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# LIFTING SURFACE THEORY FOR A HELICOPTER ROTOR IN FORWARD FLIGHT

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Abstract

A lifting surface theory has been developed for a helicopter rotor in forward flight for incompressible flow. The method utilized the concept of the linearized acceleration potential and makes use of the vortex lattice procedures. Results in terms of lift coefficient slope for several forward flight conditions are given.

## 1. INTRODUCTION

Rotating lifting surfaces are an integral part of the propulsive unit of every aeronautical and nautical vehicle, from the compressor and turbine blades of jet engines, the pumps for rocket engines, to propeller and helicopter rotors. The aerodynamics of these rotating elements has been under extensive study since the advent of the airplane and with a combination of experimental and analytical approaches, succcessful designs have been achieved. In many cases, two-dimensional theory has been used, usually modified by an assumed spanwise distribution, and inflow angles. A practical lifting surface theory has not been developed for the compressible rotating case for the general case of a helicopter in forward flights. It is the purpose of this report to present a lifting surface method within the limits of linearized theory.

The method is based on the concept of the acceleration potential, originally introduced by Hans Kussner (ref. 1). The method was first applied to an oscillating wing in uniform translatory motion by one of the authors including compressible flow in 1957 (ref. 2). The acceleration potential approach has now become standard in the determination of the unsteady aerodynamics forces for flutter studies.

The first use of the acceleration potential approach for a rotating system was made in a paper by Hanaoka (ref. 3) for the incompressible case in a paper directed at application to the marine propeller. The acceleration potential has been used in the past in studying the propeller noise problem, but in all of these noise propagation cases the problem was specialized early in the analytical development to the so-called far-field case usually with the stationary observer, whereas the lifting surface theory is essentially concerned with the details of the near-field case for a co-moving observer as well as the satisfaction of certain boundary conditions. Dat, (ref. 4), has derived a general expression for an acceleration doublet for any motion. His development has been used as a basis for this investigation. The procedure developed here involves the precise numerical integration over the surface of the rotor. It is hoped that the method lays fundamental ground work for propeller and helicopter unsteady, three dimensional compressible aerodynamic theory.

With regard to the contents of the report, the next section contains a brief derivation of the fundamental equations, including a discussion of some implications of the equations. The third section contains a description of the method of solution. Finally, the results of some preliminary calculation for the incompressible case are given.

## 2. BASIC EQUATIONS

The fundamental equation required for the solution of the lifting surface problem relates the downwash velocity (velocity normal to the lifting surface) to the unknown pressures on the rotor blade.

The acceleration potential method will be used in the analysis. The acceleration potential, $\psi$, is defined as

$$p - p_\infty = -\rho\psi \qquad (2.1)$$

Bernoulli's equation for pressure in terms of the velocity potential, $\phi$, is defined as

$$p - p_\infty = \rho\frac{D\phi}{dt} \qquad (2.2)$$

where the operation $\frac{D}{dt}$ represents the substantial derivative. For a wing translating in the x-direction with velocity $U$, the equation can be written

$$p - p_\infty = -\rho\psi = \rho\left(\frac{\partial\phi}{\partial t} + U\frac{\partial\phi}{\partial x}\right)$$

or

$$\psi = -\left(\frac{\partial\phi}{\partial t} + U\frac{\partial\phi}{\partial x}\right) \qquad (2.3)$$

For wings oscillating in harmonic motion such as in flutter, an harmonic substitution can be made to obtain

$$\psi = -\left(i\omega\,\phi + U\frac{\partial\phi}{\partial x}\right)$$

Solving for $\phi$ gives

$$\phi = \frac{e^{\frac{-i\omega x}{U}}}{U}\int_{-\infty}^{x}\psi(\tau,y,z)e^{\frac{i\omega\lambda}{U}}\,d\lambda \qquad (2.4)$$

Since the motion of a rotating helicopter blade is non-harmonic, this procedure is not practical, thus the relationship is

$$\psi = \frac{D\phi}{dt} \qquad (2.5)$$

Equation (2.5) can be integrated with respect to $t$ to give

$$\phi = \int_{-\infty}^{t}\psi dt \qquad (2.6)$$

Finally, the downwash is derived

$$w = \frac{\partial\phi}{\partial n} = \frac{\partial}{\partial n} \int_{-\infty}^{t} \psi dt \qquad (2.7)$$

## 3. ARBITRARY MOTION OF AN OSCILLATING DOUBLET IN COMPRESSIBLE FLOW

The expression for an oscillating doublet moving in an arbitrary manner in compressible flow was derived by Dat (ref. 4). A concise derivation of the doublet expression is given below.

The expression for an accelerating source is

$$\psi_s(\vec{P},t) = \frac{q(\vec{P}_0,\tau)}{4\pi\left|\vec{P} - \vec{P}_0(\tau)\right|\left|1 - \frac{\vec{V}_0(\tau)\cdot[\vec{P} - \vec{P}_0(\tau)]}{a\left|\vec{P} - \vec{P}_0(\tau)\right|}\right|} \qquad (3.1)$$

where $\vec{P}_0(\tau)$ the vector position of the source at time $\tau$, $\vec{P}$ is the position of the field point, $\vec{V}_0(\tau)$ is the velocity of the source point at time $\tau$, "a" is the speed of sound and $q$ is the strength of the source.

An auxiliary equation which relates the time interval $t - \tau$ between the time the source emits a disturbance at $\tau$ and the time at which it is received at the field point at $t$, to the distance between the two points is given below

$$t - \tau = \frac{\left|\vec{P} - \vec{P}_0(\tau)\right|}{a} \qquad (3.2)$$

A doublet may be formed by placing a source and a sink in close proximity and allowing the distance to approach zero, while at the same time maintaining the product of the distance and the source strength finite. This result may also be accomplished by performing a differentiation in the proper direction. Dat applies this second technique by first defining a vector distance $\vec{D}$

$$\vec{D} = \vec{P} - \vec{P}_o - \xi \vec{n}_o \tag{3.3}$$

where $\xi$ is a parameter and $\vec{n}_o$ is the unit vector which is in the desired direction. Substituting $\vec{D}$ for $\vec{P} - \vec{P}_o$ in equation (3.1) gives

$$\psi_s = \frac{q(\tau)}{4\pi|\vec{D}|[1 - \frac{\vec{V}_o \cdot \vec{D}}{a|\vec{D}|}]} \tag{3.4}$$

The doublet is now obtained by applying the derivative with respect to $\xi$ and letting $\xi \to 0$ or

$$\psi_D = \left(\frac{\partial \psi_s}{\partial \xi}\right)_{\xi \to 0} \tag{3.5}$$

to give

$$\psi_D = \frac{\frac{\partial q}{\partial \tau} \vec{D} \cdot \vec{n}_o}{4\pi a|\vec{D}|^2\left[1 - \frac{\vec{V}_o \cdot \vec{D}}{a|D|}\right]^2} + \frac{q(\vec{D} \cdot \frac{d\vec{V}_o}{dt})(\vec{D} \cdot \vec{n}_o)}{4\pi a^2|\vec{D}|^3\left[1 - \frac{\vec{V}_o \cdot \vec{D}}{a|D|}\right]^3}$$

$$\frac{q[(a^2 - V_o^2)\vec{D} \cdot \vec{n}_o - a|\vec{D}|(\vec{V}_o \cdot \vec{n}_o) + (\vec{V}_o \cdot \vec{D})(\vec{V}_o \cdot \vec{n}_o)]}{4\pi a^2|\vec{D}|^3\left[1 - \frac{\vec{V}_o \cdot \vec{D}}{a|D|}\right]^3}$$

$$+ \frac{q\,\vec{D}\cdot\frac{\partial\vec{n}_o}{\partial\tau}}{4\pi a|\vec{D}|^2\left[1-\frac{\vec{V}_o\cdot\vec{D}}{a|D|}\right]^2} \quad (3.6)$$

This expression checks an expression derived in a different manner in ref. 7.

4. INCOMPRESSIBLE FLOW. - The incompressible case may be obtained by letting $a \to \infty$ to get

$$\psi_D = \frac{q\vec{D}\cdot\vec{n}_o}{4\pi|\vec{D}|^3} \quad (4.1)$$

the downwash equation may now be written

$$w = \frac{\partial\psi_D}{\partial n} = \frac{1}{4\pi}\int_{-\infty}^{t} q(r_o,\tau)\left[\frac{\vec{n}\cdot\vec{n}_o}{|\vec{D}|^3} - \frac{3(\vec{n}\cdot\vec{D})(\vec{n}_o\cdot\vec{D})}{|\vec{D}|^5}\right] d\tau \quad (4.2)$$

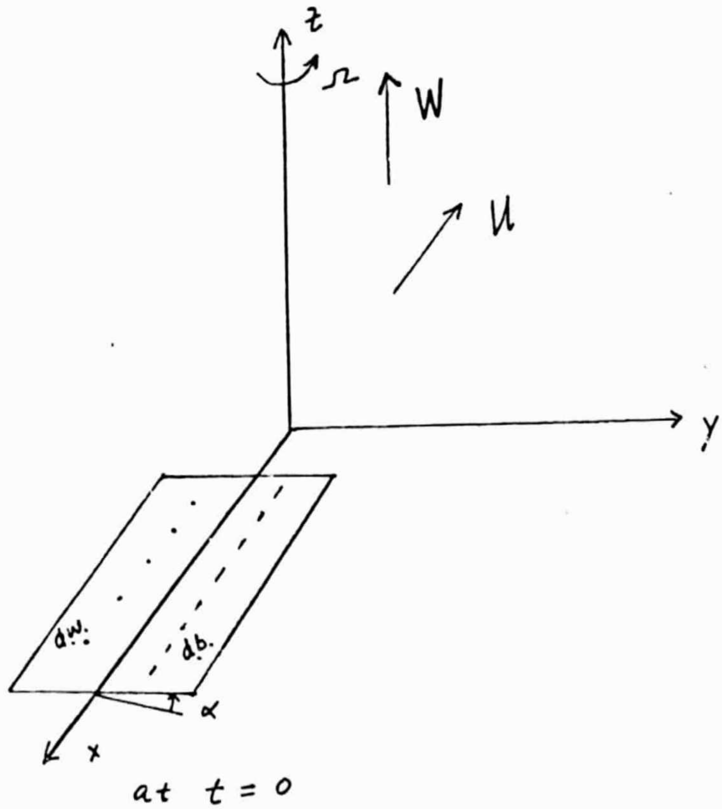
The integral equation relating the known downwash, $w(\vec{P},t)$ to the unknown pressures, $q(r_o,\tau)$, is given by the following equation for incompressible flow

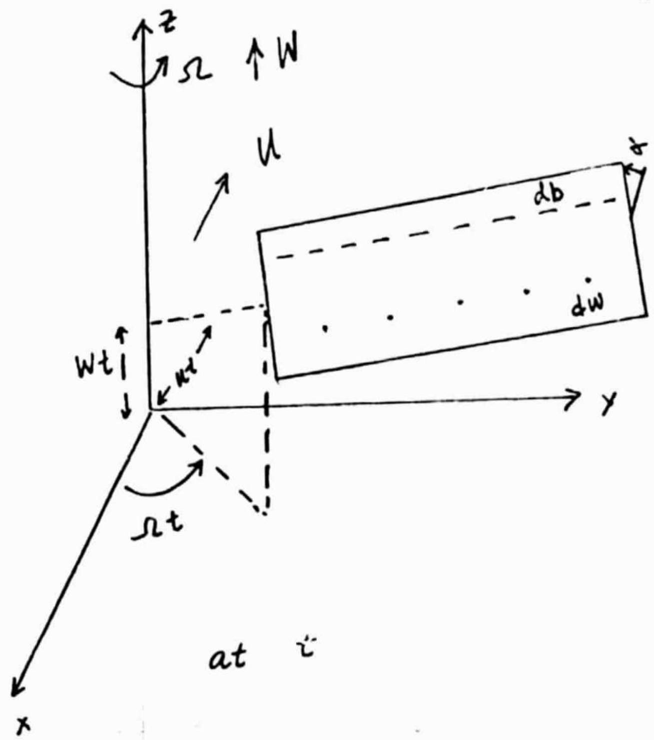
$$w(\vec{p},t) = \frac{1}{4\pi\rho}\int_{Chord}\int_{Span}\int_{-\infty}^{t} q(r_o,\tau)\left[\frac{\vec{n}_o\cdot\vec{n}}{|\vec{D}|^3} - \frac{3\,\vec{P}_o\cdot\vec{n}_o\,\vec{P}\;\vec{n}}{|\vec{D}|^5}\right] dc\,dr_o\,dt \quad (4.3)$$

In eq. (4.3), $\vec{D} = \vec{P}(r,t) - \vec{P}_o(r_o,\tau)$ denotes the distance between the position vectors of the downwash and doublet points relative to a fixed coordinate system, where $r$, $r_o$ are measured radially along the span; $t,\tau$ are the independent time variables of the downwash and doublet point respectively. $q(r_o,\tau)$ is the lift strength per unit length of span; $\rho$ is the density of the fluid. $\vec{n}$ and $\vec{n}_o$ are the unit normal vectors perpendicular to the surface at the downwash and doublet point, respectively.

The final downwash equation involves a double integration over the surface of the blade which has proved to be a stumbling block to the efficient application of lifting surface theories. Most of the research directed towards the application of the kernel function for the planar wing in uniform motion has been aimed at devising feasible and adequate means for accomplishing this integration. Of the several techniques available, the one selected for application to the rotor case is termed the doublet-lattice method, a principal assumption in the method is the placement of line loads at the 1/4 chord of the wing and the satisfaction of the downwash condition at the 3/4 chord location of each element. This would make eq. (4.3) tractable, namely by yielding a set of linear equations for $q_i$ . The total lift of the blade at the time $t$, or equivalently at certain azimuthal position will be $\sum q_i(t)$.

Integration over $\tau$, it is tantamount to considering all positions successively occupied by each lifting element in the past, which amounts to scanning the whole wake. The blade has the cord $c$ and length $R_t - R_s$, $R_s$ being the root of the blade, $R_t$ is the maximum length of the blade. Suppose at the time origin, the blade momentarily coincide with the coordinate system along the x-axis as shown in the sketch.



The blade executes a counterclockwise rotation with angular velocity $\Omega$ while moving with velocity $U$ along the negative x direction and $W$ along the positive z direction. Since the quarter line method has been adopted, the doublet point lies 1/4 c ahead and downwash point lies 1/4 c aft of the midchord. The positions of the doublet point as well as the downwash point can be established as follows. The Cartesian components of the doublet position are

$$x_0 = -U\tau + r_0 \cos(\Omega\tau) - c/4 \sin(\Omega\tau)\cos\alpha_0$$

$$y_0 = r_0 \sin(\Omega\tau) + c/4 \cos(\Omega\tau)\cos\alpha_0$$

$$z_0 = W\tau + (c/4) \sin \alpha_0 \tag{4.4}$$

With the substitution of $c \to -c$, $r_0 \to r$, $\tau \to t$ the position of the downwash point is given by

$$x = -Ut + r \cos(\Omega t) + c/4 \sin(\Omega t)\cos \alpha$$

$$y = r \sin(\Omega t) - c/4 \cos(\Omega t) \cos \alpha \tag{4.5}$$

$$z = Wt - c/4 \sin \alpha$$

In eqs. (4.4) and (4.5), the angle $\alpha, \alpha_0$ are the twist angle of the blade defined below

$$\tan\alpha = \frac{W}{U\sin(\Omega t) + r\Omega}$$

$$\tan\alpha_0 = \frac{W}{U\sin(\Omega t) + r_0\Omega} \qquad (4.6)$$

This means that the twist angle is determined by the velocity vector at the time $t$ (i.e., the final time) which implies that the angle of twist is constant in time, and is only a function of radial distance. Therefore, the reference plane (defined by the doublets and downwash points) is a twisted surface. Only when the doublet (hereafter denoted as d.b.) and the downwash point (hereafter denoted as d.w.) possess the same radial distance, $r = r_0$, then the d.b. and d.w. lie on a plane (i.e., $\vec{n} \,||\, \vec{n}_0$ at $\tau = t$).

From eq. (4.4) the doublet velocity can be computed, namely the time derivative of the position vectors.

$$\vec{V}_0 = \dot{x}_0\vec{i} + \dot{y}_0\vec{j} + \dot{z}_0\vec{k} \qquad (4.7)$$

where

$$\dot{x}_0 = -U - r_0\Omega\sin(\Omega\tau) - c/4\Omega\cos(\Omega\tau)\cos\alpha_0$$

$$\dot{y}_0 = r_0\Omega\cos\Omega\tau - c\Omega/4\sin(\Omega\tau)\cos\alpha_0$$

$$\dot{z} = W$$

To determine $\vec{n}_0$ as a function of $\tau$, write $\vec{V}_0 = \vec{V}_0' + \vec{W}$, where $\vec{V}_0'$ is the projection of doublet velocity on the horizontal plane. Write

$$\vec{n}_0 = \ell_0\vec{i} + m_0\vec{j} + n_0\vec{k} \qquad (4.8)$$

where $\ell_0$, $m_0$, $n_0$ are the directional cosine of the unit vector $\vec{n}_0$. Form the vector quantity $\vec{S}$ by demanding $\vec{S} = \vec{V}' \times \vec{W}$

$$\vec{S} = [(-U - r_0\Omega \sin(\Omega\tau) - c/4\Omega \cos(\Omega\tau)\cos \alpha_0)\vec{i} + (r_0\Omega \cos \Omega\tau$$

$$- c\Omega/4 \sin(\Omega\tau)\cos \alpha)\vec{j}] \times W\vec{k} \qquad (4.9)$$

$$= W(U + r_0 \Omega \sin(\Omega\tau) + c/4\Omega \cos(\Omega\tau)\cos \alpha_0)\vec{j} + W(r_0 \Omega \cos(\Omega\tau)$$

$$- c\Omega/4 \sin(\Omega\tau)\cos \alpha)\vec{i}$$

It is now required that $\vec{n}_0 \perp \vec{S}$

and $\vec{n}_0 \perp \vec{V}_0$ (4.10)

$$|\vec{n}_0| = 1$$

From (7), (8) and (9), there is obtained

$$\ell_0 = \frac{W(U + r_0\Omega \sin(\Omega\tau) + c/4\Omega \cos(\Omega\tau)\cos \alpha_0)}{\sqrt{W^2 V_0'^2 + V_0'^4}}$$

$$m_0 = - \frac{W(r_0\Omega \cos \Omega\tau - c\Omega/4 \sin \Omega\tau \cos \alpha_0}{\sqrt{W^2 + V_0'^2}} \qquad (4.11)$$

$$n_0 = \frac{V_0'}{\sqrt{W^2 + V_0'^2}}$$

where $V_0'$ is the magnitude of $\vec{V}_0'$

$$V_0'^2 = (U + r_0\Omega \sin(\Omega\tau) + c/4\Omega \cos(\Omega\tau)\cos\alpha_0)^2 + (r_0\Omega\cos(\Omega\tau) - c\Omega/4 \sin(\Omega\tau)\cos\alpha_0)^2 \quad (4.12)$$

By the same procedure, $\vec{n}$ is derived by making the appropriate substitutions, $c \to -c$, $\alpha_0 \to \alpha$, $\tau \to t$, etc. (i.e., $\vec{n} = \ell\vec{i} + m\vec{j} + n\vec{k}$) to get

$$\ell = \frac{W(U + r\Omega \sin(\Omega t) - c/4\Omega \cos(\Omega t)\cos\alpha)}{\sqrt{W^2 V'^2 + V'^4}}$$

$$m = \frac{-W(r\Omega \cos(\Omega t) + c\Omega/4 \sin(\Omega t)\cos\alpha)}{\sqrt{W^2 V'^2 + V'^4}} \quad (4.13)$$

$$n = \frac{V'}{\sqrt{W^2 + V'^2}}$$

where

$$V'^2 = (U + r\Omega \sin(\Omega t) - c/4\Omega \cos(\Omega t)\cos\alpha)^2 + (r\Omega\cos(\Omega t) + c\Omega/4 \sin(\Omega t)\cos\alpha)^2 \quad (4.14)$$

since

$$\vec{P} = x\vec{i} + y\vec{j} + z\vec{k}, \qquad \vec{P}_0 = x_0\vec{i} + y_0\vec{j} + z_0\vec{k} \quad (4.15)$$

a vector $\vec{D}$ is defined as $\vec{D} = \vec{P} - \vec{P}_0$; then the magnitude $|\vec{D}|$ in eq. (1) can be expressed as

$$|\vec{D}| = \{[U(\tau-t) + r\cos(\Omega t) - r_0\cos(\Omega\tau) + c/4(\sin(\Omega t)\cos\alpha + \sin(\Omega\tau)\cos\alpha_0)]^2$$
$$+ [r\sin(\Omega t) - r_0\sin(\Omega\tau) - c/4(\cos(\Omega t)\cos\alpha + \cos(\Omega\tau)\cos\alpha_0)]^2$$
$$+ [W(t-\tau) - c/4(\sin\alpha_0 + \sin\alpha)]^2\}^{1/2} \quad (4.16)$$

The final equation may now be written as

$$W = \frac{1}{4\pi\rho}\int_{R_0}\int_{-\infty} q(r_0,\tau)\left[\frac{\vec{n}\cdot\vec{n}_0 - 3(\vec{n}\cdot\vec{n}_D(\vec{n}_0\cdot\vec{n}_D)}{|\vec{D}|^3}\right] d\tau dr_0 \quad (4.17)$$

where $\vec{n}_D$ is a unit vector in the $\vec{D}$ direction, namely, $\vec{n}_D = \frac{\vec{D}}{|\vec{D}|}$

This integral equation was solved for the unknown $q(r_0,\tau)$ by using a collocation process based on the vortex lattice assumption (ref. 6). The kernel is singular when $D = 0$, and this was handled by use of the finite part technique (ref. 5). The remainder of the integrations was accomplished numerally.

## 5. RESULTS AND DISCUSSION

The foregoing procedure was applied to a rotor having the dimension shown in fig. 1. The rotor was divided into 5 equal spanwise sections. The vortex lattice approximation was used which resulted in the placement of a vortex at the 1/4 chord positon on each section and a downward point was located at the center of each span section and was placed at the 3/4 chord position as indicated on the figure.

The results of calculations are given in fig. 2-4 for three cases. For all calculation the rotor speed was held constant at 30 rad/sec and the vertical velocity was 20 ft/sec. On figure 2 is shown the overall lift curve slope, $C_{L_\alpha}$, for 100 ft/sec forward velocity plotted against azimuthal angle. For the range of angles from approximately 90 degrees to 270 degrees, the lift curve slope is nearly constant. However, for the position of the angular position between 315 degrees to 67.5° the lift curve slope is considerably lower. It could be surmised that with the rotor in a trailing position, that the effective aspect ratio of the rotor is lower than at other azimuthal positions and would accordingly have a lower left curve slope.

On figure 3, the forward velocity has been increased to 200 ft/sec. Note that a large increase in $C_{L_\alpha}$ is found at 225 degrees. This one point is suspect, in as much as the stall region is being approached and the theory is based on linear approximations. Note that the region close to 0 deg. azimuth $C_{L_\alpha}$, is again low, where the minimum $C_{L_\alpha}$ is 3.4 as compared to 3.9 for the 100 ft/sec case.

On figure 4, the case for a forward velocity of 300 ft/sec is shown. For this case, the rotor is definitely stalled in the azimuthal region $\theta = 270°$ to 315°.

## 6. CONCLUDING REMARKS

A lifting surface theory has been developed for a helicopter rotor in forward flight for incompressible flow. The method utilizes the concept of the linearized acceleration potential, and makes use of the vortex lattice procedure which has been successfully used for the non-rotating case.

Results are shown for three forward velocities, U = 100, 200 and 300 ft/sec. The vertical velocity was held constant at W = 20 ft/sec as well as the rotation speed, $\Omega$ = 30 rad/sec. For regions when the rotor blade was in a stalled condition the analytical results are not applicable and these regions are easily discernible in the plots. A significant finding in the results is the degradation in the lift curve slope, $C_{L_\alpha}$, in the neighborhood of $\theta$ = 0 deg. It should be pointed out that if two-dimensional coefficients were used in an analysis that this degradation in $C_{L_\alpha}$ would not be found.

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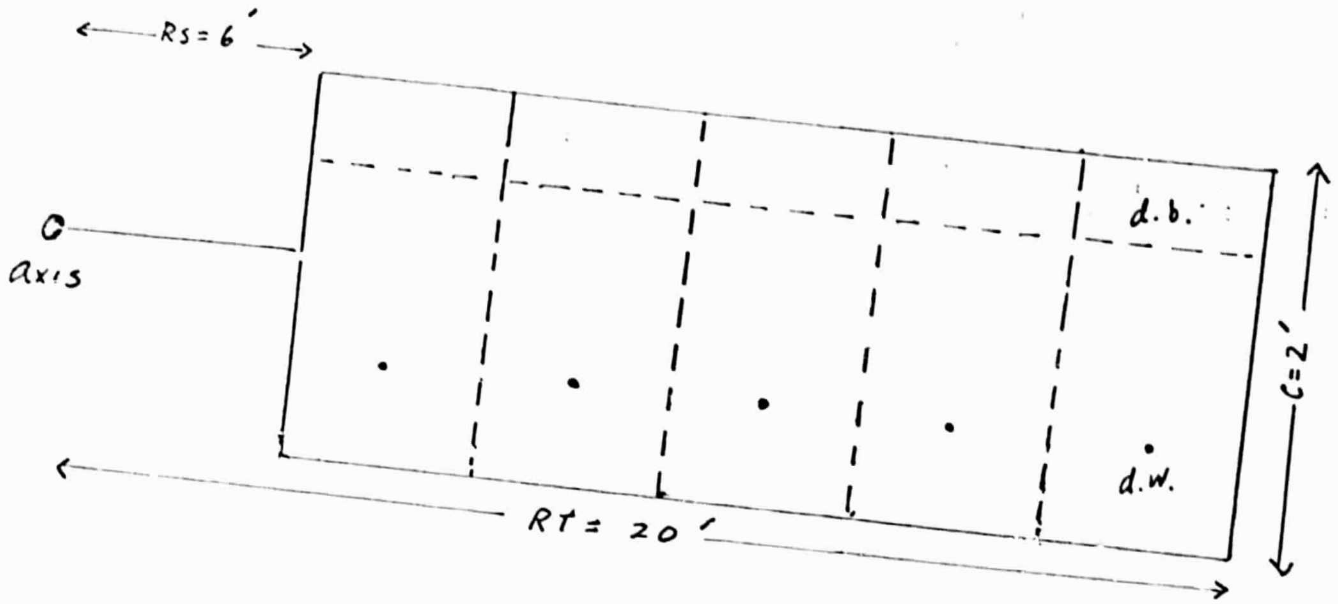


Fig. 1 Rotor Blade Dimension

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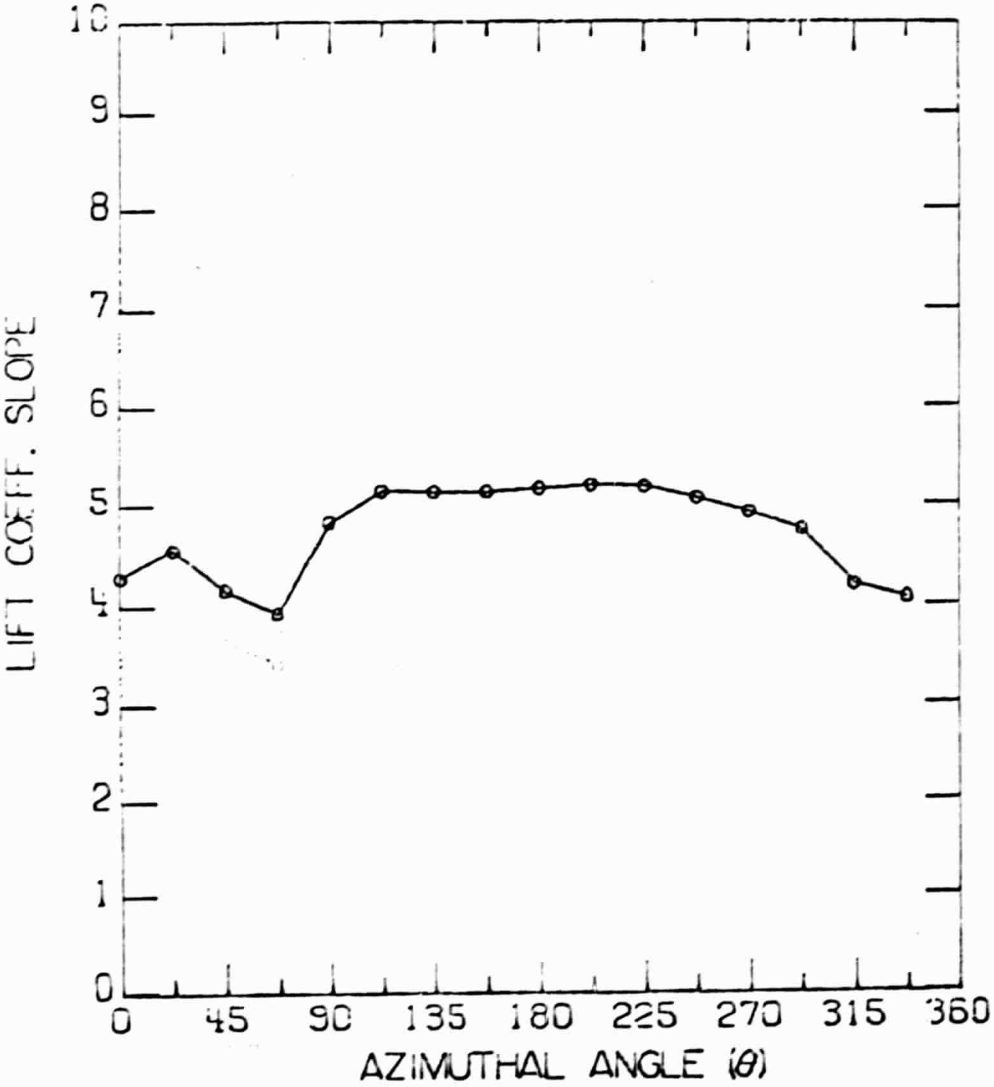


Fig. 2 Lift Coefficient Slope Versus Azimuthal Angle for
$U$ = 100 ft/sec, $W$ = 20 ft/sec, $\Omega$ = 30 rad/sec

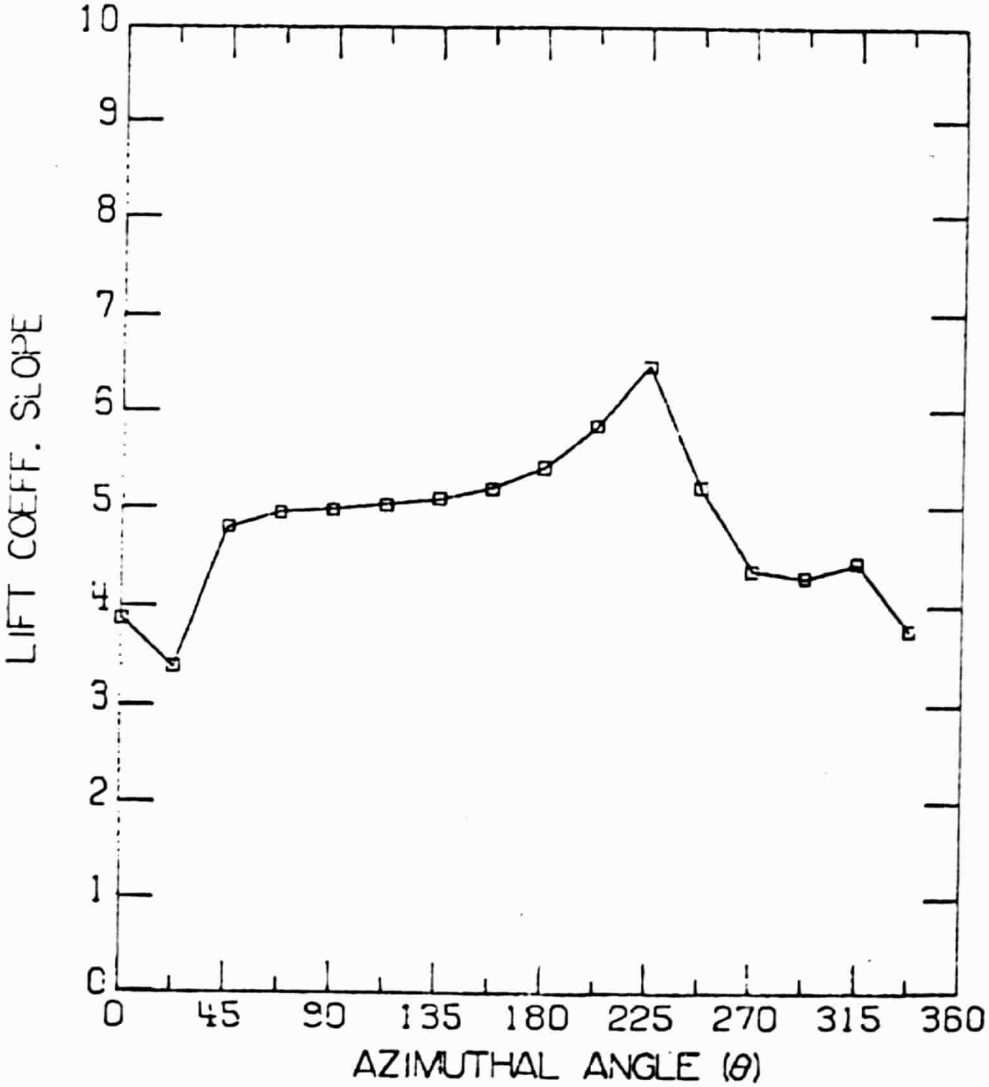


Fig. 3 Lift Coefficient Slope versus Azimuthal Angle for
U = 200 ft/sec, W = 20 ft/sec, Ω = 30 rad/sec

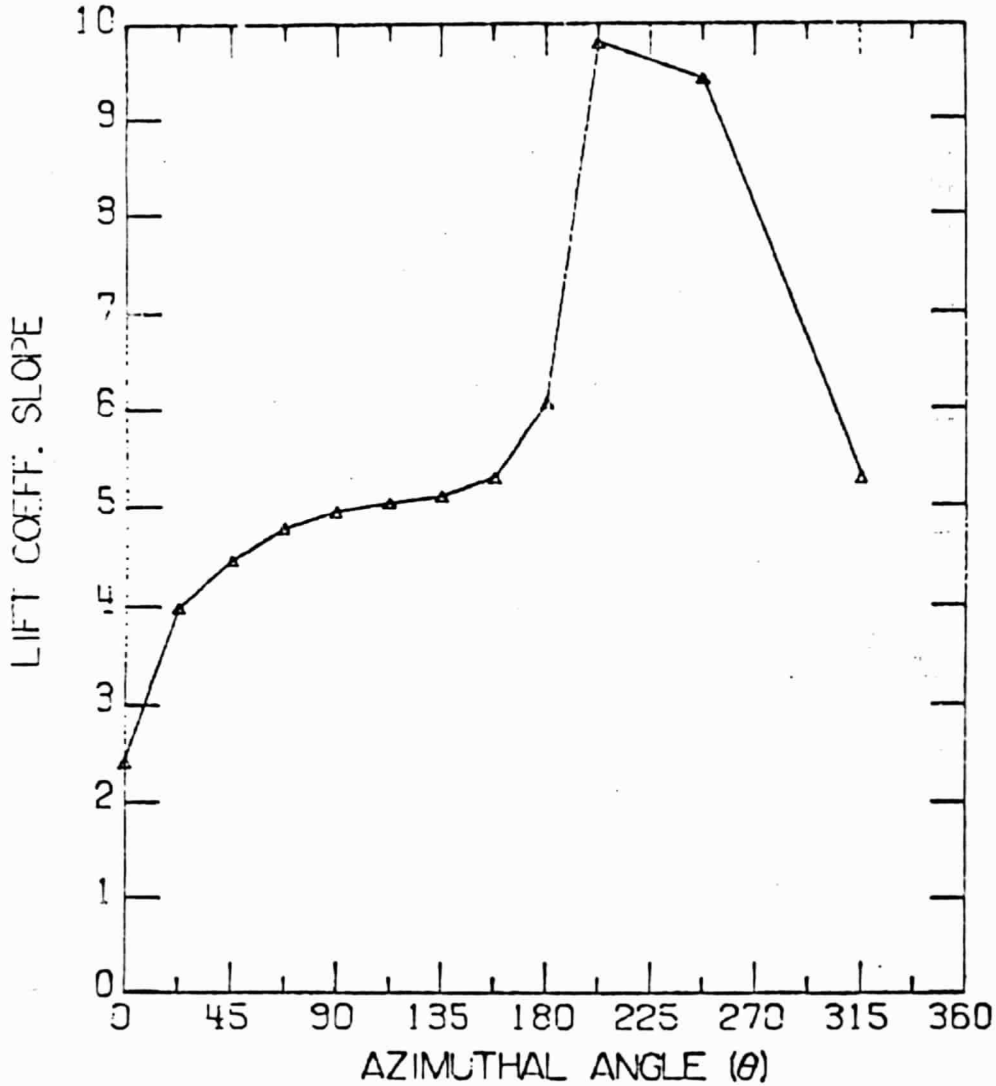


Fig. 4 Lift Coefficient Slope Versus Azimuthal Angle for
U = 300 ft/sec, W = 20 ft/sec, Ω = 30 rad/sec