616418	.600502	.584476	.568357	.552164	.535911	.519617	.503298
.486972	.470656	.454367	.438122	.421939	.405835	.389827	.373933
.358170	.342554	.327104	.311836	.296766	.281912	.267290	.252916
.238806	.224976	.211441	.198216	.185316	.172755	.160547	.148705
.137243	.126173	.115506	.105255	.095429	.086040	.077097	.068609
.060584	.053030	.045954	.039362	.033261	.027656	.022552	.017951
.013855	.010265	.007190	.004642	.002636	.001185	.000302	.000000
.000287	.001156	.002593	.004586	.007122	.010187	.013769	.017859
.022455	.027558	.033165	.039270	.045869	.052955	.060523	.068565
.077074	.086041	.095456	.105311	.115596	.126298	.137406	.148909
.160793	.173045	.185652	.198599	.211872	.225456	.239335	.253493
.267915	.282584	.297484	.312597	.327907	.343397	.359050	.374847
.390772	.406807	.422935	.439138	.455399	.471699	.488023	.504352
.520669	.536958	.553200	.569379	.585479	.601482	.617372	.633133
.648748	.664201	.679476	.694558	.709431	.724080	.738490	.752644
.766530	.780132	.793437	.806430	.819098	.831428	.843408	.855024
.866266	.877122	.887580	.897630	.907262	.916463	.925225	.933536
.941388	.948772	.955679	.962104	.968040	.973481	.978423	.982861
.986789	.990205	.993104	.995484	.997340	.998671	.999475	
.690361	.656356	.596546	.541305	.492031	.447898	.407448	.370006
.334847	.301909	.271024	.242687	.217650	.196792	.179978	.167052

Figure A.1- Continued

.157254	.149354	.142417	.135136	.126755	.117612	.108340	.099260
.090556	.082476	.075370	.069612	.065550	.063428	.063404	.065642
.070375	.078016	.089318	.105996	.135177	-.325555	-.937842	-1.060325
-1.060803	-1.059618	-1.058243	-1.056541	-1.054481	-1.052070	-1.049330	-1.046276
-1.042894	-1.039123	-1.034912	-1.030251	-1.025139	-1.019570	-1.013510	-1.006873
-.999582	-.991604	-.982923	-.973520	-.963377	-.952488	-.940854	-.928446
-.915216	-.901110	-.886058	-.869976	-.852776	-.834366	-.814625	-.793436
-.770749	-.746545	-.720675	-.692791	-.662530	-.629874	-.594864	-.557071
-.514931	-.467638	-.417005	-.362583	-.301383	-.231014	-.149003	-.053413
.057361	.187132	.342448	.526322	.729709	.928541	1.084027	1.155726
1.128870	1.012243	.841514	.655645	.480877	.328901	.199244	.087229
-.010176	-.094391	-.167231	-.230919	-.287428	-.338159	-.384273	-.426944
-.468225	-.507922	-.544009	-.576739	-.606829	-.634609	-.660613	-.685034
-.707858	-.729149	-.749073	-.767762	-.785290	-.801715	-.817107	-.831525
-.845027	-.857664	-.869466	-.880432	-.890566	-.899894	-.908441	-.916229
-.923300	-.929743	-.935646	-.941041	-.945929	-.950302	-.954170	-.957577
-.960575	-.963161	-.965297	-.966976	-.967134	-.891227	-.388176	.016617
-.013808	-.031362	-.042711	-.049855	-.053853	-.055299	-.054551	-.051908
-.047599	-.041755	-.034446	-.025751	-.015904	-.005265	.005813	.017016
.028104	.038880	.049008	.057818	.064944	.071212	.078029	.086557
.097472	.112066	.131560	.154748	.180107	.206812	.234611	.263023
.291802	.320372	.347910	.372612	.389172	.378431	.102037	
.790000							
.790000	.120000	3.300000	.500000				
.999496	.998715	.997410	.995580	.993229	.990358	.986971	.983070
.978660	.973744	.968327	.962414	.956009	.949119	.941750	.933909
.925605	.916847	.907646	.898011	.887952	.877482	.866609	.855345
.843703	.831693	.819329	.806623	.793588	.780237	.766586	.752648
.738438	.723972	.709263	.694329	.679185	.663846	.648330	.632651
.616828	.600876	.584813	.568655	.552420	.536124	.519786	.503422
.487050	.470688	.454353	.438063	.421835	.405687	.389637	.373702
.357899	.342245	.326759	.311457	.296356	.281473	.266824	.252426
.238294	.224444	.210891	.197652	.184739	.172167	.159951	.148102
.136635	.125562	.114894	.104642	.094819	.085433	.076494	.068011
.059992	.052447	.045381	.038803	.032717	.027130	.022046	.017472
.013415	.009884	.006884	.004420	.002496	.001116	.000283	.000000
.000268	.001086	.002451	.004362	.006813	.009802	.013323	.017371
.021940	.027020	.032606	.038692	.045274	.052346	.059901	.067931
.076429	.085385	.094791	.104638	.114915	.125611	.136714	.148213
.160095	.172347	.184956	.197907	.211186	.224777	.238667	.252838
.267276	.281963	.296884	.312021	.327358	.342877	.358560	.374392
.390353	.406426	.422593	.438838	.455141	.471485	.487854	.504228
.520590	.536924	.553211	.569434	.585577	.601622	.617551	.633350
.649000	.664487	.679793	.694902	.709800	.724470	.738897	.753067
.766964	.780574	.793884	.806878	.819544	.831870	.843842	.855448
.866678	.877518	.887959	.897990	.907601	.916781	.925519	.933807
.941635	.948995	.955878	.962279	.968192	.973611	.978531	.982948
.986858	.990257	.993141	.995508	.997354	.998678	.999477	
.711768	.645392	.594448	.539330	.491980	.448305	.408459	.371134
.336114	.303148	.272382	.244058	.218907	.197206	.179626	.166281
.156161	.148010	.140548	.132595	.123648	.113979	.103976	.093953
.084295	.075257	.067001	.059873	.054368	.050749	.049031	.049329
.051947	.057252	.065675	.077898	.095274	.121348	-.490433	-1.080718
-1.064847	-1.064851	-1.063964	-1.062705	-1.061028	-1.058946	-1.056476	-1.053634
-1.050407	-1.046746	-1.042604	-1.037969	-1.032838	-1.027202	-1.021021	-1.014210
-1.006692	-.998429	-.989400	-.979577	-.968933	-.957458	-.945147	-.931965
-.917864	-.902795	-.886708	-.869543	-.851238	-.831708	-.810822	-.788425
-.764397	-.738657	-.711068	-.681384	-.649333	-.614674	-.577399	-.536900
-.491623	-.441741	-.388068	-.329282	-.263655	-.189861	-.105695	-.007019
.110888	.251190	.414112	.596149	.786578	.964671	1.098841	1.157352
1.129748	1.022688	.864665	.687359	.512968	.353863	.214830	.096516
-.003612	-.089689	-.165421	-.232817	-.293064	-.347038	-.395604	-.440383
-.483482	-.524526	-.562120	-.596212	-.627279	-.656062	-.683028	-.708300
-.731889	-.753891	-.774478	-.793789	-.811900	-.828863	-.844731	-.859557
-.873395	-.886306	-.898325	-.909456	-.919705	-.929096	-.937655	-.945401
-.952372	-.958653	-.964327	-.969431	-.973972	-.977949	-.981371	-.984277

Figure A.1- Continued

-.986706	-.988653	-.990077	-.990993	-.988242	-.818706	-.292617	.036788
.005036	-.014117	-.026716	-.034750	-.039348	-.041187	-.040682	-.038166
-.033896	-.028035	-.020661	-.011844	-.001832	.008984	.020251	.031692
.043063	.054121	.064532	.073707	.081297	.088052	.095449	.104781
.116709	.132019	.151811	.175703	.201995	.229757	.258817	.288613
.319390	.350326	.381599	.410948	.435629	.435114	.297725	
.790000							
.790000	.120000	3.200000	.400000				
.999496	.998714	.997407	.995575	.993220	.990346	.986955	.983051
.978636	.973715	.968293	.962374	.955963	.949067	.941691	.933844
.925533	.916769	.907562	.897920	.887856	.877380	.866502	.855233
.843587	.831573	.819206	.806497	.793460	.780109	.766457	.752520
.738311	.723846	.709141	.694210	.679070	.663736	.648225	.632552
.616735	.600791	.584735	.568584	.552357	.536070	.519740	.503384
.487021	.470667	.454340	.438058	.421838	.405697	.389653	.373724
.357927	.342279	.326797	.311499	.296401	.281521	.266874	.252478
.238347	.224498	.210945	.197705	.184792	.172219	.160001	.148150
.136681	.125605	.114935	.104681	.094854	.085465	.076523	.068038
.060016	.052468	.045400	.038819	.032731	.027141	.022055	.017479
.013421	.009888	.006887	.004422	.002497	.001117	.000283	.000000
.000268	.001087	.002453	.004364	.006817	.009807	.013330	.017381
.021952	.027035	.032624	.038714	.045300	.052375	.059934	.067969
.076471	.085433	.094844	.104695	.114977	.125678	.136786	.148290
.160176	.172433	.185045	.198000	.211282	.224876	.238768	.252941
.267379	.282067	.296987	.312123	.327458	.342974	.358655	.374482
.390439	.406506	.422668	.438906	.455202	.471539	.487900	.504266
.520620	.536945	.553223	.569438	.585571	.601607	.617529	.633319
.648962	.664441	.679740	.694843	.709735	.724400	.738823	.752989
.766883	.780491	.793798	.806792	.819458	.831783	.843756	.855364
.866596	.877439	.887883	.897918	.907532	.916716	.925460	.933752
.941585	.948949	.955837	.962243	.968160	.973584	.978508	.982930
.986844	.990246	.993133	.995503	.997351	.998676	.999477	
.677429	.655493	.611041	.550169	.500602	.458197	.418405	.379138
.343855	.312197	.282395	.253812	.227282	.205221	.189006	.176819
.166921	.158948	.151879	.144444	.135975	.126783	.117412	.108205
.099439	.091385	.084343	.078728	.075021	.073579	.074652	.078633
.086183	.098316	.116731	.145729	-.049555	-.674497	-1.072818	-1.080633
-1.079708	-1.078437	-1.076894	-1.075026	-1.072800	-1.070217	-1.067291	-1.064035
-1.060432	-1.056426	-1.051970	-1.047047	-1.041653	-1.035777	-1.029380	-1.022375
-1.014682	-1.006265	-.997104	-.987173	-.976452	-.964937	-.952626	-.939482
-.925443	-.910435	-.894381	-.877203	-.858842	-.839246	-.818329	-.795965
-.772017	-.746325	-.718687	-.688892	-.656825	-.622521	-.585762	-.545608
-.500427	-.450233	-.396311	-.337589	-.272067	-.198396	-.114356	-.015771
.102132	.242572	.405841	.588537	.780063	.959790	1.096100	1.156927
1.131890	1.027089	.870766	.694623	.520961	.362264	.223413	.105120
.004904	-.081319	-.157220	-.224787	-.285203	-.339339	-.388045	-.432747
-.475792	-.517125	-.554957	-.589390	-.620893	-.649900	-.677045	-.702516
-.726320	-.748538	-.769332	-.788843	-.807153	-.824319	-.840398	-.855445
-.869517	-.882675	-.894955	-.906360	-.916894	-.926583	-.935454	-.943530
-.950853	-.957514	-.963603	-.969161	-.974198	-.978716	-.982730	-.986287
-.989439	-.992190	-.994515	-.996384	-.997814	-.998160	-.933117	-.396480
.052775	.022182	.004670	-.006879	-.014455	-.018950	-.020843	-.020473
-.018116	-.013983	-.008169	-.000738	.008049	.017763	.028033	.038586
.049172	.059523	.069278	.077833	.084857	.091112	.097966	.106601
.118509	.134325	.153382	.175423	.200531	.228447	.258123	.287532
.315615	.345390	.376504	.402899	.417185	.411824	.228052	
.790000							
.790000	.120000	3.100000	.300000				
.999495	.998712	.997403	.995568	.993210	.990331	.986935	.983025
.978604	.973676	.968247	.962320	.955901	.948997	.941613	.933757
.925438	.916666	.907450	.897800	.887728	.877245	.866360	.855086
.843434	.831416	.819045	.806333	.793294	.779942	.766290	.752353
.738146	.723684	.708983	.694057	.678922	.663595	.648091	.632427
.616619	.600684	.584638	.568499	.552282	.536006	.519688	.503344
.486992	.470649	.454333	.438061	.421851	.405720	.389686	.373766
.357976	.342336	.326860	.311568	.296476	.281599	.266956	.252562

Figure A.1- Continued

.238433	.224584	.211032	.197792	.184877	.172302	.160082	.148229
.136756	.125677	.115002	.104744	.094913	.085520	.076573	.068083
.060057	.052504	.045431	.038846	.032754	.027161	.022071	.017492
.013431	.009895	.006892	.004425	.002499	.001117	.000283	.000000
.000268	.001088	.002455	.004368	.006823	.009816	.013342	.017396
.021971	.027059	.032652	.038748	.045339	.052420	.059986	.068027
.076535	.085503	.094921	.104779	.115067	.125775	.136889	.148399
.160291	.172553	.185170	.198129	.211414	.225011	.238905	.253079
.267518	.282205	.297124	.312258	.327589	.343101	.358777	.374598
.390547	.406607	.422760	.438989	.455275	.471602	.487952	.504306
.520649	.536962	.553229	.569432	.585554	.601578	.617489	.633269
.648901	.664370	.679661	.694756	.709640	.724298	.738716	.752877
.766767	.780372	.793678	.806670	.819336	.831662	.843637	.855247
.866482	.877329	.887778	.897817	.907437	.916627	.925377	.933676
.941516	.948887	.955782	.962194	.968118	.973547	.978478	.982905
.986824	.990231	.993123	.995496	.997347	.998674	.999476	
.724661	.700654	.643964	.580642	.529076	.483172	.442298	.404340
.368688	.335476	.304723	.276311	.251201	.230350	.213864	.201398
.191558	.183508	.176799	.170313	.162774	.154319	.145907	.138027
.130674	.124002	.118483	.114578	.112574	.112683	.115150	.120414
.129641	.146893	.093261	-.500491	-1.055961	-1.094203	-1.095723	-1.094261
-1.092552	-1.090658	-1.088552	-1.086169	-1.083469	-1.080450	-1.077122	-1.073495
-1.069548	-1.065222	-1.060467	-1.055268	-1.049622	-1.043520	-1.036921	-1.029739
-1.021892	-1.013341	-1.004065	-.994035	-.983230	-.971643	-.959275	-.946099
-.932061	-.917099	-.901132	-.884064	-.865804	-.846263	-.825348	-.802963
-.779034	-.753444	-.725938	-.696177	-.664021	-.629609	-.592998	-.553685
-.508982	-.458304	-.404112	-.345468	-.280067	-.206535	-.122639	-.024169
.093698	.234239	.397818	.581132	.773708	.955011	1.093384	1.156410
1.133928	1.031386	.876761	.701787	.528868	.370601	.231957	.113713
.013437	-.072911	-.148959	-.216679	-.277245	-.331522	-.380348	-.424975
-.467980	-.509620	-.547717	-.582304	-.614130	-.643477	-.670842	-.696494
-.720485	-.742892	-.763875	-.783577	-.802081	-.819446	-.835734	-.851002
-.865310	-.878717	-.891260	-.902939	-.913764	-.923761	-.932961	-.941390
-.949095	-.956169	-.962705	-.968743	-.974292	-.979355	-.983950	-.988131
-.991960	-.995452	-.998597	-1.001382	-1.003802	-1.005895	-1.007818	-1.006165
-.727051	-.177580	.065036	.040480	.025063	.014731	.008196	.004717
.003825	.005236	.008719	.014053	.020969	.029122	.038051	.047281
.056614	.066001	.074961	.082505	.088406	.093964	.100378	.108286
.118684	.132672	.151370	.173692	.198227	.224457	.251493	.279116
.307133	.335201	.361003	.383012	.391899	.378809	.091006	
.790000							
.790000	.120000	3.000000	.200000				
.999495	.998710	.997398	.995559	.993196	.990312	.986909	.982991
.978562	.973626	.968187	.962250	.955822	.948907	.941512	.933645
.925316	.916533	.907306	.897646	.887565	.877072	.866179	.854898
.843239	.831216	.818840	.806125	.793084	.779730	.766078	.752143
.737939	.723481	.708785	.693865	.678738	.663420	.647926	.632273
.616477	.600554	.584522	.568396	.552194	.535932	.519628	.503299
.486962	.470634	.454333	.438075	.421879	.405761	.389739	.373830
.358051	.342420	.326954	.311669	.296583	.281712	.267074	.252683
.238556	.224709	.211157	.197916	.185000	.172423	.160199	.148342
.136865	.125781	.115101	.104836	.094999	.085599	.076646	.068149
.060117	.052557	.045478	.038887	.032789	.027190	.022095	.017511
.013445	.009906	.006899	.004430	.002502	.001118	.000284	.000000
.000269	.001089	.002458	.004374	.006832	.009828	.013359	.017419
.021999	.027093	.032693	.038795	.045394	.052484	.060057	.068106
.076624	.085600	.095027	.104894	.115191	.125906	.137029	.148546
.160445	.172713	.185336	.198299	.211589	.225189	.239084	.253259
.267698	.282383	.297299	.312429	.327756	.343262	.358930	.374742
.390682	.406731	.422873	.439089	.455363	.471675	.488011	.504351
.520678	.536976	.553227	.569415	.585522	.601532	.617428	.633195
.648814	.664271	.679550	.694635	.709510	.724160	.738571	.752726
.766611	.780213	.793516	.806507	.819173	.831501	.843478	.855092
.866331	.877183	.887638	.897684	.907312	.916510	.925268	.933576
.941425	.948805	.955709	.962130	.968062	.973500	.978438	.982873
.986798	.990212	.993109	.995486	.997342	.998672	.999475	

Figure A.1- Continued

.794188	.734961	.651940	.580487	.536725	.500650	.463153	.422861
.386712	.353493	.323188	.295173	.269754	.248643	.232176	.219981
.210926	.204083	.197950	.191193	.183721	.175977	.168081	.160238
.153020	.146792	.141623	.137865	.136224	.137165	.140625	.145853
.151211	-.444249	-1.071788	-1.112778	-1.110462	-1.108555	-1.106547	-1.104388
-1.102093	-1.099669	-1.097088	-1.094280	-1.091199	-1.087841	-1.084219	-1.080337
-1.076173	-1.071662	-1.066748	-1.061411	-1.055642	-1.049429	-1.042725	-1.035441
-1.027492	-1.018842	-1.009473	-.999365	-.988499	-.976874	-.964487	-.951310
-.937288	-.922356	-.906438	-.889449	-.871299	-.851890	-.831100	-.808799
-.784905	-.759358	-.731991	-.702465	-.670459	-.636059	-.599450	-.560154
-.516052	-.465705	-.411174	-.352555	-.287287	-.213918	-.130200	-.031888
.085890	.226471	.390285	.574131	.767656	.950421	1.090722	1.155748
1.135906	1.035752	.882932	.709221	.537127	.379365	.240992	.122855
.022566	-.063864	-.140024	-.207861	-.268543	-.322929	-.371849	-.416449
-.459127	-.501084	-.539652	-.574198	-.606283	-.635984	-.663552	-.689355
-.713497	-.736052	-.757180	-.777029	-.795689	-.813222	-.829690	-.845150
-.859661	-.873280	-.886042	-.897953	-.909023	-.919284	-.928766	-.937495
-.945517	-.952926	-.959816	-.966226	-.972169	-.977653	-.982704	-.987386
-.991770	-.995883	-.999718	-1.003268	-1.006541	-1.009588	-1.012479	-1.015237
-1.017921	-1.020385	-.872584	-.301626	.094973	.071633	.058074	.049323
.044035	.041505	.041449	.043779	.048090	.053720	.060283	.067648
.075444	.083082	.090228	.096585	.101688	.105751	.110218	.116709
.126274	.139410	.156659	.177537	.201205	.226828	.252674	.278578
.302468	.330649	.365417	.394931	.399179	.361190	.004147	

Figure A.1- Concluded

Figure A.2- Abbreviated print output for sample case

```
*************
* TEST CASE *
*************
```

```
*******************************************************
*                                                     *
*                  PROGRAM PERTURB                    *
*                                                     *
*      CALCULATES NONLINEAR MULTIPLE-PARAMETER        *
*                                                     *
*           CONTINUOUS OR DISCONTINUOUS               *
*                                                     *
*             PERTURBATION SOLUTIONS                  *
*                                                     *
*         WHICH REPRESENT CHANGES IN EITHER           *
*                                                     *
*           GEOMETRY OR FLOW CONDITIONS               *
*                                                     *
*    BY EMPLOYING A STRAINED-COORDINATE PROCEDURE     *
*                                                     *
*    UTILIZING UNIT PERTURBATIONS DETERMINED FROM     *
*                                                     *
*              PREVIOUSLY CALCULATED                  *
*                                                     *
*        "BASE" AND "CALIBRATION" SOLUTIONS           *
*                                                     *
*   DISPLACED FROM ONE ANOTHER BY SOME REASONABLE     *
*                                                     *
*       CHANGE IN GEOMETRY OR FLOW CONDITION          *
*                                                     *
*                                                     *
*                   WRITTEN BY                        *
*                                                     *
*      JAMES P. ELLIOTT AND STEPHEN S. STAHARA        *
*                                                     *
*                                                     *
*      NIELSEN ENGINEERING AND RESEARCH, INC.         *
*                                                     *
*            MOUNTAIN VIEW, CALIFORNIA                *
*                                                     *
*******************************************************
```

```
*****************************************************************************************************
*                TEST CASE - 4 PARAMETER PERTURBATION OF A SUPERCRITICAL CASCADE FLOW              *
*****************************************************************************************************


.....LIST OF INPUT PARAMETERS

     N = 191

     A = -1.0     B =  1.0

     NPARAM = 4


.....STRAINING OPTIONS

     NUMBER OF FIXED POINTS: 5

     FIXED POINTS WILL BE AUTOMATICALLY DETERMINED
     BY THE PROGRAM FOR ALL SOLUTIONS AS FOLLOWS:

          TWO END POINTS
          POINT OF MAXIMUM CP
          CPCRIT (1ST POINT)
          CPCRIT (2ND POINT)
```

Figure A.2- Continued

```
RESULTS OF COMPUTATIONS ON BASE SOLUTION:


.....MACH NUMBER,
     VALUES OF PERTURBATION PARAMETERS,
     CRITICAL VALUE OF CP:

          M0 =  .7800

          Q0(1) =  .7800     (MACH NO.)

          Q0(2) =  .1100     (   TAU  )

          Q0(3) = 3.2000     (  PITCH )

          Q0(4) =  .3000     ( ALPHA1 )

          CPCRIT =  -.4940


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)

     MINIMUM AT X =  .4220*    (POINT NO.  53)
     MAXIMUM AT X = 0.0000     (POINT NO.  96)
     2 CRITICAL POINT(S):
         1ST AT X =  .5001*    (AFTER POINT NO.  48)
         2ND AT X =  .4273     (AFTER POINT NO. 139)


.....LOCATION OF FIXED POINTS
  (* DENOTES POINT ON LOWER SURFACE)

          XFIX(1) = 1.0000*
          XFIX(2) =  .5001*
          XFIX(3) = 0.0000
          XFIX(4) =  .4273
          XFIX(5) = 1.0000
```

Figure A.2- Continued

```
RESULTS OF COMPUTATIONS ON 1ST CALIBRATION SOLN:


.....MACH NUMBER,
     VALUES OF PERTURBATION PARAMETERS,
     CRITICAL VALUE OF CP:

          M1 =  .7900

 (** DENOTES PERTURBATION FROM BASE VALUE)

        **Q1(1) =  .7900      (MACH NO.)

          Q1(2) =  .1100      (   TAU   )

          Q1(3) = 3.2000      (  PITCH  )

          Q1(4) =  .3000      ( ALPHA1  )

          CPCRIT =  -.4638


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)

     MINIMUM AT X =  .5197*    (POINT NO.  47)
     MAXIMUM AT X = 0.0000     (POINT NO.  96)
     2 CRITICAL POINT(S):
         1ST AT X =  .5788*    (AFTER POINT NO.  43)
         2ND AT X =  .5149     (AFTER POINT NO. 144)


.....LOCATION OF FIXED POINTS
  (* DENOTES POINT ON LOWER SURFACE)

          XFIX(1) = 1.0000*
          XFIX(2) =  .5788*
          XFIX(3) = 0.0000
          XFIX(4) =  .5149
          XFIX(5) = 1.0000
```

Figure A.2- Continued

```
RESULTS OF COMPUTATIONS ON 2ND CALIBRATION SOLN:


.....MACH NUMBER,
     VALUES OF PERTURBATION PARAMETERS,
     CRITICAL VALUE OF CP:

          M1 =  .7800

 (** DENOTES PERTURBATION FROM BASE VALUE)

          Q1(1) =  .7800     (MACH NO.)

        **Q1(2) =  .1200     (   TAU  )

          Q1(3) = 3.2000     (  PITCH )

          Q1(4) =  .3000     ( ALPHA1 )

          CPCRIT =  -.4940


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)

     MINIMUM AT X =  .5197*   (POINT NO.  47)
     MAXIMUM AT X = 0.0000    (POINT NO.  96)
     2 CRITICAL POINT(S):
         1ST AT X =  .5774*   (AFTER POINT NO.  43)
         2ND AT X =  .5175    (AFTER POINT NO. 144)


.....LOCATION OF FIXED POINTS
  (* DENOTES POINT ON LOWER SURFACE)

          XFIX(1) = 1.0000*
          XFIX(2) =  .5774*
          XFIX(3) = 0.0000
          XFIX(4) =  .5175
          XFIX(5) = 1.0000
```

Figure A.2- Continued

```
RESULTS OF COMPUTATIONS ON 3RD CALIBRATION SOLN:


.....MACH NUMBER,
     VALUES OF PERTURBATION PARAMETERS,
     CRITICAL VALUE OF CP:

          M1 =  .7800

 (** DENOTES PERTURBATION FROM BASE VALUE)

          Q1(1) =  .7800     (MACH NO.)

          Q1(2) =  .1100     (   TAU  )

        **Q1(3) = 3.0000     (  PITCH )

          Q1(4) =  .3000     ( ALPHA1 )

          CPCRIT =  -.4940


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)

     MINIMUM AT X =  .4382*   (POINT NO.  52)
     MAXIMUM AT X = 0.0000    (POINT NO.  96)
     2 CRITICAL POINT(S):
         1ST AT X =  .5165*   (AFTER POINT NO.  47)
         2ND AT X =  .4520    (AFTER POINT NO. 140)


.....LOCATION OF FIXED POINTS
  (* DENOTES POINT ON LOWER SURFACE)

          XFIX(1) = 1.0000*
          XFIX(2) =  .5165*
          XFIX(3) = 0.0000
          XFIX(4) =  .4520
          XFIX(5) = 1.0000
```

Figure A.2- Continued

```
RESULTS OF COMPUTATIONS ON 4TH CALIBRATION SOLN:


.....MACH NUMBER,
     VALUES OF PERTURBATION PARAMETERS,
     CRITICAL VALUE OF CP:

          M1 =  .7800

 (** DENOTES PERTURBATION FROM BASE VALUE)

          Q1(1) =  .7800     (MACH NO.)

          Q1(2) =  .1100     (   TAU  )

          Q1(3) = 3.2000     (  PITCH )

        **Q1(4) =  .5000     ( ALPHA1 )

          CPCRIT =  -.4940


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)

     MINIMUM AT X =  .4058*   (POINT NO.  54)
     MAXIMUM AT X = 0.0000    (POINT NO.  96)
     2 CRITICAL POINT(S):
         1ST AT X =  .4903*   (AFTER POINT NO.  48)
         2ND AT X =  .4360    (AFTER POINT NO. 139)


.....LOCATION OF FIXED POINTS
  (* DENOTES POINT ON LOWER SURFACE)

          XFIX(1) = 1.0000*
          XFIX(2) =  .4903*
          XFIX(3) = 0.0000
          XFIX(4) =  .4360
          XFIX(5) = 1.0000
```

Figure A.2- Continued

UNIT PERTURBATION OF CP AND UNIT STRAINING OF XBASE
FOR CALIBRATION SOLUTIONS 1 THROUGH 4

POINT	XBASE	* 1ST CALB XSTRUNIT	SOLN * CPUNIT	* 2ND CALB XSTRUNIT	SOLN * CPUNIT	* 3RD CALB XSTRUNIT	SOLN * CPUNIT	* 4TH CALB XSTRUNIT	SOLN * CPUNIT
1	.9995	.9996	-3.6772	.9996	13.0723	.9995	-.0683	.9995	-.0125
2	.9987	.9989	5.8554	.9989	10.2272	.9988	-.0697	.9987	-.0134
3	.9974	.9978	6.7075	.9978	8.9041	.9975	-.0590	.9974	-.0264
4	.9956	.9963	5.8289	.9963	8.3875	.9957	-.0538	.9955	-.0263
5	.9932	.9943	5.6358	.9943	7.9171	.9934	-.0508	.9931	-.0260
6	.9903	.9919	5.5107	.9918	7.6585	.9907	-.0492	.9901	-.0255
7	.9869	.9890	5.4424	.9890	7.4524	.9874	-.0480	.9867	-.0258
8	.9830	.9857	5.4475	.9857	7.3280	.9836	-.0472	.9827	-.0261
9	.9786	.9820	5.4801	.9819	7.2282	.9793	-.0464	.9782	-.0268
10	.9737	.9778	5.5339	.9778	7.1532	.9745	-.0454	.9732	-.0273
11	.9683	.9732	5.5788	.9732	7.0535	.9693	-.0439	.9676	-.0270
12	.9623	.9683	5.5916	.9681	6.9361	.9636	-.0421	.9616	-.0262
13	.9559	.9628	5.5484	.9627	6.7704	.9574	-.0393	.9550	-.0242
14	.9490	.9570	5.3490	.9569	6.4648	.9507	-.0349	.9480	-.0223
15	.9416	.9508	5.0069	.9506	6.0231	.9435	-.0298	.9405	-.0212
16	.9338	.9442	4.6407	.9440	5.5879	.9359	-.0256	.9325	-.0200
17	.9255	.9372	4.3709	.9370	5.2574	.9279	-.0227	.9240	-.0193
18	.9167	.9298	4.2246	.9296	5.0385	.9194	-.0211	.9150	-.0195
19	.9075	.9220	4.2128	.9218	4.9532	.9105	-.0213	.9056	-.0209
20	.8978	.9139	4.3770	.9136	5.0572	.9012	-.0238	.8958	-.0233
21	.8877	.9054	4.7068	.9051	5.3353	.8914	-.0279	.8855	-.0257
22	.8773	.8966	5.0993	.8962	5.6726	.8813	-.0317	.8748	-.0278
23	.8664	.8874	5.4668	.8870	5.9736	.8708	-.0347	.8637	-.0297
24	.8551	.8779	5.7992	.8775	6.2320	.8598	-.0374	.8522	-.0316
25	.8435	.8681	6.1138	.8677	6.4713	.8486	-.0399	.8404	-.0335
26	.8314	.8579	6.4157	.8575	6.7004	.8370	-.0422	.8281	-.0353
27	.8191	.8475	6.7070	.8470	6.9180	.8250	-.0444	.8155	-.0369
28	.8063	.8368	6.9767	.8363	7.1126	.8127	-.0461	.8025	-.0382
29	.7933	.8258	7.2083	.8253	7.2699	.8001	-.0472	.7892	-.0394
30	.7800	.8146	7.4079	.8140	7.3994	.7872	-.0483	.7756	-.0410
31	.7663	.8031	7.6082	.8024	7.5350	.7740	-.0500	.7617	-.0429
32	.7524	.7913	7.8418	.7906	7.7081	.7605	-.0523	.7475	-.0451
33	.7382	.7793	8.1228	.7786	7.9326	.7467	-.0551	.7330	-.0477
34	.7237	.7672	8.4590	.7664	8.2092	.7327	-.0586	.7182	-.0508
35	.7090	.7548	8.8524	.7540	8.5568	.7185	-.0629	.7032	-.0544
36	.6941	.7422	9.3080	.7414	8.9729	.7041	-.0682	.6880	-.0584
37	.6789	.7294	9.8246	.7286	9.4572	.6894	-.0741	.6726	-.0625
38	.6636	.7165	10.3836	.7156	9.9922	.6746	-.0803	.6570	-.0665
39	.6481	.7034	10.9615	.7025	10.5565	.6596	-.0867	.6411	-.0705
40	.6324	.6902	11.5372	.6893	11.1326	.6445	-.0934	.6252	-.0743
41	.6166	.6769	12.0919	.6759	11.7063	.6292	-.1003	.6091	-.0774
42	.6007	.6635	12.6054	.6624	12.2629	.6138	-.1075	.5928	-.0793
43	.5846	.6500	13.0992	.6489	12.7943	.5982	-.1142	.5764	-.0793
44	.5685	.6364	13.4932	.6352	13.2977	.5826	-.1197	.5600	-.0772
45	.5523	.6227	13.7283	.6215	13.6960	.5670	-.1230	.5435	-.0755
46	.5360	.6090	13.8275	.6078	14.0485	.5512	-.1238	.5269	-.0853
47	.5197	.5953	1.0263	.5940	-.2971	.5354	-.0601	.5102	-.3145
48	.5034	.5815	13.1470	.5802	16.2397	.5196	.0009	.4936	1.2524
49	.4870	.5636	6.1472	.5623	5.9650	.5030	.0937	.4774	1.4047
50	.4707	.5447	-2.8708	.5434	-9.4729	.4861	.0764	.4614	.0766
51	.4544	.5258	-2.4337	.5246	-9.1191	.4693	.0681	.4454	.1006
52	.4381	.5070	-2.2079	.5058	-8.8921	.4525	.0642	.4295	.0969
53	.4220	.4883	-2.0468	.4872	-8.7206	.4358	.0611	.4136	.0953
54	.4058	.4696	-1.9189	.4685	-8.5696	.4191	.0583	.3978	.0947
55	.3898	.4511	-1.8124	.4501	-8.4372	.4026	.0561	.3821	.0946

Figure A.2- Continued

56	.3739	.4327	-1.7337	.4317	-8.3259	.3862	.0545	.3665	.0949
57	.3582	.4145	-1.6864	.4135	-8.2398	.3699	.0533	.3511	.0955
58	.3425	.3964	-1.6684	.3955	-8.1763	.3537	.0525	.3358	.0962
59	.3271	.3785	-1.6712	.3776	-8.1262	.3378	.0518	.3206	.0969
60	.3118	.3608	-1.6843	.3600	-8.0786	.3220	.0513	.3057	.0978
61	.2968	.3434	-1.7024	.3426	-8.0281	.3065	.0510	.2909	.0986
62	.2819	.3262	-1.7170	.3255	-7.9622	.2911	.0507	.2763	.0994
63	.2673	.3093	-1.7238	.3086	-7.8822	.2760	.0505	.2620	.1003
64	.2529	.2927	-1.7277	.2920	-7.7894	.2612	.0503	.2479	.1012
65	.2388	.2764	-1.7330	.2757	-7.6871	.2466	.0503	.2341	.1021
66	.2250	.2604	-1.7435	.2598	-7.5777	.2324	.0504	.2206	.1031
67	.2115	.2447	-1.7605	.2442	-7.4611	.2184	.0505	.2073	.1041
68	.1983	.2295	-1.7843	.2289	-7.3370	.2048	.0505	.1944	.1052
69	.1854	.2146	-1.8128	.2141	-7.2045	.1915	.0503	.1817	.1065
70	.1729	.2000	-1.8416	.1996	-7.0621	.1785	.0499	.1695	.1081
71	.1607	.1859	-1.8697	.1855	-6.9127	.1659	.0493	.1575	.1101
72	.1489	.1723	-1.8914	.1719	-6.7550	.1537	.0486	.1459	.1123
73	.1374	.1590	-1.9037	.1586	-6.5767	.1419	.0482	.1347	.1143
74	.1264	.1462	-1.9117	.1459	-6.3754	.1305	.0480	.1239	.1161
75	.1157	.1339	-1.9295	.1336	-6.1490	.1195	.0482	.1134	.1174
76	.1055	.1221	-1.9753	.1218	-5.9059	.1089	.0485	.1034	.1187
77	.0957	.1107	-2.0420	.1105	-5.6470	.0988	.0484	.0938	.1204
78	.0863	.0999	-2.1030	.0997	-5.3671	.0892	.0480	.0846	.1235
79	.0774	.0896	-2.1597	.0894	-5.0885	.0799	.0477	.0759	.1282
80	.0689	.0798	-2.2410	.0796	-4.8528	.0712	.0478	.0676	.1350
81	.0609	.0705	-2.4085	.0703	-4.6923	.0629	.0490	.0597	.1398
82	.0534	.0618	-2.5574	.0616	-4.4183	.0551	.0493	.0523	.1379
83	.0463	.0536	-2.4802	.0535	-3.7783	.0479	.0469	.0454	.1362
84	.0398	.0460	-2.4087	.0459	-3.0266	.0411	.0456	.0390	.1392
85	.0337	.0390	-2.5850	.0389	-2.4638	.0348	.0469	.0330	.1443
86	.0281	.0325	-2.9590	.0324	-2.1314	.0290	.0500	.0275	.1498
87	.0230	.0266	-3.4047	.0265	-2.2073	.0237	.0537	.0225	.1533
88	.0184	.0212	-3.6285	.0212	-2.3516	.0190	.0547	.0180	.1542
89	.0142	.0165	-3.5027	.0164	-1.7427	.0147	.0519	.0140	.1553
90	.0106	.0123	-3.3397	.0123	-.0485	.0110	.0486	.0104	.1591
91	.0075	.0087	-3.4626	.0086	1.8499	.0077	.0485	.0073	.1614
92	.0049	.0056	-3.5612	.0056	2.7560	.0050	.0472	.0048	.1540
93	.0028	.0032	-3.0212	.0032	2.4403	.0029	.0388	.0027	.1317
94	.0013	.0015	-1.7192	.0014	1.5007	.0013	.0222	.0012	.0949
95	.0003	.0004	-.1863	.0004	.7268	.0003	.0033	.0003	.0504
96	0.0000	.0000	.4666	0.0000	.2299	0.0000	-.0028	0.0000	.0076
97	.0003	.0004	-.5163	.0004	-.2795	.0003	.0144	.0003	-.0330
98	.0012	.0015	-2.6946	.0015	-.6713	.0013	.0513	.0012	-.0754
99	.0027	.0033	-4.7477	.0033	-.8324	.0029	.0869	.0028	-.1090
100	.0048	.0058	-5.8699	.0058	-1.2241	.0051	.1081	.0049	-.1295
101	.0074	.0089	-6.1156	.0090	-2.3637	.0078	.1151	.0076	-.1382
102	.0105	.0127	-6.1800	.0128	-4.1376	.0111	.1184	.0108	-.1412
103	.0142	.0171	-6.4569	.0171	-5.5968	.0150	.1242	.0144	-.1429
104	.0183	.0220	-6.6293	.0221	-6.0056	.0193	.1283	.0186	-.1436
105	.0229	.0275	-6.3905	.0277	-5.7458	.0242	.1263	.0233	-.1420
106	.0280	.0337	-5.8672	.0339	-5.6281	.0296	.1206	.0285	-.1387
107	.0336	.0404	-5.3306	.0406	-5.8660	.0355	.1154	.0342	-.1351
108	.0397	.0478	-5.0064	.0480	-6.3126	.0419	.1123	.0405	-.1319
109	.0462	.0557	-4.8464	.0560	-6.7808	.0489	.1109	.0472	-.1293
110	.0533	.0642	-4.8099	.0646	-7.1860	.0564	.1111	.0544	-.1273
111	.0608	.0733	-4.8938	.0737	-7.5935	.0644	.1121	.0621	-.1259
112	.0689	.0830	-5.1069	.0834	-8.1542	.0728	.1142	.0703	-.1260
113	.0774	.0932	-5.2199	.0937	-8.6874	.0818	.1186	.0789	-.1298
114	.0863	.1040	-5.0457	.1045	-8.8902	.0913	.1203	.0880	-.1314
115	.0957	.1153	-4.7993	.1159	-8.9242	.1012	.1171	.0976	-.1257
116	.1055	.1271	-4.6563	.1278	-9.0467	.1116	.1141	.1077	-.1198
117	.1158	.1395	-4.5874	.1402	-9.2338	.1224	.1122	.1181	-.1169
118	.1264	.1524	-4.5443	.1531	-9.4244	.1337	.1120	.1290	-.1153
119	.1375	.1657	-4.5032	.1665	-9.5943	.1455	.1131	.1403	-.1149
120	.1490	.1795	-4.4525	.1804	-9.7379	.1576	.1139	.1520	-.1142
121	.1609	.1938	-4.4019	.1948	-9.8681	.1701	.1138	.1641	-.1127

Figure A.2- Continued

122	.1731	.2086	-4.3589	.2096	-9.9930	.1831	.1133	.1766	-.1108
123	.1857	.2237	-4.3241	.2248	-10.1121	.1964	.1128	.1894	-.1086
124	.1986	.2393	-4.3021	.2405	-10.2126	.2100	.1124	.2026	-.1065
125	.2118	.2553	-4.2842	.2565	-10.3195	.2241	.1123	.2161	-.1047
126	.2254	.2716	-4.2865	.2730	-10.4349	.2384	.1127	.2300	-.1031
127	.2393	.2883	-4.3048	.2897	-10.5554	.2531	.1134	.2441	-.1019
128	.2534	.3053	-4.3331	.3069	-10.6749	.2680	.1144	.2585	-.1009
129	.2678	.3227	-4.3655	.3243	-10.7876	.2832	.1155	.2732	-.0999
130	.2825	.3404	-4.4000	.3421	-10.8919	.2987	.1167	.2882	-.0991
131	.2973	.3583	-4.4403	.3601	-10.9813	.3145	.1180	.3034	-.0982
132	.3124	.3765	-4.4843	.3784	-11.0694	.3305	.1194	.3188	-.0975
133	.3278	.3949	-4.5544	.3969	-11.1784	.3467	.1213	.3344	-.0969
134	.3432	.4136	-4.6781	.4157	-11.3323	.3630	.1239	.3502	-.0967
135	.3589	.4325	-4.8839	.4346	-11.5598	.3796	.1274	.3662	-.0970
136	.3747	.4515	-5.2157	.4537	-11.9043	.3963	.1327	.3823	-.0979
137	.3906	.4707	-5.8147	.4730	-12.4596	.4131	.1353	.3986	-.0922
138	.4067	.4900	-5.6585	.4925	-11.7958	.4301	.0166	.4149	.0388
139	.4228	.5095	-15.1585	.5120	-27.5664	.4472	.8869	.4314	-.8876
140	.4390	.5248	-1.4607	.5273	-9.6817	.4631	.5598	.4475	-.6155
141	.4553	.5386	17.0301	.5410	17.2792	.4787	-.2261	.4635	.1379
142	.4716	.5524	16.5106	.5548	16.4700	.4943	-.2073	.4796	.1224
143	.4879	.5663	15.6672	.5685	15.4641	.5100	-.1856	.4957	.1075
144	.5043	.5801	14.5876	.5823	14.3404	.5256	-.1631	.5118	.0934
145	.5206	.5939	13.4548	.5961	13.1640	.5412	-.1427	.5279	.0812
146	.5369	.6077	12.3582	.6098	12.0312	.5568	-.1250	.5439	.0712
147	.5532	.6215	11.3377	.6235	10.9847	.5724	-.1100	.5600	.0630
148	.5694	.6352	10.4095	.6372	10.0417	.5879	-.0972	.5759	.0563
149	.5855	.6489	9.5781	.6507	9.2059	.6033	-.0861	.5918	.0506
150	.6015	.6625	8.8578	.6643	8.4769	.6187	-.0765	.6076	.0459
151	.6174	.6759	8.2158	.6777	7.8536	.6339	-.0684	.6232	.0419
152	.6332	.6893	7.6343	.6910	7.2955	.6490	-.0616	.6388	.0386
153	.6488	.7026	7.0962	.7041	6.7854	.6640	-.0557	.6542	.0357
154	.6643	.7157	6.5956	.7172	6.3182	.6788	-.0503	.6694	.0332
155	.6796	.7286	6.1374	.7300	5.8992	.6934	-.0451	.6845	.0308
156	.6947	.7414	5.7353	.7428	5.5417	.7078	-.0400	.6993	.0286
157	.7096	.7540	5.4001	.7553	5.2557	.7221	-.0357	.7140	.0267
158	.7242	.7664	5.1279	.7677	5.0355	.7361	-.0325	.7284	.0252
159	.7387	.7786	4.9100	.7798	4.8708	.7499	-.0302	.7426	.0241
160	.7528	.7906	4.7349	.7917	4.7580	.7635	-.0284	.7566	.0232
161	.7667	.8024	4.5996	.8035	4.6899	.7768	-.0271	.7703	.0225
162	.7803	.8139	4.4888	.8149	4.6489	.7898	-.0265	.7837	.0220
163	.7936	.8252	4.3714	.8261	4.6009	.8025	-.0264	.7968	.0217
164	.8066	.8362	4.2139	.8371	4.5114	.8150	-.0259	.8096	.0213
165	.8193	.8470	4.0082	.8478	4.3727	.8271	-.0246	.8221	.0206
166	.8316	.8574	3.7682	.8582	4.1991	.8389	-.0225	.8342	.0197
167	.8436	.8675	3.5065	.8682	4.0024	.8504	-.0200	.8460	.0187
168	.8552	.8774	3.2289	.8780	3.7856	.8615	-.0171	.8574	.0176
169	.8665	.8869	2.9227	.8875	3.5380	.8722	-.0139	.8685	.0164
170	.8773	.8961	2.5581	.8966	3.2238	.8826	-.0100	.8792	.0150
171	.8878	.9049	2.1375	.9054	2.8520	.8926	-.0048	.8895	.0133
172	.8978	.9134	1.7673	.9139	2.5354	.9022	.0009	.8994	.0116
173	.9074	.9216	1.5812	.9220	2.4056	.9114	.0046	.9088	.0106
174	.9166	.9294	1.5995	.9298	2.4788	.9202	.0054	.9179	.0108
175	.9254	.9368	1.7505	.9371	2.6960	.9286	.0039	.9265	.0117
176	.9337	.9438	1.9702	.9441	3.0096	.9365	.0008	.9347	.0131
177	.9415	.9505	2.2206	.9507	3.3759	.9440	-.0040	.9424	.0148
178	.9489	.9567	2.4506	.9569	3.7213	.9511	-.0093	.9497	.0163
179	.9558	.9625	2.5669	.9627	3.9554	.9577	-.0130	.9565	.0179
180	.9622	.9680	2.5297	.9681	4.0428	.9638	-.0145	.9628	.0194
181	.9681	.9730	2.4157	.9731	4.0564	.9695	-.0146	.9686	.0202
182	.9735	.9776	2.2248	.9777	3.9903	.9747	-.0136	.9740	.0208
183	.9785	.9818	1.9768	.9819	3.8828	.9794	-.0117	.9788	.0209
184	.9829	.9855	1.6741	.9856	3.7336	.9836	-.0089	.9832	.0209
185	.9868	.9888	1.3148	.9889	3.5268	.9874	-.0052	.9870	.0215
186	.9902	.9917	.8999	.9918	3.2608	.9907	-.0002	.9904	.0222
187	.9931	.9942	.3520	.9942	2.8816	.9934	.0064	.9932	.0241

Figure A.2- Continued

188	.9955	.9962	-.4223	.9962	2.3418	.9957	.0160	.9956	.0261
189	.9973	.9978	-1.4918	.9978	1.1609	.9975	.0313	.9974	.0283
190	.9987	.9989	-7.6871	.9989	-.9395	.9987	.0648	.9987	.0275
191	.9995	.9996	-24.6958	.9996	-1.9799	.9995	.1357	.9995	-.0299

Figure A.2- Continued

```
*********************************
* OUTPUT FOR CASE NO.  1 OF 19 *
*********************************


.....MACH NUMBER,
     VALUES OF PERTURBATION PARAMETERS,
     CRITICAL VALUE OF CP:

          M2 =  .7750

          Q2(1) =  .7750      (MACH NO.)

          Q2(2) =  .1200      (   TAU   )

          Q2(3) = 3.0000      (  PITCH  )

          Q2(4) =  .5000      ( ALPHA1  )

          CPCRIT =  -.5095


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)

     PERTURBATION SOLN:

          MINIMUM AT X =  .4771*    (POINT NO.  52)
          MAXIMUM AT X = 0.0000     (POINT NO.  96)
          2 CRITICAL POINT(S):
              1ST AT X =  .5420*    (AFTER POINT NO.  48)
              2ND AT X =  .5065     (AFTER POINT NO. 139)

     COMPARISON SOLN:

          MINIMUM AT X =  .4870*    (POINT NO.  49)
          MAXIMUM AT X = 0.0000     (POINT NO.  96)
          2 CRITICAL POINT(S):
              1ST AT X =  .5493*    (AFTER POINT NO.  45)
              2ND AT X =  .5086     (AFTER POINT NO. 144)


.....FINAL PRINTOUT AND GRAPHICAL DISPLAY OF CP

                                                                                   H = MAXIMUM  VALUE OF CP =  1.1517
                                                                                   L = MINIMUM  VALUE OF CP = -1.0527
                                                                                   * = CRITICAL VALUE OF CP =  -.5095
                                                                                   P = VALUE OF CP PREDICTED BY PERTURBATION SOLUTION
                                                                                   C = VALUE OF CP IN COMPARISON SOLUTION
                                                                                   $ = AGREEMENT BETWEEN P AND C

 PT  XBASE   CPBASE  XPERT   CPPERT  XCHEK   CPCHEK  CPPINT H-----------------------------------------------------*-----------------L

  1  .9995   .6608   .9995   .8210   .9995   .7155   .8108            P   C                                       *
  2  .9987   .5751   .9988   .6594   .9987   .6231   .6506                  PC                                    *
  3  .9974   .5086   .9976   .5706   .9974   .5555   .5626                    PC                                  *
  4  .9956   .4543   .9960   .5145   .9956   .4972   .5050                      PC                                *
  5  .9932   .4077   .9938   .4637   .9932   .4469   .4533                         $                              *
  6  .9903   .3659   .9912   .4197   .9903   .4022   .4081                         PC                             *
```

Figure A.2- Continued

7	.9849	.3274	.9881	.3792	.9869	.3619	.3667
8	.9830	.2914	.9845	.3417	.9830	.3238	.3281
9	.9786	.2575	.9805	.3063	.9786	.2879	.2918
10	.9737	.2254	.9760	.2729	.9736	.2538	.2576
11	.9683	.1952	.9711	.2412	.9682	.2216	.2254
12	.9623	.1671	.9657	.2117	.9623	.1917	.1959
13	.9559	.1417	.9598	.1847	.9558	.1649	.1694
14	.9490	.1202	.9535	.1606	.9489	.1419	.1464
15	.9416	.1031	.9468	.1400	.9415	.1235	.1278
16	.9338	.0897	.9397	.1235	.9336	.1093	.1131
17	.9255	.0790	.9321	.1104	.9253	.0982	.1012
18	.9167	.0700	.9241	.0996	.9165	.0889	.0910
19	.9075	.0615	.9157	.0900	.9073	.0800	.0814
20	.8978	.0522	.9069	.0810	.8976	.0703	.0715
21	.8877	.0413	.8977	.0716	.8876	.0591	.0605
22	.8773	.0292	.8882	.0612	.8771	.0467	.0483
23	.8664	.0163	.8783	.0497	.8662	.0335	.0353
24	.8551	.0030	.8680	.0374	.8549	.0199	.0218
25	.8435	-.0107	.8574	.0247	.8432	.0061	.0082
26	.8314	-.0245	.8464	.0118	.8312	-.0076	-.0054
27	.8191	-.0382	.8352	-.0011	.8188	-.0211	-.0188
28	.8063	-.0517	.8236	-.0138	.8061	-.0341	-.0317
29	.7933	-.0645	.8117	-.0262	.7931	-.0463	-.0439
30	.7800	-.0766	.7995	-.0381	.7797	-.0574	-.0553
31	.7663	-.0880	.7871	-.0493	.7661	-.0675	-.0658
32	.7524	-.0990	.7744	-.0597	.7521	-.0768	-.0754
33	.7382	-.1094	.7614	-.0692	.7379	-.0851	-.0841
34	.7237	-.1194	.7483	-.0780	.7235	-.0923	-.0918
35	.7090	-.1289	.7349	-.0859	.7088	-.0984	-.0984
36	.6941	-.1381	.7213	-.0929	.6939	-.1033	-.1037
37	.6789	-.1467	.7075	-.0989	.6787	-.1067	-.1075
38	.6636	-.1546	.6935	-.1038	.6634	-.1081	-.1091
39	.6481	-.1614	.6794	-.1074	.6479	-.1067	-.1080
40	.6324	-.1666	.6651	-.1091	.6323	-.1020	-.1033
41	.6166	-.1698	.6507	-.1086	.6165	-.0927	-.0939
42	.6007	-.1704	.6362	-.1052	.6006	-.0774	-.0784
43	.5846	-.1676	.6216	-.0981	.5845	-.0534	-.0555
44	.5685	-.1601	.6069	-.0860	.5684	-.0106	-.1007
45	.5523	-.1461	.5921	-.0683	.5522	-.4144	-.3249
46	.5360	-.1225	.5773	-.0435	.5359	-.9481	-.6201
47	.5197	-.0800	.5624	-.1390	.5196	-1.0476	-.9160
48	.5034	-.4036	.5476	-.4091	.5033	-1.0523	-1.0445
49	.4870	-.8588	.5303	-.7216	.4870	-1.0527	-1.0451
50	.4707	-.9641	.5126	-1.0444	.4706	-1.0523	-1.0454
51	.4544	-.9721	.4948	-1.0447	.4543	-1.0509	-1.0446
52	.4381	-.9743	.4771	-1.0456	.4381	-1.0488	-1.0428
53	.4220	-.9750	.4595	-1.0451	.4219	-1.0459	-1.0402
54	.4058	-.9746	.4419	-1.0434	.4058	-1.0422	-1.0368
55	.3898	-.9731	.4245	-1.0407	.3897	-1.0379	-1.0327
56	.3739	-.9707	.4072	-1.0372	.3738	-1.0328	-1.0277
57	.3582	-.9672	.3900	-1.0328	.3581	-1.0268	-1.0219
58	.3425	-.9628	.3730	-1.0275	.3424	-1.0200	-1.0153
59	.3271	-.9574	.3562	-1.0212	.3270	-1.0122	-1.0078
60	.3118	-.9510	.3395	-1.0140	.3117	-1.0036	-.9994
61	.2968	-.9437	.3231	-1.0059	.2966	-.9941	-.9901
62	.2819	-.9354	.3070	-.9967	.2817	-.9837	-.9799
63	.2673	-.9263	.2911	-.9866	.2671	-.9723	-.9688
64	.2529	-.9163	.2754	-.9754	.2527	-.9601	-.9568
65	.2388	-.9054	.2601	-.9632	.2386	-.9468	-.9438
66	.2250	-.8935	.2450	-.9500	.2247	-.9325	-.9297
67	.2115	-.8806	.2303	-.9357	.2112	-.9172	-.9145
68	.1983	-.8667	.2159	-.9202	.1979	-.9008	-.8981
69	.1854	-.8516	.2019	-.9034	.1850	-.8831	-.8804
70	.1729	-.8354	.1882	-.8851	.1724	-.8642	-.8613
71	.1607	-.8179	.1750	-.8655	.1602	-.8439	-.8409
72	.1489	-.7990	.1621	-.8443	.1483	-.8220	-.8190

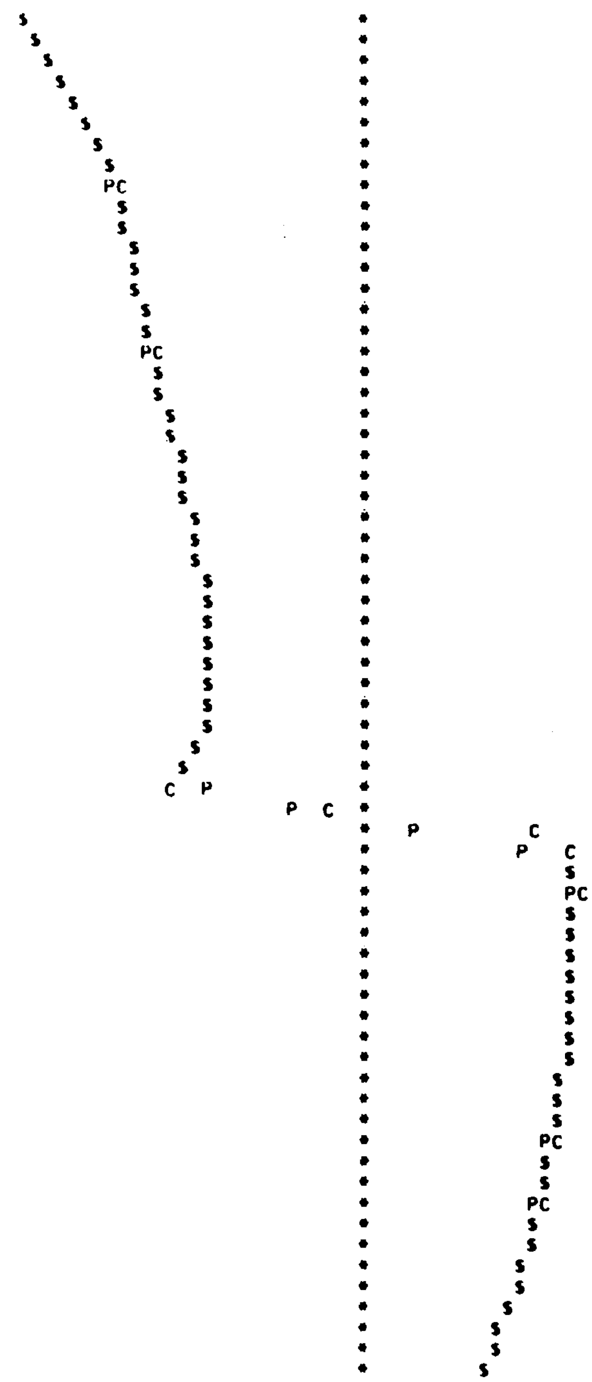


Figure A.2- Continued

73	.1374	-.7786	.1496	-.8216	.1369	-.7985	-.7957
74	.1264	-.7568	.1376	-.7973	.1258	-.7732	-.7706
75	.1157	-.7332	.1260	-.7712	.1151	-.7460	-.7435
76	.1055	-.7077	.1149	-.7429	.1048	-.7165	-.7138
77	.0957	-.6801	.1042	-.7120	.0950	-.6844	-.6815
78	.0863	-.6502	.0940	-.6782	.0856	-.6496	-.6466
79	.0774	-.6176	.0843	-.6416	.0766	-.6124	-.6087
80	.0689	-.5820	.0751	-.6019	.0681	-.5717	-.5678
81	.0609	-.5422	.0664	-.5589	.0601	-.5262	-.5233
82	.0534	-.4984	.0581	-.5120	.0526	-.4766	-.4741
83	.0463	-.4523	.0505	-.4599	.0455	-.4230	-.4199
84	.0398	-.4028	.0433	-.4023	.0389	-.3639	-.3600
85	.0337	-.3463	.0367	-.3385	.0328	-.2977	-.2923
86	.0281	-.2792	.0306	-.2657	.0272	-.2232	-.2159
87	.0230	-.1984	.0250	-.1836	.0221	-.1380	-.1291
88	.0184	-.1044	.0200	-.0898	.0175	-.0381	-.0283
89	.0142	.0004	.0155	.0212	.0134	.0814	.0924
90	.0106	.1190	.0116	.1573	.0099	.2240	.2374
91	.0075	.2642	.0081	.3226	.0069	.3897	.4065
92	.0049	.4479	.0053	.5146	.0044	.5753	.5934
93	.0028	.6643	.0030	.7224	.0025	.7699	.7855
94	.0013	.8855	.0014	.9236	.0011	.9523	.9623
95	.0003	1.0641	.0003	1.0817	.0003	1.0903	1.0948
96	0.0000	1.1497	0.0000	1.1517	0.0000	1.1513	1.1517
97	.0003	1.1194	.0004	1.1097	.0003	1.1229	1.1206
98	.0012	.9874	.0015	.9688	.0011	1.0133	1.0159
99	.0027	.7990	.0032	.7753	.0025	.8518	.8601
100	.0048	.6020	.0057	.5716	.0044	.6710	.6814
101	.0074	.4267	.0088	.3830	.0068	.4934	.5025
102	.0105	.2827	.0125	.2203	.0098	.3316	.3376
103	.0142	.1623	.0168	.0852	.0134	.1904	.1933
104	.0183	.0555	.0217	-.0258	.0174	.0705	.0710
105	.0229	-.0401	.0271	-.1193	.0220	-.0309	-.0314
106	.0280	-.1227	.0332	-.2016	.0271	-.1180	-.1188
107	.0336	-.1924	.0398	-.2745	.0327	-.1945	-.1950
108	.0397	-.2518	.0470	-.3388	.0388	-.2625	-.2633
109	.0462	-.3040	.0549	-.3956	.0454	-.3232	-.3241
110	.0533	-.3509	.0632	-.4464	.0525	-.3775	-.3783
111	.0608	-.3940	.0722	-.4931	.0601	-.4262	-.4271
112	.0689	-.4341	.0817	-.5382	.0681	-.4705	-.4717
113	.0774	-.4719	.0918	-.5824	.0766	-.5132	-.5141
114	.0863	-.5082	.1024	-.6222	.0856	-.5544	-.5552
115	.0957	-.5421	.1135	-.6559	.0950	-.5919	-.5946
116	.1055	-.5725	.1252	-.6865	.1049	-.6258	-.6298
117	.1158	-.6001	.1374	-.7153	.1152	-.6570	-.6603
118	.1264	-.6254	.1500	-.7424	.1259	-.6857	-.6882
119	.1375	-.6488	.1632	-.7678	.1370	-.7122	-.7145
120	.1490	-.6706	.1768	-.7914	.1485	-.7369	-.7392
121	.1609	-.6909	.1909	-.8129	.1604	-.7599	-.7626
122	.1731	-.7096	.2054	-.8325	.1727	-.7813	-.7843
123	.1857	-.7268	.2203	-.8506	.1853	-.8011	-.8044
124	.1986	-.7426	.2356	-.8670	.1983	-.8196	-.8229
125	.2118	-.7570	.2513	-.8822	.2116	-.8367	-.8400
126	.2254	-.7702	.2674	-.8963	.2252	-.8527	-.8558
127	.2393	-.7821	.2839	-.9092	.2391	-.8676	-.8703
128	.2534	-.7928	.3007	-.9209	.2533	-.8813	-.8839
129	.2678	-.8022	.3178	-.9314	.2677	-.8941	-.8965
130	.2825	-.8105	.3351	-.9406	.2824	-.9057	-.9080
131	.2973	-.8175	.3528	-.9484	.2973	-.9164	-.9186
132	.3124	-.8231	.3707	-.9548	.3124	-.9260	-.9281
133	.3278	-.8272	.3889	-.9598	.3278	-.9346	-.9367
134	.3432	-.8295	.4073	-.9635	.3433	-.9421	-.9442
135	.3589	-.8297	.4258	-.9657	.3589	-.9485	-.9505
136	.3747	-.8272	.4446	-.9663	.3747	-.9539	-.9559
137	.3906	-.8201	.4635	-.9611	.3907	-.9582	-.9602
138	.4067	-.7930	.4825	-.8783	.4067	-.9615	-.9634

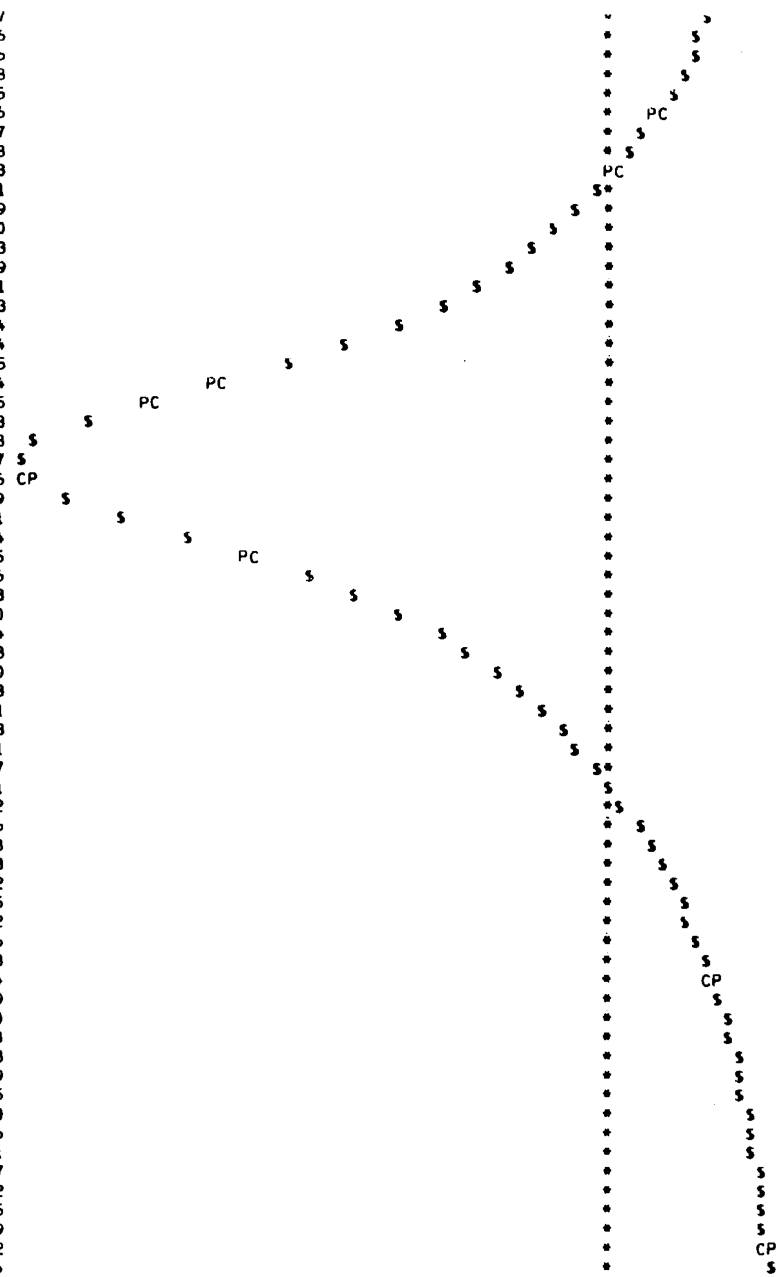


Figure A.2- Continued

139	.4228	-.5631	.5017	-.5838	.4229	-.9638	-.9654
140	.4390	-.3148	.5171	-.3465	.4391	-.9648	-.9661
141	.4553	-.2915	.5311	-.1311	.4554	-.9645	-.9633
142	.4716	-.3152	.5451	-.1671	.4717	-.9612	-.9255
143	.4879	-.3233	.5592	-.1884	.4880	-.9335	-.7937
144	.5043	-.3228	.5733	-.2011	.5044	-.6105	-.5424
145	.5206	-.3171	.5873	-.2080	.5207	-.2209	-.2912
146	.5369	-.3082	.6014	-.2104	.5370	-.1516	-.1462
147	.5532	-.2972	.6154	-.2094	.5532	-.1798	-.1794
148	.5694	-.2850	.6293	-.2059	.5694	-.1958	-.1976
149	.5855	-.2721	.6432	-.2005	.5855	-.2041	-.2071
150	.6015	-.2588	.6570	-.1938	.6015	-.2068	-.2104
151	.6174	-.2453	.6707	-.1858	.6174	-.2054	-.2089
152	.6332	-.2318	.6843	-.1769	.6332	-.2011	-.2044
153	.6488	-.2182	.6977	-.1676	.6488	-.1945	-.1978
154	.6643	-.2047	.7110	-.1578	.6643	-.1863	-.1895
155	.6796	-.1913	.7242	-.1478	.6796	-.1770	-.1800
156	.6947	-.1781	.7372	-.1376	.6946	-.1669	-.1697
157	.7096	-.1652	.7500	-.1271	.7095	-.1564	-.1589
158	.7242	-.1525	.7626	-.1163	.7242	-.1455	-.1478
159	.7387	-.1399	.7750	-.1049	.7386	-.1342	-.1365
160	.7528	-.1273	.7872	-.0930	.7527	-.1225	-.1248
161	.7667	-.1144	.7992	-.0806	.7666	-.1104	-.1126
162	.7803	-.1014	.8109	-.0676	.7802	-.0977	-.0999
163	.7936	-.0879	.8224	-.0541	.7935	-.0844	-.0865
164	.8066	-.0739	.8336	-.0404	.8065	-.0704	-.0725
165	.8193	-.0595	.8445	-.0268	.8192	-.0558	-.0579
166	.8316	-.0450	.8551	-.0134	.8315	-.0410	-.0429
167	.8436	-.0305	.8654	-.0003	.8435	-.0262	-.0280
168	.8552	-.0164	.8754	.0123	.8551	-.0116	-.0133
169	.8665	-.0026	.8851	.0242	.8663	.0026	.0009
170	.8773	.0105	.8944	.0349	.8772	.0162	.0145
171	.8878	.0227	.9034	.0442	.8876	.0288	.0271
172	.8978	.0335	.9120	.0522	.8977	.0400	.0383
173	.9074	.0427	.9203	.0600	.9073	.0494	.0478
174	.9166	.0509	.9282	.0687	.9165	.0577	.0564
175	.9254	.0596	.9358	.0793	.9253	.0666	.0655
176	.9337	.0699	.9429	.0926	.9336	.0774	.0762
177	.9415	.0828	.9497	.1092	.9414	.0908	.0898
178	.9489	.0993	.9560	.1294	.9488	.1079	.1071
179	.9558	.1201	.9619	.1530	.9557	.1298	.1285
180	.9622	.1447	.9675	.1792	.9621	.1556	.1540
181	.9681	.1718	.9726	.2072	.9681	.1840	.1826
182	.9735	.2008	.9772	.2364	.9735	.2142	.2131
183	.9785	.2313	.9815	.2668	.9784	.2460	.2451
184	.9829	.2633	.9853	.2982	.9829	.2791	.2783
185	.9868	.2967	.9887	.3307	.9868	.3136	.3128
186	.9902	.3317	.9916	.3643	.9902	.3495	.3485
187	.9931	.3686	.9941	.3992	.9931	.3863	.3856
188	.9955	.4081	.9961	.4356	.9955	.4242	.4243
189	.9973	.4504	.9977	.4688	.9973	.4638	.4610
190	.9987	.4941	.9989	.5157	.9987	.5021	.5080
191	.9995	.5272	.9995	.5977	.9995	.5144	.5889

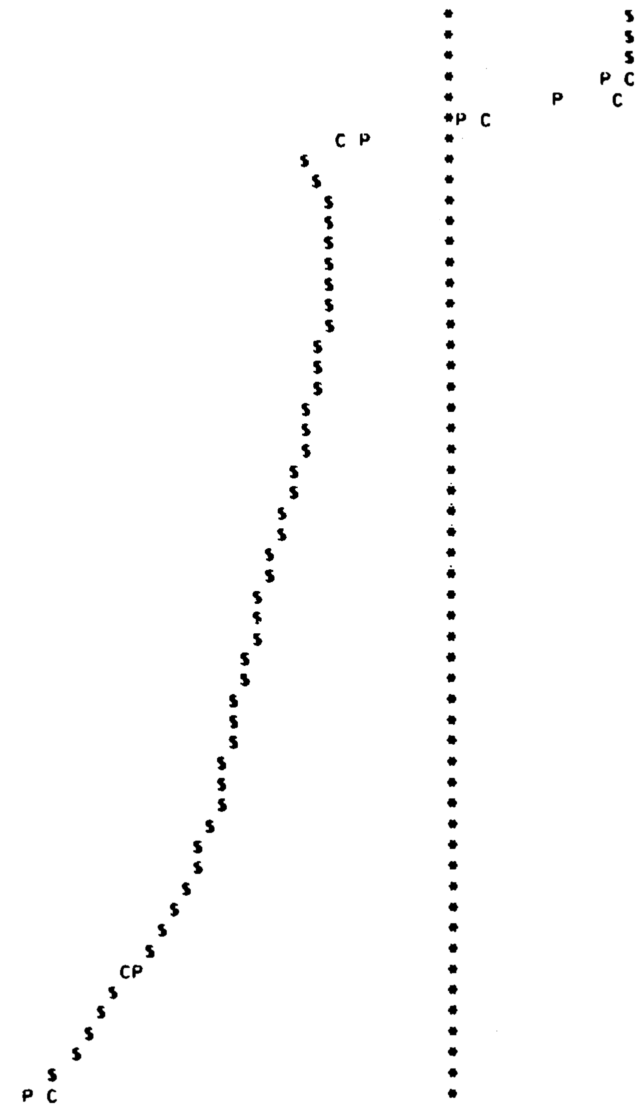


Figure A.2- Continued

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*******************************
* OUTPUT FOR CASE NO. 19 OF 19 *
*******************************


.....MACH NUMBER.
     VALUES OF PERTURBATION PARAMETERS.
     CRITICAL VALUE OF CP:

          M2 =  .7900

          Q2(1) =  .7900     (MACH NO.)

          Q2(2) =  .1200     (   TAU  )

          Q2(3) = 3.0000     (  PITCH )

          Q2(4) =  .2000     ( ALPHA1 )

          CPCRIT =  -.4638


.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.
  (* DENOTES POINT ON LOWER SURFACE)


     PERTURBATION SOLN:

          MINIMUM AT X =  .6154*    (POINT NO.  51)
          MAXIMUM AT X =  .0000*    (POINT NO.  96)
          2 CRITICAL POINT(S):
              1ST AT X =  .6726*    (AFTER POINT NO.  48)
              2ND AT X =  .6186     (AFTER POINT NO. 138)

     COMPARISON SOLN:

          MINIMUM AT X =  .6939*    (POINT NO.  36)
          MAXIMUM AT X = 0.0000     (POINT NO.  96)
          2 CRITICAL POINT(S):
              1ST AT X =  .7230*    (AFTER POINT NO.  34)
              2ND AT X =  .6903     (AFTER POINT NO. 155)


.....FINAL PRINTOUT AND GRAPHICAL DISPLAY OF CP
```

H = MAXIMUM VALUE OF CP = 1.1565
L = MINIMUM VALUE OF CP = -1.1128
* = CRITICAL VALUE OF CP = -.4638
P = VALUE OF CP PREDICTED BY PERTURBATION SOLUTION
C = VALUE OF CP IN COMPARISON SOLUTION
$ = AGREEMENT BETWEEN P AND C

PT	XBASE	CPBASE	XPERT	CPPERT	XCHEK	CPCHEK	CPPINT	H---*-------------------L
1	.9995	.6608	.9997	.7696	.9995	.7942	.7631	CP *
2	.9987	.5751	.9992	.7513	.9987	.7350	.7120	$ *
3	.9974	.5086	.9983	.6791	.9974	.6519	.6249	CP *
4	.9956	.4543	.9971	.6098	.9956	.5805	.5545	$ *
5	.9932	.4077	.9956	.5560	.9932	.5367	.4993	CP *
6	.9903	.3659	.9938	.5100	.9903	.5007	.4500	C P *

Figure A.2- Continued

7	.9869	.3274	.9916	.4686	.9869	.4632	.4053
8	.9830	.2914	.9890	.4312	.9830	.4229	.3639
9	.9786	.2575	.9862	.3965	.9786	.3867	.3257
10	.9737	.2254	.9830	.3641	.9736	.3535	.2894
11	.9683	.1952	.9795	.3330	.9682	.3232	.2544
12	.9623	.1671	.9757	.3034	.9623	.2952	.2212
13	.9559	.1417	.9715	.2752	.9558	.2698	.1945
14	.9490	.1202	.9671	.2476	.9489	.2486	.1749
15	.9416	.1031	.9623	.2215	.9415	.2322	.1614
16	.9338	.0897	.9572	.1991	.9336	.2200	.1534
17	.9255	.0790	.9519	.1818	.9253	.2109	.1486
18	.9167	.0700	.9462	.1688	.9165	.2041	.1428
19	.9075	.0615	.9403	.1595	.9073	.1980	.1347
20	.8978	.0522	.9340	.1536	.8976	.1912	.1252
21	.8877	.0413	.9275	.1499	.8876	.1837	.1150
22	.8773	.0292	.9208	.1460	.8771	.1760	.1044
23	.8664	.0163	.9137	.1406	.8662	.1681	.0933
24	.8551	.0030	.9065	.1339	.8549	.1602	.0826
25	.8435	-.0107	.8989	.1265	.8432	.1530	.0736
26	.8314	-.0245	.8912	.1186	.8312	.1468	.0671
27	.8191	-.0382	.8832	.1106	.8188	.1416	.0638
28	.8063	-.0517	.8750	.1023	.8061	.1379	.0638
29	.7933	-.0645	.8666	.0937	.7931	.1362	.0671
30	.7800	-.0766	.8580	.0853	.7797	.1372	.0738
31	.7663	-.0880	.8491	.0777	.7661	.1406	.0836
32	.7524	-.0990	.8402	.0715	.7521	.1459	.0964
33	.7382	-.1094	.8310	.0669	.7379	.1512	.1137
34	.7237	-.1194	.8216	.0641	.7235	-.4442	.1362
35	.7090	-.1289	.8122	.0632	.7088	-1.0718	.1665
36	.6941	-.1381	.8025	.0642	.6939	-1.1128	.0515
37	.6789	-.1467	.7927	.0672	.6787	-1.1105	-.2583
38	.6636	-.1546	.7829	.0719	.6634	-1.1086	-.7695
39	.6481	-.1614	.7728	.0782	.6479	-1.1065	-1.0096
40	.6324	-.1666	.7627	.0862	.6323	-1.1044	-1.1107
41	.6166	-.1698	.7525	.0960	.6165	-1.1021	-1.1113
42	.6007	-.1704	.7422	.1077	.6006	-1.0997	-1.1090
43	.5846	-.1676	.7319	.1221	.5845	-1.0971	-1.1064
44	.5685	-.1601	.7215	.1395	.5684	-1.0943	-1.1039
45	.5523	-.1461	.7110	.1603	.5522	-1.0912	-1.1010
46	.5360	-.1225	.7005	.1895	.5359	-1.0878	-1.0979
47	.5197	-.0800	.6900	-.0293	.5196	-1.0842	-1.0945
48	.5034	-.4036	.6794	-.2352	.5033	-1.0803	-1.0909
49	.4870	-.8588	.6596	-.8969	.4870	-1.0762	-1.0871
50	.4707	-.9641	.6375	-1.1105	.4706	-1.0717	-1.0830
51	.4544	-.9721	.6154	-1.1114	.4543	-1.0667	-1.0786
52	.4381	-.9743	.5934	-1.1078	.4381	-1.0614	-1.0738
53	.4220	-.9750	.5715	-1.1044	.4219	-1.0556	-1.0685
54	.4058	-.9746	.5496	-1.1006	.4058	-1.0494	-1.0625
55	.3898	-.9731	.5279	-1.0963	.3897	-1.0427	-1.0558
56	.3739	-.9707	.5064	-1.0917	.3738	-1.0354	-1.0484
57	.3582	-.9672	.4851	-1.0867	.3581	-1.0275	-1.0403
58	.3425	-.9628	.4639	-1.0813	.3424	-1.0188	-1.0316
59	.3271	-.9574	.4430	-1.0754	.3270	-1.0095	-1.0220
60	.3118	-.9510	.4223	-1.0686	.3117	-.9994	-1.0118
61	.2968	-.9437	.4019	-1.0610	.2966	-.9885	-1.0010
62	.2819	-.9354	.3818	-1.0523	.2817	-.9769	-.9894
63	.2673	-.9263	.3620	-1.0425	.2671	-.9645	-.9772
64	.2529	-.9163	.3425	-1.0317	.2527	-.9513	-.9640
65	.2388	-.9054	.3234	-1.0199	.2386	-.9373	-.9497
66	.2250	-.8935	.3047	-1.0071	.2247	-.9224	-.9346
67	.2115	-.8806	.2864	-.9934	.2112	-.9064	-.9184
68	.1983	-.8667	.2685	-.9785	.1979	-.8894	-.9012
69	.1854	-.8516	.2511	-.9625	.1850	-.8713	-.8828
70	.1729	-.8354	.2341	-.9452	.1724	-.8519	-.8629
71	.1607	-.8179	.2176	-.9266	.1602	-.8311	-.8415
72	.1489	-.7990	.2016	-.9064	.1483	-.8088	-.8189

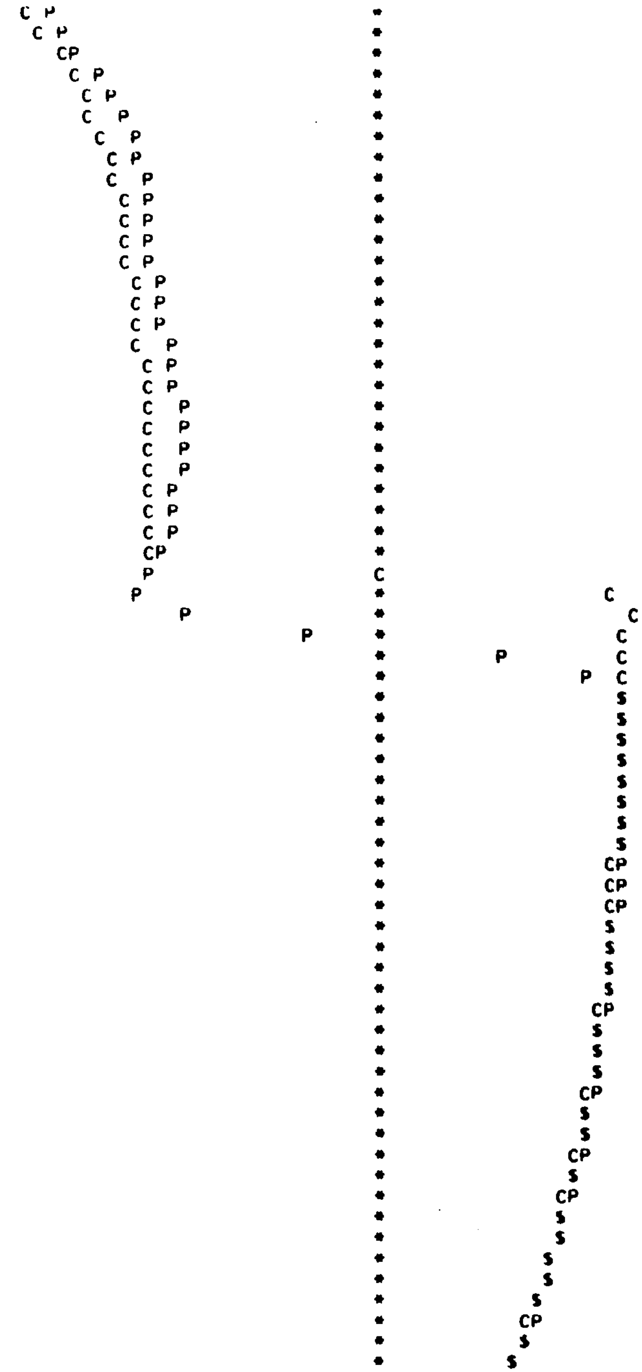

Figure A.2- Continued

73	.1374	-.7786	.1861	-.8845	.1369	-.7849	-.7948
74	.1264	-.7568	.1711	-.8608	.1258	-.7594	-.7691
75	.1157	-.7332	.1567	-.8354	.1151	-.7320	-.7417
76	.1055	-.7077	.1429	-.8081	.1048	-.7025	-.7125
77	.0957	-.6801	.1296	-.7787	.0950	-.6705	-.6812
78	.0863	-.6502	.1169	-.7468	.0856	-.6361	-.6480
79	.0774	-.6176	.1048	-.7125	.0766	-.5995	-.6109
80	.0689	-.5820	.0934	-.6760	.0681	-.5602	-.5683
81	.0609	-.5422	.0825	-.6369	.0601	-.5161	-.5209
82	.0534	-.4984	.0723	-.5918	.0526	-.4657	-.4709
83	.0463	-.4523	.0628	-.5379	.0455	-.4112	-.4196
84	.0398	-.4028	.0538	-.4802	.0389	-.3526	-.3627
85	.0337	-.3463	.0456	-.4206	.0328	-.2873	-.2988
86	.0281	-.2792	.0380	-.3551	.0272	-.2139	-.2241
87	.0230	-.1984	.0311	-.2806	.0221	-.1302	-.1345
88	.0184	-.1044	.0249	-.1905	.0175	-.0319	-.0281
89	.0142	.0004	.0193	-.0779	.0134	.0859	.0952
90	.0106	.1190	.0144	.0594	.0099	.2265	.2345
91	.0075	.2642	.0101	.2223	.0069	.3903	.3977
92	.0049	.4479	.0066	.4150	.0044	.5741	.5846
93	.0028	.6643	.0038	.6375	.0025	.7677	.7790
94	.0013	.8855	.0017	.8694	.0011	.9504	.9588
95	.0003	1.0641	.0004	1.0638	.0003	1.0907	1.0959
96	0.0000	1.1497	.0000	1.1565	0.0000	1.1557	1.1565
97	.0003	1.1194	.0004	1.1118	.0003	1.1359	1.1297
98	.0012	.9874	.0018	.9510	.0011	1.0358	1.0352
99	.0027	.7990	.0040	.7368	.0025	.8829	.8864
100	.0048	.6020	.0070	.5224	.0044	.7092	.7102
101	.0074	.4267	.0108	.3327	.0068	.5371	.5362
102	.0105	.2827	.0154	.1700	.0098	.3794	.3833
103	.0142	.1623	.0207	.0312	.0134	.2410	.2433
104	.0183	.0555	.0267	-.0821	.0174	.1229	.1176
105	.0229	-.0401	.0335	-.1726	.0220	.0226	.0071
106	.0280	-.1227	.0409	-.2479	.0271	-.0639	-.0870
107	.0336	-.1924	.0491	-.3139	.0327	-.1400	-.1623
108	.0397	-.2518	.0580	-.3743	.0388	-.2079	-.2265
109	.0462	-.3040	.0677	-.4295	.0454	-.2685	-.2840
110	.0533	-.3509	.0780	-.4803	.0525	-.3229	-.3367
111	.0608	-.3940	.0891	-.5287	.0601	-.3718	-.3859
112	.0689	-.4341	.1008	-.5770	.0681	-.4164	-.4317
113	.0774	-.4719	.1132	-.6217	.0766	-.4591	-.4735
114	.0863	-.5082	.1263	-.6585	.0856	-.5011	-.5136
115	.0957	-.5421	.1400	-.6902	.0950	-.5397	-.5533
116	.1055	-.5725	.1544	-.7204	.1049	-.5742	-.5918
117	.1158	-.6001	.1694	-.7490	.1152	-.6063	-.6273
118	.1264	-.6254	.1851	-.7759	.1259	-.6360	-.6574
119	.1375	-.6488	.2013	-.8009	.1370	-.6636	-.6833
120	.1490	-.6706	.2181	-.8239	.1485	-.6894	-.7081
121	.1609	-.6909	.2354	-.8451	.1604	-.7135	-.7319
122	.1731	-.7096	.2533	-.8647	.1727	-.7361	-.7547
123	.1857	-.7268	.2717	-.8828	.1853	-.7572	-.7764
124	.1986	-.7426	.2906	-.8996	.1983	-.7770	-.7963
125	.2118	-.7570	.3100	-.9151	.2116	-.7957	-.8150
126	.2254	-.7702	.3299	-.9296	.2252	-.8132	-.8326
127	.2393	-.7821	.3502	-.9432	.2391	-.8297	-.8491
128	.2534	-.7928	.3709	-.9556	.2533	-.8452	-.8646
129	.2678	-.8022	.3919	-.9669	.2677	-.8597	-.8789
130	.2825	-.8105	.4134	-.9769	.2824	-.8733	-.8923
131	.2973	-.8175	.4352	-.9855	.2973	-.8860	-.9049
132	.3124	-.8231	.4573	-.9928	.3124	-.8980	-.9168
133	.3277	-.8272	.4797	-.9991	.3278	-.9090	-.9281
134	.3432	-.8295	.5023	-1.0047	.3433	-.9193	-.9386
135	.3589	-.8297	.5253	-1.0099	.3589	-.9288	-.9485
136	.3747	-.8272	.5484	-1.0151	.3747	-.9375	-.9577
137	.3906	-.8201	.5717	-1.0207	.3907	-.9455	-.9662
138	.4067	-.7930	.5952	-.9748	.4067	-.9529	-.9738

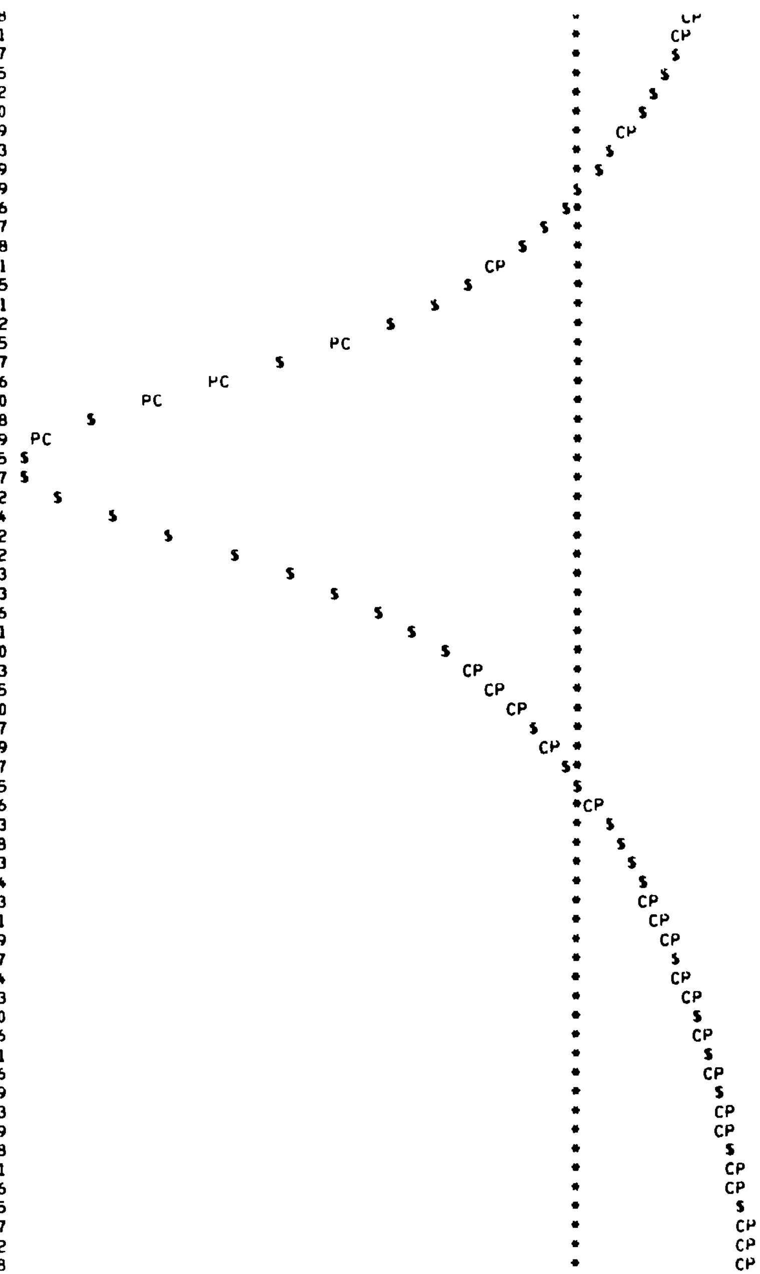

Figure A.2- Continued

139	.4228	-.5631	.6188	-.4599	.4229	-.9598	-.9806
140	.4390	-.3148	.6330	-.1491	.4391	-.9662	-.9868
141	.4553	-.2915	.6437	.0830	.4554	-.9722	-.9922
142	.4716	-.3152	.6543	.0439	.4717	-.9777	-.9968
143	.4879	-.3233	.6650	.0143	.4880	-.9827	-1.0011
144	.5043	-.3228	.6757	-.0103	.5044	-.9874	-1.0051
145	.5206	-.3171	.6864	-.0305	.5207	-.9918	-1.0088
146	.5369	-.3082	.6971	-.0464	.5370	-.9959	-1.0125
147	.5532	-.2972	.7077	-.0583	.5532	-.9997	-1.0163
148	.5694	-.2850	.7183	-.0667	.5694	-1.0033	-1.0202
149	.5855	-.2721	.7289	-.0721	.5855	-1.0065	-.9936
150	.6015	-.2588	.7393	-.0747	.6015	-1.0096	-.8358
151	.6174	-.2453	.7497	-.0751	.6174	-1.0125	-.4892
152	.6332	-.2318	.7601	-.0740	.6332	-1.0152	-.1455
153	.6488	-.2182	.7703	-.0718	.6488	-1.0179	.0642
154	.6643	-.2047	.7804	-.0688	.6643	-1.0204	.0164
155	.6796	-.1913	.7904	-.0650	.6796	-.8726	-.0175
156	.6947	-.1781	.8003	-.0602	.6946	-.3016	-.0428
157	.7096	-.1652	.8100	-.0541	.7095	.0950	-.0597
158	.7242	-.1525	.8196	-.0469	.7242	.0716	-.0697
159	.7387	-.1399	.8291	-.0385	.7386	.0581	-.0745
160	.7528	-.1273	.8383	-.0290	.7527	.0493	-.0748
161	.7667	-.1144	.8474	-.0184	.7666	.0440	-.0726
162	.7803	-.1014	.8563	-.0069	.7802	.0415	-.0689
163	.7936	-.0879	.8650	.0049	.7935	.0414	-.0635
164	.8066	-.0739	.8735	.0164	.8065	.0438	-.0563
165	.8193	-.0595	.8818	.0272	.8192	.0481	-.0472
166	.8316	-.0450	.8899	.0372	.8315	.0537	-.0360
167	.8436	-.0305	.8977	.0467	.8435	.0603	-.0229
168	.8552	-.0164	.9053	.0555	.8551	.0676	-.0085
169	.8665	-.0026	.9127	.0631	.8663	.0754	.0067
170	.8773	.0105	.9198	.0688	.8772	.0831	.0212
171	.8878	.0227	.9266	.0723	.8876	.0902	.0345
172	.8978	.0335	.9332	.0752	.8977	.0966	.0467
173	.9074	.0427	.9395	.0805	.9073	.1017	.0575
174	.9166	.0509	.9455	.0895	.9165	.1058	.0662
175	.9254	.0596	.9512	.1021	.9253	.1102	.0716
176	.9337	.0699	.9566	.1182	.9336	.1167	.0756
177	.9415	.0828	.9617	.1380	.9414	.1263	.0835
178	.9489	.0993	.9666	.1612	.9488	.1394	.0968
179	.9558	.1201	.9711	.1862	.9557	.1567	.1155
180	.9622	.1447	.9753	.2114	.9621	.1775	.1399
181	.9681	.1718	.9791	.2374	.9681	.2012	.1695
182	.9735	.2008	.9827	.2636	.9735	.2268	.2007
183	.9785	.2313	.9859	.2902	.9784	.2527	.2327
184	.9829	.2633	.9888	.3171	.9829	.2786	.2650
185	.9868	.2967	.9914	.3440	.9868	.3025	.2983
186	.9902	.3317	.9936	.3711	.9902	.3306	.3317
187	.9931	.3686	.9955	.3973	.9931	.3654	.3650
188	.9955	.4081	.9971	.4215	.9955	.3949	.3971
189	.9973	.4504	.9983	.4380	.9973	.3992	.4254
190	.9987	.4941	.9991	.3922	.9987	.3612	.4165
191	.9995	.5272	.9997	.2363	.9995	.0041	.2905

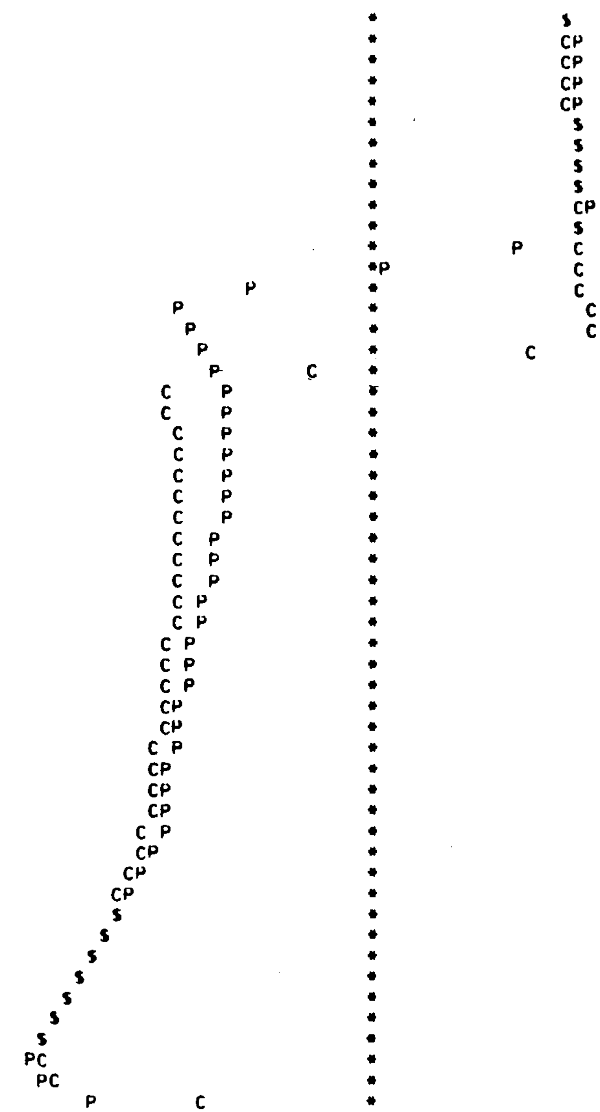

Figure A.2- Continued

TEST CASE

	MACH NO.	1ST PAR. MACH NO.	2ND PAR. TAU	3RD PAR. PITCH	4TH PAR. ALPHA1
BASE SOLUTION:	.780	.780	.110	3.200	.300
1ST CALIBRATION SOLN:	.790	.790	.110	3.200	.300
2ND CALIBRATION SOLN:	.780	.780	.120	3.200	.300
3RD CALIBRATION SOLN:	.780	.780	.110	3.000	.300
4TH CALIBRATION SOLN:	.780	.780	.110	3.200	.500
1ST PERTURBATION SOLN:	.775	.775	.120	3.000	.500
2ND PERTURBATION SOLN:	.780	.780	.115	3.000	.500
3RD PERTURBATION SOLN:	.780	.780	.120	3.100	.500
4TH PERTURBATION SOLN:	.780	.780	.120	3.000	.400
5TH PERTURBATION SOLN:	.785	.785	.110	3.000	.500
6TH PERTURBATION SOLN:	.785	.785	.115	3.100	.500
7TH PERTURBATION SOLN:	.785	.785	.115	3.000	.400
8TH PERTURBATION SOLN:	.785	.785	.120	3.200	.500
9TH PERTURBATION SOLN:	.785	.785	.120	3.100	.400
10TH PERTURBATION SOLN:	.785	.785	.120	3.000	.300
11TH PERTURBATION SOLN:	.790	.790	.105	3.000	.500
12TH PERTURBATION SOLN:	.790	.790	.110	3.100	.500
13TH PERTURBATION SOLN:	.790	.790	.110	3.000	.400
14TH PERTURBATION SOLN:	.790	.790	.115	3.100	.400
15TH PERTURBATION SOLN:	.790	.790	.115	3.000	.300
16TH PERTURBATION SOLN:	.790	.790	.120	3.300	.500
17TH PERTURBATION SOLN:	.790	.790	.120	3.200	.400
18TH PERTURBATION SOLN:	.790	.790	.120	3.100	.300
19TH PERTURBATION SOLN:	.790	.790	.120	3.000	.200

Figure A.2- Concluded

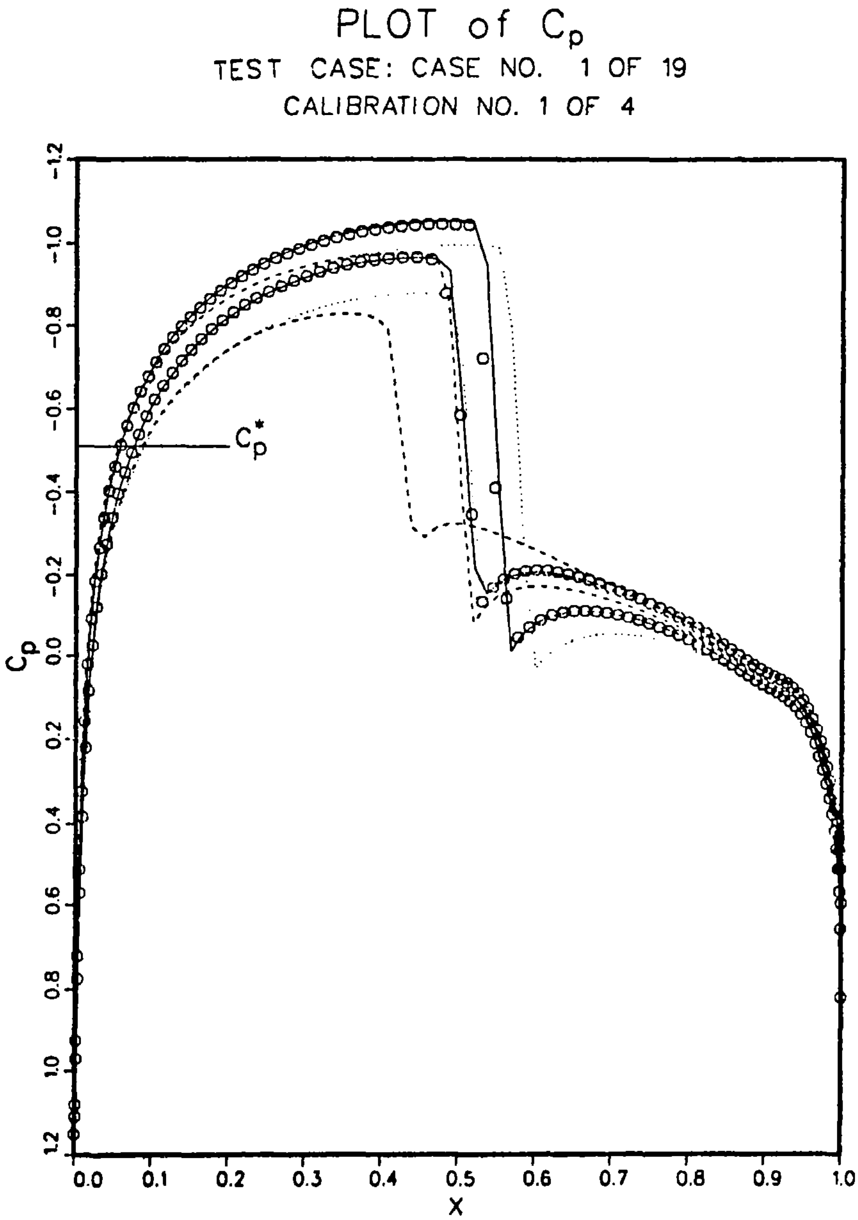

Figure A.3- Abbreviated plot output for sample case

PLOT of C_p

TEST CASE: CASE NO. 1 OF 19

CALIBRATION NO. 2 OF 4

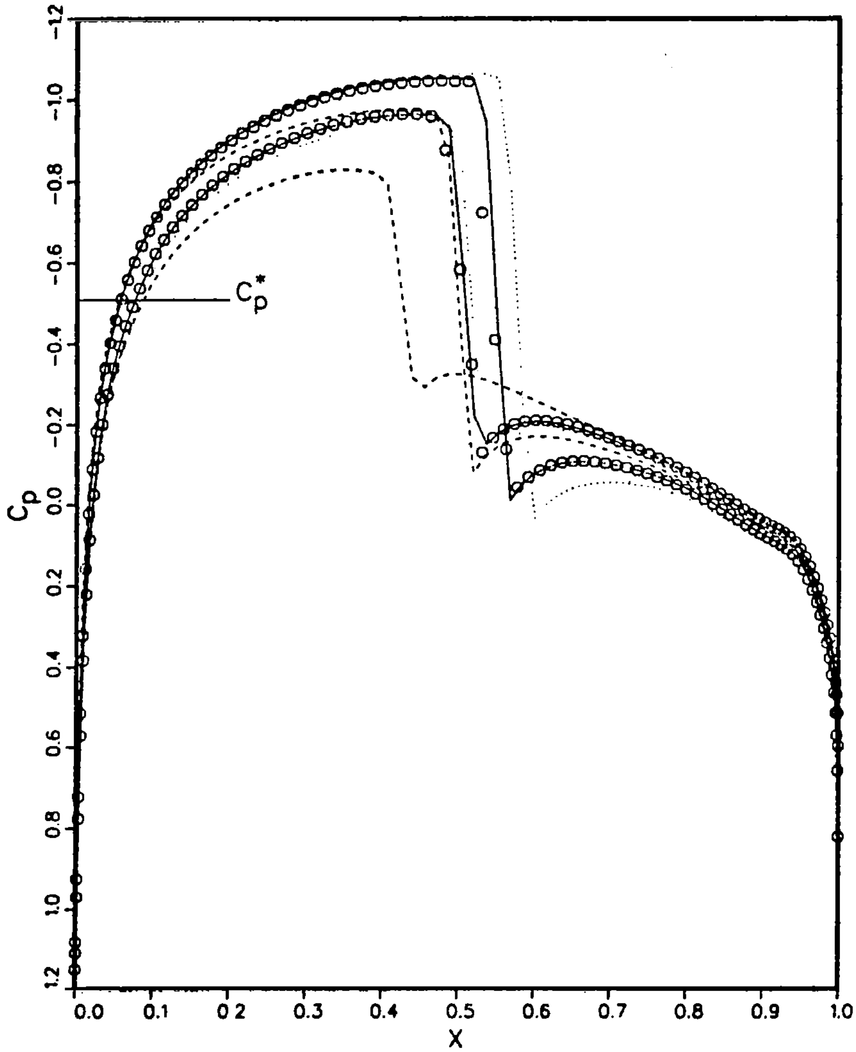

Figure A.3- Continued

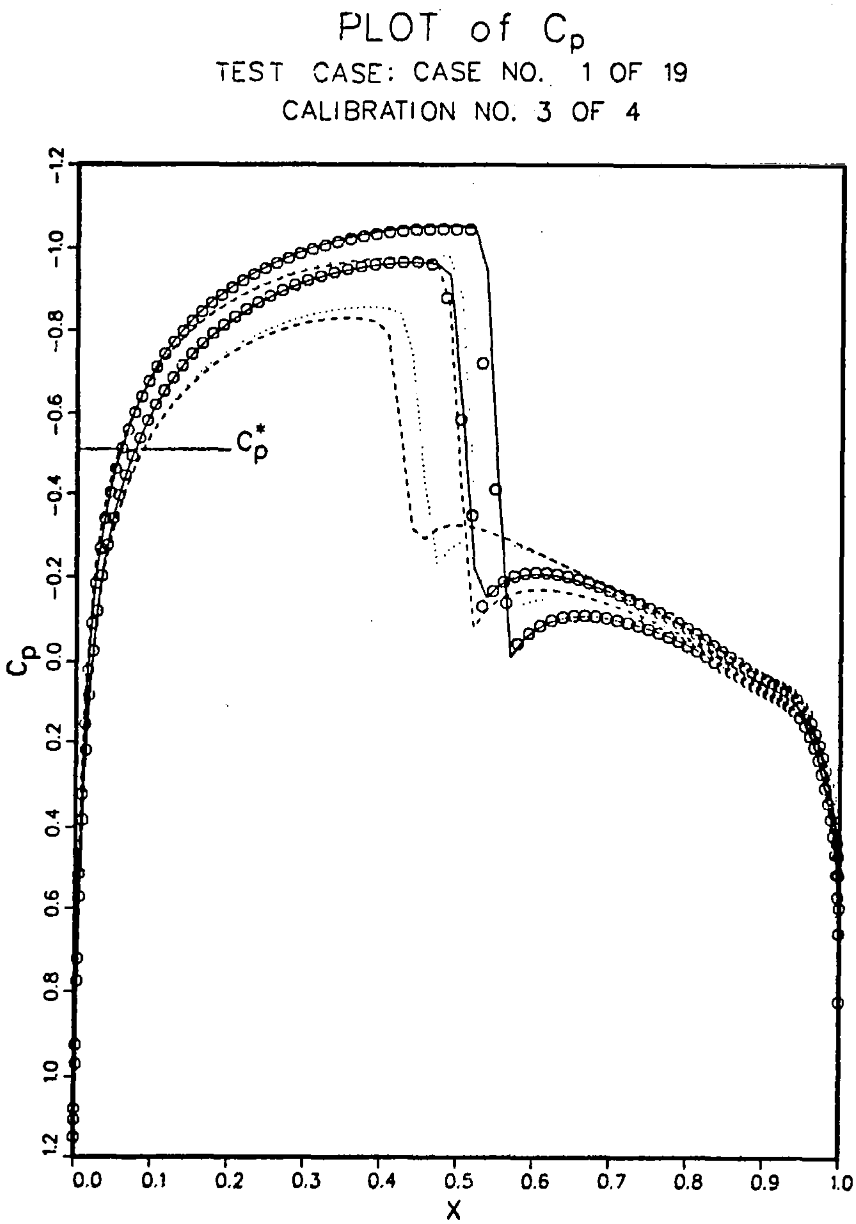

Figure A.3- Continued

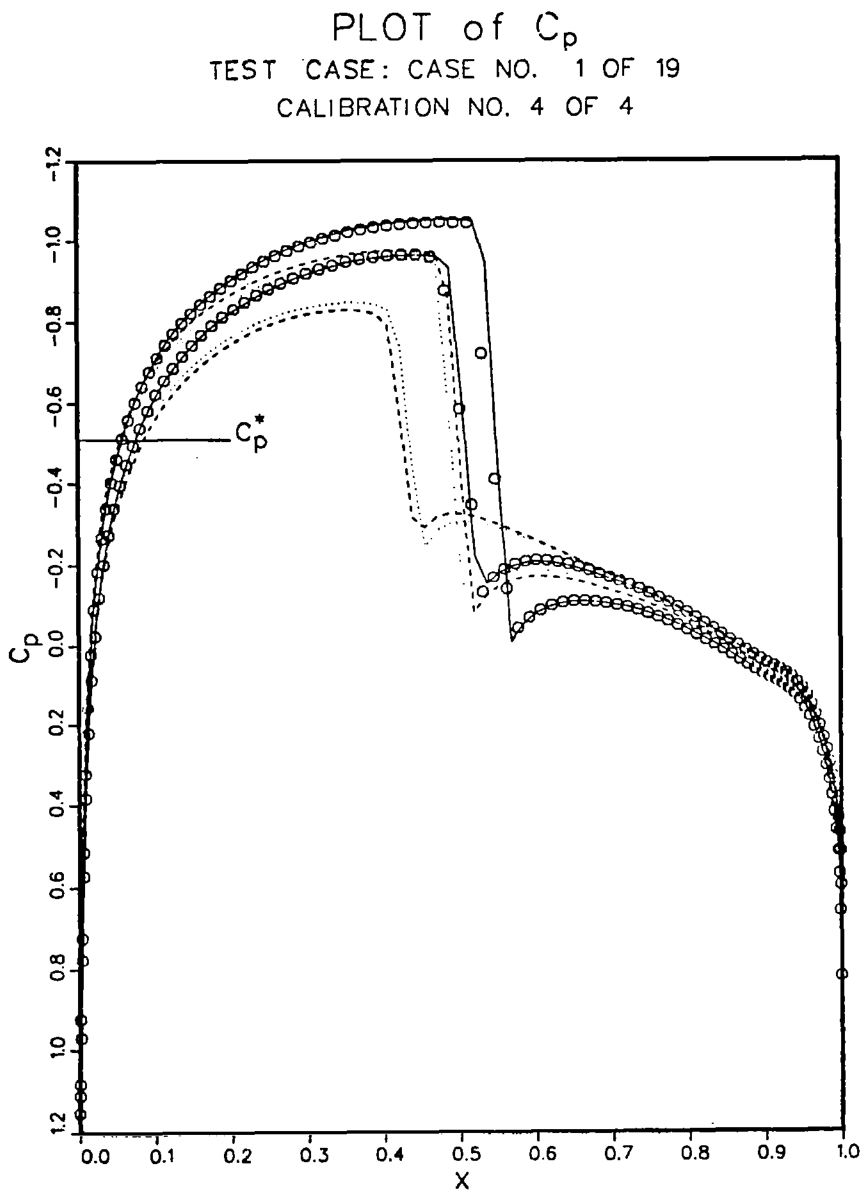

Figure A.3- Concluded

```
      PROGRAM PERTRB (INPUT,OUTPUT,TAPE1,TAPE5=INPUT,TAPE6=OUTPUT,
     % TAPE15)
C
C
C************************ MAIN PROGRAM PERTURB *************************
C
C     CALCULATES CONTINUOUS OR DISCONTINUOUS NONLINEAR PERTURBATION
C     SOLUTIONS WHICH REPRESENT A MULTIPLE-PARAMETER CHANGE IN EITHER
C     GEOMETRY OR FLOW CONDITIONS BY EMPLOYING A STRAINED-COORDINATE
C     PROCEDURE. THE METHOD UTILIZES UNIT PERTURBATIONS, DETERMINED
C     FOR EACH PARAMETER FROM A PREVIOUSLY CALCULATED COMMON BASE
C     SOLUTION, AND A CALIBRATION SOLUTION DISPLACED FROM IT BY SOME
C     REASONABLE CHANGE IN THE RELEVANT PARAMETER, TO PREDICT NEW
C     NONLINEAR SOLUTIONS OVER A RANGE OF PARAMETER VARIATION.
C
C     THIS CURRENT VERSION OF THE PROCEDURE IS CONFIGURED TO PREDICT AND
C     PLOT PRESSURE COEFFICIENTS ON A BLADE OR AIRFOIL SURFACE, AND CAN
C     ACCOUNT FOR THE MOTION OF
C                  (1) ONE OR MORE CRITICAL POINTS (SHOCK POINTS),
C                  (2) A STAGNATION POINT,
C                  (3) A MAXIMUM-SUCTION-PRESSURE POINT,
C     OR SIMULTANEOUSLY FOR ANY COMBINATION OF THESE.
C
C     THE PROGRAM IS ALSO CONFIGURED TO COMPARE THE PERTURBATION-
C     PREDICTED SOLUTIONS WITH THE CORRESPONDING EXACT SOLUTIONS
C     OBTAINED BY EMPLOYING THE SAME EXPENSIVE COMPUTATIONAL
C     PROCEDURE USED TO DETERMINE THE BASE AND CALIBRATION SOLUTIONS.
C     SEE THE SUBROUTINE INPUT FOR DETAILS.
C
C     N = NO. OF POINTS IN SURFACE PRESSURE DISTRIBUTION - ASSUMED EQUAL
C     FOR BASE, CALIBRATION, AND PREDICTED DISTRIBUTIONS.
C                           NOTE: N .LE. 200.
C
C     NPARAM = NO. OF PARAMETERS VARIED. NOTE: NPARAM .LE. 8.
C
C                          BASE               KTH CALIBRATION
C     MACH NO.              M0                      M1
C     PERT. PARAMS.   Q0(1),Q0(2),...   Q0(1),...,Q0(K-1),Q1,Q0(K+1),...
C
C     M2 = ONCOMING MACH NO. OF PREDICTED FLOW.
C     Q2(1),Q2(2),... = VALUES OF PERTURBED PARAMETERS IN PREDICTED
C                       FLOW.
C
C     COORDINATE STRAINING IS PIECEWISE LINEAR WITH END POINTS AND ONE
C     OR MORE USER-SELECTED INTERIOR POINTS HELD INVARIANT.
C
C     THE PROGRAM LOCATES MINIMUM, MAXIMUM, AND ALL CRITICAL POINTS
C     (SHOCK POINTS) IN THE BASE AND CALIBRATION SOLUTIONS, AND STORES
C     THESE IN THE ARRAYS XLOCO AND XLOC1 (IT IS ASSUMED THAT THE NUMBER
C     OF CRITICAL POINTS DOES NOT EXCEED FOUR) AS FOLLOWS
C
C               BASE                              CALIBRATION
C
C        XLOCO(1) = MINIMUM PT.             XLOC1(1) = MINIMUM PT.
C        XLOCO(2) = MAXIMUM PT.             XLOC1(2) = MAXIMUM PT.
C        XLOCO(3) = CRITICAL PT. NO. 1      XLOC1(3) = CRITICAL PT. NO. 1
C        ...      = ...                     ...      = ...
C        XLOCO(6) = CRITICAL PT. NO. 4      XLOC1(6) = CRITICAL PT. NO. 4
C
C     THE NUMBER OF POINTS SELECTED FROM THESE IS SPECIFIED BY NSELCT.
C     THE CORRESPONDING SUBSCRIPTS OF XLOCO AND XLOC1 ARE SPECIFIED IN
C     THE FIRST NSELCT ELEMENTS OF THE ARRAY LSELCT, E.G. TO SELECT THE
C     MAXIMUM POINT AND THE FIRST AND THIRD CRITICAL POINTS, ONE
C     SPECIFIES
C
C                              NSELCT = 3
C                              LSELCT(1) = 2
C                              LSELCT(2) = 3
C                              LSELCT(3) = 5
C
C     PROVISION IS ALSO MADE TO ALLOW THE USER TO SPECIFY THE POINTS IN
C     THE BASE AND CALIBRATION SOLUTIONS THAT WILL BE HELD INVARIANT, TO
C     A MAXIMUM OF SIX. SEE THE SUBROUTINE INPUT FOR DETAILS.
C
C*********************************************************************
C
      DIMENSION XCSAVE(8,200),YCSAVE(8,200),XUSAVE(8,200),YUNIT(8,200)
      DIMENSION XBASE(200),XCALB(200),XCHEK(200),XPERT(200),XUNIT(200),
     %          DELX(200)
      DIMENSION YBASE(200),YCALB(200),YPERT(200),YCHEK(200),YINTP(200),
     %          YPRTI(200)
      DIMENSION XLOC0(6),XLOC1(6),XLOC2(6),XLOC3(6),XFIX0(8),XFIX1(8)
      DIMENSION LCR0(4),LCR1(4),LCR2(4),LCR3(4),LOC0(6),LOC1(6),
     %          ISEQ0(8),ISEQ1(8),LSELCT(6)
      DIMENSION Q0(8),Q2(8),DEL1(8),XOUT(8),ORD(8),PARNAM(8)
      DIMENSION HEAD0(5),HEAD1(5),HEAD2(5),HEAD3(5)
      DIMENSION VNAM(2),FLAG(8),STRING(96),STRUNI(96)
      DIMENSION M1SAVE(8),M2SAVE(25),Q1SAVE(8),Q2SAVE(8,25)
      REAL M0,M1,M2,M1SAVE,M2SAVE
      COMMON /COEFF/ C(8,7),D(8,7)
      COMMON /HEAD/ TITLE(80)
      COMMON /PARAM/ PARNAM,LOC0,LOC1,LSELCT,N,NCASE,LSPEC,LUNIT,
     %               LCHEK,LPLOT,NSELCT,A,B,NPARAM,VNAM
      COMMON /PERT/ Q0,Q2,M0,M1,M2,Q1,YCR0,YCR1,YCR2,YMIN,YMAX
      COMMON /SAVE/ XCSAVE,YCSAVE
      COMMON /TBL/ HEAD0,HEAD1,HEAD2,M1SAVE,M2SAVE,Q1SAVE,Q2SAVE
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK,YPRTI
      DATA LTERM /0/, LCORR /0/
      DATA HEAD0 /4HBASE,4H SOL,4HUTIO,4HN:  ,4H    /,
     %     HEAD1 /4HCALI,4HBRAT,4HION ,4HSOLN,4H:   /,
     %     HEAD2 /4HPERT,4HURBA,4HTION,4H SOL,4HN:  /,
     %     HEAD3 /4HCOMP,4HARIS,4HON S,4HOLN:,4H    /
      DATA ORD /4H1ST ,4H2ND ,4H3RD ,4H4TH ,
     %          4H5TH ,4H6TH ,4H7TH ,4H8TH /
C
C*********************************************************************
C
C     USER-SUPPLIED STATEMENT FUNCTION YCRIT(Z) DETERMINES CRITICAL
C     VALUES OF FLOW VARIABLE YCRIT AS FUNCTION OF FLOW PARAMETER Z.
C     IGRAD (+1 OR -1) IS THE USER-SUPPLIED ALGEBRAIC SIGN OF DYCRIT/DX
C     USED IN LOCATING THE CRITICAL POINT.
C
C     IN THE PRESENT VERSION OF THE CODE, YCRIT REPRESENTS THE FULL-
C     POTENTIAL CRITICAL PRESSURE COEFFICIENT FOR AIR (GAMMA = 1.4), Z
```

```
C     IS THE FREE STREAM MACH NUMBER, AND IGRAD CORRESPONDS TO POSITIVE
C     PRESSURE GRADIENT (+1).
C
      YCRIT(Z)=2.0*(((2.0+0.4*Z**2)/2.4)**(1.4/0.4)-1.0)/(1.4*Z**2)
      IGRAD=1
C
C***********************************************************************
C
C.....PRINT BANNER PAGE.
C
      CALL BANNER
C
C.....READ LECHO AND ECHO INPUT DECK IF LECHO .EQ. 1.
C
      CALL ECHINP
C
C.....INPUT CONTROL, GEOMETRY, AND STRAINING PARAMETERS.
C
      CALL INPUT (1)
C
C.....WRITE TITLE AND INPUT PARAMETERS.
C
      WRITE (6,1000) TITLE
      WRITE (6,1010) N,A,B,NPARAM
C
      NFIX=NSELCT+2
      NSEG=NFIX-1
C
C.....PRINT INFORMATION REGARDING STRAINING TO BE USED.
C
      WRITE (6,1020) NFIX
      IF (LSPEC .EQ. 0) GO TO 10
      WRITE (6,1030)
      GO TO 30
   10 CONTINUE
      WRITE (6,1040)
      DO 20 I=1,NSELCT
      IF (LSELCT(I) .EQ. 1) WRITE (6,1050) VNAM
      IF (LSELCT(I) .EQ. 2) WRITE (6,1060) VNAM
      IF (LSELCT(I) .LE. 2) GO TO 20
      LCORR=1
      LPR=LSELCT(I)-2
      WRITE (6,1070) VNAM,ORD(LPR)
   20 CONTINUE
   30 CONTINUE
C
C.....BEGIN CALCULATIONS ON BASE SOLUTION.
C
      CALL INPUT (2)
      YCR0=YCRIT(M0)
      WRITE (6,1080) HEAD0
      WRITE (6,1090) VNAM
      WRITE (6,1100) M0
      IF (NPARAM .EQ. 1) WRITE (6,1110) Q0(1),PARNAM(1)
      IF (NPARAM .GT. 1) WRITE (6,1120) (K,Q0(K),PARNAM(K),K=1,NPARAM)
      WRITE (6,1130) VNAM,YCR0
C
C.....NORMALIZE X COORDINATES AND LOCATE MINIMUM, MAXIMUM, AND CRITICAL
C     POINTS FOR BASE SOLUTION.
C
      CALL SCALE (N,XBASE,1,A,B)
      CALL LOCATE (N,XBASE,YBASE,YCR0,IGRAD,LMN0,LMX0,NCR0,LCR0,XLOC0)
      YBCMIN=YBASE(LMN0)
      YBCMAX=YBASE(LMX0)
      WRITE (6,1140)
      WRITE (6,1150)
      CALL UPLOW (A,B,XLOC0,6,NCR0+2,XOUT,FLAG)
      WRITE (6,1160) XOUT(1),FLAG(1),LMN0,XOUT(2),FLAG(2),LMX0
      IF (NCR0 .GT. 0) WRITE (6,1170) NCR0,
     $      (ORD(I),XOUT(I+2),FLAG(I+2),LCR0(I),I=1,NCR0)
C
C.....LOAD SELECTED STRAINING POINTS INTO FIXED-POINT ARRAY FOR BASE
C     SOLUTION.
C
      XFIX0(1)=0.0
      XFIX0(NFIX)=1.0
      IF (LSPEC .EQ. 1) WRITE (6,1180)
      DO 50 I=1,NSELCT
      IF (LSPEC .EQ. 0) GO TO 40
      XFIX0(I+1)=XBASE(LOC0(I))
      WRITE (6,1190) LOC0(I)
      GO TO 50
   40 CONTINUE
      XFIX0(I+1)=XLOC0(LSELCT(I))
   50 CONTINUE
C
C.....ARRANGE SELECTED FIXED POINTS IN A MONOTONE SEQUENCE.
C
      CALL SORT (NFIX,XFIX0,ISEQ0)
      WRITE (6,1200)
      WRITE (6,1150)
      CALL UPLOW (A,B,XFIX0,8,NFIX,XOUT,FLAG)
      WRITE (6,1210) (I,XOUT(I),FLAG(I),I=1,NFIX)
C
C.....END CALCULATIONS ON BASE SOLUTION.
C
C***********************************************************************
C
C.....BEGIN CALCULATIONS ON CALIBRATION SOLUTIONS.
C
      DO 180 K=1,NPARAM
      CALL INPUT (3)
      M1SAVE(K)=M1
      Q1SAVE(K)=Q1
      YCR1=YCRIT(M1)
      DEL1(K)=Q1-Q0(K)
      CALL COPY (1,N,K,XCALB,XCSAVE)
      CALL COPY (1,N,K,YCALB,YCSAVE)
      IF (NPARAM .EQ. 1) WRITE (6,1080) HEAD1
      IF (NPARAM .GT. 1) WRITE (6,1220) ORD(K),HEAD1
      WRITE (6,1090) VNAM
      WRITE (6,1230) M1
      IF (NPARAM .GT. 1) WRITE (6,1240)
      DO 60 KK=1,NPARAM
```

```
      IF (NPARAM .EQ. 1) WRITE (6,1250) Q1,PARNAM(1)
      IF (NPARAM .GT. 1 .AND. KK .EQ. K) WRITE (6,1260) KK,Q1,PARNAM(KK)
      IF (KK .NE. K) WRITE (6,1270) KK,Q0(KK),PARNAM(KK)
   60 CONTINUE
      WRITE (6,1130) VNAM,YCR1
C
C.....NORMALIZE X COORDINATES AND LOCATE MINIMUM, MAXIMUM, AND CRITICAL
C     POINTS FOR KTH CALIBRATION SOLUTION.
C
      CALL SCALE (N,XCALB,1,A,B)
      CALL LOCATE (N,XCALB,YCALB,YCR1,IGRAD,LMN1,LMX1,NCR1,LCR1,XLOC1)
      YTMIN=YCALB(LMN1)
      YTMAX=YCALB(LMX1)
      IF (YTMIN .LT. YBCMIN) YBCMIN=YTMIN
      IF (YTMAX .GT. YBCMAX) YBCMAX=YTMAX
      WRITE (6,1140)
      WRITE (6,1150)
      CALL UPLOW (A,B,XLOC1,6,NCR1+2,XOUT,FLAG)
      WRITE (6,1160) XOUT(1),FLAG(1),LMN1,XOUT(2),FLAG(2),LMX1
      IF (NCR1 .GT. 0) WRITE (6,1170) NCR1,
     $     (ORD(I),XOUT(I+2),FLAG(I+2),LCR1(I),I=1,NCR1)
C
C.....CHECK FOR INVALID STRAINING SPECIFICATION IF LSPEC = 0.
C
      IF (LSPEC .EQ. 1) GO TO 80
      ICOUNT=0
      DO 70 I=1,NSELCT
      IF (LSELCT(I) .LE. 2) GO TO 70
      ICOUNT=ICOUNT+1
      IF (NCR0 .NE. NCR1) LTERM=1
   70 CONTINUE
C
C.....STOP EXECUTION IF CRITICAL POINTS ARE TO BE USED IN STRAINING AND
C     NUMBER OF CRITICAL POINTS IN BASE AND CALIBRATION SOLUTIONS ARE
C     UNEQUAL.
C
      IF (LTERM .EQ. 1) GO TO 900
C
C.....STOP EXECUTION IF NUMBER OF CRITICAL POINTS SELECTED EXCEEDS
C     NUMBER ACTUALLY LOCATED.
C
      IF (ICOUNT .GT. NCR0) GO TO 905
C
   80 CONTINUE
C
C.....LOAD SELECTED STRAINING POINTS INTO FIXED-POINT ARRAY FOR KTH
C     CALIBRATION SOLUTION.
C
      XFIX1(1)=0.0
      XFIX1(NFIX)=1.0
      IF (LSPEC .EQ. 1) WRITE (6,1180)
      DO 100 I=1,NSELCT
      IF (LSPEC .EQ. 0) GO TO 90
      XFIX1(I+1)=XCALB(LOC1(I))
      WRITE (6,1190) LOC1(I)
      GO TO 100
   90 CONTINUE
      XFIX1(I+1)=XLOC1(LSELCT(I))
  100 CONTINUE
C
C.....ARRANGE SELECTED FIXED POINTS IN A MONOTONE SEQUENCE.
C
      CALL SORT (NFIX,XFIX1,ISEQ1)
      WRITE (6,1200)
      WRITE (6,1150)
      CALL UPLOW (A,B,XFIX1,8,NFIX,XOUT,FLAG)
      WRITE (6,1210) (I,XOUT(I),FLAG(I),I=1,NFIX)
C
C.....STOP EXECUTION IF ORDER OF OCCURRENCE OF CRITICAL POINTS IN BASE
C     AND CALIBRATION SOLUTIONS DOES NOT CORRESPOND.
C
      IF (LSPEC .EQ. 1) GO TO 120
      DO 110 I=1,NFIX
      IF (ISEQ0(I) .NE. ISEQ1(I)) GO TO 910
  110 CONTINUE
  120 CONTINUE
C
C.....COMPUTE COEFFICIENTS IN KTH UNIT STRAINING OF XBASE
C
C           XSTR = C(K,I) + D(K,I)*XBASE,  I=1,2, ... ,NSEG,
C
C     WHERE NSEG IS THE NUMBER OF LINEAR SEGMENTS.
C
      DO 130 I=1,NSEG
      CNUM=XFIX1(I)*XFIX0(I+1)-XFIX1(I+1)*XFIX0(I)
      DNUM=XFIX1(I+1)-XFIX1(I)
      DENOM=XFIX0(I+1)-XFIX0(I)
      C(K,I)=CNUM/DENOM
      D(K,I)=DNUM/DENOM
  130 CONTINUE
C
C.....DETERMINE KTH UNIT STRAINING OF XBASE.
C
      CALL STRAIN (N,K,NSEG,XFIX0,XBASE,1.0,DELX)
      DO 140 I=1,N
  140 XUNIT(I)=XBASE(I)+DELX(I)
C
C.....INTERPOLATE CALIBRATION SOLUTION TO BASE FLOW POINTS CORRESPONDING
C     TO UNIT STRAINING.
C
      CALL INTERP (N,XCALB,YCALB,XUNIT,YINTP)
C
C.....CORRECT VALUES ON EITHER SIDE OF CRITICAL POINTS, IF THESE ARE
C     USED IN STRAINING.
C
      IF (LCORR .EQ. 0) GO TO 160
      DO 150 I=1,NCR1
      YINTP(LCR0(I))=YCALB(LCR1(I))
      YINTP(LCR0(I)+1)=YCALB(LCR1(I)+1)
  150 CONTINUE
  160 CONTINUE
C
C.....DETERMINE THE KTH UNIT PERTURBATION.
C
```

```
      DO 170 I=1,N
  170 YUNIT(K,I)=(YINTP(I)-YBASE(I))/DEL1(K)
C
C.....SAVE UNIT STRAINING IF REQUIRED FOR LATER PRINTOUT.
C
      IF (LUNIT .EQ. 0) GO TO 180
      CALL SCALE (N,XUNIT,2,A,B)
      CALL COPY (1,N,K,XUNIT,XUSAVE)
  180 CONTINUE
C
C.....END CALCULATIONS ON CALIBRATION SOLUTIONS.
C
C***********************************************************************
C
C.....PRINT UNIT PERTURBATION(S) AND UNIT STRAINING(S) IF LUNIT .NE. 0.
C
      IF (LUNIT .EQ. 0) GO TO 240
      CALL SCALE (N,XBASE,2,A,B)
      IRPT=0
      IF (NPARAM .GT. 4) IRPT=1
      KSTART=1
      KSTOP=4
      IF (KSTOP .GT. NPARAM) KSTOP=NPARAM
      GO TO 200
  190 KSTART=5
      KSTOP=NPARAM
  200 CONTINUE
      WRITE (6,1280) VNAM
      IF (NPARAM .GT. 1) WRITE (6,1290) KSTART,KSTOP
      IF (NPARAM .EQ. 1) WRITE (6,1300)
      IF (NPARAM .EQ. 1) GO TO 210
      NUM=KSTOP-KSTART+1
      IF (NUM .EQ. 1) WRITE (6,1310) (ORD(K),K=KSTART,KSTOP)
      IF (NUM .EQ. 2) WRITE (6,1320) (ORD(K),K=KSTART,KSTOP)
      IF (NUM .EQ. 3) WRITE (6,1330) (ORD(K),K=KSTART,KSTOP)
      IF (NUM .EQ. 4) WRITE (6,1340) (ORD(K),K=KSTART,KSTOP)
  210 CONTINUE
      CALL FILL (1,0,STRUNI)
      KLAST=20*(KSTOP-KSTART+1)
      WRITE (6,1350) (STRUNI(K),K=1,KLAST)
      WRITE (6,1360)
      DO 220 I=1,N
  220 WRITE (6,1370) I,XBASE(I),(XUSAVE(K,I),YUNIT(K,I),K=KSTART,KSTOP)
      IF (IRPT .EQ. 0) GO TO 230
      IRPT=0
      GO TO 190
  230 CALL SCALE (N,XBASE,1,A,B)
  240 CONTINUE
C
C.....CONSTRUCT PERTURBATION SOLUTIONS FOR TEST CASES (AND COMPARE WITH
C     EXACT SOLUTION, IF AVAILABLE).
C
      DO 330 ICASE=1,NCASE
      CALL INPUT (4)
      M2SAVE(ICASE)=M2
      YCR2=YCRIT(M2)
      YCR3=YCR2
C
C.....INITIALIZE STRAINED COORDINATE AND PERTURBATION SOLUTION.
C
      DO 250 I=1,N
      XPERT(I)=XBASE(I)
  250 YPERT(I)=YBASE(I)
C
C.....ADD IN CONTRIBUTIONS FROM ALL PERTURBATIONS.
C
      DO 270 K=1,NPARAM
      Q2SAVE(K,ICASE)=Q2(K)
      DEL2=Q2(K)-Q0(K)
      DEL21=DEL2/DEL1(K)
      CALL STRAIN (N,K,NSEG,XFIX0,XBASE,DEL21,DELX)
      DO 260 I=1,N
      XPERT(I)=XPERT(I)+DELX(I)
  260 YPERT(I)=YPERT(I)+DEL2*YUNIT(K,I)
  270 CONTINUE
C
C.....ADJUST VALUES NEAR CRITICAL POINT FOR MONOTONE BEHAVIOR.
C
      IF (LCORR .EQ. 1) CALL MONO (NCR0,LCR0,XPERT,YPERT)
C
C.....LOCATE MINIMUM, MAXIMUM, AND CRITICAL POINTS IN PERTURBATION
C     SOLUTION.
C
      CALL SCALE (N,XPERT,2,A,B)
      CALL SCALE (N,XPERT,1,A,B)
      CALL LOCATE (N,XPERT,YPERT,YCR2,IGRAD,LMN2,LMX2,NCR2,LCR2,XLOC2)
      YMIN=YPERT(LMN2)
      YMAX=YPERT(LMX2)
      YPCMIN=YMIN
      YPCMAX=YMAX
      WRITE (6,1380) ICASE,NCASE
      WRITE (6,1090) VNAM
      WRITE (6,1390) M2
      IF (NPARAM .EQ. 1) WRITE (6,1400) Q2(1),PARNAM(1)
      IF (NPARAM .GT. 1) WRITE (6,1410) (K,Q2(K),PARNAM(K),K=1,NPARAM)
      WRITE (6,1130) VNAM,YCR2
      WRITE (6,1140)
      WRITE (6,1150)
      CALL UPLOW (A,B,XLOC2,6,NCR2+2,XOUT,FLAG)
      WRITE (6,1420) HEAD2,XOUT(1),FLAG(1),LMN2,XOUT(2),FLAG(2),LMX2
      IF (NCR2 .GT. 0) WRITE (6,1430) NCR2,
     $      (ORD(I),XOUT(I+2),FLAG(I+2),LCR2(I),I=1,NCR2)
      CALL SCALE (N,XBASE,2,A,B)
C
C.....IF LCHEK .NE. 0 READ IN DATA FOR COMPARISON SOLUTION AND LOCATE
C     MINIMUM, MAXIMUM, AND CRITICAL POINTS.
C
      IF (LCHEK .EQ. 0) GO TO 290
      CALL INPUT (5)
      CALL SCALE (N,XCHEK,1,A,B)
      CALL LOCATE (N,XCHEK,YCHEK,YCR3,IGRAD,LMN3,LMX3,NCR3,LCR3,XLOC3)
      YMIN=YCHEK(LMN3)
      YMAX=YCHEK(LMX3)
      IF (YMIN .LT. YPCMIN) YPCMIN=YMIN
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```
      IF (YMAX .GT. YPCMAX) YPCMAX=YMAX
      CALL UPLOW (A,B,XLOC3,6,NCR3+2,XOUT,FLAG)
      WRITE (6,1420) HEAD3,XOUT(1),FLAG(1),LMN3,XOUT(2),FLAG(2),LMX3
      IF (NCR3 .GT. 0) WRITE (6,1430) NCR3,
     %     (ORD(I),XOUT(I+2),FLAG(I+2),LCR3(I),I=1,NCR3)
      CALL INTERP (N,XPERT,YPERT,XCHEK,YPRTI)
      CALL LOCATE (N,XCHEK,YPRTI,YCR3,IGRAD,LMN3,LMX3,NCR3,LCR3,XLOC3)
      YTMIN=YPRTI(LMN3)
      YTMAX=YPRTI(LMX3)
      IF (YTMIN .LT. YMIN) YMIN=YTMIN
      IF (YTMAX .GT. YMAX) YMAX=YTMAX
      CALL SCALE (N,XPERT,2,A,B)
      CALL SCALE (N,XCHEK,2,A,B)
      CALL FILL (2,0,STRING)
      WRITE (6,1440) VNAM
      WRITE (6,1450) VNAM,YMAX,VNAM,YMIN,VNAM,YCR2,VNAM,VNAM
      WRITE (6,1460) VNAM,VNAM,VNAM,VNAM,(STRING(I),I=1,72)
      DO 280 I=1,N
      CALL FILL (3,I,STRING)
  280 WRITE (6,1470) I,XBASE(I),YBASE(I),XPERT(I),YPERT(I),
     % XCHEK(I),YCHEK(I),YPRTI(I),(STRING(II),II=1,72)
      GO TO 310
  290 CONTINUE
C
      CALL SCALE (N,XPERT,2,A,B)
      CALL FILL (2,0,STRING)
      WRITE (6,1440) VNAM
      WRITE (6,1480) VNAM,YMAX,VNAM,YMIN,VNAM,YCR2,VNAM
      WRITE (6,1490) VNAM,VNAM,STRING
      DO 300 I=1,N
      CALL FILL(3,I,STRING)
  300 WRITE (6,1500) I,XBASE(I),YBASE(I),XPERT(I),YPERT(I),STRING
  310 CONTINUE
C
C.....IF LPLOT .NE. 0 GENERATE PERIPHERAL PLOT OF PERTURBATION AND
C     COMPARISON SOLUTIONS.
C
      IF (LPLOT .EQ. 0) GO TO 320
      YMIN=YBCMIN
      YMAX=YBCMAX
      IF (YPCMIN .LT. YMIN) YMIN=YPCMIN
      IF (YPCMAX .GT. YMAX) YMAX=YPCMAX
      CALL DRVPLT (ICASE,N,NCASE,NPARAM,YMIN,YMAX,YCR2)
  320 CONTINUE
      CALL SCALE (N,XBASE,1,A,B)
  330 CONTINUE
C
C.....PRINT FINAL TABLE SUMMARIZING CALCULATIONS.
C
      CALL TABLE (NPARAM,NCASE,PARNAM,M0,Q0)
      GO TO 999
C
C.....ABNORMAL TERMINATION OF COMPUTATION.
C
  900 WRITE (6,9000)
      GO TO 999
  905 WRITE (6,9050)
      GO TO 999
  910 WRITE (6,9100)
C
  999 WRITE (6,9500)
      STOP
C
C.....I/O FORMAT STATEMENTS FOLLOW.
C
 1000 FORMAT (1H1,132(1H*)/
     %        1X,1H*,25X,80A1,25X,1H*/
     %        1X,132(1H*)///)
 1010 FORMAT (1X,29H.....LIST OF INPUT PARAMETERS//
     %        6X,3HN =,I4//
     %        6X,3HA =,F5.1,4X,3HB =,F5.1//
     %        6X,8HNPARAM =,I2///)
 1020 FORMAT (1X,22H.....STRAINING OPTIONS//
     %        6X,23HNUMBER OF FIXED POINTS:,I2/)
 1030 FORMAT (6X,40HFIXED POINTS WILL BE PRESELECTED BY USER/
     %        6X,37HAND LISTED BELOW IN PRINTOUT FOR BASE/
     %        6X,26HAND CALIBRATION SOLUTIONS.///)
 1040 FORMAT (6X,45HFIXED POINTS WILL BE AUTOMATICALLY DETERMINED/
     %        6X,44HBY THE PROGRAM FOR ALL SOLUTIONS AS FOLLOWS://
     %        11X,14HTWO END POINTS)
 1050 FORMAT (11X,16HPOINT OF MINIMUM,1X,2A1)
 1060 FORMAT (11X,16HPOINT OF MAXIMUM,1X,2A1)
 1070 FORMAT (11X,2A1,6HCRIT (,A4,6HPOINT))
 1080 FORMAT (1H1,26HRESULTS OF COMPUTATIONS ON,1X,5A4///)
 1090 FORMAT (1X,17H.....MACH NUMBER,/
     %        6X,34HVALUES OF PERTURBATION PARAMETERS,/
     %        6X,17HCRITICAL VALUE OF,1X,2A1,1H:/)
 1100 FORMAT (11X,4HM0 =,F7.4/)
 1110 FORMAT (11X,4HQ0 =,F7.4,5X,1H(,A8,1H)/)
 1120 FORMAT (11X,3HQ0(,I1,3H) =,F7.4,5X,1H(,A8,1H)/)
 1130 FORMAT (11X,2A1,6HCRIT =,F8.4///)
 1140 FORMAT (1X,47H.....LOCATIONS OF MIN., MAX., AND CRITICAL PTS.)
 1150 FORMAT (3X,34H(* DENOTES POINT ON LOWER SURFACE)/)
 1160 FORMAT (6X,14HMINIMUM AT X =,F7.4,A1,3X,10H(POINT NO.,I4,1H)/
     %        6X,14HMAXIMUM AT X =,F7.4,A1,3X,10H(POINT NO.,I4,1H))
 1170 FORMAT (6X,I1,1X,18HCRITICAL POINT(S):/
     %        (10X,A4,6HAT X =,1X,F6.4,A1,3X,
     %             16H(AFTER POINT NO.,I4,1H)))
 1180 FORMAT (1X,5(1H*),1X,32HFIXED POINTS PRESELECTED BY USER,
     %        1X,5(1H*)/)
 1190 FORMAT (6X,2HX(,I3,1H))
 1200 FORMAT (///1X,29H.....LOCATION OF FIXED POINTS)
 1210 FORMAT (11X,5HXFIX(,I1,3H) =,F7.4,A1)
 1220 FORMAT (1H1,26HRESULTS OF COMPUTATIONS ON,1X,6A4///)
 1230 FORMAT (11X,4HM1 =,F7.4/)
 1240 FORMAT (2X,41H(** DENOTES PERTURBATION FROM BASE VALUE)/)
 1250 FORMAT (11X,4HQ1 =,F7.4,5X,1H(,A8,1H)/)
 1260 FORMAT (9X,5H**Q1(,I1,3H) =,F7.4,5X,1H(,A8,1H)/)
 1270 FORMAT (11X,3HQ1(,I1,3H) =,F7.4,5X,1H(,A8,1H)/)
 1280 FORMAT (1H1,20HUNIT PERTURBATION OF,1X,2A1,1X,
     %             27HAND UNIT STRAINING OF XBASE)
 1300 FORMAT (26HFOR CALIBRATION SOLUTIONS,I2,1X,7HTHROUGH,I2)
 1310 FORMAT (///19X,1(1H*,1X,A4,11HCALB SOLN *,3X))
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 1320 FORMAT (///19X,2(1H*,1X,A4,11HCALB SOLN *,3X))
 1330 FORMAT (///19X,3(1H*,1X,A4,11HCALB SOLN *,3X))
 1340 FORMAT (///19X,4(1H*,1X,A4,11HCALB SOLN *,3X))
 1350 FORMAT (1X,5HPOINT,4X,5HXBASE,4X,96A1)
 1360 FORMAT (1X)
 1370 FORMAT (1X,I4,1X,11F10.4)
 1380 FORMAT (1H1,32(1H*)/
     $        1X,21H* OUTPUT FOR CASE NO.,I3,4H OF ,I2,2H */
     $        1X,32(1H*)///)
 1390 FORMAT (11X,4HM2 =,F7.4/)
 1400 FORMAT (11X,4HQ2 =,F7.4,5X,1H(,A8,1H)/)
 1410 FORMAT (11X,3HQ2(,I1,3H) =,F7.4,5X,1H(,A8,1H)/)
 1420 FORMAT (/6X,5A4//
     $         11X,14HMINIMUM AT X =,F7.4,A1,3X,10H(POINT NO.,I4,1H)/
     $         11X,14HMAXIMUM AT X =,F7.4,A1,3X,10H(POINT NO.,I4,1H))
 1430 FORMAT (1H ,10X,I1,1X,18HCRITICAL POINT(S):/
     $        (15X,A4,6HAT X =,F7.4,A1,3X,
     $             16H(AFTER POINT NO.,I4,1H)))
 1440 FORMAT (///1X,44H.....FINAL PRINTOUT AND GRAPHICAL DISPLAY OF,1X,
     $            2A1)
 1450 FORMAT (/72X,21HH = MAXIMUM  VALUE OF,1X,2A1,1X,1H=,F8.4/
     $          72X,21HL = MINIMUM  VALUE OF,1X,2A1,1X,1H=,F8.4/
     $          72X,21H* = CRITICAL VALUE OF,1X,2A1,1X,1H=,F8.4/
     $          72X,12HP = VALUE OF,1X,2A1,1X,
     $              34HPREDICTED BY PERTURBATION SOLUTION/
     $          72X,12HC = VALUE OF,1X,2A1,1X,
     $              22HIN COMPARISON SOLUTION/
     $          72X,29H$ = AGREEMENT BETWEEN P AND C)
 1460 FORMAT (/2X,2HPT,2X,5HXBASE,3X,2A1,4HBASE,2X,5HXPERT,3X,2A1,
     $            4HPERT,2X,5HXCHEK,3X,2A1,4HCHEK,2X,2A1,4HPINT,1X,72A1/)
 1470 FORMAT (1X,I3,7F8.4,1X,72A1)
 1480 FORMAT (/60X,21HH = MAXIMUM  VALUE OF,1X,2A1,1X,1H=,F8.4/
     $          60X,21HL = MINIMUM  VALUE OF,1X,2A1,1X,1H=,F8.4/
     $          60X,21H* = CRITICAL VALUE OF,1X,2A1,1X,1H=,F8.4/
     $          60X,12HP = VALUE OF,1X,2A1,1X,
     $              34HPREDICTED BY PERTURBATION SOLUTION)
 1490 FORMAT (/2X,2HPT,2X,5HXBASE,3X,2A1,4HBASE,2X,5HXPERT,3X,2A1,
     $            4HPERT,1X,96A1/)
 1500 FORMAT (1X,I3,4F8.4,1X,96A1)
C
C.....ABNORMAL TERMINATION FORMATS FOLLOW.
C
 9000 FORMAT (///1X,28HNUMBER OF CRITICAL POINTS IN/
     $            1X,30HBASE AND CALIBRATION SOLUTIONS/
     $            1X,31HARE UNEQUAL - CALCULATION ENDED)
 9050 FORMAT (///1X,25HNUMBER OF CRITICAL POINTS/
     $            1X,23HSELECTED EXCEEDS NUMBER/
     $            1X,30HACTUALLY LOCATED - CALCULATION/
     $            1X,5HENDED)
 9100 FORMAT (///1X,28HORDER OF SPECIFIED POINTS IN/
     $            1X,30HBASE AND CALIBRATION SOLUTIONS/
     $            1X,39HDOES NOT CORRESPOND - CALCULATION ENDED)
 9500 FORMAT (1H1)
      END
```

```
      SUBROUTINE BANNER
C
C.....PRINTS TWO BANNER PAGES WITH JOB KEYWORD IN UPPER RIGHT CORNER.
C
      COMMON /HEAD/ TITLE(80)
      READ (5,1000) TITLE
      DO 10 IPAGE=1,2
      WRITE (6,1020) (TITLE(I),I=1,9)
      WRITE (6,1030)
      WRITE (6,1040)
   10 CONTINUE
      RETURN
 1000 FORMAT (80A1)
 1020 FORMAT (1H1,119X,13(1H*)/120X,2H* ,9A1,2H */120X,13(1H*))
 1030 FORMAT (////////38X,55(1H*)/38X,1H*,53X,1H*/
     $ 38X,1H*,19X,15HPROGRAM PERTURB,19X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,7X,39HCALCULATES NONLINEAR MULTIPLE-PARAMETER,
     $                                 7X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,13X,27HCONTINUOUS OR DISCONTINUOUS,
     $                                13X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,15X,22HPERTURBATION SOLUTIONS,16X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,10X,33HWHICH REPRESENT CHANGES IN EITHER,
     $                                10X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,13X,27HGEOMETRY OR FLOW CONDITIONS,
     $                                13X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,4X,44HBY EMPLOYING A STRAINED-COORDINATE PROCEDURE,
     $                                 5X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,4X,44HUTILIZING UNIT PERTURBATIONS DETERMINED FROM,
     $                                 5X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,16X,21HPREVIOUSLY CALCULATED,
     $                                16X,1H*/38X,1H*,53X,1H*)
 1040 FORMAT (38X,1H*,9X,34H"BASE" AND "CALIBRATION" SOLUTIONS,
     $                                10X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,4X,45HDISPLACED FROM ONE ANOTHER BY SOME REASONABLE,
     $                                 4X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,8X,36HCHANGE IN GEOMETRY OR FLOW CONDITION,
     $                                 9X,1H*/38X,1H*,53X,1H*/
     $                                        38X,1H*,53X,1H*/
     $ 38X,1H*,21X,10HWRITTEN BY,22X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,7X,39HJAMES P. ELLIOTT AND STEPHEN S. STAHARA,
     $                                 7X,1H*/38X,1H*,53X,1H*/
     $                                        38X,1H*,53X,1H*/
     $ 38X,1H*,7X,38HNIELSEN ENGINEERING AND RESEARCH, INC.,
     $                                 8X,1H*/38X,1H*,53X,1H*/
     $ 38X,1H*,14X,25HMOUNTAIN VIEW, CALIFORNIA,14X,1H*/38X,1H*,53X,1H*/
     $                                38X,55(1H*))
      END
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```
      SUBROUTINE DRVPLT (ICASE,N,NCASE,NPARAM,YMIN,YMAX,YCR2)
      DIMENSION HLINE2(3),HLINE3(3)
      DIMENSION XCSAVE(8,200),YCSAVE(8,200)
      DIMENSION XBASE(200),XCALB(200),XPERT(200),XCHEK(200),YBASE(200),
     $          YCALB(200),YPERT(200),YCHEK(200),YPRTI(200)
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```
      COMMON /HEAD/ ZTITLE(80)
      COMMON /SAVE/ XCSAVE,YCSAVE
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK,YPRTI
      DATA NPLOT /0/
      IF (NPLOT .EQ. 0) CALL BETA
      MIN=10.0*(YMIN-0.1)
      MAX=10.0*(YMAX+0.1)
      YMIN=0.1*MIN
      YMAX=0.1*MAX
      ENCODE (28,1000,HLINE2) (ZTITLE(I),I=1,9),ICASE,NCASE
      DO 20 K=1,NPARAM
      NPLOT=NPLOT-1
      ENCODE (22,1010,HLINE3) K,NPARAM
      CALL BGNPL (-1)
      CALL MIXALF ("L/CST")
      CALL MX3ALF ("INSTR","$")
      CALL SIMPLX
      CALL TITLE (1H ,1,1HX,1,"$E0.5)C$EX(P)$",100,6.0,8.0)
      CALL HEADIN ("PLOT (OF) C$L0.25H0.7(P)$",100,3,3)
      CALL HEADIN (HLINE2,28,2,3)
      CALL HEADIN (HLINE3,22,2,3)
      CALL GRAF (0.0,"SCALE",1.0,YMAX,"SCALE",YMIN)
      CALL FRAME
      IF (YCR2 .GT. YMAX) GO TO 10
      CALL RLVEC (0.0,YCR2,0.2,YCR2,0000)
      CALL RLMESS ("C$L(P$BE(*)$",100,0.21,YCR2)
   10 CONTINUE
      CALL DASH
      CALL CURVE (XBASE,YBASE,N,0)
      CALL RESET ("DASH")
      CALL DOT
      CALL COPY (2,N,K,XCALB,XCSAVE)
      CALL COPY (2,N,K,YCALB,YCSAVE)
      CALL CURVE (XCALB,YCALB,N,0)
      CALL RESET ("DOT")
      CALL MARKER (1)
      CALL CURVE (XPERT,YPERT,N,-1)
      CALL RESET ("MARKER")
      CALL CURVE (XCHEK,YCHEK,N,0)
      CALL ENDPL (NPLOT)
   20 CONTINUE
 1000 FORMAT (9A1,11H: CASE NO. ,I2,4H OF ,I2)
 1010 FORMAT (16HCALIBRATION NO. ,I1,4H OF ,I1)
      RETURN
      END


      SUBROUTINE ECHINP
      DIMENSION CARD(20)
      READ (5,1000) LECHO
      IF (LECHO .EQ. 0) GO TO 10
      WRITE (6,1010)
      BACKSPACE 5
      BACKSPACE 5
   10 READ (5,1020) CARD
      IF (EOF(5)) 30,20
   20 WRITE (1,1020) CARD
      IF (LECHO .EQ. 1) WRITE (6,1030) CARD
      GO TO 10
   30 REWIND 1
      IF (LECHO .EQ. 0) RETURN
      READ (1,1020) CARD
      READ (1,1020) CARD
      RETURN
 1000 FORMAT (I5)
 1010 FORMAT (1H1,25(1H*)/
     $        1X,1H*,1X,21HLISTING OF INPUT DECK,1X,1H*/
     $        1X,25(1H*)////)
 1020 FORMAT (20A4)
 1030 FORMAT (1X,20A4)
      END


      SUBROUTINE FILL (ICALL,I,STRING)
C
C.....FILLS ARRAY STRING WITH CHARACTERS FOR TABLE HEADINGS AND PRINTER
C     PLOTS.
C
      DIMENSION XBASE(200),XCALB(200),XCHEK(200),XPERT(200)
      DIMENSION YBASE(200),YCALB(200),YPERT(200),YCHEK(200),YPRTI(200)
      DIMENSION LOCO(6),LOC1(6),LSELCT(6),Q0(8),Q2(8)
      DIMENSION STRING(96),VNAM(2),UNIT(20),PARNAM(8)
      REAL M0,M1,M2
      COMMON /PARAM/ PARNAM,LOC0,LOC1,LSELCT,N,NCASE,LSPEC,LUNIT,
     $               LCHEK,LPLOT,NSELCT,A,B,NPARAM,VNAM
      COMMON /PERT/ Q0,Q2,M0,M1,M2,Q1,YCR0,YCR1,YCR2,YMIN,YMAX
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK,YPRTI
      DATA IENT /0/
      DATA STAR/1H*/, P/1HP/, C/1HC/, DASH/1H-/, H/1HH/, EL/1HL/,
     $     BLANK/1H /, DOLLAR/1H$/
      DATA UNIT /1HX,1HS,1HT,1HR,1HU,1HN,1HI,1HT,1H ,1H ,
     $           1H ,1H ,1H ,1HU,1HN,1HI,1HT,1H ,1H ,1H /
      GO TO (10,30,50), ICALL
   10 IENT=IENT+1
      IF (IENT .GT. 1) RETURN
      UNIT(12)=VNAM(1)
      UNIT(13)=VNAM(2)
      DO 20 J=1,20
      DO 20 K=1,4
      II=J+20*(K-1)
   20 STRING(II)=UNIT(J)
      RETURN
   30 NY=96
      IFLAG=0
      IF (YCR2 .GT. YMAX .OR. YCR2 .LT. YMIN) IFLAG=1
      IF (LCHEK .EQ. 1) NY=72
      RANGE=YMAX-YMIN
      DO 40 II=1,NY
   40 STRING(II)=DASH
      STRING(1)=H
      STRING(NY)=EL
      IF (IFLAG .EQ. 1) RETURN
      NSTAR=1+(YMAX-YCR2)/RANGE*(NY-1)
      STRING(NSTAR)=STAR
      RETURN
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   50 CONTINUE
      DO 60 II=1,NY
   60 STRING(II)=BLANK
      IF (IFLAG .EQ. 0) STRING(NSTAR)=STAR
      YP=YPERT(I)
      IF (LCHEK .EQ. 1) YP=YPRTI(I)
      NPERT=1+(YMAX-YP)/RANGE*(NY-1)
      STRING(NPERT)=P
      IF (LCHEK .EQ. 0) RETURN
      YC=YCHEK(I)
      NCHEK=1+(YMAX-YC)/RANGE*(NY-1)
      STRING(NCHEK)=C
      IF (NPERT .EQ. NCHEK) STRING(NPERT)=DOLLAR
      RETURN
      END


      SUBROUTINE INPUT (ICALL)
C
C     WITH THE EXCEPTION OF THE TITLE AND THE ECHO PARAMETER (ITEMS 1
C     AND 2), ALL INPUT FOR PROGRAM PERTURB IS READ BY THIS SUBROUTINE,
C     AND IS REQUIRED IN THE FOLLOWING ORDER (FOR DETAILS, REFER TO
C     ACCOMPANYING MANUAL).
C
C**** ITEM NO. 1 - ONE CARD (8A10) ************************************
C
C         TITLE      IDENTIFIES JOB - PRINTED AS HEADLINE ON FIRST PAGE
C                    OF OUTPUT. FIRST NINE CHARACTERS ARE PRINTED IN
C                    UPPER RIGHT CORNER OF BANNER PAGE, AND IN UPPER
C                    LEFT CORNER OF SUMMARY PAGE.
C
C**** ITEM NO. 2 - ONE CARD (I5) **************************************
C
C         LECHO      CONTROLS WHETHER OR NOT INPUT DECK IS PRINTED.
C
C                    LECHO = 0 ... NO OUTPUT
C                    LECHO = 1 ... OUTPUT
C
C**** ITEM NO. 3 - ONE CARD (16I5) ************************************
C
C         N          NUMBER OF DATA POINTS IN BASE AND CALIBRATION
C                    SOLUTIONS.
C
C         NCASE      NUMBER OF CASES FOR WHICH PERTURBATION SOLUTIONS ARE
C                    TO BE COMPUTED.
C
C         NPARAM     NUMBER OF PARAMETERS PERTURBED.
C
C         NSELCT     NUMBER OF POINTS (IN ADDITION TO END POINTS) TO BE
C                    HELD INVARIANT IN STRAINING.
C                          NOTE: 1 .LE. NSELCT .LE. 6.
C
C         LSPEC      CONTROLS HOW INVARIANT POINTS IN STRAINING ARE
C                    SPECIFIED.
C
C                    LSPEC = 0 ... INVARIANT POINTS SELECTED FROM AMONG
C                                  THOSE LOCATED BY THE PROGRAM
C                                  (SEE ITEM NO. 4)
C                    LSPEC = 1 ... INVARIANT POINTS PRESELECTED BY USER
C                                  (SEE ITEMS NOS. 12 AND 16)
C
C         LUNIT      CONTROLS WHETHER OR NOT UNIT COORDINATE STRAINING(S)
C                    AND UNIT PERTURBATION(S) ARE PRINTED.
C
C                    LUNIT = 0 ... NO OUTPUT
C                    LUNIT = 1 ... OUTPUT
C
C         LCHEK      SPECIFIES WHETHER OR NOT PERTURBATION SOLUTION IS TO
C                    BE CHECKED AGAINST AN EXACT COMPARISON SOLUTION.
C                    A PRINTER PLOT OF SOLUTION IS MADE IN EITHER CASE.
C
C                    LCHEK = 0 ... NO COMPARISON
C                    LCHEK = 1 ... COMPARISON
C
C         LPLOT      SPECIFIES WHETHER OR NOT AN ADDITIONAL PLOT BY A
C                    PERIPHERAL DEVICE IS TO BE MADE (SOFTWARE MUST BE
C                    SUPPLIED BY USER IN SUBROUTINE PLOT).
C
C                    LPLOT = 0 ... NO PERIPHERAL PLOT
C                    LPLOT = 1 ... PERIPHERAL PLOT
C
C**** ITEM NO. 4 - ONE CARD (16I5) ************************************
C
C     ----- THIS ITEM TO BE OMITTED IF LSPEC = 1 -----
C
C         LSELCT(I) ...
C                    ARRAY OF LENGTH 6 OF WHICH NSELCT ELEMENTS ARE READ
C                    IN. SPECIFIES NATURE OF POINTS TO BE HELD INVARIANT
C                    ACCORDING TO THE CODE
C
C                          1 ... MINIMUM PT. HELD INVARIANT
C                          2 ... MAXIMUM PT. HELD INVARIANT
C                          3 ... 1ST CRITICAL PT. HELD INVARIANT
C                          4 ... 2ND CRITICAL PT. HELD INVARIANT
C                          5 ... 3RD CRITICAL PT. HELD INVARIANT
C                          6 ... 4TH CRITICAL PT. HELD INVARIANT
C
C                    NOTE THAT THE CODE NUMBERS CAN BE ASSIGNED IN ANY
C                    ORDER, E.G.
C
C                                   LSELCT(1) = 1
C                                   LSELCT(2) = 3
C                                   LSELCT(3) = 4
C
C                    IS EQUIVALENT TO
C
C                                   LSELCT(1) = 4
C                                   LSELCT(2) = 1
C                                   LSELCT(3) = 3
C
C                    BOTH CORRESPONDING TO NSELCT = 3 WITH THE MINIMUM,
C                    AND FIRST AND SECOND CRITICAL POINTS HELD INVARIANT.
C
C**** ITEM NO. 5 - ONE CARD (2A1) *************************************
C
C         VNAM       CHARACTER STRING OF LENGTH 2 WHICH SYMBOLIZES
C                    DEPENDENT VARIABLE, E.G. CP FOR PRESSURE
```

```
C                   COEFFICIENT.
C
C**** ITEM NO. 6 - ONE CARD (10A8) ************************************
C
C        PARNAM(K) ...
C                   ARRAY OF 8-CHARACTER STRINGS WHICH IDENTIFY THE
C                   PARAMETERS VARIED. NPARAM ELEMENTS OF THE ARRAY
C                   ARE READ IN.
C
C**** ITEM NO. 7 - ONE CARD (8F10.6) **********************************
C
C        A          SCALING PARAMETER (A = -X(1), WHERE X(1) IS FIRST
C                   DATA POINT ON LOWER SURFACE ... SEE MANUAL).
C
C        B          SCALING PARAMETER (B = X(N), WHERE X(N) IS LAST DATA
C                   POINT ON UPPER SURFACE ... SEE MANUAL).
C
C**** ITEM NO. 8 - ONE CARD (8F10.6) **********************************
C
C        M0         ONCOMING MACH NUMBER IN BASE SOLUTION.
C
C**** ITEM NO. 9 - ONE CARD (8F10.6) **********************************
C
C        Q0(K)      ARRAY OF LENGTH 8 GIVING VALUES OF PERTURBATION
C                   PARAMETERS IN BASE SOLUTION. NPARAM ELEMENTS OF THE
C                   ARRAY ARE READ IN.
C
C**** ITEM NO. 10 - ONE SET OF C CARDS (8F10.6), C = 1 + INT(N/8) ******
C
C        XBASE(I), I=1,N ...
C                   X COORDINATE IN BASE SOLUTION.
C
C**** ITEM NO. 11 - ONE SET OF C CARDS (8F10.6), C AS IN ITEM NO. 10 ***
C
C        YBASE(I), I=1,N ...
C                   DEPENDENT VARIABLE IN BASE SOLUTION.
C
C**** ITEM NO. 12 - ONE CARD (16I5) ***********************************
C
C     ----- THIS ITEM TO BE OMITTED IF LSPEC = 0 -----
C
C        LOC0(I)    ARRAY OF LENGTH 6 OF WHICH NSELCT ELEMENTS ARE READ
C                   IN. SPECIFIES SUBSRIPTS OF THOSE BASE FLOW POINTS
C                   WHICH ARE TO BE HELD INVARIANT.
C
C**********************************************************************
C
C        ********   ONE SET OF ITEMS 13 THROUGH 16 IS
C        * NOTE *   REQUIRED FOR EACH OF THE NPARAM
C        ********   CALIBRATION SOLUTIONS.
C
C**** ITEM NO. 13 - ONE CARD (8F10.6) *********************************
C
C        M1         ONCOMING MACH NUMBER IN KTH CALIBRATION SOLUTION.
C
C        Q1         VALUE OF KTH PERTURBATION PARAMETER IN KTH
C                   CALIBRATION SOLUTION.
C
C**** ITEM NO. 14 - ONE SET OF C CARDS (8F10.6), C AS IN ITEM NO. 10 ***
C
C        XCALB(I), I= 1,N ...
C                   X COORDINATE IN KTH CALIBRATION SOLUTION.
C
C**** ITEM NO. 15 - ONE SET OF C CARDS (8F10.6), C AS IN ITEM NO. 10 ***
C
C        YCALB(I), I=1,N ...
C                   DEPENDENT VARIABLE IN KTH CALIBRATION SOLUTION..
C
C**** ITEM NO. 16 - ONE CARD (16I5) ***********************************
C
C     ----- THIS ITEM TO BE OMITTED IF LSPEC = 0 -----
C
C        LOC1(I)    ARRAY OF LENGTH 6 OF WHICH NSELCT ELEMENTS ARE READ
C                   IN. SPECIFIES SUBSCRIPTS OF THOSE POINTS IN KTH
C                   CALIBRATION FLOW WHICH ARE TO BE HELD INVARIANT.
C
C**********************************************************************
C
C        ********   ONE SET OF ITEMS 17 THROUGH 20 IS
C        * NOTE *   REQUIRED FOR EACH OF THE NCASE
C        ********   SOLUTIONS TO BE COMPUTED.
C
C**** ITEM NO. 17 - ONE CARD (8F10.6) *********************************
C
C        M2         ONCOMING MACH NUMBER IN SOLUTION TO BE COMPUTED.
C
C**** ITEM NO. 18 - ONE CARD (8F10.6) *********************************
C
C        Q2(K)      ARRAY OF LENGTH 8 GIVING VALUES OF PERTURBATION
C                   PARAMETERS IN SOLUTION TO BE COMPUTED. NPARAM
C                   ELEMENTS OF ARRAY ARE READ IN.
C
C**** ITEM NO. 19 - ONE SET OF C CARDS (8F10.6), C AS IN ITEM NO. 10 ***
C
C     ----- THIS ITEM TO BE OMITTED IF LCHEK = 0 -----
C
C        XCHEK(I), I= 1,N ...
C                   X COORDINATE IN COMPARISON SOLUTION.
C
C**** ITEM NO. 20 - ONE SET OF C CARDS (8F10.6), C AS IN ITEM NO. 10 ***
C
C     ----- THIS ITEM TO BE OMITTED IF LCHEK = 0 -----
C
C        YCHEK(I), I=1,N ...
C                   DEPENDENT VARIABLE IN COMPARISON SOLUTION.
C
C**********************************************************************
C
      DIMENSION LOC0(6),LOC1(6),LSELCT(6)
      DIMENSION XBASE(200),XCALB(200),XPERT(200),XCHEK(200),
     %          YBASE(200),YCALB(200),YPERT(200),YCHEK(200),YPRT1(200)
      DIMENSION Q0(8),Q2(8)
      REAL M0,M1,M2
      DIMENSION VNAM(2),PARNAM(8)
```

```
      COMMON /PARAM/ PARNAM,LOC0,LOC1,LSELCT,N,NCASE,LSPEC,LUNIT,
     $               LCHEK,LPLOT,NSELCT,A,B,NPARAM,VNAM
      COMMON /PERT/ Q0,Q2,M0,M1,M2,Q1,YCR0,YCR1,YCR2,YMIN,YMAX
      COMMON /XY/ XBASE,XCALB,XPERT,XCHEK,YBASE,YCALB,YPERT,YCHEK,YPRTI
      GO TO (10,20,30,40,50), ICALL
   10 READ (1,1000) N,NCASE,NPARAM,NSELCT,LSPEC,LUNIT,LCHEK,LPLOT
      IF (LSPEC .EQ. 0) READ (1,1000) (LSELCT(I),I=1,NSELCT)
      READ (1,1010) VNAM
      READ (1,1020) (PARNAM(K),K=1,NPARAM)
      READ (1,1030) A,B
      RETURN
   20 READ (1,1030) M0
      READ (1,1030) (Q0(K),K=1,NPARAM)
      READ (1,1030) (XBASE(I),I=1,N)
      READ (1,1030) (YBASE(I),I=1,N)
      IF (LSPEC .EQ. 1) READ (1,1000) (LOC0(I),I=1,NSELCT)
      RETURN
   30 READ (1,1030) M1,Q1
      READ (1,1030) (XCALB(I),I=1,N)
      READ (1,1030) (YCALB(I),I=1,N)
      IF (LSPEC .EQ. 1) READ (1,1000) (LOC1(I),I=1,NSELCT)
      RETURN
   40 READ (1,1030) M2
      READ (1,1030) (Q2(K),K=1,NPARAM)
      RETURN
   50 READ (1,1030) (XCHEK(I),I=1,N)
      READ (1,1030) (YCHEK(I),I=1,N)
      RETURN
 1000 FORMAT (16I5)
 1010 FORMAT (2A1)
 1020 FORMAT (10A8)
 1030 FORMAT (8F10.6)
      END


      SUBROUTINE INTERP (N,X,Y,XI,YI)
C
C.....GIVEN THE SET OF POINTS X(I), Y(I), I=1,N, AND THE SET XI(J),
C     J=1,N, USES LINEAR INTERPOLATION TO COMPUTE THE SET YI(J), J=1,N.
C
      DIMENSION X(200),Y(200),XI(200),YI(200)
      NM1=N-1
      JSTART=1
      DO 70 I=1,N
      IF (XI(I) .LE. X(1)) GO TO 10
      IF (XI(I) .GE. X(N)) GO TO 20
      GO TO 30
   10 J=1
      GO TO 60
   20 J=N-1
      GO TO 60
   30 CONTINUE
      DO 50 J=JSTART,NM1
      IF (XI(I) .NE. X(J)) GO TO 40
      YI(I)=Y(J)
      GO TO 70
   40 IF (XI(I) .GT. X(J) .AND. XI(I) .LT. X(J+1)) GO TO 60
   50 CONTINUE
   60 SLOPE=(Y(J+1)-Y(J))/(X(J+1)-X(J))
      YI(I)=Y(J)+SLOPE*(XI(I)-X(J))
      JSTART=J
   70 CONTINUE
      RETURN
      END


      SUBROUTINE LOCATE (N,X,Y,YCRIT,IGRAD,LMIN,LMAX,NCRIT,LCRIT,XLOC)
C
C.....OPERATES ON THE INPUT ARRAY Y, LOCATING MINIMUM AND MAXIMUM
C     VALUES, AND ALL CRITICAL POINTS (Y=YCRIT) FOR WHICH DY/DX (IN
C     PHYSICAL COORDINATES) HAS ALGEBRAIC SIGN GIVEN BY IGRAD. NCRIT
C     IS NUMBER OF CRITICAL POINTS. POINTS FOUND ARE STORED IN THE ARRAY
C     XLOC AS FOLLOWS
C
C                           XLOC(1) = MINIMUM PT.
C                           XLOC(2) = MAXIMUM PT.
C                           XLOC(3) = CRITICAL PT. NO. 1
C                           ...     = ...
C                           XLOC(6) = CRITICAL PT. NO. 4
C
      DIMENSION X(200),Y(200),LCRIT(4),XCRIT(4),XLOC(6)
      COMMON /FLOREV/ IREV
      IFLOW=-1
      LMIN=1
      LMAX=1
      ISTART=2
      IF (IREV .EQ. 0) GO TO 10
      LMIN=2
      LMAX=2
      ISTART=3
   10 CONTINUE
      NCRIT=0
      DO 30 I=ISTART,N
      IF (IREV .NE. 0 .AND. I .EQ. N) GO TO 20
      IF (Y(I) .GT. Y(LMAX)) LMAX=I
      IF (Y(I) .LT. Y(LMIN)) LMIN=I
   20 CONTINUE
      IF ((Y(I) .GT. YCRIT .AND. Y(I-1) .GT. YCRIT) .OR.
     $    (Y(I) .LT. YCRIT .AND. Y(I-1) .LT. YCRIT)) GO TO 30
      IF (I .GT. IREV) IFLOW=1
      IF ((Y(I)-Y(I-1))*FLOAT(IFLOW*IGRAD) .LT. 0.0) GO TO 30
      NCRIT=NCRIT+1
      LCRIT(NCRIT)=I-1
      SLOPE=(X(I)-X(I-1))/(Y(I)-Y(I-1))
      XCRIT(NCRIT)=X(I-1)+SLOPE*(YCRIT-Y(I-1))
   30 CONTINUE
      XLOC(1)=X(LMIN)
      XLOC(2)=X(LMAX)
      IF (NCRIT .EQ. 0) RETURN
      DO 40 I=1,NCRIT
   40 XLOC(I+2)=XCRIT(I)
      RETURN
      END
```

```
      SUBROUTINE MONO (N,L,X,Y)
C
C.....CHECKS POINTS IN VICINITY OF A CRITICAL POINT FOR MONOTONE
C     BEHAVIOR, AND ADJUSTS VALUES IF NECESSARY TO GIVE A LINEAR
C     PROFILE.
C
      DIMENSION L(4),X(200),Y(200)
      DO 10 I=1,N
      LS=L(I)
      Y1=Y(LS-1)
      Y2=Y(LS)
      Y3=Y(LS+1)
      Y4=Y(LS+2)
      IF ((Y1 .LT. Y2) .AND. (Y2 .LT. Y3) .AND. (Y3 .LT. Y4)) GO TO 10
      IF ((Y1 .GT. Y2) .AND. (Y2 .GT. Y3) .AND. (Y3 .GT. Y4)) GO TO 10
      X1=X(LS-1)
      X2=X(LS)
      X3=X(LS+1)
      X4=X(LS+2)
      SLOPE=(Y4-Y1)/(X4-X1)
      Y(LS)=Y1+SLOPE*(X2-X1)
      Y(LS+1)=Y1+SLOPE*(X3-X1)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SCALE (N,X,M,A,B)
C
C.....ENTRY WITH M = 1 CONVERTS FROM PHYSICAL X (0 TO -A ON LOWER
C     SURFACE, 0 TO B ON UPPER SURFACE) TO NORMALIZED X (0 .LT. X .LT.
C     1) ENTRY WITH M=2 REVERSES THE PROCESS. NZ (DETERMINED WHEN M=1)
C     CORRESPONDS TO POINT AT NOSE OF BLADE OR AIRFOIL.
C
      COMMON /FLOREV/ NZ
      DIMENSION X(200)
      IF (M .EQ. 2) GO TO 30
      CONTINUE
      NZ=0
      DO 10 I=2,N
      IF (X(I) .LT. X(I-1)) NZ=I
   10 CONTINUE
      DO 20 I=1,N
      IF (I .LE. NZ) T=-X(I)
      IF (I .GT. NZ) T=X(I)
      X(I)=(T-A)/(B-A)
   20 CONTINUE
      RETURN
   30 DO 40 I=1,N
      X(I)=ABS((B-A)*X(I)+A)
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SORT (N,X,ISEQ)
C
C.....ARRANGES THE SET X(1), X(2), ... , X(N) IN A MONOTONE INCREASING
C     SEQUENCE. ISEQ GIVES ORDER OF SUBSCRIPTS IN REARRANGED SEQUENCE.
C
      DIMENSION X(8),ISEQ(8)
      NM1=N-1
      DO 10 I=1,N
   10 ISEQ(I)=I
   20 ITEST=0
      DO 30 I=1,NM1
      IF (X(I) .LE. X(I+1)) GO TO 30
      XSAVE=X(I)
      X(I)=X(I+1)
      X(I+1)=XSAVE
      ISAVE=ISEQ(I)
      ISEQ(I)=ISEQ(I+1)
      ISEQ(I+1)=ISAVE
      ITEST=1
   30 CONTINUE
      IF (ITEST .EQ. 1) GO TO 20
      RETURN
      END
```

```
      SUBROUTINE STRAIN (N,K,NSEG,XFIX,XIN,PARM,DELX)
C
C.....COMPUTES STRAINING INCREMENT DELX FROM INPUT ARRAY XIN, USING
C     PIECEWISE LINEAR STRAINING WITH NSEG LINEAR SEGMENTS. FOR UNIT
C     STRAINING, INPUT VALUE OF PARM IS 1.0; FOR GENERAL CASE,
C
C                     PARM = (Q2(K)-Q0(K))/(Q1-Q0(K)).
C
      DIMENSION XFIX(8),XIN(200),DELX(200)
      COMMON /COEFF/ C(8,7),D(8,7)
      JSTART=1
      DO 30 I=1,N
      DO 10 J=JSTART,NSEG
      IF (XIN(I) .GE. XFIX(J) .AND. XIN(I) .LE. XFIX(J+1)) GO TO 20
   10 CONTINUE
   20 DELX(I)=PARM*(C(K,J)+(D(K,J)-1.0)*XIN(I))
      JSTART=J
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TABLE (NPARAM,NCASE,PARNAM,M0,Q0)
      DIMENSION PARNAM(8),Q0(8)
      DIMENSION ORD(25),HEAD0(5),HEAD1(5),HEAD2(5),M1SAVE(8),
     $          M2SAVE(25),Q1SAVE(8),Q2SAVE(8,25)
      DIMENSION QWRITE(8)
      REAL M0,M1SAVE,M2SAVE
      COMMON /HEAD/ TITLE(80)
```

```
      COMMON /TBL/ HEAD0,HEAD1,HEAD2,M1SAVE,M2SAVE,Q1SAVE,Q2SAVE
      DIMENSION STAR(127),BLANK(126),STRNG1(100),STRNG2(28),END(6)
      DATA STAR /127*1H*/, BLANK /126*1H /, STRNG2 /1H*,27*1H-/,
     %     STRNG1 /100*1H-/
      DATA END /1HP,1HA,1HR,1H.,1H ,1H*/
      DATA ORD /5H 1ST ,5H 2ND ,5H 3RD ,5H 4TH ,5H 5TH ,5H 6TH ,5H 7TH ,
     %          5H 8TH ,5H 9TH ,5H10TH ,5H11TH ,5H12TH ,5H13TH ,5H14TH ,
     %          5H15TH ,5H16TH ,5H17TH ,5H18TH ,5H19TH ,5H20TH ,5H21ST ,
     %          5H22ND ,5H23RD ,5H24TH ,5H25TH /
      DO 10 I=1,10
      II=11*I-10
   10 STRNG1(II)=STAR(1)
      DO 50 IPAGE=1,2
      WRITE (6,1000) (TITLE(I),I=1,9)
      IMAX1=11*NPARAM+12
      IMAX2=IMAX1+28
      WRITE (6,1010) (BLANK(I),I=1,28),(STAR(I),I=1,IMAX1)
      WRITE (6,1020) (ORD(K),K=1,NPARAM)
      IMAX=IMAX2-6
      WRITE (6,1030) (BLANK(I),I=1,IMAX),END
      IMAX=IMAX1-11
      WRITE (6,1040) (STRNG1(I),I=1,IMAX)
      WRITE (6,1050) (PARNAM(K),K=1,NPARAM)
      IMAX=IMAX2-1
      WRITE (6,1030) (BLANK(I),I=1,IMAX),STAR(1)
      WRITE (6,1010) (STAR(I),I=1,IMAX2)
      WRITE (6,1060) HEAD0,M0,(Q0(K),K=1,NPARAM)
      WRITE (6,1030) (BLANK(I),I=1,IMAX),STAR(1)
      WRITE (6,1010) (STAR(I),I=1,IMAX2)
      WRITE (6,1070) (STAR(I),I=1,IMAX2)
      DO 30 K=1,NPARAM
      DO 20 KK=1,NPARAM
   20 QWRITE(KK)=Q0(KK)
      QWRITE(K)=Q1SAVE(K)
      WRITE (6,1080) ORD(K),HEAD1,M1SAVE(K),(QWRITE(KK),KK=1,NPARAM)
      WRITE (6,1030) (BLANK(I),I=1,IMAX),STAR(1)
      IF (K .LT. NPARAM) WRITE (6,1010) STRNG2,(STRNG1(I),I=1,IMAX1)
   30 CONTINUE
      WRITE (6,1010) (STAR(I),I=1,IMAX2)
      WRITE (6,1070) (STAR(I),I=1,IMAX2)
      DO 40 ICASE=1,NCASE
      WRITE (6,1080) ORD(ICASE),HEAD2,M2SAVE(ICASE),
     %               (Q2SAVE(K,ICASE),K=1,NPARAM)
      WRITE (6,1030) (BLANK(I),I=1,IMAX),STAR(1)
      IF (ICASE .LT. NCASE) WRITE (6,1010) STRNG2,(STRNG1(I),I=1,IMAX1)
   40 CONTINUE
      WRITE (6,1010) (STAR(I),I=1,IMAX2)
   50 CONTINUE
      RETURN
 1000 FORMAT (1H1,4X,13(1H*)/5X,2H* ,9A1,2H */5X,13(1H*)///)
 1010 FORMAT (5X,128A1)
 1020 FORMAT (33X,1H*,10X,8(1H*,A5,5HPAR. ))
 1030 FORMAT (1H+,4X,128A1)
 1040 FORMAT (33X,11H* MACH NO. ,89A1)
 1050 FORMAT (33X,1H*,10X,8(2H* ,A8,1X))
 1060 FORMAT (5X,3H*  ,5A4,5X,9(3H*  ,F6.3,2X))
 1070 FORMAT (1H0,4X,128A1)
 1080 FORMAT (5X,2H* ,A5,5A4,1X,9(3H*  ,F6.3,2X))
      END
```

```
      SUBROUTINE UPLOW (A,B,XIN,K,N,XOUT,FLAG)
C
C.....CONVERTS NORMALIZED ARRAY XIN TO PHYSICAL ARRAY XOUT AND FLAGS
C     POINTS ON LOWER SURFACE WITH A "*".
C
      DIMENSION XIN(K),XOUT(8)
      DIMENSION FLAG(8)
      DATA BLANK/1H /, STAR/1H*/
      XNOSE=-A/(B-A)
      DO 10 I=1,N
      FLAG(I)=BLANK
      IF (XIN(I) .LT. XNOSE) FLAG(I)=STAR
      XOUT(I)=ABS((B-A)*XIN(I)+A)
   10 CONTINUE
      RETURN
      END
```

APPENDIX C

LIST OF SYMBOLS

C	blade chord, m
DV_i	design variable coefficient of profile shape function; eq. (22)
i	invariant point index; eq. (5); also, index for surface shape functions; eqs. (22,23)
k	dummy index; eq. (24)
L	two-dimensional full potential operator; eq. (1)
L_1	linear operator representing first-order perturbation of two-dimensional full potential equation; eq. (3)
L_2	linear operator representing first-order perturbation terms arising from coordinate straining; eq. (6)
M	number of independent flow or geometrical variables to be perturbed
M_∞	absolute inlet Mach number
n	total number of shock points and high-gradient maxima points; eq. (21)
N	total number of invariant points, equal to $n + 2$; eqs. (13,14)
q_j	j^{th} arbitrary geometric or flow parameter to be perturbed; eq. (8)
q_{c_j}	calibration flow value of q_j; eq. (8)
q_{o_j}	base flow value of q_j; eq. (2)
Q	approximate flow solution for arbitrary flow quantity; eq. (7)
Q_{c_j}	calibration flow solution for value q_{c_j} of arbitrary parameter; eq. (7)
Q_o	base flow solution for values q_{o_j} of arbitrary parameters; eq. (7)
Q_{1_j}	j^{th} perturbation solution per unit change of perturbed parameter q_j; eq. (7)

(s,t)	strained (x,y) coordinates; eq. (4)
t	gap to chord spacing ratio
(x,y)	nondimensional blade-fixed orthogonal coordinates; eq. (7), normalized by C
$(\bar{x}_j,\bar{y}_j)$	nondimensional blade-fixed orthogonal coordinates related to j^{th} calibration solution; eq. (7)
(x_1,y_1)	straining functions associated with (x,y) coordinates; eq. (4)
(x_{1_i},y_{1_i})	straining functions associated with ith invariant point; eq. (5)
α	angle oncoming flow makes with blade chord line
$(\delta x_i,\delta y_i)$	unit displacements in (x,y) directions associated with ith invariant point; eqs. (5,8)
ε_j	desired perturbation change of j^{th} geometric or flow parameter; eq. (8)
$\bar{\varepsilon}_j$	perturbation change of j^{th} geometric or flow parameter between base and calibration flows; eq. (8)
τ	thickness ratio of blade
Φ	nondimensional total velocity potential; eq. (1), normalized by CV_∞
Φ_o	nondimensional base flow velocity potential; eq. (2), normalized by CV_∞
Φ_{1_j}	nondimensional j^{th} perturbation velocity potential; eq. (2), normalized by CV_∞

Subscripts

i	denotes quantities associated with i^{th} invariant point
j	denotes perturbation quantities

Superscripts

o	denotes base flow quantities
c	denotes quantities associated with calibration flows

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TABLE 1

COMPARISON OF FINAL DESIGN VARIABLES AND OBJECTIVE FUNCTION WHEN EMPLOYING FULL NONLINEAR TSONIC SOLUTIONS OR PERTURBATION METHOD FOR DIFFERENT CHOICES OF CALIBRATION SOLUTION MATRIX FOR SIX DESIGN VARIABLE SUBCRITICAL OPTIMIZATION CASE STUDY USING MAXIMUM SUCTION SURFACE VELOCITY DIFFUSION OBJECTIVE

Design Variables	KOCR	T	ZM	P	TMX	THLE	Objective Function
				INITIAL			
Baseline	-10.0000	0.2500	0.4500	1.50000	0.0500	0.0050	1.8400
Upper Bound	0.0000	0.6000	0.5500	4.0000	0.1000	0.0120	
Lower Bound	-15.0000	0.2000	0.2000	0.5000	0.0300	0.0030	
				FINAL			
				TSONIC SOLUTIONS ONLY RESULTS			
CDC 7600	-7.1106	0.2000	0.5500	0.9401	0.0300	0.0064	1.6748
IBM 3033	-8.8659	0.2400	0.5500	0.7628	0.0300	0.0051	1.6752
				PERTURBATION SOLUTION RESULTS			
CASE 1							
Calibration	-7.0000	0.2000	0.5500	0.9400	0.0300	0.0064	
Final	-9.2223	0.2327	0.5500	0.9281	0.0359	0.0052	1.6904
CASE 2							
Calibration	-9.0000	0.2300	0.5500	0.8000	0.0300	0.0060	
Final	-8.8714	0.2228	0.5500	0.9523	0.0315	0.0051	1.6908
CASE 3							
Calibration	-12.0000	0.3000	0.4000	1.2500	0.0400	0.0070	
Final	-8.9865	0.3001	0.5500	0.8776	0.0300	0.0050	1.6974
CASE 4							
Calibration	-8.0000	0.3500	0.5000	2.0000	0.0600	0.0060	
Final	-8.2180	0.3257	0.5500	1.4527	0.0385	0.0049	1.7628
CASE 5							
Calibration	-9.0000	0.3000	0.5000	2.5000	0.0400	0.0040	
Final	-5.9055	0.4440	0.5500	1.3325	0.0412	0.0037	1.7486
CASE 6							
Calibration	-11.0000	0.2300	0.3500	1.2500	0.0600	0.0040	
Final	-9.4297	0.2522	0.5500	0.8036	0.0300	0.0052	1.6807

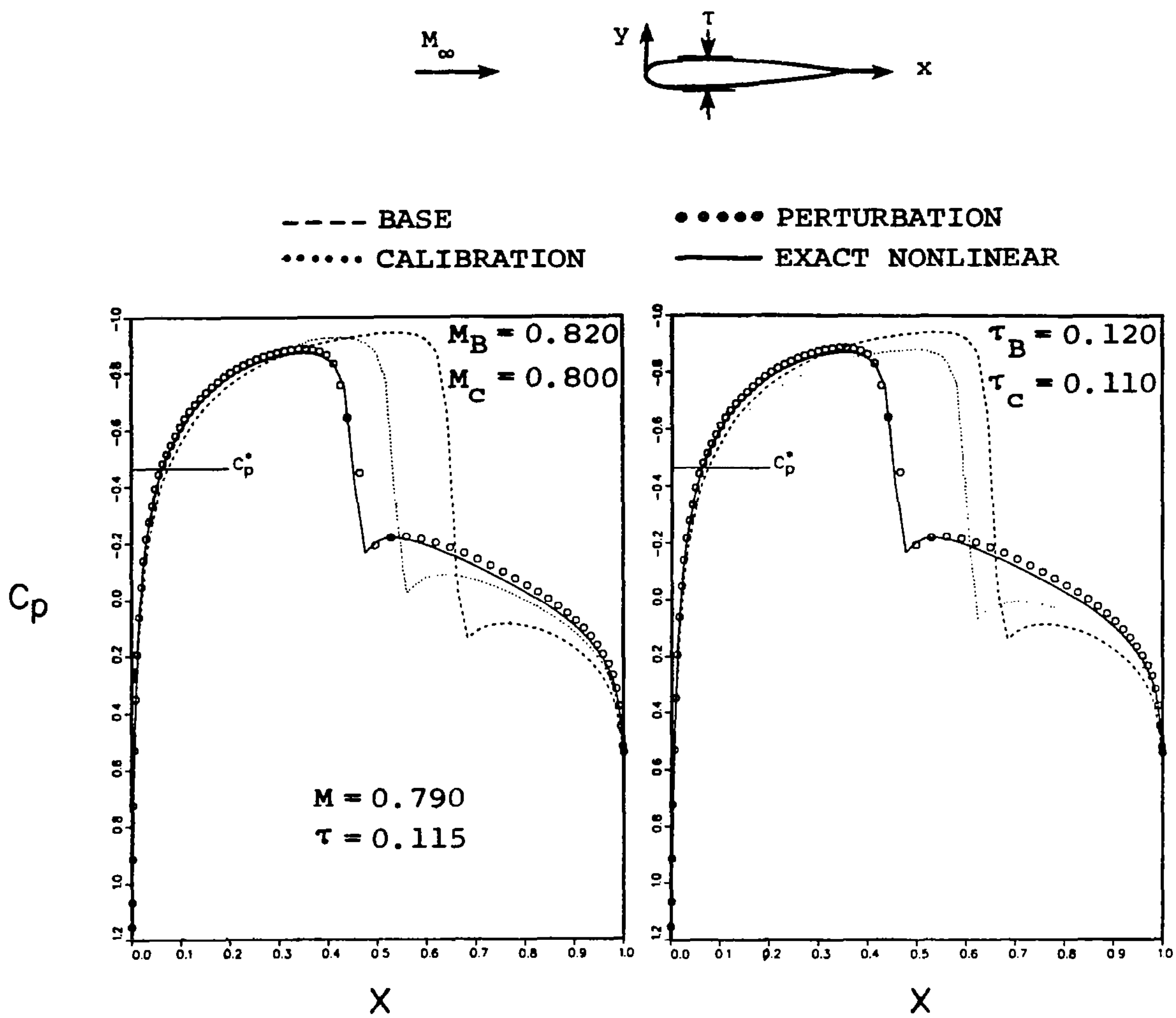

Figure 1.- Comparison of perturbation (○) and nonlinear (—) surface pressures for the simultaneous two-parameter perturbation of (M_∞, τ) for nonlifting strongly supercritical flows past isolated NACA 00XX blade profiles.

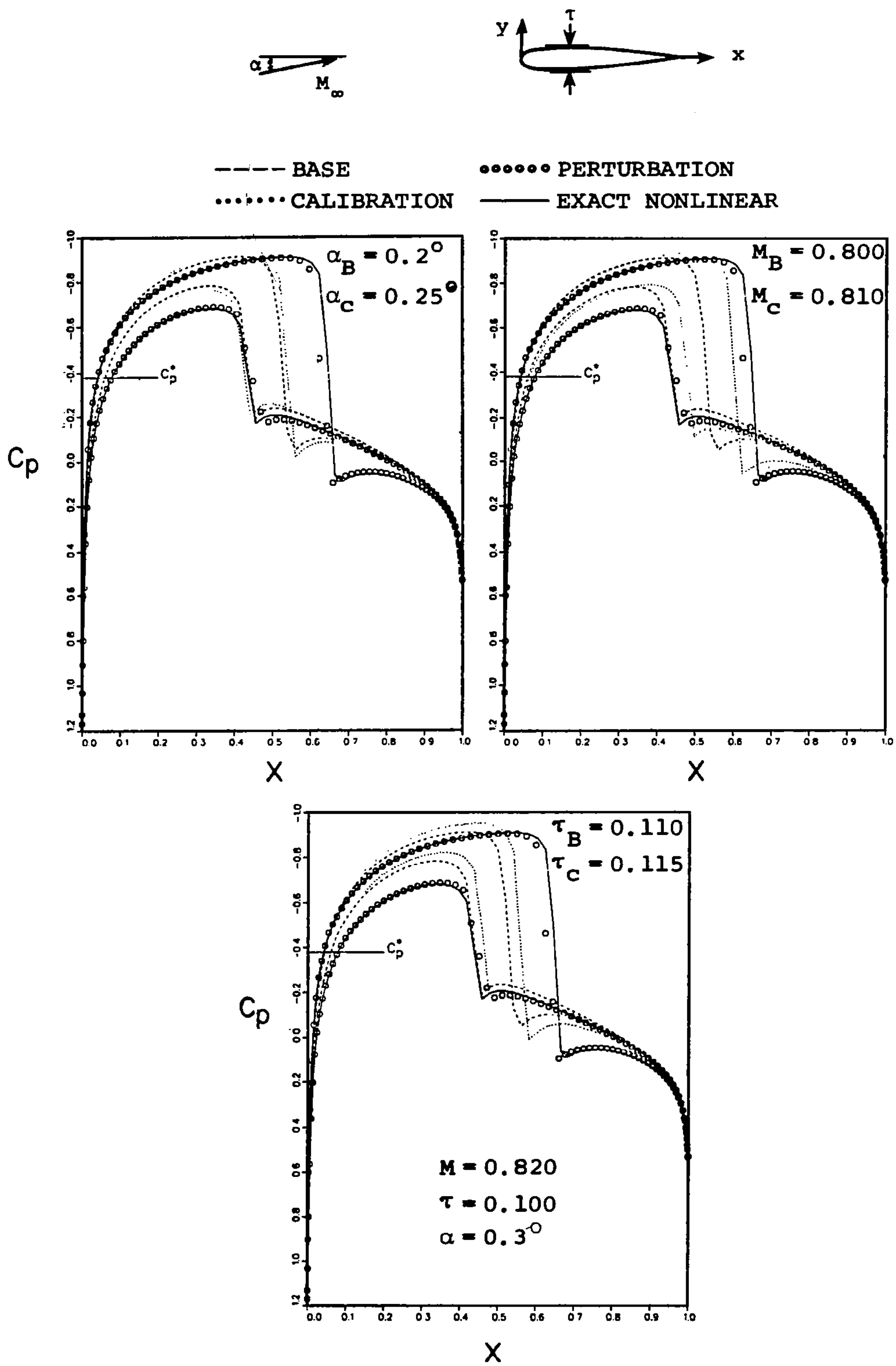

Figure 2.- Comparison of perturbation (○) and exact nonlinear (—) surface pressures for the simultaneous three-parameter perturbation of (α, M_∞, τ) for strongly supercritical flows past isolated NACA 00XX blade profiles.

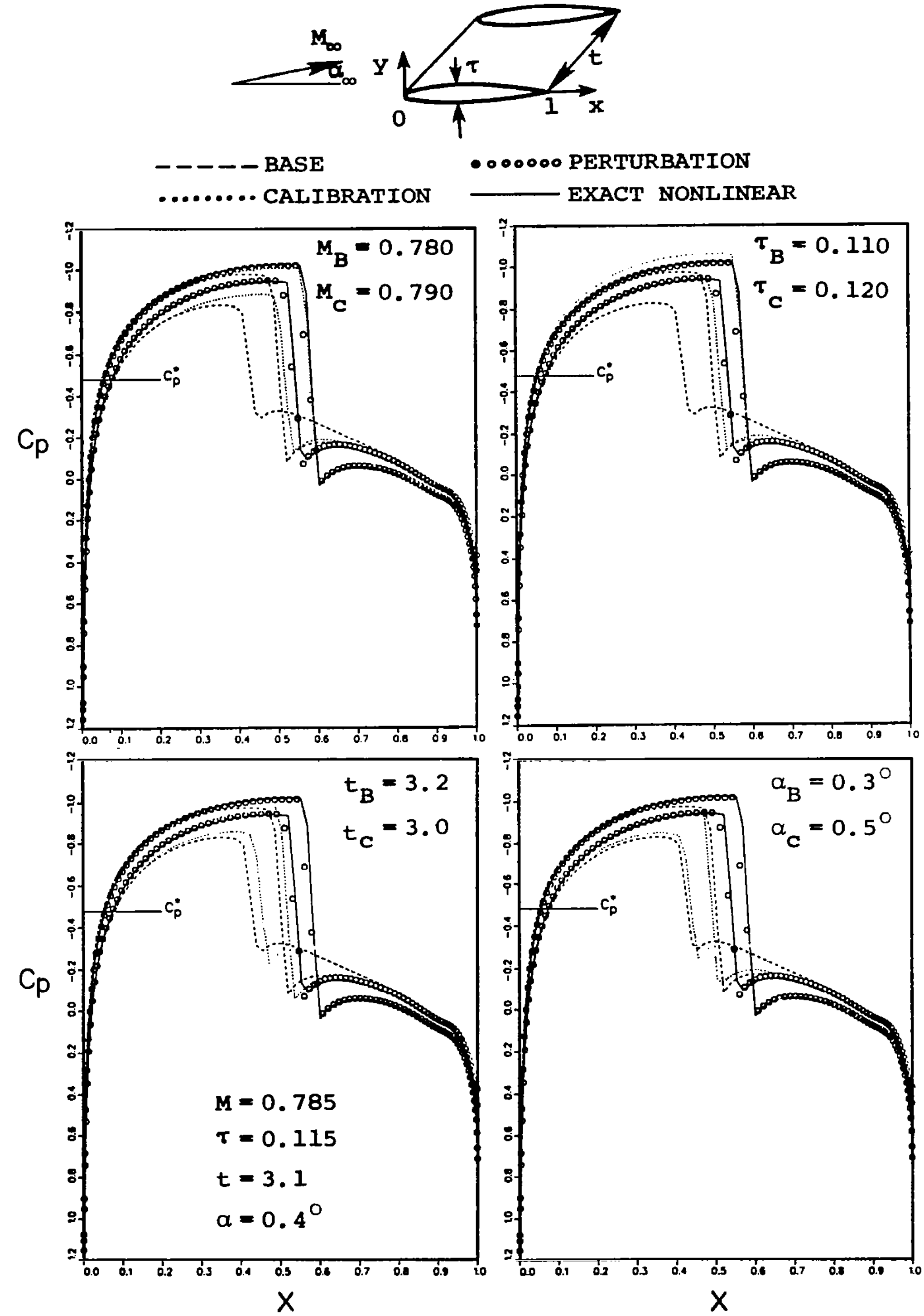

Figure 3.- Comparison of perturbation (O) and exact nonlinear (—) surface pressures for the simultaneous four-parameter perturbation of $(M_\infty, \tau, t, \alpha_\infty)$ for strongly supercritical flows past a cascade of NACA 00XX blade profiles.

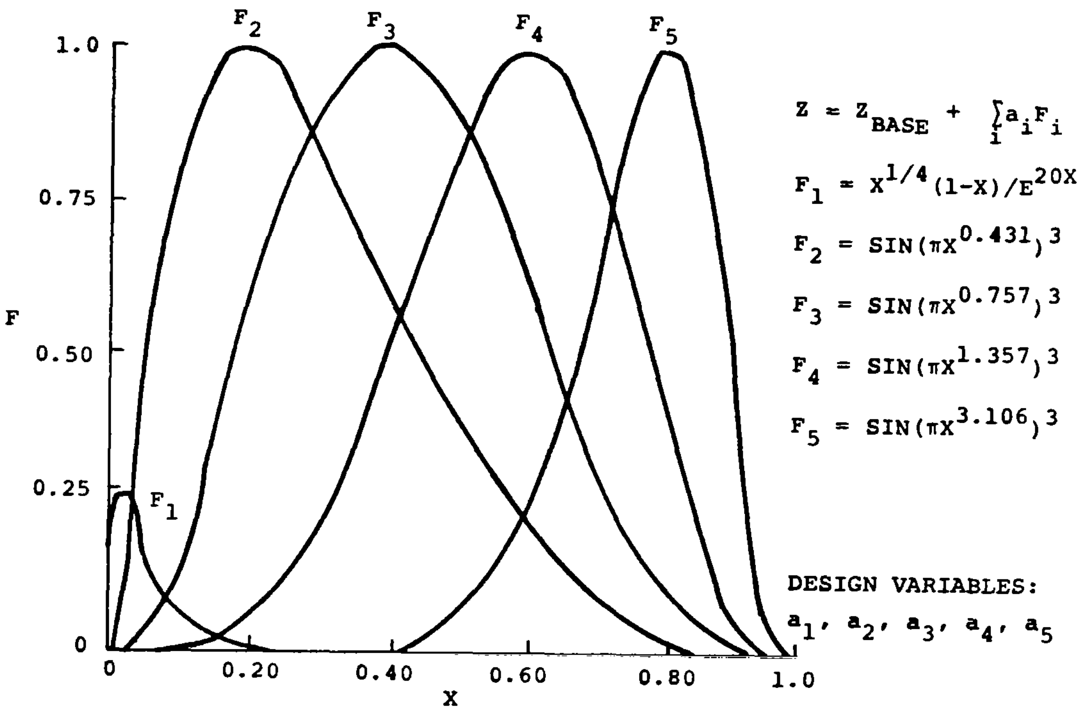

Figure 4.- Illustration of typical ordinate shape functions F_i employed in blade contour alteration optimization problems.

OBJECTIVE IS TO MINIMIZE:

$$\Sigma\left(C_{P_{Predicted}} - C_{P_{Desired}} \right)^2$$

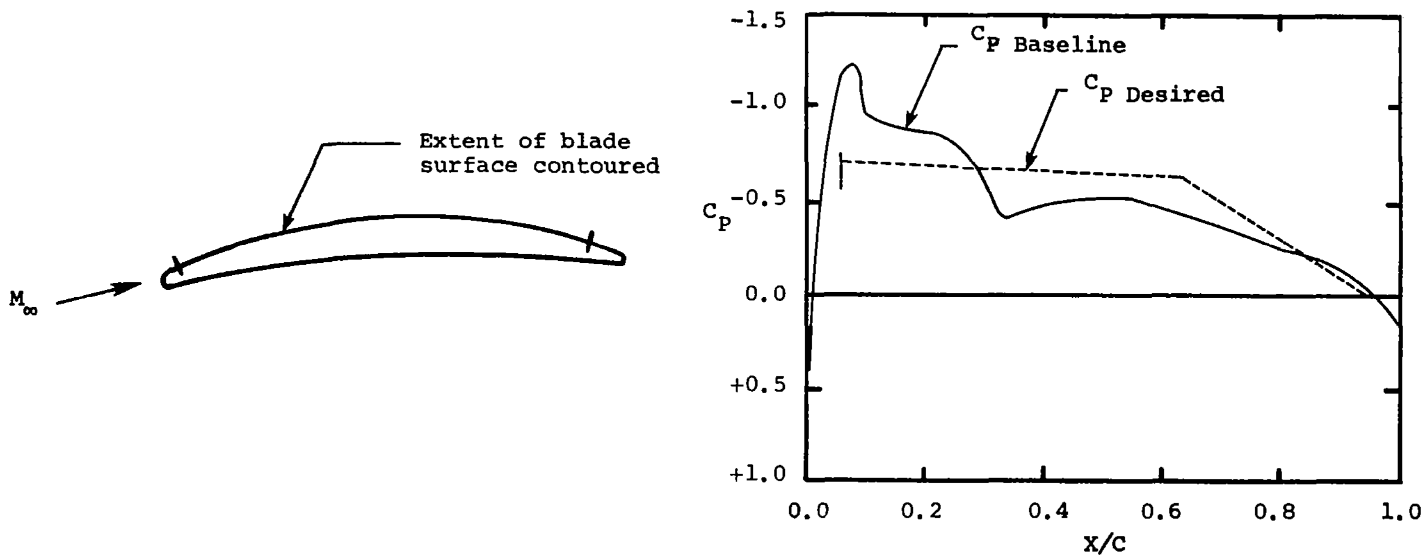

Figure 5.- Illustration of physical basis of optimization problem involving blade surface contouring to taylor the surface pressure distribution to a desired distribution.

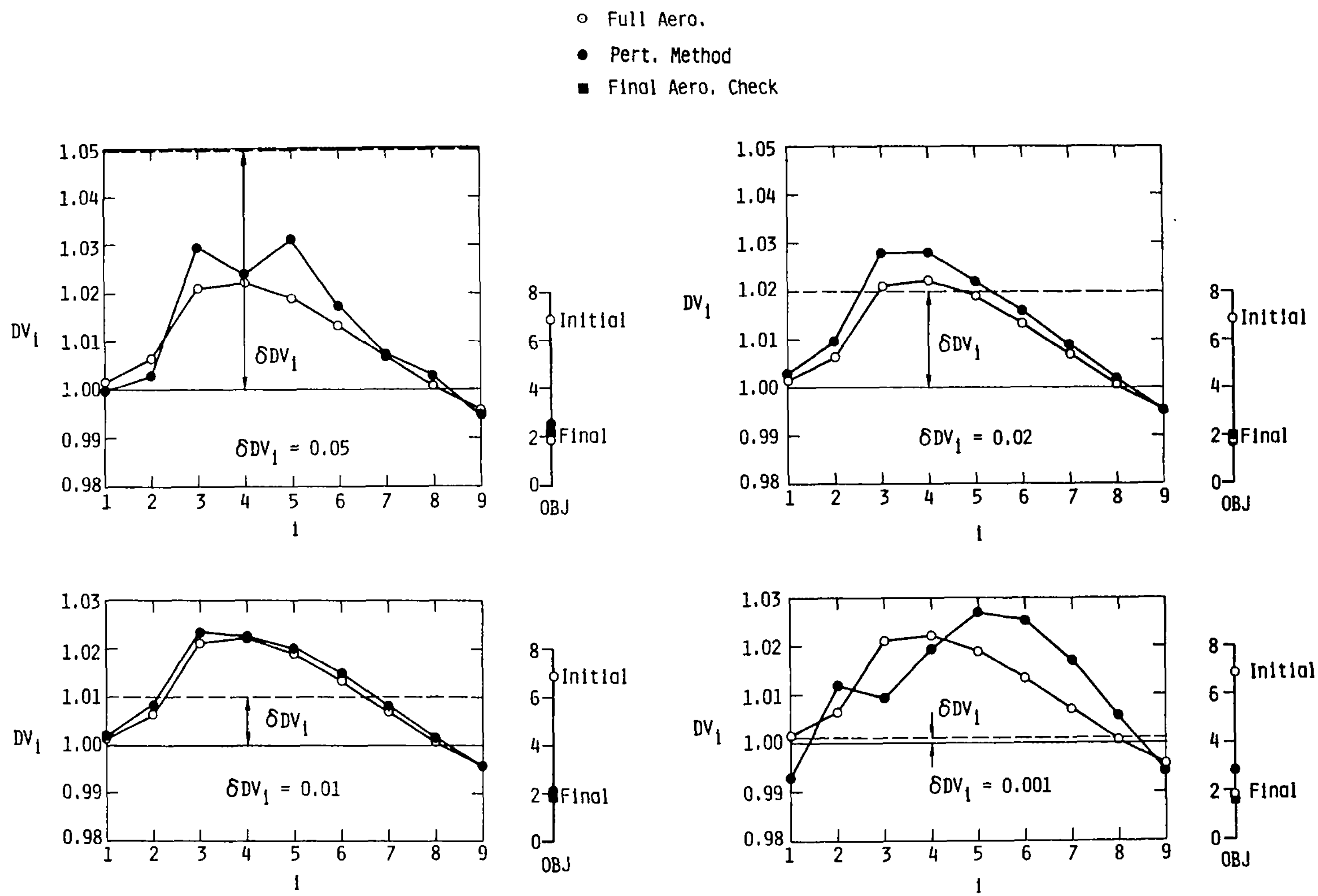

Figure 6.- Comparison of final design variables and objective function when employing the perturbation method (●) in lieu of the full nonlinear aerodynamic solver (⊙) for various choices of initial design variable stepsize for nine design variable subcritical optimization case study with a surface pressure tailoring objective.

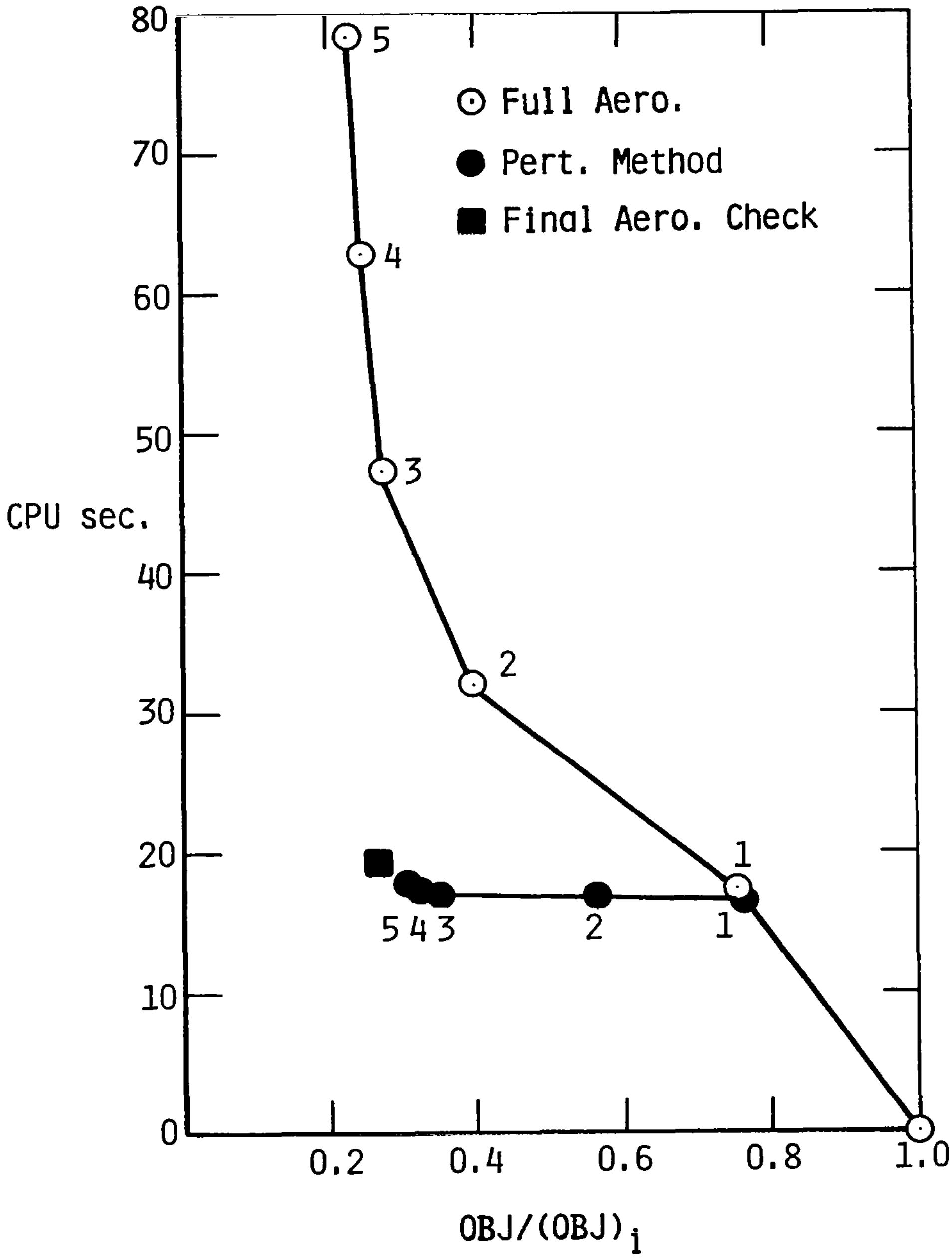

Figure 7.- Comparison of computational work and objective function reduction per optimization search cycle when employing perturbation method (●) or full nonlinear aerodynamic solver (⊙) for nine design variable subcritical optimization case study using a surface pressure tailoring objective.

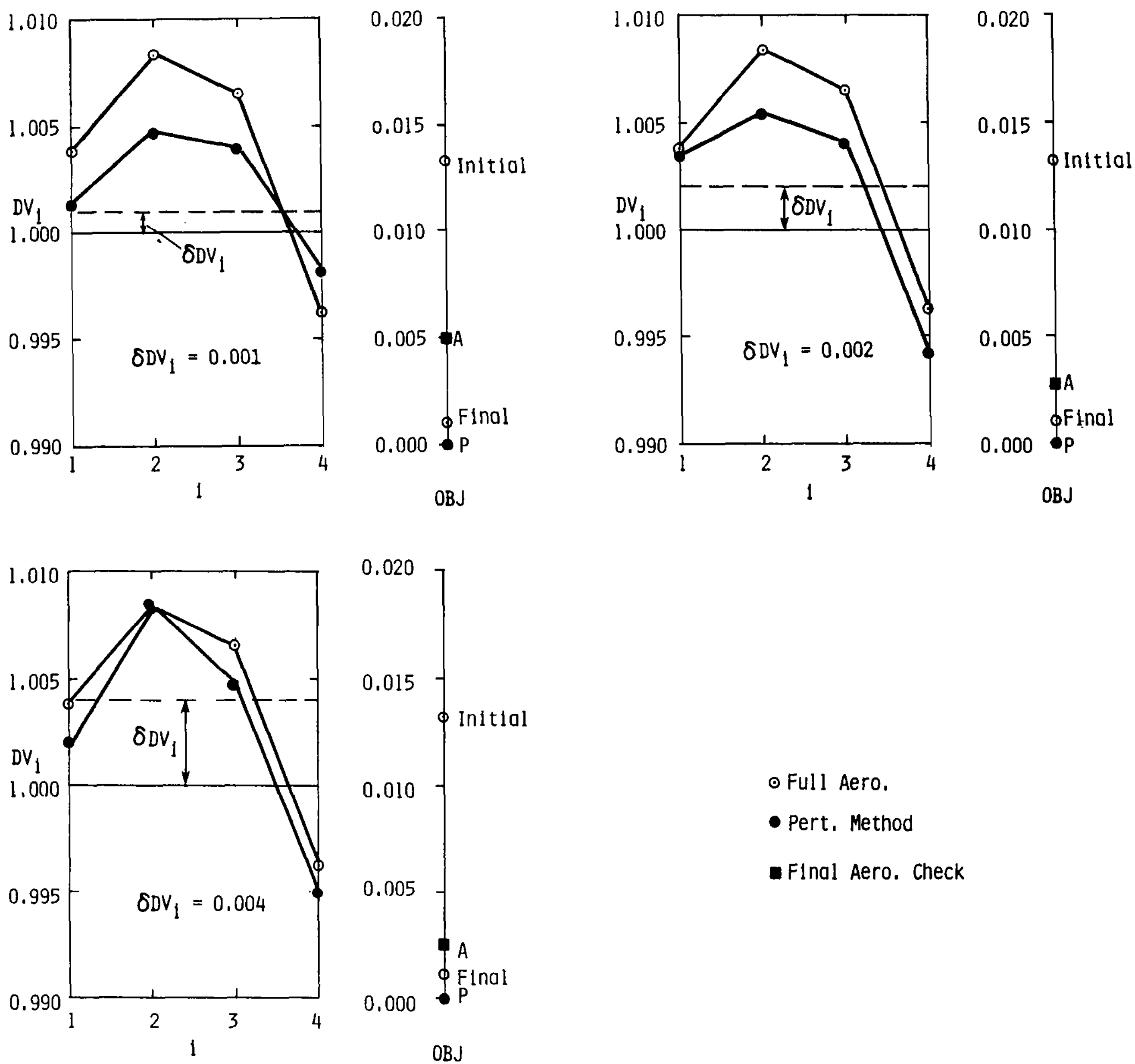

Figure 8.- Comparison of perturbation-predicted final design variables and objective function for various choices of initial design variable stepsize for four design variable supercritical optimization case study with drag minimization objective.

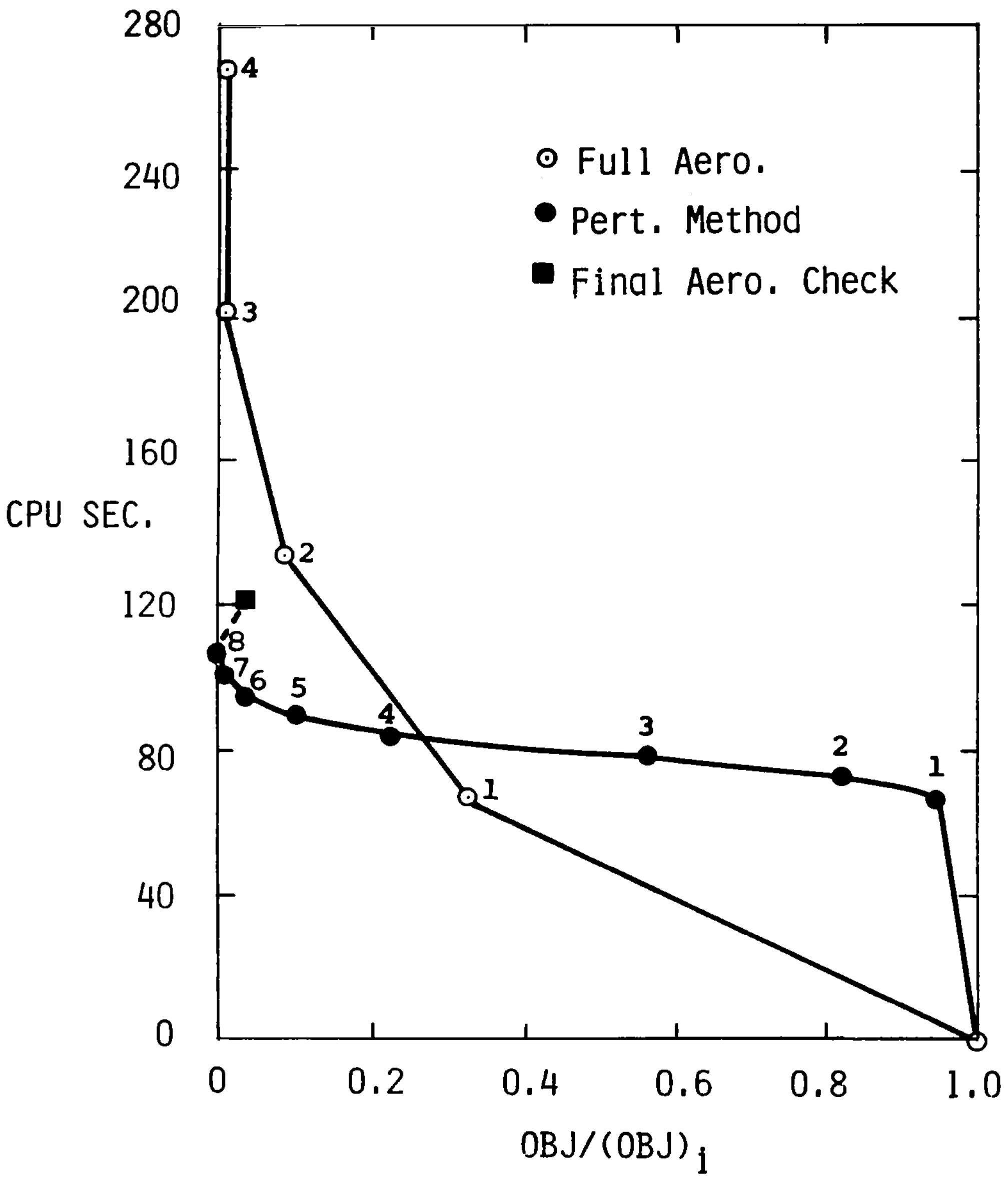

Figure 9.- Comparison of computational work and objective function reduction per optimization search cycle when employing perturbation method (●) or full nonlinear aerodynamic solver (⊙) for four design variable supercritical optimization case study using a drag minimization objective.

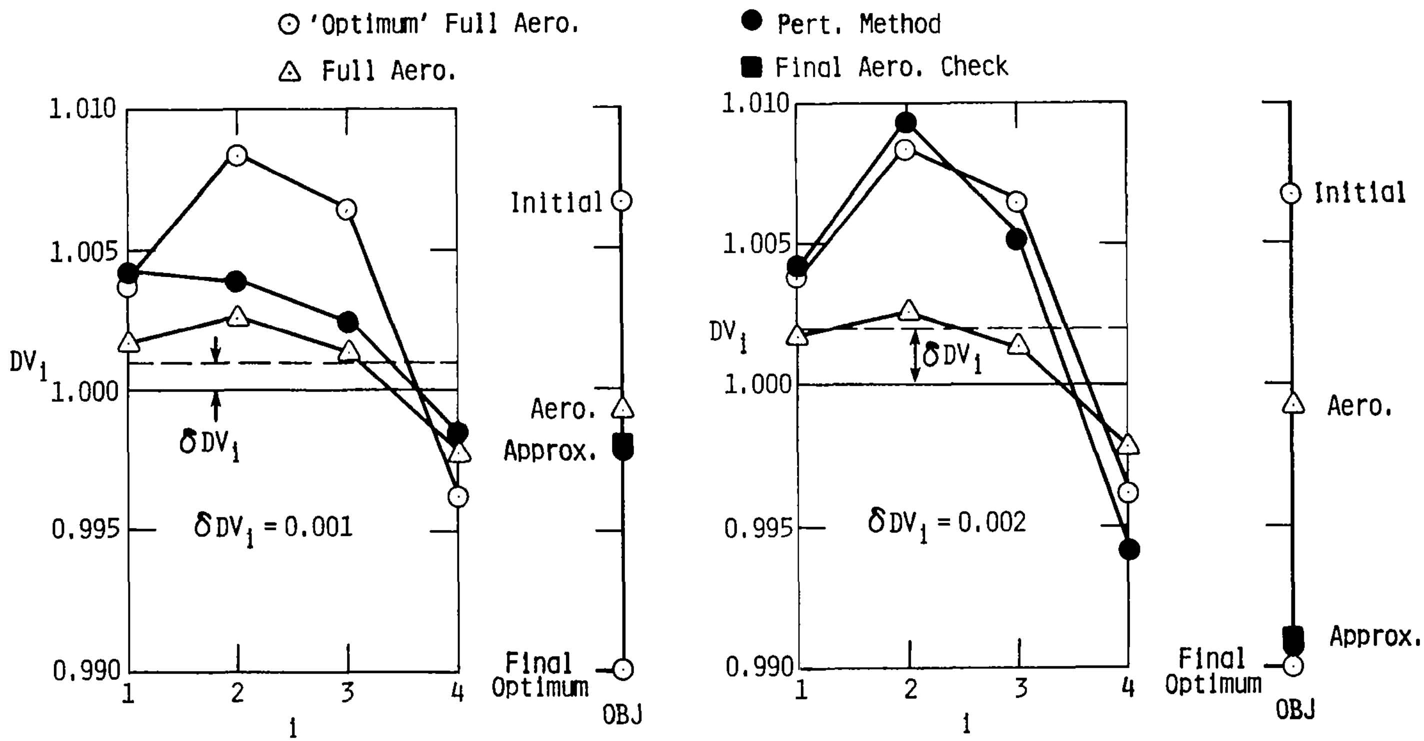

Figure 10.- Comparison of final design variables and objective function when employing perturbation method (●) or full aerodynamic solver (△) as compared with 'optimum' full aerodynamic result (⊙) for different choices of initial design variable stepsize for four design variable supercritical optimization case study using a surface pressure tailoring objective.

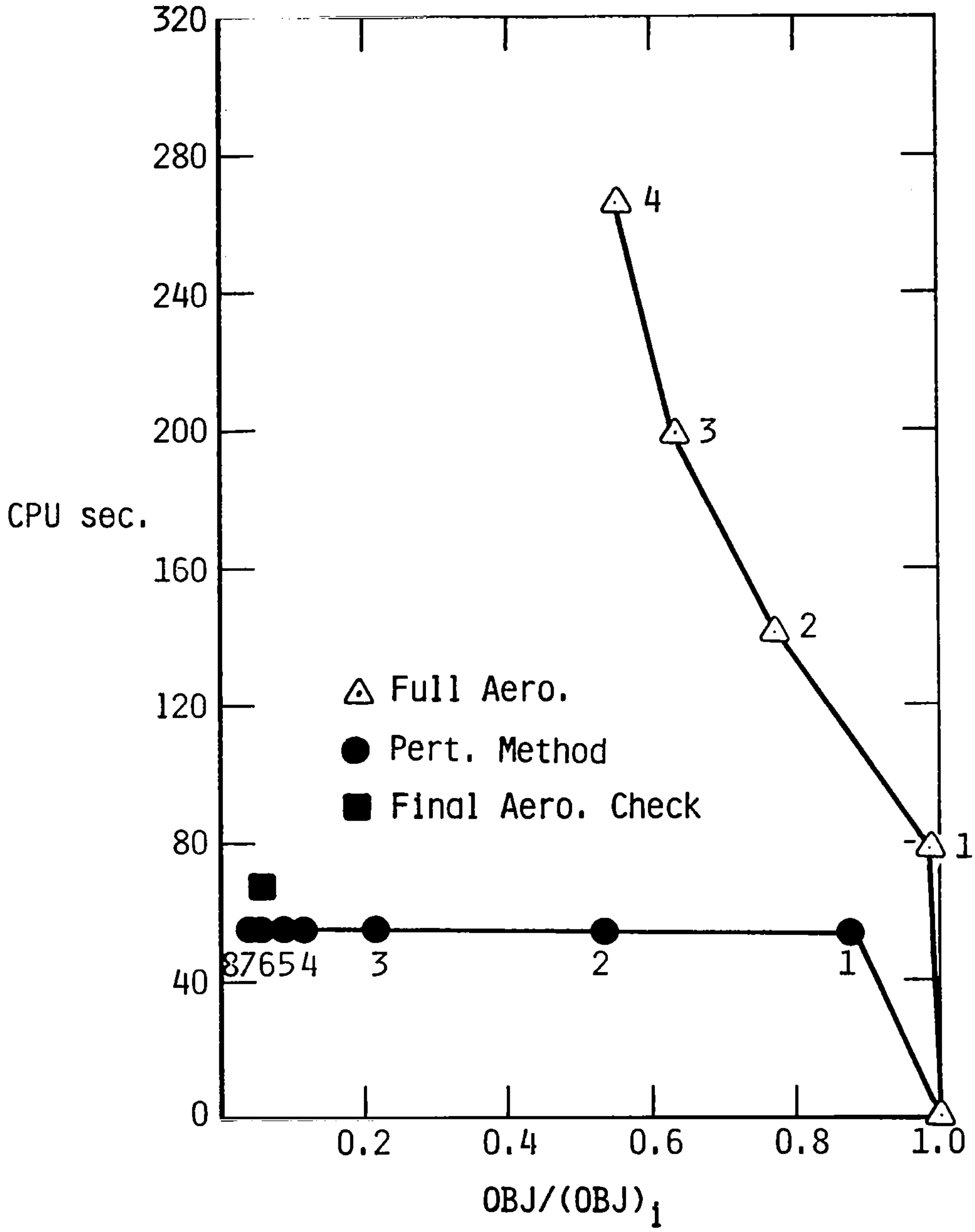

Figure 11.- Comparison of computational work and objective reduction per optimization search cycle when employing perturbation method (●) or full nonlinear aerodynamic solver (⊙) for four design variable supercritical optimization case study using a surface pressure tailoring objection.

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16. Abstract

An investigation was conducted to continue the development of perturbation procedures and associated computational codes for rapidly determining approximations to nonlinear flow solutions, with the purpose of establishing a method for minimizing computational requirements associated with parametric design studies of transonic flows in turbomachines. The results reported here concern the extension of the previously-developed successful method for single-parameter perturbations to simultaneous multiple-parameter perturbations, and the preliminary application of the multiple-parameter procedure in combination with an optimization method to blade design/optimization problem. In order to provide as severe a test as possible of the method, attention is focused in particular on transonic flows which are highly supercritical. Flows past both isolated blades and compressor cascades, involving simultaneous changes in both flow and geometric parameters, are considered. Comparisons with the corresponding 'exact' nonlinear solutions display remarkable accuracy and range of validity, in direct correspondence with previous results for single-parameter perturbations. Initial applications of the perturbation method combined with an optimization procedure demonstrate the ability of the multiple-parameter method to work accurately in a design environment and establish its potential for reducing the computational work required in such applications by an order of magnitude.

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