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National Aeronautics and
Space Administration

Goddard Space Flight Center
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(GEM-L2)

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ABSTRACT

A refined gravity field model, Goddard Earth Model GEM-L2, has been derived using the Lageos orbital data yielding better baseline measurements for the analysis of tectonic plate motion. This field also contributes to an improved understanding of long wavelength features, such as the sea slope across broad ocean basins, through its significant improvement of the long wavelength geoid (through degree and order 4). The geoid for these terms has an accuracy estimated at ± 8 cm in GEM-L2. GEM-L2, as in all recent Goddard Earth Models, relies heavily on the precise near-Earth satellite laser ranging data, in this case provided by NASA's Crustal Dynamics Program. Two and a half years of Lageos laser data acquired from over 20 well-distributed stations were combined with the existing data from the best satellite-derived model, GEM-9, to develop the new Lageos model. Testing shows that the Lageos gravity field error at long wavelengths is less than half that for GEM-9. Independent tests using well determined longitude accelerations of 24-hour satellites have verified the improved accuracy of the new model. A comparison of global laser "base" stations from independent data sets of alternating 15 day data segments over two years of Lageos show total inter-station positioning to ± 1.8 cm when using this new field. The same comparison using the 1979 versus the 1980 Lageos data yields ± 5.2 cm; this difference in agreement reflects the change in data distribution and other systematic errors along with the tectonic motion which has occurred between these chronologically distinct data

sets. Five day average polar motion values with a precision of 10 cm and change in length of day values accurate to better than .5 msec have been derived in the solution. The adjustment of these earth orientation parameters are necessary to achieve the accurate stations and geopotential results in GEM-L2.

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1.0 INTRODUCTION

The Lageos spacecraft is unique in several respects. In addition to its well-designed shape as a laser target, the satellite's high density and high altitude orbit virtually eliminate any error from the uncertainty arising in modeling non-gravitational forces. At an altitude of nearly one earth radius, Lageos experiences a strong gravitational force from the longest wavelength portion of the geopotential, with the effects falling off quickly at short wavelength due to the attenuation of the field at Lageos' altitude.

Coupled with an extensive and highly accurate laser tracking campaign, Lageos data have provided a rich resource for improving our knowledge of the long wavelength gravity field. Since the use of <u>S</u>atellite <u>L</u>aser <u>R</u>anging (SLR) for measuring crustal dynamics requires the proper modeling of orbital dynamics, the gravity studies for Lageos have borne dual results: 1) an improved ability to estimate intersite distances for tectonic motion studies, and 2) a greatly enhanced knowledge of the gravity field through degree and order four.

2.0 DATA

The Lageos spacecraft was placed into a high altitude (5900 km), nearly circular orbit on May 4, 1976. Typical mean Keplerian elements for Lageos are shown below:

semi-major axis:	12267.7 km
eccentricity:	.002
inclination:	109°.84

To date, data from a global network of laser stations was acquired over a time span of six years on Lageos. Figure 1 presents a synopsis of the data available over the first four years (1976 through 1979) of Lageos tracking. As is apparent from this figure, there was a significant increase in data availability, improved global distribution and many more participating NASA stations commencing in 1979. Of note, also, is the improvement of the SAO systems (systems with 7900 number designation) which began about 1979. Consequently, our gravity modeling activities utilized Lageos data beginning with February of 1979.

The new model, GEM-L2, has incorporated 2 1/2 years of Lageos based data in combination with satellite tracking data taken on 30 other satellites as in GEM 9 (Lerch et al., 1979). This solution, complete in spherical harmonics through degree and order 20, contained well over 600,000 laser measurements, more than half of which were taken from Lageos. Lageos laser data are accurate down to about 8 cm and formed the predominant weight in the solution. However, because of the attenuation of the gravitational field at

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FIGURE 1

LAGEOS DATA SUMMARY FOR GRAVITY MODEL IMPROVEMENT

NUMBER OF PASSES/STATION

YEAR	NO. OF ARCS	7907	7921	7929	7943	7051	7062	7063	7065	7067	7068	7069	7082	7084	7085	7086	7090	7091	7096	7101	7102	7103	7104	7114	7115	7210	7833
1976	10	61	96	32	●	7	34	9	2	2	●	●	30	●	●	●	●	●	●	●	●	●	●	●	●	●	●
1977	15	101	74	24	129	●	17	8	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	●	6
1978	14	46	61	9	37	●	18	44	●	9	4	●	●	14	27	●	●	22	●	●	●	●	●	●	●	●	●
1979	24	151	20	10	70	13	34	89	●	●	●	7	36	●	●	44	15	53	12	7	22	15	13	71	57	42*	●
TOTAL	63	359	251	75	236	20	103	150	2	11	4	7	66	14	27	44	15	75	12	7	22	15	13	71	57	42*	6

TOTAL NUMBER OF PASSES: 1662

*Data has not been used in normal equation generation.

high altitude the effective strength of the Lageos data falls off rapidly for the high degree terms and resulted in the GEM 9 data controlling the adjustment for the harmonics beyond degree 7. For Lageos approximately 20 laser stations with a global distribution contributed data in the GEM-L2 solution although more than half of these stations were located in the United States. A description of the data (1979 through 1981) is given by station for each of the orbital arcs (15 days in length) in Tables 1a through 1c. The data on Lageos were weighted in the solution to provide for a balance both by station and by orbital arc as described in Table 2.

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	SAO																			
	ARELAS	NATLAS	HOPLAS	ORRLAS	STALAS	GSFC	GSFC	SANDIE	FT. DAVIS	YARRAGADEE	HAYSTACK	AM. SAMOA	GSFC	OWENS VALLEY	GOLDSTONE	HOLLAS	BEAR LAKE	GSFC	RAMLAS	QUINCY
	7907	7929	7921	7943	7063	7103	7104	7062	7086	7090	7091	7096	7102	7114	7115	7210	7082	7101	7069	7051
1979																				
201	x	x	x	x	x	x	x	x										x		
216	x	x	x	x				x												
303	x	x	x	x	x	x		x									x			
318	x		x	x	x	x		x					x				x	x	x	x
402	x	x	x	x	x			x					x				x		x	x
417	x		x	x	x	x	x	x					x				x		x	x
502	x	x	x	x	x		x	x									x			x
517	x			x	x			x									x			x
603	x	x		x																
618	x			x																
703	x			x													x			
718	x			x												x	x			
802	x	x	x	x							x	x					x			
817	x	x		x								x				x			x	
901	x			x	x						x	x		x						
916	x			x	x				x		x			x	x					
1001	x	x		x	x				x	x	x	x	x	x					x	
1016	x			x	x				x	x	x	x	x	x						
1031	x			x	x				x		x	x	x	x	x					
1115	x	x		x	x				x		x	x	x	x	x					
1130	x	x		x	x				x	x	x	x	x	x	x					
1215	x	x			x	x			x	x		x	x	x	x					

Table 1a. LAGEOS Data Summary for 1979

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	SAO																					
	ARELAS	NATLAS	HOPLAS	ORRLAS	STALAS	GSFC	GSFC	SANDIE	FT. DAVIS	YARRAGADEE	HAYSTACK	AM. SAMOA	GSFC	OWENS VALLEY	GOLDSTONE	HOLLAS	BEAR LAKE	GSFC	RAMLAS	QUINCY	KWAJ 802	MLO110
	7907	7929	7921	7943	7063	7103	7104	7062	7086	7090	7091	7096	7102	7114	7115	7210	7082	7101	7069	7051	7092	7120
1980																						
102	x	x			x				x	x	x		x	x	x							
117	x	x			x				x	x	x			x	x				x			
201	x	x		x	x				x	x	x	x		x	x				x			
216	x			x	x				x	x	x	x	x	x	x				x			
302	x	x		x	x				x	x	x	x	x	x	x				x			
317	x	x		x	x				x	x	x	x	x	x	x				x			
401	x	x		x	x				x	x	x	x	x	x	x							
416	x	x		x	x				x	x	x	x	x	x	x	x			x			
501	x	x		x					x	x		x		x	x							
516	x	x		x	x					x	x	x	x	x	x							
600	x	x		x							x	x	x		x	x						
615	x	x		x							x		x			x						
630	x	x		x					x	x	x	x									x	
716	x	x		x	x				x	x	x	x	x	x	x						x	
731	x	x		x	x				x	x	x	x		x	x				x		x	
815	x	x		x	x					x	x	x		x	x						x	
830	x	x		x	x					x	x	x	x	x	x						x	x
914	x	x		x	x					x	x	x	x	x	x						x	
929	x	x		x	x					x	x	x	x	x	x						x	
1014	x	x		x	x					x	x	x	x	x	x						x	x
1029	x	x		x						x	x	x	x	x	x						x	x
1112	x	x		x						x	x			x	x							
1128	x	x		x	x					x				x	x							x
1213	x	x		x	x					x			x	x	x							

Table Ib. LAGEOS Data Summary for 1980

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1981	7907 ARELAS	7929 NATLAS SAO	7943 ORRLAS	7051 QUINCY	7063 STALAS	7082 BEARLK	7090 YARRAGADEE	7112 ML0212	7115 GOLDSTONE	7102 ML0402	7114 OWENS VALLEY	7120 HAWXAA	7105 ML0705
801228	X	X	X				X		X	X	X		
810112	X	X	X		X		X			X			
127		X	X		X		X			X			
211		X	X	X	X		X	X	X			X	
226	X	X	X	X	X		X	X	X	X			X
313	X	X			X	X	X		X	X		X	X
328	X	X	X	X	X	X	X	X	X				
412	X	X	X	X	X	X	X	X		X			
427	X	X		X	X	X	X	X					
512	X	X		X	X		X	X					
527	X	X					X	X					
611	X	X	X		X		X	X		X		X	
626	X	X	X				X	X				X	

Table 1c. LAGEOS Data Summary for 1981

TABLE 2

DATA WEIGHTING IN GEM--L2

DATA	WEIGHTING USED	TOTAL OBS
GEM 9 OPTICAL	2 ARC SEC	150,000
GEM 9 ELECTRONIC	10 M(4 CM/SEC)	477,000
GEM 9 LASER	4 M (AVE.)	213,000
(LAG'79) LAGEOS 1979	50 CM	105,000
(LAG'80) LAGEOS 1980	50 CM	251,000*
(LAG'81) LAGEOS 1981	50 CM	87,000*
GEM--L2	50 CM	443,000*

*1980 AND 1981 DATA SCALED TO HAVE SAME OVERALL WEIGHT AS 1979 LAGEOS DATA.

3.0 METHOD

The potential coefficients $(C_{\ell m}, S_{\ell m})$ for GEM-L2 refer to the definition of the gravitational potential as

$$V = \frac{\mu}{r}\left| 1 + \sum_{\ell=2}^{\infty} \sum_{m=0}^{\ell} \left(\frac{r_e}{r}\right)^{\ell} \overline{P}_{\ell m} (\sin\phi) \right. \left. [C_{\ell m} \cos m\lambda + S_{\ell m} \sin m\lambda] \right|$$

where r, ϕ, and λ are the distance to the center of mass, geocentric latitude and longitude. $\overline{P}_{\ell m}$ is the fully normalized associated Legendre function of degree ℓ and order m, and r_e is the earth's mean radius. The GEM-L2 model is complete to degree and order 20, with Lageos being evaluated for its contribution complete out through degree and order 16, and beyond for the zonals and certain other orders. Figure 2 shows the coefficients selected for Lageos' geopotential contribution when combined with GEM-9. An estimate of the orbital perturbations on Lageos arising from the gravity field is contained, also, in Figure 2. The complete set of the GEM-L2 coefficients are given in Table 3.

Even though a large Lageos data set has been used in GEM-L2, this satellite only solution, like GEM-9 (Lerch et al., 1979), still requires the utilization of a modified least squares method to achieve stability in its recovered

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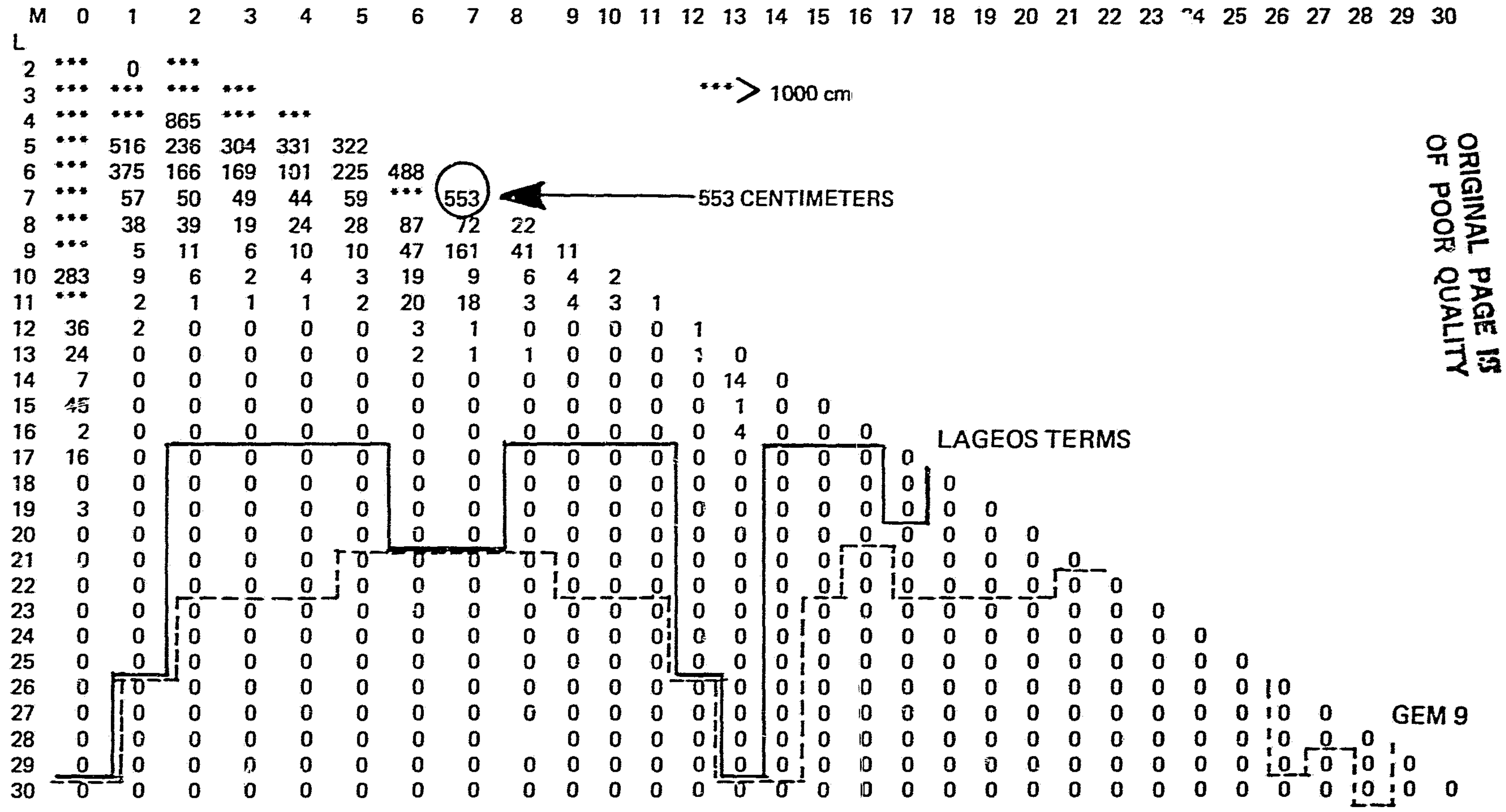
FIGURE 2

ORBIT PERTURBATION OF LAGEOS POSITION
DUE TO GRAVITY FIELD HARMONICS



ORDER

DEGREE

***> 1000 cm

553 CENTIMETERS

LAGEOS TERMS

GEM 9

M L	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
2	***	0	***																												
3	***	***	***	***																											
4	***	***	865	***	***																										
5	***	516	236	304	331	322																									
6	***	375	166	169	101	225	488																								
7	***	57	50	49	44	59	***	553																							
8	***	38	39	19	24	28	87	72	22																						
9	***	5	11	6	10	10	47	161	41	11																					
10	283	9	6	2	4	3	19	9	6	4	2																				
11	***	2	1	1	1	2	20	18	3	4	3	1																			
12	36	2	0	0	0	0	3	1	0	0	0	0	1																		
13	24	0	0	0	0	0	2	1	1	0	0	0	1	0																	
14	7	0	0	0	0	0	0	0	0	0	0	0	0	14	0																
15	45	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0															
16	2	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0														
17	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0													
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0												
19	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0											
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0										
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0									
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0								
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0							
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0					
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			
28	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 3
GEM–L2 NORMALIZED COEFFICIENTS
(x 10^6)

ZONALS

INDEX		VALUE	INDEX		VALUE	INDEX		VALUE	INDEX		VALUE	INDEX		VALUE
N	M		N	M		N	M		N	M		N	M	
2	0	-484.16499	3	0	0.95772	4	0	0.54127	5	0	0.06924	6	0	-0.15063
7	0	0.09237	8	0	0.05088	9	0	0.02769	10	0	0.05334	11	0	-0.04849
12	0	0.03852	13	0	0.04288	14	0	-0.02045	15	0	0.00286	16	0	-0.00723
17	0	0.01486	18	0	0.01162	19	0	0.00073	20	0	0.02689	21	0	-0.00002
22	0	-0.00239	23	0	-0.02047	24	0	-0.00282	25	0	0.00151	26	0	0.00813
27	0	0.01097	28	0	-0.02263	29	0	-0.01127						

SECTORIALS AND TESSERALS

INDEX		VALUE		INDEX		VALUE		INDEX		VALUE	
N	M	C	S	N	M	C	S	N	M	C	S
2	1	-0.00106	-0.00307	3	1	2.02882	0.24969	4	1	-0.53520	-0.46629
5	1	-0.05400	-0.09510	6	1	-0.07489	0.01773	7	1	0.26915	0.09374
8	1	0.02321	0.06138	9	1	0.15543	0.01703	10	1	0.08261	-0.13442
11	1	0.00068	-0.01344	12	1	-0.06461	-0.05435	13	1	-0.03591	0.03623
14	1	-0.01763	0.05171	15	1	-0.01791	0.00030	16	1	0.02899	-0.00991
17	1	-0.00871	-0.02421	18	1	0.00265	-0.00214	19	1	-0.03652	-0.00351
20	1	0.00570	-0.02189	21	1	-0.01301	0.00741	22	1	0.00281	0.00524
23	1	0.00285	0.00927	24	1	-0.00684	-0.00285	25	1	0.01246	-0.00529
2	2	2.43791	-1.39918	3	2	0.90403	-0.61594	4	2	0.35465	0.66083
5	2	0.65096	-0.33063	6	2	0.05028	-0.35499	7	2	0.33473	0.10701
8	2	0.07422	0.04809	9	2	0.02584	-0.03132	10	2	-0.08041	-0.01170
11	2	0.04616	-0.09373	12	2	0.00715	-0.01034	13	2	0.02318	-0.03652
14	2	-0.03199	0.04065	15	2	0.01423	-0.02580	16	2	-0.00275	0.00841
17	2	-0.00775	0.01031	18	2	0.00749	0.02250	19	2	0.01145	-0.00576
20	2	-0.00090	0.00143	21	2	0.01694	0.00744	22	2	-0.00077	-0.01265
3	3	0.72315	1.41520	4	3	0.99325	-0.20270	5	3	-0.45002	-0.21279
6	3	0.06030	0.00178	7	3	0.24609	-0.21509	8	3	-0.00560	-0.08863
9	3	-0.15615	-0.09936	10	3	-0.01610	-0.16891	11	3	-0.02191	-0.13229
12	3	0.06697	0.03004	13	3	-0.02825	0.07916	14	3	0.01665	-0.01102
15	3	0.02950	0.01047	16	3	-0.00701	-0.02406	17	3	-0.01148	0.00962
18	3	-0.00422	-0.00173	19	3	0.00045	0.00023	20	3	-0.01551	0.00928
21	3	0.00460	0.00318	22	3	0.00058	0.01083	4	4	-0.19210	0.30701
5	4	-0.29163	0.04959	6	4	-0.10190	-0.46823	7	4	-0.28677	-0.12730
8	4	-0.24414	0.07443	9	4	-0.00997	0.00946	10	4	-0.09734	-0.06744
11	4	-0.04491	-0.05961	12	4	-0.07182	-0.00841	13	4	-0.02524	0.00968

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TABLE 3. (CONTINUED)

SECTORIALS AND TESSERALS (CONTINUED)

INDEX		VALUE		INDEX		VALUE		INDEX		VALUE	
N	M	C	S	N	M	C	S	N	M	C	S
14	4	-0.01379	0.00930	15	4	-0.04430	-0.00687	16	4	0.04104	0.01358
17	4	0.00450	-0.00448	18	4	0.02833	0.01689	19	4	-0.00323	-0.00967
20	4	-0.00166	-0.00468	21	4	0.00256	-0.00296	22	4	-0.00780	0.00739
5	5	0.17799	-0.66068	6	5	-0.26580	-0.53659	7	5	0.01208	0.04169
8	5	-0.01470	0.08008	9	5	0.00314	-0.06886	10	5	-0.06239	-0.03986
11	5	0.04797	0.08457	12	5	0.04567	0.01427	13	5	0.07560	0.04919
14	5	0.01761	-0.01990	15	5	-0.01052	-0.00634	16	5	-0.01438	0.00103
17	5	-0.02360	0.00927	18	5	0.01235	0.01468	19	5	-0.00090	-0.00176
20	5	-0.00230	0.00689	6	6	0.00870	-0.23801	7	6	-0.36046	0.15172
8	6	-0.06813	0.31230	9	6	0.04725	0.22226	10	6	-0.03388	-0.08941
11	6	0.00435	0.04243	12	6	0.00735	0.03337	13	6	-0.03617	0.00706
14	6	-0.00211	-0.00766	15	6	0.02715	-0.03243	16	6	0.00311	-0.03344
17	6	0.00003	-0.00976	18	6	0.01601	-0.01367	19	6	0.00043	-0.00079
20	6	-0.00563	0.00187	7	7	0.00919	0.02759	8	7	0.06709	0.07413
9	7	-0.10256	-0.08868	10	7	0.00508	0.00393	11	7	0.01439	-0.08383
12	7	-0.00999	0.04509	13	7	-0.00202	0.00021	14	7	0.01813	-0.03150
15	7	0.06865	0.01295	16	7	0.00778	-0.00257	17	7	0.01837	0.01082
18	7	-0.00384	-0.00028	19	7	-0.00622	0.00776	20	7	-0.00978	-0.00322
8	8	-0.12224	0.12976	9	8	0.18658	-0.01314	10	8	0.04009	-0.08124
11	8	0.00269	0.02225	12	8	-0.02500	0.02469	13	8	-0.02569	-0.00398
14	8	-0.04494	0.00494	15	8	-0.01833	0.02496	16	8	-0.00746	0.00811
17	8	0.01423	-0.00082	18	8	0.00993	-0.01356	19	8	0.00686	-0.01824
20	8	-0.00214	-0.00324	9	9	-0.05569	0.09397	10	9	0.12351	-0.04977
11	9	-0.03015	0.03431	12	9	0.04378	0.01404	13	9	0.02602	0.04295
14	9	0.04251	0.01453	15	9	0.01402	0.04282	16	9	-0.01646	-0.03521
17	9	-0.00876	-0.01985	18	9	-0.02329	0.02532	19	9	0.00259	0.01826
20	9	0.00551	0.02069	21	9	0.00304	-0.00339	22	9	0.00422	-0.00307
10	10	0.10891	-0.02522	11	10	-0.06075	-0.00632	12	10	0.00693	0.04739
13	10	0.03117	-0.03419	14	10	0.05922	0.01020	15	10	0.02971	0.00611
16	10	0.01674	0.00776	17	10	0.01204	0.00997	18	10	-0.00314	-0.00890
19	10	-0.01406	-0.00843	20	10	-0.01468	-0.00957	21	10	-0.00039	0.00208
22	10	0.00141	-0.00021	11	11	0.04004	-0.08095	12	11	0.01593	-0.00680
13	11	-0.04370	-0.01857	14	11	0.01777	-0.03419	15	11	-0.00104	-0.00418
16	11	0.02037	0.01117	17	11	-0.03894	-0.00199	18	11	-0.00288	0.01764
19	11	-0.00188	0.02721	20	11	0.01092	-0.01022	21	11	-0.00568	-0.00994
22	11	-0.00574	-0.02402	12	12	-0.00776	-0.01244	13	12	-0.03219	0.09036
14	12	0.00815	-0.03131	15	12	-0.03325	0.01656	16	12	0.01743	0.00944
17	12	0.03141	0.02155	18	12	-0.04014	-0.02431	19	12	-0.01021	0.00576
20	12	-0.01117	0.02646	21	12	0.00153	0.02238	22	12	-0.01614	-0.01805
23	12	0.01738	0.01004	24	12	-0.00853	-0.01525	25	12	-0.01241	0.00101

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TABLE 3. (CONTINUED)

SECTORIALS AND TESSERALS (CONTINUED)

INDEX		VALUE		INDEX		VALUE		INDEX		VALUE	
N	M	C	S	N	M	C	S	N	M	C	S
13	13	-0.06036	0.07051	14	13	0.02851	0.04251	15	13	-0.02297	-0.00174
16	13	0.01243	-0.00720	17	13	0.01436	0.01985	18	13	-0.01142	-0.03714
19	13	-0.01285	-0.02993	20	13	0.02423	0.00444	21	13	-0.01666	0.01429
22	13	-0.02936	0.00848	23	13	-0.00229	-0.00146	24	13	0.00616	-0.00586
25	13	0.01509	-0.00833	26	13	-0.00332	-0.00814	27	13	-0.00823	-0.01032
28	13	0.02059	0.00836	29	13	-0.01111	-0.00871	14	14	-0.05101	-0.00578
15	14	0.00400	-0.02458	16	14	-0.01947	-0.03788	17	14	-0.01584	0.01085
18	14	-0.00866	-0.00921	19	14	-0.00559	-0.01259	20	14	0.01228	-0.01004
21	14	0.01937	0.01022	22	14	0.00958	0.00652	23	14	0.00863	-0.00534
24	14	-0.01684	0.00360	25	14	-0.02421	0.01692	26	14	0.00745	0.00118
27	14	0.02345	0.00589	28	14	-0.00535	-0.01057	29	14	-0.01098	0.00998
15	15	-0.02184	-0.00570	16	15	-0.01312	-0.02596	17	15	0.00097	0.00481
18	15	-0.05446	-0.01612	19	15	-0.02227	-0.01532	20	15	-0.02379	0.00874
21	15	0.01043	0.01149	22	15	0.02408	0.00008	23	15	0.01346	0.00344
24	15	0.00586	-0.00217	25	15	-0.00247	-0.00172	16	16	-0.02921	0.01225
17	16	-0.02391	0.01726	18	16	0.01111	0.01945	19	16	-0.03540	-0.01307
20	16	-0.01393	0.00097	21	16	-0.00008	-0.00598	22	16	-0.00408	-0.00471
17	17	-0.01190	-0.00003	18	17	0.02115	0.01066	19	17	0.03042	-0.00578
20	17	-0.01306	-0.00728	21	17	-0.00267	-0.00293	22	17	-0.00531	-0.00377
18	18	0.00199	-0.01274	19	18	0.03385	-0.02103	20	18	-0.00318	0.02954
21	18	0.01250	-0.01386	22	18	0.00909	-0.00950	19	19	0.00971	-0.00210
20	19	0.01721	0.00871	21	19	0.01192	-0.00890	22	19	-0.01801	-0.02282
20	20	-0.00618	-0.00553	27	26	-0.01117	-0.00610	28	26	0.00742	-0.00231
29	26	-0.01032	-0.01690	27	27	0.00010	-0.00300	28	27	-0.00831	0.00257
28	28	0.00425	0.00586	30	28	-0.01835	-0.03486				

coefficients of high degree. The modified least squares method which is employed, is fully described in Lerch et al., 1977. An elaborate force model was utilized in the analysis of the Lageos data. The solution is referred to a speed of light of 299792.5 m/sec because of the large data set in GEM-9 which used this pre-existing value. The Earth's μ has been adjusted in the GEM-L2 solution (simultaneously with the gravity harmonics and station positions) yielding a value of 398600.607 km^3/sec^2.[1] This value is in good agreement with other recent findings (e.g., Lerch et al., 1978). Solid earth tides were also modeled with (Love's numbers) $h_2 = .60$, $\ell_2 = .075$ and $k_2 = .29$. A solid earth phase angle of 2.018° was obtained in the solution.

Also polar motion and A1-UT1 variations were simultaneously estimated from 5 day segments of the Lageos data. A solar radiation pressure coefficient, C_r, was adjusted over each 15 day arc. The Jacchia (1971) atmospheric density model does not contain values at the altitude of Lageos and therefore an along track acceleration is also permitted to adjust in order to account for "drag-like" effects discussed by Rubincam (1980) and also Smith and Dunn (1980).

[1] If the speed of light value of 299792.458 m/sec were to be used, the value of GM would be changed by -.167, equalling 398600.440 km^3/sec^2.

4.0 EVALUATION OF THE RESULTS

GEM-9 was the gravity model adopted initially in our LAGEOS analysis. GEM-9 (for the 1979 time frame, which coincided with the beginning of our effort) was considered the best available gravity model developed from a data set strictly composed of conventional satellite observations. GEM-9 did, however, use an augmented laser ranging observation set which included some of the newest NASA systems tracking GEOS-3 and Starlette. Therefore, GEM-9 was a larger and considerably improved field over preceding odd-numbered (which designate "satellite-only") GEM fields like that of GEM-7. Figure 3 presents the estimated uncertainty of the coefficients found in GEM-9. These error estimates were obtained from the scaled formal uncertainties of the solution obtained through an analysis against independent data (see Lerch et al., 1975 and 1977). Based upon a calibration with surface gravity a scale factor of $\sqrt{10}$ was derived to relate our formal noise only error estimates to the "true" uncertainty in the fields. Similar tests made with altimeter data confirmed that this scale factor was acceptable but somewhat conservative. The scaling factor was further confirmed from orbital data through the analysis of satellite resonant effects (Wagner and Lerch, 1978).

Based upon this scale factor ($\sqrt{10}$) the estimated accuracy of GEM-L2 is compared to GEM-9 in Table 4 for the longest wavelength terms in the model through degree and order 4. The actual coefficient differences between GEM-L2 and GEM-9 are also shown for these same terms. The highly accurate laser tracking data, together with Lageos' unique

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FIGURE 3

ESTIMATED ERRORS FOR GEM 9 (UNITS = 10^{-9} FOR FULLY NORMALIZED HARMONICS

DEGREE	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
2	1	3	3																										
3	1	5	8	9																									
4	1	5	5	4	6																								
5	2	7	9	10	9	13																							
6	2	7	8	7	8	6	8																						
7	2	9	13	13	12	12	10	16																					
8	2	9	12	10	11	8	10	7	10																				
9	2	13	15	18	16	15	13	14	11	9																			
10	2	10	17	14	15	12	12	10	11	7	10																		
11	3	15	17	22	21	19	17	16	13	10	10	16																	
12	3	11	18	19	18	17	17	12	13	7	10	5	6																
13	4	14	18	21	22	19	19	17	14	10	11	11	3	3															
14	4	14	15	20	18	18	18	16	14	9	11	6	5	2	2														
15	5	13	19	19	20	20	19	19	17	11	13	11	4	2	1	5													
16	4	13	16	16	19	16	19	18	15	11	12	9	5	3	2	5	12												
17	5	14	17	19	19	20	19	19	18	14	15	11	5	3	1	5	5	12											
18	3	12	15	18	16	19	18	18	18	15	14	11	6	3	2	6	12	17	21										
19	4	14	17	16	18	17	18	17	17	15	15	11	9	4	1	4	5	11	11	15									
20	4	12	14	15	16	16	16	16	16	14	15	11	7	4	2	7	11	12	14	16	17								
21	4	15	14	15	15					14	15	12	9	3	2	5	8	10	9	9									
22	5	11	13	14	14					13	14	11	9	4	3	9	11	11	11	11									
23	5	12											15	4	3	6													
24	6	11											21	6	5	10													
25	8	11											23	7	7	8													
26	7													6	5														
27	7													10	9												7	11	
28	9													10	6												1	7	7
29	8													10	8												7		
30																													2
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28

ORDER

sensitivity to this portion of the field, accounts for the improvements indicated in Table 4. However, since GEM-L2 contains the GEM-9 data set, an independent verification of these accuracies was sought. An independent verification of these accuracies (mostly for terms where ℓ-m is even) has been performed at NGS/NOAA (Wagner, 1982). A comparison of 24 hour satellite longitude accelerations observed through the study of mean elements was compared to those calculated from GEM-L1 (Lerch and Klosko, 1981) and GEM-L2 as shown in Table 5. The results for each of the eight 24 hour satellites are shown in Table 6.

The low degree and order portion of the gravity field cause long period changes in the longitude of deeply resonant (librating) 24 hour satellites. These perturbations commonly are seen as either long period librations of the longitude of the spacecraft from its nearly stationary position or as a secular drift in longitude. By evaluating several year histories of the spacecraft's mean Kepler coordinates and comparing these with the calculated trajectories from given geopotential models, one obtains a very good independent measure of the accuracy of the fields at lowest degree and order where ℓ-m is even thereby satisfying the resonance condition (principally terms $CS_{2,2}$, $CS_{3,1}$ and $CS_{3,3}$). Figure 4 shows the orbital evolution of the SYNCOM2 satellite (Wagner, 1972) indicating that the magnitude of the effect of $CS_{2,2}$ on the longitude of a stationary orbit can be several degrees over a few years.

TABLE 4

ESTIMATED INDIVIDUAL LOW DEGREE AND ORDER COEFFICIENT ACCURACY FROM GODDARD GRAVITY MODELS IN CENTIMETERS

COEFFICIENT	UNCERTAINTY GEM–9	UNCERTAINTY GEM–L2	ACTUAL COEFFICIENT DIFFERENCE (GEM–9) MINUS (GEM–L2)	
(ℓ,m)	$\sigma J_{\ell,m}$	$\sigma J_{\ell,m}$	ΔC	ΔS
C2,0	0.4	0.3	– .3	
C3,0	1.0	0.1	.5	
C4,0	0.8	0.7	.2	
C,S2,1	2.3	1.3	.5	– .6
2,2	3.1	0.9	– 2.5	.9
3,1	4.8	2.3	– .4	1.[illegible]
3,2	7.2	3.4	– 7.7	–4.1
3,3	8.6	3.2	–13.1	–2.4
4,1	4.5	3.7	1.5	.8
4,2	4.4	3.3	– 1.3	1.2
4,3	4.1	2.4	– 3.0	– .3
4,4	5.6	2.4	– 2.9	–5.1

TOTAL: RSS GEOID ACCURACY FOR:

4x4	15.9 cm	8.1 cm	17.7 cm

$$\sigma J_{\ell,m} = \left[\sigma C^2_{\ell,m} + \sigma S^2_{\ell,m}\right]^{1/2}$$

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TABLE 5

UPDATED EVALUATION OF RECENT GRAVITY MODELS USING THE OBSERVED LONGITUDE ACCELERATIONS OF EIGHT 24–HOUR SATELLITES

	SAO SE2 (1970)	GEM–9 (1979)	GRIM–3 (1981)	GEM–L1 (1981)	GEM–L2 (1982)
Weighted RMS Residual Accelerations	15.7	5.0	21.6	1.0	.80

Where the weighted residual acceleration is $(A_o - A_c/\sigma_{A_o})^*$

* A_o	is observed longitude acceleration
A_c	is the calculated accelerations from gravity model
σ_{A_o}	is the estimated uncertainty ascribed to the observed longitude acceleration ($\sim 1. \times 10^{-8}$ r/d^2)

CONTRIBUTION OF GEOPOTENTIAL BY DEGREE TO ACCELERATION

ℓ	CONTRIBUTION FROM FULL FIELD	CONTRIBUTION FROM ESTIMATED ERROR OF GEM–L2
2	3000 X 10^{-8}	2.6 X 10^{-8}
3	400	1.0
4	40	0.2
5	4	0.1
6	0.7	0.0

TABLE 6

GRAVITY MODEL EVALUATION USING DATA FROM 24 HOUR SATELLITES

			RESIDUAL ($A_{OBS}-A_{FIELD}$)				
Satellite Name	Synchronous East Long.	(A) Acceleration ( ) (10^{-8}rad; d^2)	GRIM 3 (1981)	Standard Earth (1970)	GEM 9 (1977)	GEM L1 (1981)	GEM L2 (1982)
Skynet 1, 1	41.43	3004.3 (2.2)	− 2.3	12.9	3.8	−0.17	−0.56
Skynet 1, 4	45.64	2840.9 (1.6)	− 6.8	15.8	6.3	1.50	1.00
Skynet 1, 5	47.88	2720.0 (1.4)	−12.0	14.7	5.6	0.57	0.00
Skynet 1, 8	50.54	2550.63 (1.11)	−21.0	13.8	4.9	−0.71	−1.42
INTELSAT 2F2/SYNCOM3	166.40	502.40 (1.56)	10.8	20.1	3.0	1.29	0.81
ATS 5	254.96	−11.2 (0.4)	−22.0	0.0	8.2	−1.00	−0.25
INTELSAT 2F3–4	347.40	−104.77 (0.64)	60.5	37.3	−8.9	−1.27	−0.57
INTELSAT 2F3–1	350.40	182.31 (2.00)	19.4	12.8	−3.9	−1.96	−1.75
(wt. rms) (residual)			21.6	15.7	5.0	1.0	0.80

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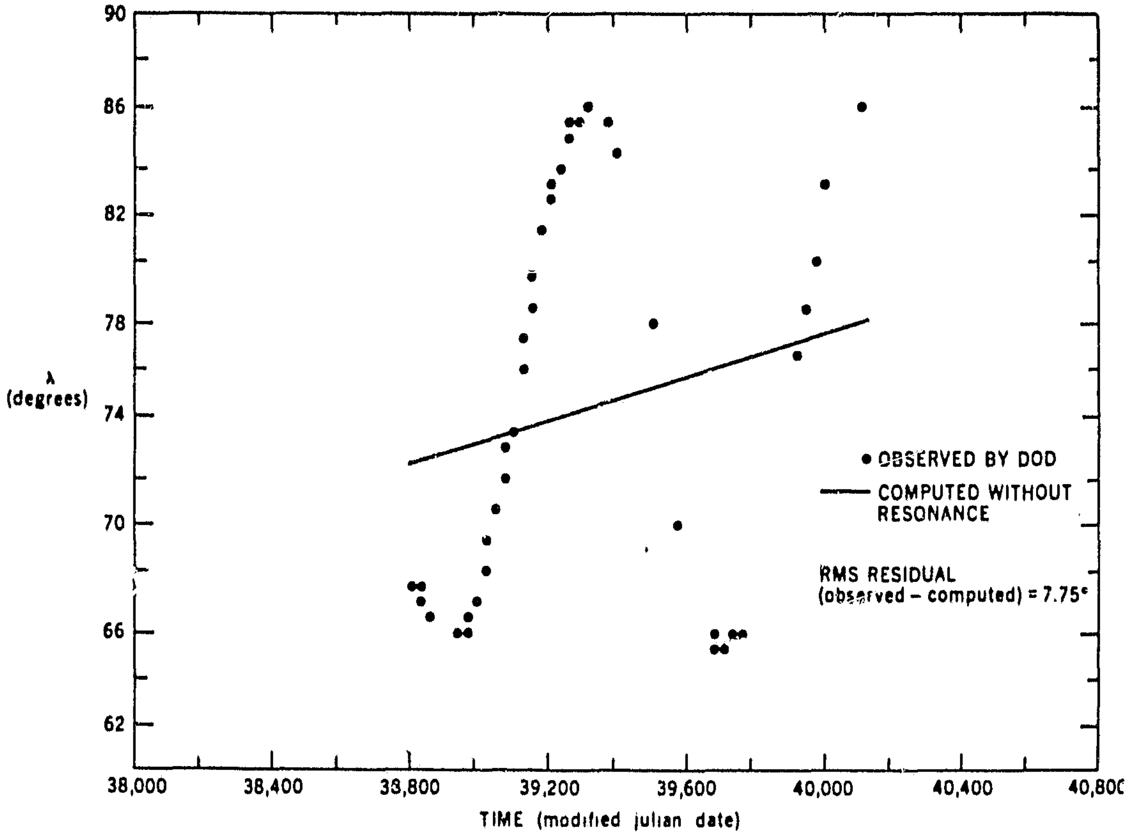
FIGURE 4
THE LONGITUDE OF SYNCOM 2



(From Wagner, 1972)

GEM-L2 overall satisfies these accelerations to their requisite accuracy. This is shown in Table 5 where the weighted RMS acceleration residual is equal to 1.0 for GEM-L1 and 0.80 for GEM-L2. The contribution of the geopotential by degree to the longitude accelerations are shown on the bottom of Table 5 for the full values of the harmonic coefficients and also for the error estimates of Table 4. They show that while the accelerations are sensitive to the harmonic coefficients as high as degree 6, the GEM-L2 error estimates beyond degree 3 or 4 are not directly tested by these longitude accelerations. Hence, the accuracy of GEM-L2's higher degree terms cannot be tested by this method for they exceed the accuracy of the longitude accelerations themselves. However, since no longitude acceleration residuals came close to approaching a 2.6 sigma error for the degree 2 contribution (see Table 5), it was concluded that our stated accuracies for the 6x6 portion of GEM-L2 are possibly conservative.

The simultaneous estimation of polar motion and A1-UT1 variations from the a priori 5-day mean values[2] made an important contribution to both the stability of the gravity solution as well as improved station positioning. Separate gravity fields were made utilizing the 1979 and 1980 Lageos data, and were intercompared between themselves and with GEM-L2. The new polar motion and A1-UT1 values made an improvement of 30% in the RSS coefficient difference to

[2] Bureau International De L'Heure (BIH) Circular D 5-day mean values using 90 day smoothing. A1-UT1=(A1-UTC)+(UTC-UT1) where (A1-UTC) is from the U.S. Naval Observatory Bulletin and (UTC-UT1) is from BIH.

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4x4 over fields holding the polar motion and A1-UT1 values fixed at BIH (e.g., the RSS coefficient difference between the 1979 vs. 1980 solutions decreased from 11 to 8 cm with the adjustment of polar motion - see Table 7). The 5-day mean polar motion values themselves were well determined in GEM-L2 and have a precision of about 10 cm. Figures 5 and 6 show examples of the corrections made to BIH (Circular D values) polar motion in GEM-L2 for the X and Y coordinates during 1980. Adjustments of up to 96 cm are seen from the a priori values. The full set of GEM-L2 polar motion and ΔLOD (changes in length of day) computed from the A1-UT1 values are found in Tables 8 and 9.

In the procedure with laser data, even after one station's longitude is held fixed, a singularity still exists for each orbital arc between the positioning of the satellite's longitude (node) and the earth's rotation (A1-UT1). The three 5-day mean values of A1-UT1 in each 15-day arc are good to within an undetermined constant and hence one value is held fixed at the a priori value. The three mean values are differenced to provide two accurate values of 5-day mean ΔLOD and are listed for each arc in Tables 8 and 9. In order to benefit from the adjusted A1-UT1 values in the computation of orbits, the specific arcs used in GEM-L2 or arcs contained within these 15-day intervals must be employed due to the discontinuity. Using this procedure the values recovered in GEM-L2 were used in our orbital analysis and provided the means for correcting the a priori values of A1-UT1 in our test arcs. The use of this corrected information greatly improved our orbital results as subsequently shown.

TABLE 7

THE EFFECT ON THE GRAVITY FIELD OF SOLVING FOR POLAR MOTION

(VALUES ARE RSS COEFFICIENT DIFFERENCES TO 4x4 IN CM)

	WITHOUT POLAR MOTION	WITH POLAR MOTION	(IMPROVEMENT %)
LAG'79 V. LAG'80	10.8 cm	8.2 cm	32%
LAG'79 V. GEM–L2	6.7 cm	5.1 cm	31%
LAG'80 V. GEM–L2	5.9 cm	4.8 cm	23%

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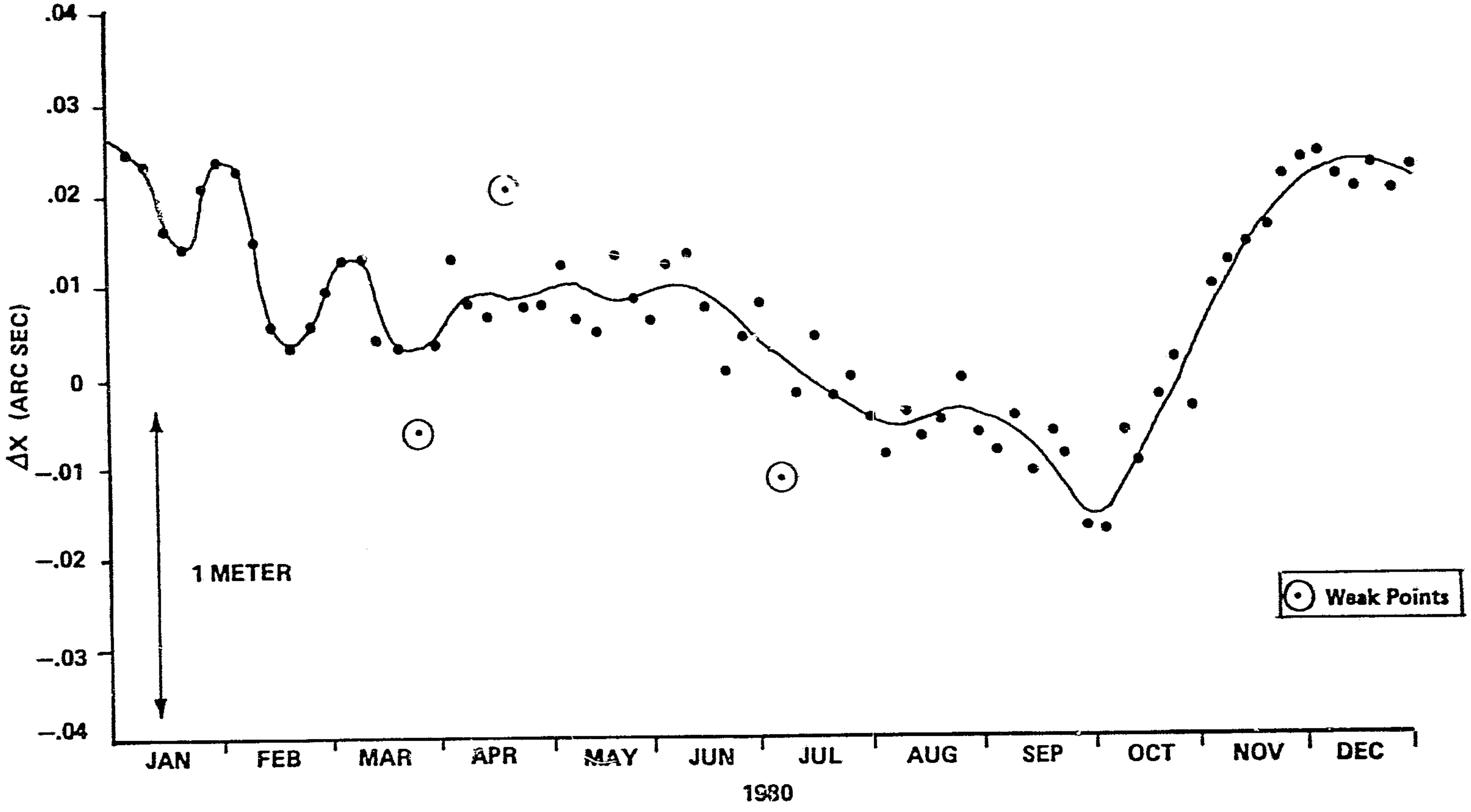
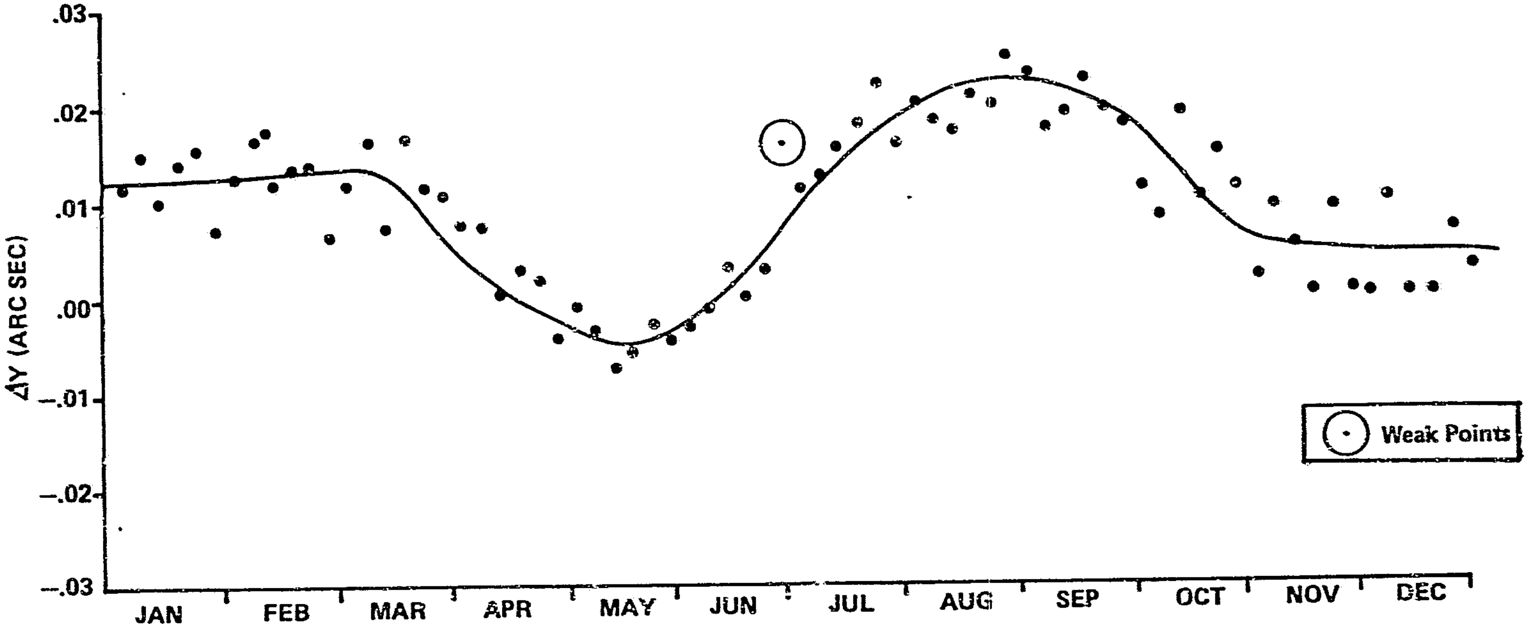
FIGURE 5

1980 POLAR MOTION: ΔX
(GEM–L2 — BIH*)



*Circular D 90–day smooth values

FIGURE 6
1980 POLAR MOTION: ΔY
(GEM–L2 – BIH*)



*Circular D 90–day smooth values

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Satellite positioning accuracy directly reflects on the capabilities of using SLR to monitor tectonic motions and deformations. GEM-L2, by improving the force modeling on the Lageos orbit, yields a more accurate ephemeris than previously available, which in turn improves the accuracy of the estimated station coordinates and their inter-station distances. The improvements in orbital accuracy are discussed at length in the next section. Typically, the post-fit RMS of the range residuals (normal points) on Lageos when using GEM-L2 in a 15 day arc is on the order of 10 to 15 cm. Consequently polar motion and A1-UT1 errors, which produce range residuals of this magnitude or in excess of it, map into the positioning of the stations with respect to the orbit and must be accounted for if stable station position solutions are to result. This is clearly shown in the results presented in Table 10.

Table 10 shows a comparison of laser interstation distances for 8 "base" stations using two independent solutions. The values in Table 10 are the average error in the magnitude of the baseline differences. The Lageos data set (from 1979 through December of 1980) has been divided into two parts where each single month's data is split into two 15 day segments and each alternating segment contributes to one of these two independent solutions. Therefore, while the two solutions contain independent data, they both span the same time interval, have nearly equal representation for each station, and should average out (for the purposes of this comparison) any resulting plate motion over these baselines. The impact of the GEM-9 and GEM-L2 force models on station positioning are intercompared as well as the

TABLE 8
POLAR MOTION FROM GEM-L2

EPOCH TIME (YYMMDD)	END TIME (YYMMDD)	X OF POLE (ARC SEC)	Y OF POLE (ARC SEC)
790201	790206	0.04236	0.06581
790206	790211	0.03006	0.07124
790211	790216	0.00962	0.06823
790216	790221	-.01840	0.07566
790221	790226	0.00731	0.06682
790226	790303	-.00234	0.06454
790303	790308	-.02427	0.08268
790308	790313	-.04352	0.10821
790313	790318	-.03390	0.10377
790318	790323	-.07209	0.09730
790323	790328	-.10478	0.12547
790328	790402	-.10689	0.13757
790402	790407	-.11646	0.14281
790407	790412	-.12197	0.16053
790412	790417	-.12915	0.16711
790417	790422	-.13187	0.17149
790422	790427	-.13487	0.20250
790427	790502	-.13713	0.20528
790502	790507	-.14700	0.23164
790507	790512	-.14232	0.23456
790512	790517	-.14868	0.23370
790517	790522	-.15254	0.25760
790522	790527	-.15193	0.26936
790527	790601	-.14911	0.27508
790603	790608	-.14744	0.29402
790608	790613	-.13265	0.28258
790613	790618	-.13930	0.31097
790618	790623	0.12884*	0.24472*
790623	790628	-.12911*	0.34322*
790628	790703	-.14598*	0.41153*
790703	790708	-.09748	0.34779
790708	790713	-.09682	0.36196
790713	790718	-.10153	0.37976
790718	790723	-.08346	0.38751
790723	790728	-.06815	0.38973
790728	790802	-.06076	0.39404
790802	790807	-.04733	0.41488
790807	790812	-.02678	0.42905
790812	790817	-.01391	0.42019
790817	790822	-.00727	0.41940
790822	790827	0.01385	0.42772
790827	790901	0.01165*	0.42513*
790901	790906	0.07187	0.41025
790906	790911	0.05219	0.43264
790911	790916	0.05721	0.43222
790916	790921	0.07387	0.42335
790921	790926	0.07021	0.43368
790926	791001	0.09166	0.42970
791001	791006	0.09349	0.40731
791006	791011	0.10094	0.41176
791011	791016	0.11009	0.39841

* Insufficient data; poorly determined values

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TABLE 8 (CONTINUED)

POLAR MOTION FROM GEM-L2

EPOCH TIME (YYMMDD)	END TIME (YYMMDD)	X OF POLE (ARC SEC)	Y OF POLE (ARC SEC)
791016	791021	0.11346	0.38981
791021	791026	0.11931	0.39255
791026	791031	0.13151	0.38211
791031	791105	0.13902	0.37152
791105	791110	0.14515	0.36059
791110	791115	0.14919	0.35074
791115	791120	0.15053	0.34341
791120	791125	0.13841	0.33747
791125	791130	0.14077	0.32170
791130	791205	0.14970	0.31194
791205	791210	0.14695	0.30421
791210	791215	0.14422	0.29476
791215	791220	0.14492	0.29006
791220	791225	0.14258	0.27804
791225	791230	0.15174	0.26835
800102	800107	0.15036	0.25186
800107	800112	0.14440	0.24786
800112	800117	0.13140	0.24356
800117	800122	0.12183	0.23096
800122	800127	0.12263	0.22665
800127	800201	0.11801	0.22046
800201	800206	0.11028	0.20601
800206	800211	0.09687	0.20555
800211	800216	0.08302	0.20398
800216	800221	0.07669	0.19529
800221	800226	0.07467	0.19269
800226	800302	0.07474	0.18940
800302	800307	0.07374	0.18160
800307	800312	0.06761	0.18773
800312	800317	0.05152	0.19367
800317	800322	0.04221	0.18745
800322	800327	0.02451	0.19994
800327	800401	0.02357	0.19849
800401	800406	0.02056	0.19245
800406	800411	0.00381	0.20385
800411	800416	-.00724	0.20956
800416	800421	-.00151	0.20750
800421	800426	-.02221	0.21768
800426	800501	-.02738	0.22346
800501	800506	-.02865	0.22351
800506	800511	-.03924	0.23294
800511	800516	-.04372	0.23690
800516	800521	-.03841	0.23761
800521	800526	-.04502	0.24491
800526	800531	-.04830	0.25296
800531	800605	-.04194	0.25609
800605	800610	-.05277	0.26201
800610	800615	-.04652	0.26776
800615	800620	-.05282	0.27515
800620	800625	-.04612	0.27612
800625	800630	-.03831	0.28083

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TABLE 8 (CONTINUED)

POLAR MOTION FROM GEM-L2

EPOCH TIME (YYMMDD)	END TIME (YYMMDD)	X OF POLE (ARC SEC)	Y OF POLE (ARC SEC)
800701	800706	-.05511	0.29827
800706	800711	-.04163	0.29676
800711	800716	-.03260	0.30141
800716	800721	-.03657	0.30816
800721	800726	-.03092	0.31439
800726	800731	-.03368	0.32159
800731	800805	-.03553	0.31654
800805	800810	-.02807	0.32481
800810	800815	-.03024	0.32520
800815	800820	-.02572	0.32693
800820	800825	-.01953	0.33423
800825	800830	-.02583	0.33547
800830	800904	-.02577	0.34264
800904	800909	-.01982	0.34304
800909	800914	-.02404	0.34017
800914	800919	-.01789	0.34540
800919	800924	-.01904	0.35156
800924	800929	-.02451	0.35282
800929	801004	-.00803	0.35642
801004	801009	-.01127	0.35375
801009	801014	-.01399	0.35480
801014	801019	-.00681	0.37008
801019	801024	-.00160	0.36526
801024	801029	-.00945	0.37419
801029	801103	0.00243	0.37585
801103	801108	0.00587	0.36965
801108	801113	0.01022	0.38007
801113	801118	0.01466	0.37843
801118	801123	0.02706	0.37407
801123	801128	0.03433	0.38633
801128	801203	0.04107	0.37814
801203	801208	0.04463	0.37785
801208	801213	0.05022	0.38555
801213	801218	0.05987	0.37261
801218	801223	0.06229	0.37020
801223	801228	0.07107	0.37347
801228	810102	0.07320	0.36536
810102	810107	0.07508	0.36159
810107	810112	0.07839	0.37329
810112	810117	0.08586	0.35664
810117	810122	0.07958	0.35986
810122	810127	0.08426	0.35575
810127	810201	0.08829	0.34396
810201	810206	0.09155	0.33750
810206	810211	0.09535	0.33137
810211	810216	0.09981	0.33657
810216	810221	0.09405	0.33510
810221	810226	0.09683	0.31882
810226	810303	0.10090	0.31807
810303	810308	0.08837	0.32041
810308	810313	0.09328	0.30899

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TABLE 8 (CONTINUED)

POLAR MOTION FROM GEM-L2

EPOCH TIME (YYMMDD)	END TIME (YYMMDD)	X OF POLE (ARC SEC)	Y OF POLE (ARC SEC)
810313	810318	0.09790	0.31154
810318	810323	0.10133	0.30774
810323	810328	0.11215	0.28541
810328	810402	0.09394	0.30943
810402	810407	0.09140	0.29810
810407	810412	0.10261	0.28539
810412	810417	0.09087	0.29124
810417	810422	0.09544	0.27951
810422	810427	0.10647	0.26483
810427	810502	0.09547	0.27798
810502	810507	0.10013	0.26164
810507	810512	0.09482	0.25310
810512	810517	0.09274	0.26566
810517	810522	0.10048	0.25009
810522	810527	0.09377	0.24716
810527	810601	0.11247	0.24930
810601	810606	0.10014	0.24549
810606	810611	0.08707	0.23571

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TABLE 9

CHANGE IN LENGTH OF DAY FROM GEM-L2

MIDPOINT DATE (YYMMDD)	DELTA LOD (SEC)
790206	.00280
790211	.00276
790221	.00306*
790226	.00360*
790308	.00283
790313	.00281
790323	.00356
790328	.00370
790407	.00277
790412	.00316
790422	.00374
790427	.00330
790507	.00299
790512	.00298
790522	.00309
790527	.00259
790608	.00241
790613	.00222*
790623	.00108*
790628	.00292*
790708	.00221
790713	.00246
790723	.00151
790728	.00182
790807	.00248
790812	.00266
790822	.00199*
790827	.00209
790906	.00232
790911	.00260
790921	.00247
790926	.00229
791006	.00321
791011	.00264

* Insufficient data; poorly determined value.

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TABLE 9 (CONTINUED)

CHANGE IN LENGTH OF DAY FROM GEM-L2

DPOINT DATE (YYMMDD)	DELTA LOD (SEC)
791021	.00280
791026	.00258
791105	.00269
791110	.00214
791120	.00251
791125	.00246
791205	.00235
791210	.00248
791220	.00243
791225	.00242
800107	.00247
800112	.00249
800122	.00289
800127	.00215
800206	.00222
800211	.00199
800221	.00272
800226	.00222
800307	.00221
800312	.00221
800322	.00260
000327	.00241
800406	.00248
800411	.00270
800421	.00229
800426	.00255
800506	.00238
800511	.00286
800521	.00215
800526	.00214
800605	.00214
800610	.00219
800620	.00176
800625	.00160

TABLE 9 (CONTINUED)

CHANGE IN LENGTH OF DAY FROM GEM-L2

MIDPOINT DATE (YYMMDD)	DELTA LOD (SEC)
800706	.00166
800711	.00137
800721	.00153
800726	.00137
800805	.00161
800810	.00132
800820	.00155
800825	.00190
800904	.00188
800909	.00217
800919	.00218
800924	.00290
801004	.00215
801009	.00259
801019	.00266
801024	.00322
801103	.00262
801108	.00230
801118	.00274
801123	.00251
801203	.00249
801208	.00211
801218	.00256
801223	.00214
810102	.00224
810107	.00197
810117	.00252
810122	.00214
810201	.00178
810206	.00224
810216	.00195
810221	.00219
810303	.00260
810308	.00350

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TABLE 9 (CONTINUED)

CHANGE IN LENGTH OF DAY FROM GEM-L2

MIDPOINT DATE (YYMMDD)	DELTA LOD (SEC)
810318	.00255
810323	.00257
810402	.00301
810407	.00301
810417	.00310
810422	.00262
810502	.00324
810507	.00254
810517	.00245
810522	.00228
810601	.00295
810606	.00218

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a priori modeling of polar motion and A1-UT1 with that obtained from LAGEOS for their contribution to station positioning. An excellent 1.8 cm agreement is obtained for the baseline differences of these two solutions when using the gravity and Lageos earth orientation parameters of the GEM-L2 solution. The 28 baseline-comparison from the GEM-L2 model is shown as a histogram in Figure 7.[3] All 28 baseline differences from these alternating 15-day arcs are shown in Figure 8. GEM-9 with a priori polar motion and A1-UT1 values, which reflects the state of the art prior to the Lageos analysis as performed by ourselves and many others, yields about a four-fold worse agreement. These results clearly show that the new LAGEOS polar motion and A1-UT1 and the gravity of GEM-L2 are needed to achieve the 1.8 cm baseline comparison.

A comparison was also made dividing the data chronologically into a 1979 versus a 1980 set of solutions. In an ideal environment, the sole difference between this comparison and that described in Table 10 would be the plate motion which has resulted over this year. However, in this later case, data distribution and station participation also varied and must be considered. However, the results are still quite satisfying and are shown in Table 11. A 6 cm average has been obtained for the total position differences over these two years when using GEM-

[3] The laser system at Westford, Mass. (station number 7091) seems to be somewhat inconsistent in longitude as compared with the other sites. The longitude coordinate is most effected by any range bias. See Christodoulidis and Smith, 1981.

TABLE 10
BASELINE COMPARISON FROM TWO INDEPENDENT LAGEOS DATA SETS SPANNING 1979 THROUGH 1980*

	GEM–9		GEM–L2		
	w/BIH** POLAR MOTION AND A1–UT1	w/LAGEOS POLAR MOTION AND A1–UT1	w/BIH POLAR MOTION AND A1–UT1	w/LAGEOS POLAR MOTION & BIH A1 – UT1	w/LAGEOS POLAR MOTION AND A1–UT1
BASELINE AGREEMENT FOR EIGHT "BASE" STATIONS (ALL 28 BASELINES)	7.2 cm	6.0 cm	6.9 cm	4.7 cm	1.8 cm

"BASE STATIONS ARE:

7063	GSFC, MD
7086	FT. DAVIS, TEXAS
7090	YARRAGADEE, AUSTRALIA
7091	WESTFORD, MA
7115	GOLDSTONE, CA
7114	OWENS VALLEY, CA
7907	AREQUIPA, PERU
7943	ORRORAL, AUSTRALIA

*THE DATA HAS BEEN DIVIDED INTO ALTERNATING 15 DAY SEGMENTS AND THEREFORE BOTH SOLUTIONS HAVE SIMILAR AND UNIFORM DATA DISTRIBUTION FROM THESE SITES.

**BIH CIRCULAR D 90–DAY SMOOTHED VALUES

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L2. With GEM-L2 the baseline stability over the United States is 4.1 cm between 1979 and 1980 solutions, again, a good result.

An error analysis has been performed to give an estimate of the expected baseline error due to the estimated uncertainties in the GEM-L2 model for these 28 "base station" baselines. In this analysis, the actual normal matrix obtained from 2 years of LAGEOS tracking was employed. Table 12 presents these results indicating that the gravity field error in GEM-L2 is the cause of 1.7 cm average error on these baselines. However, 2 years of data is averaged (in a geometrical sense about these stations) to form these results. Also the gravity field error should become somewhat worse for more limited data distributions found in annual or monthly solutions.

The tracking station coordinates obtained from GEM-L2 are presented in Table 13. These coordinates refer to the optical axes of the laser instruments. These station coordinates, unlike the ones discussed above, represent values derived from the entire 2 1/2 years of Lageos tracking. As such, they most likely average out any interceding plate motion over 1979 to 1981. They, however, represent a starting set of consistent coordinates to be utilized with GEM-L2.

FIGURE 7
HISTOGRAM OF BASELINE AGREEMENT FOR 8 BASE STATIONS
(AVG = 1.8 cm) WHEN USING GEM–L2 GRAVITY AND POLAR MOTION

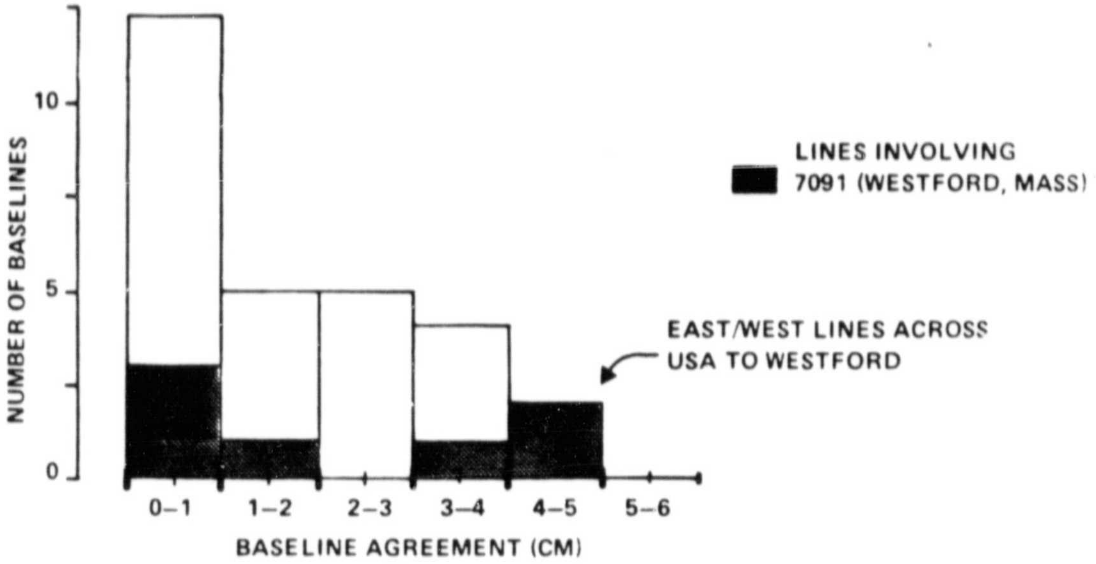

FIGURE 8
A MEASURE OF SLR BASELINE PRECISION FROM ALTERNATING ARCS OF 1979 THROUGH 1980 LAGEOS DATA

STATION NO.

STATION NO.	7086	7090	7091	7114	7115	7907	7943
7063	2.2	−0.5	3.0	2.6	−0.5	−2.4	−0.4
7086		3.6	4.9	2.0	−1.0	−1.3	3.3
7090			−0.7	1.1	1.0	−0.0	−3.7
7091				4.5	1.6	−0.5	−0.1
7114					1.4	−1.0	0.3
7115						−3.5	0.8
7907							2.3

ACTUAL BASELINE DIFFERENCES (CM)

BASELINE AGREEMENT FOR 8 BASE STATIONS (AVG = 1.8 cm) WHEN USING GEM–L2 GRAVITY AND LAGEOS POLAR MOTION

TABLE 11
GEM—9 VERSUS GEM—L2 STATION POSITIONING COMPARISON

SOLUTION (a) 1979 LAGEOS DATA
(b) 1980 LAGEOS DATA

TOTAL POSITION DIFFERENCES FOR 8 BASE STATIONS (80—79 SOLUTIONS)

STATIONS	GEM—9 w/BIH POLAR MOTION*	w/LAGEOS POLAR MOTION*	GEM—L2 w/BIH POLAR MOTION*	w/LAGEOS POLAR MOTION*
STALAS (GSFC)	40.9 cm	17.3 cm	18.1 cm	7.1 cm
FT. DAVIS (TEXAS)	17.7	13.1	10.2	8.3
YARLAS (AUSTRALIA)	24.8	18.9	13.8	8.5
HAYSTK (MASS.)	38.0	21.9	18.3	4.5
OWNSVL (CAL.)	34.1	26.4	15.0	2.3
GOLDST (CAL.)	44.4	31.4	12.1	5.1
ARELAS (PERU)	12.1	7.3	19.5	5.8
ORRLAS (AUSTRALIA)	27.7	9.8	24.1	6.8
(AVG)	30.0	18.3	16.4	6.0
BASELINE DIFFERENCES				
ALL GLOBAL AVERAGE BASELINE DIFFERENCE (28)	14.6	12.9	8.0	5.3
USA BASELINE AVERAGE DIFFERENCE (10)	11.3	10.2	5.2	4.1

*AND A1—UT1 FROM RESPECTIVE SOURCES. BIH CIRCULAR D 90—DAY SMOOTH VALUES

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TABLE 12

BASELINE UNCERTAINTIES COMPUTED FROM GEM–L2 GRAVITY FIELD ERRORS

	FT. DAVIS 7086	YARLAS 7090	HAYSTACK 7091	OWENS VALLEY 7114	GOLDSTONE 7115	ARELAS 7907	ORRLAS 7943
7063	2.2	1.4	0.5	2.2	2.1	1.8	1.5
7086		1.4	2.2	1.2	1.2	2.1	1.9
7090			1.1	1.7	1.8	1.3	2.5
7091				2.3	2.0	2.0	1.5
7114					0.5	2.4	1.8
7115						2.5	1.8
7907							2.0

(Centimeters)

AVG = 1.7 cm

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TABLE 13. GEM–L2 STATION COORDINATES

NAME	NUMBER	X (METERS)	Y (METERS)	Z (METERS)
QUIN079	7051	-2516892.543	-4198849.621	4076414.163
QUIN081	7052	-2516892.536	-4198849.345	4076414.005
SANTIE	7062	-2428024.909	-4799755.613	3417274.698
STALAS	7063	1130718.148	-4831370.304	3994089.550
RAMLAS	7069	917962.324	-5548373.708	2998778.273
BEAR79	7082	-1735995.810	-4425052.079	4241433.323
BEAR81	7083	-1735995.266	-4425053.706	4241432.565
FTDAVS	7086	-1330123.387	-5328531.275	3236152.478
TLRSFD	7087	-1330123.944	-5328531.908	3236151.293
YARLAS	7090	-2389010.270	5043331.356	-3078527.399
HAYSTK	7091	1492457.214	-4457281.176	4296818.441
ML0802	7092	-6143451.870	1364697.593	1034163.840
SAMOA1	7096	-6100049.239	-996207.189	-1568977.450
ML06XX	7101	1131243.048	-4831182.602	3994150.974
ML0402	7102	1130690.048	-4831355.833	3994113.001
ML0601	7103	1130688.703	-4831351.571	3994118.648
ML0701	7104	1131099.348	-4831199.503	3994173.037
GSFCLS	7105	1130723.076	-4831353.187	3994108.774
COLRLS	7112	-1240676.533	-4720467.396	4094483.412
OWNSV2	7114	-2410421.637	-4477807.289	3838689.350
GLDST2	7115	-2350860.745	-4655550.950	3661000.415
HAWNAA	7120	-5466000.857	-2404413.491	2242229.839
HOLLAS	7210	-5466428.652	-2404100.627	2241560.366
ARELAS	7907	1942795.076	-5804077.481	-1796919.935
HOPLAS	7921	-1936757.663	-5077708.928	3331923.314
NATLAS	7929	5186470.620	-3653854.157	-654322.535
ORRLAS	7943	-4447550.496	2677132.438	-3694997.182

NOTE:

STATION 7052 IS THE 1981 OCCUPANCY OF QUINCY
STATION 7083 IS THE 1981 OCCUPANCY OF BEAR LAKE
STATION 7087 IS THE TLRS OCCUPANCY OF FORT DAVIS

GM = 398600.607 KM**3/SEC**2 WITH C = 299792.5 M/SEC.

5.0 ORBITAL ACCURACIES ON LAGEOS

Dynamic satellite geodesy implies that improvement in the modeling of the forces acting upon the satellite should yield less error in the estimation of station positions and parameters describing the location of the earth with respect to the satellite's orbital plane. With laser systems routinely producing range observations at the 10 cm level of accuracy, a great deal of improvement was possible in our modeling of the geopotential to fully exploit this data to improve the capability of measuring plate tectonic activity. This was the principal direction of our work. Initial error analysis mapping the uncertainties found in each of the coefficients of GEM-9 revealed that LAGEOS orbital errors of ± 1m or more could be expected. This analysis was made using the linear perturbation theory developed by Kaula (1966, pp 40 and 49). Each coefficient's uncertainty was used to estimate the magnitude of the resulting total perturbation error on LAGEOS for each harmonic constituent. Since most of the simple spherical harmonics give rise to a number of perturbations at different frequencies on LAGEOS' orbit, these were combined as their RSS error for each term. Figure 9 shows the results of this analysis for the GEM-9 uncertainties. In Figure 9, the low degree and order field comes through as the strongest source of error with the exception of the resonance terms of order 6 and 7 (corresponding to LAGEOS's 6.4 orbital revolutions per day). The total RSS of all of the errors shown in Figure 9 is approximately ± 1m. This analysis was repeated at the conclusion of the development of GEM-L2 and is shown for the estimates of the GEM-L2

FIGURE 9

LAGEOS ORBIT ERRORS (CENTIMETERS)

DUE TO ERRORS IN GEM 9 HARMONICS

	ORDER										
DEGREE	0	1	2	3	4	5	6	7	8	9	10
2	1		㉔								
3	1	13	9	12							
4		㉖	9	7	8						
5		2	5	7	7	10					
6		9	5	4	3	4	13				
7		3	3	3	2	3	(65)	㊹			
8		3	4	2	2	1	6	3	1		
9			2	1	2	1	6	⑳	4	1	
10		1	1		1		2	1	1		
11							4	3			1

RSS = 1 meter

coefficient errors in Figure 10. Based upon this analysis, LAGEOS orbital accuracies lie in the vicinity of 35 cm when using GEM-L2.

A direct measure of the orbital improvement gained through the utilization of GEM-L2 has been obtained through a series of tests using actual LAGEOS data. Some of these tests are very severe -- they are designed to give a magnified insight into orbit errors arising from the gravity field. Therefore, the accuracies which are seen may be considered as worse case phenomena, for they are tests where large data gaps have been imposed on the Lageos observation set. In the tests shown in Table 14, a fifteen day span of LAGEOS data is selected. The middle five days of data is then deleted so that the data which is utilized for the original orbit computations consist solely of the data available from the first and last five day data spans. This fifteen day test orbit over this middle five day interval is therefore completely independent of the data taken during this interval. This independent five day span of data is then used to calculate a second orbit. This five day orbit is differenced over its interval with that of the original fifteen day arc in three components -- radially, along track and across track. Table 14 shows a sample (representative) of the results we have obtained. Over the two year period tested, GEM-L2 outperforms GEM-9 in all cases and gives results averaging 26 cm in total orbit position error which closely reflects the magnitude of error expected from our error analysis using the linear perturbation theory. These improved orbital tests of necessity incorporated the GEM-L2 polar motion and A1-UT1 values since the a priori values were corrected for these 15 day arcs. It is this

FIGURE 10

LAGEOS ORBIT ERRORS (CENTIMETERS)

DUE TO ERRORS IN GEM-L2 HARMONICS

	ORDER										
DEGREE	0	1	2	3	4	5	6	7	8	9	10
2	0		6								
3	0	6	3	3							
4		19	6	4	3						
5		1	4	4	4	4					
6		8	4	3	2	2	3				
7		3	3	3	2	2	6	15			
8		3	3	2	2	1	3	1	1		
9			2	1	2	1	4	15	3	1	
10		1	1		1		2	1	1		
11							3	3			1

RSS = 37 cm

improvement in the modeling of the gravitational forces on LAGEOS which enhances the utilization of LAGEOS laser data for the estimation of the tectonic plate motion and dynamics of the earth's rotation and polar wandering.

A second series of runs was made showing (Table 15) the RMS of fit to "normal" point laser observations created from the original range observation set. The use of GEM-L2 and the LAGEOS polar motion/A1-UT1 shows a factor of two improvement in our ability to fit these nearly "noiseless" observations. A typical RMS of fit to the normal points in 15 day arcs is about 10 cm when using GEM-L2 with LAGEOS polar motion and A1-UT1.

TABLE 14

LAGEOS ORBIT TEST:

15 DAY ARC WITH MIDDLE 5 DAY'S DATA DELETED

VS.

ORBIT FROM MIDDLE 5 DAY

		RMS SATELLITE POSITION DIFFERENCES			
EPOCH DATE	GRAVITY MODEL	RADIAL (cm)	ACROSS TRACK (cm)	ALONG TRACK (cm)	TOTAL (cm)
3/80	GEM–9	7.9	81.7	59.7	101.5
	GEM–L2	3.4	17.9	27.1	32.7
2/80	GEM–9	20.4	78.0	270.0	281.8
	GEM–L2	2.2	7.6	13.1	15.3
12/79	GEM–9	17.5	15.2	73.0	76.6
	GEM–L2	10.7	4.6	23.0	25.8
11/79	GEM–9	5.1	31.8	82.6	88.7
	GEM–L2	10.7	32.9	41.8	54.3
10/79	GEM–9	4.6	51.1	205.0	211.3
	GEM–L2	5.4	31.9	41.9	52.9

Estimated error in each orbit is 26 cm. As an example, since the error of 32.7 cm for the first case reflects errors from two orbits, the estimated error for each of the orbits (assuming they are equal) would be $32.7 \div \sqrt{2}$.

TABLE 15
15 DAY RMS OF FIT TO NORMAL POINT OBSERVATIONS ON LAGEOS

EPOCH	GRAVITY MODEL	POLAR MOTION A1–UT1	NO. OBS.	RMS OF FIT (CM)
791215	GEM–9	BIH*	602	17.3
	GEM–L2	LAGEOS	602	8.0
790901	GEM–9	BIH	203	12.2
	GEM–L2	LAGEOS	203	6.6
800416	GEM–9	BIH	663	33.6
	GEM–L2	LAGEOS	663	16.7
800716	GEM–9	BIH	771	39.5
	GEM–L2	LAGEOS	771	11.3
800830	GEM–9	BIH	1376	36.7
	GEM–L2	LAGEOS	1376	13.8

*BIH Circular D 90–day smooth values was used as a priori.

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6.0 SUMMARY

A new gravity model GEM-L2 has been derived as part of the analysis being undertaken on Lageos for the NASA Crustal Dynamics Program. Lageos' unique contribution toward the resolution of the long wavelength (for terms through degree and order 4) geopotential has resulted in a two-fold improvement in our knowledge of these terms. This improvement is confirmed through an analysis of 24-hour synchronous satellite orbits. Over 400,000 Lageos ranges taken by 20 laser tracking stations over a 2 1/2 year period have been utilized in the new field. GEM-L2 contains a consistent set of parameters including a geopotential field (20 x 20), tracking station coordinates (GEM 9 and Lageos), 5-day polar motion and ΔLOD values, and GM = 398600.607 km^3/sec^2 based upon c = 299792.5 m/sec.

GEM-L2 provides for better baseline recovery, orbital modeling, and station positioning from Lageos tracking. The adjustment of earth orientation parameters was essential for improving the geopotential and producing better interstation distances. The accuracy of the station positioning results (± 6 cm in total position globally and ± 1.8 cm in baselines) of this report confirms the utility of using SLR for the measurement of tectonic plate motion and inter-plate deformations. These results are summarized in Table 16.

TABLE 16
SUMMARY OF GEM–L2

- GEM–L2 CONSISTS OF:
    1. (1) A COMBINATION OF 1979, 1980 AND 1981 LAGEOS DATA WITH GEM–9. HARMONICS COMPLETE TO (20x20)
    2. (2) LASER TRACKING STATION COORDINATES
    3. (3) 5 DAY POLAR MOTION VALUES AND EARTH ROTATION
    4. (4) GM = 398600.440 km^3/sec
- 4x4 PORTION OF THE FIELD NOW HAS ACCURACY ESTIMATED TO BE BETTER THAN 10 CM
- LAGEOS ORBIT ERRORS HAVE IMPROVED FROM 1M WITH GEM–9 TO ABOUT 30 CM WITH GEM–L2
- ORBITAL RANGE RESIDUALS USING 2 MINUTE NORMAL POINTS ARE NOW ABOUT 10 CM RMS FOR 15 DAY ARCS
- GEM–L2 SIGNIFICANTLY IMPROVES BASELINE DETERMINATION CAPABILITIES (7 CM FOR GEM–9 TO 1.8 CM FOR GEM–L2

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