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ANALYTIC DETERMINATIONS OF SINGLE-FOLDING OPTICAL POTENTIALS

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NOMENCLATURE

A	nuclear mass number
$B(e)$	average slope parameter of nucleon-nucleon scattering amplitude, fm^2
c	Woods-Saxon surface diffuseness, fm
e	two-nucleon kinetic energy in their center of mass frame, GeV
J	defined in Eq. (21)
m	nucleon mass, kg
R	Woods-Saxon half-density radius, fm
$\vec{r}$	position vector, fm
r_u	radius of equivalent uniform distribution, fm
t	average two-nucleon transition amplitude, MeV
t_o	defined in Eq. (3)
V_o	defined in Eq. (16)
V_{opt}	optical potential (defined in Eq. (15)), MeV
$\vec{x}$	relative position vector of projectile, fm
$\vec{y}$	two-nucleon relative position vector, fm
$\vec{z}$	position vector of projectile in beam direction, fm
$\alpha(e)$	average ratio of real part to imaginary part of nucleon-nucleon scattering amplitude
β	defined in Eq. (6)
$\delta(\vec{r})$	Dirac delta function
ρ	nuclear density, fm^{-3}
ρ_o	normalization constant in Eq. (20), fm^{-3}
$\sigma(e)$	average nucleon-nucleon total cross section, mb

Subscripts:

P projectile

T target

Arrows over symbols indicate vectors.

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SUMMARY

A simple analytic method for calculating nucleon-nucleus optical potentials using a single-folding of a Gaussian two-body interaction with an arbitrary nuclear distribution is presented. When applied to proton-lead elastic scattering, the predicted real part of the Woods-Saxon potential is in substantial agreement with the experimentally-determined phenomenological potential, although there are no adjustable parameters. In addition, the volume integrals of both real potentials are nearly identical.

INTRODUCTION

In order to properly assess the optimum shield requirements for future manned space efforts, especially missions of long duration, accurate methods for predicting the interactions of cosmic rays with spacecraft materials and inhabitants are required. Although theories capable of accurately predicting high-energy nucleon-nucleus and nucleus-nucleus cross sections exist (refs. 1 through 6), disagreements between theory and experiment are evident at low energies. These are probably due, in large part, to the inapplicability of the eikonal approximation at low energies (refs. 1 and 2). In a recent work (ref. 7), substantial improvement in the agreement, at low energies, for light and medium nuclei was noted when partial wave analyses, utilizing complex WKB (Wentzel-Kramers-Brillouin) phase shifts, were performed. The method outlined in reference 7, however, appears to be suitable only for collisions involving light and medium nuclei whose matter densities are Gaussian or harmonic well distributions, since their nuclear optical potentials can be analytically determined (ref. 8). For heavier nuclei whose density distributions are neither Gaussian or harmonic well shapes, their optical potentials are not readily reduced to analytic forms (ref. 1). In this work, a method is presented for obtaining an approximate analytic expression for the nucleon-nucleus optical potential, involving the single-folding of an arbitrary target density distribution with a Gaussian two-nucleon interaction. As such it represents an initial effort toward discovery of approximate analytic methods for eventual use in evaluating double-folding optical potentials involving arbitrary nuclear density distributions. When applied to 1 GeV proton-lead collisions, this method yields a predicted real part of the nuclear potential which is in excellent agreement with the available phenomenological results obtained from elastic scattering experiments (ref. 9).

THEORETICAL ANALYSIS

In previous work (ref. 2) it was shown that the optical potential approximation to the exact nucleus-nucleus multiple-scattering series is

$$V(\vec{x}) = A_P A_T \int d^3\vec{z}\ \rho_T(\vec{z}) \int d^3\vec{y}\ \rho_P(\vec{x} + \vec{y} + \vec{z})\ t(e,\vec{y}) \tag{1}$$

where the two-body transition amplitude, averaged over constituent types (ref. 1) is

$$t(e,\vec{y}) = t_o \exp[-y^2/2B(e)] \tag{2}$$

with

$$t_o = -(e/m)^{\frac{1}{2}} \sigma(e)[\alpha(e) + i]\ [2\pi B(e)]^{-\frac{3}{2}} \tag{3}$$

and ρ_T and ρ_P are the target and projectile single-particle number (matter densities). In equation (3), e is the two-nucleon kinetic energy in their center of mass frame, and $\sigma(e)$, $\alpha(e)$, and $B(e)$ are the usual nucleon-nucleon scattering parameters (refs. 1 and 2).

For nucleon-nucleus scattering involving a target nucleus of mass number A_T, the projectile single-particle density is

$$\rho_P(\vec{x} + \vec{y} + \vec{z}) = \delta(\vec{x} + \vec{y} + \vec{z}) \tag{4}$$

with $A_P = 1$. Equation (1) then reduces, for nucleon-nucleus scattering, to

$$V_{opt}(\vec{x}) = A_T \int d^3\vec{z}\ \rho_T(\vec{z})\ t(e,\ \vec{x} + \vec{z}) \tag{5}$$

Inserting equation (2) into (5) and letting

$$\beta^2 = [2B(e)]^{-1} \tag{6}$$

yields

$$V_{opt}(\vec{x}) = 2\pi t_o A_T \int_0^\infty z^2 dz\ \rho_T(\vec{z})\ \exp[-\beta^2\ (x^2 + z^2)] \int_\pi^0 \exp[-2xz\ \beta^2 \cos\theta]\ \sin\theta d\theta \tag{7}$$

The angular integration is of the form

$$\int_0^\pi \exp(-q \cos \Theta) \sin \Theta d\Theta = [\exp(q) - \exp(-q)]/q \quad (8)$$

which, upon collecting exponents, yields

$$V_{opt}(\vec{x}) = (\pi t_0 A_T/\beta^2 x) \int_0^\infty zdz \, \rho_T(z) \{\exp[-\beta^2 (z - x)^2] - \exp[-\beta^2 (z + x)^2]\} \quad (9)$$

Evaluating the integrals in equation (9) gives

$$\int_0^\infty zdz \, \rho_T(z)\exp[-\beta^2 (z - x)^2] = \beta^{-1} \int_{-\beta x}^\infty (x + \beta^{-1} s)\rho_T(x + \beta^{-1} s)\exp(-s^2)ds \quad (10)$$

and

$$\int_0^\infty zdz \, \rho_T(z)\exp[-\beta^2 (x + z)^2] = \beta^{-1} \int_{\beta x}^\infty (\beta^{-1} s - x) \, \rho_T(\beta^{-1} s - x)\exp(-s^2)ds \quad (11)$$

Incorporating the results from equations (10) and (11) into (9), and assuming that the density distribution is spherically symmetric yields, after some algebra,

$$V_{opt}(\vec{x}) = \pi t_0 A_T \beta^{-3} \, \{(\beta \;\; x)^{-1} \int_0^\infty sds \, \exp(-s^2) \, [\rho_T(x + \beta^{-1} s) - \rho_T(x - \beta^{-1} s)] + \int_{-\infty}^\infty ds \, \exp(-s^2) \, \rho_T(x + \beta^{-1} s)\} \quad (12)$$

The integrals in equation (12) are of the same form as those in equation (86) of reference 1, where a similar method was utilized to extract matter densities from nuclear charge densities.

The first integral, is generally smaller than the second since it contributes only when x is near the nuclear edge. For the extreme case, where ρ_T is a finite uniform distribution, the ratio of the first integral to the second, at the uniform radius, r_u, is

$$\text{Ratio} = r_u^{-1}[2B(e)/\pi]^{-\frac{1}{2}} \tag{13}$$

Noting that this ratio is a maximum for large B(e) and small r_u, which is the situation for light target nuclei and high energies, an estimate of the ratio is made. Choosing r_u = 3 fm (a nominal value for a lithium nucleus from reference 10) and B = .5 fm^2 (ref. 11) gives a maximum ratio of 19 percent. It is expected that the error resulting from neglect of the first integral, for realistic densities, will be substantially less than this value.

Neglecting the first term in equation (12) yields an approximate nuclear optical potential

$$V_{opt}(x) = \pi t_0 A_T \beta^{-3} \int_{-\infty}^{\infty} ds \, \exp(-s^2) \, \rho_T(x + \beta^{-1} s) \tag{14}$$

For an arbitrary density distribution, the integral can be approximated by a two-point Gauss-Hermite quadrature formula (ref. 12) to yield an analytic nuclear optical potential

$$V_{opt}(\vec{x}) = (V_0 A_T/2)[\rho_T(x + \sqrt{B}) + \rho_T(x - \sqrt{B})] \tag{15}$$

with

$$V_0 = -(e/m)^{\frac{1}{2}} \quad \sigma(e)[\alpha(e) + i] \tag{16}$$

Should the need arise to include the first integral in equation (12), it can be similarly approximated by changing variables such that

$$\int_0^{\infty} s ds \, \exp(-s^2)[\rho_T(x + \beta^{-1} s) - \rho_T(x - \beta^{-1} s)]$$

$$= \frac{1}{2} \int_0^{\infty} dp \, \exp(-p)[\rho_T(x + (p/\beta^2)^{-\frac{1}{2}}) \tag{17}$$

$$- \rho_T(x - (p/\beta^2)^{-\frac{1}{2}})]$$

This allows the integral to be easily approximated by a Laguerre quadrature formula (ref. 12).

RESULTS FOR PROTON-LEAD ELASTIC SCATTERING

To illustrate the application of equation (15), the real part of the nuclear optical potential for proton-lead elastic scattering at an incident kinetic energy of 1.04 GeV is calculated. Thus, taking the real part of equation (15) gives

$$V_{opt}(\vec{x}) = -[(e/m)^{\frac{1}{2}} \sigma(e) \alpha(e) A_T/2][\rho_T(x + \sqrt{B}) + \rho_T(x - \sqrt{B})] \quad (18)$$

which, for 1.04 GeV protons colliding with lead yields

$$V_{opt}(\vec{x}) = 9641.5 [\rho_T(x + .49) + \rho_T(x - .49)] \quad (19)$$

For ρ_T given in fm^{-3} (normalized to unity), $V_{opt}(\vec{x})$ in equation (19) is given in MeV. Choosing ρ_T of lead to be a Woods-Saxon charge density using the methods in references 1 and 2 yields.

$$\rho_T(\vec{r}) = \rho_0 \{1 + \exp[(r - R)/c]\}^{-1} \quad (20)$$

where R = 6.624 fm, c = 0.860 fm, and $\rho_0 = 7.059 \times 10^{-4}\ fm^{-3}$. The results obtained from equations (19) and (20) are plotted in figure 1. Also plotted in figure 1 are the phenomenological Woods-Saxon potential results obtained from elastic scattering experiments (ref. 9). The agreement between theory and experiment is impressive particularly since there are no arbitrarily adjustable parameters in the theory. In addition, the volume integrals, J , of both real potentials

$$J = 4\pi \int_0^\infty r^2\, V(r)\, dr \quad (21)$$

agree to within 0.02 percent.

Finally, we note from equation (13) that the maximum expected error due to the neglect of the first integral in equation (12) is less than 6 percent. The actual error, for the density used in the calculations (eq. (20)), was found by numerical methods to be less than 5 percent (typically 2-3 percent) for values of x up to 12 fm.

CONCLUDING REMARKS

In this work a simple analytic method for approximating nuclear optical potential integrals involving the single-folding for a Gaussian interaction with arbitrary nuclear distributions was presented. Applying the method to 1.04 GeV proton-lead collisions, the real part of the predicted potential was found to be in remarkably good agreement with the phenomenological results obtained from elastic scattering experiments. The resulting Woods-Saxon radial shape, for the potential, obtained when a Woods-Saxon target density was utilized also suggests that there may be some credibility in the usual assumption that the spatial dependence of the nuclear potential should follow that of the associated nuclear density distribution. It is anticipated that the analytic methods described herein will be useful for determining complex WKB solutions to low energy scattering problems and for other nuclear potential calculations where purely numerical methods may not be appropriate.

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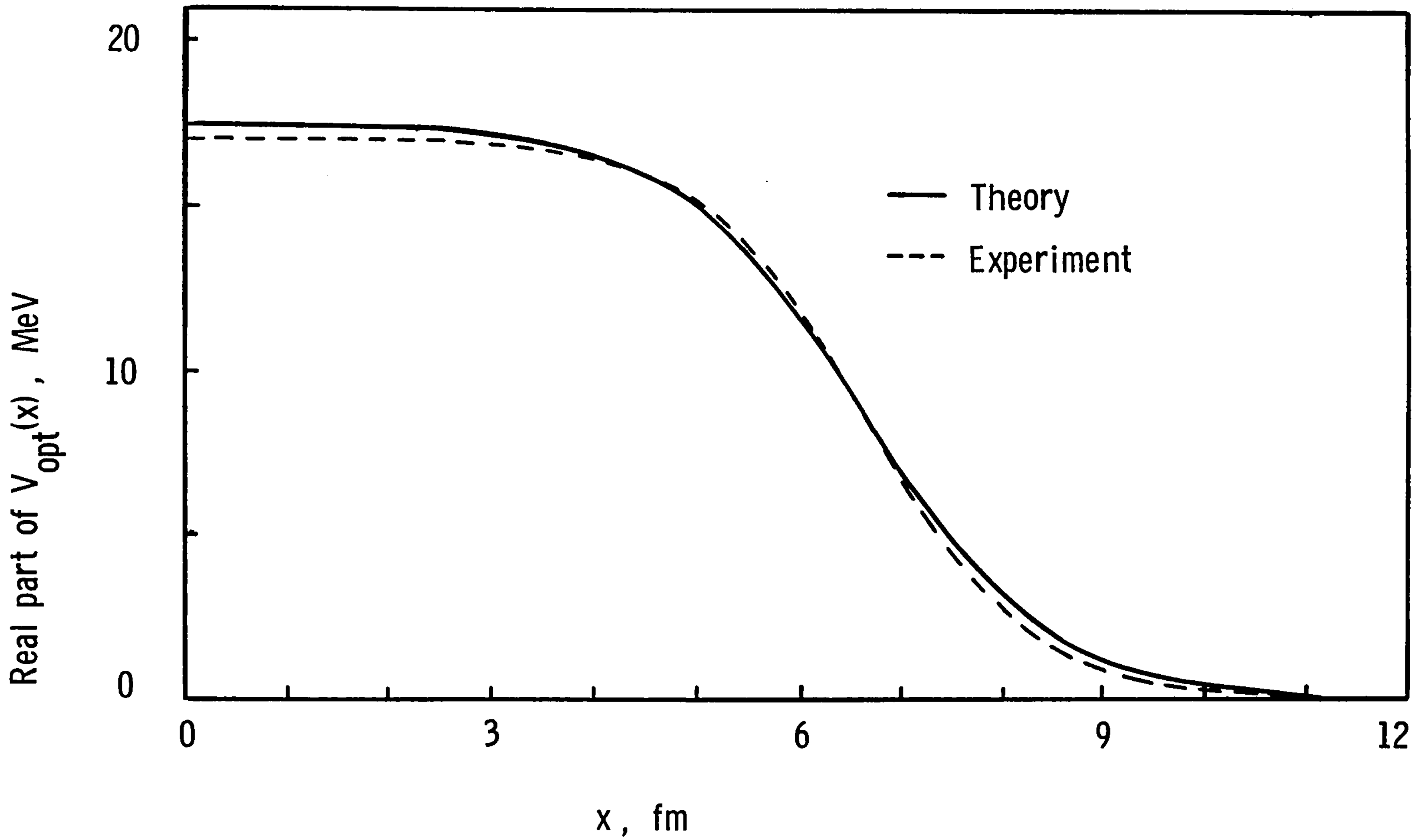

Figure 1. - Real part of the proton - lead optical potential at 1.04 GeV incident kinetic energy.

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