Large Space Structure Damping Design

Final Report

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Walter D. Pilkey
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NASA Grant NAG-1-137-1
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER UVA/5-28201/MAE 83/101	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Large Space Structure Damping Design		5. TYPE OF REPORT & PERIOD COVERED 1/16/81-1/15/83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) W. D. Pilkey J. Kenneth Haviland		8. CONTRACT OR GRANT NUMBER(s) NASA Grant NAG-1-137-1
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Virginia Dept. of Mechanical Engineering Charlottesville, VA 22901		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE February 1983
		13. NUMBER OF PAGES 127
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Unlimited

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Minimum Constrained Frequency Criterion (MCFC)
damped system
eigenvalue problem
spill-over
damper prototype
electromagnetic actuator

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Several FORTRAN subroutines and programs were developed which compute complex eigenvalues of a damped system using different approaches, and which rescale mode shapes to unit generalized mass and make rigid bodies orthogonal to each other. An analytical proof of a Minimum Constrained Frequency Criterion (MCFC) for a single damper is presented. A method to minimize the effect of control spill-over for large space structures is proposed. The characteristic equation of an undamped system with a generalized control law is derived using reanalysis theory. This equation can be implemented in computer programs for efficient eigenvalue analysis or control gain synthesis. Methods to control vibrations in large space structures are reviewed and analyzed. The resulting prototype, using electromagnetic actuator, is described.

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I. INTRODUCTION

It is expected that large space structures will be placed into orbit in the not-too-distant future. Such structures will lack the damping forces due to ground reactions or to hydraulic or aerodynamic forces available to earth-bound structures, thus, if they are excited dynamically by docking maneuvers or by control reactions, they could be expected to continue vibrating for hours or even for days.

It is this problem which is addressed in this report. The work to be discussed divides roughly into two areas. In the first, the questions such as how much damping is required, how are the dampers to be located, and how their required performance can be specified are examined. In the second, the design of the dampers is investigated, and is supported by the demonstration of a prototype damper.

Work on damping requirements resulted in five computer programs and proofs of two criteria for the optimum location of dampers. Also, a method of minimizing "control spill over" by locating actuators was demonstrated, and the characteristic equation of an undamped system using reanalysis theory was derived.

Work on damper design included a review of possible systems. Since hydraulic and pneumatic systems were eliminated for poor reliability, only electromagnetic or piezo-electric systems were considered. Passive dampers were eliminated because they need very fine tuning. Thus, only linear or rotational dampers using active electromagnetic or piezoelectric drives were left. A prototype linear damper using a moving magnet and a fixed coil was built and demonstrated using a current feedback proportional to local structure velocity.

II. TECHNICAL DEVELOPMENTS

1. Computer Programs

Several FORTRAN subroutines and programs were developed under this grant. These are documented in Appendix A. The capabilities of the programs are enumerated below.

Subroutines ASMD1 and ASMD2 compute the complex eigenvalues of a damped system using the undamped modes of the structure as the assumed modes (assumed mode method). In subroutine ASMD1, a diagonal damping matrix must be used, whereas in ASMD2, a general damping matrix may be used.

Subroutine SPSTGN solves the damped eigenvalue problem using a reanalysis approach. It assumes a diagonal damping matrix.

Program COPZ computes the optimal (i.e., minimum) gains of a diagonal damping matrix for specified damping ratios.

Program NORMAL is designed to rescale the mode shapes to unit generalized mass and to make the rigid bodies orthogonal to each other with respect to the mass matrix.

These programs are stored in the Langley computer.

2. Proof of Optimal Damper Location

In previous papers [Ref. 1 and 2] we have proposed two criteria for the optimal location of a damper. They are the Minimum Constrained Frequency Criterion (MCFC) for a single damper [Ref. 1] and the Maximum Frequency Separation Criterion (MFSC) for multiple dampers [Ref. 2]:

MCFC: The optimal damper location is where the constrained frequency a_ℓ is a minimum.

MFSC: The optimal locations of dampers are where the constrained frequency associated with the damper location has the largest separation from the corresponding undamped natural frequency of the system.

In Ref. 1 and 2, these criteria are demonstrated using the Langley beam and grillage models. These demonstrations show that in choosing between sets of positions of dampers, the one that gives the best results is the one that gives the greatest separation between the undamped natural frequency and the corresponding constrained frequency. In this section, an analytical proof of MCFC for a single damper will be presented. The proof of the more general MFSC is still under investigation.

Proof of MCFC

The proof of MCFC is based on the existence of fixed points in the frequency response curve when a single damper is introduced into an n-dof undamped system. This is an extension of the classical damped vibration absorber theory [Ref. 3], in which Den Hartog showed that when a damped vibration absorber is attached to a undamped SDOF main system, there are points on the frequency response curve which are independent of damping. These points are the fixed points [Ref. 3]. In this section, we will first show the existence of fixed points when a single damper is introduced into an undamped system. The responses at the fixed points can then be evaluated from the undamped equation of motion. The MCFC can then be proved using the characteristics of the resonance curve of the undamped system near resonance.

Consider the case of an n-dof undamped system under sinusoidal excitation

$$[M]\{x\} + [K]\{x\} = \{F\}e^{j\omega t} \tag{1}$$

where [M], [K] are mass and stiffness matrices and {x} is the displacement vector and {F} is the magnitude vector of the forcing function. When a single damper at dof J is introduced into this system, the equation of motion becomes

$$[M]\{x\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\}e^{j\omega t} \tag{2}$$

where

$$[C] = c\{e_J\}\{e_J\}^T \tag{3}$$

$$\{e_J\}^T = [0 \; \ldots \; 0 \;\; \underset{\substack{\uparrow \\ J\text{th component}}}{1} \;\; 0 \; \ldots \; 0]$$

Let the steady state solution of (2) be

$$\{x\} = \{X\}e^{j\omega t} \tag{4}$$

Then, use (3) and (4) the steady state response {X} can be computed from

$$[Z(\omega)]\{X\} = \{F\} - c\omega j\{e_J\}\{e_J\}^T\{X\} \quad (5)$$

where

$$[Z(\omega)] = -\omega^2[M] + [K]$$

Define

$$[R(\omega)] = [Z(\omega)]^{-1} \quad (6)$$

and note that

$$\{e_J\}^T\{x\} = x_J \quad (7)$$

$$[R]\{e_J\} = \{R_J\}$$

where $\{R_J\}$ is the Jth column of [R]. Premultiply (5) by [R] and use (7) to obtain

$$\{X\} = \{X_o\} - j\omega c\ R_J\ X_J \quad (8)$$

Note that

$$\{X_o\} = [R(\omega)]\{F\}$$
= steady state solution of the undamped system.

The Jth equation of (8) is

$$X_J = X_{oJ} - j\omega c\ R_{JJ}\ X_J$$

or

$$X_J = \frac{X_{oJ}}{1 + j\omega c\ R_{JJ}} \quad (9)$$

Equation (8) together with (9) are the general frequency response reanalysis equations when a damper c is introduced at dof J. It is interesting to note that at the frequency a_ℓ at which

$$R_{JJ}(a_\ell) = 0 \quad (9a)$$

the response X_J is independent of damper gain c. Thus, we have shown the existence of fixed points for the response curve $X_J(\omega)$. Furthermore, those frequencies are the antiresonant frequencies for dof J. A typical frequency response curve for X_J (or $\omega^2 X_J$, the acceleration response) is shown in Fig. 2.1. In Fig. 2.1, point A is the fixed point corresponding to the smallest root of Eq. (9a). By definition of fixed point, all response curves pass through A

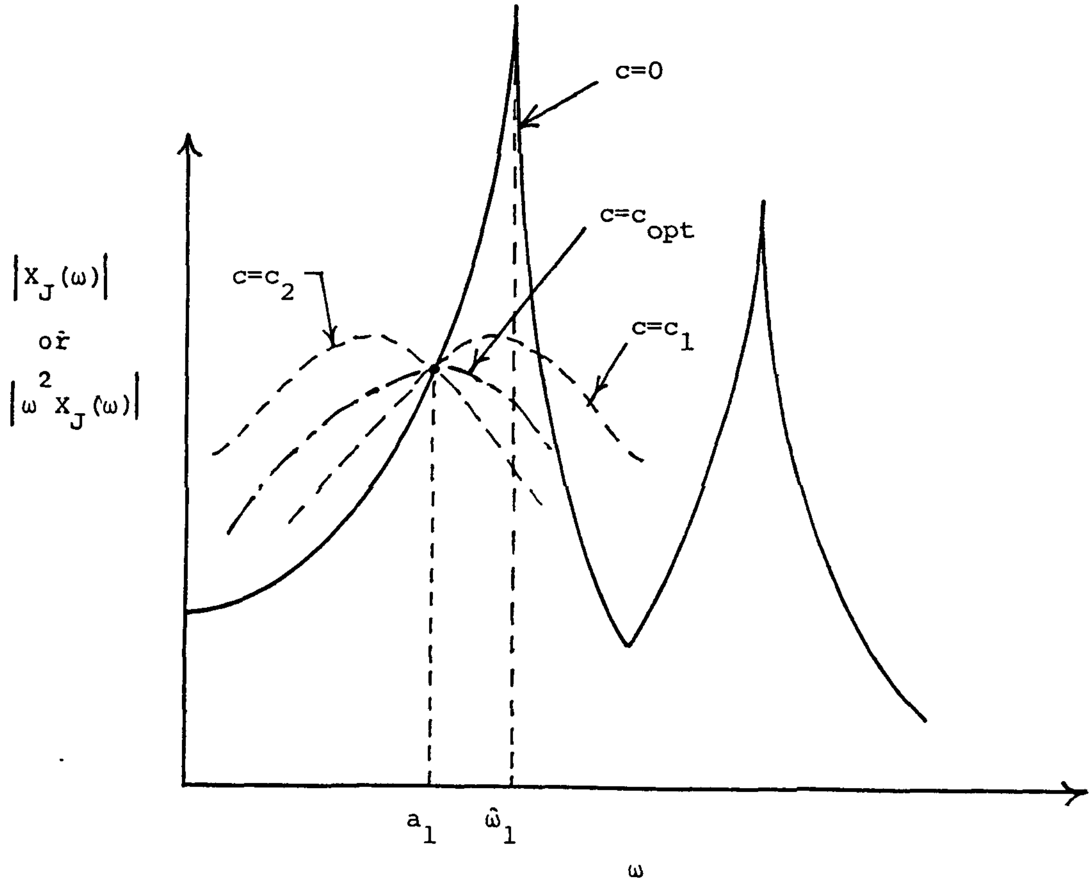

Fig. 2.1 Frequency Response Curve for $|X_J|$ Showing fixed point A. a_1 is the frequency corresponding to point A.

regardless of the damper gain c. Thus, it is obvious the optimal c (in terms of the smallest response) should produce a response curve that peaks at point A. In other words, the optimal response curve has a slope zero at the fixed point A.

From the above discussion, we know that once a damper location is selected, the minimum response X_J is determined by the undamped response at the fixed point frequency. Since near resonance, the undamped response is monotonically increasing as the frequency is approaching the undamped natural frequency from below, (see Fig. 2.2), we conclude that the optimal location for lowest amplification for a particular mode with undamped natural frequency ω_n is where $(\omega_n - a_\ell)$ is a maximum.

This proves MCFC for minimum frequency response. Our original MCFC pertains to the modal damping ratio. Since modal damping is closely related to the amplification factor we conclude that MCFC will produce a design with maximum damping ratio.

Discussion

(1) In general, there are (n-1) fixed points for response X_J, where n is the number of elastic modes of the system.
(2) The responses X_J $(I \neq J)$ exhibits fixed points at frequencies where

$$|X_I|_{c=0} = |X_I|_{c=\infty} \tag{10}$$

or from (8) and (9), condition (10) becomes

$$|X_{OI}| = |X_{OI} - R_{IJ}(\omega)X_{OJ}| \tag{11}$$

which corresponds to the following two equations

$$X_{OI} = X_{OI} - R_{IJ}(\omega)X_{OJ} \tag{12a}$$

and

$$X_{OI} = -X_{OI} + R_{IJ}(\omega)X_{OJ} \tag{12b}$$

(3) Our proof only pertains to X_J, however, experience showed that once $X_J(\omega)$ is minimized, the other $X_I(\omega)$'s are almost optimal.
(4) The proof cannot be extended to multiple dampers since no fixed points exist for the multiple damper case.

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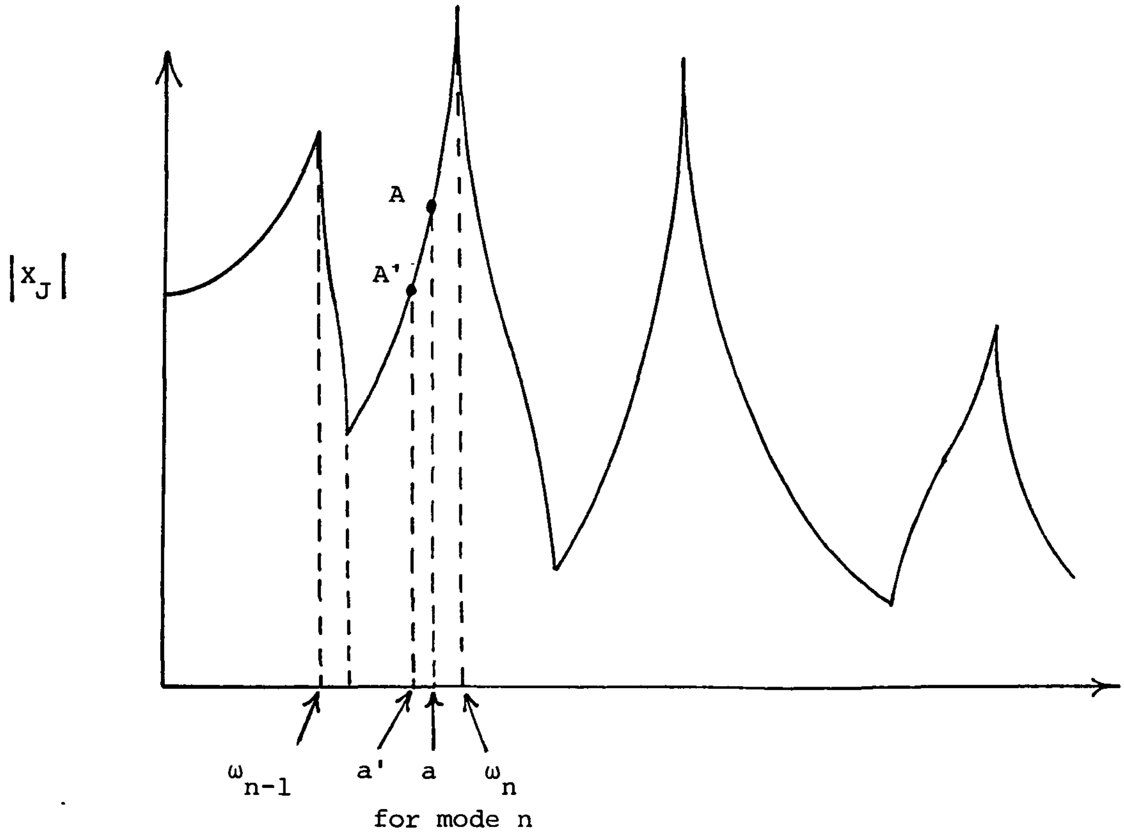

Fig. 2.2 Frequency response of undamped n-dof system. $|X_J(\omega)|$ increases monotonically as $\omega \to \omega_n$.

3. Locating Actuators for Minimum Spill-Over

A frequently discussed problem in the control of LSS is that of "control spill-over". A method of minimizing the effect of control spill-over is proposed in this section. It is shown that to control n_c modes using n_a actuators, the spill over effect can be minimized by insuring that n_s secondary modes are not excited. This requires $n_a > n_s$. The method described allows only $n' = n_a - n_s$ of the n_a actuators to be controlled independently. Conditions of actuator placement is also indicated.

Consider a proportionally damped system whose finite element model can be written as

$$\underline{M}\,\underline{x} + \underline{C}\,\dot{\underline{x}} + \underline{K}\,\underline{x} = \underline{B}\,\underline{u} \tag{13}$$

where $\underline{M}$, $\underline{C}$, and $\underline{K}$ are mass, damping, and stiffness matrices of the system respectively, and $\underline{x}$ is the system displacement vector, $\underline{u}$ is the control vector and $\underline{B}$ is the actuator influence matrix.

In modal coordinates, (13) becomes

$$\ddot{\underline{q}} + 2\underline{E}\,\underline{\omega}\,\dot{\underline{q}} + \underline{\omega}^2\,\underline{q} = \underline{\Phi}^T\,\underline{B}\,\underline{u} \tag{14}$$

where Φ is the modal matrix and is the solution of the undamped eigenvalue problem

$$\omega_i^{\,2} M\,\rho_i = K\,\rho_i \tag{15}$$

$$E = \Phi^T\,C\,\underline{\Phi} = \begin{bmatrix} \zeta_1 & & 0 \\ & \ddots & \\ 0 & & \zeta_n \end{bmatrix}$$

$$\underline{\omega}^2 = \begin{bmatrix} \omega_1^{\,2} & & & \\ & \omega_2^{\,2} & & \\ & & \ddots & \\ & & & \omega_n^{\,2} \end{bmatrix}$$

In general, only a few of the important modes are controlled. Designate these modes as $\underline{q}_c$ and the remaining modes as $\underline{q}_r$, the residue modes. Then Eq. (14) can be written as

$$\ddot{q}_c + 2E_c\,\omega_c\,\dot{q}_c + \omega_c^2\,q_c = \Phi_c^T\,B\,u \qquad (16)$$

and

$$\ddot{q}_r + 2E_r\,\omega_r\,\dot{q}_r + \omega_r^2\,q_r = \Phi_r^T\,B\,u \qquad (17)$$

Now the physical response is given by

$$\underline{x} = \Phi_c q_c + \Phi_r q_r \qquad (18)$$

Hence, if $q_r \neq 0$, the physical response is influenced by the uncontrolled mode. This effect is called control spill-over. Examination of (17) shows that $q_r \neq 0$ if $\Phi_1^T\,B\,u \neq 0$.

It would appear then if we choose B such that

$$\underline{\Phi}_r\underline{B} = \underline{0} \qquad (19)$$

the control spill over problem is solved. Unfortunately, condition (19) can not be met in general.

In the following, a simple technique of reducing the control spill-over effect is described. Partition the residue modes into a set of secondary modes ρ_s and higher modes, with the higher modes having negligible effects on the system. In this way, we have

$$\underline{x} = \underline{\Phi}_c\underline{q}_c + \underline{\Phi}_s\underline{q}_s \qquad (20)$$

Now, to minimize control spill-over, minimize $\underline{q}_s$, or ultimately make $\underline{q}_s = 0$. The response q_s can be solved from

$$\ddot{q}_s + 2E_s\omega_s\dot{q}_s + \omega_s^2\,q_s = \Phi_s^T\,B\,u \qquad (21)$$

Now if

$$\Phi_s^T B\,u = 0 \qquad (22)$$

then

$$q_s = 0$$

Rewrite (22) as

$$\underline{A}\,\underline{u} = 0 \quad \text{where} \quad A = \Phi_s^T B \tag{23}$$

Assume $n_s < n_a$, then partition $\underline{A}$ and $\underline{u}$ into

$$\underline{A} = [\underline{A}_i \; \vdots \; A_d], \text{ with } \underline{A}_d \text{ nonsingular}$$

$$\underline{u} = \begin{bmatrix} u_i \\ u_d \end{bmatrix} \tag{24}$$

Using (24), (23) becomes

$$A_i u_i + A_d u_d = 0 \tag{25}$$

From (25), solve for

$$\underline{u}_d = -\underline{A}_d^{-1} \underline{A}_i \, \underline{u}_i \tag{26}$$

Thus, if

$$\underline{u} = \begin{bmatrix} \underline{u}_i \\ \underline{u}_d \end{bmatrix} = \begin{bmatrix} I \\ -A_d^{-1} A_i \end{bmatrix} \underline{u}_i \tag{27}$$

Then the condition (22) will be met. Check

$$\Phi_s^T \underline{B}\, \underline{u} = \underline{A}\, \underline{u} = [A_i \quad A_d] \begin{bmatrix} I \\ -A_d^{-1} A_i \end{bmatrix} \underline{u}_i$$

$$= (A_i - A_i) u_i = 0$$

Equation (25) implies that out of the n_a actuators, only $n' = n_a - n_s$ of them can be controlled independently. Define

$$\underline{D} = \begin{bmatrix} I \\ -A_d^{-1} A_i \end{bmatrix} \tag{28}$$

then

$$\underline{u} = \underline{D}\, \underline{u}_i \tag{29}$$

Substitute (29) into (16) to get

$$\ddot{q}_c + 2E_c\omega_c\dot{q}_c + \omega_c^2 q_c = \underset{n_c x n}{\underline{\Phi}_c^T} \ \underset{n x n_a}{\underline{B}} \ \underset{n_a x n'}{\underline{D}} \ \underset{n' x 1}{\underline{u}_i} \tag{30}$$

The design problem is then to choose $\underline{u}_i$ to control $\underline{q}_c$ in

$$q_c + 2E_c\omega_c \dot{q}_c + \omega_c^2 q_c = \underline{\Phi}_c^T \underline{B}' \underline{u}' \tag{31}$$

where

$$\underline{B}' = \underline{B}\ \underline{D} = \underline{B} \begin{bmatrix} I \\ -A_d^{-1} A_i \end{bmatrix}$$

$$\underline{u} = \underline{u}_i$$

$$\underline{A} = [\underline{\Phi}_s^T B]$$

Note that the placement on actuator should be such that one can find a non-singular $n_s x n_s$ matrix $\underline{A}_d$ from the matrix $\underline{A}$.

4. Characteristic Equations for Undamped System with Generalized Control Law

The characteristic equation of an undamped system with a generalized control law is derived using reanalysis theory in this section. This equation can be implemented in the computer programs for efficient eigenvalue analysis or control gain synthesis.

Consider an undamped n-dof system with n_a-actuators, the equation of motion can be written as

$$\underline{M}\ \underline{x} + \underline{K}\ \underline{x} = \underline{B}\ \underline{u} \tag{32}$$

where $\underline{M}$, $\underline{K}$ are the nxn mass and stiffness matrices respectively, $\underline{u}$ is a $n_a x1$ vector, i.e.,

$$\underline{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_{n_a} \end{bmatrix}$$

$\underline{B}$ is the nxn_a Boolean distribution matrix with n_a nonzero rows. We will designate $\underline{X}_a$ and $\underline{X}_s$ as the dof locations for actuator and sensors respectively. Now define

$$\underline{X}_a = \underline{B}_a^T \underline{x} = n_a x1 \text{ vector}$$

$$\underline{X}_s = \underline{B}_s^T \underline{x} = n_s x1 \text{ vector} \tag{33}$$

then we conclude that

$$\underline{B} = \underline{B}_a \tag{34}$$

Furthermore, assume $\underline{X}_a$ is contained in $\underline{X}_s$, and partition $\underline{X}_s$ as

$$\underline{x}_s = \begin{bmatrix} \underline{x}_a \\ \underline{x}_r \end{bmatrix} \tag{35}$$

Now, the system dof can be partitioned either as

$$\underline{x} = \begin{bmatrix} \underline{x}_a \\ \underline{x}_r \end{bmatrix} \quad \text{or} \quad \underline{x} = \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \\ \underline{x}_c \end{bmatrix}$$

Note that

$$\underline{x}_r = \begin{bmatrix} \underline{x}_b \\ \underline{x}_c \end{bmatrix}$$

Also note that

$$n = n_a + n_r = n_a + n_b + n_c$$

Now, express

$$\underline{x}_s = \underline{B}_a \underline{x} + \underline{B}_b \underline{x} \tag{36}$$

Assume the control law is

$$\underline{u} = -(\underline{c}_a \dot{\underline{x}}_a + \underline{c}_b \dot{\underline{x}}_b + \underline{k}_a \underline{x}_a + \underline{k}_b \underline{x}_b) \tag{37}$$

Substitute (6) into (1), so that for the closed loop system

$$\underline{M}\,\underline{x} + \underline{K}\,\underline{x} = -\underset{n \times n_a}{\underline{B}_a} \, (\underset{n_a \times n_b}{\underline{c}_a} \dot{\underline{x}}_a + \underset{n_a \times n_b}{\underline{c}_b} \dot{\underline{x}}_b + \underset{n_a \times n_b}{\underline{k}_a} \underline{x} + \underset{n_a \times n_b}{\underline{k}_b} \underline{x}_b) \tag{38}$$

or, assuming

$$\underline{x} = \underline{X}e^{st} \tag{39}$$

and letting $\underline{R} = (s^2M + K)^{-1}$ equation (38) becomes:

$$\underline{x} = -\underline{R}\,\underline{B}_a(s\underline{c}_a\,\underline{x}_a + s\underline{c}_b\,\underline{x}_b + \underline{k}_a\,\underline{x}_a + \underline{k}_b\,\underline{x}_b)$$

or

$$\underline{x} = -\underline{R}\,\underline{B}_a[(s\underline{c}_a + \underline{k}_a)\underline{x}_a + (s\underline{c}_b + \underline{k}_b)\underline{x}_b] \tag{40}$$

and finally

$$\underline{x} = -\underline{R}\,\underline{B}_a[(s\underline{c}_a + \underline{k}_a) \quad (s\underline{c}_b + \underline{k}_b)] \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix}$$

Now, premultiply (40) by $\underline{B}_a^T$ to produce (41) and premultiply (40) by $\underline{B}_b^T$ to produce (42).

$$\underline{B}_a^T\underline{x} = -\underline{B}_a^T\underline{R}\,\underline{B}_a\,[\underline{H}_a \quad \underline{H}_b] \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix} \tag{41}$$

$$\underline{B}_b^T\underline{x} = -\underline{B}_b^T\underline{R}\,\underline{B}_a\,[\underline{H}_a \quad \underline{H}_b] \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix} \tag{42}$$

Use the definition

$$\underline{x}_a = \underline{B}_a^T\underline{x}$$

$$\underline{x}_b = \underline{B}_b^T\,\underline{x}$$

$$\underline{H} = [\underline{H}_a \quad \underline{H}_c] = [s\underline{c}_a + \underline{k}_a \quad s\underline{c}_B + \underline{k}_b]$$

$$\hat{\underline{x}} = \begin{bmatrix} \underline{x}_a \\ \underline{x}_b \end{bmatrix}$$

Equations (41) and (42) become

$$\underset{n_a x 1}{\underline{x}_a} = -\underset{n_a x n_a}{\underline{R}_{ac}} \; \underset{n_a x n'}{\underline{H}} \; \underset{n' x 1}{\hat{\underline{x}}} \qquad n' = n_a + n_b$$

$$\underset{n_b x 1}{\underline{x}_b} = -\underset{n_b x n_b}{\underline{R}_{bc}} \; \underset{n_b x n'}{\underline{H}} \; \underset{n' x 1}{\hat{\underline{x}}} \tag{43}$$

where

$\underline{R}_{aa}$ = portions of $\underline{R}$ associated with $\underline{x}_a$ only

$\underline{R}_{ba}$ = portion of $\underline{R}$ associated with the coupling between $\underline{x}_a$ and $\underline{x}_b$

Rewrite (43) as

$$\begin{bmatrix} x_a \\ x_b \end{bmatrix} = \begin{bmatrix} -R_{aa} & H \\ -R_{ba} & H \end{bmatrix} \hat{\underline{x}}$$

or

$$\hat{\underline{x}} = \begin{bmatrix} -R_{aa} & H \\ -R_{ba} & H \end{bmatrix} \hat{\underline{x}}$$

$$\det[I + \begin{bmatrix} R_{aa} & \underline{H} \\ R_{bb} & \underline{H} \end{bmatrix}] = 0$$

thus

$$\det \begin{bmatrix} I + R_{aa}\underline{H}_a & R_{aa}\underline{H}_b \\ R_{ba}\underline{H}_a & [I + R_{ba}\underline{H}_b] \end{bmatrix} = 0 \tag{44}$$

is the characteristic equation of the closed loop system where

$$\underline{H}_a = s\underline{c}_a + \underline{k}_a \qquad \underline{H}_b = s\underline{c}_B + \underline{k}_b$$

Example

For the 2-dof system shown in Figure (4.1)

$$u_1 = -(c_a\dot{x}_1 + c_b\dot{x}_2)$$

$$H_a = sc_a \qquad H_b = sc_b \qquad \underline{x}_a = x_1 \qquad \underline{x}_b = x_2$$

$$R_{aa} = R_{11} \qquad R_{bb} = R_{22} \qquad R_{ba} = R_{21}$$

Thus, Eq. (14) becomes

$$\det \begin{bmatrix} 1 + sc_a R_{11} & sc_b R_{11} \\ sc_a R_{21} & sc_b R_{21} \end{bmatrix} = 0$$

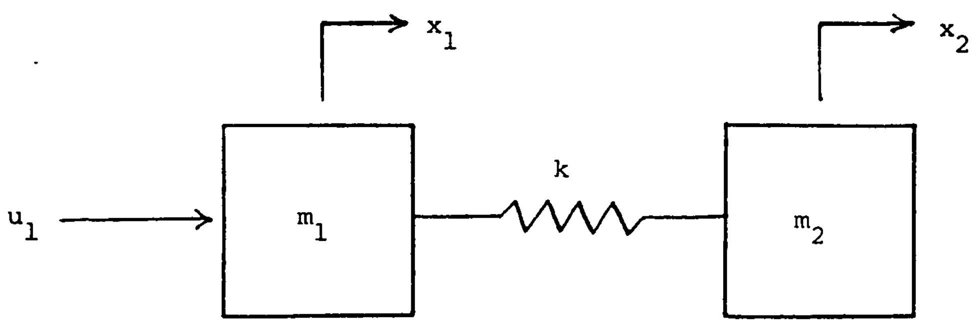

Fig. 4.1

5. Damper Design

Possible methods of controlling vibrations on large space structures have been reviewed. Two basic approaches are identified:

1. Modification of the control system so that the poles are well damped. This approach is based on the assumption that the major, if not only, cause of vibrations is the control system itself.
2. Addition of a separate damping system, which might be:
   2.1 Passive damping
   2.2 Active damping

The working medium of a ground based damper of either kind might be electromagnetic, but it would more probably be hydraulic or pneumatic, or, in the case of passive systems, magnetic or frictional. For space applications, hydraulic, pneumatic, and frictional systems must be ruled out as too unreliable in the space environment when little or no maintenance is possible. This leaves electromagnetic, magnetic, and possibly new systems based on the piezoelectric or other effects.

A list of possible systems is given in Table 5.1. The active damping systems using electromagnetics are assumed to incorporate coils moving in the fields of permanent magnets. The current in a coil would be provided by an electrical circuit in response to the output from one or more sensors. For the purpose of this study, such a circuit is assumed to introduce a negligible weight penalty, whereas the permanent magnet is considered to be a significant weight item. Similarly, the permanent magnet in the eddy current damper of a passive system is also considered to be a significant weight item. For systems which require a moving mass, such as the seismic mass of a passive system, or the mass in an active inertial system, it would therefore seem to be advantageous to combine the roles of moving mass and magnet in order to save weight. On this basis, the gyro concept might have a considerable weight disadvantage, because it would be impossible to combine the magnet and the flywheel.

Table 5.1. Possible Damping Systems

1.0	Modification of Control System
2.0	Damping System
2.1	Passive Damping: Magnet and conductive strip
2.1.1	Seismic mass with stationary magnet
2.1.2	Magnet as seismic mass
2.1.3	Gyro
2.1.4	Two-force member
2.2	Active Damping: Electromagnetic
2.2.1	Separate mass and magnet
2.2.1.1	Linear
2.2.1.2	Rotating (inertia wheel)
2.2.2	Moving magnet
2.2.2.1	Linkage system
2.2.2.2	Rack and pinion
2.2.3	Gyro
2.2.4	Two-force member
2.3	Active Damping: Piezoelectric

In comparing active against passive damping, the relative disadvantage of requiring an electrical system for the former must be weighed against the need to tune the seismic mass to a given frequency, so that the latter is essentially a narrow band device, whereas the active damper is relatively broadband.

The systems mentioned in Table 5.1 are discussed in more detail below.

Modification of Control System (1.0)

Modern control theory prescribes methods for designing control circuits having poles in any desired location, thus active damping can be achieved directly at the major source of disturbances. A very simple example is illustrated in Figure (5.1) which shows a one-dimensional system consisting of two masses connected by a spring, having one rigid-body translational mode, and one vibrational mode. Suppose that the impulse from a thruster is applied at one of the two masses. The vibrational mode will be excited and the system will move off with one mass coming to rest every half cycle. Now suppose that there are two thruster impulses applied one half-period apart, then the system will move without any internal vibration after the first half-period. Applied on a very much larger and more complex scale, such an approach could be used to maneuver a large space structure without any residual vibrations of serious amplitude.

Passive Damping (2.1)

The only reliable method of passive damping appears to be through the use of a magnet and a conductive strip. When the strip is passed between the poles of a magnet, eddy currents are induced in a direction normal to the velocity and to the magnetic field, and the effectiveness of such a damper is directly proportional to the product of electrical conductivity, magnetic field strength, and pole area. Unfortunately, the damping forces so produced are relatively small at frequencies of interest.

Four possible systems are shown in Figures (5.2) to (5.5). The first, system 2.1.1 in Figure (5.2), is a seismic damper, while for system 2.1.2 in Figure (5.3) the magnet doubles as the seismic mass. For system 2.1.3 in Figure (5.4), the magnetic damper is used in conjunction with a gyro to provide damping against angular motion of the structure. Finally, for system 2.1.4 in Figure (5.5), the magnetic damper is used in a two-force member spanning part of the large structure.

Active Damping: Electromagnetic (2.2)

In the active damping system, energy is removed from the structure by the action of an electromagnetic actuator or an electric motor. For the purpose of illustration, a voice-coil shaker of the type used for vibration testing has been assumed for the analyses of the active systems in the appendices. Such shakers are relatively heavy, because they contain powerful permanent magnets, although careful redesign for space applications might result in considerable

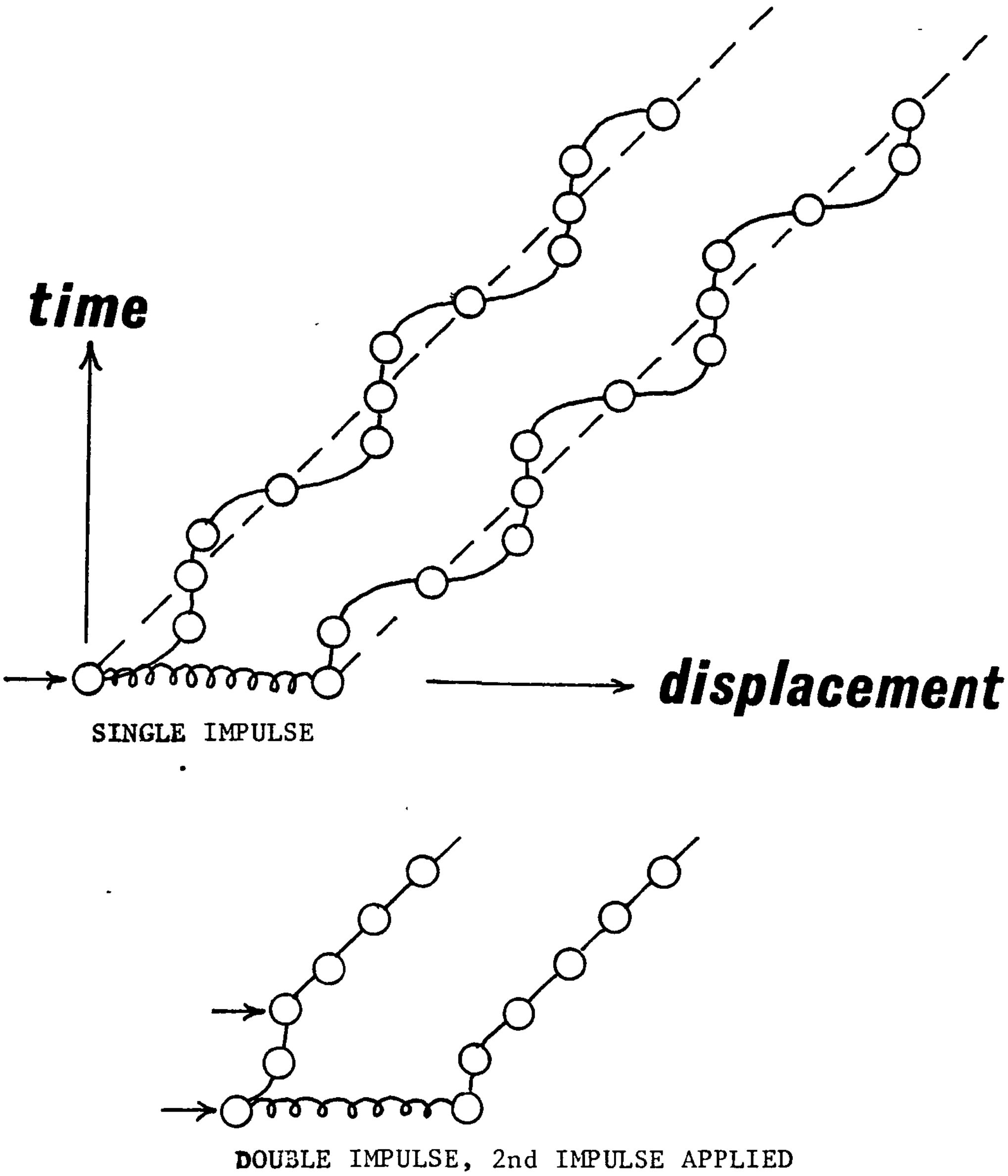

Fig. (5.1) 1.0 Modification of Control System

Figs. (5.2)-(5.5) Passive Damping Systems

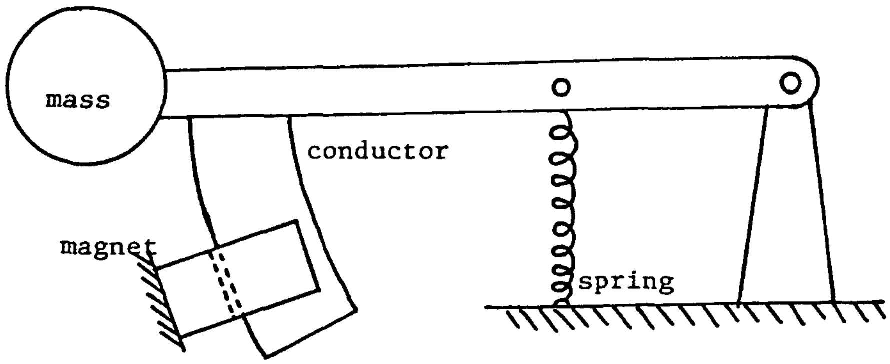

Fig. (5.2) Seismic Mass, Stationary Mass (2.1.1)

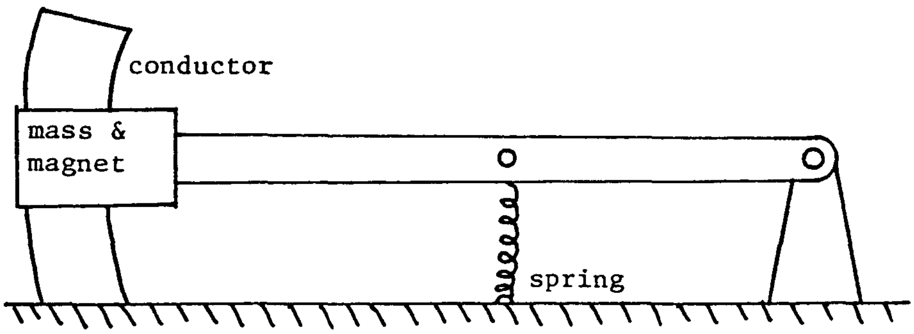

Fig. (5.3) (2.1.2) Magnet as Seismic Mass

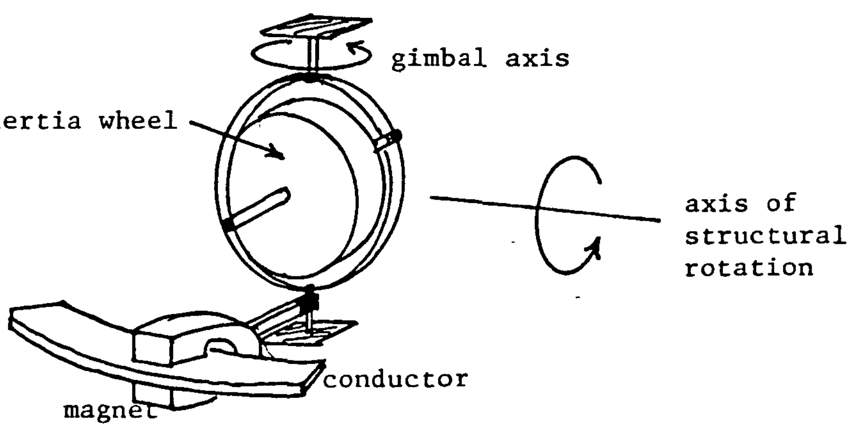

Fig. (5.4) Gyro System

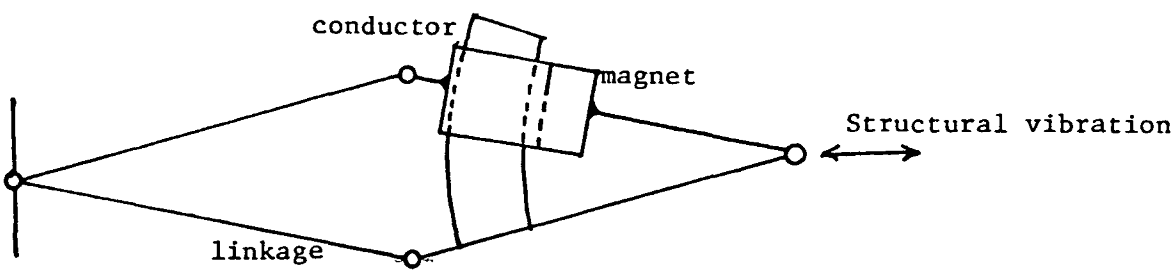

Fig. (5.5) (2.1.4) Two-Force Member

weight reductions. However, the weight penalty is directly proportional to the maximum energy which can be removed per cycle, as is shown in Appendix B. The design problem with a damper of this kind is that of ensuring that this maximum energy reduction per cycle is in fact available.

The system would be driven through a current amplifier from the difference between two integrated accelerometer signals, one from the structure, and one from the moving mass. Thus a force would be applied to the structure which would be directly proportional to the local velocity of the structure.

The major problem to be faced is how to provide an appropriate reaction on the structure so that as much energy is removed per cycle as is possible. One approach, shown as system 2.2.1.1 in Figure (5.6), employs a moving mass. As is shown in Appendix B, this system can be designed for peak performance at a given frequency. Above this frequency, performance is limited by the maximum available shaker force, while below this frequency, performance is limited by the maximum displacement available. An analogous rotarysystem, 2.2.1.2, shown in Figure (5.7) uses an electric motor driving an inertia wheel.

System 2.2.2.1 in Figure (5.8) is an improvement on the previous system, in which the shaker magnet doubles as the moving mass. A second version is shown in Figure (5.9). In Figure (5.8), the actuator motion is amplified by a "lazy tong" linkage. Such a linkage would not be practical, but could be used in a laboratory demonstration. In the second system, 2.2.2.2., Figure (5.9). the shaker is replaced by an electric motor which drives itself up and down a shaft with a rack-and-pinion gear.

System 2.2.3 shown in Figure (5.10) is similar to system 2.1.3 shown in Figure (5.4) except that the passive damper has been replaced by an active electromagnetic damper, while system 2.2.4 shown in Figure (5.11) is a two force member corresponding in a similar way with system 2.1.4 in Figure (5.5).

Active Damping: Piezoelectric (2.3)

Acoustical transducers have been made from polyvinylidene fluoride (PVF_2) sheet, aluminized on both sides. The sheet is polarized. so that when a voltage differential is applied between the aluminized coatings. the material strains in one direction. Conversely, when it is strained, a voltage differential is induced. If the material is bonded to the surface of a structural element, a surface shearing force can be induced which will load the structure, and, if properly controlled, is capable of introducing damping. Proper use of such a material evidently depends on new approaches to structural dynamic analysis, but is certainly worthy of consideration.

Electronics

The active systems would be driven by feedback circuits. Such a circuit is shown in Figure (5.12), which repeats system 2.2.1.2 of Figure (5.8) with the addition of a circuit which compares md^2y/dt^2 and

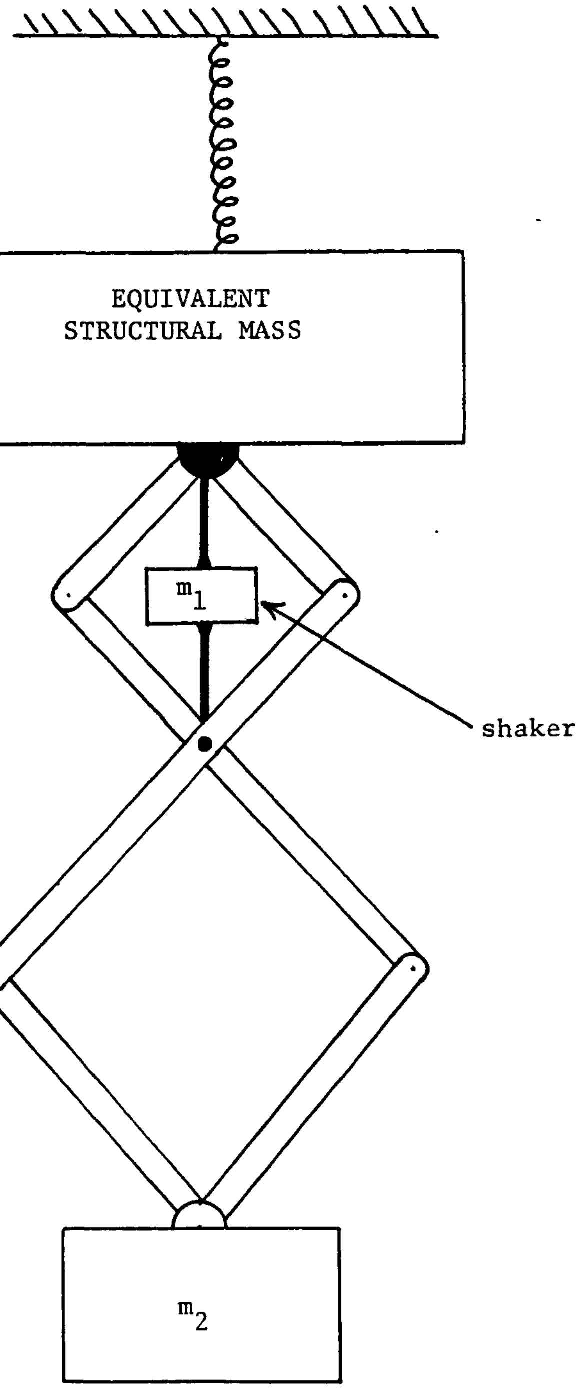

Fig. (5.6) System (2.2.1) Active Damper

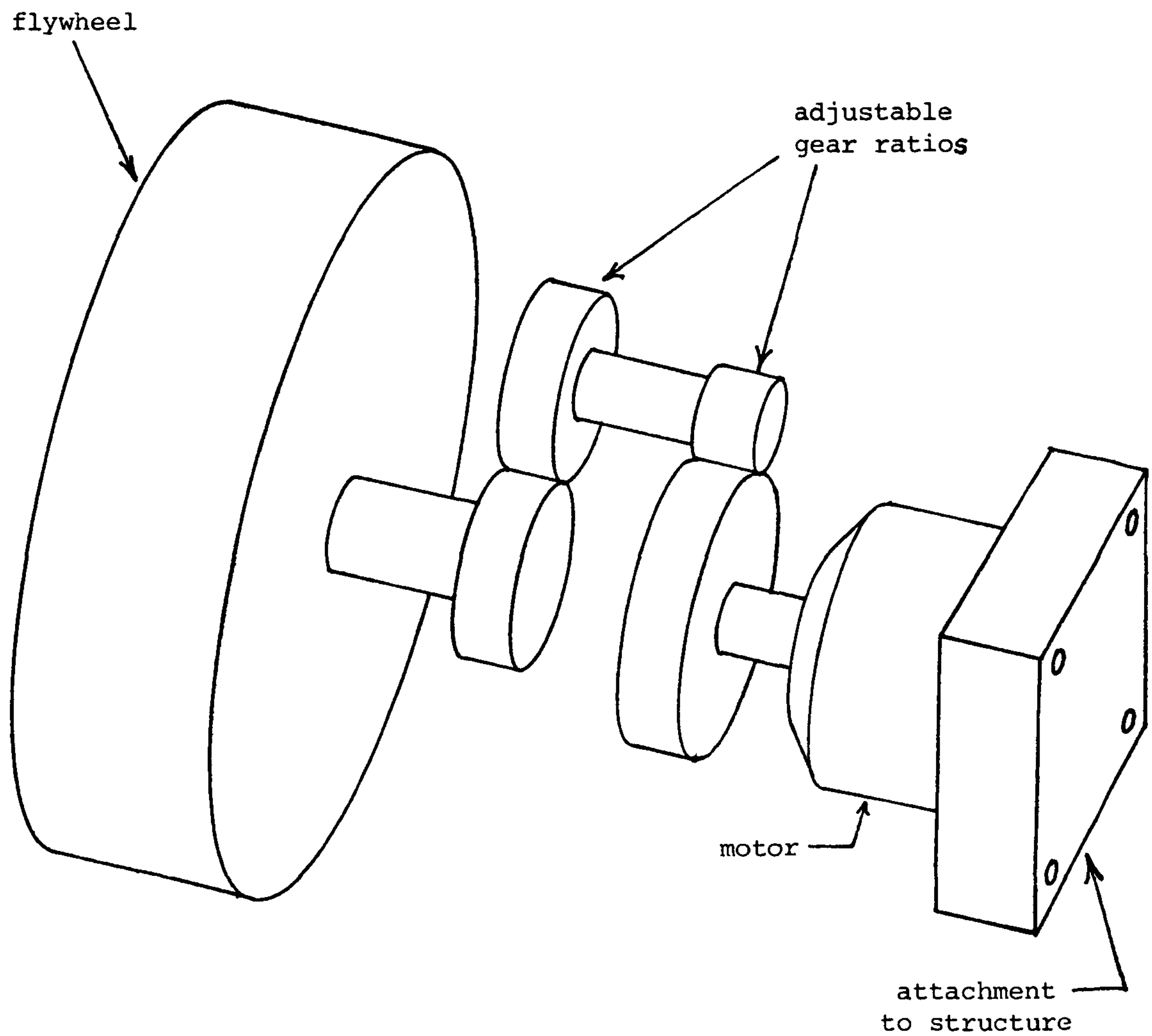

Fig. (5.7) Inertia Wheel System 2.2.1.2

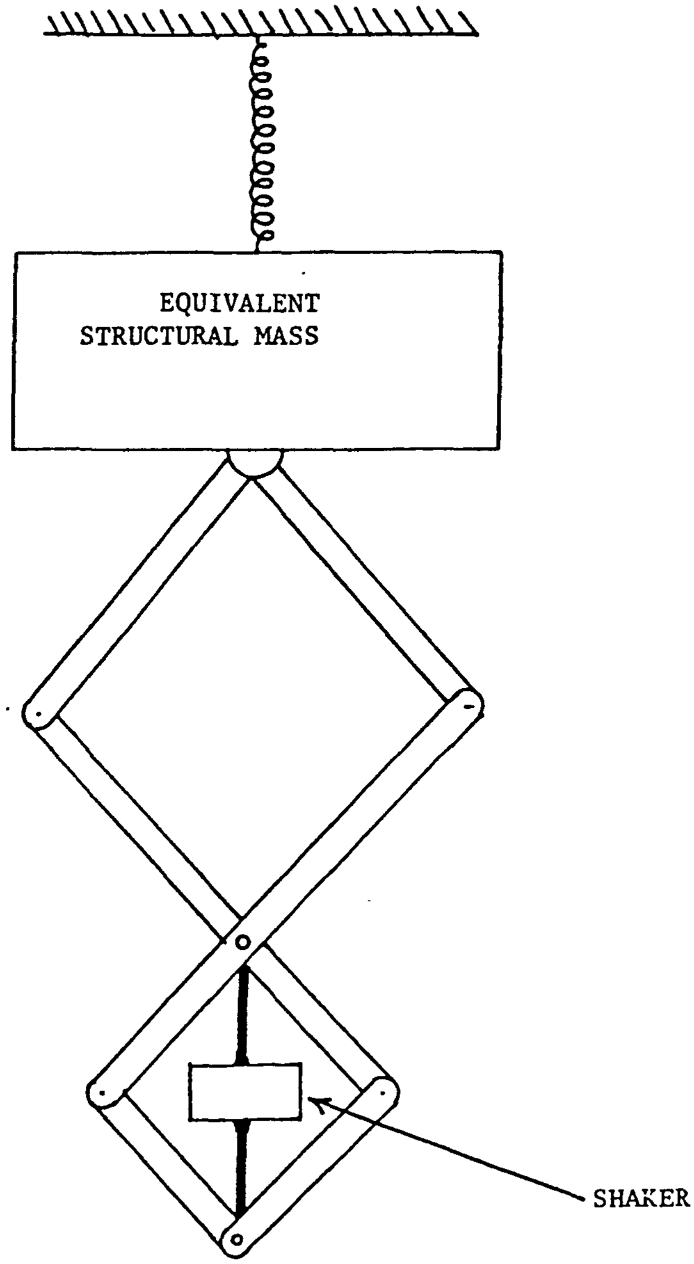

Fig. (5.8) System 2.2.2.1 Active Damper

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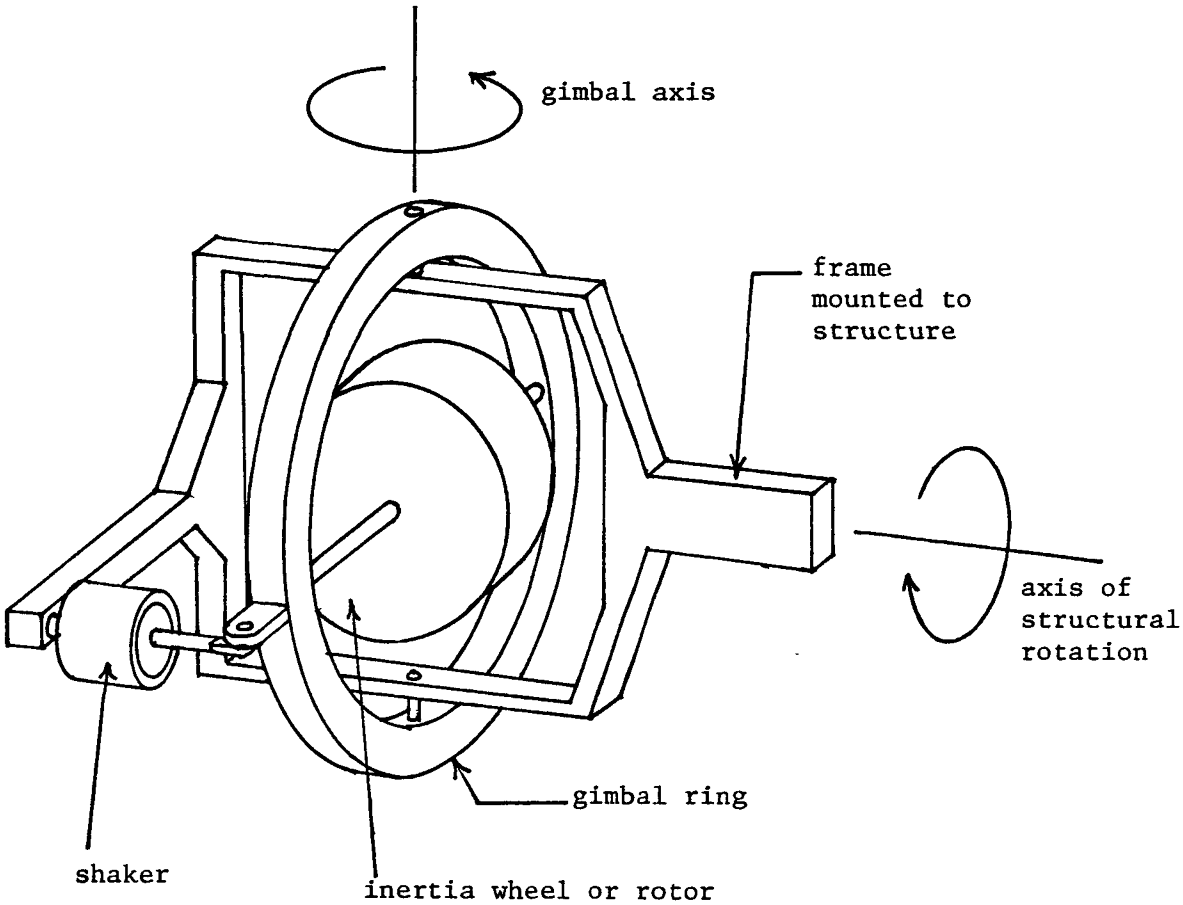

Fig. (5.10) System 2.2.3 Actively Damped Inertia Wheel

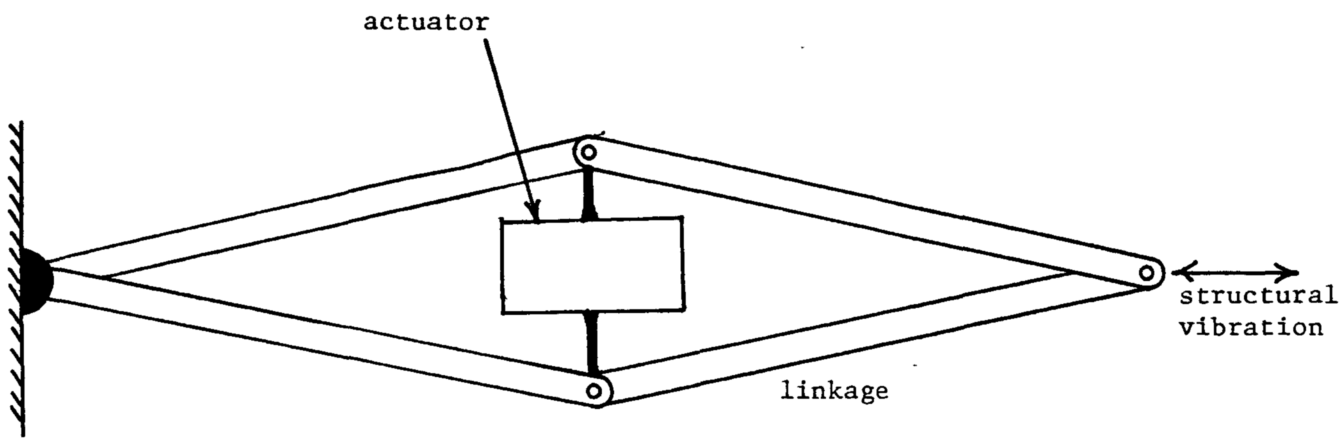

Fig. (5.11) System 2.2.4 Two Force Member Active Damper

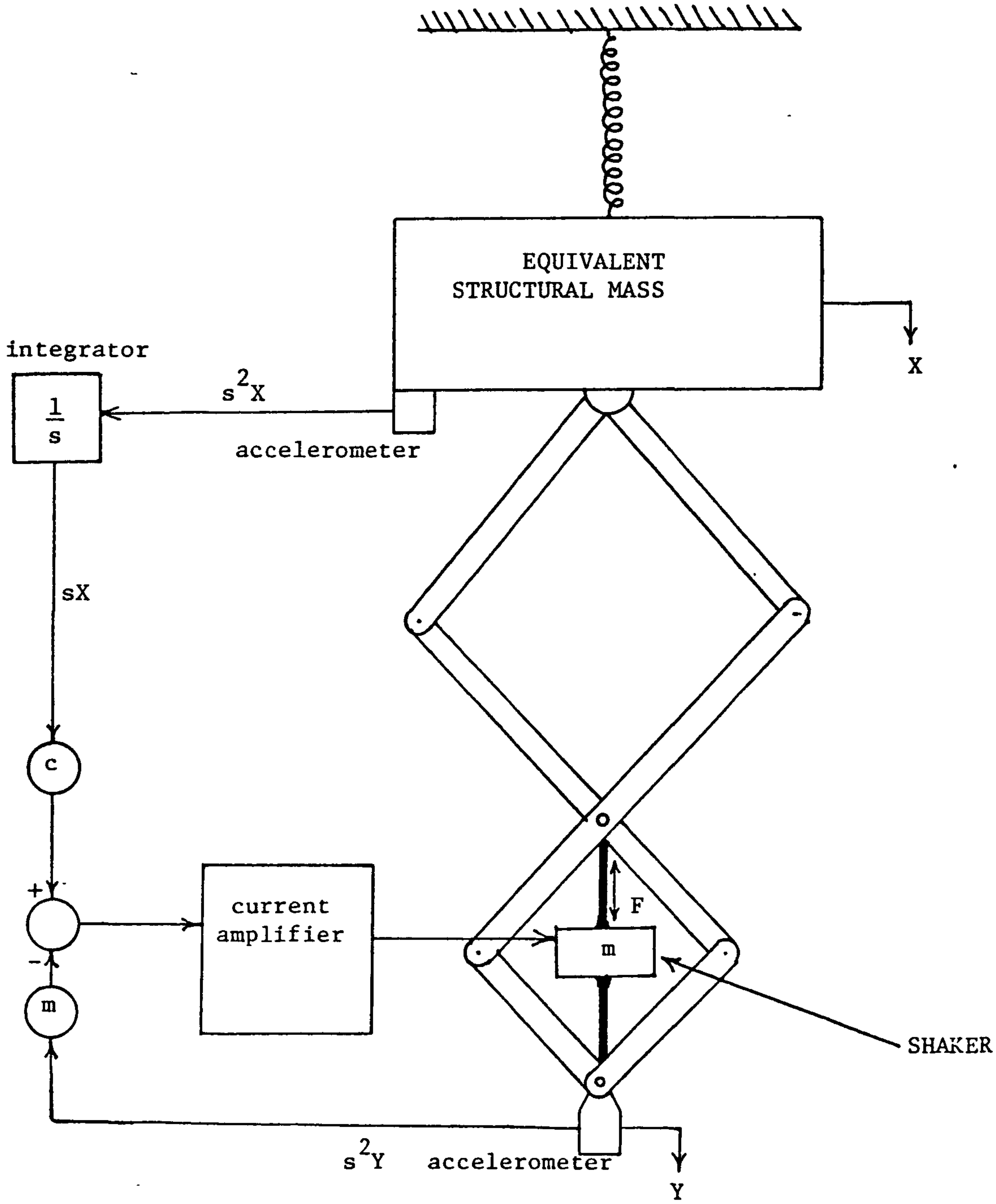

FIG. 12 SYSTEM 2.2.2.1 ACTIVE DAMPER Showing possible control loop added to system to yield desired characteristics

cdx/dt as derived from two accelerometers, the difference being used to drive the voice-coil current through a amplifier.

Comparative Performance

For a preliminary comparison of performance capability, two figures of merit can be considered.

1. Energy removed per cycle per unit mass at design frequency, W_D/m_T
2. Bandwidth

In Appendix B, the value for W_D/m_T achieved with system 2.2.2.1 in Figure (5.8) under design conditions is shown to be .0619 J/kg, which is 79% of the maximum possible capability of a voice coil shaker. This was twice as good as the best system 2.2.1 arrangement of Figure (5.6). Therefore, it is tentatively concluded that other active damping systems, such as the inertia wheel, in Fig. (5.7), or the two force member, in Fig. (5.11), cannot be much better if they are based on electromagnetic action. Passive systems, being based on magnets, are probably no better, although this has not been investigated.

Performance of the active system, based on rate of energy removal, is constant above the design frequency, but rolls off below it. In contrast, the passive system is narrow-band and has to be tuned to a given frequency.

It is shown in Appendix C that the PVF_2 piezo-electric material has the potential for work per cycle of as much as 4 J/kg, which is more than 30 times that of the voice-coil shaker. On the other hand, using the minimum property values available, the work per cycle could be as low as .005 J/kg, which is less than one tenth as good as a voice-coil shaker. One of the most critical properties is the dielectric field strength, E_M. Since a dielectric breakdown causes an explosive failure, it might be necessary to allow quite a large safety margin, so that PVF_2 might prove no better than voice-coil shakers.

Conclusion of Review

It is concluded from this series that an electromagnetic actuator, suspended in such a way that the magnet forms part of the moving mass, is the most promising approach for immediate application.

Design of Experimental Damper

It was decided, as a result of the review, that an electromagnetic damper should be investigated experimentally, and that one should be developed for use on the scale model grillage at the NASA Langley Laboratory. The following design criteria were used initially

Maximum Force = 1 lbf = 4.45 N
Design Amplitude = 0.5 in. = 0.0127 m.
Design Frequency = 0.36 Hz

Instrumentation

Accelerometer on moving mass - NASA supplied
Accelerometer on grillage - NASA supplied
Electronics - off grillage - NASA supplied

A design which meets these requirements is shown in Fig. (5.13).

Description of Damper

The magnetic flux is supplied by the two toroidal samarium-cobalt magnets. The remainder of the magnetic circuit is temporarily completed with mild steel, which has a high saturation flux, and can therefore be designed for minimum weight. However, pure iron will be substituted when it becomes available. The annular gap is large to accomodate the windings of the fixed coil, this dictates a design which minimizes flux leakage at the gap. For example, two magnets are used where one would suffice if the gap were small, also, the internal diameter is larger than is required merely for clearance.

The entire magnet assembly moves along a hardened steel shaft on linear bearings, with one inch of useful travel. An accelerometer is attached to the moving magnet to feed back the acceleration of its mass, which is directly proportional to the damping force. The fixed coil has a Delrin core, and is designed to take ten layers of 26 gage magnet wire.

An outer case of polycarbonate tubing is used to keep the linear bearings clean. It also holds a track in which a small ball race moves to prevent the magnet from turning and thereby twisting the leads to the accelerometer. The end cap of the cover is removable so that the accelerometer leads can be connected to terminals.

Magnet Analysis

The samarium cobalt magnets have the following specifications:

Outside Diameter = 0.75 in.
Inside Diameter = 0.43 in.
Magnetic Induction = 8000 Gauss
= 0.8 Tesla

The following is calculated:

Magnetic Flux = 153 micromaxwells

The remaining circuit is designed so that the magnetic flux density in the mild steel does not exceed 66% of the 2.1 Tesla saturation flux density.

For the purpose of the subsequent analyses, the total 153 uM flux is assumed to cut the coil windings.

N.A.S.A. Active Damper Actuator Prototype

1 Grillage Structure
2 Aluminum Base
3 1/8-inch Steel Shaft
4 Plastic Coil Frame and Coil Windings 4A
5 Iron Pole Piece
6 Iron Flux Loop

7 Samarium Cobalt Ring Magnets
8 Linear Bearings
9 Stainless Steel Keeper
10 Accelerometer
11 Accelerometer Mount
12 3" External Casing and End Cap
13 Plastic Bumper
14 Linear Slide Bearings

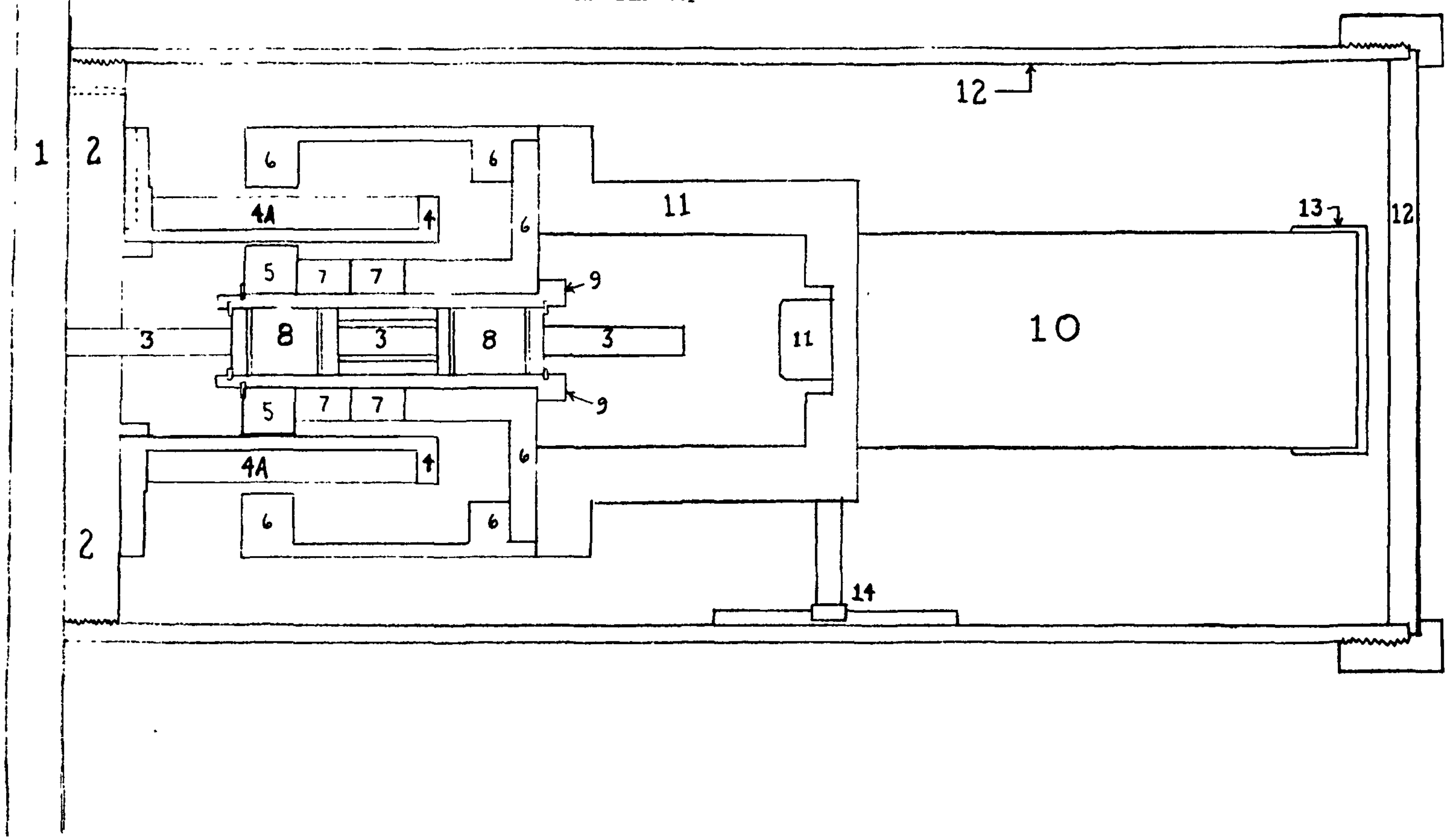

Fig. (5.13) N.A.S.A. Active Damper Actuator Prototype

Analysis of Coil

It is assumed that each of the ten layers of 26 gage wire contains 67 turns, based on the following analysis

Spool length = 1.25 in.
Diameter of bare wire = 0.015945 ins
Diamater of enamelled wire (measured) = 0.01772 ins
Winding efficiency = 67 x 0.01772/1.25 = 0.95 = 95%
Turns per meter = 10 x 67 x 39.37/1.25 = 21100 m^{-1}

The relationship between the force acting, F, the flux Φ, the turns per meter, n, and the coil current A is developed as follows. Assume an effective coil diameter D, and a gap width w_g, then the flux in the gap is

$$B_{gap} = \Phi/\pi D w_g \tag{5.1}$$

The length of wire immersed in the gap is

$$L_{gap} = \pi D n w_g \tag{5.2}$$

and the force acting between the coil and the magnet is

$$F = I B_{gap} L_{gap} = \Phi n I \tag{5.3}$$

For a force of one lbf, or 4.45 N

$$I = F/n\,\Phi = (4.45)/(21100)(153 \times 10^{-6}) = 1.38 \text{ A}$$

In operation, this would be a peak value, the actual DC current recommended for 26 gage wire is 0.51 A, based on 254 circular mils (in^2) at 500 per A. Thus a damper operating continuously at maximum amplitude would be limited to a maximum force of

$$F_{max} = \sqrt{2}\ (0.51/1.38) = 0.52 \text{ lb}$$

However, during tests, peak amplitude will vary, so that the damping constant can be set to correspond to a much higher maximum force without causing excessive heating in the coil. The resistance of the coil, based on 0.0410 Ohms/ft., and a mean diameter of 1.2 ins, is

$$R = (0.0410)(10)(67)(1.2)\pi/(12) = 8.63 \text{ Ohms}$$

Other estimates, based on different values for resistivity, have been as high as 23 Ohms. Based on 8.63 Ohms, the peak voltage is

$$Vmax = (1.38)(8.63) = 11.9\ V$$

$$Vmin = -11.9\ V$$

<u>Analysis of Damping Constants</u>

If we redefine K so that it is the ratio of structural amplitude $|x_o|$ to damper amplitude, D_M, i.e.,

$$K = |X_o|/D_m \tag{5.4}$$

then, from Eq. (B.2) of Appendix B

$$1/K^2 = 1 + c^2/\omega^2 m_2^2 \tag{5.5}$$

also, with the linkage ratio R set to unity, Eq. (B.1) can be written as

$$F/F_m = \omega c\ D_m K/F_m \tag{5.6}$$

For the present design, the following values are anticipated

$$m_2 = 0.428\ kg$$
$$D_m = 1/2\ inch = 0.0127\ m$$
$$F_m = 1\ lbf = 4.45\ N$$

From Eq. (5.5), we can construct the following table for the damping constants, c.

Table 5.2 Values of Damping Constants

K	$\|X_o\|$ (m)	c/ω (kg)	c (Ns/m) f = .35Hz	f = 1 Hz	f = 3.5Hz
.8	.0102	.321	.7059	2.017	7.059
.5	.0064	.7413	1.630	4.658	16.3
.3	.0038	1.361	2.993	8.551	29.93
.1	.0013	4.259	9.37	26.76	93.7

and from Eq. (5.6) we can construct the following table for the corresponding force ratios, F/F_M.

Table 5.3 Force Ratios

K	$\lvert X_o \rvert$ (m)	F/F_M f = .35Hz	f = 1 Hz	f = 3.5Hz
.8	.0102	.00354	.0289	.354
.5	.0064	.00511	.0418	.511
.3	.0038	.00563	.046	.563
.1	.0013	.00588	.0480	.588

Note that the full force capability of the actuator is not required until the frequency exceeds 3.5 Hz.

In the proposed design, the damping constant c will be set by the electronic circuit. Suppose we pick a value of 2 Ns/m and examine the corresponding values for K and F/F_M as given by Eqs. (5.5) and (5.6). Note that a value for K of unity could correspond to a peak-to-peak structural amplitude of one inch while a value for F/F_m of unity would correspond to a maximum force of one lbf, the design value. The results are given in Table 5.4.

Table 5.4 Values for K and F/F_M when C = 2 Ns/m, (Eqns. 2 and 3)

f(Hz)	K	F/F_M
.1	.133	.00048
.2	.261	.00185
.3	.374	.00403
.35	.426	.00535
.4	.474	.00678
.5	.558	.01006
.8	.732	.0210
1.0	.802	.0288
1.5	.899	0.0484
2.0	.937	.0672
2.5	.958	.0859
3.0	.971	.1045
5.0	.989	.1773

Note that the maximum force is not approached even at 5 Hz. Had a larger damping constant been used, these forces would have been larger, but the value for K would have been smaller. For example, Table 5.5 shows how K varies with c at frequencies of 0.35, 1.0 and 3.5 Hz.

Table 5.5 K vs. c (Eqn. 2)

c	K		
Ns/m	0.35 Hz	1.0 Hz	3.5 Hz
0.1	.994	.999	1.000
0.2	.978	.997	1.000
0.3	.953	.994	1.000
0.4	.920	.989	.999
0.6	.843	.976	.998
0.8	.762	.958	.996
1.0	.685	.937	.994
1.5	.532	.873	.988
2.0	.426	.802	.978
2.5	.352	.732	.966
3.0	.299	.607	.953
5.0	.185	.474	.883
10.0	.094	.260	.685

None of the examples shown in Table 5.5 correspond to a value for F/F_m exceeding unity. Thus, the present design, if achieved, should be adequate for all of the cases shown, provided that c is set at 2 Ns/m or less.

Preliminary Tests

A prototype unit was built from the design shown in Figure 5.13. a photograph of this unit is shown in Figure 5.14, while the analog circuit used in an initial test of the unit attached to the NASA grillage is shown in Figure 5.15.

Because of the possibility of hitting the stops when the moving mass is not centralized, a spring was added to the system which gave it a natural frequency of about 2 Hz. Using switch no. 2, the system was excited at a natural frequency close to 4 Hz., then it was switched to feedback control, and the resulting damping was observed. Through switch no. 1, the inner feedback loop incorporating the accelerometer mounted on the moving mass could be included at will. When it was excluded, rapid damping of the 4 Hz. mode was observed, but when the inner loop was included, the system at first damped, and then went unstable at a frequency lower than 2 Hz.

It was concluded that the accelerometer on the inner loop could be discarded, but that an inner loop should be incorporated which included an LVDT to measure position. This way, a very low natural frequency could be simulated for the system, while a bias voltage could be applied to offset gravitational effects. At the same time, studies should continue on a system which incorporates an inner loop accelerometer.

Conclusions

An experimental damper has been designed in which a mass of .428 kg (.95 lbm) moves over a peak-to-peak amplitude of one inch under a programmed force which can be as much as one lbf. The design includes an accelerometer attached to the mass.

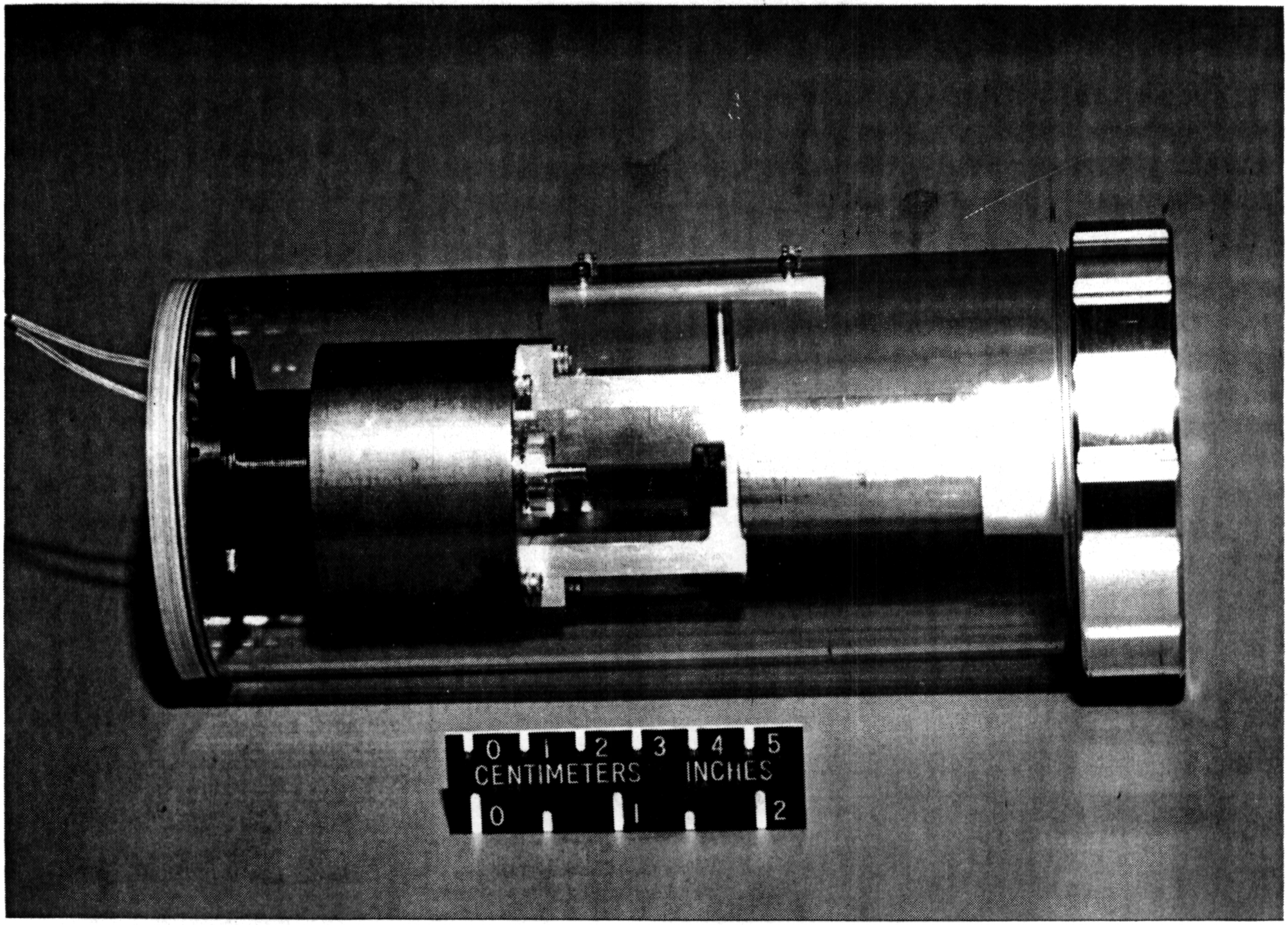

Fig. 5.14 Prototype Damper

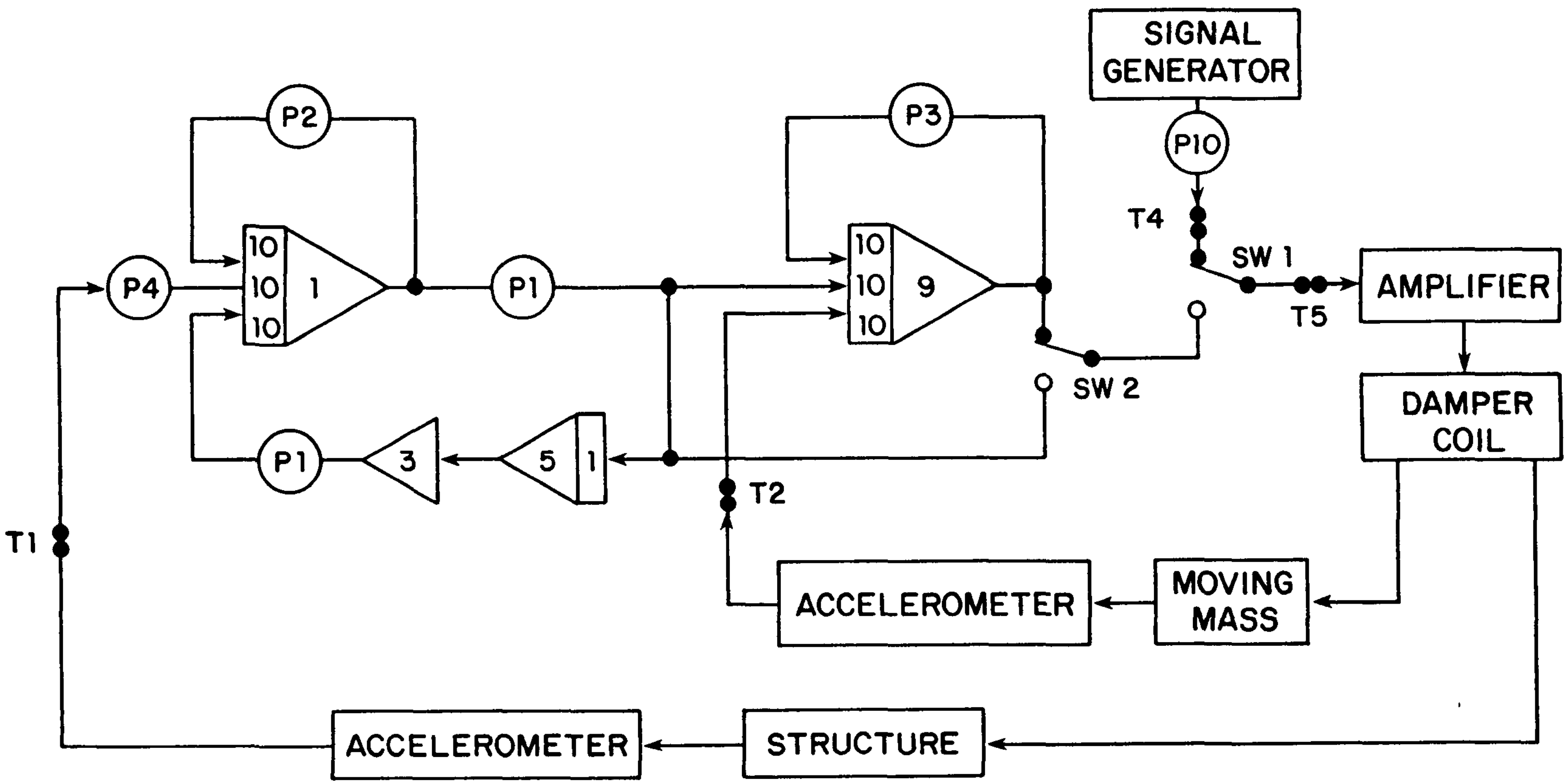

Fig. 5.15 Control Circuit for Prototype Damper

The programmed force is to be produced from the generation of a current in an amplifier driven by the difference between a signal proportional to the structural velocity and one proportional to the acceleration of the mass. Thus a damping force c will be generated on the structure.

Tentatively, the following design values are suggested

Damping constant = 2 Newton sec/meter

Design frequency = 0.35 Hz

Structural Amplitude = .43 inches peak-to-peak

From early experience with the prototype, it is concluded that the inner-loop accelerometer should be replaced by an LVDT if an early working system is required. However, research should continue into the use of the inner accelerometer, and an alternative to the LVDT should be found if dampers with longer strokes are found to be desirable.

III. CONCLUSIONS

In conclusion, this work has covered two aspects of the problems involved in damping large space structures. On the one hand, the analytical problem of locating dampers has been investigated, while on the other, the problems of damper design have been reviewed. These considerations are summarized below

(1) Five computer programs have been developed. They are

- (i) ASMD1 - Assumed mode method with diagonal damping matrix
- (ii) ASMD2 - Assumed mode method with full damping matrix
- (iii) SPSTGN - Damped eigenvalues using reanalysis
- (iv) COPZ - Optimization of damper gain
- (v) NORMAL - Orthonormatization of mode shapes

(2) Proof has been presented for two criteria for the optimal location of dampers. They are:

- (i) MCFC - That the optimal single damper location is where the constrained frequency is a minimum
- (ii) MFSC - That the optimal locations of several dampers are where the constrained frequency associated with the damper location has the largest separation from the corresponding undamped natural frequency of the system.

(3) A method of minimizing the effect of control "spill-over" has been proposed and demonstrated.

(4) The characteristic equation of an undamped system with a generalized control law has been derived using reanalysis theory.

(5) A review of possible damper designs has been conducted. It was concluded that the most promising designs are active systems, using electromagnetic or piezoelectric actuators with linear or rotational motion.

(6) A prototype linear electromagnetic damper was built and demonstrated using a moving permanent magnet. The damper is driven by feedback from an accelerometer, mounted in the structure, and integrated to provide a feedback force proportional to structural velocity. It was found necessary to incorporate a centering spring which will be replaced in future designs by a position feedback from an LVDT or other device.

IV. Bibliography of Papers Published under this Contract

Optimal Damping for the Vibration Control of a Two-Dimemsional Structure
B.P. Wang, W.D. Pilkey, G. Horner
AIAA Paper No. 81-1845

Optimal Damper Location in the Vibration Control of Large Space Structures
B.P. Wang, W.D. Pilkey, presented at the Third VPI/AIAA Symposium on Dynamics and Control of Large Flexible Spacecraft.

APPENDICES

Appendix A-1

Subroutine ASMD1

Assumed Mode Method - Diagonal Damping Matrix

I. INTRODUCTION

ASMD1 is a FORTRAN subroutine, based on the assumed mode method, that can be used to compute the eigenvalues of a damped system. The eigenvectors of the undamped system are used as the assumed modes. When all modes of the undamped system are used, the method is equivalent to a direct solution. Experience shows that retaining 1/3 to 1/2 of the modes of the original system in the assumed mode method usually leads to accurate eigenvalues while providing considerable savings in computer time.

II. ASSUMED MODE FORMULATION

Let the free vibration of a damped system be described by

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\} \qquad \text{(A1.1)}$$

where [c] is a diagonal damping matrix. Let $\{\rho_i\}$, ω_i be the solution of the corresponding undamped problem

$$\omega^2[m]\{\rho\} = [k]\{\rho\} \qquad \text{(A1.2)}$$

Furthermore, assume the mode shapes are normalized to unit generalized mass, i.e.,

$$\{\rho_i\}^T[m]\{\rho_i\} = 1.0 \qquad \text{(A1.3)}$$

Define

$$\underset{n\times 1}{\{x\}} = \sum_{\ell=1}^{L} \{\rho_\ell\}q_\ell = \underset{n\times L}{[\Phi]}\ \underset{L\times 1}{\{q\}} \qquad \text{(A1.4)}$$

where L is the number of modes used. Substitute (A1.4) into (A1.1) and premultiply $[\Phi]^T$ to get

$$[\ I\ ]\{\ddot{q}\} + [\bar{c}]\{\dot{q}\} + [\ \Lambda\ ]\{q\} = \{0\} \qquad \text{(A1.5)}$$

where

$[\ I\ ]$ = LxL identity matrix

$[\ \Lambda\ ] = [\ \omega_\ell^{\,2}\ ]$ = LxL diagonal matrix

$[c] = [\Phi]^T[c][\Phi]$ = LxL full matrix

Set

$$\{z\} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}_{2L\times 1} = \begin{bmatrix} \dot{q} \\ q \end{bmatrix} \tag{A1.6}$$

Then, (A1.5) becomes

$$\{\dot{z}_1\} = -[\bar{c}]\{z_1\} - [\ \Lambda\ ]\{z_2\} \tag{A1.7a}$$

By definition (A1.6)

$$\{\dot{z}_2\} = \{z_1\} \tag{A1.7b}$$

Place (A1.7a), (A1.7b) together

$$\{\dot{z}\} = [A]\{z\} \tag{A1.8}$$

where

$$[A] = \begin{bmatrix} -[c] & -[\ \Lambda\ ] \\ [I] & [0] \end{bmatrix} = \begin{bmatrix} -[\Phi]^T[c][\Phi] & -[\ \omega_\ell^{\,2}\ ] \\ [I] & [0] \end{bmatrix}$$

A standard eigenvalue problem can be formulated from (8) and solved using the routine RESV in the NASA Langley library.

III. INPUT PARAMETERS

III.1 Formal Parameters

Description of the 6 formal parameters

N: number of DOF's of the original system
NC: number of dampers

IPRT: Dynamical matrix ([A] in (8)) printing flag,
= 0 no printing
= 1 print

III.2 Common Blocks

In addition to the formal parameters, the following variables enter the subroutine ASMD through the common block set contained in file COMASMD. This file must be placed in the main program.

NTITLE(I): a holerith array of up to 80 columns which is used to define the title of the job
JC(I): damper location (DOF) of the Ith damper
C(I): initial damper values
WN(I): undamped natural frequency of the ith mode
PHI(J,I): mode shape coefficient at dof J of the Ith mode
NM: number of modes used in the assumed mode method

IV. OTHER SUBROUTINES USED

ASMD calls NASA Library routine RESV to compute the eigenvalues of the dynamical matrix [A]. The computed complex eigenvalues are stored in array ER(I) and EI(I), which are the real and imaginary part of the Ith eigenvalue of matrix [A].

V. USING SUBROUTINE ASMD

The following is a sample program that calls routine ASMD1.

```
      PROGRAM TESTAS1(INPUT,OUTPUT,TAPE12)
C
C
          DIMENSION G(88)
      COMMON / DATA1 /C(50)
      COMMON / DATA2 / PHI(99,99),WN(99)
      COMMON / DATA3 / EIGDATA(40,20,4),DAMP(40)
      COMMON / DATA4 / NTITLE(80),JC(50)
      COMMON / DATA5 / NM
      COMMON / DATA6 / NDP
      COMMON /DATA7/ER(99),EI(99)
C
C
        DO 1 I=1,50
1          C(I)=0.0
C
C        INPUT SECTION
C
        READ  10,(NTITLE(I),I=1,80)
       PRINT  10,(NTITLE(I),I=1,80)
10      FORMAT(80A1)
       READ *,N,NM,NC
      PRINT *,N,NM,NC
       DO 20 I=1,NC
        READ *,JC(I),C(I)
       PRINT *,JC(I),C(I)
20        CONTINUE
       READ *,IPRT
      PRINT *,IPRT
C
       DO 5010 I =1,NM
       READ*,    WN(I),G(I)
          PRINT*,WN(I),G(I)
       READ*, (PHI(J,I),J=1,N)
          PRINT*,(PHI(J,I),J=1,N)
5011   FORMAT(5E16.9)
5010    CONTINUE
          PRINT*, 'N   WN  G'
          DO 99 I=1,N
99        PRINT*,I,WN(I),G(I)
          PRINT*,'PHI'
          DO 999 J=1,N
999       PRINT*,(PHI(J,I),I=1,N)
C
C
C
C
          CALL ASMD1(N,NC,IPRT)
C
C         END
```

VI. PROGRAM LISTING

```
C
         SUBROUTINE ASMD1(N,NC,IPRT)
C
C
C        THIS PROGRAM USES THE ASSUMED MODE METHOD TO COMPUTE
C        COMPLEX EIGENVALUES OF A DAMPED SYSTEM.
C         DIAGONAL DAMPING MATRIX.
C        THE PROGRAM IS A MODIFICATION OF PROGRAM CEIG 4/13/82.
C
C
       COMPLEX CMCOEIG
     COMMON / DATA1 /C(50)
     COMMON / DATA2 / PHI(99,99),WN(99)
     COMMON / DATA3 / EIGDATA(40,20,4),DAMP(40)
     COMMON / DATA4 / NTITLE(80),JC(50)
     COMMON / DATA5 / NM
     COMMON / DATA6 / NDP
     COMMON /DATA7/ER(99),EI(99)
C
     DIMENSION DYEXAC(99,99),V(99,5),WK(270)
C
C
         MAX=99
         NMD=2*NM
         N=NMD
C
C        FORM DYNAMICAL MATRIX
C
       NN=2*NM
       DO 100 I=1,NN
       DO 100 J=1,NN
100        DYEXAC(I,J)=0.0
       DO 200 I=1,NM
       IN=I+NM
       DYEXAC(I,IN)=-WN(I)*WN(I)
       DYEXAC(IN,I)=1.0
       DO 200 J=1,NM
       DO 200 L=1,NC
       K=JC(L)
      DYEXAC(I,J)=DYEXAC(I,J)-C(L)*PHI(K,I)*PHI(K,J)
200    CONTINUE
C
         PRINT*,'*******************      OUTPUT   ******************'
         PRINT 11
      PRINT  10,(NTITLE(I),I=1,80)
10     FORMAT(80A1)
11       FORMAT(///)
       IF(IPRT.EQ.0) GO TO 210
C
       PRINT 201
201     FORMAT(1H1,///,*            DYNAMICAL MATRIX IS *///)
       DO 202 I=1,NN
202   PRINT 205, (DYEXAC(I,J),J=1,NN)
205     FORMAT(/,5X,5E15.6)
210      CALL RESV(MAX,N,DYEXAC,ER,EI,0,0,V,WK,IERR)
```

```
C
        PRINT 1020,NM
C
        PRINT*,'*****  DAMPERS  *****'
        PRINT*,' PLACEMENT(DOF)    VALUE'
        DO 666 I=1,NC
666     PRINT*,'             ',JC(I),'         ',C(I)
1020   FORMAT(/,10X,* ASSUMED MODE METHOD, USE *,I5,,*  MODES*/)
        PRINT 11
        PRINT*,'THE DAMPED EIGENVALUES:'
C
        DO 333 II=1,NMD
333     PRINT*,II,ER(II),'+',EI(II),'J'
        PRINT 11
        PRINT 11
9999   CONTINUE
      END
```

VII NUMERICAL EXAMPLE

To illustrate the use of ASMD1, the 2 dof system of Fig. (A1.1) is used. A damper of c = 2.3 is attached to dof 2. The input data and control cards for this sample problem are shown in Table 1. The output is given in Table 2.

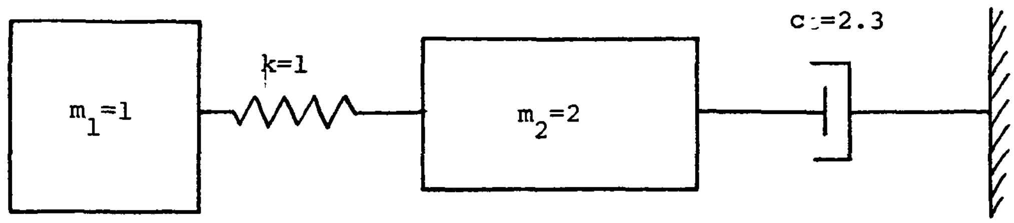

Fig. A1.1 A Damped 2-dof system

Table 1 Control Cards and Input Data for Sample Problem

The test program TESTAS1 can be run through the use of the following job file:

```
/JOB                                          1293B    UVA
BPW,CM300000,T1000.
USER,677121E.
CHARGE,102428,LRC.
DELIVER. 1293B D  UVA
ATTACH,FTNMLIB/UN=LIBRARY.
GET,TESTAS1.
MAP(OFF).
FTN,I=TESTAS1,L=0,OPT=1,B=F.
LDSET,LIB=FTNMLIB.
F.
REWIND,OUTPUT.
COPYEI,OUTPUT,OTTAS1.
REPLACE,OTTAS1.
DAYFILE,L=DATAS1.
REPLACE,DATAS1.
EXIT.
REWIND,OUTPUT.
COPYEI,OUTPUT,XOTTAS1.
REPLACE,XOTTAS1.
DAYFILE,L=XDATAS1.
PACK,XDATAS1.
REPLACE,XDATAS1.
/EOR
TEST: 2-DOF FREE SYSTEM,TEST ASMD1 <---- Title
 2, 2,1 <---------------------------- 2 DOF, 2 MODES USED, 1 DAMPER
2,2.3 <------------------------------ DAMPER: DOF 2, C=2.3
 0 <--------------------------------- DYN. MAT. NOT PRINTED
 0,1 <------------------------------- ω1=0, G1=1
 0.577, 0.577 <---------------------- φ1
 1.2247,1 <-------------------------- ω2=1.2247, G2=1
 0.316,-0.400 <---------------------- φ2
 EOI ENCOUNTERED.
/
```

Table 2 Output of Sample Problem

```
TEST: 2-DOF FREE SYSTEM,TEST ASMD1

          ASSUMED MODE METHOD, USE      2  MODES

 *****  DAMPERS  *****
  PLACEMENT(DOF)     VALUE
            2         2.3


 THE DAMPED EIGENVALUES:
 1 0.+0.J
 2 -.8999940753023+0.J
 3 -.1243049123488+1.122804973927J
 4 -.1243049123488+-1.122804973927J
```

Appendix A-2

Subroutine ASMD2

Assumed Mode Method - Full Damping Matrix

I. INTRODUCTION

ASMD2 is a FORTRAN subroutine, based on the assumed mode method, that can be used to compute the eigenvalues of a damped system. The eigenvectors of the undamped system are used as the assumed modes. When all modes of the undamped system are used, the method is equivalent to a direct solution. Experience shows that retaining 1/3 to 1/2 of the modes of the original system in the assumed mode method usually leads to accurate eigenvalues while providing considerable savings in computer time. ASMD2 allows a full damping matrix to be used.

II. ASSUMED MODE FORMULATION

Let the free vibration of a damped system be described by

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\} \qquad \text{(A2.1)}$$

where [c] is a diagonal damping matrix. Let $\{\rho_i\}$, ω_i be the solution of the corresponding undamped problem

$$\omega^2[m]\{\rho\} = [k]\{\rho\} \qquad \text{(A2.2)}$$

Furthermore, assume the mode shapes are normalized to unit generalized mass, i.e.,

$$\{\rho_i\}^T[m]\{\rho_i\} = 1.0 \qquad \text{(A2.3)}$$

Define

$$\underset{n\times 1}{\{x\}} = \sum_{\ell=1}^{L} \{\rho_\ell\}q_\ell = \underset{n\times L}{[\Phi]}\ \underset{L\times 1}{\{q\}} \qquad \text{(A2.4)}$$

where L is the number of modes used. Substitute (A2.4) into (A2.1) and premultiply by $[\Phi]^T$ to get

$$[\ I\ ]\{\ddot{q}\} + [\bar{c}]\{\dot{q}\} + [\ \Lambda\ ]\{q\} = \{0\} \qquad \text{(A2.5)}$$

where

$[\,I\,]$ = LxL identity matrix

$[\,\Lambda\,] = [\,\omega_\ell^2\,]$ = LxL diagonal matrix

$[\bar{c}] = [\Phi]^T[c][\Phi]$ = LxL full matrix

Set

$$\{z\} = \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix}_{2L\times 1} = \begin{Bmatrix} \dot{q} \\ q \end{Bmatrix} \tag{A2.6}$$

Then, (A2.5) becomes

$$\{\dot{z}_1\} = -[\bar{c}]\{z_1\} - [\,\Lambda\,]\{z_2\} \tag{A2.7a}$$

By definition (A2.6)

$$\{\dot{z}_2\} = \{z_1\} \tag{A2.7b}$$

Place (7a), (7b) together

$$\{\dot{z}\} = [A]\{z\} \tag{A2.8}$$

where

$$[A] = \begin{bmatrix} -[\bar{c}] & -[\,\Lambda\,] \\ [I] & [0] \end{bmatrix} = \begin{bmatrix} -[\Phi]^T[c][\Phi] & -[\,\omega_\ell^2\,] \\ [I] & [0] \end{bmatrix}$$

A standard eigenvalue problem can be formulated from (A2.8) and solved using the routine RESV in the NASA Langley library.

III. FORMAL PARAMETERS AND CALLING ASMD2

The subroutine ASMD2 is called with the statement:

```
CALL  ASMD2(N,NM,NROWC,C,PHI,WN,ER,EI,NTITLE,IPRT)
```

The formal parameters are defined as follows:

```
N=  N.D.O.F.  IN THE SYSTEM
NM=  NO. MODES USED IN THE ASSUMED MODE CALC.
NROWC=  NO. OF ROWS IN THE DIM. STATEMENT FOR C,PHI,WN,ER,EI
       IN THE CALLING PROGRAM
C=  THE DAMPING MATRIX
PHI=  THE MODAL MATRIX-EIGENVECTORS NORMALIZED TO UNIT MASS.
WN=  VECTOR OF UNDAMPED EIGENVALUES.
ER,EI=  VECTORS CONTAINING THE REAL AND IMAG. PARTS OF THE EIGEN values
       OF THE DAMPED SYSTEM.
NTITLE=  A VECTOR(DIM=80) CONTAINING THE TITLE OF THE RUN.
IPRT=  A FLAG TO ALLOW PRINTING THE DYNAMICAL MAT.(1-PRINT, 0-NO
```

IV. OTHER SUBROUTINES USED

ASMD2 calls NASA Library routine RESV to compute the eigenvalues of the dynamical matrix [A]. The computed complex eigenvalues are stored in array ER(I) and EI(I), which are the real and imaginary part of the Ith eigenvalue of matrix [A].

V. LISTING THE SUBROUTINE ASMD2

```
      SUBROUTINE ASMD2(N,NM,NROWC,C,PHI,WN,ER,EI,NTITLE,IPRT)
C
C
C        THIS PROGRAM USES THE ASSUMED MODE METHOD TO COMPUTE
C        COMPLEX EIGENVALUES OF A DAMPED SYSTEM.
C        A FULL DAMPING MATRIX MAY BE USED.
C
C        THE PROGRAM IS A MODIFICATION OF PROGRAM CEIG 4/13/82.
C
C
C        FORMAL PARAMETERS:
C
C        N=  N.D.O.F.  IN THE SYSTEM
C        NM=  NO. MODES USED IN THE ASSUMED MODE CALC.
C        NROWC=  NO. OF ROWS IN THE DIM. STATEMENT FOR C,PHI,WN,ER,EI
C                IN THE CALLING PROGRAM
C        C=  THE DAMPING MATRIX
C        PHI=  THE MODAL MATRIX-EIGENVECTORS NORMALIZED TO UNIT MASS.
C        WN=  VECTOR OF UNDAMPED EIGENVALUES.
C        ER,EI=  VECTORS CONTAINING THE REAL AND IMAG. PARTS OF THE EIGENVALUE
C                OF THE DAMPED SYSTEM.
C        NTITLE=  A VECTOR(DIM=80) CONTAINING THE TITLE OF THE RUN.
C        IPRT=  A FLAG TO ALLOW PRINTING THE DYNAMICAL MAT.(1=PRINT, 0=NO PRINT
C
C
      DIMENSION C(NROWC,1),PHI(NROWC,1),WN(NROWC),ER(NROWC),
     1 EI(NROWC),NTITLE(80)
C
      DIMENSION DYEXAC(99,99),V(99,5),WK(270),A(99,99)
C
C
      MAX=99
      NDOF=N
      NMD=2*NM
C
C     FORM DYNAMICAL MATRIX
C
      NN=2*NM
      DO 100 I=1,NN
      DO 100 J=1,NN
      A(I,J)=0.0
100   DYEXAC(I,J)=0.0
      DO 200 I=1,NM
      IN=I+NM
      DYEXAC(I,IN)=-WN(I)*WN(I)
      DYEXAC(IN,I)=1.0
200   CONTINUE
C
C
      DO 12 J=1,NDOF
      DO 12 L=1,NM
      DO 13 K=1,NDOF
13    A(J,L)=A(J,L)+C(J,K)*PHI(K,L)
12    CONTINUE
```

```
C
      DO 14 K=1,NM
      DO 14 L=1,NM
      DO 15 J=1,NDOF
15      DYEXAC(K,L)=DYEXAC(K,L)-PHI(J,K)*A(J,L)
14    CONTINUE
C
      PRINT*,'*******************      OUTPUT   *****************'
      PRINT 11
    PRINT  10,(NTITLE(I),I=1,80)
10    FORMAT(80A1)
C     PRINT 11
11    FORMAT(//)
     IF(IPRT.EQ.0) GO TO 210
C
     PRINT 201
201   FORMAT(1H1,///,*                DYNAMICAL MATRIX IS */// )
     DO 202 I=1,NN
202  PRINT 205, (DYEXAC(1,J),J=1,NN)
205   FORMAT(/,5X,5E15.6)
210   CALL RESV(MAX,NMD,DYEXAC,ER,EI,0,0,V,WK,IERR)
C
      PRINT 1020,NM
      PRINT*,'THE DAMPING MATRIX IS:'
      DO 666 I=1,NDOF
666   PRINT*,(C(I,J),J=1,NDOF)
C
1020  FORMAT(/,10X,* ASSUMED MODE METHOD, USE *,I5,,*  MODES*/)
      PRINT 11
      PRINT*,'THE DAMPED EIGENVALUES:'
C
      DO 333 II=1,NMD
333   PRINT*,II,ER(II),'+',EI(II),'J'
      PRINT 11
      PRINT 11
2999  CONTINUE
```

VI. CONTROL CARDS AND TEST PROGRAM

The test program TESTAS4 can be run through the use of the job file of Table 1. Table 2 is a listing of the test program TESTAS4.

Table 1 Control Cards and Input Data for Sample Problem

```
/JOB                                        1293B    UVA
BPW,CM300000,T1000.
USER,697121E.
CHARGE,102428,LRC.
DELIVER. 1293B D  UVA
ATTACH,FTNMLIB/UN=LIBRARY.
GET,TESTAS4.
MAP(OFF).
FTN,I=TESTAS4,L=0,OPT=1,B=F.
LDSET,LIB=FTNMLIB.
F.
REWIND,OUTPUT.
COPYEI,OUTPUT,OTTAS4.
REPLACE,OTTAS4.
DAYFILE,L=DATAS4.
REPLACE,DATAS4.
EXIT.
REWIND,OUTPUT.
COPYEI,OUTPUT,XOTTAS4.
REPLACE,XOTTAS4.
DAYFILE,L=XDATAS4.
PACK,XDATAS4.
REPLACE,XDATAS4.
/EOR
TEST: 2-DOF FREE SYSTEM  <----- Title
 2, 2  <----- 2 DOF, 2 modes used in calc.
 1.3,-.5,0,0  <----- Damping matrix [1.3  -0.5 ; 0  0]
 0  <----- No printing of Dyn. mat.
 0,1  <----- ω1 = 0, G1 = 1
 0.577,-0.577  <----- φ1
 1.2247,1  <----- ω2 = 1.2247, G2 = 1
 0.816,-0.408  <----- φ2
 0
 EOI ENCOUNTERED.
/
```

Table 2 Listing of program TESTAS4

```
   P*
      PROGRAM TESTAS4(INPUT,OUTPUT)
C
C
          DIMENSION G(88),C(99,99),PHI(99,99),WN(99),ER(99),EI(99)
          DIMENSION NTITLE(80)
C
C

        DO 1 I=1,50
            DO 1 J=1,50
1          C(I,J)=0.0
C
C        INPUT SECTION
C
        READ  10,(NTITLE(I),I=1,80)
       PRINT  10,(NTITLE(I),I=1,80)
10      FORMAT(80A1)
599    READ *,N,NM
      PRINT *,N,NM
C
          DO 20 I=1,N
            READ*,(C(I,J),J=1,N)
20        PRINT*,(C(I,J),J=1,N)
       READ *,IPRT
      PRINT *,IPRT
C
C
       DO 5010 I =1,NM
       READ*,    WN(I),G(I)
          PRINT*,WN(I),G(I)
       READ*, (PHI(J,I),J=1,N)
          PRINT*,(PHI(J,I),J=1,N)
5010    CONTINUE
          PRINT*, 'N    WN   G'
          DO 99 I=1,N
99        PRINT*,I,WN(I),G(I)
          PRINT*,'PHI'
          DO 999 J=1,N
999       PRINT*,(PHI(J,I),I=1,N)
C
C
          CALL ASMD2(N,NM,99,C,PHI,WN,ER,E1,NT1TLE,IPRT)
          PRINT*,'1=REPEAT,0=STOP'
          READ*,ITEST
          IF (1TEST.EQ.1) GO TO 599
C
C
C
          END
C
```

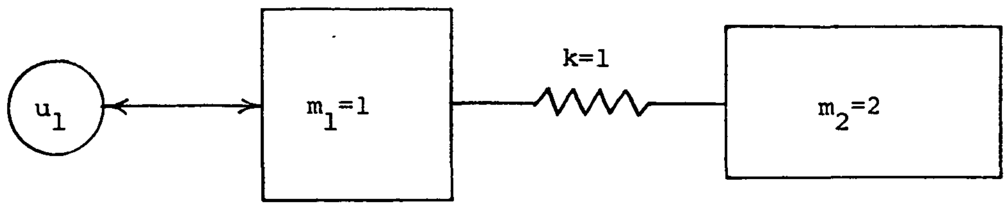

Fig. A2.1 Actively Controlled 2-dof system

VII SAMPLE PROBLEM

Figure (A2.1) shows an actively controlled 2-dof free-free system. The contorl law is

$$u_1 = -1.3\ \dot{x}_1 + 0.5\ \dot{x}_2$$

The input data to test program TESTAS4 is shown in Table 1. Table 3 shows the output.

Table 3 Output of Sample Problem

```
******************** OUTPUT ****************

TEST: 2-DOF FREE SYSTEM

ASSUMED MODE METHOD, USE 2 MODES

THE DAMPING MATRIX IS:
1.3 -.5
0. 0.

THE DAMPED EIGENVALUES:
1 0.+0.J
2 -.3403281689367+0.J
3 -.4790459155316+.971771301425J
4 -.4790459155316+-.971771301425J
```

Appendix A-3

Subroutine SPSTGN Documentation

Damped Eigenvalues Using Reanalysis

I. Introduction

SPSTGN is a FORTRAN subroutine that can be used to compute the damped eigenvalues when damping is introduced into an originally undamped system through the introduction of a diagonal viscous damping matrix. Based on the modal data of the undamped structure, the damped eigenvalues are computed using an efficient reanalysis formulation. It is the purpose of this document to summarize the formulation and usage of the subroutine SPSTGN.

II. Summary of the Reanalysis Formulation of the Damped Eigenvalue Problem

Given an undamped system described by the following normalized modal data

$$\omega_i, \quad \{\rho_i\}, \quad i = 1 \text{ to } L \leq \text{Number DOF} = \text{NELEM}$$

where

ω_i = natural frequency of the ith mode, rad/sec
= OMEGA(I)

$\{\rho_i\}$ = ith mode shape

$\rho_{j,i}$ = mode shape coefficient, jth component of the ith mode
= PHI(J,I)

The normalization requirements are

$$\{\rho_i\}^T[M]\{\rho_j\} = \begin{cases} 0 & \text{if} \quad i \neq j \\ 1 & \text{if} \quad i = j \end{cases}$$

where [M] is the system mass matrix.

With the introduction of n_c damper at dof $J_1, J_2, \ldots J_{n_c}$, the damped eigenvalue problem can be formulated as the solution of the characteristic equation:

$$\det(I + \hat{R}(s)\hat{C}) = 0 \tag{A3.1}$$

where

$[\hat{R}]$ = n_c x n_c condensed receptance matrix with elements

$$\hat{R}_{ij} = R_{J_i J_j} \qquad \text{for } i = 1, n_c$$
$$\qquad\qquad\qquad \text{for } j = 1, n_c$$

The element R_{ij} are computing using

$$R_{ij} = \sum_{\ell=1}^{L} \frac{\rho_{i\ell}\, \rho_{j\ell}}{s^2 + \omega_\ell^2} \qquad \text{(A3.2)}$$

The following nomenclature is helpful in understanding the program

L = no. of modes used in the receptance calculation ($\leqslant$ number of DOF's)

n_c = NSTAT = no. of dampers introduced

J_i = ISTATNO(i) = ith damper location (DOF in the model) for i = 1 to NSTAT

$$[\hat{C}] = \begin{bmatrix} c_1 & & & 0 \\ & c_2 & & \\ & & \ddots & \\ 0 & & & c_{n_c} \end{bmatrix} = \text{condensed diagonal damping matrix}$$

c_i = CARRAY(I) = ith damper value, for i = 1 to n_c [located at J_i (or ISTATNO(i)]

Two types of variation of damping constants are allowed. For no variation, set NCD = 1, IDOPT = 0, and CDINC = 0.

A. IDOPT = 0, all dampers vary at the same time. For this case the initial damper values are c_i = CARRAY(I). Then all the c_i's are increased by CDINC after each pass through the code. There are a total of NCD increments as specified by the user.

B. IDOPT = 1, only one damper value changes, all other values remain constant. The initial damper values are input through the array CARRAY(I), and the initial damper values are

c_i = CARRAY(i) for i = 1 to n_c.

C_{n_d} will increase by CDINC each time the damper constants are updated.

C_{n_d} = CARRAY(n_d) + CDINC

n_d = NODEPL = the number of the particular damper in ISTATNO(I) being varied.

III Input Parameters

III.1 Formal Parameters

Description of the 11 formal parameters:

NELEM: number of DOF's in the original system
NMOD : the number of modes requested for the printout $\leq$ L
RUNNO: your run number identifier
IDOPT: 0 = all C values vary
1 = only one value varies (see NODEPL)
EPS : convergence parameter (used in the test for convergence in the Muller's Method root finding routine)
NDAM : number of modified system modes desired ($<$ L)
NUMITER: maximum number of iterations per eigenvalue
CDINIT: initial damper value (units should be consistent with mass and stiffness, e.g. lb sec/in)
CDINC : increment in damper vlaue
NCD : total number of damper increments
NODEPL: the number of the varied damper in ISTATNO(I). NODEPL is an element of 1,2,...,NSTAT.

Note: if IDOPT = 0,

$c_i = c_{io} + \Delta c$ for L = 1,2,...,NSTAT

if IDOPT = 1

$c_i = c_{io}$ $i \neq$ NODEPL

$c_i = c_{io} + \Delta c$, i = NODEPL

III.2 Common Blocks

In addition to these variables, the following variables enter the subroutine through the common blocks and must be evaluated prior to calling SPSTGN. Also, the common block set contained in the file COMSPST must be placed in the main program.

NTITLE(I): a holerith array with a title of the job
L: the number of modes used in the receptance calculation
OMEGA(I): a vector containing the L values of the original ω_ℓ
PHI(J,I): a matrix whose L columns are the modeshapes of the original system (DIM: NELEM x L)
Note: These modeshapes must be normalized to unit generalized mass.
NSTAT: number of stations where dampers are placed
ISTATNO(I): an array giving the location (DOF) of the attached dampers I = 1 to NSTAT
CARRAY(I): an array containing the damper value for position ISTATNO(I), I = 1 to NSTAT

IV. Using Subroutine SPTGN

The data required are input through formal parameters (Section III.1) and through the common block set COMSPST, into SPSTGN (Section III.2). It should be noted that mode shapes have to be normalized to unit generalized mass. That is,

$$\{\rho_i\}^T[M]\{\rho_i\} = 1.0$$

V. SAMPLE PROBLEM

The Model is the NASA 88 D.C.F. grid.
The model data is read from TAPE 12 is NASSAGG2, which contains data for all 88 modes, including 3 rigid body modes.
Four dampers are placed at D.O.F. 1, 11, 77, 88.
The initial damper values are 0.2.
The approximate receptance elements are to be calculated on the basis of using L(= 30**) modes of the 88 modes.

The first 2 modes are to be printed out (NMOD = 2).

All the damper values are to vary (IDOPT = 0) with increments of CDINC = 0.1 for NCD 5 trials.
The convergence criterion for eigenvalues is $|\det(A)| < \epsilon = 10^{-5}$.

The test program TESTSP is designed to interactively accept the above data and then call subroutine SPSTGN to do the calculations and print out the results.

The following input values were given (the remark number corresponds to the number in the following TESTSP printout).

Remark 3 TEST SPST (title)
2 88, 2, 30, 1 (88 DOF, 2 modes to be printed, use 30 modes, Run No. = 1)
3 0 (all damper values change)
4 The modal data was read from TAPE 12 NASAGG2
5 4, 1.E-S, 2, 50 (4 dampers ϵ (EPS) = 10^{-5}, computes 2 damped eigenvalues, max 50 iterations)
6 1, 11, 77, 88 (damper locations)
7 0.2, 0.2, 0.2, 0.2 (initial damper values)
8 0.2, 0.1, 5, 1 (initial damper values, increment, 5 increments, all dampers vary)

**An arithmatic underflow condition will appear if a larger L is used. We have had such an experience by using L = 88. This is due to the formulation used in teh receptance calculation. It can be improved if so desired. Our experience indicates that using L = 30 provides very good results.

VI. Program Listing and Sample Run

The subroutine SPSTGN is listed in Table 1. Table 2 lists a sample main program TESTSP which calls SPSTGN. The output of the sample problem is given in Table 3. The control cards necessary to run this sample problem is shown in Table 4.

Table 1 Listing of Subroutine SPSTGN

```
      SUBROUTINE SPSTGN(NELEM,NMOD,RUNNO,IDOPT,EPS,NDAM,NUMITER,
     &         CDINIT,CDINC,NCD,NODEPL)
C
C
C
C     SPSTG WRITTEN BY DR. A. PALAZZOLO  AND DR. B. P. WANG MAY , 1981
C      COMPUTER CODE TO GENERATE THE COMPLEX EIGENVALUES USING
C      REANALYSIS FORMULATION.
C
C
C
C     INPUT CARD DESCRIPTION
C
C
C     NELEM: NO. OF ELEMENTS
C     NMOD: NO. OF MODES REQUESTED FOR PRINTOUT AND DAMPER SENSITIVITY
C          ANALYSIS AND PLOTTING   @ 10 AND @ L
C     RUNNO: RUN NUMBER IDENTIFIER
C     IDOPT:  0=ALL C VALUES VARY
C             1= ONLY ONE VALUE VARIES
C     EPS: CONVERGENCE PARAMETER
C     NDAM: NUMBER OF MODIFIED SYSTEM MODES DESIRED  @ L-2 AND @20
C     NUMITER: MAXIMUM NUMBER OF ITERATIONS PER EIGENVALUE
C     CDINIT: INITIAL DAMPER VALUE FOR ROOT LOCUS PLOT  LP. SEC./IN
C         THIS VALUE WILL OVERIDE THE VALUE IN POSITION NODEPL , ENTERED INTO
C             CARRAY
C     CDINC: DAMPER INCREMENT FOR ROOT LOCUS PLOT   LP.SEC./IN
C     NCD: TOTAL NUMBER OF DAMPER INCREMENTS FOR ROOT LOCUS PLOT
C     NODEPL: THE NUMBER OF THE VARIED DAMPER IN ISTATNO(I)
C        I.E. NODEPL IS AN ELEMENT OF 1,2,...NSTAT.
C           THIS TELLS WHETHER NODEPL IS FIRST , SECOND ETC. IN ISTATNO
C
C
C
C  IN ADDITION THE FOLLOWING VARIABLES ENTER THE SUBROUTINE THROUGH THE
C   COMMON BLOCK:
C
C
C     L: NO. OF MODES FOR RECEPTANCE CALCULATION
C     ISTATNO(I): THE NSTAT DAMPER LOCATIONS (NODE NUMBERS)
C     CARRAY(I): THE DAMPER VALUES CORRESPONDING TO ISTATNO(I)
C     NSTAT: NUMBER OF STATIONS WHERE DAMPERS ARE ATTACHED
C     PHI(I,J): AN ARRAY CONTAINING THE UNDAMPED SYSTEMS EIGENVECTORS
C                NORMALIZED TO UNIT GENERALIZED MASS
C     OMEGA(I):  A VECTOR CONTAINING THE FREQUENCIES OF THE UNDAMPED SYSTEM
C     NTITLE:  AN ALPHA ARRAY CONTAINING THE TITLE OF THE PROJECT
C
C
C
      INTEGER ISTATNO
      DIMENSION IROW(100),TEMP(100)
      COMPLEX CIGEN,SLAMIN,VLAM,RLAM,COEIG,SLAMNXT,CMCOEIG,FII
      COMPLEX FHAT
      LOGICAL TEST
```

Table 1 Continued

```
C
C
      COMMON/BLK1/AEJ(50),ARLJ(50),ATJ(50),ARHJ(50),ARO(50)
      COMMON / BLK2 / SK(1,1)
      COMMON/BLK3/SSTR(88),SSROT(88)
      COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
     6 SM(1,1)
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
      COMMON/BLK8/L
      COMMON/BLK10/SMINV(1,1)
      COMMON/BLK12/WREX(105),WIEX(105)
      COMMON/BLK13/CMCOEIG(105)
      COMMON/BLK14/SENSD(1,1)
      COMMON/BLK15/A(1,1)
      COMMON/BLK16/FMR1
      COMMON/BLK20/IAUTO,REMAX,AIMAX
      COMMON/BLK21/STOM(83)
      COMMON/BLK22/FACT
      COMMON/BLK23/ISTATNU(16),NSTAT,CARRAY(16)
      COMMON/BLK24/R1VEC(16)
      COMMON/BLK25/IME
      COMMON/BLK26/FHAT(10,10)
      COMMON / BLK27 / NTITLE(30)
C         ********
C         *******************
           GM=1.0
           BETA=1.0
           IDIAG=0
C     READ *,NELEM,NMOD,NRIG,L,IFLAG,R,BETA,RUNNO
C     PRINT 1,NELEM,NMOD,NRIG,L,IFLAG,R,BETA,RUNNO
C   1 FORMAT(5I10,3F10.3)
      N=NELEM
      NNN=NELEM+1
C      READ *,INOPT,IDOPT
C     PRINT 75,INOPT,IDOPT
       IF(INOPT.EQ.0) GO TO 140
C      GO TO 250
140         CONTINUE
C
C
       DO 5010 I=1,L
C      READ(12,5011) OMEGA(I),GM
        STOM(I)=OMEGA(I)
C      READ(12,5011) (PHI(J,I),J=1,N)
5010    CONTINUE
5011   FORMAT(5E16.9)
5011    FORMAT(5X,2E16.9)
        GO TO 240
250         CONTINUE
      DO 5 I=1,NNN
C     READ *,SSTR(I),SSROT(I)
C     PRINT 6,SSTR(I),SSROT(I)
C   6 FORMAT(2F10.3)
C   5 CONTINUE
C     DO 10 I=1,NELEM
```

Table 1 Continued

```
C       READ *,AEJ(I),ARLJ(I),ATJ(I),ARHJ(1),ARO(I)
C       PRINT 15,AEJ(I),ARLJ(I),ATJ(I),ARHJ(I),ARO(I)
C    15 FORMAT(5F10.3)
C    10 CONTINUE
240          CONTINUE
C       READ *,NSTAT,EPS,NDAM,NUMITER,IDIAG
C       PRINT 20,NSTAT,EPS,NDAM,NUMITER,IDIAG
C    20 FORMAT(I10,E10.3,3I10)
C       READ 115,TMDFLT
C   115 FORMAT(I10)
C       READ *,(ISTATNO(I),I=1,NSTAT)
C       PRINT 75,(ISTATNO(I),I=1,NSTAT)
C    75 FORMAT(16I5)
C       READ *,(CARRAY(I),I=1,NSTAT)
C       PRINT 77,(CARRAY(I),I=1,NSTAT)
C    77 FORMAT(8F10.3)
C       READ *,CDINIT,CDINC,NCD,NODEPL
C       PRINT 55,CDINIT,CDINC,NCD,NODEPL
C    55 FORMAT(2F10.3,2I10)
C    65 FORMAT(2I10,2F10.3)
C
C

        PRINT 101,T
   101 FORMAT(1H-,10X,*TIME 1 - *,F10.3)
C            **********
C            ***********
C             REANALYSIS SECTION
C            **************
C       CARFAY(NODEPL)=CDINIT-CDINC
C             CALL INPR2(L,EPS,NDAM)
        DO 30 I=1,NCD
C       CARRAY(NODEPL)=CARRAY(NODEPL)+CDINC
              CALL INPR2(L,EPS,NDAM)
        DO 50 INDEIG=1,NDAM
        FACT=1.0
        CALL INITIAL(SLAMIN,INDEIG,BETA,I)
        DO 60 ITER=1,NUMITER
        CALL MULLER(SLAMNXT)
        CALL UPDATE(SLAMNXT,INDEIG)
        CALL CONVCHK(ITER,EPS,IEST)
        IME=ITER
        IF(IEST) GO TO 70
    60 CONTINUE
    70 CALL EIGSTOR(INDEIG)
        CALL NEXMODE(SLAMIN,INDEIG)
    50 CONTINUE
        CALL APEISOT(NDAM)
        CALL APEIPRT(NDAM)

        PRINT 103,T
          IF(IDOPT.EQ.0) GO TO 1100
         CARRAY(NODEPL)=CARRAY(NODEPL)+CDINC
          GO TO 30
```

Table 1 Continued

```
1100        DO 1000 ID=1, NSTAT
1000           CARRAY(ID)=CARRAY(ID)+CDINC
  103 FORMAT(1H ,2X,*TIME 3 = *,F10.3)
   30 CONTINUE
                 RETURN
      END
C
C
C
      SUBROUTINE INFR2(L,EPS,NDAM)
      INTEGER ISTATNO
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      PRINT 1
    1 FORMAT(1H ,40X,*BEGIN REANALYSIS PORTION OF CODE*)
      PRINT 20
   20 FORMAT(1H0,5X,*NODE*,10X,*DAMPER VALUE*)
      DO 30 I=1,NSTAT
      PRINT 35,ISTATNO(I),CARRAY(I)
   35 FORMAT(1H0,6X,I2,16X,E10.3)
   30 CONTINUE
      PRINT 3,L,EPS
    3 FORMAT(1H ,10X,*NO. OF MODES FOR RECEP. CALC.,L = *,
     3 I5,5X,*EPS = *,E10.3)
      PRINT 10,NDAM
   10 FORMAT(1H ,10X,*NUMBER OF MODIFIED SYSTEM MODES DESIRED = *,I3)
      RETURN
      END
      SUBROUTINE INITIAL(SLAMIN,INDEIG,BETA,I)
C        CALCULATE THREE STARTING VALUES FOR MULLER ITERATION
      INTEGER ISTATNO
      COMPLEX CIGEN,SLAMIN,VLAM,FLAM,COEIG,SLAM,RL,CMPLX,EE
      COMMON/BLK7/VLAM(3),FLAM(3),COEIG(20)
      COMMON/BLK8/L
      COMMON/BLK21/STOM(30)
      COMMON/BLK22/FACT
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      COMMON/BLK27/IME
            NPIG 0
            IPIAG 0
      ICE INDEIG
      IF(NKI .EQ.1) ICE INDEIG+1
      IF(IPIAG.EQ.0) GO TO 10
      PRINT 1,ICE
    1 FORMAT(1H ,40X,*ITERATION FOR EIGENVALUE *,I3)
   10 CONTINUE
      ZZ=STOM(ICE)
      EE CMPLX(-0.5,ZZ)
      EE EE/BETA
      IF(I.EQ.1) SLAMIN=EE
      IF(I.NE.1) SLAMIN=COEIG(INDEIG)/BETA
      VLAM(3)=SLAMIN*(1.0,0.001)
      VLAM(2)=1.001*SLAMIN
      VLAM(1)=SLAMIN
      SLAM=VLAM(3)
      CALL FOFLA(SLAM,RL,INDEIG)
```

Table 1 Continued

```
      RLAM(3)=RL
      SLAM=VLAM(2)
      CALL ROFLA(SLAM,RL,INDEIG)
      RLAM(2)-RL
      SLAM=VLAM(1)
      CALL ROFLA(SLAM,RL,INDEIG)
      RLAM(1)=RL
      AB=CABS(RLAM(1))
      TP=L
      TP=1.0/TP
      CB=AB**TP
      FACT=1.0/CB
      RLAM(3)=RLAM(3)/AB
      RLAM(2)-RLAM(2)/AB
      RLAM(1)=RLAM(1)/AB
      RETURN
      END
      SUBROUTINE MULLER(SLAMNXT)
C        CALCULATE NEXT EIGENVALUE GUESS BY MULLER METHOD
      COMPLEX SLAMNXT,VLAM,RLAM,COEIG,CSQRT,FZJ3J2,FZJ1J2,FZJ1J3,
     6 FJ1J2J3,W,SQ,AC1,AC2,DEN
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
      FZJ3J2=(RLAM(2)-RLAM(3))/(VLAM(2)-VLAM(3))
      FZJ1J2=(RLAM(2)-RLAM(1))/(VLAM(2)-VLAM(1))
      FZJ1J3-(RLAM(3)-RLAM(1))/(VLAM(3)-VLAM(1))
      FJ1J2J3-(FZJ3J2-FZJ1J2)/(VLAM(3)-VLAM(1))
      W=FZJ1J2+FZJ1J3-FZJ3J2
      SQ=W**2-4.0*RLAM(1)*FJ1J2J3
      SQ=CSQRT(SQ)
      AC1=W+SQ
      AC2=W-SQ
      T1=CABS(AC1)
      T2-CABS(AC2)
      IF(T1.GE.T2) DEN-AC1
      IF(T2.GT.T1) DEN-AC2
      SLAMNXT VLAM(1)-2.0*RLAM(1)/DEN
      RETURN
      END
      SUBROUTINE UPDATE(SLAMNXT,INDEIG)
      COMPLEX SLAMNXT,VLAM,RLAM,COEIG,RL
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
C        UPDATE THE LAMBDA (VLAM) AND F OF LAMBDA (RLAM) ARRAYS
      VLAM(3)=VLAM(2)
      VLAM(2)=VLAM(1)
      VLAM(1)=SLAMNXT
      CALL ROFLA(SLAMNXT,RL,INDEIG)
      RLAM(3) RLAM(2)
      RLAM(2)=RLAM(1)
      RLAM(1)-RL
      RETURN
      END
      SUBROUTINE CONVCHK(ITER,EPS,TEST)
      LOGICAL TEST
      COMPLEX VLAM,RLAM,COEIG,SLAM,TEST1,TEST2,TEST3,FII
```

Table 1 Continued

```
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
      COMMON/BLK25/IME
C         TEST FOR CONVERGENCE IN MULLER METHOD
           IDIAG=0
      EPS2=0.01*EPS
      SLAM=VLAM(1)
      TEST1=VLAM(1)-VLAM(2)
      TEST2=RLAM(1)
      TEST3=VLAM(3)-VLAM(1)
      CHECK1=CABS(TEST1)
      CHECK2=CABS(TEST2)
      CHECK3=CABS(TEST3)
      TEST=.FALSE.
      IF(CHECK1.LT.EPS.AND.CHECK2.LT.EPS) TEST=.TRUE.
      IF(CHECK1.LT.EPS2) TEST=.TRUE.
      IF(CHECK3.EQ.0.0) TEST=.TRUE.
      IF(IDIAG.EQ.0) GO TO 5
      PRINT 1,ITER,VLAM(1),RLAM(1),CHECK1,CHECK2,CHECK3
    1 FORMAT(1H ,1X,I2,1X,E12.5,1X,E12.5,3X,E12.5,1X,E12.5,3(5X,E12.5))
    5 CONTINUE
      RETURN
      END
      SUBROUTINE EIGSTOR(INDEIG)
      COMPLEX VLAM,RLAM,COEIG
C         STORE MODIFIED SYSTEM EIGENVALUE
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
      COEIG(INDEIG)=VLAM(1)
      RETURN
      END
      SUBROUTINE NEXMODE(SLAMIN,INDEIG)
      COMPLEX SLAMIN,VLAM,RLAM,COEIG
                                      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(
      COMMON/BLK25/IME
C         THIS SUBROUTINE IS CURRENTLY NOT BEING USED.
C         SUBROUTINE INITIAL IS NOW MAKING THE INITIAL GUESS
      ICE=INDEIG

           IDIAG=0
           NPIG=0
      IF(NPIG.EQ.1) ICE=INDEIG+1
C         CALCULATE INITIAL CENTRAL EIGENVALUE GUESS FOR NXT MODE
      SLAMIN=1.001*COEIG(INDEIG)
C     PRINT 5,ICE,IME
    5 FORMAT(1H ,50X,*EIGENVALUE *,I3,5X,*NO. OF ITERATIONS = *,I5)
      RETURN
      END
      SUBROUTINE FOFLA(SLAM,FL,INDEIG)
       INTEGER ISTATNO
      COMPLEX SLAM,FL,VLAM,RLAM,COEIG,FII,H,CONJG,PHAT,ESC(10,10)
C         SUBROUTINE TO EVALUATE THE ITERATED FUNCTION
C     COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
      COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
     6 SM(1,1)
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
      COMMON/BLK8/L
```

Table 1 Continued

```
      COMMON/BLK22/FACT
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      COMMON/BLK26/FHAT(10,10)
C           PRINT 320,FACT
320        FORMAT(10X,*FACT=*,2E16.5)
C          PRINT 100,SLAM
100           FORMAT(10X,* SLAM + *,2E16.5/)
         NRIG=0
         BETA=1.0
      CALL RECEP(SLAM)
C       PRINT 100,SLAM
      DO 20 I=1,NSTAT
      DO 30 J=1,NSTAT
   30 BDC(I,J)=FHAT(I,J)*CARRAY(I)*SLAM*BETA
      BDC(I,I)=BDC(I,I)+(1.0,0.0)
   20 CONTINUE
      ISTAT=NSTAT
      IF(NSTAT.EQ.1) G=BDC(1,1)
      IF(NSTAT.EQ.1) GO TO 50
      CALL CDETER(ISTAT,BDC,G)
   50 CONTINUE
C          PRINT 100,SLAM
C               PRINT 310,G
310         FORMAT (10X,* G   *,2E16.5/)
      RL=G
      IF(NRIG.EQ.1) RL=G/SLAM
C         PRINT 310,G
C       PRINT 330,BETA
330           FORMAT(10X,*BETA=*,2E16.5)
      DO 1 I=1,L
C             PRINT 340,I
340    -      FORMAT(10X,* I=*,I5)
    1 RL=RL*(SLAM**2*(BETA**2)+OMEGA(I)**2)*FACT
      IF(INDEIG.EQ.1) GO TO 10
      INT=INDEIG-1
      DO 2 I=1,INT
C               PRINT 200,I,COEIG(I)
200         FORMAT(10X,*I,COEIG(I)*,I5,2E16.5)
      RL=RL/(SLAM-COEIG(I))
      RA=AIMAG(COEIG(I))
      IF(RA.EQ.0.0) GO TO 2
C             PRINT 110,CONJG(COEIG(I))
110        FORMAT(10X,*CONJG =*,2E16.5)
      RL=RL/(SLAM-CONJG(COEIG(I)))
    2 CONTINUE
   10 CONTINUE
      RETURN
      END
      SUBROUTINE RECEP(SLAM)
C        COMPUTE RECEPTANCES
C        MODES NORMALIZED TO UNIT MODAL MASS
      INTEGER ISTATNO
      COMPLEX SLAM,FII
      COMPLEX DEN,FHAT
```

Table 1 Continued

```
C     COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
      COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
     6 SM(1,1)
      COMMON/BLK8/L
      COMMON/BLK16/PMR1
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      COMMON/BLK26/FHAT(10,10)
            BETA=1.0
      DO 5 I=1,NSTAT
      IP=ISTATNO(I)
      DO 10 J=1,NSTAT
      FHAT(I,J)=(0.0,0.0)
      JP=ISTATNO(J)
      DO 20 K=1,L
   20 FHAT(I,J)=FHAT(I,J)+PHI(IP,K)*PHI(JP,K)/
     6 (SLAM**2*(BETA**2)+OMEGA(K)**2)
   10 CONTINUE
    5 CONTINUE
      RETURN
      END
      SUBROUTINE APEISOT(NDAM)
      COMPLEX VLAM,RLAM,COEIG,DEN
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
C        ARRANGE MODIFIED SYSTEM EIGENVALUES  (APPROX)
C        ACCORDING TO INCREASING IMAG. PARTS. DE- TIME SCALE EIGENVALUES
            BETA=1.0
      DO 10 I=1,NDAM
   10 COEIG(I)=COEIG(I)*BETA
      NPC=NDAM-1
      DO 1 I=1,NPC
      JSTOR=0
      TMIN=1000000.0
      IST=I
      DO 2 J=IST,NDAM
      AI=AIMAG(COEIG(J))
      AI=ABS(AI)
      IF(AI.LE.TMIN) TMIN=AI
      IF(AI.LE.TMIN) JSTOR=J
    2 CONTINUE
      DEN=COEIG(I)
      COEIG(I)=COEIG(JSTOR)
      COEIG(JSTOR)=DEN
    1 CONTINUE
      RETURN
      END
      SUBROUTINE APEIPRT(NDAM)
      COMPLEX VLAM,RLAM,COEIG
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
C        PRINT THE MODIFIED SYTEM APPROX. EIGENVALUES
      PRINT 1
    1 FORMAT(1H ,22X,*MODIFIED SYSTEM APPROXIMATE EIGENVALUES*,12X,
     6 *ZETA*,13X,*OMEGAN SEC-1*)
            NRIG=0
      DO 10 I=1,NDAM
      ICE=I
```

Table 1 Continued

```
      IF(NRIG.EQ.1) ICE=I+1
      OMEGAN=CABS(COEIG(I))
      ZETA=-REAL(COEIG(I))/OMEGAN
      PRINT 20,ICE,COEIG(I),ZETA,OMEGAN
   20 FORMAT(1H ,10X,I5,10X,E12.5,3X,E12.5,13X,F13.4,11X,E12.5)
   10 CONTINUE
      RETURN
      END
      SUBROUTINE CDETER(NST,A,D)
C
C        DETERMINANT OF AN NST BY NST COMPLEX MATRIX WHERE NST>1
      COMPLEX A(10,10),D
      COMPLEX PMULT,BIG,DET,BG,TEMP
      NSIZ=NST
      ICOUNT=0
      NUMSYS=NSIZ-1
      DO 14 I=1,NUMSYS
C
C        SCAN FIRST COLUMN OF CURRENT SYSTEM FOR LARGEST ELEMNT
C         CALL THE ROW CONTAINING THIS ELEMENT, ROW NBGRW
C
      NN=I+1
      BIG=A(I,I)
      NBGRW=I
      DO 5 J=NN,NSIZ
      IF(CABS(BIG)-CABS(A(J,I))) 6,5,5
    6 BIG=A(J,I)
      NBGRW=J
    5 CONTINUE
      BG=1.0/BIG
C
C        SWAP ROW I WITH ROW NBGRW UNLESS I=NBGRW
C
      IF(NBGRW.NE.I) ICOUNT=ICOUNT+1
      IF(NBGRW-I) 7,10,7
C
C        SWAP A MATRIX ROWS
C
    7 DO 3 J=I,NSIZ
      TEMP=A(NBGRW,J)
      A(NBGRW,J)=A(I,J)
    3 A(I,J)=TEMP
C
C        ELIMINATE UNKNOWNS FROM FIRST COLUMN OF CURRENT SYSTEM
C
   10 DO 13 K=NN,NSIZ
C
C         COMPUTE PIVOTAL MULTIPLIER
C
      PMULT=-A(K,I)*BG
C
C        APPLY PMULT TO ALL COLUMNS OF THE CURRENT A MARRIX ROW
      DO 11 J=NN,NSIZ
   11 A(K,J)=PMULT*A(I,J)+A(K,J)
   13 CONTINUE
```

Table 1 Continued

```
   14 CONTINUE
      D=(1.0,0.0)
      DO 100 I=1,NSIZ
  100 D=D*A(I,I)
      JCOUNT=(ICOUNT/2)*2
      IF(JCOUNT.NE.ICOUNT) D=-D
      RETURN
      END
      SUBROUTINE HEADNG
      INTEGER ISTATNO
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      PRINT 1
    1 FORMAT(1H1,40X,*EISPACK(EXACT) SOLUTION OF THE MODIFIED SYSTEM*)
      PRINT 20
   20 FORMAT(1H0,5X,*NODE*,10X,*DAMPER VALUE*)
      DO 30 I=1,NSTAT
      PRINT 35,ISTATNO(I),CARRAY(I)
   35 FORMAT(1H0,6X,I2,16X,E10.3)
   30 CONTINUE
      RETURN
      END
      SUBROUTINE LABRT(ISW,LHOL,INX)
      DIMENSION LHOL(5)
      LOGICAL PS,TS
      IF((ISW.EQ.0).OR.(ISW.GT.5))RETURN
       GO TO (1,2,3,4,5) ISW
      DATA NP/10/,PS/.TRUE./,TS/.FALSE./
1     IF(PS.AND.(NP.GT.0)) PRINT 27,LHOL,INX
27    FORMAT(1H0,9X,5A10,3X,O6)
      NP=NP-1
      IF(TS) CALL EXIT
      RETURN
2     PS=.FALSE.
      RETURN
3     PS=.TRUE.
      NP=10
      RETURN
4     TS=.TRUE.
      RETURN
      TS=.FALSE.
      RETURN
      END
```

Table 2 Listing of Program TESTSP

```
      PROGRAM TESTSP(INPUT,OUTPUT,TAPE12)
C        SPSTG WRITTEN BY DR. A. PALAZZOLO  AND DR. B. P. WANG MAY , 1981
C        COMPUTER CODE TO GENERATE ROOT LOCUS PLOTS FOR A EULER BERNOULLI BEAM
C        MODEL WITH MULTIPLE DAMPERS.
C        THE BEAM IS SUPPORTED BY TRANSLATIONAL AND ROTATIONAL SPRINGS.
C        THIS CODE WAS DEVELOPED UNDER THE NASA LANGLEY SPACE STRUCTURE CONTRACT
      INTEGER ISTATNO
      DIMENSION IROW(100),TEMP(100)
      COMPLEX EIGEN,SLAMIN,VLAM,RLAM,COEIG,SLAMNXT,CMCOEIG,FII
      COMPLEX FHAT
      LOGICAL TEST
      COMMON/BLK1/AEJ(50),ARLJ(50),ATJ(50),ARHJ(50),ARO(50)
C     COMMON/BLK2/SK(88,88)
      COMMON / BLK2 / SK(1,1)
      COMMON/BLK3/SSTR(88),SSROT(88)
C     COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
      COMMON/BLK6/WR(88),Z(1,1),PHI(88,88),OMEGA(88),DYN(1,1),
     & SM(1,1)
      COMMON/BLK7/VLAM(3),RLAM(3),COEIG(20)
      COMMON/BLK8/L
      COMMON/BLK10/SMINV(1,1)
      COMMON/BLK12/WREX(105),WIEX(105)
      COMmON/BLK13/CMCOEIG(105)
C     COMMON/BLK14/SENSD(1,1)
      COMMON/BLK14/SENSD(1,1)
      COMMON/BLK15/A(1,1)
      COMMON/BLK16/PMR1
      COMMON/BLK20/IAUTO,PEMAX,AIMAX
      COMMON/BLK21/STOM(88)
      COMMON/BLK22/FACT
      COMMON/BLK23/ISTATNO(16),NSTAT,CARRAY(16)
      COMMON/BLK24/R1VEC(16)
      COMMON/BLK25/IME
      COMMON/BLK26/FHAT(10,10)
      COMMON / BLK27 / NTITLE(80)
C        ********
C     ***INPUT DATA SECTION
C        ****************
 (1)   READ 10,(NTITLE(I),I=1,80)
      PRINT 40,(NTITLE(I),I=1,80)
10        FORMAT(80A1)
 (2)  READ 1,NELEM,NMOD,L,RUNNO
      PRINT *,"NELEM,NMOD,L,RUNNO :",NELEM,NMOD,L,RUNNO
    1 FORMAT(2I10,3F10.3)
      N=NELEM
      NNN=NELEM+1
 (3)   READ 4,IDOPT
       PRINT 75,IDOPT
C
C       READ MODAL DATA FROM TAPE12
C
 (4)  DO 5010 I=1,L
      READ(12,6011) OMEGA(I),GM
       STOM(I)=OMEGA(I)
      READ(12,5011) (PHI(J,I),J=1,N)
5010   CONTINUE
5011  FORMAT(5E16.9)
6011   FORMAT(5X,2E16.9)
```

Table 2 Continued

```
(5)   READ *,NSTAT,EPS,NDAM,NUMITER
      PRINT *,'NSTAT,EPS,NDAM,NUMITER :',NSTAT,EPS,NDAM,NUMITER
 20   FORMAT(I10,E10.3,3I10)
115   FORMAT(I10)
(6)   READ *,(ISTATNO(I),I=1,NSTAT)
      PRINT 75,(ISTATNO(I),I=1,NSTAT)
 75   FORMAT(16I5)
(7)   READ *,(CARRAY(I),I=1,NSTAT)
      PRINT 77,(CARRAY(I),I=1,NSTAT)
 77   FORMAT(9F10.3)
(8)   READ *,CDINIT,CDINC,NCD,NODEPL
      PRINT 35,CDINIT,CDINC,NCD,NODEPL
 30   FORMAT(2F10.3,2I10)
 40   FORMAT(2I10,2F10.3)
C
C
          CALL DESIGN(NELEM,NMOD,RUNNO,IDOFT,EPS,NDAM,NUMITER,
     *                CDINIT,CDINC,NCD,NODEPL)
C
C
      END
```

Table 3 Output of Sample Problem

```
? TEST SPSI
TEST SPSI
? 88,2,34,1
 NELEM,NNOD,L,RUNNO :88 2 34 1
? 0
     0
? 4,1.E-5
? 2,50
 NSTAT,EPS,NDAM,NUMITER :4 .00001 2 50
? 1,11,78,88
     1    11    78    88
? .2,.2,.2,.2
       .200       .200       .200       .200
? .2,.1,5,1
       .200       .100          5          1
             TIME 1 =          -1
                                          BEGIN REANALYSIS PORTION OF CODE
0      NODE          DAMPER VALUE
0        1                  .200E+00
0       11                  .200E+00
0       78                  .200E+00
0       88                  .200E+00
             NO. OF MODES FOR RECEP. CALC.,L =    30      EPS =    .100E-04
             NUMBER OF MODIFIED SYSTEM MODES DESIRED =     2
                         MODIFIED SYSTEM APPROXIMATE EIGENVALUES                    ZETA              OMEGAN SEC-1
                 1          -.10938E-04    -.61737E-09                        1.0000             .10938E-04
                 2           .16491E-04     .81334E-05                        -.8967             .15578E 01
    TIME 3          -I
                                          BEGIN REANALYSIS PORTION OF CODE
0      NODE          DAMPER VALUE
0        1                  .300E+00
0       11                  .300E+00
0       78                  .300E+00
0       88                  .300E+00
             NO. OF MODES FOR RECEP. CALC.,L =    30      EPS =    .100E-04
             NUMBER OF MODIFIED SYSTEM MODES DESIRED =     2
                         MODIFIED SYSTEM APPROXIMATE EIGENVALUES                    ZETA              OMEGAN SEC-1
                 1          -.99563E-07    -.18945E-08                         .9998             .99581E-07
                 2           .14869E-06    -.10327E-06                        -.8213             .18104E-06
    TIME 3 =        -I
```

Table 3 Continued

```
                                        BEGIN REANALYSIS PORTION OF CODE
0      NODE            DAMPER VALUE
0        1                   .400E+00
0       11                   .400E+00
0       78                   .100E+00
0       88                   .100E+00
            NO. OF MODES FOR RECEP. CALC.,L =     30      EPS =    .100E-04
            NUMBER OF MODIFIED SYSTEM MODES DESIRED =      2
                       MODIFIED SYSTEM APPROXIMATE EIGENVALUES                    ZETA              OMEGAN SEC-1
                1            .27372E-07     -.15135E-07                          -.8773              .31540E-07
                2           -.19009E-07      .26869E-07                           .8777              .56067E-07
   TIME 3                 -I
                                        BEGIN REANALYSIS PORTION OF CODE
0      NODE            DAMPER VALUE
0        1                   .500E+00
0       11                   .500E+00
0       78                   .500E+00
0       88                   .500E+00
            NO. OF MODES FOR RECEP. CALC.,L =     30      EPS =    .100E-04
            NUMBER OF MODIFIED SYSTEM MODES DESIRED =      2
                       MODIFIED SYSTEM APPROXIMATE EIGENVALUES                    ZETA              OMEGAN SEC-1
                1           -.21993E-07      .14222E-07                           .8397              .26191E-07
                2            .16118E-07     -.16081E-07                          -.7086              .22789E-07
   TIME 3                 -I
                                        BEGIN REANALYSIS PORTION OF CODE
0      NODE            DAMPER VALUE
0        1                   .600E+00
0       11                   .600E+00
0       78                   .600E+00
0       88                   .600E+00
            NO. OF MODES FOR RECEP. CALC.,L =     30      EPS =    .100E-04
            NUMBER OF MODIFIED SYSTEM MODES DESIRED =      2
                       MODIFIED SYSTEM APPROXIMATE EIGENVALUES                    ZETA              OMEGAN SEC-1
                1           -.14834E-07      .55449E-08                           .9368              .15046E-07
                2            .12597E-07     -.79000E-08                          -.8491              .14956E-07
   TIME 3                 -I
     3.821 CP SECONDS EXECUTION TIME.
```

Table 4 Control Cards for Using SPSTGN

```
 COPY-[illegible]
/JOB
SFW,CM300000,T1000.
USER,[illegible].
CHARGE,[illegible].
GET,SPSTGN
GET,TAPE12=NASADC2.
GET,DATA=SPDI.
SPSTGN(DATA).
REWIND,OUTPUT.
COPYEI,OUTPUT,SPOUT.
PACK,SPOUT.
REPLACE,SPOUT.
DAYFILE,L=SPDAY
REPLACE,SPDAY
EXIT.
REWIND,OUTPUT.
COPYEI,OUTPUT,XSPOUT.
PACK,XSPOUT.
REPLACE,XSPOUT.
DAYFILE,L=XSPDAY
REPLACE,XSPDAY.
 EOR
 EOI ENCOUNTERED.
```

Appendix A-4

Program COPZ Documentation

Damper Gain Optimization with Fixed Modal Damping Ratios

I. INTRODUCTION

COPZ is a damping optimization program to compute minimum damping gains for specified modal damping ratios using a diagonal damping matrix. Subroutine COMMIN is used in the optimization process.

II. PROBLEM FORMULATIONS

When a diagonal damping matrix [C] is introduced into an originally undamped system, the damped eigenvalue can be found by solving

$$f(s) = \det[I + [\hat{R}(s)][\hat{C}]] = 0 \qquad \text{(A4.1)}$$

where

s = compelx eigenvalue of the damped system

$[\hat{C}]$ = submatrix of $[\Delta C]$ that contains only the non-zero terms of $[\Delta C]$

$[\hat{R}]$ = corresponding sub-matrix of the receptance matrix R

$$\hat{R}_{ij} = R_{J_i J_j} = \sum_{\ell=1}^{n} \frac{\varphi_{J_i \ell}\,\varphi_{J_j \ell}}{S^2 + \omega_\ell^2}$$

Let

$$[\hat{C}] = \begin{bmatrix} C_1 & & 0 \\ & \ddots & \\ 0 & & C_{N_C} \end{bmatrix}$$

The optimization problem is:

find c_i, $i = 1$ to N_c

such that $J_1 = \sum_{i=1}^{N_c}$ is minimized subject to the constraint

$$f(S_\ell) = 0, \quad \ell = 1 \text{ to } N_E$$

where

$$S_\ell = -\zeta_\ell \omega_{n_\ell} \pm j\sqrt{1 - \zeta_\ell^2}\, \omega_{n_\ell}$$

To convert the above constrained optimization problem into one without constraints, a penalty function approach is used. The problem solved by COPZ can then be rephrased as

find c_i, $i = 1$ to N_c

such that

$$J = \sum_{i=1}^{N_c} c_i + \sum_{\ell=1}^{N_E} W(f(S_\ell))$$

is minimized, where W is weighting function. Experience shows W = 1000 yields good results.

There are $N_c + N_E$ unknowns in the optimized problems. The first N_E of the unknowns are the unknown ω_{n_ℓ} in S_ℓ (with δ_ℓ specified). The last N_c unknowns are the damper gains.

II. INPUT TO COPZ

A. Input from terminal (or input from TAPE 5). All are free format.

1. Title. One line description of the problem.
2. NDOF,NMODE,NE,NC
   NDOF = no. of system dof
   NMODE = no. of modes to be used
   NE = no. of specificied ζ
   NC = no. of dampers.
3. JD(I),I = 1,NC)
   JD(I) = Ith damper location
4. X(I), I = 1,NC
   initial damper constant
5. NM(I), ETA(I), (I = 1 to NE)
   NM(I) = mode no.
   ETA(I) = ζ_i = specified damping ratio
6. WEIGHT
   WEIGHT = W is objective function J = 1000 usually

B. Modal Data Needed: (same as routine SPSTGN)

Read from TAPE 12:

For I = 1 to NMODE

WN(I),GM (Format 5X,2E16.9)

(PHI(JJ,I),I=1,NDOF) (Format 5E16.9)

Note: WN(I) = ω_i, GM(noticed), PHI(J,I) = $\rho_{j,i}$

IV. SAMPLE RUN OF COPZ

Table 1 shows the job file to run COPZ, include data for a sample problem. For this sample problem, we place 6 dampers at dof 1, 11, 39, 50, 78, and 88 of the 88-dof grillage model. We specify the model dampings to be 0.7 and 0.6 respectively for mode 4 and 5 (the first vibration modes). The results are:

$$C_1 = 0.214, \quad C_{11} = 0.268, \quad C_{39} = 0.146$$
$$C_{50} = 0.146, \quad C_{78} = 0.207, \quad C_{88} = 0.267$$

These results can be found in the end of Table 2.

Table 1 CONTROL Cards for use of Program COPZ

```
[illegible]
[illegible]
RFL,200000,T1000.
USER,4971215,
CHARGE,102928,LRC,
DELIMTR,1253...T...UUA
ATTACH,FTNMLIB/UN=LIBRARY
GET,...PEID=NAPASSL
GET,PSIMCON/UN=862E10N.
GET,ENEWC/UN=0126864.
GET,COPZ.
FTN,I=COPZ,PL=10000,B=F.
LDSET,LIB=FTNMLIB.
LOAD(F,PSIMCON,ENEWC)
EXECUTE.
REWIND,OUTPUT
COPYBF,OUTPUT,OTCOPZ.
PACK,OTCOPZ,
REPLACE,OTCOPZ
DAYFILE,DACOPZ.
REPLACE,DACOPZ.
EXIT.
REWIND,OUTPUT
COPYBF,OUTPUT,XOTCOPZ
PACK,XOTCOPZ.
REPLACE,XOTCOPZ.
DAYFILE,L=XDACOPZ.
REPLACE,XDACOPZ.
*EOR
      NASA GRILLAGE MODEL DAMPING OPTIMIZATION,VARY NMODE
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]
```

Table 2 COPZ Sample Run Output

```
  M100
S
            INPUT PROBLEM TITLE
     NASA GRILLAGE MODEL DAMPING OPTIMIZATION,VARY NMODE
            INPUT: NDOF,NMODE,NE,NC
 39 64 2 6
            INPUT DAMPER LOCATIONS
 1 11 39 50 78 98
            INPUT INITIAL DAMPER CONSTANTS
 .1 .1 .1 .1 .1 .1
            INPUT PAIRS OF: MODE NO. , DESIRED ZETA
 1 .7
 5 .6
            INPUT WEIGHT
 1000.


             COMPLETED READ MODAL DATA

1




                * * * * * * * * * * * * * * * * * * * * * * * * * * * *
                *                                                     *
                *                   S I M C O N                       *
                *                                                     *
                *              FORTRAN PROGRAM FOR                    *
                *                                                     *
                *            SIMPLIFIED CONMIN USAGE                  *
                *                                                     *
                * * * * * * * * * * * * * * * * * * * * * * * * * * * *



                   INPUT PARAMETERS

                   NUMBER OF DESIGN VARIABLES,  NDV     =     6
                   NUMBER OF CONSTRAINTS,       NCON    =     0
                   MAXIMUM ITERATIONS,          ITMAX   =    50
                   PRINT CONTROL,               IPRINT  =     5
                   SCALING PARAMETER,           NSCAL   =     1
                   CONVERGENCE CRITERIA,        DELFUN    .10000E-02
                   CONVERGENCE CRITERIA,        DABFUN    .10000E-04
```

Table 2 Continued

DESIGN VARIABLES

I	LOWER BOUND VLB(I)	INITIAL VALUE X(I)	UPPER BOUND VUB(I)
1	.10000E-02	.13740E+02	.27509E+02
2	.10000E-02	.14369E+02	.32771E+02
3	.10000E-02	.10000E+00	.10000E+05
4	.10000E-02	.10000E+00	.10000E+05
5	.10000E-02	.10000E+00	.10000E+05
6	.10000E-02	.10000E+00	.10000E+05
7	.10000E-02	.10000E+00	.10000E+05
8	.10000E-02	.10000E+00	.10000E+05

STORAGE REQUIREMENTS FOR WORK ARRAYS

ARRAY	DIMENSION	REQUIRED	USED
WK(NWK)	2000	272	306
IWK(NIWK)	200	46	47

```
 * * * * * * * * * * * * * * * * * * * * * * * * * * *
 *                                                     *
 *                   C O N M I N                       *
 *                                                     *
 *             FORTRAN PROGRAM FOR                     *
 *                                                     *
 *    CONSTRAINED FUNCTION MINIMIZATION                *
 *                                                     *
 * * * * * * * * * * * * * * * * * * * * * * * * * * *
X(I)
 .1374E+02     .1437E+02     .1000E+00     .1000E+00     .1000E+00
 .1000E+00     .1000E+00     .1000E+00
[illegible]
-- [illegible]
          FOR II=    1     DET= .1566E+00 -.1011E-01
X(I)
 .1374E+02     .1477E+02     .1000E+00     .1000E+00     .1000E+00
 .1000E+00     .1000E+00     .1000E+00
[illegible]
-9.0215773474·13.79540514630
          FOR II=    2     DET= .1524E+00 -.3977E-01
```

Table 2 Continued

```
CONSTRAINED FUNCTION MINIMIZATION

CONTROL PARAMETERS
?? P100
CONTROL PARAMETERS

IPRINT   NDV     ITMAX     NCON     NSIDE   ICNDIR    NSCAL    NFDG
  5       8       20        0        1        9         9        0

LINOBJ   ITRM     N1        N2       N3       N4        N5
  0       3       10        16       17       17        34

     CT                CTMIN              CTL              CTLMIN
 -.10000E+00        .40000E-02        -.10000E-01        .10000E-02

    THETA               PHI              DELFUN            DABFUN
  .10000E+01        .50000E+01         .10000E-03        .10000E-04

    FDCH               FDCHM             ALPHAX            ABOBJ1
  .10000E-02        .10000E-03         .10000E+00        .10000E+00

LOWER BOUNDS ON DECISION VARIABLES (VLB)
  1)      .10000E-02   .10000E-02   .10000E-02   .10000E-02   .10000E-[illegible]
  7)      .10000E-02   .10000E-02

UPPER BOUNDS ON DECISION VARIABLES (VUB)
  1)      .27209E+02   .32771E+02   .10000E+05   .10000E+05   .10000E+0[illegible]
  7)      .10000E+05   .10000E+05


INITIAL FUNCTION INFORMATION

OBJ        .32[illegible]E+0[illegible]

DECISION VARIABLES (X-VECTOR)
  1)      .137[illegible]E-02   .1[illegible]E+0[illegible]   .10000E+00   .10000E+00   .10000E+0[illegible]
  7)      .10000E+00   .10000E+00



BEGIN ITERATION NUMBER     1

NEW SCALING VECTOR (SCAL
   .1[illegible]E+02    .1[illegible]E+0[illegible]    .1000E+00    .1000E+00    .1000E+00    .10[illegible]
   .1000E+0[illegible]
```

Table 2 Continued

```
X(I)=
  .1375E+02     .1637E+02     .1000E+00     .1000E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.627771052219,9.822291127974)
            FOR II=    1     ,DET=  .1668E+00 -.4041E-01
X(I)=
  .1375E+02     .1637E+02     .1000E+00     .1000E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.821533574?1,13.09540516632)
            FOR II=    2     ,DET=  .1524E+00 -.3935E-01
X(I)=
  .1374E+02     .1639E+02     .1000E+00     .1000E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.618152899932,9.812475649325)
            FOR II=    1     ,DET=  .1668E+00 -.4044E-01
X(I)=
  .1374E+02     .1639E+02     .1000E+00     .1000E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.831375428615,13.10850057149)
            FOR II=    2     ,DET   .1524E+00 -.3945E-01
X(I)=
  .1374E+02     .1637E+02     .1001E+00     .1000E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.618152899932,9.812479649325)
            FOR II=    1     ,DET=  .1667E+00 -.4043E-01
X(I)=
  .1374E+02     .1637E+02     .1001E+00     .1000E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.82152357474,13.09[illegible])
            FOR II=    2     ,DET   .1523E+00 -.3934E-01
X(I)=
  [illegible]   .1637E+02     .1000E+00     .1001E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
[illegible]
            FOR II=    1     ,DET=  .1667E+00 -.4047E-01
X(I)=
  .1371E+02     .1637E+02     .1000E+00     .1001E+00     .1000E+00
  .1000E+00     .1000E+00     .1000E+00
SI=
(-9.821[illegible]7471,13.0954[illegible])
            FOR II=    2     ,DET   .1522E+00 -[illegible]
X(I)=
  [illegible]   [illegible]   .1000E+00     .1000E+00     .1001E+00
  [illegible]   .1000E+00     .1000E+00
SI=
(-9.618152899932,9.812478649325)
            FOR II=    1     ,DET=  .1665E+00 -.4014E-01
```

Table 2 Continued

```
X(I) =
  .1374E+02      .1637E+02      .1000E+00      .1000E+00      .1001E+00
  .1000E+00      .1000E+00      .1000E+00
=> P10=
  .1000E+00      .1000E+00      .1000E+00
=I=
(-9.8215538747 4,13.0954051 6432)
             FOR II=     2      -DET=   .1524E+00 -.3935E-01
X(I) =
  .1374E+02      .1637E+02      .1000E+00      .1000E+00      .1000E+00
  .1001E+00      .1000E+00      .1000E+00
Q1
( 9.8181028992 0,-9.812479 6453 25)
             FOR II=     1      -DET=   .1668E+00 -.1044E-01
X(I) =
  .1374E+02      .1637E+02      .1000E+00      .1000E+00      .1000E+00
  .1001E+00      .1000E+00      .1000E+00
Q1
( 9.8215538747 1,13.0954051 6432)
             FOR II=     2      -DET=   .1524E+00 -.3935E-01
X(I) =
  .1374E+02      .1637E+02      .1000E+00      .1000E+00      .1000E+00
  .1000E+00      .1001E+00      .1000E+00
Q1
[illegible]
             [illegible]
X(I) =
  .1374E+02      .1637E+02      .1000E+00      .1000E+00      .1000E+00
  .1000E+00      .1001E+00      .1000E+00
Q1 =
(-9.8215538747 1,13.0954051 6432)
             FOR II=     2      -DET=   .1523E+00 -.3931E-01
X(I) =
  [illegible]
  [illegible]

[illegible]
             [illegible]
X(I)
  [illegible]
  [illegible]

[illegible]
             [illegible]

  [illegible]  ACTIVE SIZE CONSTRAINTS

  [illegible]
    [illegible]
    [illegible]

  SEARCH DIRECTION  S VECTOR
    1)     .27301E+00    .11216E-01    .99563E+00    .10000E+01    .17605E+0
    2)     .99506E+00    .99630E+00

  ONE-DIMENSIONAL SEARCH
  INITIAL SLOPE =  -.7099E+00  PROPOSED ALPHA =   .1444E-01
```

Table 2 Continued

```
* * * CONSTRAINED ONE-DIMENSIONAL SEARCH INFORMATION * * *

     PROPOSED DESIGN
     ALPHA =   .46436E-01
     X-VECTOR
     .1398E+02   .1638E+02   .1046E+00   .1046E+00   .1007E+00   .1007E+00
 X(I)-
    .1398E+02      .1638E+02      .1046E+00      .1046E+00      .1007E+00
   .1007E+00      .1046E+00      .1046E+00
 SI
 (-9.7849745927[illegible],9.9826708184[illegible])
            FOR II=     1       -DET=   .1506E+00 -.3830E-01
 X(I)-
    .1398E+02      .1638E+02      .1046E+00      .1046E+00      .1007E+00
    .1007E+00      .1046E+00      .1046E+00
 SI
 (-9.[illegible],10.[illegible])
            FOR II=     2       -DET=   .1349E+00 -.3740E-01

     OBJ =    .2879E+03

     TWO-POINT INTERPOLATION


     PROPOSED DESIGN
     ALPHA =   .73219E+00
     X-VECTOR
     .1197E+02   .1642E+02   .1232E+00   .1232E+00   .1036E+00   .1036E+00
 X(I)-
    [illegible]      [illegible]      [illegible]      [illegible]      [illegible]
    [illegible]      [illegible]      [illegible]
 SI
 [illegible]
            FOR II=     1       -DET=   [illegible] - [illegible]
 X(I)-
    [illegible]      [illegible]      [illegible]      [illegible]      [illegible]
    [illegible]      [illegible]      [illegible]
 SI
 [illegible]
            FOR II=     [illegible]       -DET=   [illegible]   [illegible]

 [illegible]

            FOR II=     [illegible]       -DET= - [illegible] - [illegible]
 [illegible]
 [illegible]
```

Table 2 Continued

```
B'
Y(I)=
 .1709E+02      .1631E+02      .2143E+00      .2679E+00      .1457E+00
 .1459E+00      .2671E+00      .2141E+00
SI=
(-11.9613645795 6,12.2030327307 8)
          FOR II=    1      ,DET= -.5981E-03  .4868E-03
Y(I)
 .1709E+02      .1631E+02      .2143E+00      .2679E+00      .1457E+00
 .1150E+00      .2671E+00      .2141E+00
SI=
(-9.7884740322 77,13.0512987377)
          FOR II=    2      ,DET= -.2748E-05 -.9243E-06

   OBJ =   .20290E+01

   TWO-POINT INTERPOLATION


   PROPOSED DESIGN
   ALPHA     .17001E+05
   X-VECTOR
 .1709E+02    .1631E+02    .2143E+00    .2679E+00    .1457E-00    .1458E+00
Y(I)
 .1709E+02      .1631E+02      .2143E+00      .2679E+00      .1457E+00
 .1158E+00      .2671E+00      .2141E+00
SI
(-11.9613729708 2,12.2030412926)
          FOR II=    1      ,DET= -.5981E-03  .4968E-03
Y(I)=
 .1709E+02      .1631E+02      .2143E+00      .2679E+00      .1457E+00
 .1459E+00      .2671E+00      .2141E+00
[illegible]
          [illegible]               ,DET= [illegible]E-05 -.[illegible]E-06

   [illegible]   [illegible]E+01

   [illegible] INTERPOLATION


   [illegible] DESIGN
   ALPHA     .5612[illegible]E-01
   X-VECTOR
 [illegible]    .1631E+02    .2143E+00    .2679E+00    [illegible]E+00    .1458E+00
[illegible]
 [illegible]      .1631E+02      [illegible]E+00      [illegible]E+00      .1457E+00
 [illegible]      .2671E+00      .2141E+00

 [illegible]-02,12.2030[illegible]734,
          [illegible] II=    1      ,DET  -.5981E-03  .4868E-03
```

Table 2 Continued

```
X(I)
  .1709E+02      .1631E+02      .2115E+00 = C1   .2679E+00 = C11   .1457E+00 = C39
  .1458E+00      .2671E+00      .2141E+00
       = C50          = C78          = C88
SI=
(-9.788516344936,13.05135E12=61)
         FOR II=    2      ,DET= -.2650E-05 -.9123E-06

    OBJ =    .20290E+01

    * * * END OF ONE-DIMENSIONAL SEARCH

    CALCULATED ALPHA =    .33881E-20

    OBJ =    .20289?E+01     NO CHANGE ON OBJ

    DECISION VARIABLES (X-VECTOR)
      1)     .17098E+02   .16314E+02   .21446E+00   .26794E+00   .14565E+0
      7)     .26712E+00   .21408E+00


    FINAL OPTIMIZATION INFORMATION

    OBJ =    .20289?E+01
                                                C1            C11           C39
    DECISION VARIABLES (X-VECTOR)
      1)     .17098E+02   .16314E+02   .21446E+00   .26794E+00   .14565E+0
      7)     .26712E+00   .21408E+00
                C78          C88
    THERE ARE    0 ACTIVE SIDE CONSTRAINTS
```

V. Listing of Program COPZ

```
COPY-COPZ
          PROGRAM  COPZ(INPUT=DU,PUT,TAPE5=INPUT,TAPE6=OUTPUT, [illegible]
C
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      NOTE: THIS IS A MODIFICATION OF C[illegible]
C
C
C    [illegible]     COPZ : OBJ = SUM ( DAMPING ) + SUM ( WEIGHT * DE ) )
C                              NO CONSTRAINTS
C
C
C                              9/14/81        3PM
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

          DIMENSION  [illegible]
         [illegible]
          COMMON  [illegible]
          COMMON  [illegible]
          COMMON  [illegible]
          COMMON  [illegible]
          COMMON  [illegible]
          COMMON  [illegible]
          COMMON  [illegible]
C
C
          COMPLEX H
C
C
            [illegible]
C

[illegible]
[illegible]

            [illegible]
            [illegible]
            [illegible]
          [illegible]
          FORMAT( [illegible]

          [illegible]
          [illegible]

          [illegible]
          [illegible]

          [illegible]
          [illegible]
          READ(5,*)(JD(I),I=1,NC)
          WRITE(6,*)(JD(I), [illegible]
```

Program Listing Continued

```
C
C
      PRINT 10
10    FORMAT(10X,*INPUT INITIAL DAMPER CONSTANTS*)
C
C
      NE1=NE+1
      NDV=NE-NP
      READ(5,*) (X(I),I=NE1,NDV)
      WRITE(6,*)(X(I),I=NE1,NDV)
C
C
      PRINT 10
10    FORMAT(10X,*INPUT PAIRS OF MODE NO. & DESIRED DATA*)
      DO 55 I=1,NE
      READ(5,*)NM(I),ETA(I)
55    WRITE(6,*)NM(I),ETA(I)
C
C
      PRINT 10
10    FORMAT(10X,*INPUT WEIGHT*)
      READ(5,*)WEIGHT
      WRITE(6,*)WEIGHT
[illegible]
C
C     INPUT MODAL DATA
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 5010 I=1,NMODE
      READ(10,5011)WN(I),GM
5010  READ(10,5011) [illegible]
5011  FORMAT(E11.5)
[illegible]
[illegible]
[illegible]
C
[illegible]
C
[illegible]
C
C
[illegible]
C
C
      DO 300 I=1,NE
      TI=NM(I)
[illegible]
[illegible]
```

Program Listing Continued

```
300       VLB(I)=0.001
C
C
          DO 400 I=NE1,NDV
          VLB(I)=0.001
400       VUB(I)=10000.
C
C
          IGMAX=0
          CALL SIMCON(NDV,X,VLB,VUB,IGMAX,30,E-1,0.0001,0.00001,FUN,GR,
     *     2000,IWK,300,OBJ,G,0,0.001,0.0001,IER)
          STOP
          END
C
C
                SUBROUTINE RSOP(B,Y)
C
C
          DIMENSION B(*)
          COMMON /DATA2/B(NI,EB),W(88)
          COMMON /DATA/NE,NL,WEIGHT
          COMMON /PAINT/[illegible]
          [illegible]
          COMMON /[illegible]/[illegible]
C
C
          DO 100 I=1,NE
            DO 100 J=1,NE
            [illegible]
            H(I,J)=0.
            [illegible]
              [illegible]
          [illegible]
          [illegible]
 [illegible] [illegible]
            [illegible]
100         [illegible]
C
C
          [illegible]
            [illegible]
            [illegible]
200       H(I,J)=[illegible]
C
C
          DO [illegible] I=1,NE
 [illegible] [illegible]
          [illegible]
          [illegible]
C
```

Program Listing Continued

```
          SUBROUTINE FUN [illegible]-Y,NCON,OBJ-B)
C
C
        COMPLEX SI,DET,H
        COMMON /DATA2/F(30,88),WN(28)
        COMMON /DATA3/ETA(30),HM(30)
        COMMON /DATA4/NE,NC,WEIGHT
            DIMENSION X(30)
        DIMENSION B(1
        COMMON /DATA5/EPS
        COMMON /DATA7/H(6,6)
C
C
C
        IG=0
      OBJ=0
        NTT NC+NE
        DO 1000 II=1,N
        A=ETA(II)
        TM HM(II)
        SIR [illegible]
        SII [illegible]
        SI [illegible]
C
        [illegible]
        [illegible]
 [illegible]   [illegible]
        PRINT [illegible]
        WRITE(6,*) SI
        CALL [illegible](SI,X)
C
C   EVALUATE COMPLEX DETERMINANT
C
        [illegible]
 [illegible]    [illegible]
 [illegible]    [illegible]
        CALL [illegible]
        [illegible]
        [illegible]

        [illegible]
        [illegible]
 [illegible]    [illegible]
C
        [illegible]
        [illegible]
 [illegible]    [illegible]
          RETURN
        END
C
C
```

```
      SUBROUTINE IDETUR(NST,D)
C
C        DETERMINANT OF AN NST X NST COMPLEX MATRIX WHERE NST [illegible]
          COMMON /DATA/A(6,6)
      COMPLEX D,A
      COMPLEX PMULT,SIG,DET,SS,TEMP
      NSIZ=NST
C       DO 300 I=1,NST
C300             PRINT 310,(A(I,J),J=1,NST)
C310         FORMAT(10X,'A(I,J)= ',4AE14.7)
      ICOUNT=0
      NUMSYS=NSIZ-1
      DO 14 L=1,NUMSYS
C
C        SCAN FIRST COLUMN OF CURRENT SYSTEM FOR LARGEST ELEMENT
C         CALL THE ROW CONTAINING THIS ELEMENT, ROW NBGRW
C
      [illegible]
      [illegible]
      [illegible]
      [illegible]
      [illegible]
    [illegible]
      [illegible]
    [illegible] CONTINUE
      [illegible]
C
C        [illegible]
C
      IF(NBGRW.NE.I) ICOUNT=ICOUNT+1
      IF(NBGRW-I) 7,10,7
C
C        SWAP A MATRIX ROWS
C
    [illegible]
      [illegible]
      [illegible]
    [illegible]
C
C        [illegible]
C
      [illegible]
C
C         [illegible]
C
      [illegible]
C
C        [illegible]
C
    [illegible]
    [illegible]
    [illegible]
        [illegible]
       [illegible]
C             PRINT 200,I,A(I,I),D
C200          FORMAT(20X,'I,A(I,I),D = ',I3,4E14.7
```

Program Listing Continued

```
100 D=D*A(I,I)
    JCOUNT=(ICOUNT/2)*2
    IF(JCOUNT.NE.ICOUNT) D=-D
    RETURN
    END
EOI ENCOUNTERED.
```

Appendix A-5

Program NORMAL Documentation

Mode Shape Ortho Normalization

I. INTRODUCTION

NORMAL is a FORTRAN program that can be used to orthonormalize the mode shapes computed by EISPACK routines (such as RESV). For zero-frequency rigid body modes, Gram-Schmidt orthogonalization procedure (see the Appendix A5.1) is used to make the rigid body modes orthogonal to each other with respect to the mass matrix. All modes are then normalized to unit generalized mass. Currently up to 3 rigid body modes can be accepted.

II. INPUT TO NORMAL

The following data is needed to use NORMAL:

1. Input N,NR,L where N = no. of dof of the system

   NR = no. of rigid body
   (≤ 3) mode
   L = number of elastic modes to be normalized

2. Read from TAPE 20

   For I = 1 to L
   WN(I), (V(J,I),J = 1,N)

where

WN(I) = ω_i = natural frequency of the ith mode, rad/sec
V(J,I) = ϕ_{ji} = mode shape coefficient at dof j of ith mode

3. Read from TAPE 10

   NJ,(AM(I), I = 1,NJ)
   NJ = no. of system dof

where

AM(I) = m_i = lumped mass of ith dof

Note: Both TAPE 10 and TAPE 20 are created by unformatted WRITE in a prior analysis

III. PROGRAM LISTING

```
COPY NORM
      PROGRAM NORM(INPUT,OUTPUT,TAPE10,TAPE12-TAPE20)
C
C
C        N=NO. OF THE SYSTEMS D.O.F.(UNNORMALIZED),
C        NR=NO. OF RIGID BODY MODES(<=3)
C        L=NO. OF ELASTIC MODES TO BE NORMALIZED TO UNIT GEN. MASS
C        NATURAL FREQ.(RAD./SEC.) AND MODE SHAPES ARE STORED ON TAPE12
C          (USE FREE FORMAT)
C        DIAGONAL ELEMENTS OF THE MASS MATRIX ARE STORED ON TAPE10.
C        THE NORMALIZED MODAL MATRIX IS WRITTEN ON TAPE12.
C
C
C
      DIMENSION WN(88),V(88,88),Z(88,3),AM(88),PHI(88,88),TT(88)
C
C
         READ*,N,NR,L
      DO 300 I=1,L
300   READ(20) WN(I),( V(J,I),J=1,N )
      READ(10) NJ, ( AM(I),I=1,NJ )
C
C
C
C
C        NORMALIZE MODAL MATRIX
C
C
      DO 1000 I=1,N
      G2 = 0.0
      DO 2000 J=1,N
2000  G2=G2 + V(J,I)*V(J,I)*AM(J)
      GM = SQRT(G2)
      DO 3000 J=1,N
3000  V(J,I) = V(J,I)/GM
1000  CONTINUE
C
C
C        ORTHONORMALIZE RIGID BODY MODES
C
C
         IF (NR.LE.1) GO  TO 9999
      DO 4100 I=1,N
4100  Z(I,1) = V(I,1)
      A = 0
      DO 4200 I=1,N
4200  A= A - V(I,1)*AM(I)*V(I,2)
      G2 = 0
      DO 4300 I=1,N
      Z(I,2) = A*V(I,1) + V(I,2)
4300  G2 = G2 + Z(I,2)*Z(I,2)*AM(I)
      GM = SQRT(G2)
      DO 4400 I=1,N
```

```
4400   V(I,2) = Z(I,2)/GM
          IF (NR.LT.3) GO TO 9999
       C2 = 0
       C1 = 0
       DO 5100 I=1,N
       C1 = C1 - Z(I,1)*AM(I)*V(I,3)
5100   C2 = C2 - Z(I,2)*AM(I)*V(I,3)
       G2 = 0
       DO 5200 I=1,N
       Z(I,3) = C1*Z(I,1) + C2*Z(I,2) + V(I,3)
5200   G2 = G2 + Z(I,3)*Z(I,3)*AM(I)
       GM = SQRT(G2)
       DO 5300 I=1,N
5300   V(I,3) = Z(I,3)/GM
          PRINT * , WN(1),(V(J,1),J=1,N)
       GM = 1.0
       DO 6000 I=1,N
       WRITE(12)  WN(I), GM
6000   WRITE(12) ( V(J,I),J=1,N )
5011   FORMAT(5E16.9)
6011   FORMAT(5X,2E16.9)
9999      CONTINUE
       STOP
       END
```

APPENDIX A5.1

Gram-Schmidt Orthogonalization Prodecure

n independent vectors $\{v_i\}$ can be transformed into vectors $\{p_i\}$ orthogonal with respect to a matrix [m] using the following recursion equations:

$$\{p\}_1 = \{r\}_1$$
$$\{p\}_i = \{r\}_i + \alpha_{i1}\{p\}_i + \alpha_{i2}\{p\}_2 + \ldots + \alpha_{i,i-1}\{p\}_{i-1}$$

where

$$\alpha_{ij} = -\frac{\{p\}_j^T[m]\{r\}_i}{\{p\}_j^T[m]\{r\}_j}$$

APPENDIX B

Active Damper Analysis - Linear

Figure (B.1) shows system 2.2.2.1, it is essentially the same as Figure (5.6) except that definitions of displacement coordinates have been added. Note that the concept is essentially different to that used in the prototype design, which uses a long-stroke actuator without a linkage.

Definitions

- C – Damping constant
- D – Voice coil, i.e., shaker, displacement
- f – Frequency
- F – Shaker force
- K – Ratio of maximum structural amplitude to maximum voice-coil amplitude at design conditions
- m_1 – shaker mass
- m_2 – Moving mass
- P – Rate of energy absorption
- R – Linkage ratio
- W – Energy absorbed from structure
- x – Displacement of structure
- y – Displacement of m_2
- z – Relative displacement
- ω – Radial frequency

Subscripts

- AL – Amplitude limited
- CH – Characteristic
- C – Per cycle
- D – Design
- FL – Frequency limited
- M – Maximum
- O – Complex amplitude
- T – Total

Referring to Figure (B.1), the basic damping equation is

$$F/R = c\,dx/dt = m_2 d^2y/dt^2 = m_2(d^2x/dt^2 + d^2z/dt^2) \qquad (B.1)$$

and, assuming SHM with complex amplitudes of x_0, z_0, respectively

$$|z_0| = |x_0|\{1 + (C^2/\omega^2 m_2{}^2)\}^{1/2} \qquad (B.2)$$

If motion is amplitude-limited, so that

$$|z_0|_{AL} = RD_M \qquad (B.3)$$

then

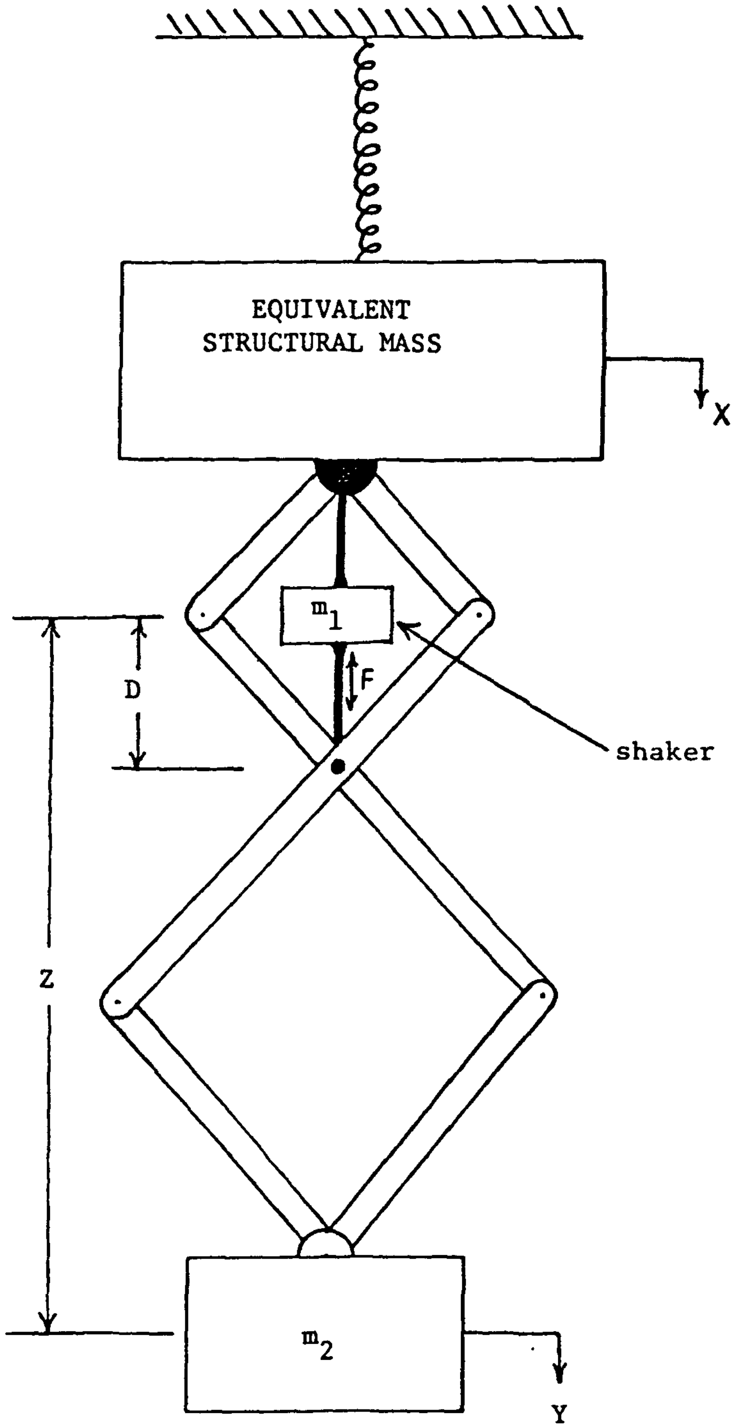

Fig. (B.1) System 2.2.1 Active Damper

$$|x_o|_{AL} = RD_M/\{1 + (C^2/\omega^2 m_2{}^2)\}^{1/2} \tag{B.4}$$

whereas, if the motion is force-limited, we get, from B1

$$|x_o|_{FL} = F_M/\omega cR \tag{B.5}$$

Defining design conditions as those for which amplitude limits and force limits coincide, so that

$$|x_o|_{AL} = |x_o|_{FL} = |x_o|_D \tag{B.6}$$

and introducing the ratio

$$K = |x_o|_D/D_M \tag{B.7}$$

one can eliminate c from (B.4) and (B.5) to get the biquadratic equation

$$(R/K)^4 - (R/K)^2 - (f_{CH}/Kf_D)^4 = 0 \tag{B.8}$$

where f_{CH} is the characteristic frequency given by

$$f_{CH} = \frac{1}{2\pi} \{F_M/m_2 D_M\}^{1/2} \tag{B.9}$$

and f_d is the design frequency. Solving A8

$$(R/K)^2 = 1/2 + 1/2\{1 + 4(f_{CH}/Kf_D)^4\}^{1/2} \tag{B.10}$$

thus, assuming that K, f_D, and f_{CH} are given, the linkage ratio R can be found, and the linkage can be designed. The damping constant c can be found by substituting back into (B.4) and assuming design conditions

$$C = 2\pi f_D m_2 \{(R^2/K^2) - 1\}^{1/2} \tag{B.11}$$

Note that the electronic system is designed so that the damping constant c is a constant regardless of frequency. Thus, on rearranging (B.4), and substituting from (B.11)

$$\frac{|x_o|_{FL}}{|x_o|} = \frac{R/K}{\{(R/K^2 - 1)(f_D^2/f^2) + 1\}^{1/2}} \; ; \; f < f_D \tag{B.12}$$

Equation (B.12) defines the ratio of structural amplitude at frequency f to the design amplitude. It is less then unity. For frequencies above the design frequency, the structural amplitude is force limited, so that, according to (B.1)

$$\frac{|x_o|_{FL}}{|x_o|_D} = \frac{f_D}{f} \; ; \quad f > f_D \tag{B.13}$$

The above two equations define structural amplitudes for which the damping system would remain linear under conditions of SHM. If the structure were to vibrate at a single frequency, they would constitute an envelope of permissible amplitudes. The situation with complicated structural responses would be much more difficult to analyze.

The energy removed from the structure per cycle is

$$W_c = \oint c\dot{x} \, dx = 2\pi^2 f c |x_o|^2 \tag{B.14}$$

thus, at design conditions

$$W_D = 2\pi^2 f_D c |x_o|^2_D \tag{B.15}$$

and, on substitution from (B.5) and (B.7)

$$W_D = W_m(R/K) \tag{B.16}$$

where

$$W_M = \pi F_M D_M \tag{B.17}$$

is the maximum energy which can be removed by the shaker. Since R/K is always greater than unity, W_D is always less than W_M, the discrepancy being due to phase differences between the structural and damper motions.

From (B.14) and (B.15)

$$\frac{W_c}{W_D} = \frac{f}{f_D} \quad \frac{|x_o|^2}{|x_o|^2}$$

thus, for the amplitude limited case of (B.12)

$$\frac{W_c}{W_D} = \frac{(f/f_D)(R^2/K^2)}{(R^2/K^2-1)(f_D^2/f^2) + 1} \; ; \; f < f_D \tag{B.19}$$

while for the force-limited case of (B.13)

$$\frac{W_c}{W_D} = \frac{f_D}{f} \; ; \; f > f_D \tag{B.20}$$

The rate of removal of energy is given by

$$P = f\ W_c \tag{B.21}$$

thus, above the design frequency

$$P = P_D = f_D W_D; \quad f > f_D \tag{B.22}$$

while, below the design frequency, we have from (B.19)

$$P/P_D = \frac{(f^2/f_D^2)(R^2/K^2)}{(R^2/K^2-1)(f_D^2/f^2) + 1} \tag{B.23}$$

Taking the Goodman V102 shaker as representative of current practice, we use the following values as typical:

mass, $m_1 = 2\ lb_m = 0.9072$ kg

maximum force, $F_M = 2\ lb_f = 8.890$ N

P-P deflection, $2D_M = 0.2" = 0.00508$ m

Max work per cycle, W_M (from (B.17)) = 0.0709 Joules

Max work per cycle per unit mass, $W_M/m = 0.0782$ J/kg

As an example, consider a design condition of one inch structural amplitude, $|x_0|_D$, at a frequency f_D of one Hertz. Then from (B.7)

$$K = |x_o|_D/D_M = 10$$

The following three sets of calculations are for three values of the moving mass m_2

Case	1	2	3	Dimensions
m_2	0.09072	0.9072	9.072	kg
f_{CH} (from A9)	31.26	9.886	3.126	Hz
R/K (from A10)	3.207	1.264	1.005	
c (from A11)	1.737	4.407	5.707	Ns/m
W_D (from A16)	0.0221	0.0561	0.0705	J
$m_T = m_1 + m_2$	0.9979	1.8144	9.9792	kg
W_D/m_T	0.0221	0.0309	0.0071	J/kg
P/m_T (from A22)	0.0221	0.0309	0.0071	W/kg

It will be noted that case 2, for which the magnet and moving mass are equal, gives the best performance in terms of watts of energy removed per kg for frequencies above one Hz. The damping constant c achieved is also close to the maximum. If system 2.2.2.1 in Figure (5.7) were used instead, the total mass m_T would be 0.0972 kg, and the performance of the system would otherwise be identical to case 2 above. Thus, values for W_D/m_T and P/m_T would be 0.0619 J/kg and 0.619 w/kg respectively. This value for W_D/m_T is close to the maximum value of W_M/m obtainable with this voice-coil shaker.

APPENDIX C

Use of Piezoelectric Materials

Polyvinylidene flouride (PVF_2) exhibits the following relationship between strain e_1, normal electric field, E_3, and stress σ_1 (see Figure (C.1))

$$e_1 = -d_{31}E_3 + (1/C^E_{11})\sigma_1 \qquad (C.1)$$

where d_{31} = transverse piezoeletric charge coefficient

C^E_1 = modulus of elasticity

Assuming SHM with maximum values of e_M, E_M and σ_M, such that e_1 and σ_1 are 90° out of phase with each other, and 45° out of phase with E_3.

$$\sigma_M = d_{31}C^E_{11}\ E_M/\sqrt{2} \qquad (C.2)$$

$$e_M = d_{31}\ E_M/\sqrt{2} \qquad (C.3)$$

and the maximum energy absorbed per unit volume is

$$w_M = \pi\sigma_M e_M = \pi\ d^2_{31}\ C^E_{11}\ E_M{}^2/2 \qquad (C.4)$$

while

$$W_M/m = w_M/\rho = \pi\ d^2_{31}\ C^E_{11}\ E_M{}^2/2\rho \qquad (C.5)$$

Typical values are

d_{31} = 5 to 37 x 10^{-12} m/V

C^E_{11} = 1.6 to 3.8 x 10^9 N/m^2

E_M = 12 to 30 x 10^6 V/m

ρ = 1.8 x 10^3 kg/m^3

which gives the following range of values

W_M/m = .005 to 4.1 J/Kg

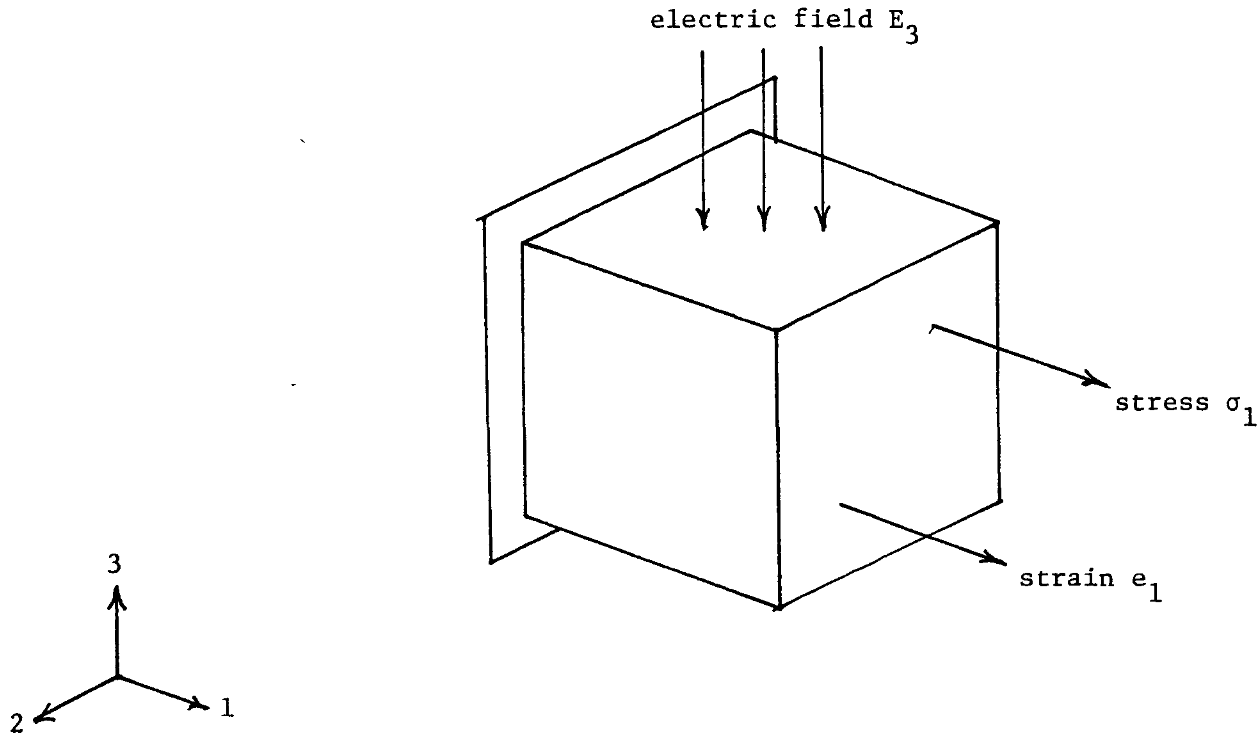

Fig. (C.1) Unit Cube of Polyvinylidene Flouride (PVF2) Piezoelectric Material

The most critical property is that of E_M. Unless a considerable safety margin is allowed, there is a severe danger of dielectric breakdown in a space application, particular after a micrometeorite has damaged the material.

APPENDIX D

Simulation of NASA Tests

To help the actuator hardware design, a model of the prototype actuators will be put on the finite element model to simulater a vibration control test. Initially, a linear simulation will be performed. The purpose is to determine the magnitude of the displacement and the actuator force under designed conditions. The assumptions of this simulation are

(1) the system is linear
(2) the FEM of the NASA grillage (or beam) will be used as the undamped structure
(3) the actuator dynamics will be ignored. That is, the actuator will be assumed to be a perfect device capable of producing force f = cx under all conditions.
(4) the weight of each damper will be included as a discrete mass added to the system
(5) the excitation will be an initial displacement and/or initial velocity of the FEM.

Formulations

Let [M], [K] be the mass and stiffness of the undamped structure. Introducing n_c dampers at dof $J_1, \ldots J_{nc}$, the damping matrix is

$$[c] = [B][\hat{c}][B]^T \tag{D.1}$$

where

$$[B] = [\{e_{J_1}\} \mid \{e_{J_2}\} \ldots \mid \{e_{J_{nc}}\}]_{n \times n_c}$$

$$[\hat{c}] = \begin{bmatrix} c_1 & & & \underline{0} \\ & c_2 & & \\ & & \ddots & \\ \underline{0} & & & c_{J_{n_c}} \end{bmatrix}$$

and

$$\{e_J\} = \text{null vector except the jth element has a value of 1}$$

In expanded form,

$$[c] = \begin{bmatrix} 0 & & & & & & \\ & \ddots & & & & & \\ & & c_1 & & & & \\ & & & \ddots & & & \\ & & & & c_2 & & \\ & & & & & \ddots & \\ & & & & & & c_{J_{n_c}} \end{bmatrix} \qquad (D.2)$$

The contribution to the mass matrix due to the mass of the damper is

$$[\Delta M] = [B][\hat{\Delta M}][B]^T$$

or

$$[\Delta M] = \begin{bmatrix} 0 & & & & & & \\ & \ddots & & & & & \\ & & \Delta m & & & & \\ & & & \ddots & & & \\ & & & & \Delta m & & \\ & & & & & \ddots & \\ & & & & & & \Delta m \\ & & & & & & & \ddots \end{bmatrix} \qquad (D.3)$$

where we assume all dampers are identical and have a mass of Δm.

The equation of motion of the system is

$$[\bar{M}]\{x\} + [c]\{\dot{x}\} + [K]\{x\} = \{0\} \qquad (D.4)$$

where

$$[\bar{M}] = [M] + [\Delta M]$$

The initial conditions are

$$\begin{aligned} \{x(0)\} &= \{x_0\} \\ \{\dot{x}(0)\} &= \{\dot{x}_0\} \end{aligned} \qquad (D.5)$$

Equation (D.4) can be solved numerically using any of the numerical solution techniques, e.g. Wilson's θ method or Newmark β method. Alternatively, a modal solution of (4) can be obtained by using the complex mode of the system (see Hurty and Rubinstein (Ref. 1). Once the modes are known an analytical expression can be written for each mode in this approach.

REFERENCE

1. W.C. Hurty and M.F. Rubenstein, Dynamics of Structures, Prentice-Hall, Inc. Englewood Cliffs, N.J. 1964 (ch. 9).

APPENDIX E

1. Bounds on Transient Responses

2. Time Domain Optimal Design

1. Bounds on Transient Responses

The disturbances acting on an Large Space Structure is mission dependent. However, one can say that these disturbances are usually of a transient type, e.g. firing of control thrusters for attitude control etc. The vibration control system must be designed to suppress these transient vibrations. One way of measuring the performance of a vibration control system is to evaluate how fast the control system can bring the response from some initial disturbed state to within an "acceptable" level. The response could be the displacement, velocity, or acceleration at certain "critical" locations of the LSS. For a large structure, it may not be easy to define which specific response is critical and the designer usually resorts to some aggregation of responses as the performance measure. One commonly used criterion is the minimization of

$$V(\{z\}) = \{z\}^T[P]\{z\}; \qquad z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{E.1}$$

where [P] is the weighting matrix. The estimation of transient behavior for the performance index can be derived according to Lyapunov's 2nd method, given a linear system governed by $\{\dot{z}\} = [A]\{z\}$ with initial state $\{z_o\}$ at $t = t_o$ and where $V(\{z\},t_o)$ is given, the system attains a value of v given by $V(\{z\},t) = V(\{z_o\},t_o)e^{-\eta(t-t)}$ in $t-t_o$ seconds using Lyapunov's second method (Ref. 1,2). The following is a description of the use of Lyapunov's second method in estimating transient responses (Ref. 6) and then indicate how it is applied to the control of LSS. Finally, some research issues will be raised.

Given initial states $\{z_o\}$ and the initial value $V(\{z_o\},t_o)$, the system reaches the region

$$V(\{z\},t) = V(x_o,t_o)e^{-\eta(t-t_o)} \tag{E.2}$$

in $t-t_o$ seconds.

For a linear time invariant system governed by the following state equations

$$\{\dot{z}\} = [A]\{z\} \tag{E.3}$$

the parameter η in Eq. (E.2) is the minimum eigenvalue of the matrix $[Q][P]^{-1}$ where [P] and [Q] are related by

$$[A]^T[P] + [P][A] = -[Q] \tag{E.4}$$

and [P] is the weighting matrix in the performance index, Eq. (E.1). Note that if [Q] is symmetric, so will be [P].

The above results can be applied to the control of LSS by letting

$$[A] = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1}k & -M^{-1}c \end{array}\right] \tag{E.5}$$

where [A] is the dynamical matrix of the <u>closed loop</u> system. That is, in Eq. (E.5) the mass matrix [M] include the mass of the control devices; [k] is the stiffness matrix of the system plus the control law that is proportion to the displacement ($\{x\}$) and [c] is the damping matrix which corresponds to the portion of control law that is proportional to rate ($\{\dot{x}\}$), see Appendix F for detailed formulation.

It should be noted that the use of Lyapunov's second method requires that all the eigenvalues of the matrix [A] have negative real parts. For a free-free structure with no position feedback, the system matrix [A] has some zero eigenvalues. Thus, to use the above results directly we need to include positive feedback terms in the control law.

The Lyapunov's second method provides a sound basis for transient performance estimates. To apply it to the design of vibration control of LSS, the following research needs to be carried out.

(1) matrix [Q] must be positive definite, otherwise its eigenvalue could be zero or negative which renders the estimate (Eq. (E.2)) useless. This implies the performance index $V(\{z\},t)$ is restricted to quadratic form with weight matrix [P] that leads to a positive definite [Q]. Thus, [P] may not be specified arbitrarily. The question of how to pick matrix [P] needs further research.

(2) Alternatively, we can specify a positive definite [Q] and solve for [P] by Eq. (E.4). This requires the solution of a set of n(n+1)/2 equation, where n is the order of matrix [A]. An efficient solution scheme of Eq. (E.4) needs to be develoepd (or found in the literature).

(3) modify the formulation to allow it to accept LSS whose control law consists of rate feedback only. This may be achieved by formulating equations of motion for elastic modes only by filtering out the rigid body motion.

(4) application of the formulation in a truncated modal space. This will reduce the computational burden drastically.

(5) Formulate a synthesis procedure using the basic results. This may be achieved by iteractive methods or by a direct formulation. The results of the above researches can also be used in the next section.

2. Time Domain Optimal Design

Traditionally, time optimal design using optimal control theory with quadratic performance results in the solution of a higher order Riccatti equation, (Ref. 3). This will result in a control law with full state feedback which may be difficult to implement. For a vibration control system, it may be advisable to take another approach. That is one could fix the form of the control law then find the pairs that optimize some performance index. In general, nonlinear programming approaches can be used to find the optimal gains. This will be extremely costly since many transient responses are to be evaluated. If we limit ourself to a quadratic performance index, then the Lyapunov's second method can be used to evaluate the performance index when the system is driven by initial conditions. The general results (Ref. 6) will be stated first and then their application to LSS will be indicated.

Consider the system

$$\{\dot{z}\} = [A]\{z\} \tag{E.6}$$

where all eigenvalue sof [A] have negative real parts, or the original {0} is asympototically stable. The design problem is to adjust the elements in matrix [A] such that the performance index

$$J = \int_0^\infty \{z\}^T[Q]\{z\}dt$$

is minimized. It has been shown that

$$J = \int_0^\infty \{z\}^T[Q]\{z\}dt = \{z_o\}^T[P]\{z_o\} \tag{E.7}$$

where [P] is the solution of the equation

$$[A]^T[P] + [P][A] = -[Q] \quad (E.8)$$

Note that [P] is symmetrical. The significance of Eq. (E.7) is that to evaluate the performance index, no transient analysis has to be performed at all. Rather, we just solve a set of n(n+1)/2 linear Eqs. (E.8) and then perform matrix multiplication. Since the elements of [P] is an implicit function of matrix [A] through Eq. (3), the following optimal procedure can be proposed:

(1) from trial value of design parameters, form matrix [A]
(2) solve [P] using Eq. (E.8)
(3) evaluate J using Eq. (E.7)
(4) repeat the above process until J is minimized.

When applied to LSS, we simply use the dynamic [A] as defined in Appendix F. The results of the research is used in the previous section is applicable here also.

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2. K. Ogata, Modern Control Engineering, Prentice-Hall, Inc. Englewood Cliffs, New Jersey.
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APPENDIX F

Formulation of Dynamical Matrix for LSS with General Control Law

Consider the actively controlled LSS

$$[M][\ddot{x}] + [k_o]\{z\} = [B]\{u\} \qquad (F.1)$$

where

- $[k_o]$ is the stiffnes matrix of the uncontrolled structure (nxn)
- $[M]$ is the mass matrix of the controlled system including the mass and inertia of the control devices (sensors and actuators, etc) (nxn)
- $[B]$ is the control distribution matrix (NxN_u)
- $\{u\}$ is the control input ($N_n x1$)

Let the control law be

$$\{u\} = -[\hat{\Delta}k]\{y\} - [\hat{\Delta}c]\{\dot{y}\} \qquad (F.2)$$

where

$$\{y\} = [B_d]\{x\} = \text{displacement measurement } (N_d x1)$$
$$\{\dot{y}\} = [B_u]\{\dot{x}\} = \text{velocity measurement } (N_u x1) \qquad (F.3)$$

$[\hat{\Delta}K]$ = displacement gain matrix ($N_u x N_d$)

$[\hat{\Delta}c]$ = velocity gain matrix ($N_u x N_u$)

$[B_d]$ = displacement measurement matrix ($N_d xN$)

$[B_u]$ = velocity measurement matrix ($N_u xN$)

Substitute (F.3) into (F.2)

$$\{u\} = -[\hat{\Delta}k][B_d]\{x\} - [\hat{\Delta}c][B_u]\{\dot{x}\} \qquad (F.4)$$

Substitute (F.4) into (F.1)

$$[M]\{\ddot{x}\} + [K_o]\{x\} = -[B\hat{\Delta}kB_d]\{x\} - [B\hat{\Delta}c\ B_u]\{\dot{x}\} \qquad (F.5)$$

Equation (F.5) can be simplified to

$$[M]\{x\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\} \qquad (F.6)$$

where

$$\begin{aligned} [c] &= [B][\hat{\Delta}c][B_v] \\ [K] &= [K_o] + [\Delta K] \\ [\Delta K] &= [B][\hat{\Delta}k][B_d] \end{aligned} \qquad (F.7)$$

Note that in general, matrix [K] and [C] are arbitrary matrices, they may not be symmetric.

Finally, we can put (F.6) in state space form as

$$\{\dot{z}\} = [A]\{z\} \qquad (F.8)$$

where

$$[A] = \left[\begin{array}{c|c} 0 & I \\ \hline -M^{-1}K & -M^{-1}c \end{array}\right] \qquad (F.9)$$

$$\{z\} = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

Appendix G

Proposed Approaches for Actuator Failure

Detection

The actuator failure modes can be classified as:

(a) total failure: actuator does not produce a force in response to a command signal

(b) random failure: actuator produces random force for a given input signal.

The total failure mode may be considered in the design of electronics for the actuator.

The random failure mode is more complex and may not be detected using hardware alone. In this case, a monitoring system to keep track of the performance of each actuator may be needed. Through the performance monitoring an adaptive control approach may be derived to detect andisolate failed actuators. Such an approach has been used in the detection of rate gyro failure in flight control systems [Ref. 1]. The feasibility of this approach to our problem is being investigated. Literature search will be made to look for other approaches.

References

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