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Report to NASA on

# Mixed Time Integration Methods For Transient Thermal Analysis of Structures*

by


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Summary

The computational methods used to predict and optimize the thermal-structural behavior of aerospace vehicle structures are reviewed. In general, two classes of algorithms, implicit and explicit, are used in transient thermal analysis of structures. Each of these two methods has its own merits. Due to the different time scales of the mechanical and thermal responses, the selection of a time integration method can be a difficult yet critical factor in the efficient solution of such problems.

Therefore mixed time integration methods for transient thermal analysis of structures are being developed. The computer implementation aspects and numerical evaluation of these mixed time implicit-explicit algorithms in thermal analysis of structures are presented. A computationally-useful method of estimating the critical time step for linear quadrilateral element is also given. Numerical tests confirm the stability criterion and accuracy characteristics of the methods. The superiority of these mixed time methods to the fully implicit method or the fully explicit method is also demonstrated.

Over the last two decades, significant attention has been devoted to the development of lightweight, durable thermal protection systems (TPS) for future space transportation systems. Research programs are currently underway at the Langley Research Center to investigate various metallic TPS concept [1]. One of the proposed candidates is the titanium multiwall tile (see [2] and references therein for a discussion). Early design procedures of the TPS concept involved both analytical and experimental studies. In particular, a degree of confidence has been established in the TPS concept due to the design studies by Jackson and Dixon [3] and Blair et al. [4].

A titanium multiwall tile consists of alternating layers of superplastically formed dimpled sheets and flat septum sheets of titanium foil. As described in reference [3], this multiwall concept impedes all three modes of heat transfer——conduction, radiation and convection. The superplastically formed dimpled sheets and the long thin conduction path tend to minimize heat conduction. The flat septum sheets of titanium foil impede radiation. The small individual volumes created by the dimpled layers virtually eliminate air convection. The optimal design of such thermal protection systems requires effective techiques in coupled thermal and stress analyses. Finite element methods offer the greatest potential in modeling such complicated problems. However, the resulting semi-discrete equations may involve many thousand degrees of freedom. Since the problem to be solved is transient and nonlinear, the selection of an appropriate time integration method is an essential step in the solution of such a complicated problem. Adelman and Hafka [5] recently conducted a survey study on the performance of explicit and implicit algorithms for transient thermal analysis of structures. Calculations were carried out using the SPAR finite element computer program [6] and a special purpose finite element program incorporating the GEARB and GEARIB algorithms. Based upon their studies, they concluded that, generally, implicit algorithms are preferable to explicit algorithms for "stiff" problems, though non-convergence and/or wide-banding of the resulting matrix equations may decrease the advantage of the implicit methods.

These difficulties are similar to those found in fluid-structure problems. Over the past few years, several remedies have been proposed for these difficulties. Belytschko and Mullen [7] have proposed an explicit-implicit method where the mesh is partitioned into domains by <u>nodes</u> and the partitions are simultaneously integrated by explicit and implicit methods. Hughes and Liu [8] have proposed an alternate implicit-explicit finite element method where the mesh is partitioned into domains by <u>elements</u> and this element partition concept simplifies the computer-implementation and enhances its compatibility with the general purpose finite element software.

Although the implicit-explicit method has been proven to be very successful in some fluid-structure interaction problems (see e.g., [8-10]), the size and complexity of the program are increased because of the addition of the implicit method. To overcome these difficulties, Belytschko and Mullen [11] have proposed an $E^m$-E partition, in which explicit time integration is used throughout. However, different time steps within different parts of the mesh can be employed simultaneously. Partitioned and adaptive algorithms for explicit time integration have also been proposed by Belytschko [12].

Recently, Liu and Belytschko [13] put forward a general mixed time implicit-explicit partition procedure within a linear context. It incorporates the mentioned algorithms as special cases and is shown to have better stability properties than that in $E^m$-E partition [11]. Similar concepts can also be used in transient conduction forced-convection analysis (see Liu and Lin [14]).

In the present report, we extend these implicit-explicit concepts (nodes and elements) to transient thermal analysis of structures where different time integration methods with different time steps can be used in each element group. The aim of this approach is to achieve the attributes of the various time integration methods.

For example, in transient structural analysis, explicit methods require the size of the time step to be proportional to the length of the shortest element; while in transient thermal analysis, explicit methods require the step size to be porportional to the the square of the length of the shortest element. So it is more advantageous to employ this mixed time implicit-explicit technique for transient thermal analysis of structures since the $E^m$-E partition proposed in [11,12] is often inefficient for this kind of problem though it is very efficient in structural analysis.

In Appendix 1, we present the stability analysis of mixed time integration schemes for transient thermal analysis. This chapter is now published in Numerical Heat Transfer Vol. 5, pp. 211-222, 1982.

In Appendix 2, we present the computer implementation aspects of these mixed time partition procedures. This chapter has been accepted for publication in International Journal of Numerical Methods in Engineering.

In Appendix 3, we present the improvement of mixed time implicit-explicit algorithms for thermal analysis of structures. In particular, three numerical examples are presented to evaluate the performance (i.e. accuracy and stability behavior, computer storage and solution time, etc.) of these mixed time finite element algorithms. This chapter has been accepted for publication in Computer Methhods in Applied Mechanics and Engineering.

In Appendix 4, we present a method for performing efficient and stable finite element calculations of heat conduction with quadrilaterals using one-point quadrature. Comparison with finite difference formulas has shown that various values of the stabilized parameter, the 5-point and 9-point molecules can be obtained. It is found that a combination of this one-point quadrature element and the mixed itme implicit-explicit methods may be an effective compromise. This chapter has been submitted to International Journal of Numerical Methods in Engineering for possible publication.

Interim report is presented in chapter 5.

In summary, the most important work in progress is the development of nonlinear mixed time integration methods for transient thermal analysis of structures suitable for incorporation into most finite element computer codes. Currently, continuous efforts are being made to include three dimensional and nonlinear thermal analysis.

It should be emphasized that although the presently developed method has been applied to TPS (Thermal Protection Systems), the same technique could be applied to a wide variety of heat transfer fluid flow problems such as transient conduction forced-convection analysis, radiation and compressible fluid flow problems.

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