

IDENTIFICATION AND CONTROL OF SPACECRAFT

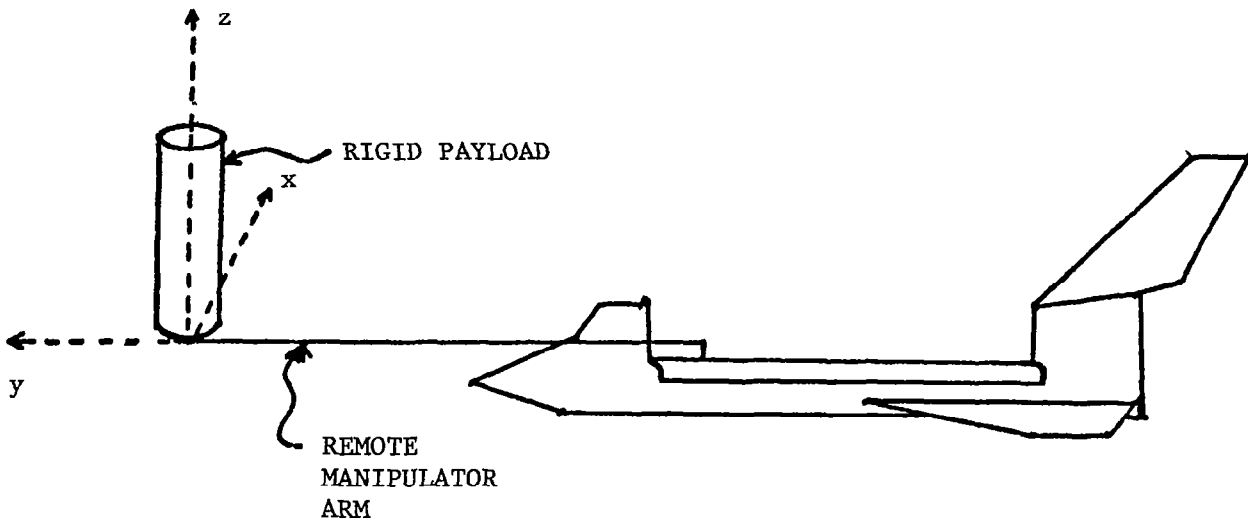
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IDENTIFICATION & CONTROL OF SPACECRAFT

* THE PROBLEM

* CONTROL
- CLASSICAL
- MODERN

* IDENTIFICATION



CLASSICAL CONTROL

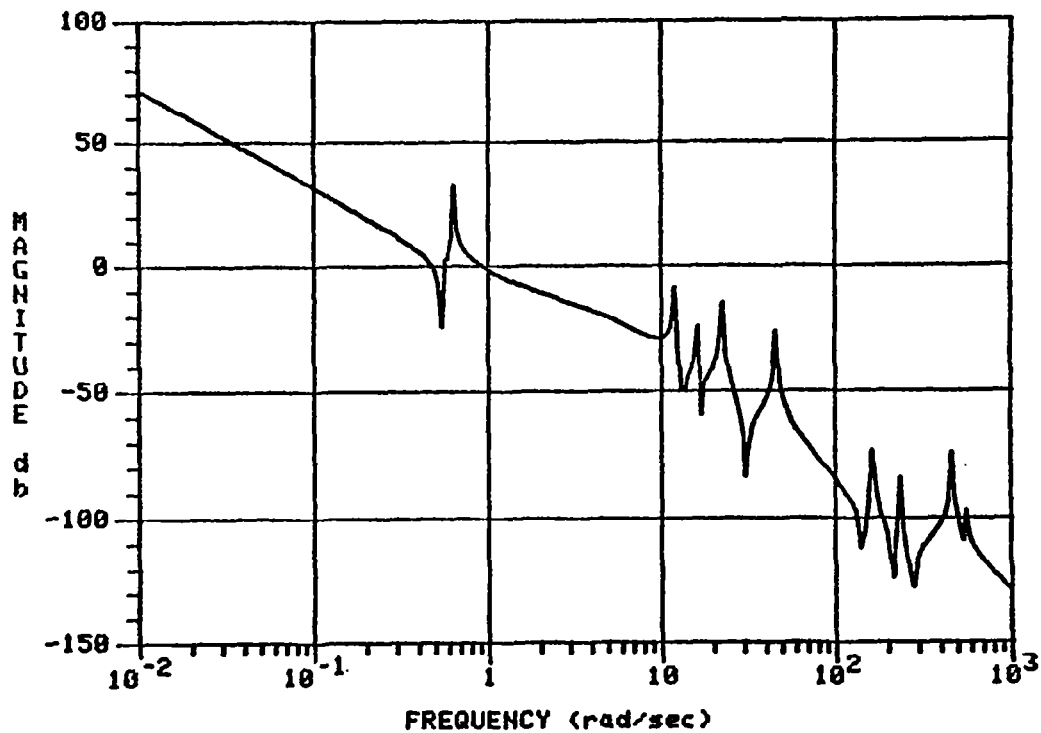
* SCALAR FEEDBACK DESIGN

* LEAD-LAG CONTROLLER

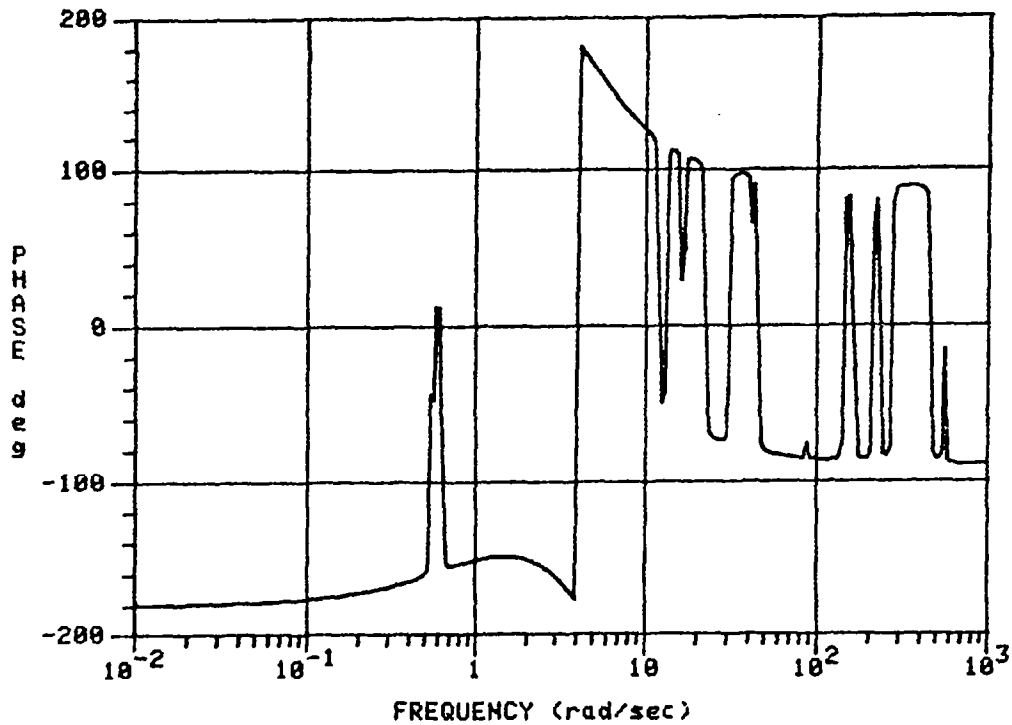
$$K(s) = \frac{k(s+a)}{(s^2+2\zeta b+b^2)}$$

* BANDWIDTH AROUND 1 RAD/SEC

SHUTTLE AND PAYLOAD: FROM ALPHA TO Y-ATTITUDE



SHUTTLE AND PAYLOAD: FROM ALPHA TO Y-ATTITUDE



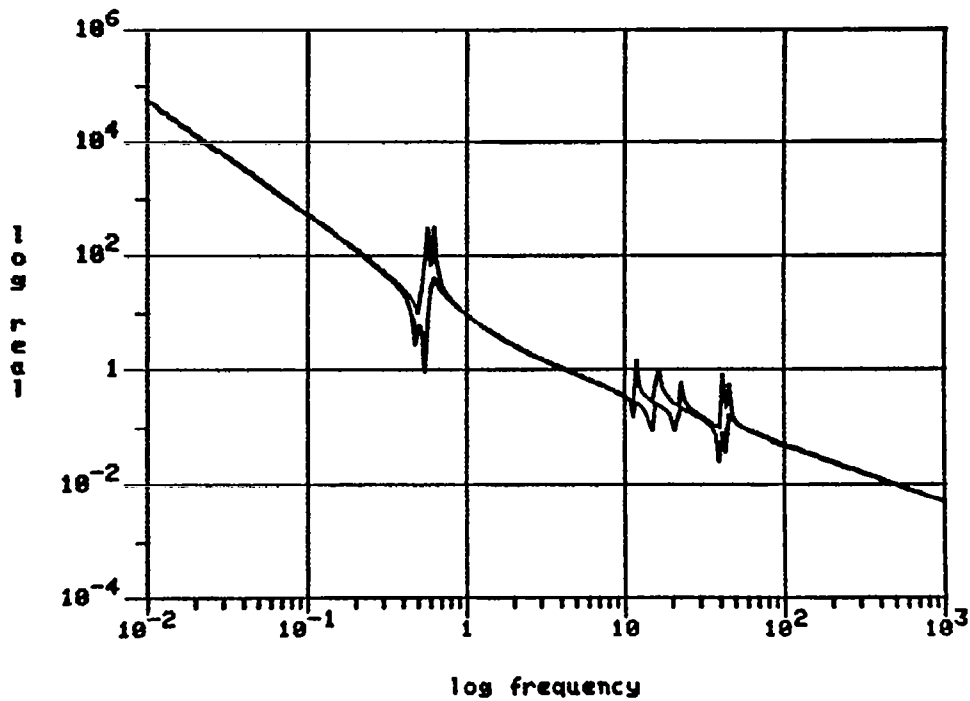
MODERN CONTROL

- * START WITH FULL-STATE DESIGN
 - GOAL: MINIMIZE PAYLOAD ATTITUDE ERRORS
 - ITERATE ON CONTROL PENALTY TO ACHIEVE BW OF 5 r/s

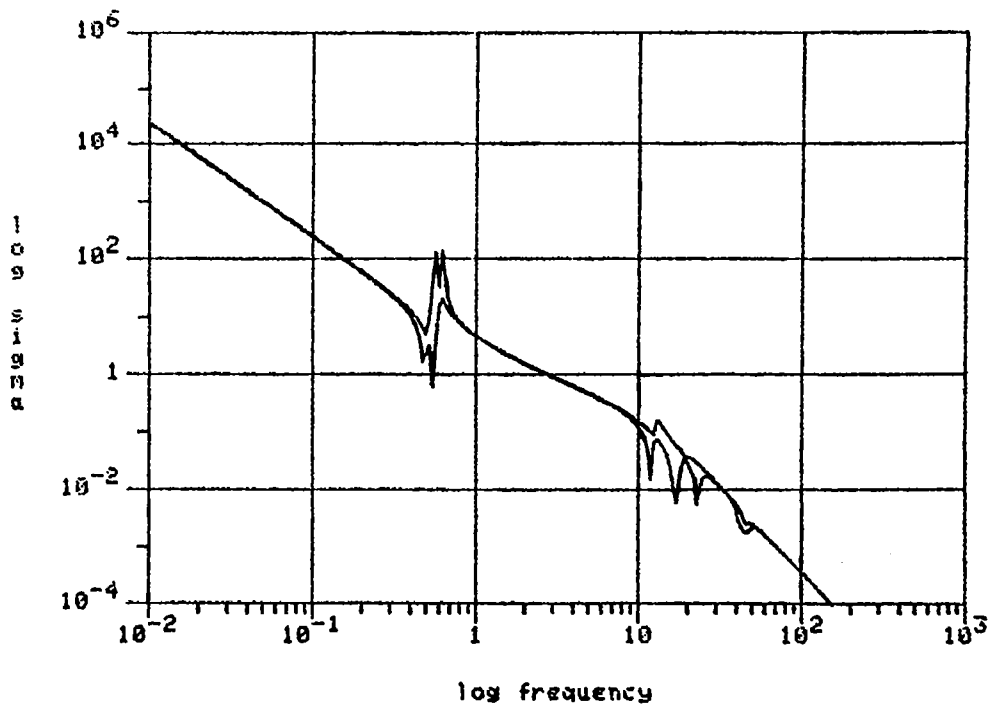
- * DESIGN FILTER TO RECOVER LQ RESPONSE
 - USE STEIN/DOYLE ROBUSTNESS RECOVERY RESULTS

- * TEST ROBUSTNESS

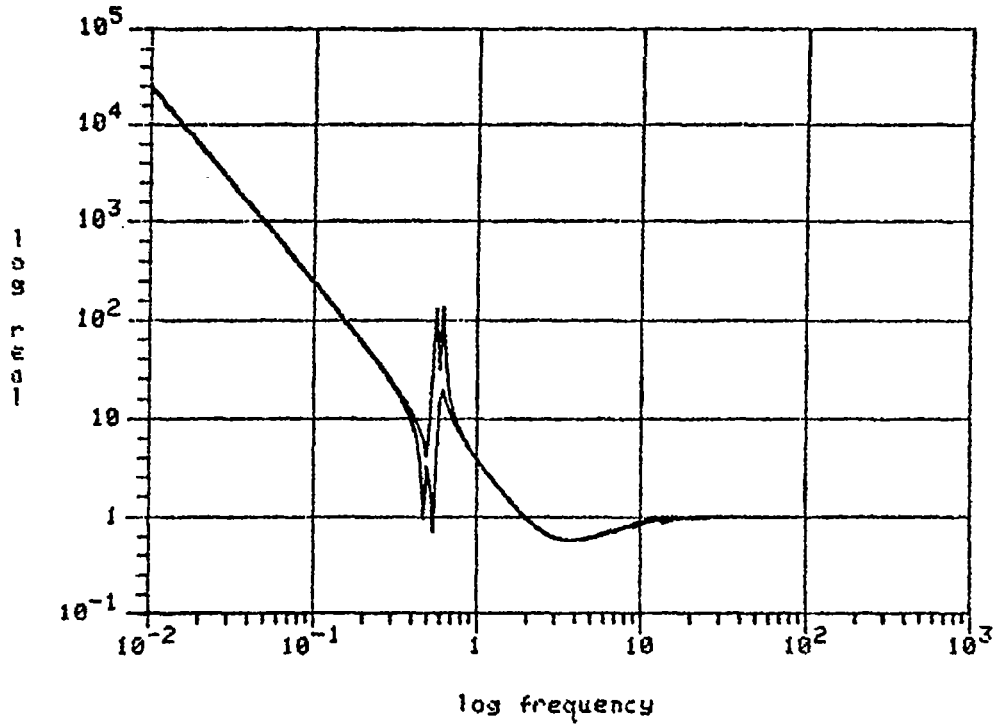
STATE FEEDBACK LQ



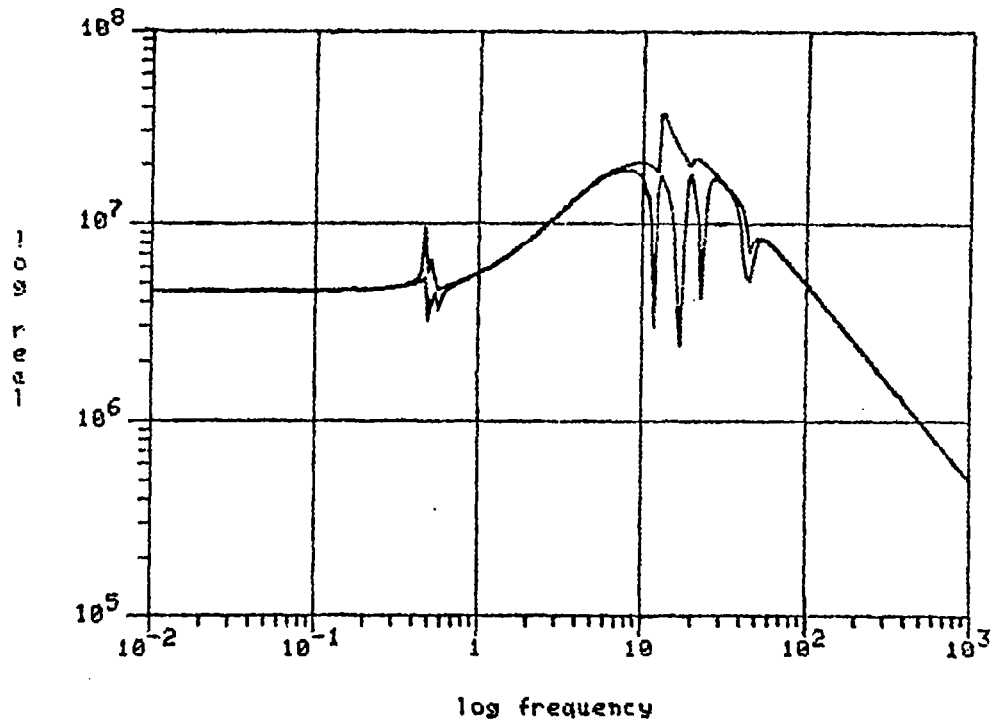
plant.r: Singular values of $K(s)*G(s)$ $q=1E8$



I + KG

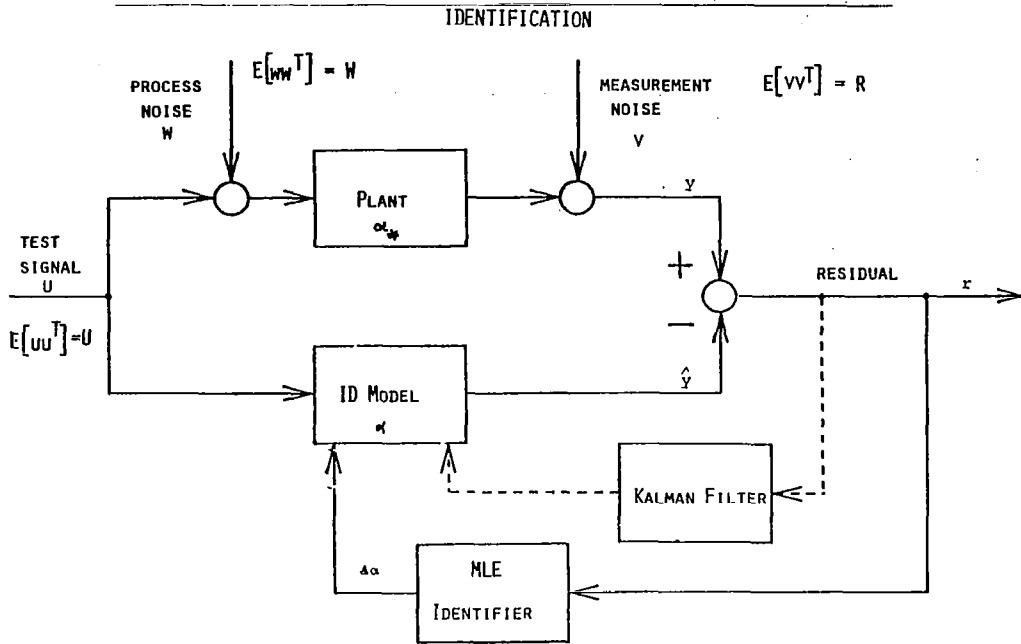


CONTROLLER



IDENTIFICATION

MAXIMUM LIKELIHOOD ESTIMATION (MLE)



MODEL STRUCTURE

- STATE SPACE

n_* MODES (2x2 BLOCKS)

m INPUTS

p OUTPUTS

$$\begin{bmatrix} \vdots \\ \dot{x}_i \\ \vdots \\ \dot{x}_i \\ \vdots \end{bmatrix} (t) = \begin{bmatrix} \ddots & & & 0 \\ & 0 & 1 & \\ & -\omega_i^2 & -2\zeta_i\omega_i & \\ & 0 & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ x_i \\ \vdots \\ x_i \\ \vdots \end{bmatrix} (t) + \begin{bmatrix} \vdots \\ 0 \\ b_i \\ \vdots \end{bmatrix} [u(t) + w(t)]$$

$$[y] (t) = \begin{bmatrix} \dots & \vdots & c_i & 0 & \vdots & \dots \end{bmatrix} \begin{bmatrix} \vdots \\ x_i \\ \vdots \\ x_i \\ \vdots \end{bmatrix} (t) + [v] (t)$$

- FREQUENCY DOMAIN

$$y(s) = (G_f(s)(u(s) + w(s)) + v(s))$$

$$G_f(s) = \sum_{i=1}^{n_*} \frac{c_i b_i^T}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}$$

- PARAMETER VECTOR

$$\alpha_* = \{\omega_{*i}^2, 2\zeta_{*i}\omega_{*i}, b_{*i}, c_{*i}; i = 1, \dots, n_*\}$$

MLE IDENTIFICATION SETUP

- RESIDUAL DEFINITION

$$r_k \triangleq y(kT) - \hat{y}(kT)$$

- LIKELIHOOD FUNCTION (NEGATIVE LOG)

$$\begin{aligned} L(\alpha) &\triangleq \sum_{k=0}^N L_k(\alpha) \\ &\triangleq \sum_{k=0}^N \frac{1}{2} \left[\text{LOG DET } S_k + r_k^T S_k^{-1} r_k \right] \end{aligned}$$

WHERE

$$\alpha \triangleq \{\omega_i^2, 2\zeta_i \omega_i, b_i, c_i; i = 1, \dots, n\} = \text{UNKNOWN PARAMETERS}$$

$$S_k \triangleq E_{\alpha} \{r_k r_k^T\} = \text{PREDICTED RESIDUAL COVARIANCE}$$

MLE IDENTIFICATION SOLUTION

- PARAMETER ESTIMATE (THEORETICAL)

$$\hat{\alpha} \triangleq \text{ARG} \left\{ \text{MIN}_{\alpha} L(\alpha) \right\} = \text{PARAMETER ESTIMATE}$$

- ITERATIVE ALGORITHMS

GRADIENT: $\hat{\alpha}_{j+1} = \hat{\alpha}_j - \epsilon_j \nabla L(\hat{\alpha}_j)$

NEWTON-RHAPSON: $\hat{\alpha}_{j+1} = \hat{\alpha}_j - \left[\nabla^2 L(\hat{\alpha}_j) \right]^{-1} \nabla L(\hat{\alpha}_j)$

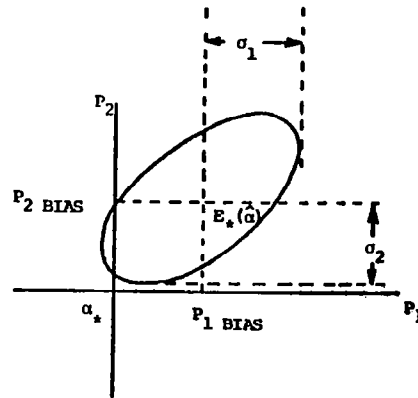
WHERE

$$\nabla L(\alpha) \triangleq \frac{\partial L}{\partial \alpha}(\alpha)$$

$$\nabla^2 L(\alpha) \triangleq \frac{\partial^2 L}{\partial \alpha^2}(\alpha)$$

IDENTIFICATION ACCURACY ISSUES

- SYSTEMATIC ERRORS: $E_*(\hat{\alpha}) - \alpha_*$
 - MODEL-ORDER MISMATCH
 - TEST SIGNAL AMPLITUDE AND SHAPING
 - SYSTEMATIC DISTURBANCES
 - SENSOR/ACTUATOR MODEL ERRORS



- STOCHASTIC ERRORS: $\hat{\alpha}_\alpha \sim \frac{1}{\sqrt{TN}}$
 - RANDOM DISTURBANCES AND SENSOR NOISE
 - TEST SIGNAL AMPLITUDE AND SHAPING
 - IDENTIFICATION TIME INTERVAL

STEADY-STATE IDENTIFIABILITY ANALYSIS (YARED)

- EXPECTED LIKELIHOOD FUNCTION

$$\begin{aligned}
 I^*(\alpha) &= E_* \left\{ l_k(\alpha) \right\} \\
 &= \frac{1}{2} \left[\text{LOG DET } \mathbf{S} + \text{TR}(\mathbf{S}^{-1} \mathbf{S}_*) \right]
 \end{aligned}$$

- RESIDUAL COVARIANCES

$$\mathbf{S} = E_\alpha \left\{ r_K r_K^T \right\} = \text{KALMAN FILTER PREDICTED RESIDUAL COVARIANCE MATRIX}$$

$$\mathbf{S}_* = E_* \left\{ r_K r_K^T \right\} = \text{ACTUAL RESIDUAL COVARIANCE MATRIX}$$

NOTE: \mathbf{S}_* AND $I^*(\alpha)$ CAN ONLY BE COMPUTED WHEN THE TRUE PLANT PARAMETERS ARE KNOWN.

EXPECTED MLE IDENTIFICATION SOLUTION

- EXPECTED PARAMETER ESTIMATE (THEORETICAL)

$$\hat{\alpha}_* \triangleq E_* \left\{ \alpha \right\} = \text{ARG} \left\{ \underset{\alpha}{\text{MIN}} I^*(\alpha) \right\}$$

- ITERATIVE ALGORITHMS

GRADIENT: $\hat{\alpha}_{*j+1} = \hat{\alpha}_{*j} - \epsilon_j \nabla I^*(\hat{\alpha}_{*j})$

NEWTON-RHAPSON: $\hat{\alpha}_{*j+1} = \hat{\alpha}_{*j} - \left[\nabla^2 I^*(\hat{\alpha}_{*j}) \right]^{-1} \nabla I^*(\hat{\alpha}_{*j})$

WHERE $\nabla I^*(\alpha) = \frac{\partial I^*(\alpha)}{\partial \alpha}$
 $\nabla^2 I^*(\alpha) = \frac{\partial^2 I^*(\alpha)}{\partial \alpha^2}$

STEADY STATE IDENTIFICATION ACCURACY

- SYSTEMATIC ERRORS (BIASES)

- PARAMETER ERRORS

$$\alpha_{\text{BIAS}} \triangleq \hat{\alpha}_* - \alpha_* = 0 \text{ WHEN NO MODEL MISMATCH}$$

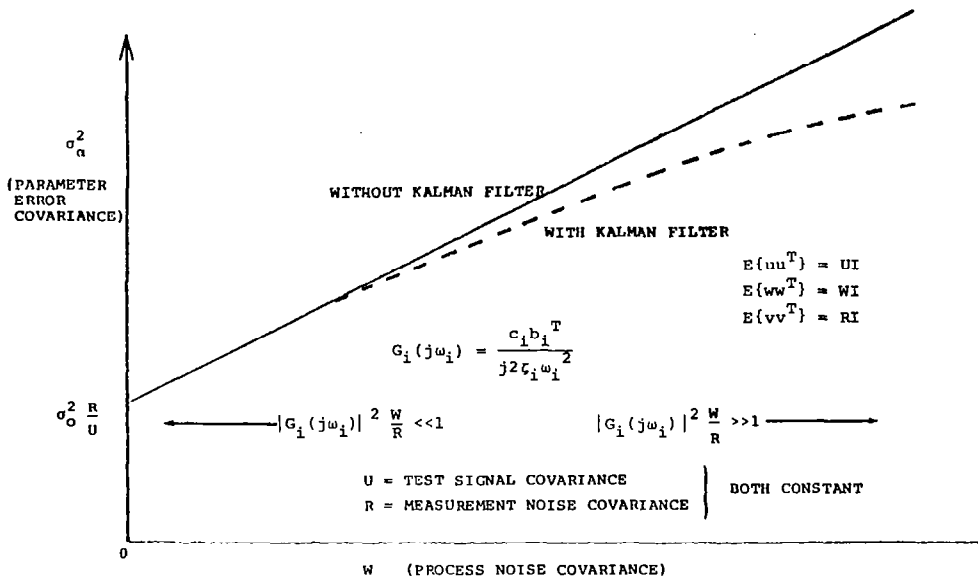
- INFORMATION MEASURE (YARED)

$$I(\alpha_*; \hat{\alpha}_*) \triangleq I^*(\hat{\alpha}_*) - I^*(\alpha_*) \geq 0$$

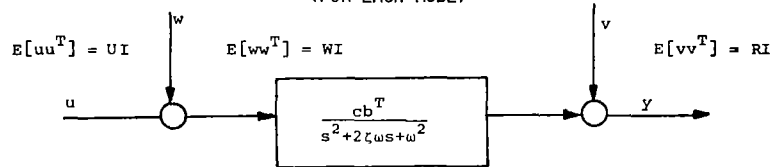
- STOCHASTIC ERRORS

$$\begin{aligned} C_{\text{OV}}\{\hat{\alpha}\} &\triangleq \lim_{N \rightarrow \infty} E_* \left\{ (\hat{\alpha} - \alpha_*) (\hat{\alpha} - \alpha_*)^T \right\} \\ &\triangleq \left[\nabla^2 I^*(\alpha_*) \right]^{-1} E_* \left\{ \left[\frac{\partial I}{\partial \alpha}(\hat{\alpha}_*) \right] \left[\frac{\partial I}{\partial \alpha}(\hat{\alpha}_*) \right]^T \right\} \left[\nabla^2 I^*(\alpha_*) \right]^{-1} \frac{1}{(N+1)^2} \\ &= \left[\nabla^2 I^*(\alpha_*) \right]^{-1} \frac{1}{(N+1)} \text{ WHEN NO MODEL MISMATCH} \end{aligned}$$

STOCHASTIC ERROR WITH PROCESS NOISE



SIMPLIFIED IDENTIFICATION ACCURACY ANALYSIS (FOR EACH MODE)



$$\alpha \triangleq (\omega^2, 2\zeta\omega, b_1, \dots, b_m, c_1, \dots, c_p) - \{b_L \text{ or } c_M\}$$

$$\frac{\sigma_{\omega}^2}{(\omega^2)^2} = \frac{8\zeta^2}{\text{SNR}} \frac{1}{(N+1)T} \quad ; \quad \rho \triangleq \max_{\ell, m} \left\{ \frac{b_{\ell}^2}{|b|^2}, \frac{c_m^2}{|c|^2} \right\} \leq 1$$

$$\frac{\sigma_{2\zeta\omega}^2}{(2\zeta\omega)^2} = \frac{4}{\text{SNR}} \frac{1}{(N+1)T} \quad ; \quad \text{SNR} \triangleq \begin{cases} \frac{|c|^2|b|^2}{4\zeta\omega^3} \frac{U}{R} & \text{(MEAS. NOISE)} \\ \zeta\omega \frac{U}{W} & \text{(PROC. NOISE)} \end{cases}$$

$$\frac{\sigma_{b_{\ell}}^2}{b_{\ell}^2} = \frac{1}{\text{SNR}} \left[\frac{|b|^2}{b_{\ell}^2} + \rho^{-1} \right] \frac{1}{(N+1)T}$$

$$\frac{\sigma_{c_m}^2}{c_m^2} = \frac{1}{\text{SNR}} \left[\frac{|c|^2}{c_m^2} + \rho^{-1} \right] \frac{1}{(N+1)T}$$

NOTE: THIS ANALYSIS ASSUMES THAT $\omega T \ll 1$, $\zeta \ll 1$ AND APPLIES FOR EACH MODE

NUMERICAL RESULTS

- PROBLEM SIZE
 - 12 MODES (=22 - 8R.B. - 2 SMALL)
 - 2 INPUTS (α , β GIMBAL ANGLES)
 - 2 OUTPUTS (x , y ATTITUDES)
 - 60 PARAMETERS (=12 MODES \times 5 PARAMETERS/MODE)

- TEST SIGNAL, NOISE STATISTICS
 - SAMPLE TIME: $T = 0.1$ SEC
 - TEST SIGNAL: $U = 4000(\text{IN-LB})^2$
 - PROCESS NOISE: $W = 40(\text{IN-LB})^2$
 - MEASUREMENT NOISE: $R = 4 \times 10^{-12} \text{ RAD}^2$

- WORST-CASE RELATIVE ERRORS AT TIME (MODE 9):

<u>PARAMETER</u>	<u>1 SEC</u>	<u>14 SEC</u>	<u>0.39 HRS</u>
ω^2	0.0265	0.007	0.0007
$2\zeta\omega$	3.75	1.0	0.1
b_1	13.6	3.6	0.36
c_1	2.7	0.7	0.07
c_2	11.7	3.1	0.31

SUMMARY

- CONTROL PROBLEM
 - MODERN LQ CONTROL DESIGN WITH ROBUSTNESS RECOVERY
PRODUCES ROBUST CONTROLLERS FOR LSS
 - ACCURATE ID ALLOWS A FIVE-FOLD INCREASE IN LOOP BW
- IDENTIFICATION PROBLEM
 - STRUCTURAL MODES MAY BE IDENTIFIED ONE AT A TIME FOR
SMALL DAMPING
 - LSS ID W/O KF
 - GREATLY REDUCES PARAMETER BIASES
 - GIVES ONLY MODEST INCREASE IN STOCHASTIC ERRORS
 - RELATIVE ERRORS IN PARAMETERS AFTER ID ARE SMALLER FOR
FREQUENCY THAN FOR DAMPING OR MODE SHAPES
- OPEN ISSUE: HOW ACCURATE MUST ID BE FOR ROBUST CONTROL DESIGN?