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APPLICATION OF A SYSTEMATIC FINITE-ELEMENT MODEL MODIFICATION TECHNIQUE TO DYNAMIC ANALYSIS OF STRUCTURES

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APPLICATION OF A SYSTEMATIC FINITE-ELEMENT MODEL MODIFICATION TECHNIQUE TO DYNAMIC ANALYSIS OF STRUCTURES

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Abstract

A systematic finite-element model modification technique has been applied to two small problems and a model of the main wing box of a research drone aircraft. The procedure determines the sensitivity of the eigenvalues and eigenvector components to specific structural changes, calculates the required changes and modifies the finite-element model. Good results were obtained where large stiffness modifications were required to satisfy large eigenvalue changes. Sensitivity matrix conditioning problems required the development of techniques to insure existence of a solution and accelerate its convergence. A method is proposed to assist the analyst in selecting stiffness parameters for modification.

Nomenclature

a_j^i	modal contribution to eigenvector in equation (3)
k	spring stiffness
$\underline{K}$	stiffness matrix
$\overline{K}$	stiffness term in equation (9)
m	lumped mass
M	mass matrix
N	diagonal scaling matrix in equation (5)
P	value of structural parameter
P_f	value of flexibility parameter
P_m	value of mass parameter
S_{ee}	diagonal matrix of standard deviation terms in equation (5)
S_{rr}	diagonal matrix of covariance terms in equation (5)
T	sensitivity matrix - rectangular matrix of eigenvalue or eigenvector component changes due to a unit change in a structural parameter in equation (4)
$\|T\|$	determinant of sensitivity matrix
X	eigenvalue or eigenvector component in equation (1)
Y	eigenvector
λ	eigenvalue
Δ	prefix indicating incremental change

Introduction

In the Drones for Aerodynamic and Structural Testing (DAST) Program[1] at the Langley Research Center and Dryden Flight Research Facility of the Ames Research Center, several wings are being designed and fabricated for a Firebee drone aircraft for research purposes. The first Aeroelastic Research Wing (ARW-1) installed on the drone is shown in figure 1. Accurate values of the natural frequencies and mode shapes of the wing are required for the design of an onboard active flutter control system. Ground testing of one wing center box revealed a large discrepancy (25 percent) between analytical and experimental values of one of the natural frequencies.[2] In the wing (fig. 2), bonding of the relatively flexible fiberglass skins to the much stiffer spar and stiffener flanges greatly reduced the width of the skins subjected to inplane shear. This increased the shear stiffness of the wing compared to the finite-element model where elements modeling the skins extended from the front to the rear spar web. Modification of the shear properties of the skins in the finite-element model to reflect this increased stiffness changed the analytical frequencies to produce satisfactory agreement with measured frequencies. While intuitive modification was satisfactory in this case, a systematic method to modify a finite-element model to produce desired changes in the natural frequencies and mode shapes is desirable. Such a capability was included in the SPAR Structural Analysis System[3] by Engineering Information Systems, Inc. (EISI) under contract to the NASA Marshall Space Flight Center and is available also in the EISI Engineering Analysis Language (EAL) System.[4] It is the purpose of this paper to evaluate this capability for stiffness changes, establish methods for its efficient use and demonstrate it on two small problems and the ARW-1 center box model.

Structural Modification Process

The structural modification process is initiated by an analyst when calculated vibration frequencies or mode shapes of a structural finite-element model do not agree with: (1) characteristics desired by the analyst, such as a minimum natural frequency; or (2) vibration frequencies or mode shapes measured on an actual structure. The tasks involved are selection of the model properties to be modified, estimating the magnitude of the property changes required and calculating natural frequencies and mode shapes of the modified structure (fig. 3).

The structural modification process is complicated by a number of factors. The relationship between the change in a particular structural property and the change in eigen characteristics is frequently nonlinear and relatively expensive to determine, at least as compared to the evaluation of displacements and stresses for an additional static load condition. The term eigen characteristic is used herein to mean an eigenvalue (frequency squared in rad^2/sec^2) or a component of an eigenvector (mode shape). The selection of the particular structural properties to be modified is difficult because the number of eigen characteristics to be changed is usually small compared to the total number of structural elements in the model, which means that several combinations of elements may exist. Furthermore, as shown by the problem with the ARW-1[2], in some cases only part of the element properties are affected. Considering the

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aforementioned difficulties, it would seem that the quality of the solution is strongly dependent upon the structural analyst, and a method that gives the analyst the greatest amount of insight into the problem is desirable. For this reason, the use of physical properties of the individual finite elements of the model, singly or in combination, may be desirable even though it may require more computation than methods that directly modify the total structure's mass and stiffness matrices.[5]

Property Change Estimation

In both SPAR[3] and EAL[4] estimation of the change in structural properties necessary to change particular eigen characteristics of the structure is accomplished in a program module called the Structural Modification (SM) processor. To estimate the structural property changes, a linear relationship

$$[T]\{\Delta P\} = \{\Delta X\} \tag{1}$$

is assumed where T is a rectangular sensitivity matrix. The coefficients in a column of T are the eigen characteristic changes caused by a "unit" value of a structural parameter (specified increment in one or more structural properties). The estimated parameter change, ΔP, is the parameter multiple necessary to satisfy the equation, and ΔX is the required change in eigen characteristics (target (desired) value minus existing value).

Terms of the sensitivity matrix are computed using a method in which the undamped modal equations are modified by applying increments to the stiffness matrix and (diagonal) mass matrix in a manner consistent with a change in the structural properties of one or more finite elements. The analyst must select the elements to be modified. The change in the eigenvalue (frequency squared), $\Delta\lambda_i$, caused by a structural parameter[3] is

$$\Delta\lambda_i = \{Y_i\}^T[\Delta K]\{Y_i\} - \lambda_i\{Y_i\}^T[\Delta M]\{Y_i\} \tag{2}$$

where λ_i is the i^{th} eigenvalue, Y_i is the i^{th} eigenvector and ΔK and ΔM are increments in the stiffness and diagonal mass, respectively, caused by the structural parameter. Eigenvector changes are approximated by

$$\{\Delta Y_i\} = a_i^1\{Y_1\} + a_i^2\{Y_2\} + .. + a_i^n\{Y_n\} \tag{3}$$

where

$$a_j^i = \frac{1}{(\lambda_i - \lambda_j)}(\{Y_j\}^T[\Delta K]\{Y_i\} - \lambda_i\{Y_j\}^T[\Delta M]\{Y_i\})$$

$$\text{for } i \neq j \tag{4}$$

$$a_j^i = \tfrac{1}{2}\{Y_i\}^T[\Delta M]\{Y_i\} \text{ for } i=j$$

Eigenvectors are scaled to produce a generalized mass of unity.

The structural properties that can be modified are beam areas and moments of inertia, spring constants, structural weights or mass/(area or length), shell section constitutive relations and lumped rigid masses. All the parameters are scaled directly with the exception of the shell section constitutive relations which are scaled inversely because these relations are stored as flexibilities in the data base. The consequences of this exception to direct scaling are discussed in appendix A. The structural parameters used in the program prohibit the specification of a nonlinear relationship between bending stiffness and element dimensions.

SM Processor Functioning

The SM processor performs the structural modification in four phases. They are: (1) ΔX generation (right-hand side of (1)); (2) sensitivity matrix, T, calculation; (3) solution of equation (1) for the predicted parameter changes, ΔP; and (4) modification of the structural properties in the data base. The analyst may select the phases to be executed in any given SM execution. The first phase determines the right-hand side of equation (1) by calculating the difference between target and actual values of the selected eigenvalues and eigenvector components.

Sensitivity matrix. The sensitivity matrix, T, is formed for the structural parameters specified by the user in the second phase of the SM processor execution. The changes in the stiffness and mass matrices due to a structural parameter are calculated by reforming the stiffness and mass matrices for the incremented properties and subtracting the original matrices therefrom. While it would be easier to ratio the individual element matrices, the method used is more general since it permits modification of the various properties of an element. Formation of the sensitivity matrix is by far the most expensive operation during SM execution. The changes in the stiffness and mass matrices due to an individual parameter may be stored in the data base at the option of the user.

Change in structural parameters. The changes in structural properties required to produce the desired changes in the eigenvalue or eigenvector components are calculated by solving equation (1). The solution method used in SPAR and EAL is basically the same as that described in reference 6 which is a least-squares solution with additional statistical terms described below. The equation that is solved in SM is

$$\{\Delta P\} = ([S_{rr}][NT]^T([NT][S_{rr}][NT]^T + [S_{ee}])^{-1}[N]\{\Delta X\} \tag{5}$$

where T, ΔP, and ΔX are the previously described sensitivity matrix, estimated parameter change and eigen characteristic change, respectively. N is an optional diagonal matrix which scales the sensitivity matrix by the inverse of the original values of the eigen characteristics. S_{rr} and S_{ee} are user selected diagonal matrices. S_{rr} is a covariance matrix of the ΔP's with all positive values. The terms of S_{ee} may be positive or zero and are interpreted as the standard deviations of the product of N and a target tolerance vector. This phase of the SM execution is relatively inexpensive, and the user may execute it several times with various values of S_{rr} and S_{ee} to determine their effect upon the calculated values of the parameter changes.

Since the calculated values of the parameter change may be so large as to cause unrealistic structural modifications, such as negative values of stiffness or mass, the processor requires that the analyst specify limits on the parameter changes. These limits may be applied to cause scaling of the parameter changes or reduction of only those that exceed the limits. The fourth phase of the SM execution actually modifies the structural property values stored in the data base.

Eigenvalue Recalculation

The estimation of structural property changes, described in the preceding section, is a linear approximation of a nonlinear process and the actual values of the eigen characteristics of the modified structure must be determined by calculating the eigenvalues and vectors for that structure. The operations shown in the lower part of figure 3 are relatively expensive and therefore it is desirable: (1) to reduce the cost of each iteration; and (2) to perform the minimum number of iterations.

Since the change in structural properties of the model is relatively small per iteration, the use of eigenvectors from the previous iteration as initial approximations for eigenvalue extraction is cost effective and simple to implement. The execution cost of the SM processor may be reduced by using previously saved changes in stiffness and mass matrices, but significant data manipulation is required to insure that the parameter changes and data sets modified are compatible with the retained stiffness and mass changes. Efforts to reduce the number of iterations by the judicious selection of parameter limits and variation of the terms of the S_{ee} matrix used in phase 3 of SM will be discussed in the next section. Application of a Rayleigh-Ritz technique to approximate the eigen characteristics for the ARW-1 example problem is described in Appendix B.

Structural Parameter Selection

The preceding discussion of the SM processor has been based on the assumption that the structural parameters required to satisfy a model change were known. In general, this is not usually the case and some method to assist the analyst in selecting the structural properties to be modified is desirable. For this purpose, a method is proposed which assumes that each element is used as a structural parameter in a preliminary analysis. Inasmuch as the group of parameters may not be unique and, as shown by the ARW-1 example, only part of the element properties may be affected, the method cannot identify absolutely the necessary parameters. It is assumed, however, that the analyst can select structural parameters by evaluating the modeling of those elements which show relatively large parameter changes in this preliminary analysis. Due to the large number of elements in a practical problem, target values are restricted to eigenvalues because of the effort required to evaluate equations (3) and (4).

From equation (2), it can be seen that the change in the eigenvalue due to a stiffness change is equal to twice the modal strain energy due to that change and the change in the eigenvalue due to a mass change is equal to twice the modal kinetic energy due to the mass change. For the present study only stiffness changes are considered, and the modal strain energies for each element are available from EAL (they are computed in SPAR but not available without program modification). The existing matrix manipulation capability in the arithmetic utility (AUS) processor is used to solve equation (5) for the required changes in element stiffnesses. The sensitivity matrix, T, consists of twice the elemental modal strain energies. When solving equation (5) using AUS, it is convenient to use an identity matrix for the sealing matrix, N, and a zero value for the standard deviation, S_{ee} but any positive value could be used.

Elemental modal kinetic energies are more difficult to obtain if elemental masses are changed. Sensitivity matrix terms due to rigid mass changes are nodal quantities and nodal modal kinetic energies can be readily calculated.

Results and Discussion

Two example problems were selected to evaluate the functioning of the SM processor before trying to solve the larger ARW-1 center box with its large stiffness changes. The problems include a simple two degree-of-freedom spring-mass system and a small idealized cantilever box beam. As stated previously, only changes in element stiffness properties were investigated. The problems solved in references 3 and 6 were modeled with bending elements while the ARW-1 center box is modeled primarily with elements having inplane stiffness.

Two Degree-of-Freedom Spring-Mass System

A simple two degree-of-freedom spring-mass system (fig. 4) was studied to determine that the SM processor produces a good approximation of the eigenvalue changes caused by a structural property change. For an undamped linear system the two eigenvalues (frequency squared) for the model shown in figure 4 are

$$\lambda = \frac{1}{2}\left(\frac{k_1+K_2}{m} \mp \sqrt{\left(\frac{k_1}{m}\right)^2 + 4\left(\frac{k_2}{m}\right)^2}\right) \qquad (6)$$

For this small problem where the eigenvalues are expressible algebraically, the terms of the sensitivity matrix due to stiffness changes are the derivatives of the eigenvalues with respect to the two stiffnesses, k_1 and k_2. These derivatives are

$$\frac{\partial(\lambda_{1,2})}{\partial k_1} = \frac{1}{2m}\left(1 \mp \frac{k_1}{(k_1^2+4k_2^2)^{1/2}}\right) \qquad (7)$$

$$\frac{\partial(\lambda_{1,2})}{\partial k_2} = \frac{1}{m}\left(1 \mp \frac{2k_2}{(k_1^2+4k_2^2)^{1/2}}\right) \qquad (8)$$

The sensitivity matrix values produced by the SM processor are the same as those obtained from *equations (7) and (8) for particular values* of k_1 and k_2 indicating that SM produces a satisfactory approximation numerically.

The accuracy of the sensitivity matrix terms insures that equation (1) will predict the change in eigenvalues caused by small known changes in structural properties. The normal situation, however, is that the sensitivity matrix terms and the required eigenvalue changes, ΔX, are known and the change in structural parameters, ΔP, is unknown. This requires the solution of equation (1), which is possible only if the *determinant of the square sensitivity matrix* is nonzero. For this very small problem, the determinant of the sensitivity matrix can be readily evaluated. Defining

$$\overline{K} = (k_1^2+4k_2^2)^{1/2} \qquad (9)$$

the determinant may be expressed as

$$\left|T\right| = \frac{1}{(2m)^2}\begin{vmatrix} 1-k_1/\overline{K} & 2-4k_2/\overline{K} \\ 1+k_1/\overline{K} & 2+4k_2/\overline{K} \end{vmatrix} \qquad (10)$$

Evaluation of the determinant gives

$$\left|T\right| = \frac{1}{m^2\overline{K}}(2k_2-k_1) \qquad (11)$$

Thus, for $k_1 = 2k_2$ the determinant of the sensitivity matrix is zero and equation (1) cannot be solved for the required parameter change.

The fact that equation (1) may become singular for this small problem indicates that there may be more difficulties to solving equation (1) than those caused by the *nonlinearities of the* problem. In the SM processor, however, the solution for the unknown parameters, ΔP, is accomplished using equation (5), and the matrix to be inverted consists of a matrix product plus an additional diagonal matrix with all positive terms. When the matrices are square, the determinant of the matrix product term must always be zero or greater since the determinant of a matrix product equals the product of the determinants[7] and the determinant of the covariance matrix, S_{rr}, must be positive since it is diagonal with all positive terms. Therefore, equation (5) can always be solved if the terms of the standard deviation matrix, S_{ee}, are large enough to overcome a singularity check in the program.

Cantilever Box Beam

A small cantilever box-beam problem was formulated to provide an example on which to experiment (fig. 5). The beam is 76.2 cm (30 in) long, 10.2 cm (4 in) wide and 5.1 cm (2 in) deep with four large masses of 175.1 kg (1.0 lb sec^2/in) at the tip. The material and dimensions of the members are shown in figure 5. The model has 16 joints and 36 unconstrained translational degrees of freedom, and was modeled using rods and isotropic quadrilateral membrane elements for the spars and ribs. The upper and lower cover skins were modeled with quadrilateral elements having orthotropic properties.

Beam Eigenvalues. The eigenvalues (frequency squared) *for the original* beam are listed in the second column of Table 1. The target beam has 10 percent stiffer skins in all three bays than the original beam. The differences between the eigenvalues of the target and original beams are listed in the third column of Table 1. These values represent the values of ΔX in equation (1) for the first six modes for this problem. The stiffness properties of the upper and lower skins in each bay of the box beam were assumed to be the three variable structural properties used as parameters. Each parameter represents a 10 percent increase in the skin stiffness in one bay, e.g., *parameter 1 is a 10 percent increase in the* skins of the bay nearest the beam tip. The changes in eigenvalues (columns 4 through 6) between the incremented beam and the original beam are the terms of a sensitivity matrix obtained by finite differences.

Parameter Changes for Eigenvalue Differences. Considering only the first three modes, equation (1) becomes

$$\begin{bmatrix} 0.1146 & 1.0181 & 2.8357 \\ 37.6527 & 38.0525 & 38.6075 \\ 117.2399 & 90.2831 & 26.1319 \end{bmatrix}\begin{Bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta P_3 \end{Bmatrix} = \begin{Bmatrix} 3.9746 \\ 118.6821 \\ 235.6705 \end{Bmatrix} \qquad (12)$$

Solution for the ΔP's using the SM processor (equation (5)) with all the terms of S_{rr} equal to 1.0 and all the terms of S_{ee} having equal but variable values produces the curves shown in figure 6. The determinant of the coefficient matrix in equation (12) is not near zero; hence, the differences between the correct value of one for each of the ΔP's (a 10 percent increase in stiffness) and those shown for $S_{ee} = 0.0$ in figure 6 *are due to the nonlinearities* of the problem. It appears that the selection of S_{ee} values anywhere within the range plotted in figure 6 produces a better estimate of the ΔP's (nearer to one for all ΔP's) than for S_{ee} =0.0 which is equivalent to the direct solution of equation (12). The curves shown decrease monotonically to zero as the value of S_{ee} is increased above that plotted in figure 6. Since the solution of the sensitivity equation (phase 3) in the SM processor is relatively inexpensive, it would appear that selection of an advantageous value of S_{ee} to speed convergence of the process is worthwhile.

SM Processor Results. The cantilever box beam was analyzed with the SM processor using a 10 percent increase in the stiffness properties of the beam skins as discussed in the section on beam eigenvalues. The resulting sensitivity matrix is given in Table 2. Comparison of the SM calculated sensitivity terms in Table 2 with the eigenvalue difference terms in equation (12) shows less than a 10 percent difference between any two corresponding terms. The SM calculated parameter changes (with N in equation (5) being the reciprocal of the eigenvalues and S_{rr} terms equal to 1) are shown as functions of the standard deviation, S_{ee}, in figure 7. Comparison of figures 6 and 7 show vastly different estimates of the required parameter change for S_{ee} = 0.0 which indicates that the sensitivity matrix is sufficiently ill-conditioned that relatively small changes in the magnitude of the matrix terms cause large differerences in the equation solution. Again it appears that the solution using values of S_{ee} anywhere within the range plotted in figure 7 produces better results.

The preceding discussion deals with the estimated parameter change for the first iteration of the structural modification process but, as mentioned previously, several iterations are necessary for convergence. Therefore, a study of the effect of the value of the terms of S_{ee} on the structural modification process was made. Three eigenvalue targets (three lowest modes) were used; no limits were placed on the magnitudes of the parameter changes and the terms of S_{rr} were taken equal to 1.0. The values of the calculated parameter changes and the resulting eigenvalue changes for S_{ee} = 0.0 are shown in figure 8(a). The first iteration causes a large increase in two of the eigenvalues. These decrease slowly over the remaining iterations to the proper value at the sixth iteration. After the first iteration the parameter changes are much smaller and generally decreasing in size. The fact that the maximum parameter change at the sixth iteration is almost 20 percent of the applied change (0.02 change in stiffness) indicates that improvement in the iterative process is desirable.

For S_{ee} = 0.0001 (fig. 8(b)), convergence of the eigenvalues is much better than that shown in figure 8(a) for S_{ee} = 0.0, but the change in the structural parameters continues to be appreciable through the fifth iteration. For S_{ee} = 0.001 (fig. 8(c)) convergence of both eigenvalues and parameter changes is essentially complete by the third iteration. For the larger values of S_{ee} used in figures 8(d) and 8(e), the damping effect of S_{ee} on the solution becomes apparent and appears to affect the final values to which convergence will occur. The results shown in figure 8 indicate that there is an optimum value or range of values for S_{ee} which maximizes the rate of convergence of the structural modification process. All of the preceding results were calculated using the default mode of solving equation (5) where the terms of N are equal to the reciprocal of the eigenvalues. If the terms of N are set equal to unity by the user, the values of S_{ee} would be different but the same effect should occur.

The results presented in figure 8 are for three target eigenvalues and three structural parameters. In practice the number of targets and structural parameters may be different. Considering the difficulty: (1) in measuring large numbers of modal frequencies; and (2) selecting a small number of parameters to effect the desired changes, it would appear that a common situation would be for the number of targets to be smaller than the number of structural parameters. If the number of targets is greater than the number of structural parameters, it would appear that exact satisfaction of all targets is not possible. To evaluate the effect of using different numbers of structural parameters and eigenvalue targets, several calculations were made with three structural parameters and different numbers of eigenvalue targets. For one target eigenvalue, the third eigenvalue of the incremented structure was chosen because it has the largest change of the lowest three eigenvalues. The results for S_{ee} = 0.001 are shown in figure 9. Convergence of the third eigenvalue to the target value occurs in 2 iterations but the first two eigenvalues converge to different values than those obtained with three targets. This implies that a unique solution is not possible in this case.

For two target eigenvalues, the second and third eigenvalues of the incremented structure were chosen. The results for S_{ee} = 0.001 are essentially the same as those shown in figure 8(c). Again convergence is rapid but in this case all three eigenvalues and structural parameters converge to those obtained with three targets suggesting that a unique solution is possible with fewer targets then parameters, at least for this example problem.

Application of Structural Parameter Selection Method. The method of structural parameter selection proposed in the section on the structural modification process was applied to the cantilever box beam. The sensitivity matrix, T, consists of twice the elemental modal strain energies, the scaling matrix, N, is an identity matrix and the standard deviation, S_{ee}, is zero. Solutions were obtained using two through six modes (target eigenvalues) and all 27 elements. The resulting ΔP's for three modes are shown in Table 3. These results show that the change in box-beam skins is by far the largest for any group of elements and quite close to the 10 percent stiffness change added to get the target eigenvalues. Similar results were obtained for the solutions using 2, 4, 5 and 6 modes. Thus, it can be concluded that for the cantilever box beam the solution of equation (5) for all the elements in the structure clearly identifies which elements will provide good choices for structural parameters and gives a good estimate of the changes required.

Parameter Change Limits. Stiffness parameter changes calculated for the cantilever box beam in the preceding discussion always remained within physically possible bounds. For problems with larger stiffness changes or more poorly-conditioned sensitivity matrices, the calculated parameter changes may exceed physical bounds and their use would create negative stiffness or mass properties. For this reason the program requires that the analyst place limits on the size of the parameter changes. These limits must not be too small, however, or the number of iterations to convergence will increase. Equation (6) shows that for a two degree-of-freedom system with a fixed mass that the eigenvalues will vary directly with the stiffness if both stiffnesses change by the same factor. If the stiffnesses do not change by the same factor, the increase in one of the stiffnesses will always be greater than the maximum change in the eigenvalue. Therefore, the parameter change limit should be at least as large as the maximum value of the difference between the original and target eigenvalues. For the cantilever box beam all the altered stiffnesses were changed the same amount (10 percent) while the maximum change in the first six eigenvalues was only 6.7 percent for mode 3. This result indicates that while the modified stiffnesses were changed the same amount, there are other unchanged stiffnesses which affect the third eigenvalue. This observation suggests that for structures where the structural properties to be modified affect only part of the total structure that the limits on structural parameter changes should always be larger than the largest change in the eigenvalues considered.

ARW-1 Center Box

The ARW-1 center box (fig. 2) is the primary wing structure outboard of the carry-through section. It consists of the front and rear spars, ribs, partial span stiffeners and upper and lower center box skins. The spars are made of 17-4PH steel and provide the largest part of the wing bending stiffness. The aluminum ribs support the center box skins and the leading and trailing edge skins (not included in the study). Aluminum stringers, not shown in the figure, are located in the inboard bays to prevent buckling of the center box skins. The center box cover skins are 0/90° fiberglass. The DAST research program required the 0/90° fiberglass layup to limit the wing stiffness. In rebuilding the wing center box after a research flight failure[2], the skins were bonded to the spars and stiffeners in addition to the single row of countersunk rivets. Dynamic testing of the wing center box in a cantilever configuration gave frequencies that differed from analytical results by a maximum of about 25 percent in the first torsion mode.

Finite-Element Model. An overall view of the ARW-1 center box finite-element model is shown in figure 10. It has 128 joints with a maximum of 6 d.o.f. per joint and 324 elements. The spar flanges are modeled as beams offset from the joints defining the quadrilateral membrane elements that model the spar webs. The rib webs are also modeled by quadrilateral membrane elements. The center box skins are modeled by quadrilateral elements having orthotropic bending and extensional properties. The skin elements are offset vertically from the joints defining the spar webs to more accurately model the wing bending and torsion properties.

The original model did not consider any increase in the shear stiffness of the skins due to bonding to the spar flanges or the aluminum access cover on the upper surface of the first bay. When experimental results indicated that a torsional frequency and, hence, torsional stiffness of the finite element model was too low, the shear stiffnesses of the skins were modified to reflect the reduced shear path through the fiberglass and the presence of the aluminum access cover.[2] The analytical results from the modified model matched the experimental frequencies well and are the target properties used in this study. The eigenvalues for the original and the modified models are listed in Table 4.

SM Processor Results. The wing center box was analyzed with the SM processor using 8 stiffness parameters representing a 1 percent increase in the shear flexibility of the wing center box skins. The first parameter was applied to the upper skin of the inboard bay, the second to the lower skin of the inboard bay and parameters three through eight applied to both skins of bays 2 through 7, respectively. Phases 1 and 2 of the SM processor were executed to generate the difference between the original and the target eigenvalues and the sensitivity matrix for the eight parameters. Then equation (5) was solved with the terms of the scaling matrix, N, equal to the reciprocal of the eight eigenvalues of the original structure and the value of S_{rr} equal to 1.0. The results of these calculations for S_{ee} equal to zero and varying from 10^{-7} to 1 by powers of 10 are shown in figure 11. These results show that for $S_{ee} = 0.0$, the results are of no value and that a physically realistic solution (less than a 100 percent decrease in the flexibility of any of the parameters) occurs only for S_{ee} greater than 10^{-3}. The results of the solution for S_{ee} from 10^{-3} to 10^{-2} (fig.12) show that minimum value of S_{ee} to produce realistic results for the parameter changes is about 1.5×10^{-3}. Comparison of the parameter values required to produce the correct stiffness difference between the original and target ARW-1 models (shown on the left in fig. 12) and the SM calculated values in figure 12 shows that there is no value of S_{ee} that produces good agreement. The SM solution produces changes having the proper sign for S_{ee} slightly greater than 0.002. The shape of the curves shown in figure 12 would seem to indicate that for values of S_{ee} greater than 0.004, S_{ee} dominates the solution of the equation.

Considering the effect of S_{ee} on the first iteration for the wing center box and on the cantilever box beam analysis discussed previously, a value of $S_{ee} = 0.002$ was selected for analysis of the wing center box. Again, considering the relation between eigenvalue and stiffness changes for the box beam, a limit of 45.0 percent was placed on the change in flexibilities. This is equivalent to a stiffness increase of 1.5 times the relative change in the fourth eigenvalue which is 0.54. The results of this calculation showed that convergence occurs in four iterations but that the effects of the standard deviation, S_{ee}, prevent converging to the exact values of the target eigenvalues. The problem was then solved with the terms of S_{ee} equal to 0.002 for two iterations, 0.0001 for one iteration, and 0.00001 for four iterations. The results are shown in figure 13. The convergence of the eigenvalues is excellent. The shear flexibilities from the shell section property tables for the original structure, the target structure and the final values calculated by SM are shown in Table 5. While large changes were made in the flexibilities, their convergence is not as good as that for the eigenvalues and several more iterations with a smaller value of

S_{ee} would probably improve the flexibility values. Comparison of the results presented for these studies of the structural modification of the cantilever box beam and ARW-1 center box suggests that the use of artificial values for S_{ee} will speed the convergence of the process. Also, larger values of S_{ee} are required for larger relative differences between the target and original eigenvalues. As the relative eigenvalue differences decrease during the modification process, the value of S_{ee} must be decreased to assure analytical convergence. The minimum value of S_{ee} is limited, of course, by practical considerations. Finally, the limits placed on the allowable magnitude of the structural parameter changes can be related to the maximum relative change in the eigen characteristics.

Structural parameter selection. The method for selecting the structural parameters discussed previously was applied to the ARW-1 center box model. This represents an attempt to determine which of the 324 elements have the greatest effect on the first eight eigenvalues. As might be expected, the results are not clear as those for the much smaller box beam problem. The estimated maximum stiffness increase is 1.26 for one of the elements in the box skins. The only elements having estimated stiffness changes greater than 10 percent of the maximum element change are located in the skins and the spar flanges and webs. The estimated stiffness changes, averaged over a bay, are shown in Table 6 for skins and spars. The results for the skin elements indicate that those elements are the best candidates for structural modification parameters. In addition, there are some elements in the spars which have large estimated changes. In a practical problem, the analyst would have to consider these elements and make a choice based on his knowledge of the structure.

Concluding Remarks

A method for estimating the changes to a structure required to attain specified values of eigenvalues or eigenvector components has been investigated for stiffness changes. The method is contained in a program segment (Structural Modification (SM) processor) in the SPAR and EAL structural analysis systems.

The SM processor functions in four steps:

(1) establishes difference, ΔX, between model eigen characteristics and user specified target values.

(2) determines linear approximation of eigen characteristic changes due to changes in user specified structural parameters (sensitivity matrix (T)).

(3) calculates estmated values of structural property changes using a method that provides for inclusion of user estimates of structural parameter covariance and the standard deviation of ΔX.

(4) changes structural and mass properties of the finite-element model. Recalculation of the eigenvalues and eigenvectors of the structure is required to verify the accuracy of the estimated parameter changes.

A small cantilever box beam problem was solved using several values for the standard deviation of ΔX to determine its effect on convergence. The method was also applied to a research drone aircraft wing model with excellent results. Study results show that the sensitivity matrix may be poorly conditioned which results in a poor estimate of the parameter changes or, for a particular combination of stiffnesses, it may be singular. Use of one of the statistical properties included in the solution method, the standard deviation of ΔX, provides: (1) a method by which the equations may be solved, and (2) a means for accelerating the solution.

One of the primary problems associated with the use of the processor is the selection of the parameters to be modified. For stiffness changes, a method is proposed which uses modal elemental strain energies as terms of a sensitivity matrix to provide an estimate of the required change in properties for all elements. This estimate was shown to provide the analyst some assistance in selecting the stiffness parameters to be modified.

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Appendix A

Shell Section Scaling

Storage of the shell section constitutive relations as flexibilities requires that corrections be made between the stiffness and mass increments specified for a given parameter if the calculated parameter change, ΔP, is large. Mass increments are expressed as

$$\Delta M = (M)(P_m)(\Delta P_m) \quad (14)$$

where P_m is the value specified in the parameter definition for the mass change. Therefore, the revised mass is

$$M_{new} = (1+P_m*\Delta P_m)M_{old} \quad (15)$$

and for flexibilities, which are the inverse of stiffnesses,

$$F_{new} = (1+P_f*\Delta P_f)F_{old} \quad (16)$$

If stiffness is assumed to vary directly with mass, then the revised stiffness equals

$$K_{new} = (1+P_m*\Delta P_m)K_{old} \quad (17)$$

For parameter calculation, the ΔP's equal one and P_f takes on a value that causes the stiffness change to be the same as the mass change or

$$K_{old}(1+P_m) = \frac{1}{F_{old}(1+P_f)} = \frac{K_{old}}{(1+P_f)} \quad (18)$$

from which

$$P_f = \frac{1}{1+P_m} - 1 \quad (19)$$

Thus, if $P_m = .01$, $P_f = -0.00901$.

If the ΔP calculated by the program for these values of P_m and P_f is 20 (a 20 percent increase in mass), then the increase in shell section stiffness is

$$\frac{\Delta K}{K_{old}} = \frac{1}{(1+P_f*\Delta P_f)} - 1 = \frac{1}{(1-.009901*20.)} - 1 = 0.2469 \quad (20)$$

which is in error by 4.69 percent.

Two difficulties result from the error. First, the calculated parameter change in phase 3 will be in error. Inasmuch as the parameter change is an estimate and the estimate will be corrected in successive iterations this is not a serious problem. Secondly, the shell section constitutive relations will be scaled so that the mass and stiffness are no longer consistent. The error in the properties will carry through successive iterations without correction. Proper scaling of the shell section properties can be accomplished by increasing the mass term by a factor of 1.235 (0.2469/0.200) so that $P_f = -0.009901$ and $P_m = 0.01235$ in the proper parameter specification between phases 3 and 4 of two SM processor executions. The original values of P_m should be replaced before another SM execution. The analyst should check the printout of the shell section properties at the completion of a run to insure that the mass and stiffness are consistent if large parameter changes have been made.

Appendix B

Rayleigh-Ritz Procedure

The operational expense of the SM processor and its use of modal techniques plus the availability of the results of intermediate calculations makes the application of a Rayleigh-Ritz (R-R) approximation attractive. R-R techniques provide a method to extract approximate eigenvalues and eigenvectors with much less effort than a direct solution. Furthermore, the availability of additional mode shapes from the SPAR eigenvalue extraction process, even though these modes may not be converged or orthogonal, provides additional degrees of freedom in a R-R solution. The fact that the calculated mode shapes are combinations of the original mode shapes makes recalculation of the sensitivity matrix possible but this is not done in the runstream in figure 14. The quadratic dependence of the sensitivity matrix on mode shapes limits the parameter changes that can be accommodated in the R-R procedure presented herein. Therefore, the R-R procedure results should be used with periodic conventional SM executions with complete eigenvalue analysis to accelerate the convergence.

Procedure Overview

The procedure is based on the direct scaling of the increments of stiffness (ΔK) and mass (ΔM) due to the structural parameters. Modal eigenvalue extraction is accomplished with the STRP processor which was formulated to extract the eigenvalues for substructured problems. Phases 1 and 3 of the SM processor are used to determine the parameter changes at each step. The procedure requires detailed data manipulation such as creating stiffness and mass matrices in labeled element or "strip" format for the STRP processor. The following detailed description is tailored to the ARW-1 problem.

Procedure Details

Input data for the procedure consists of the following data (with identifying name) from an EAL execution (fig. 14).

(a) stiffness (K) and total mass (structural plus rigid masses (M+RM)) matrices

(b) eigenvalues (VIBR EVAL) and eigenvectors (VIBR MODE)

(c) data created by first three phases of an SM execution

(1) SENS MATR 0 1 - sensitivity matrix

(2) XDKX's - modal formulation of stiffness increments

(3) DP's or DPX's - unconstrained or constrained parameter changes; whichever is smaller.

The procedure is executed in two phases where the first phase generates the required modal matrices and extracts the eigenvalues for the first structural modification and the second phase is executed repeatedly in an attempt to attain convergence to the required eigen targets. The contents of the data sets used in the first phase are:

(a) INT SYN is a "strip" format triangular (upper half+diagonal) matrix for 12 modes containing the coefficient addresses and zero coefficient values.

(b) FULK is the reduced stiffness matrix (12 d.o.f.) for the original structure.

(c) DKS is a multiblock data set of modal stiffness increments due to the eight parameters.

(d) TDK is the modal stiffness increment due to the DPX's (estimated parameter changes) calculated by SM.

(e) SYN K is the incremented (FULK+TDK) stiffness matrix in "strip" format for input to the STRP processor.

(f) SYN M is the mass matrix including rigid masses in "strip" format.

The first phase is completed by the extraction of eigenvalues using the STRP processor.

The second phase of the procedure consists of the following steps:

(a) create a data set VIBR EVAL 1 1 required by SM from the eigenvalue output of the STRP processor. The value of S_{ee} used in SM can be changed at this point if desired by the user.

(b) execute phases 1 and 3 of SM to create new structural property change estimate.

(c) manipulate DP (unscaled) and DPX (scaled) structural property changes to insure proper value is used. (COPYing a nonexistent data set to another library causes no action except printing a warning message.)

(d) create stiffness matrix increment (TDK) and add it to the stiffness matrix (SYN K). Update the total parameter change (DPTO).

(e) extract eigenvalues of the incremental structure using STRP processor.

The user can execute R-R phase 2 until satisfactory convergence results or it becomes apparent that no further improvement is possible.

The results of a R-R solution for the ARW-1 center box are shown in figure 15. The R-R procedure was initiated after the first SM execution and the results obtained can be compared to those for the second iteration in figure 13. The R-R results did not converge in the six iterations shown in figure 15. At the second iteration, however, eigenvalues produced by the R-R analysis are closer to the target values than those produced by one iteration of the direct solution. One iteration of the direct solution took 120 central processor (CP) seconds while the R-R procedure required 8.5 CP seconds for phase 1 and about 1.8 CP seconds for each iteration thereafter (R-R phase 2). Updating the sensitivity matrix in each R-R iteration improves the R-R results but does not converge to the correct result. Considering the lower cost per iteration of the R-R procedure, it is probably worthwhile to use several R-R iterations before using a direct solution for more accurate eigenvalue analysis.

Table 1 Cantilever box beam eigenvalues and eigenvalue differences for various amounts of stiffening $(rad/sec)^2$

MODE	EIGENVALUE	EIGENVALUE DIFFERENCES			
		BAYS WITH 10% STIFFER SKINS			
	ORIGINAL BEAM	ALL (TARGET)	TIP (PARA 1)	CENTER (PARA 2)	INBOARD (PARA 3)
1	1113.81	3.97	0.11	1.02	2.83
2	2067.42	118.68	37.65	38.05	38.60
3	3512.64	235.67	117.24	90.28	26.13
4	380652.97	1403.66	465.38	467.74	466.31
5	480963.63	30639.73	30127.91	1288.15	64.33
6	529884.76	14919.31	3451.54	4672.05	5926.17

Table 2 Sensitivity matrix terms for cantilever box beam calculated by SM processor

MODE	SENSITIVITY MATRIX TERM FOR +10% INCREASE IN SKIN STIFFNESS		
	TIP BAY (PARA 1)	CENTER BAY (PARA 2)	INBOARD BAY (PARA 3)
1	0.11513	1.0212	2.8398
2	40.676	41.076	41.619
3	120.04	91.435	26.413
4	467.04	469.53	468.08
5	29251.0	1339.1	67.073
6	3720.6	5009.0	6385.9

Table 3 Estimated elemental stiffness changes for three eigenvalue targets calculated by proposed method for box beam

BAY	ΔP SPAR CAP	RIB	SPAR WEB	SKIN
TIP	-0.00003	0.0001	-0.0042	.1017
CENTER	-0.0002	0.00002	-0.0014	.0977
INBOARD	0.0005	0.00002	0.0052	.0882

NOTE - WHERE MORE THAN 1 ELEMENT OCCURS IN A BAY. ALL ELEMENTS IN THAT BAY HAD THE SAME ESTIMATED CHANGE.

Table 4 Eigenvalues of the original and target ARW-1 center box models $(rad/sec)^2$

MODE	EIGENVALUES ORIGINAL	TARGET
1	8180	8215
2	104979	115198
3	143572	145314
4	172967	266797
5	632744	830863
6	804227	857799
7	865412	1134209
8	902629	1245954

Table 5 Shell section flexibilities for ARW-1 center box

PARAMETER	FLEXIBILITY ORIGINAL MODEL	TARGET MODEL	SM CALCULATION
1	.16154-2	.5103-3	.5098-3
2	.16154-2	.10929-2	.10752-2
3	.14475-2	.9367-3	.9558-3
4	.14475-2	.8890-3	.8688-3
5	.14475-2	.8262-3	.8003-3
6	.12889-2	.6676-3	.7240-3
7	.11396-2	.7483-3	.7091-3
8	.86909-3	.5268-3	.5454-3

Table 6 Estimated averaged elemental stiffness changes for ARW-1 center box

	ΔP SPAR CAPS		SPAR WEBS		SKINS	
BAY	FRONT	REAR	FRONT	REAR	UPPER	LOWER
1 (INBD)	-0.529	0.447	-0.575	0.050	0.403	0.346
2	-0.083	-0.175	-0.397	-0.165	0.726	0.864
3	-0.049	0.363	-0.221	-0.076	0.742	1.009
4	-0.275	0.341	-0.067	0.040	0.721	0.878
5	-0.165	-0.215	-0.014	0.065	0.568	0.673
6	0.147	0.054	0.015	0.038	0.102	0.456
7 (TIP)	-0.209	0.292	0.018	-0.002	-0.125	0.168

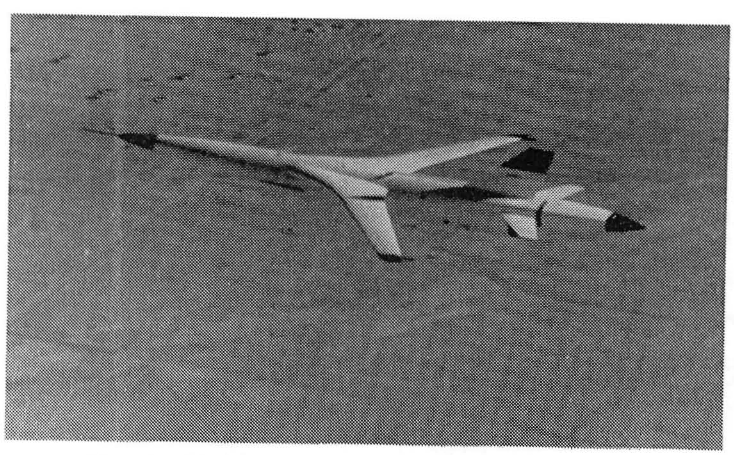


Fig. 1 Firebee drone with Aeroelastic Research Wing.

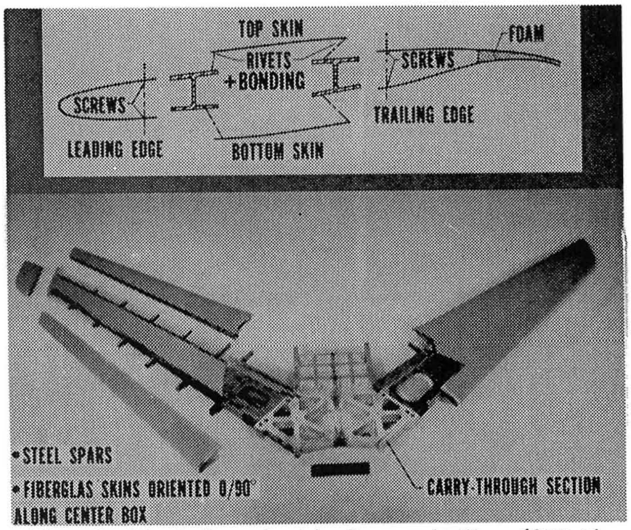


Fig. 2 First Aeroelastic Research Wing (ARW-1) structure.

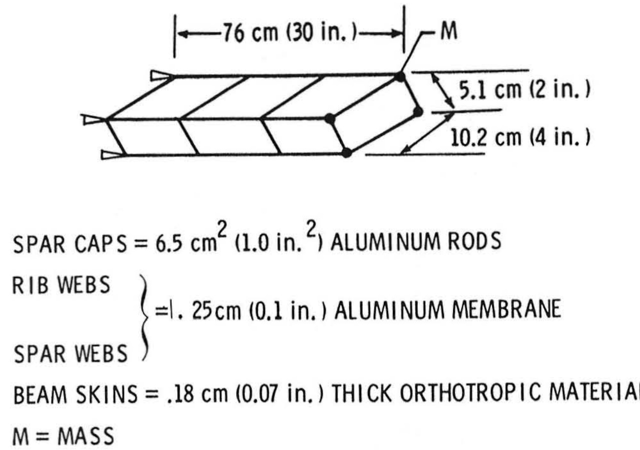


Fig. 5 Finite element model of cantilever box beam.

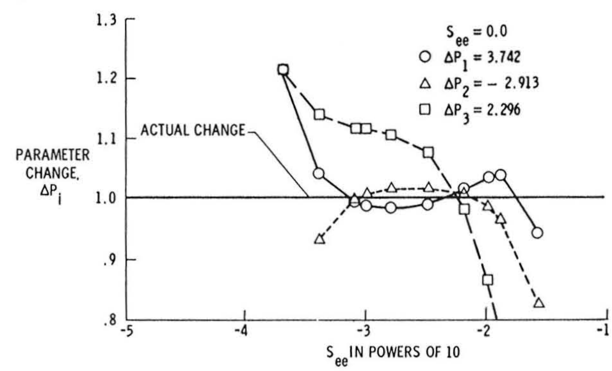


Fig. 6 Initial parameter changes for cantilever box beam using eigenvalue differences.

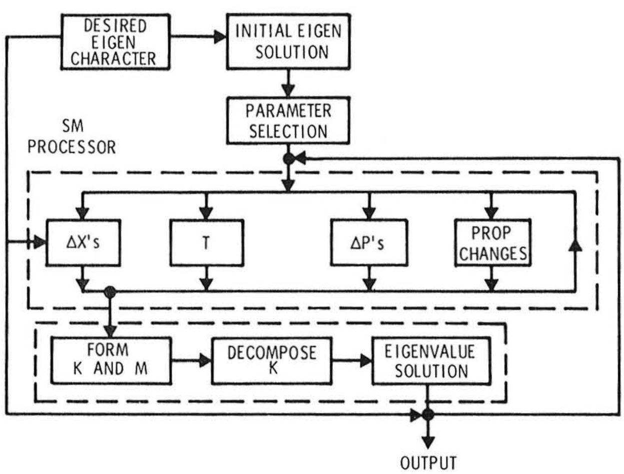
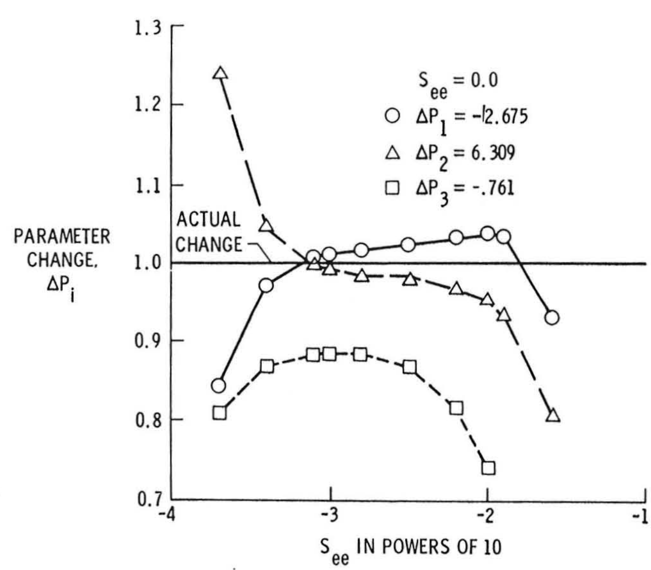


Fig. 3 Diagram of modification process for structural model.



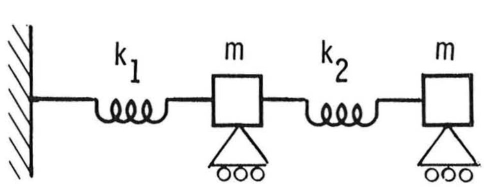


Fig. 4 Two degree-of-freedom spring-mass system.

Fig. 7 Initial parameter changes for cantilever beam using Structural Modification Processor.

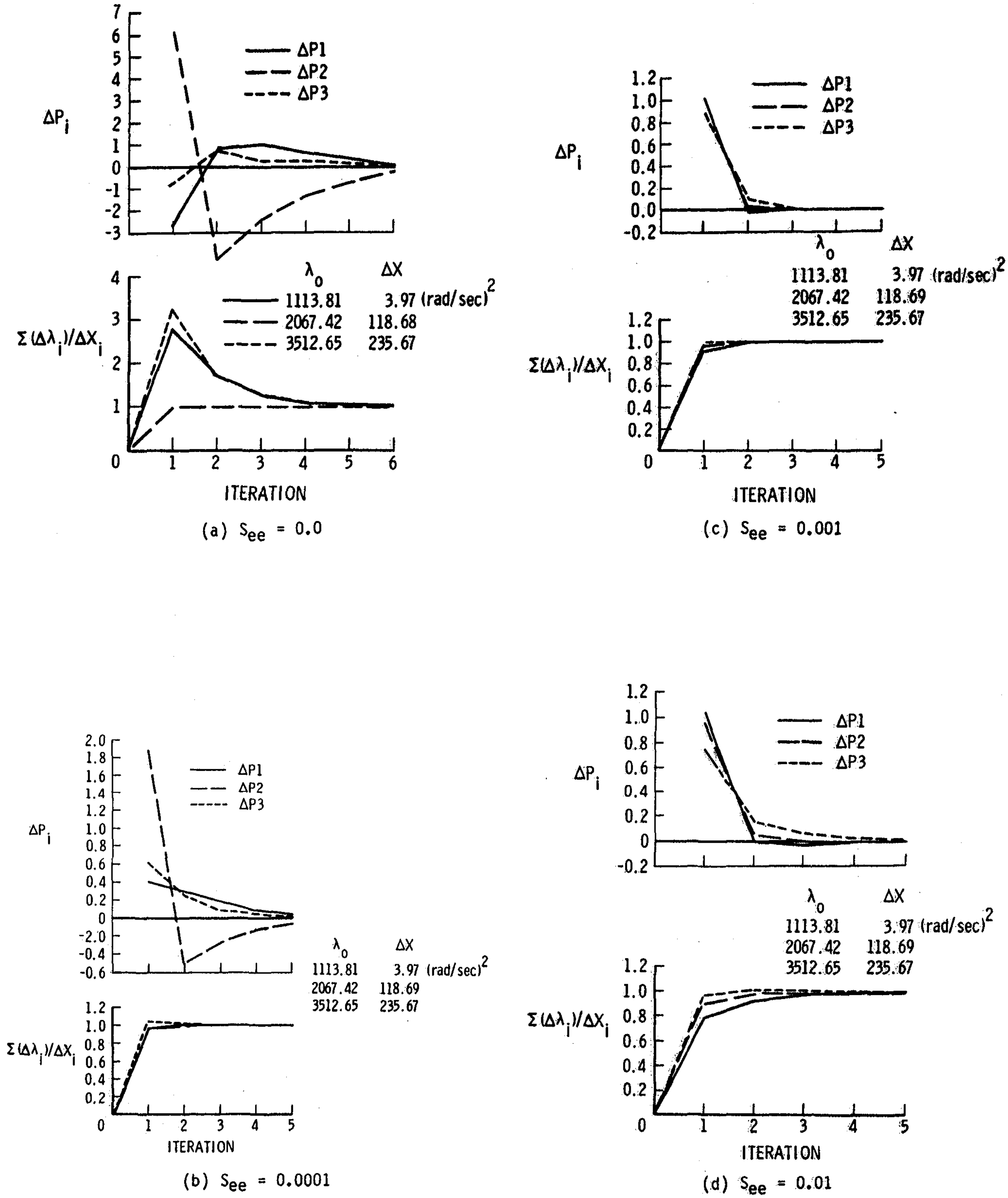
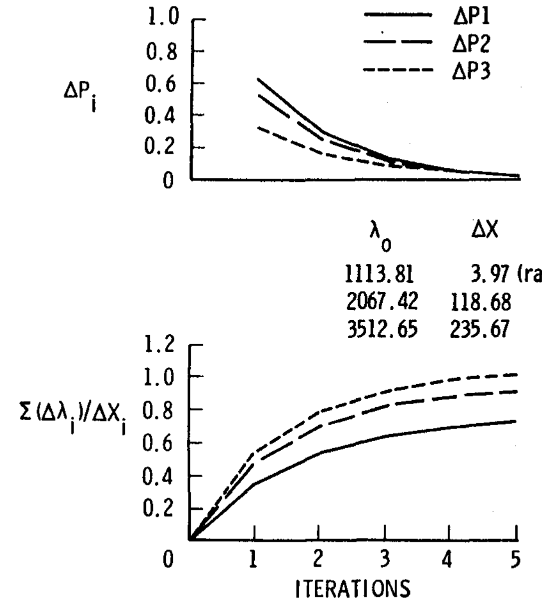


Fig. 8 *Parameter modification for cantilever box beam with no limit on parameter change magnitude.*



(e) S_{ee} = 0.05

Fig. 8 Concluded.

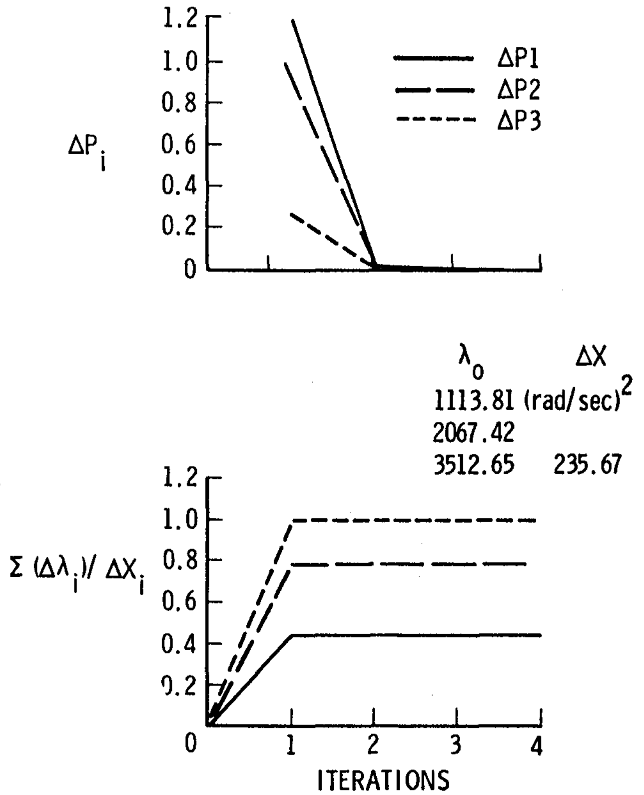


Fig. 9 Parameter modification for cantilever box beam, 1 target (3rd eigenvalue), S_{ee} = 0.01.

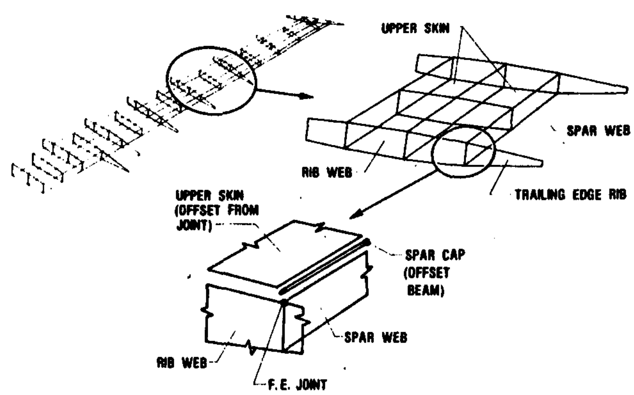


Fig. 10 Finite element model of ARW-1 center box.

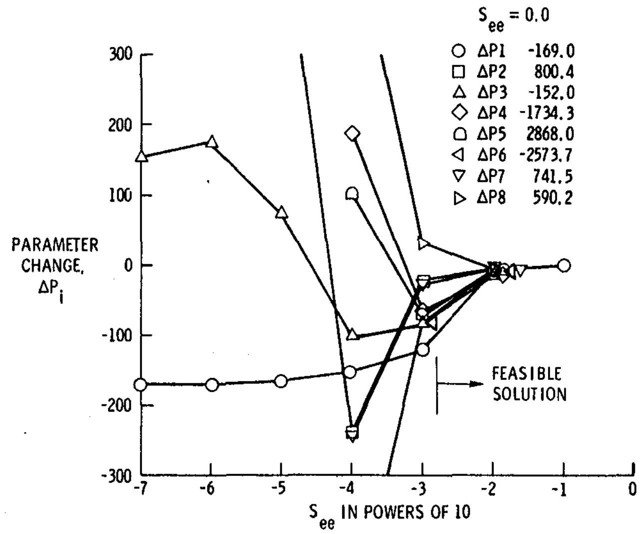


Fig. 11 Initial parameter changes for ARW-1 center box, 8 parameters, $S_{ee} = 10^{-7}$ to 1.

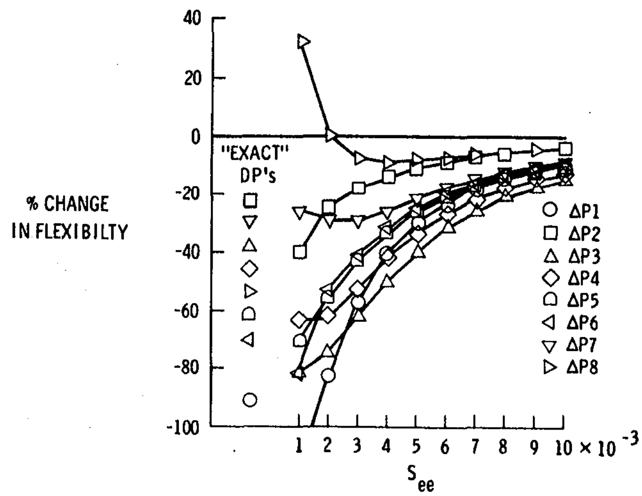


Fig. 12 Initial parameter changes for ARW-1 center box, 8 parameters, $S_{ee} = 10^{-3}$ to 10^{-2}.

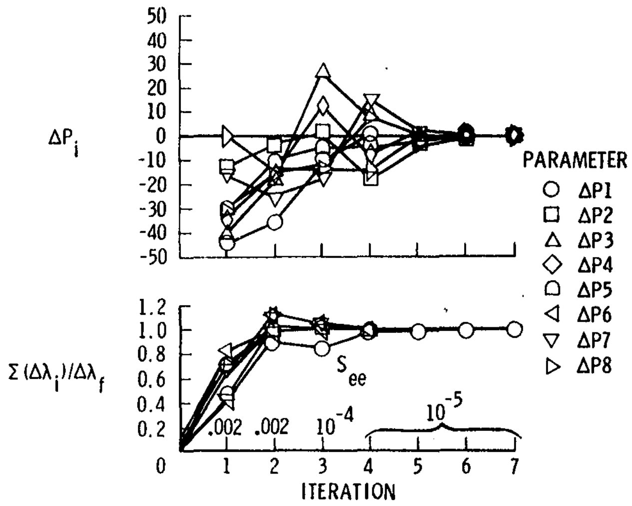


Fig. 13 Parameter modification for ARW-1 center box, 8 targets, 8 parameters = +0.01 shear flexibility, parameter change limit = ±45.

```
* CHAR $[I*!;?
$$$$ USES 1ST EIG AND SM RESULTS
$$$$ SET UP MATRICES FOR RAYLEIGH-RITZ PROC
[XOT AUS
 TABLE(NI=8,NJ=1);DPTO
  I=1;J=1;0.
TABLE(RMODE=2,TYPE=-1,NI=2,NJ=78);INT SYNT
  I=1;J=1,78
  10001; 10002; 20002; 10003; 20003; 30003
  10004; 20004; 30004; 40004; 10005; 20005
  30005; 40005; 50005; 10006; 20006; 30006
  40006; 50006; 60006; 10007; 20007; 30007
  40007; 50007; 60007; 70007; 10008; 20008
  30008; 40008; 50008; 60008; 70008; 80008
  10009; 20009; 30009; 40009; 50009; 60009
  70009; 80009; 90009; 10010; 20010; 30010
  40010; 50010; 60010; 70010; 80010; 90010
 100010; 10011; 20011; 30011; 40011; 50011
  60011; 70011; 80011; 90011;100011;110011
  10012; 20012; 30012; 40012; 50012; 60012
  70012; 80012; 90012;100012;110012;120012
 DEFI VM1=VIBR MODE 1 1
 KX=PROD(K,VM1)
 MX=PROD(M+RM,VM1)
 FULK K 10000 12=XTYSYM(VM1,KX)
 DEFI DK1=1 XDKX TRI 1 1
 DEFI DK2=1 XDKX TRI 2 1
 DEFI DK3=1 XDKX TRI 3 1
 DEFI DK4=1 XDKX TRI 4 1
 DEFI DK5=1 XDKX TRI 5 1
 DEFI DK6=1 XDKX TRI 6 1
 DEFI DK7=1 XDKX TRI 7 1
 DEFI DK8=1 XDKX TRI 8 1
 DKS=UNION(DK1,DK2,DK3,DK4,DK5,DK6,DK7,DK8)
 DEFI DPSM=DPX REV
 DPTN=SUM(DPTO,DPSM)
 DPTO=UNION(DPTN)
 TDK=CBR(DKS,DPSM)
SYN K 10000 12=UNION(INT)
 TABLE,U;SYN K 10000 12
TRAN(SOUR=TDK,ILIM=1,JLIM=78,DBAS=1,DSKI=1)
TRAN(SOUR=FULK,ILIM=1,JLIM=78,DBAS=1,DSKI=1)
 SNM=XTYSYM(VM1,MX)
SYN M 10000 12=UNION(INT)
 TABLE,U;SYN M 10000 12
TRAN(SOUR=SNM,ILIM=1,JLIM=78,DBAS=1,DSKI=1)
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$$$$ END OF PROCEDURE INITIALIZATION
$$$$ ITERATION LOOP
[XOT AUS
 DEFI SYEV=SYS EVAL
 VIBR EVAL 1 1=UNION(SYEV)
[XOT SM
 RESET NUVX 14
 RESET NPARAS=8,NUDP=2,G=386.4
 RESET KDPX=0 DEFAULT  - SCALE ALL TERMS
 OPER= 1 0 1 1
[XOT DCU
 CHAN 2 DPX REV 1 1 DP SM 1 1
 COPY 2 14 DP SM 1 1
 PRIN 14 DP SM
 CHAN 2 DP SM 1 1 REV DP 1 1
[XOT AUS
 DEFI DPSM=14 DP SM
 TDK=CBR(DKS,DPSM)1
 TABLE,U;SYN K 10000 12
TRAN(SOUR=TDK,ILIM=1,JLIM=78,DBAS=1,DSKI=1)
 DPTN=SUM(DPTO,DPSM)
 DPTO=UNION(DPTN)
[XOT STRP
[XOT EXIT
```

Fig. 14. EAL input for Rayleigh-Ritz procedure.

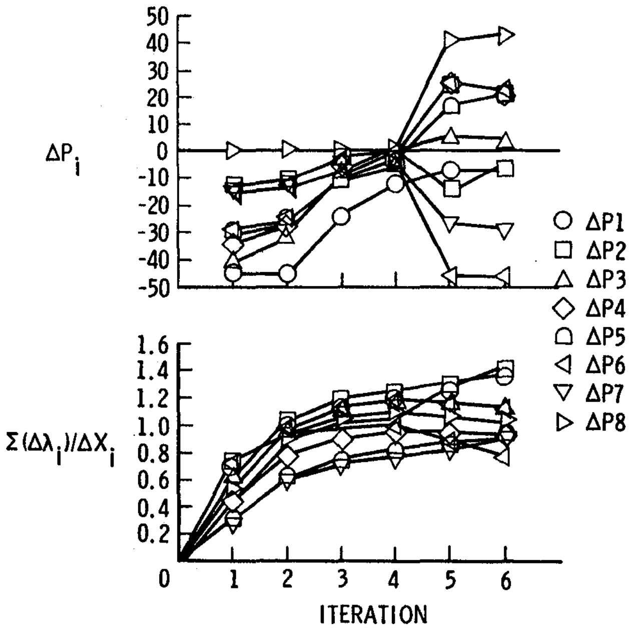

Fig. 15 Rayleigh-Ritz procedure results for ARW-1 center box, 8 targets, 8 parameters = +0.01 shear flexibility, parameter change limit = ±45.

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16. Abstract

A systematic finite-element model modification technique has been applied to two small problems and a model of the main wing box of a research drone aircraft. The procedure determines the sensitivity of the eigenvalues and eigenvector components to specific structural changes, calculates the required changes and modifies the finite-element model. Good results were obtained where large stiffness modifications were required to satisfy large eigenvalue changes. Sensitivity matrix conditioning problems required the development of techniques to insure existence of a solution and accelerate its convergence. A method is proposed to assist the analyst in selecting stiffness parameters for modification.

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