STRUCTURAL DESIGN FOR DYNAMIC RESPONSE REDUCTION

Brantley R. Hanks
NASA Langley Research Center
Hampton, Virginia

OBJECTIVE: STUDY STIFFNESS AUGMENTATION BY MATHEMATICAL DESIGN

APPROACH: APPLY LINEAR REGULATOR THEORY WITH PROPORTIONAL FEEDBACK

JUSTIFICATION: STIFFNESS IS READILY AVAILABLE TO DESIGNER AS PREDICTABLE PASSIVE CONTROL

TIME-INVARIANT LINEAR REGULATOR---GENERAL

SYSTEM:

$$\dot{x} = Ax + Bu + Dw$$

CONTROLLED VARIABLES:

$$y = Cx$$

OBJECTIVE:

$$\underset{u}{\text{Min}}\ J \quad \text{where} \quad J = x_f^T S_f x_f + \int_0^{t_f} \left[y^T Q y + u^T R u \right] dt$$

OPTIMAL CONTROL (ASSUMING w IS RANDOM):

$$u = -R^{-1} B^T Px$$

WHERE P IS SOLUTION TO

$$\dot{P} = -PA - A^T P + PBR^{-1} B^T P - C^T QC \qquad P(t_f) = S_f$$

IF $t_f \rightarrow \infty$, GET STEADY-STATE P (AND U) FROM

$$0 = -PA - A^T P + PBR^{-1} B^T P - C^T QC$$

POSITIVE DEFINITE P EXISTS IF

- A IS DETECTABLE IN C, STABILIZABLE IN B
- RESPONSE WEIGHTING MATRIX, Q, IS POSITIVE SEMIDEFINITE
- CONTROL WEIGHTING MATRIX, R, IS POSITIVE DEFINITE

LINEAR REGULATOR ADAPTED TO STRUCTURES

SYSTEM:

$$\begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}G \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{Bmatrix} 0 \\ M^{-1}B \end{Bmatrix} u + \begin{Bmatrix} 0 \\ M^{-1}D \end{Bmatrix} w \qquad w \sim N(0_1\sigma^2)$$

OBJECTIVE FUNCTION:

$$J = \int_0^{t_f} \left[\begin{pmatrix} x^T & \dot{x}^T \end{pmatrix} \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + (u^T)[R](u) \right] dt$$

ASSUME:

- RANDOM INITIAL CONDITIONS
- COMPLETE STATE FEEDBACK WITH NO ROTATIONAL COUPLING
- $t_f \rightarrow \infty$ (TIME-INVARIANT STRUCTURAL CHANGE)

CONTROL:

$$\begin{Bmatrix} 0 \\ M^{-1}B \end{Bmatrix} u = \begin{bmatrix} 0 & 0 \\ -M^{-1}BR^{-1}B^T\, M^{-1}\, {}^T P_{21} & -M^{-1}BR^{-1}B^T\, M^{-1}\, {}^T P_{22} \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

WHERE P_{21} AND P_{22} ARE SOLUTIONS TO

$$P_{21}^T A_{21} + A_{21}^T P_{21}^T - P_{21}^T M^{-1} BR^{-1}B^T(M^{-1})^T P_{21} + C_1^T Q_1 C_1 = 0 \qquad (1)$$

AND

$$P_{22}A_{22} + A_{22}^T P_{22} - P_{22}M^{-1}BR^{-1} B^T(M^{-1})^T P_{22} + (P_{21} + P_{21}^T + C_2 Q_2 C_2) = 0 \qquad (2)$$

IN THESE EQUATIONS $A_{21} = M^{-1} K$ AND $A_{22} = M^{-1} G$

NOTE THAT (1) IS NOT SYMMETRIC; ALSO THAT (1) IS INDEPENDENT OF (2).

Q WEIGHTING MATRIX CONSIDERATIONS

$$\min_u J \quad \text{where} \quad J = \int_0^\infty \left[y^T Q y + u^T R u \right] dt$$

$$y^T Q y = \left(x^T \; \dot{x}^T \right) \begin{bmatrix} C_{11}^T & C_{21}^T \\ C_{12}^T & C_{22} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

- If rate and displacement considered independently and Q chosen so as not to couple x and $\dot{x}$

$$y^T Q y = \left(x^T \; \dot{x}^T \right) \begin{bmatrix} C_{11}^T Q_{11} C_{11} & 0 \\ 0 & C_{22}^T Q_{22} C_{22} \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

- For design, selection of C is governed by desired minimum response points. Hence, C and Q may be assigned similar functions.
- Diagonal C and Q minimizes weighted square response at selected coordinates.
- Choice of $Q_n = K$ and $Q_{22} = M$ minimizes sum of strain and kinetic energy at locations determined and (optionally) weighted by C.

REGULATOR FOR STRUCTURES--MODAL COORDINATES

TRANSFORMATION $x = \phi q$ WHERE $q = q_k e^{(\sigma_i + j\omega_i)t}$

WHERE ϕ IS NORMALIZED $\phi^T M \phi = I$

AND σ_i IS ASSUMED PROPORTIONAL TO ω_i (I.E., $\sigma_i = -2\xi_i\omega_i$ OR $\phi^T G \phi = \left[\diagdown -2\xi_i\omega_i \diagdown\right]$)

OBJECTIVE FUNCTION BECOMES

$$J = \int_0^{t_f} \left[\left(q^T \dot{q}^T\right) \begin{bmatrix} \phi^T K \phi & 0 \\ 0 & \phi^T M \phi \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} + \left(u^T \mathcal{R} u\right) \right] dt$$

NOTE THAT

$$\phi^T K \phi = \left[\diagdown \omega_i^2 \diagdown\right] = \left[\diagdown \Omega^2 \diagdown\right]$$

HENCE, WEIGHTING MATRIX $Q = \begin{bmatrix} \diagdown \Omega^2 & 0 \\ 0 & I \diagdown \end{bmatrix}$

RICCATI EQUATIONS BECOME

$$\mathcal{P}_{21}^T \Omega^2 + \Omega^2 \mathcal{P}_{21} + \mathcal{P}_{21}^T \mathcal{B} \mathcal{R}^{-1} \mathcal{B}^T \mathcal{P}_{21} - \mathcal{C}_1^T \Omega^2 \mathcal{C}_1 = 0 \tag{3}$$

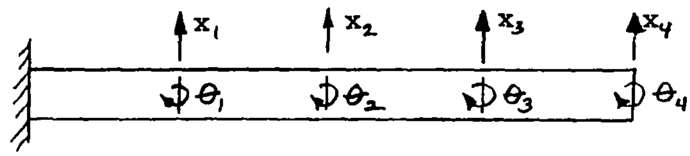
AND

$$\mathcal{P}_{22} \left[\diagdown 2\xi\Omega \diagdown\right] + \left[\diagdown 2\xi\Omega \diagdown\right] \mathcal{P}_{22} + \mathcal{P}_{22} \mathcal{B} \mathcal{R}^{-1} \mathcal{B}^T \mathcal{P}_{22} - \left(\mathcal{P}_{21} + \mathcal{P}_{21}^T + \mathcal{C}_2^T \mathcal{C}_2\right) = 0 \tag{4}$$

WHERE $\mathcal{P}$, $\mathcal{B}$, $\mathcal{R}$, AND $\mathcal{C}$ ARE MODAL EQUIVALENTS OF P, B, R, AND C.

BY CHOOSING $\mathcal{P}$, $\mathcal{B}$, $\mathcal{R}$, AND $\mathcal{C}$ DIAGONAL, WE DECOUPLE THE SOLUTION AND GET PURE "MODAL CONTROL."

CANTILEVER BEAM MODEL

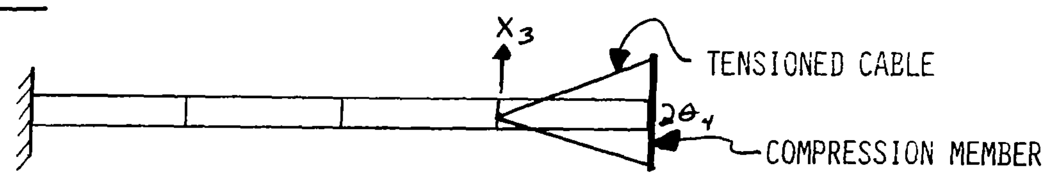


ASSUMED:

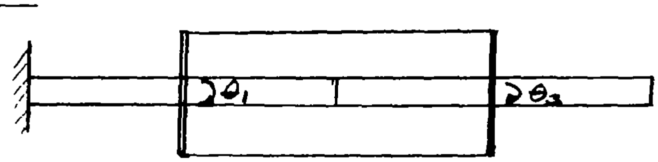
- CONSISTENT MASS FINITE ELEMENTS
- UNIFORM INITIAL STIFFNESS & MASS DISTRIBUTION
- FIRST NATURAL FREQUENCY = .047 Hz (.297 RAD/SEC)

PHYSICAL IMPLEMENTATION OF STIFFNESS CONTROL

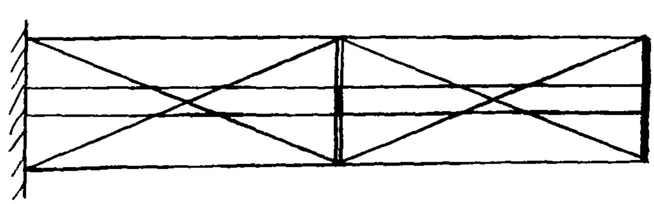
X_3, θ_4 COUPLING



θ_1, θ_3 COUPLING



PRACTICAL OPTIMUM?



CONTROL WEIGHTING EFFECTS ON DESIGN

UNDAMPED NATURAL FREQUENCIES

MODE	INITIAL FREQ., RAD/SEC	FINAL FREQUENCY, RAD/SEC R = 10I	R = I	R = .1 I	R = .01 I
1	.297	.359	.557	.972	1.725
2	1.867	1.880	1.989	2.619	4.538
3	5.262	5.267	5.309	5.684	7.711
4	10.382	10.384	10.406	10.615	12.233

DAMPING RATIOS

MODE	INITIAL DAMPING % C/C_R	FINAL DAMPING, % C/C_{CR} R = 10I	R = I	R = .1 I	R = .01 I
1	2	59.3	108	176	298
2	2	12.0	35.0	78.1	131
3	2	4.7	13.4	38.6	82.9
4	2	2.9	7.1	21.0	55.4

*NOTE: SOLUTIONS OBTAINED SEPARATELY FOR STIFFNESS AND DAMPING COMPARED EXACTLY TO FULL ORDER CONTROLLER SOLUTION

STIFFNESS MATRIX COMPARISON (ASSUMED CONSTANT MASS)

ORIGINAL K

X_1	θ_1	X_2	θ_2	X_3	θ_3	X_4	θ_4	
125	-1250	-125	-1250	0	0	0	0	X_1
	16667	1250	8333	0	0	0	0	θ_1
		250	0	-125	-1250	0	0	X_2
			33333	1250	8333	0	0	θ_2
				250	0	-125	-1250	X_3
					33333	1250	8333	θ_3
						250	0	X_4
							33333	θ_4

$\omega = .297$ rad/sec

FINAL K FOR R = .1 I

X_1	θ_1	X_2	θ_2	X_3	θ_3	X_4	θ_4	
129.7	-1261	-127.8	-1241	-1.74	-3.83	-.166	-.71	X_1
	16719	1254	8278	6.41	12.85	.84	2.94	θ_1
		260.3	-5.22	-127	-1239	-1.58	-3.84	X_2
			33528	1236	8249	2.45	4.58	θ_2
				246.3	-2.06	-124.6	-1239	X_3
					33540	1234	8252	θ_3
						268	-3.37	X_4
							33543	θ_4

$\omega = .972$ rad/sec

R WEIGHTING EFFECT ON STIFFNESS MATRIX
(FIRST ROW ONLY SHOWN)

ORIG. K_{ij}	125	-1250	-125	-1250	0	0	0	0
R = 10 I	125.1	-1250	-125	-1250	-.008	-.009	-.002	-.005
R = I	125.7	-1252	-125.1	-1249	-.21	-.27	-.06	-.13
R = .1 I	129.7	-1261	-127.8	-1241	-1.74	-3.83	-17	-.71
R = .01 I	150.8	-1310	-150.3	-1220	-3.19	-24.0	-1.8	-2.9

RELATED SPONSORED RESEARCH

- KAMAN AEROSPACE CORPORATION - AUTOMATED MATH MODEL IMPROVED FOR MATCHING EXPERIMENTAL DATA.
- INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING - IDENTIFICATION OF EQUIVALENT PDE SYSTEMS TO MATCH MEASURED DATA.

SUMMARY

- COMPUTER PROGRAM FOR REDESIGNING STRUCTURAL MODES TO REDUCE RESPONSE HAS BEEN INITIATED.
- LINEAR REGULATOR APPROACH IN MODAL COORDINATES HAS BEEN IMPLEMENTED. TRANSFORMATION OF SOLUTION TO PHYSICAL STRUCTURE IS A MAJOR PROBLEM.
- SOLUTION OF STIFFNESS EQUATIONS AND DAMPING EQUATIONS CAN BE DONE SEPARATELY AS NXN SET OF (MATRIX RICCATI) EQUATIONS.

PLANNED EFFORT FOR '82

- INCLUDE MASS OF CONTROL
- STUDY WEIGHTING TO MINIMIZE OR SELECT CROSS-TERMS
- IMPLEMENT PHYSICAL COORDINATE SOLUTION
- STUDY POTENTIAL FOR "BENEFICIAL" CROSS TERMS