

Progress Report

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MECHANICALLY FASTENED JOINTS  
IN WOVEN FABRIC COMPOSITES

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## RESEARCH OBJECTIVES

A comprehensive investigation of the behavior of bolted joints in woven fabric composites was proposed as outlined by the following research objectives:

- 1) The experimental evaluation of the effects of fabric construction, material properties, laminate configuration and joint geometry on the strength and failure mode of woven fabric composites through a joint effort with NASA Langley Research Center.
- 2) The formulation of failure models consistent with the failure characteristics exhibited for each of the three primary modes, net tension, shearout and bearing.
- 3) The development of a complete bolted joint failure analysis for woven fabric composites through the coupling of mode specific failure criteria with an accurate stress analysis.

This interim report summarizes the progress attained in meeting these research objectives during the first six months of the research grant.

## INTRODUCTION

Traditionally, the design and analysis of composite bolted joints has relied heavily on empirical data for the specific material system, laminate configuration and joint geometry desired. For years, substantial effort has been devoted to the development of more sophisticated stress analyses, while few satisfactory advances in failure criteria have been proposed. Recently, significant advances have been made in the area of failure criteria for composite bolted joints [1-10], with resultant improvements in the accuracy of strength analyses. The ultimate objective is the development of a bolted joint strength analysis which requires only intrinsic material properties, thus eliminating the need to develop large empirical bolted joint data bases.

The development of any new strength analysis requires verification by comparison with experimental results. The current research program has been designed to simultaneously develop analytical failure criteria for the three primary failure modes (net tension, bearing and shearout), and generate the experimental data for the verification of these failure models over meaningful ranges of the parameters which influence bolted joint strength.

## EXPERIMENTAL PROGRAM

The goal of this portion of the research program is experimental characterization of the behavior of bolted joints in woven fabric composites. The overall program involves the generation of three data sets. First, basic material properties must be characterized. Second, the effects of fabric construction, laminate configuration and joint geometry on strength and failure mode must be determined experimentally. Lastly, the notched strength behavior of the system must be experimentally evaluated. The results from these three test programs form the empirical data base necessary for the bolted joint strength analysis program and its verification.

## TEST PROGRAM

The experimental program was fully detailed in the program proposal. For convenience, the summary of the test matrix is reproduced in Tables 1 and 2. Table 1 outlines the bolted joint tests which investigate the following important parameters: laminate configuration, stacking sequence, fastener diameter, edge distance, fastener half spacing, laminate thickness and fastener torque. Table 2 summarizes the notched strength test program which investigates the influence of laminate configuration, notch size and laminate thickness on strength.

Table 1. Proposed Bolted Joint Test Program

<u>Test Variable</u>	<u>Configuration</u>
Laminate Configuration	[0/90/±45/±45/0/90] <sub>ns</sub> [±45/0/90/0/90/±45] <sub>ns</sub>
Fastener Diameter	0.500 in. (12.7 mm) 0.375 in. ( 9.5 mm) 0.188 in. ( 4.8 mm)
Edge Distance (e/D)	4.0
Fastener Half Spacing (w/D)	3.0 6.0
Fastener Torque	15 in-lb 30 in-lb

Table 2. Notched Strength Test Program

<u>Laminate</u>	<u>Thickness (in.)</u>	<u>Hole Size (in.)</u>	<u>No. Specimens</u>
[0/90/±45/±45/0/90] <sub>2s</sub>	.128	.125	3
	.128	.250	3
	.128	.375	3
	.128	.500	3
[0/90/±45/±45/0/90] <sub>6s</sub>	.384	.125	3
	.384	.250	3
	.384	.375	3
	.384	.500	3
[±45/0/90/0/90/±45] <sub>2s</sub>	.128	.125	3
	.128	.250	3
	.128	.375	3
	.128	.500	3
[±45/0/90/0/90/±45] <sub>6s</sub>	.384	.125	3
	.384	.250	3
	.384	.375	3
	.384	.500	3

## SPECIMEN FABRICATION

At the request of NASA, Hercules AS4/2220-3 rubber toughened graphite epoxy material was chosen for the program. Plain weave fabric (3k fiber tows) prepreg has been procured for laminate fabrication using standard layup techniques.

Based on the test matrix, which calls for 264 tests, the set of laminates summarized in Table 3 have been designed to meet the test program requirements. In addition, a set of [0/90]\* and [+45]\* laminates were fabricated for the basic property characterization tests. About 60-70 percent of the bolted joint and notched strength laminate panels have been fabricated (laid up and cured), and another 10-15 percent laid up.

Processing of the AS4/2220-3 fabric-based system encountered problems not initially mentioned by the manufacturer. The standard cure had to be modified to a "no bleed" process in order to obtain the desired laminate quality and surface smoothness. Aluminum caul plates were used on top and bottom surfaces. The modified cure schedule and bagging

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\*These laminates were laid up such that warp and fill directions were always aligned in the same direction.



Table 3. Stacking Sequence/Cured Thickness

<u>Laminate Code</u>	<u>Stacking Conf.</u>	<u>Cured Thickness</u>
I <sub>a</sub>	[0/90/±45/±45/0/90] <sub>s</sub>	0.064
I <sub>b</sub>	[0/90/±45/±45/0/90] <sub>2s</sub>	0.128
I <sub>c</sub>	[0/90/±45/±45/0/90] <sub>3s</sub>	0.192
I <sub>d</sub>	[0/90/±45/±45/0/90] <sub>4s</sub>	0.256
I <sub>e</sub>	[0/90/±45/±45/0/90] <sub>5s</sub>	0.320
I <sub>f</sub>	[0/90/±45/±45/0/90] <sub>6s</sub>	0.384
I <sub>g</sub>	[0/90/±45/±45/0/90] <sub>9s</sub>	0.576
II <sub>a</sub>	[±45/0/90/0/90/±45] <sub>s</sub>	0.064
II <sub>b</sub>	[±45/0/90/0/90/±45] <sub>2s</sub>	0.128
II <sub>c</sub>	[±45/0/90/0/90/±45] <sub>3s</sub>	0.192
II <sub>d</sub>	[±45/0/90/0/90/±45] <sub>4s</sub>	0.256
II <sub>e</sub>	[±45/0/90/0/90/±45] <sub>5s</sub>	0.320
II <sub>f</sub>	[±45/0/90/0/90/±45] <sub>6s</sub>	0.384
II <sub>f</sub>	[±45/0/90/0/90/±45] <sub>9s</sub>	0.576

technique was discussed with Hercules and found to be identical to practices used by them for the cure of woven fabric 2220-3 composite systems. The cure schedule is given below:

1. Place prepreg in vacuum bag, then in autoclave, and apply full vacuum.
2. Apply pressure to 70-80 psi.
3. Vent vacuum to atmosphere at approximately 20 psi.
4. Begin heating at 3-5°F/min.
5. Hold temperature at 350°F for 2 hours.
6. Cool to 200°F in 40 min. or more under pressure (pressure may be decayed as temperature decreases).
7. Release pressure and remove from autoclave.

Individual test specimens are fabricated from the panels using standard techniques. Diamond wafering saws and diamond core bits are employed in trimming specimens to size and boring fastener holes. Specimen fabrication for the material property study has been completed. Fabrication is ongoing for the bolted joint and notched strength studies.

## TEST METHODS

The material property characterization study was conducted to measure the basic strength and stiffness of the AS4/2220-3 material in the warp and fill directions and in shear. Standard ASTM methods were employed [11-13] for the tension, compression and shear testing. Three replicates were tested for each property. Results are given in Table 4 for the tension and compression tests. The shear response tests have not been completed yet.

Only slight differences were measured between the warp and fill directions for either tension or compression properties. It is significant to note that the compressive properties, stiffness and strength are significantly lower than the tensile properties. The difference can be attributed to the woven fabric undulations, which amplify instability of the fibers and reduce the properties. The lower strength coupled with lower stiffness could result in interesting effects on the bearing strength of the material in a bolted joint.

None of the bolted joint testing has been completed yet. Bolted joint strength and failure modes will be measured using a single fastener, double lap joint configuration

Table 4. Tension and Compression Properties of the  
AS4/2220-3 Plain Weave Fabric

<u>Description</u>	<u>Modulus Msi</u>	<u>Strength Msi</u>	<u><math>\nu_{12}</math></u>	<u>Max. Strain Est. %</u>
Tension (warp)				
BJT1A	9.52	116.3	.063	1.24
BJT1C	8.83	122.8	.036	1.38
BJT1D	9.52	120.6	.050	1.28
Tension (fill)				
BJT2A	9.10	122.9	0.027	1.33
BJT2B	9.49	126.5	0.050	1.36
BJT2C	9.23	123.6	0.031	1.34
Compression (warp)				
BJC1A	8.04	97.1	-	
BJC1B	7.86	89.5	-	
BJC1C	7.97	108.1	-	
Compression (fill)				
BJC2C	7.93	85.3	-	
BJC2D	7.77	80.9	-	
BJC2E	7.85	94.6	-	

(Figure 1). A sketch of the fixture to be employed is shown in Figure 2. Metal plates are used to transmit load between the load cell and the bolted joint. Washers of known area will be placed between the metal links and the specimen. A fastener inserted through the double lap joint will be tightened to one of the two specified torques. The fit between the washer and the fastener and the specimen and the fastener will be closely controlled and quantified. The joint will be monotonically loaded to failure while recording the load versus extension behavior. The failure load will be designated as the maximum load carried by the joint before the material ceases to carry load.

Notched strength tests have not yet been performed, but will be conducted using standard tension test methods. Each specimen has been fabricated to the 2 inch wide by 9 inch long geometry shown in Figure 3. Specimens will be monotonically loaded to failure while recording the load extension characteristics.

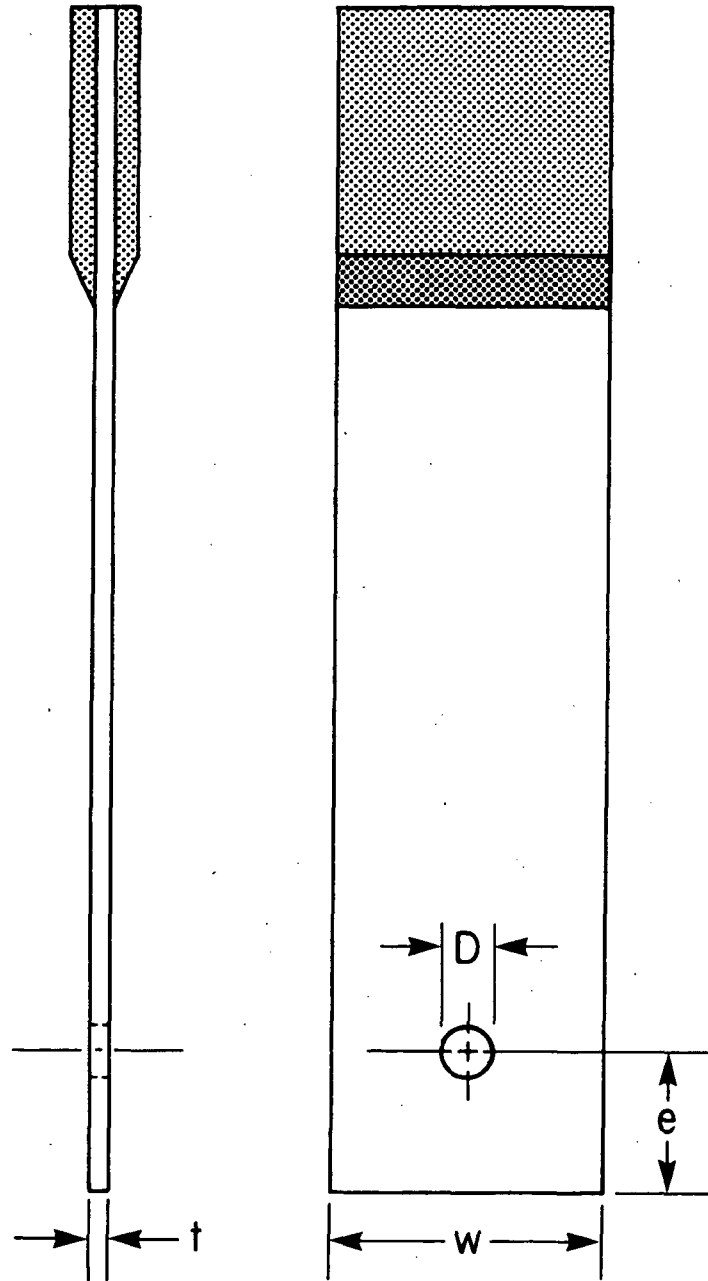


Figure 1. Bolted Joint Specimen

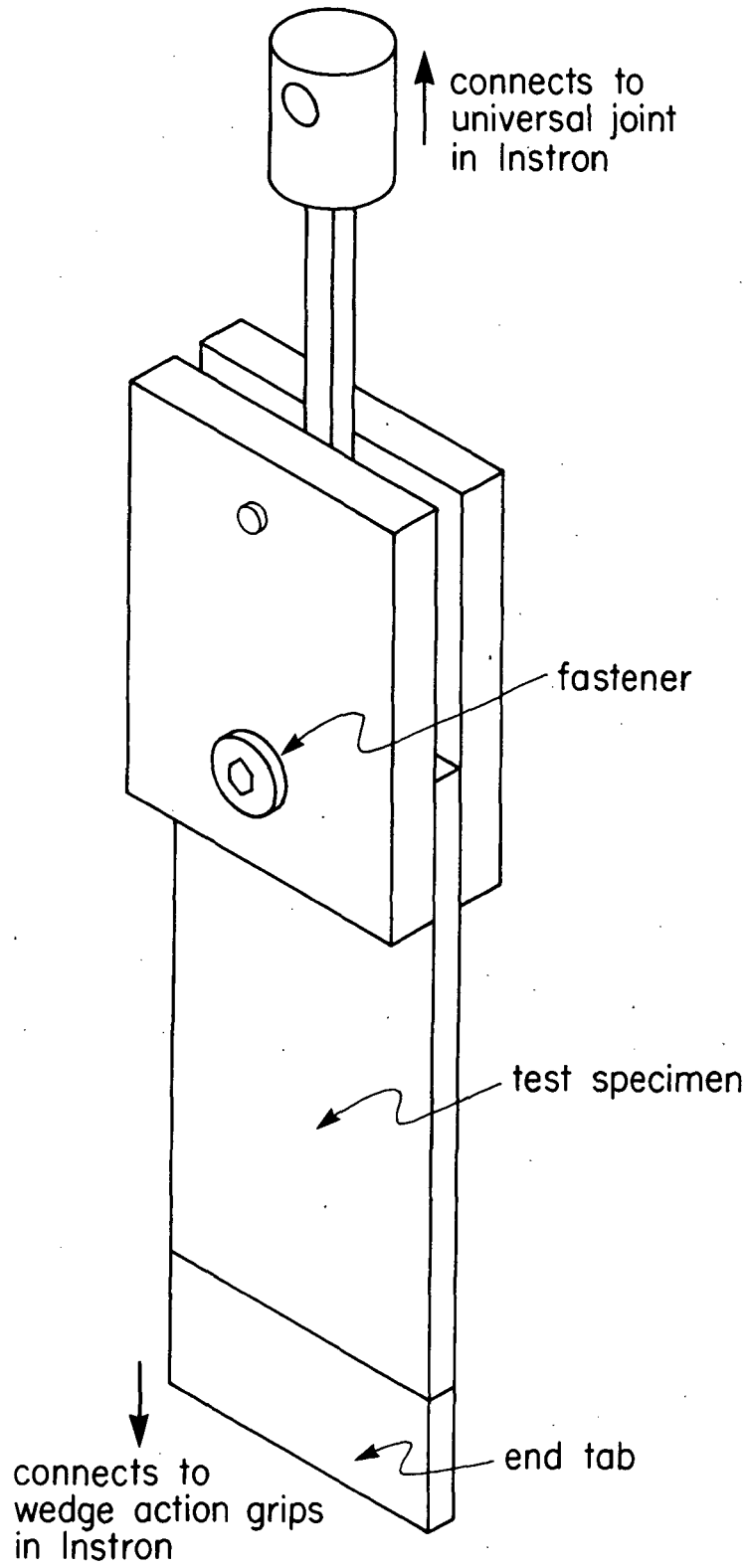


Figure 2. Bolted Joint Test Fixture

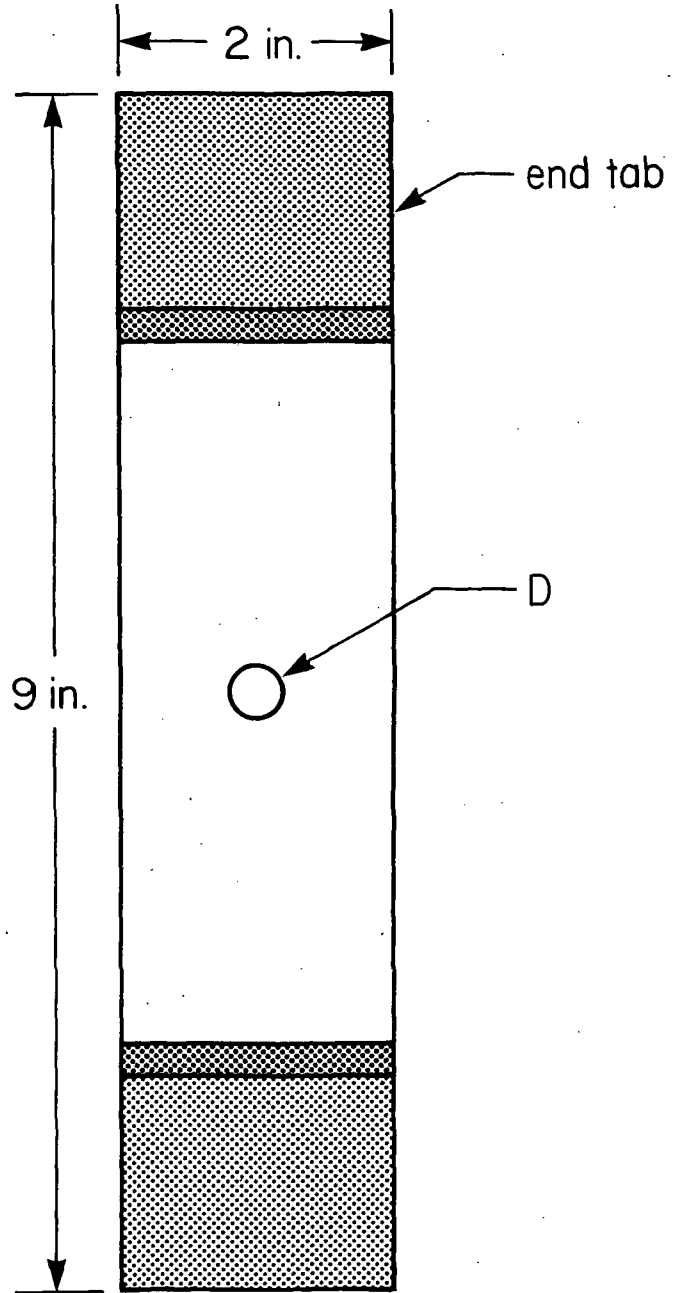


Figure 3. Notched Specimen



## BOLTED JOINT STRENGTH ANALYSIS

Strength analysis for composite bolted joints involves the mating of a stress analysis with an appropriate mode specific failure criterion for each of the primary failure modes. The stress analysis and failure criteria are independent of each other, and can be manipulated separately in order to optimize the strength analysis package formed by their coupling. Two methods of stress analysis were proposed, an approximate elasticity plane-stress formulation, and finite element analysis. The objective is to determine the simplest effective stress analysis which provides reliable results in combination with the empirically based failure criteria.

Both two-dimensional plane stress and pseudo three-dimensional finite element analyses were proposed. In order to more fully understand the bearing failure mechanism, a full three-dimensional finite element analysis has been substituted for the pseudo three-dimensional analysis. Both the 2-D and 3-D elements employ orthotropic material properties.

The mesh has been developed for the two-dimensional plane stress analysis and is shown in Figure 4. Mechanical

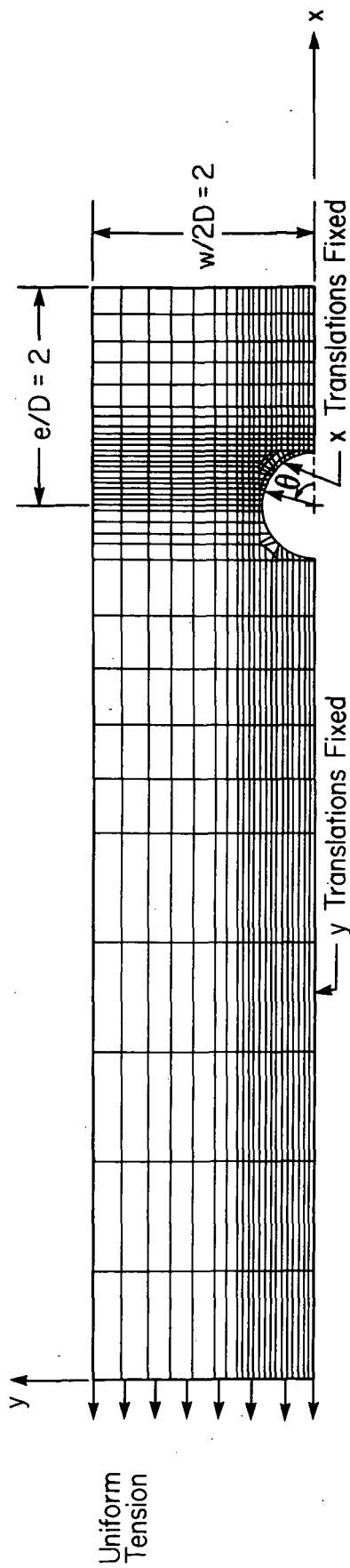


Figure 4. Finite Element Model Used to Analyze the Bolted Joint.

loads are applied axially along the  $x=0$  edge, and the bolt is simulated by fixing the  $x$  translations of the nodes along the hole boundary, excluding the two nodes near the  $90^\circ$  position. Symmetry was used to reduce the number of degrees of freedom necessary for solution of the problem. The mesh and boundary conditions are a refinement of those employed for previous bolted joint studies [14-15], and have been shown to provide accurate results for stresses along the net tension and shearout planes.

The three-dimensional finite element analysis will employ the same mesh, only contain elements through the thickness. The inplane boundary conditions remain similar. A compressive uniform pressure distribution will be employed to model constraint by the washer and the transverse compressive loads introduced by torque on the bolt. This analysis will only be used for a limited number of case studies directed at characterizing the bearing failure mechanism.

A literature survey has identified several approximate elasticity solutions [1,2,4,16,17] for composite bolted joint stress analysis. The solutions all treat the laminate as a homogeneous orthotropic plate of infinite width loaded by a rigid circular inclusion. A solution also exists for the case of an infinite orthotropic plate containing a

circular cutout loaded in tension. The superposition of these two solutions allows the analysis of bolt-bearing bypass loading conditions, such as experienced when rows of fasteners are employed.

Finite width correction factors must be employed to these analyses to account for fastener interactions or interactions between the fastener and edge of the plate. In addition, the loading induced by the fastener is assumed to be a cosine distribution, an assumption which breaks down for some combinations of geometry and anisotropy. Even with these limitations, the accuracy has been deemed satisfactory enough to base several bolted joint analysis packages on this form of stress analysis [1,2,10].

A version of Waszczak's solution, which uses anisotropic elasticity theory, has been developed and programmed into the computer to form a stress analysis module. In this solution, the complex stress function

$$F(x,y) = 2R_e [F_1(z_1) + F_2(z_2)]$$

is used to express the components of stress as follows:

$$\sigma_x = \partial^2 F / \partial y^2$$

$$\sigma_y = \partial^2 F / \partial x^2$$

$$\tau_{xy} = \partial^2 F / \partial x \partial y$$

The complex stress functions are analytic functions of the complex characteristic coordinates  $z_1$  and  $z_2$ , where

$$z_1 = x + \mu_1 y$$

$$z_2 = x + \mu_2 y$$

and should not be confused with the complex physical coordinates  $z$  which are given by

$$z = x + iy$$

Compatibility requires that the following characteristic equation be satisfied by the  $\mu_k$  values.

$$B_{ij} \mu^4 - 2B_{16} \mu^3 + (2B_{12} + B_{66}) \mu^2 - 2B_{26} \mu + B_{22} = 0$$

Thus, the  $\mu_k$  are found in terms of the material compliance,  $B_{ij}$ . For an anisotropic material, four distinct roots are obtained. A mapping function is employed to map the internal boundary of the fastener hole into a unit circle in the transformed plane. The mapping function is

$$\zeta_k = \frac{z_k \pm \sqrt{z^2 - a^2 - \mu_k^2 a^2}}{a(1 - i\mu_k)}$$

where only one of the two values of  $\zeta_k$  lies inside the unit

circle, the other maps to a point outside the unit circle. The point outside the unit circle is the one of interest.

Making the following simplification in notation by letting

$$\phi_1(z_1) = dF_1(z_1)/dz_1$$

$$\phi_2(z_2) = dF_2(z_2)/dz_2$$

the stress components become

$$\sigma_x = 2\text{Re} [u_1^2 \phi_1'(z_1) + u_2^2 \phi_2'(z_2)]$$

$$\sigma_y = 2\text{Re} [\phi_1'(z_1) + \phi_2'(z_2)]$$

$$\tau_{xy} = 2\text{Re} [u_1 \phi_1'(z_1) + u_2 \phi_2'(z_2)]$$

Because of the mapping function, the dimensionality of the problem has been reduced by one, and the boundary conditions can be introduced by a complex Fourier series in terms of the single variable  $\theta$ .

Requiring that all stress components be real and single valued, the function  $\phi_k$  for a loaded hole in an infinite plate has been shown to be of the following form.

$$\phi_k(\zeta_k) = A_k/u_k \zeta_k + \sum_{m=1}^{\infty} A_{km} \zeta_k^{-m} \quad k = 1, 2$$

By applying the boundary conditions and imposing the equilibrium conditions, the constants  $A_k$  and  $A_{km}$  can be determined for a known load. Once the  $\phi_k$  is known, the stress components can be determined.

The computer code developed to perform this stress analysis is currently being verified and debugged.

## FAILURE CRITERIA

Ideally, the failure criterion should use a basic material property to predict failure of the composite laminate. For the case of unidirectional continuous fiber based laminates, the basic material properties are failure stresses and/or strains of a single ply in the principal directions (normally  $x_1^T$ ,  $x_1^C$ ,  $x_2^T$ ,  $x_2^C$ ,  $S^6$ , etc.). When the laminate consists of a combination of unidirectional and off-axis ply orientations, the failure of each ply will depend upon its orientation with respect to the principal loading axis. Most failure criteria perform a ply-by-ply analysis and designate first ply failure as failure of the laminate. The drawbacks to that philosophy are obvious if one considers a laminate with  $90^\circ$  plies. The inaccuracy of the first ply failure analysis has been circumvented by many investigators by using progressive-failure analysis; that is, redistributing the load upon ply failure and continuing until ultimate failure occurs.

Another option has been to use laminate based failure parameters instead of ply properties. The result is better accuracy, but more data must be collected since each laminate configuration demands a new set of data. However,



even this type of failure criterion is far less data intensive than bolted joint characterization.

Woven fabrics do not exhibit strong differences in strength as a function of direction, like unidirectional systems. Therefore, the first ply failure problem should be less of a factor in laminate failure prediction. Woven fabrics are not as anisotropic in properties as unidirectional tape composites, making the first ply failure less significant in laminate strength analysis. In view of these considerations, both ply-by-ply and laminate based failure criteria will be evaluated.

Candidate failure criteria have been identified for each failure mode based on current knowledge of bolted joint failure mechanisms. Comparisons of more than one criteria will be made in cases where appropriate. The criteria which will be investigated for each failure mode are listed below.

Net Tension:	Point Stress Criterion Quadratic Interaction Criterion
Shearout:	Point Stress Criterion Quadratic Interaction Criterion
Bearing:	Collings Criterion Quadratic Interaction Criterion (full 3-D)

The essence of the point stress criterion can be stated mathematically as follows:

$$\sigma_i|_{x=r+d_0} = \sigma_0$$

where  $\sigma_i$  = stress component adjacent to the hole for the failure mode under consideration

$\sigma_0$  = unnotched strength of the laminate for the failure mode under consideration

$d_0$  = is the critical distance parameter

The critical distance parameter is a function of hole size and has the following form [22].

$$d_0 = \frac{1}{c} \left( \frac{R}{R_0} \right)^m$$

Constants  $c$  and  $m$  are found from empirical data by determining  $d_0$  for two notch sizes under bolted joint loading conditions.

The quadratic interaction criterion is expressed in the following form to predict strength of composites subjected to multiaxial states of stress.

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j \geq 1 \quad i = 1, 3$$

where the  $F_i$  and  $F_{ij}$  are strength tensors of 2nd and 4th rank, and the  $\sigma_i$  are the stress components. The strength tensors must be determined from experiment for each material system

considered. Failure occurs when the sum of the tensor products exceeds unity.

The bearing failure criterion developed by Collings [6] was based on careful observation of bearing and compression failures of laminates. Collings determines the average bearing stresses in basic subunits of the laminate such as:

$$\sigma_{b_1} = \frac{1}{A}(\sigma_o A_o + \sigma_{b_{45}} A_{45}) \quad \text{for a } 0/\pm 45 \text{ sublaminates}$$

$$\sigma_{b_2} = \frac{1}{A}(\sigma_{90} A_{90} + \sigma_{b_{45}} A_{45}) \quad \text{for a } 90/\pm 45 \text{ sublaminates}$$

$$\sigma_{b_3} = \frac{1}{A}(\sigma_{b_o} A_o + \sigma_{90} A_{90}) \quad \text{for a } 0/90 \text{ sublaminates}$$

where  $\sigma_o$  is the average bearing stress in  $0^\circ$  plies at failure

$\sigma_{90}$  is the average bearing stress in the  $90^\circ$  plies at failure

$\sigma_{b_{45}}$  is the average bearing strength of a  $\pm 45^\circ$  laminate

$\sigma_{b_o}$  is the average bearing strength of a  $0^\circ$  laminate

$A_o$ ,  $A_{90}$ ,  $A_{45}$  are the projected areas of the  $0^\circ$ ,  $90^\circ$  and  $\pm 45^\circ$  plies, respectively

The interaction effects (resulting from the presence of other plies) on bearing strength of  $0^\circ$  and  $90^\circ$  plies in multidirectional laminates expressed through ply bearing factors.

These factors are defined as

$$k_o = \frac{\sigma_c}{\sigma_o}$$

and

$$k_{90} = \frac{\sigma_{TC}}{\sigma_{90}}$$

where  $\sigma_c$  is the longitudinal compression strength

$\sigma_{TC}$  is the constrained transverse compression strength

Using these bearing factors in conjunction with the preceding equations for bearing stresses, bearing strength expressions are formed.

For 0/±45 laminates

$$\sigma_{b_1} = \frac{1}{t} \left[ \frac{t_o \sigma_c}{k_o} + t_{45} \sigma_{b_{45}} \right]$$

for 90/±45 laminates

$$\sigma_{b_2} = \frac{1}{t} \left[ \frac{t_{90} \sigma_{TC}}{k_{90}} + t_{45} \sigma_{b_{45}} \right]$$

and for 0/90 laminates

$$\sigma_{b_3} = \frac{1}{t} \left[ \frac{t_{90} \sigma_{TC}}{k_{90}} + t_o \sigma_{b_o} \right]$$

When laminates contain more than three fiber directions, the above criteria can be superposed to give the bearing strength of the new laminate.

These failure criteria have all been used with varying degrees of success in composite bolted joint strength analysis. From the present research, their applicability to woven fabric composites will be assessed. The most accurate criteria for each failure mode will be incorporated into a bolted joint failure analysis package.

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