

# Comparison of Spur Gear Efficiency Prediction Methods

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The prediction of spur-gear efficiency has been the subject of several investigations in the last 20 years (refs. 1 to 5). The methods proposed are necessarily approximate because of the difficulty in modelling the gear mesh and its associated losses. Efficiency prediction techniques to date have either utilized a friction coefficient deduced from gear power-loss measurements or adapted disk machine data to simulate the gear contact conditions. When using disk machine data, either an integration (averaging) technique or selection of a representative operating point along the path of contact must be accomplished to accurately model the actual gear mesh. The instantaneous values of power loss along the path of contact provided by the disk machine data must be converted into an average power loss for the entire mesh.

The five methods, Anderson and Loewenthal (ref. 1), Buckingham (ref. 2), Chiu (ref. 3), Merritt (ref. 4) and Shipley (ref. 5), use various approaches to spur gear efficiency. The major differences among these methods are the friction coefficient used for sliding power loss and the number of additional terms included to account for sources of power loss other than sliding. In reference 6 Martin provides a summary of the many friction coefficients available in the literature and the lubrication conditions under which they apply. Each of the five efficiency prediction methods (refs. 1 to 5) uses a different friction coefficient expression.

Of the five methods only Anderson and Loewenthal and Chiu include an additional expression for rolling or pumping power loss. This is the additional power required to form the elastohydrodynamic (EHD) oil film that separates the gear teeth during engagement. Rolling loss was shown to be significant, particularly at higher operating speeds, in reference 1. This loss combines with windage losses to form the tare, or no-load, loss of a gearset. Expressions for windage loss were given only by Anderson and Loewenthal and Shipley. Bearing losses were also found to be a significant portion of the total gearset loss in reference 1, where an approximate ball-bearing loss expression was used (ref. 11). Only gear losses will be considered in the present work, however.

Three sources of spur-gear power-loss data were located to allow comparison of the five theories with actual measurements (refs. 7 to 9). These data include pitch-line velocities from 1 to 20 m/sec (200 to 4000 ft/min) and K-factor (ref. 10) loading conditions from 17 to 1600. The data selected for comparison purposes were limited to jet-lubricated and ground gears.

It is the objective of this paper to compare the accuracy of the five gear efficiency methods in references 1 to 5 using three sources of test data (refs. 7 to 9) over a wide range of operating conditions.

## Symbols

CR	contact ratio
$C_1$ to $C_{14}$	constants of proportionality; see table I
$D$	pitch circle diameter, m (in.)
$E$	modulus of elasticity, N/m <sup>2</sup> (lbf/in <sup>2</sup> )
$E'$	equivalent modulus of elasticity, $2/[(1 - \gamma_g^2)/E_g + (1 - \gamma_p^2)/E_p]$ , Pa (lbf/in <sup>2</sup> )
$F$	face width of tooth, m (in.)
$f$	coefficient of friction
$H_S$	specific sliding at start of approach action
$H_T$	specific sliding at end of recess action

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$h$	length of path of approach, m (in.)
$\bar{h}$	isothermal central film thickness, m (in.)
$K$	gear capacity factor (ref. 10)
$l_T$	length of path of contact, m (in.)
$m_g$	gear ratio, $N_g/N_p$
$N$	number of gear teeth
$n$	rotational speed, rpm
$P$	power loss
$\Phi$	diametral pitch
$R$	pitch circle radius or radius in general, m (in.)
$R_{eq}$	equivalent rolling radius, m (in.)
$s$	length of path of approach, m (in.)
$T$	pinion torque, N-m (in-lbf)
$\bar{V}$	velocity, m/sec (ft/min)
$\bar{V}_S$	average sliding velocity, $V_g - V_p$ , m/sec (in/sec)
$\bar{V}_T$	average rolling velocity, $V_g + V_p$ , m/sec (in/sec)
$W$	average gear contact, normal load, N (lbf)
$W_n$	gear contact, normal load, N (lbf)
$x$	coordinate along path of contact, m (in.)
$\beta_a$	arc of approach, rad
$\beta_r$	arc of recess, rad
$\theta$	gear tooth pressure angle, deg
$\mu, \eta$	lubricant absolute viscosity, $10^{-3}$ N sec/m <sup>2</sup> (lbf sec/in <sup>2</sup> )
$\nu$	lubricant kinematic viscosity, 10 <sup>2</sup> cm <sup>2</sup> /sec (ft <sup>2</sup> /sec)
$\rho$	equivalent rolling radius as defined in ref. 3, m (in.)
$\sigma$	surface roughness, $\mu\text{m}$ , ( $\mu\text{in}$ )

Subscripts:

$a$	approach
$B$	Buckingham
$b$	base
$e$	entraining
$g$	gear
$M$	Merritt
$P$	pitch line
$p$	pinion
$R$	rolling
$r$	recess
$S$	sliding
$t$	tip
$W$	windage

## Spur Gear Efficiency Theories

Five spur-gear efficiency prediction methods were used to predict the efficiency of three gear geometries from the three experimental studies (refs. 7 to 9). Selected for comparison were the methods of Anderson and Loewenthal (ref. 1), Buckingham (ref. 2), Chiu (ref. 3), Merritt (ref. 4), and Shipley (ref. 5).

All of the methods are similar in that each considers gear-tooth sliding losses through the use of a friction coefficient. The methods differ in the choice of the friction coefficient, the form of the

efficiency equation, and the number of additional terms included to account for gear pumping and windage power loss. Only the methods of Anderson and Loewenthal and Chiu include terms for rolling traction or pumping loss, while windage loss expressions are only given by Anderson and Loewenthal and Shipley.

### Method of Anderson and Loewenthal

The method of Anderson and Loewenthal considers three components of gear power loss: sliding, rolling, and windage. In reference 1 the sliding and rolling losses were calculated by numerically integrating the instantaneous values of these losses over the path of contact. The friction coefficient used to calculate sliding loss was based on disk machine data generated by Benedict and Kelley (ref. 12). This friction coefficient expression is considered to be applicable to gear sliding loss calculations in the EHD lubrication regime where some asperity contact occurs, that is, for  $\lambda$  ratios less than 2 ( $\lambda$  = ratio of minimum EHD film thickness to composite surface roughness). In reference 1 rolling losses were based on disk machine data generated by Crook (ref. 13). Crook found that the rolling loss was simply a constant value multiplied by the EHD central film thickness. Gear tooth film thickness was calculated by the method of Hamrock (ref. 14) and adjusted for thermal effects using Cheng's thermal reduction factor (ref. 15). At high pitch-line velocities the isothermal equations, such as Hamrock's, will predict an abnormally high film thickness since inlet shear heating of the lubricant is not considered. Cheng's thermal reduction factor will account for the inlet shear heating and reduce the film thickness accordingly. Inlet starvation effects at high speeds were not considered, however.

In reference 16 a simplified version of this analysis was developed to study the sensitivity of gear power loss to changes in tooth geometry and operating variables. It was found to agree with the numerically integrated solution to within 0.1 percentage point of efficiency except at combinations of light load and high speed. Under these conditions the EHD thermal reduction factor, which was omitted for simplicity, is needed to properly predict the rolling losses. The simplified version will be used for comparison with the other theories herein.

In reference 16 the sliding and rolling loss equations were

Sliding loss,

$$\bar{P}_S = C_1 f \bar{W} \bar{V}_S \quad (1)$$

Rolling loss,

$$\bar{P}_R = C_2 \bar{h} \bar{V}_T F C R \quad (2)$$

where  $f$ ,  $\bar{V}_S$ ,  $\bar{h}$ , and  $\bar{V}_T$  are evaluated at a point halfway between the pitch point and the start of engagement along the path of contact. The constants  $C_1$  to  $C_{14}$  can be found in table I.

Pinion and gear windage expressions were given as

$$P_{W,p} = C_3 \left(1 + 2.3 \frac{F}{R_p}\right) n_p^{2.8} R_p^{4.6} (0.028 \mu + C_4)^{0.2} \quad (3)$$

$$P_{W,g} = C_3 \left(1 + 2.3 \frac{F}{R_g}\right) \left(\frac{n_p}{m_g}\right)^{2.8} R_g^{4.6} (0.028 \mu + C_4)^{0.2} \quad (4)$$

The required supporting equations are as follows:

Length of path of contact,

$$l_T = 0.5 \left\{ \left[ \left( D_p + \frac{2}{C_5 \phi} \right)^2 - (D_p \cos \theta)^2 \right]^{1/2} + \left[ \left( D_g + \frac{2}{C_5 \phi} \right)^2 - (D_g \cos \theta)^2 \right]^{1/2} - (D_p + D_g) \sin \theta \right\} \quad (5)$$

TABLE I. - CONSTANTS USED IN GEAR POWER LOSS EQUATIONS

Constant	Value for SI unit	Value for U.S. customary unit
C <sub>1</sub>	2x10 <sup>-3</sup>	3.03x10 <sup>-4</sup>
C <sub>2</sub>	9x10 <sup>4</sup>	1.970
C <sub>3</sub>	2.82x10 <sup>-7</sup>	4.05x10 <sup>-13</sup>
C <sub>4</sub>	0.019	2.86x10 <sup>-9</sup>
C <sub>5</sub>	39.37	1.0
C <sub>6</sub>	29.66	45.94
C <sub>7</sub>	2.05x10 <sup>-7</sup>	4.34x10 <sup>-3</sup>
C <sub>8</sub>	196.9	1.0
C <sub>9</sub>	1x10 <sup>-3</sup>	1.515x10 <sup>-4</sup>
C <sub>10</sub>	1.54x10 <sup>-5</sup>	5.738x10 <sup>-6</sup>
C <sub>11</sub>	1.0	0.0254
C <sub>12</sub>	1.43x10 <sup>-3</sup>	1.383x10 <sup>-5</sup>
C <sub>13</sub>	0.0114	0.180
C <sub>14</sub>	9.226x10 <sup>8</sup>	1.0

Average sliding velocity,

$$\bar{V}_S = 0.262 n_p \frac{1 + m_g}{m_g} l_T \quad (6)$$

Average rolling velocity,

$$\bar{V}_T = 0.1047 n_p \left[ D_p \sin \theta - \frac{l_T}{4} \left( \frac{m_g - 1}{m_g} \right) \right] \quad (7)$$

Average normal load,

$$\bar{W} = \frac{T_p}{D_p \cos \theta} \quad (8)$$

Friction coefficient from Benedict and Kelley (ref. 12),

$$f = 0.0127 \log \left( \frac{C_6 \bar{W}}{F \mu \bar{V}_S \bar{V}_T} \right) \quad (9)$$

(where  $f$  is limited to a minimum of 0.01 and a maximum of 0.2).

Equivalent contact radius,

$$R_{eq} = \frac{\left[ D_p (\sin \theta) + \frac{l_T}{2} \right] \left[ D_g (\sin \theta) - \frac{l_T}{2} \right]}{2(D_p + D_g) \sin \theta} \quad (10)$$

Central EHD film thickness,

$$\bar{h} = C_7 (\bar{V}_T \mu)^{0.67} \bar{W}^{(-0.067)} R_{eq}^{0.464} \quad (11)$$

Contact ratio,

$$CR = \frac{C_5 l_T \mathcal{P}}{\pi \cos \theta} \quad (12)$$

### Method of Buckingham

In reference 2 Buckingham developed a gear efficiency expression based solely on sliding friction loss. Noting that the coefficient of friction changes along the path of contact, he set forth a final expression that was an average coefficient of friction during the arc of approach  $\beta_a$  and another average value during the arc of recess  $\beta_r$ :

$$\text{Efficiency} = 1 - \left[ \frac{1 + (1/m_g)}{\beta_a + \beta_r} \right] \left( \frac{f_a}{2} \beta_a^2 + \frac{f_r}{2} \beta_r^2 \right) \quad (13)$$

where

$$\beta_a = \frac{(R_{ig}^2 - R_{bg}^2)^{1/2} - R_g \sin \theta}{R_{bp}} \quad (14)$$

and

$$\beta_r = \frac{(R_{ip}^2 - R_{bp}^2)^{1/2} - R_p \sin \theta}{R_{bp}} \quad (15)$$

Since the present comparison is considering only hardened steel gears, the average coefficients of friction during approach and recess were taken to be equal as described in reference 2.

$$f_a = f_r = \frac{2}{3} f \quad (16)$$

where

$$f = \frac{0.05}{e^{(C_8 0.125 V_{SB})}} + 0.002 (C_8 V_{SB})^{1/2} \quad (17)$$

and

$$V_{SB} = \frac{V_P}{2} \left( 1 + \frac{N_P}{N_G} \right) \beta_r \cos \theta \quad (18)$$

This friction coefficient was derived from several sets of data, including reference 17 by Buckingham, by solving equation (13) for  $f$  and introducing measured data. Because neither equation (13) nor (17) is a function of load, the predicted efficiencies are independent of load.

### Method of Chiu

The method of Chiu (ref. 3), like that of Anderson and Loewenthal, used a numerical integration of instantaneous values of sliding and rolling loss to obtain an average loss across the path of contact. However, windage losses were not included.

$$\text{Power loss} = P_S + P_R \quad (19)$$

where  $P_S$  and  $P_R$  are sliding and rolling losses:

$$P_S = C_9 \frac{W_n}{(h+s)} \int_{-s}^h V_S f dx \quad (20)$$

$$P_S = C_{10}(C_{11}\sigma + 0.55)W_n V_P^{1/2} \rho^{-1/2} \mu^{-1/8} I_1 \quad (21)$$

$$P_R = \frac{C_{12} \mu^{0.71} (E')^{0.29} F}{(h+s)} \int_{-s}^h V_e^{1.71} \rho^{0.29} dx \quad (22)$$

$$P_R = C_{12} \sin^2 \theta V_P \left( \frac{\mu V_P}{E' \rho} \right)^{0.71} E' \rho F I_2 \quad (23)$$

The constants  $I_1$  and  $I_2$  can be found in graphical form in reference 3 or they can be evaluated by integration. Values for  $I_1$  and  $I_2$  were taken from the graphical data for the present comparison.

The friction coefficient used by Chiu was that of O'Donoghue and Cameron (ref. 18). Friction coefficients predicted by this method are substantially greater than those predicted by Benedict and Kelley (ref. 12). The rolling loss expression used by Chiu was based on work done by Dowson and Higginson (ref. 19).

### Method of Merritt

In reference 4 Merritt provides a simple expression for the calculation of spur gear efficiency:

$$\text{Percent loss} = \frac{f}{2} \pi \left( \frac{1}{N_p} + \frac{1}{N_g} \right) 100 \quad (24)$$

where

$$f = C_{13} \left( \frac{1.6}{v^{0.15} V_e^{0.15} V_{SM}^{0.35} R_M^{0.5}} \right) \quad (25)$$

and where

$$V_e = 2V_P \sin \theta \quad (26)$$

$$V_{SM} = V_P \left[ \frac{\pi}{2} \cos \theta \left( \frac{1}{N_p} + \frac{1}{N_g} \right) \right] \quad (27)$$

$$R_M = \frac{D_p D_g}{(D_p + D_g)} \frac{\sin \theta}{2} \quad (28)$$

This equation was derived from more complete equations for instantaneous loss similar to those used by Anderson and Loewenthal and Chiu. Merritt made several assumptions to reduce the general equations to the simplified form given above: (1) power loss is evaluated at a point whose distance from the pitch point is equal to 1/4<sup>th</sup> the base pitch along the path of contact, (2) the diametral pitch equals 1, (3) the contact ratio equals 1.5, and (4) the tooth loading diagram is trapezoidal. The friction coefficient is based on the Benedict and Kelley expression and others. Merritt realized that his gear loss expression was approximate and suggested that this analysis might serve as a starting point for further investigation.

### Method of Shipley

Shipley's method appears in references 5 and 20. The method used here is that published in reference 5 since it is more recent and more complete. (It should be noted that there are several differences in these two works). This method considers both sliding and windage losses but not rolling loss:

$$\text{Percent sliding loss} = \frac{50 f}{\cos \theta} \frac{(H_S^2 + H_T^2)}{(H_S + H_T)} \quad (29)$$

where  $H_S$  is the specific sliding at the start of approach action, such that

$$H_S = (m_g + 1) \left\{ \left[ \left( \frac{R_{tg}}{R_g} \right)^2 - \cos^2 \theta \right]^{1/2} - \sin \theta \right\} \quad (30)$$

where  $H_T$  is the specific sliding at the end of recess action, such that

$$H_T = \frac{m_g + 1}{m_g} \left\{ \left[ \left( \frac{R_{tp}}{R_p} \right)^2 - \cos^2 \theta \right]^{1/2} - \sin \theta \right\} \quad (31)$$

and where

$$\text{Windage loss} = \frac{C_{14} n_3 D^5 F^{0.7}}{100 \times 10^{15}} \quad (32)$$

In Shipley's method (ref. 5) the friction coefficient is given in graphical form as a function of "K-factor" (a nondimensional loading term, ref. 10), pitch-line velocity, and type of oil (light or medium weight). The K-factor gives an indication of gearset loading and thus adds the effect of load to the power-loss calculation. Limitations placed by Shipley on the windage equation are that gear diameters are to be less than 0.51 m (20 in.) and that gear width to diameter ratios are approximately equal to 0.5.

## Gear Power-Loss Data

Although there is a scarcity of published gear power loss data, three sources were located for comparison with the predictions from the five methods described earlier. In reference 7 Fletcher and Bamborough used a power recirculating test rig (four-square rig) to determine the effects of oil-jet location, lubricant flow rate, lubricant viscosity, and gear face width on gear power loss. Care was taken to accurately calibrate the 28-kW (37.5-hp) dynamometer to  $\pm 0.07$  N-m ( $\pm 0.6$  in-lbf) (ref. 21). The gear geometry and operating conditions used in these tests are shown in table II. The data presented here are for a lubricant flow rate of 1.9 to 11.4 l/min (0.5 to 3.0 gal/min) with the supply jet located at the into mesh locations.

Ohlendorf (ref. 9) also used a recirculating power test rig to generate spur gear power loss data. Ohlendorf conducted an extensive test program on the effects of type of lubricant, lubrication method, gear fabrication method, tooth shape, and operating conditions on the power loss of spur gears. The data presented here are only for one of Ohlendorf's many test conditions—a jet lubricated gearset with geometry and operating conditions as described in table II.

Yada, in references 8 and 22, took a different approach to the measurement of gear power loss. He submerged his test gears in oil. By comparing the time-temperature characteristics of the oil surrounding the loaded gearset with that of the same gearset rotating in an unloaded condition but with a known amount of electrical power heating the oil, the gear-mesh power losses were deduced with high accuracy, according to Yada (ref. 22). This method automatically calibrates out any windage loss since the temperature rise in the oil is due only to losses in the gear mesh. In addition to speed and torque, Yada also investigated the effects of various fabrication techniques. The data used for comparison here include gears that were ground by Niles, Reishauer, and Maag machines.

## Discussion of Results

In figures 1 to 3 the power-loss predictions of Anderson and Loewenthal, Buckingham, Chiu, Merritt, and Shipley are compared with three sets of test data. Power loss is shown as a function of pinion torque at constant speed. It is apparent that, although these methods vary widely in the prediction of power loss, some give consistently good agreement with the three sets of data.

TABLE II. - TEST GEAR DATA AND OPERATING CONDITIONS

	Fletcher and Bamborough (ref. 7)	Yada (ref. 8)	Ohlendorf (ref. 9)
Pinion diameter, m (in.)	0.152 (6)	0.076 (3)	0.089 (3.5)
Ratio	1.67	1.07	1.0
Face width to diameter ratio	0.26	0.06	0.23
Diametral pitch	8	5	5.6
Pressure angle, deg	20	20	20
Oil viscosity, cP	54	32	39
K-factor range	17 - 137	400 - 1600	30 - 449
Pitch line velocity range, m/sec (ft/min)	2.0 - 15.2 (400 - 3000)	1.5 - 8.1 (300 - 1600)	1.0 - 20.3 (200 - 4000)

### Fletcher and Bamborough Data

In figure 1 a comparison is made with the Fletcher and Bamborough data. The data shown are for into mesh lubrication at 1.9- to 11.4-l/min (0.5- to 3.0-gal/min) flow rate. At the lowest speed most methods tend to overestimate the power loss, but as speed increases most tend to underestimate the losses. The method of Chiu consistently overestimates the losses not only for the gearset of Fletcher and Bamborough but also for the gears of Yada and Ohlendorf as will be shown. This is due to the use of the O'Donoghue and Cameron friction coefficient which predicts abnormally high values.

In figures 1(b) and (c) a significant distinction can be made among these methods. Only the methods of Anderson and Loewenthal and Chiu predict a tare or no-load power loss as suggested by the data. The methods of Merritt, Buckingham, and Shipley cannot predict this tare loss since all three use only a friction coefficient term to predict the power loss (Shipley does not predict any windage loss at these speeds). Since the power loss is proportional to transmitted load, at zero load the power loss must also equal zero. Overall, the method of Anderson and Loewenthal provides the best estimates of power loss for this gearset.

### Yada Data

In figure 2 the five methods are compared with the data of Yada. The three data points at each torque represent three manufacturing methods. None of the prediction methods are sufficiently refined to include surface finish effects. Both the Anderson and Loewenthal and Shipley methods provide good agreement with the data. Again Chiu's predictions are greater than the others. Although the methods of Merritt and Buckingham show good agreement, at times, these methods do not consistently follow the data.

In these data no tare loss was measured or predicted by the methods of Anderson and Loewenthal or Chiu. This is thought to be due to the narrowness of the gear, which reduces the rolling loss and also to the lower pitch-line velocities, which minimize the windage loss.

The torque range of these test data shows a span in K-factor of 400 to 1600. This variation in K-factor represents a moderate to a heavy load situation (ref. 10) and thus is a good test of the load dependence of these prediction methods.

### Ohlendorf Data

The five methods are compared with the data of Ohlendorf in figure 3. The range in pitch-line velocity shown is from 1 to 20 m/sec (200 to 4000 ft/min). Again, the methods of Anderson and Loewenthal and Shipley show the best agreement with the data over the range shown. Buckingham's prediction is fair at the lower speeds but is very poor at 4211 rpm. Merritt's prediction is unsatisfactory at all speeds.

The Ohlendorf data show a rather small tare loss for the pitch-line velocity and gear width that was used. Both Anderson and Loewenthal and Chiu predict higher tare losses than that indicated by the data.



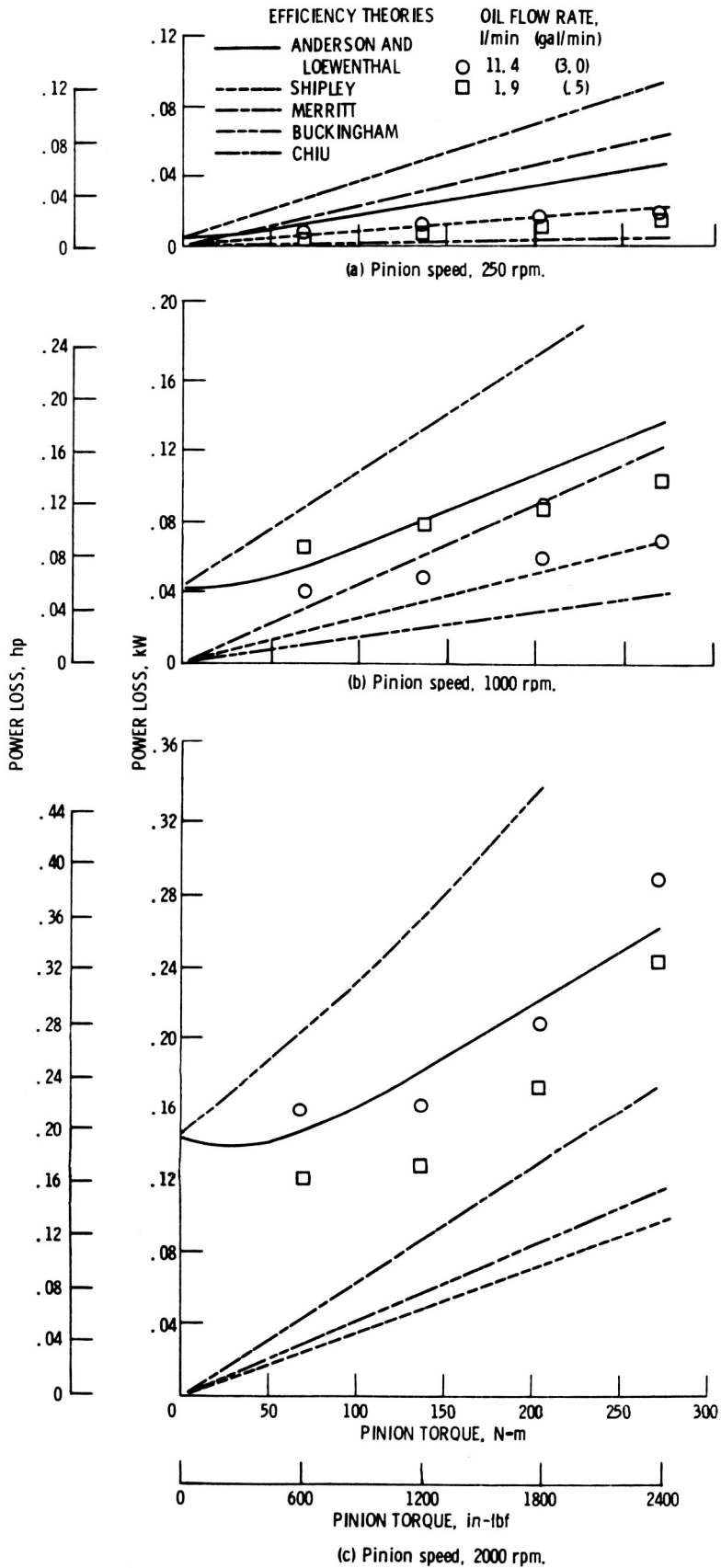


Figure 1. - Comparison of five power-loss prediction methods with the data of Fletcher and Bamborough.

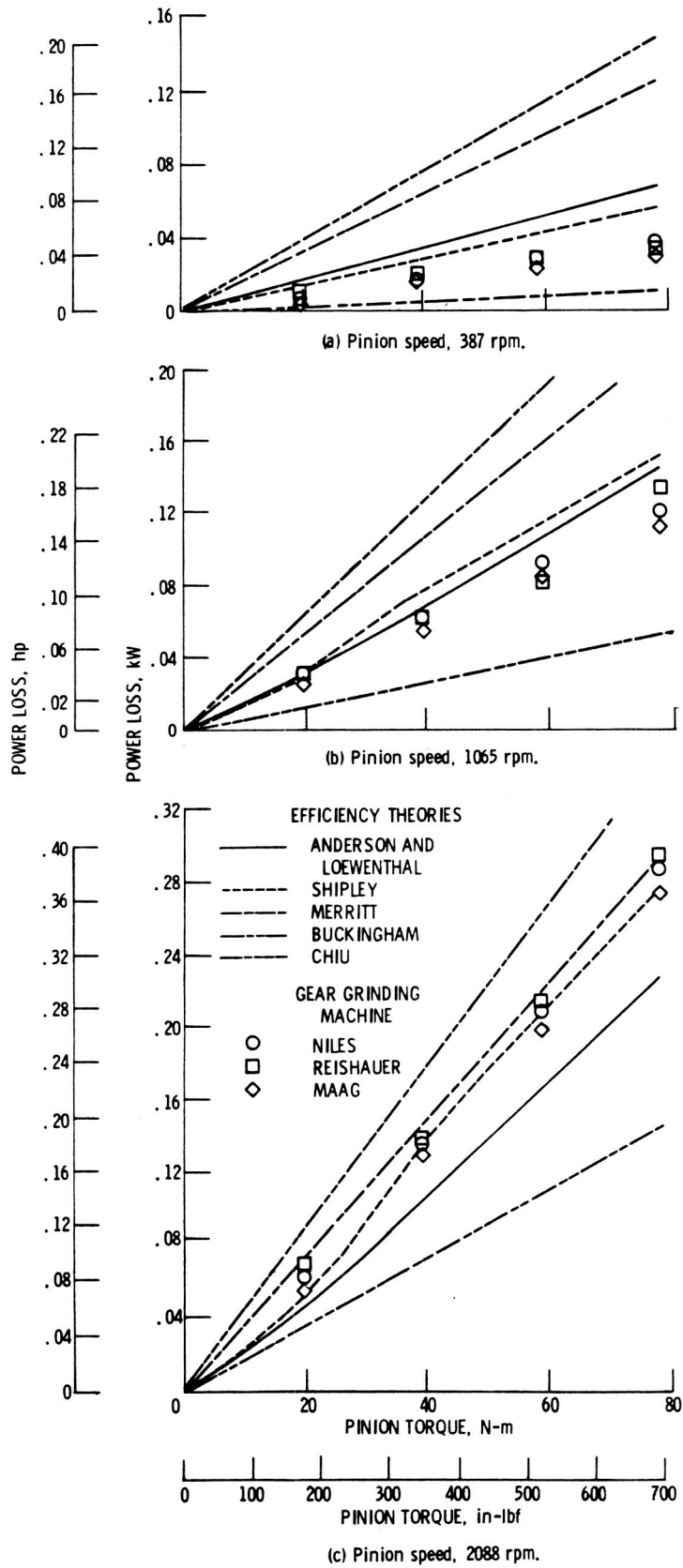


Figure 2. - Comparison of five power-loss prediction methods with the data of Yada.

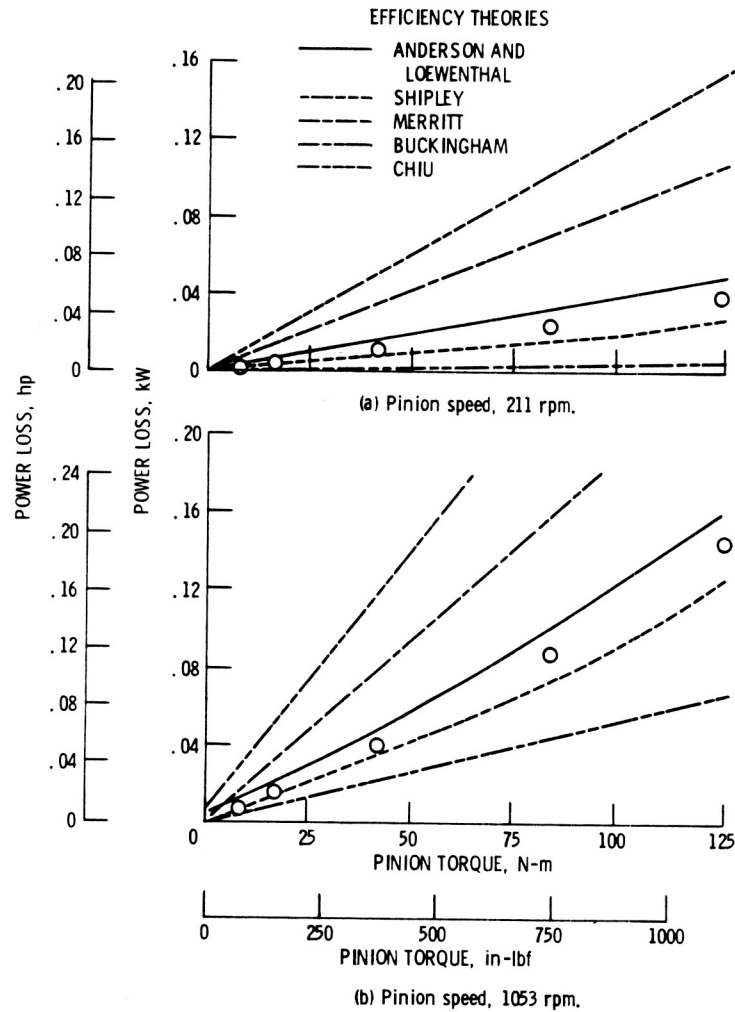


Figure 3. - Comparison of five power-loss prediction methods with the data of Ohlendorf.

## Tare Losses

Figure 4 summarizes the tare loss measurements and predictions from all sources as shown in the previous figures. Tare losses are shown as a function of pitch-line velocity. Figure 4 shows that the methods of Anderson and Loewenthal and Chiu accurately predict the tare losses for the Fletcher and Bamborough data but tend to overestimate the tare losses for Ohlendorf's gearset. The other methods predict no tare losses at all. No comparisons were made with Yada's data since no tare loss was predicted by any method and none was measured by Yada.

## Comparison of Sliding Loss Equations

As stated earlier, each of the five investigations arrived at a different expression for sliding power loss as well as a different friction coefficient. The question then arises as to whether the difference in power-loss prediction lies in the friction coefficient used or in the form of the equations relating the friction coefficient to the power loss. To make this determination, the Benedict and Kelley friction coefficient, as defined in the method of Anderson and Loewenthal, was used in place of the coefficient specified in each method. The results of this substitution (shown in fig. 5) are typical for all other gearsets and speeds examined.

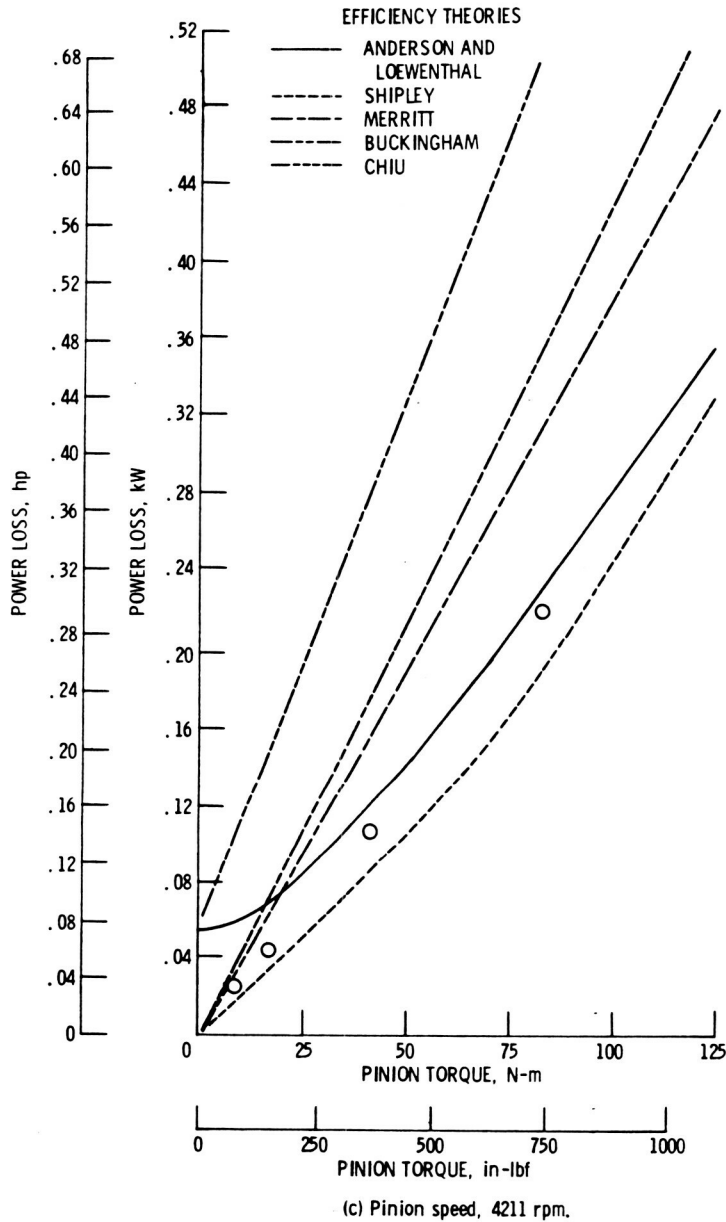


Figure 3. - Concluded.

All methods except Merritt's predict the same sliding power loss when the same friction coefficient is used. This result has several implications. First, all methods, except Merritt's, show basic agreement in calculating sliding power loss. Secondly, the choice of a friction coefficient is crucial. In the methods that use only a sliding loss term, the friction coefficient must account for rolling and windage losses as well as the sliding loss. Thus, the friction coefficient that is used in these methods will not agree with a true sliding loss coefficient such as that determined from disk machine data. This procedure is acceptable except at light loads where the friction coefficient method alone cannot predict tare losses.

The last observation that can be drawn from figure 5 is that the results of Merritt's sliding loss expression do not agree with the other methods. This is probably due to the assumptions Merritt made to simplify the loss equation. It is the simplest expression, but it lacks the flexibility to handle the various gear geometries accurately.

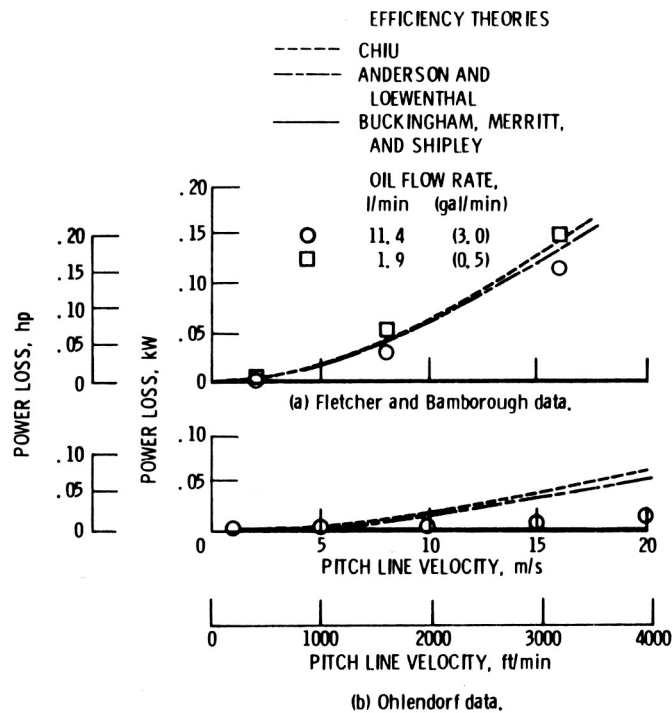


Figure 4. - Comparison of predicted tare (no-load) loss with data.

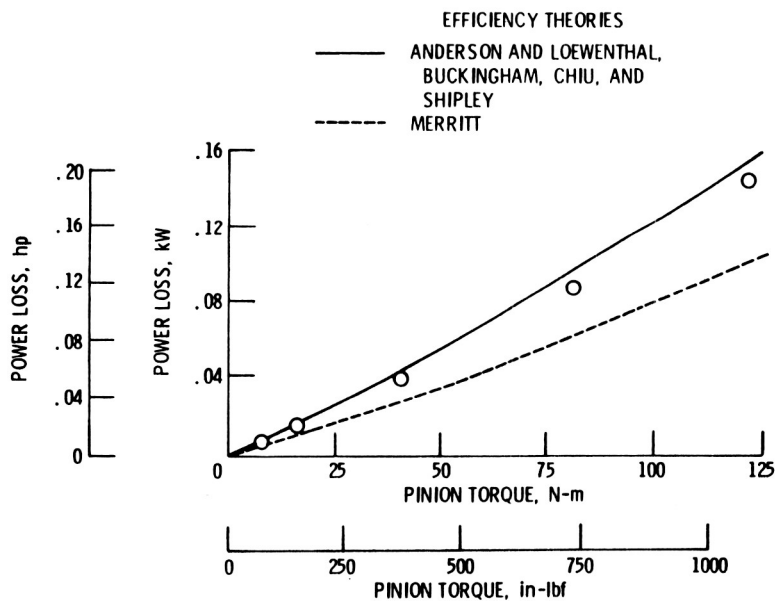


Figure 5. - Comparison of sliding power-loss equations from five power-loss prediction methods, all using the Benedict and Kelley friction coefficient (eq. (9)). Ohlendorf's gearset at a pinion speed of 1053 rpm was used for this comparison.

## Efficiency Comparison

In figures 6 to 8 the power-loss data are shown as efficiency versus torque for constant values of speed. The data shown in this manner bring out several points that are not obvious from the power-loss curves. First, the methods of Buckingham and Merritt show no variation in efficiency with load. Their friction coefficients lack a load factor, thus the loss is a constant percentage of the transmitted load. This results in a constant efficiency at any torque.

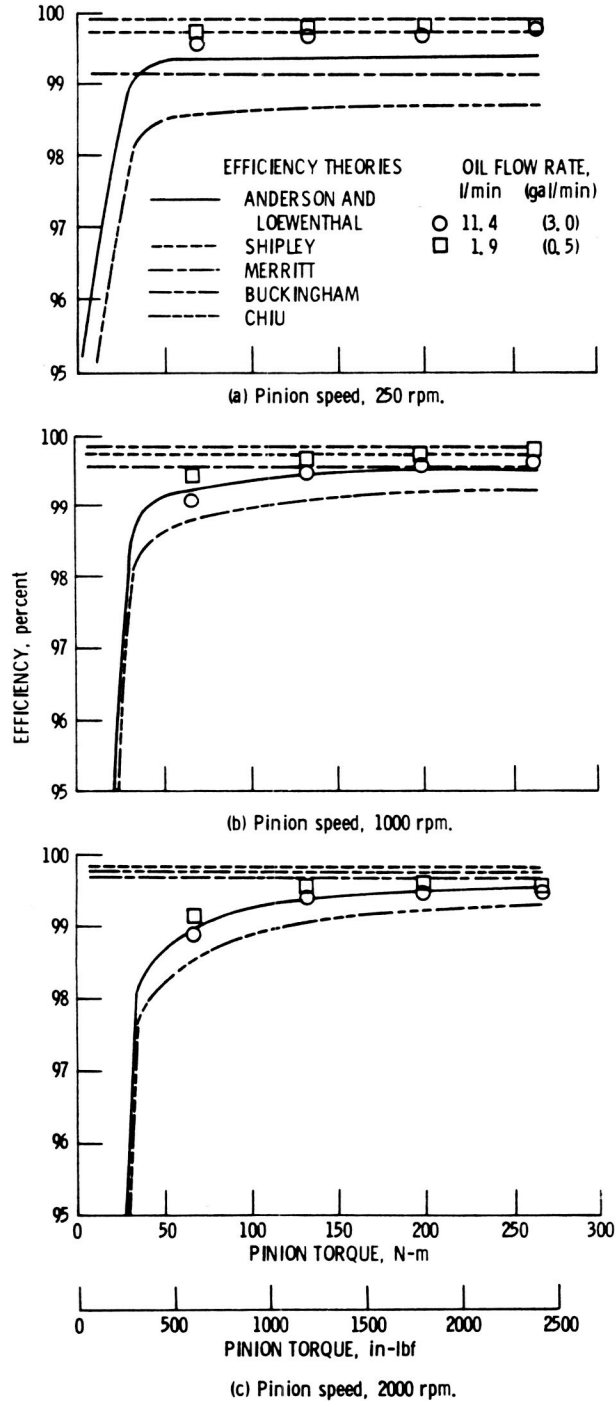


Figure 6. - Gearset efficiency predictions of five methods compared with data of Fletcher and Bamborough.

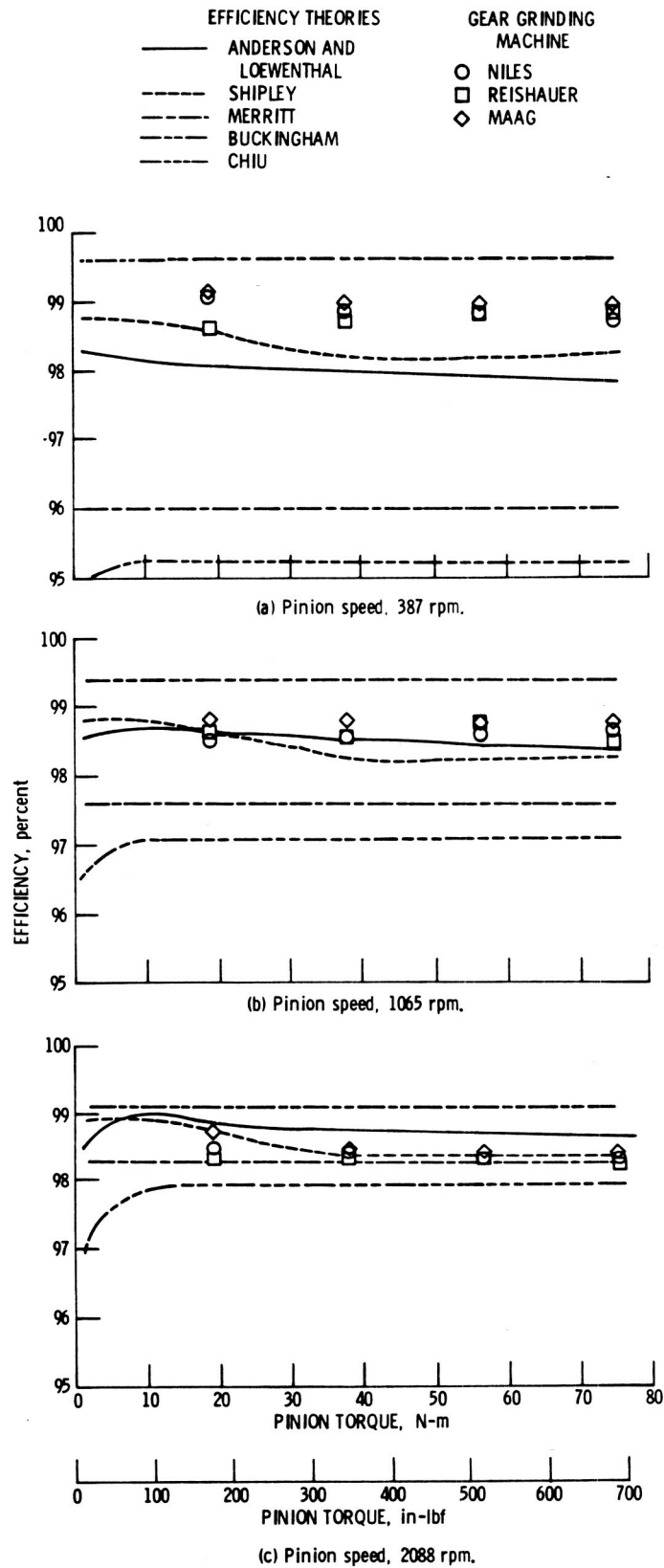


Figure 7. - Gearset efficiency predictions of five methods compared with data of Yada.

EFFICIENCY THEORIES

- ANDERSON AND LOEWENTHAL
- - - SHIPLEY
- - - MERRITT
- - - BUCKINGHAM
- - - CHIU

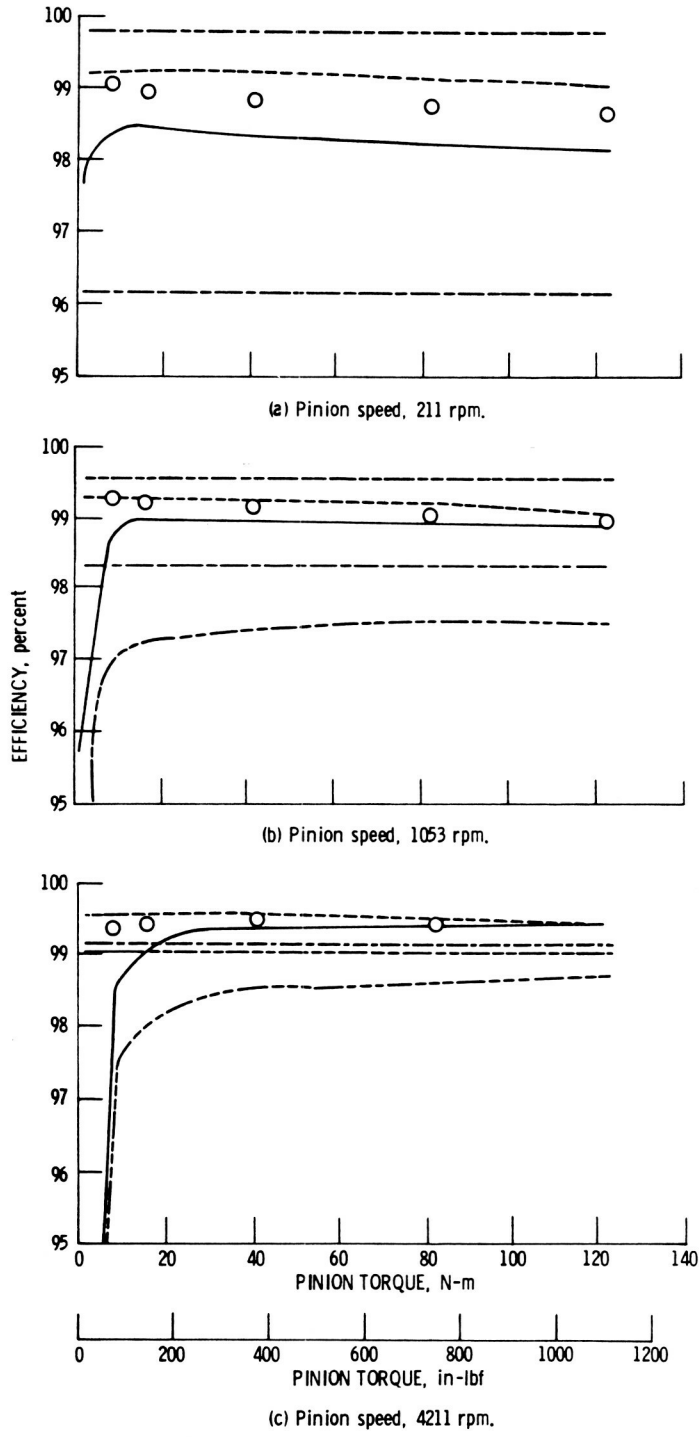


Figure 8. - Gearset efficiency predictions of five methods compared with data of Ohlendorf.



As noted earlier only the methods of Anderson and Loewenthal and Chiu predict no-load, or tare, losses. In figure 6 a decrease in efficiency at lower torques occurs because of the tare losses. The methods that do not predict tare losses show constant efficiency as torque levels are reduced to an unloaded condition. The data of Ohlendorf at 4211 rpm in figure 8(c) show the same effect but to a lesser degree.

In general, the largest variation in predicted efficiency occurs at low speeds. As speed is increased the variations are not as great. For example, at the maximum test speed and torque in each of the three sets of data, the variation in the five prediction techniques was less than 1 percentage point in efficiency. At this same set of conditions the Anderson and Loewenthal and Shipley predictions were within 0.3 percentage point of each other and within 0.5 percentage point of the test data.

It is instructive to note that the measured gear-mesh losses (excluding support bearings) at medium to heavy loads ranged from approximately 0.3 percentage point in the Fletcher and Bamborough test gears (at low speeds) to about 1.7 percentage points for Yada's test gears at high speeds. The Ohlendorf gears showed measured losses from about 0.6 to 1.4 percent. This wide variation in measured efficiency underscores the risk of the common practice of allowing 0.5 percent loss for each spur gear mesh. It should be kept in mind that these data do not include support bearing losses which can often be as great or greater than the mesh losses.

The five methods were rated for closeness of fit to each of the nine sets of experimental data (three gearsets at three speeds). The methods of Anderson and Loewenthal and Shipley gave the best overall correspondence with these data. Buckingham's method showed the next best correlation with data followed by Merritt and Chiu. The largest error at any data point was 1 percentage point for Anderson and Loewenthal, Shipley, and Buckingham, 3 percentage points for Merritt, and 5 percentage points for Chiu.

## Summary of Results

The predictions of five spur-gear efficiency calculation methods were compared with three sets of test data generated by different investigators using different gear geometries. The prediction methods were those of Anderson and Loewenthal, Buckingham, Chiu, Merritt, and Shipley. The test data were those of Fletcher and Bamborough, Ohlendorf, and Yada. The data and the analysis methods were limited to jet lubricated, ground, spur gears. The data covered a range in pitch-line velocity of 1 to 20 m/sec (200 to 4000 ft/min) and K-load factor range of 17 to 1600. The following results were obtained:

1. Of the five calculation methods only the method of Anderson and Loewenthal was able to make consistently good prediction of full and part load losses for all sets of tests data.
2. The method of Shipley gave results comparable with those of Anderson and Loewenthal when tare (no-load) losses were not significant. Buckingham's method showed the next best agreement with the data followed by Merritt and Chiu.
3. Tare losses were predicted only by the methods of Anderson and Loewenthal and Chiu. The other methods which use a friction coefficient expression alone to predict mesh losses, cannot account for tare losses.
4. All expressions for sliding power loss gave the same result when the same friction coefficient was used. The only exception was Merritt's expression.
5. Measured mesh losses (without bearing losses) varied from approximately 0.3 to 1.7 percentage points for the loaded spur test gears.

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