COMPONENT NUMBER AND PLACEMENT

IN LARGE SPACE STRUCTURE CONTROL

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PURPOSE:

PROVIDE AN OBJECTIVE MEANS OF ASSISTING THE DESIGNER OF THE CONTROL SYSTEM FOR A LARGE FLEXIBLE SPACE STRUCTURE IN HIS CHOICE OF HOW MANY ACTUATORS AND SENSORS TO INCORPORATE IN THE SYSTEM, AND WHERE TO LOCATE THEM ON THE STRUCTURE.

WHAT WE NEED IS:

- A QUANTITATIVE MEASURE OF HOW WELL A SYSTEM CAN BE CONTROLLED WITH A SPECIFIED SET OF ACTUATORS
- A QUANTITATIVE MEASURE OF HOW WELL A SYSTEM CAN BE OBSERVED WITH A SPECIFIED SET OF SENSORS
- A MEANS OF RECOGNIZING THE EFFECTS OF COMPONENT FAILURES IN THESE MEASURES
- A MEANS OF OPTIMIZING THE LOCATIONS OF ACTUATORS AND SENSORS SO AS TO MAXIMIZE THESE MEASURES

A MEASURE OF THE DEGREE OF CONTROLLABILITY

1. FIND THE MINIMUM CONTROL ENERGY STRATEGY FOR DRIVING THE SYSTEM FROM A GIVEN INITIAL STATE TO THE ORIGIN IN A PRESCRIBED TIME.

2. DEFINE THE REGION OF INITIAL STATES WHICH CAN BE RETURNED TO THE ORIGIN WITHIN SPECIFIED LIMITS ON CONTROL ENERGY AND TIME USING THE OPTIMAL STRATEGY.

3. DEFINE THE DEGREE OF CONTROLLABILITY TO BE SOME MEASURE OF THE SIZE OF THIS REGION.

STEP 1. MINIMUM ENERGY CONTROL

Problem statement: $\text{Min } E = \frac{1}{2}\int_{o}^{T} \underline{u}^T R\, \underline{u}\, dt$

subject to $\dot{\underline{x}} = A\underline{x} + B\underline{u}$

$\underline{x}(0)$, T given

$\underline{x}(T) = 0$

Solution: $\underline{u}(t) = R^{-1} B^T \Phi_{pp}(t)\, \Phi_{xp}(T)^{-1}\, \Phi_{xx}(T)\, \underline{x}(0)$

The $\Phi_{ij}(t)$ are partitions of the transition matrix for the system

$$\frac{d}{dt}\begin{bmatrix}\underline{x}\\ \underline{p}\end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ o & -A^T \end{bmatrix}\begin{bmatrix}\underline{x}\\ \underline{p}\end{bmatrix}$$

STEP 2. THE RECOVERY REGION

Using optimal control, the cost to return to the origin can be expressed:

$$E = \frac{1}{2}\, \underline{x}(0)^T\, V(0)^{-1}\, \underline{x}(0)$$

$$\text{with} \qquad \dot{V} = AV + VA^T - BR^{-1}B^T \qquad\qquad V(T) = 0$$

For specified values of time, T, and control energy, E_s, the recovery region is the interior of the space bounded by the surface

$$\underline{x}(0)^T\, V(0)^{-1}\, \underline{x}(0) = 2\, E_s$$

STEP 3. THE SIZE OF THE RECOVERY REGION

First scale the state variables such that equal displacements in all directions are equally important.

$$\underline{z} = D\underline{x}$$

$$D = \begin{bmatrix} \frac{1}{x_{1_{min}}} & & & 0 \\ & \frac{1}{x_{2_{min}}} & & \\ & & \ddots & \\ 0 & & & \frac{1}{x_{n_{min}}} \end{bmatrix}$$

where $x_{i_{min}}$ is the minimum initial value of x_i one would like to be able to drive to the origin with constrained time and control energy.

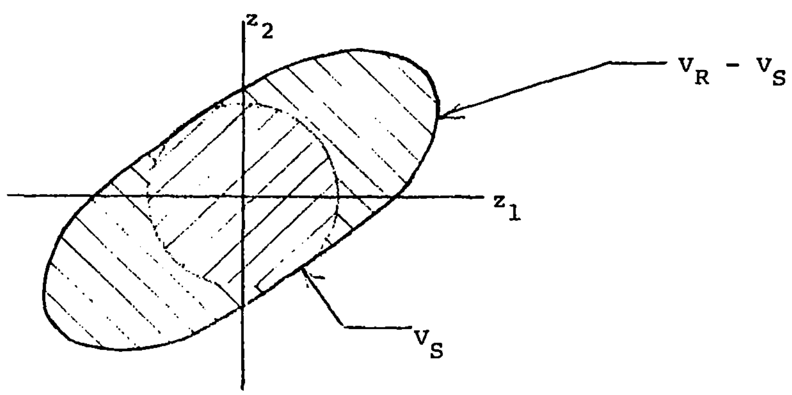
STEP 3 (CONTINUED)

Then define a weighted measure of the volume of the recovery region in the scaled space.

$$Vol = V_S + \frac{V_S}{V_R}(V_R - V_S)$$

V_S = volume of the largest sphere which can be inscribed in the elliptical boundary of the recovery region

V_R = volume of the recovery region



The Degree of Controllability is defined to be the n^{th} root of this weighted volume.

$$DC = \sqrt[n]{Vol}$$

Also $$V_R \approx \prod_i (\lambda_i)^{-1/2}$$

where the λ_i are the eigenvalues of $D^{-1}\ V(0)^{-1}\ D^{-1}$.

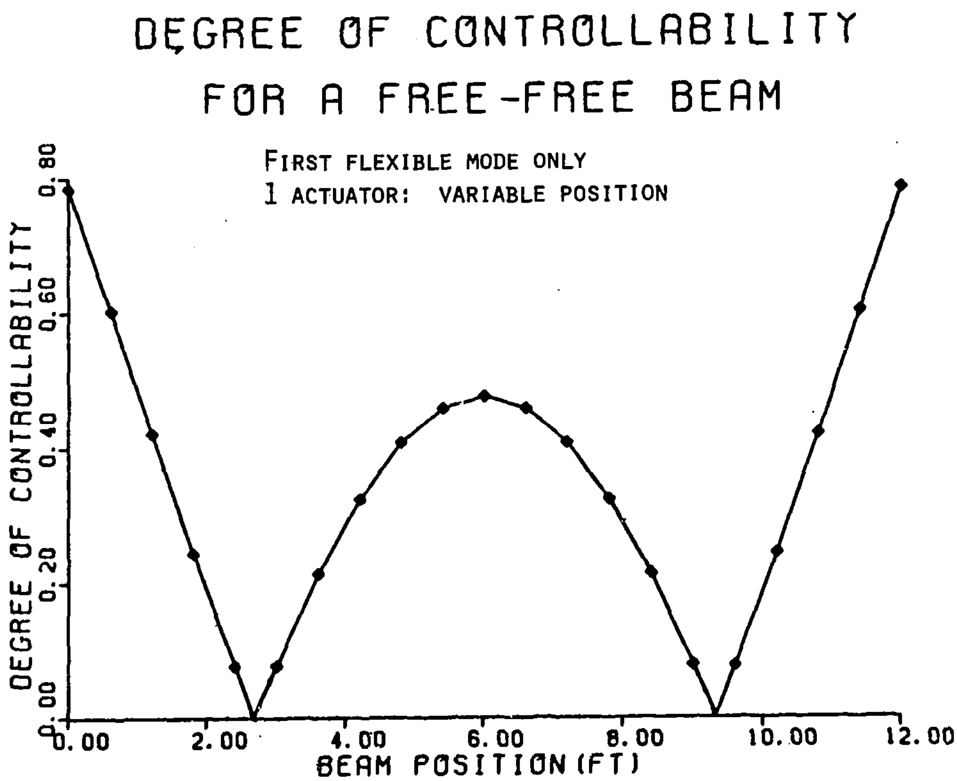
Alternatively

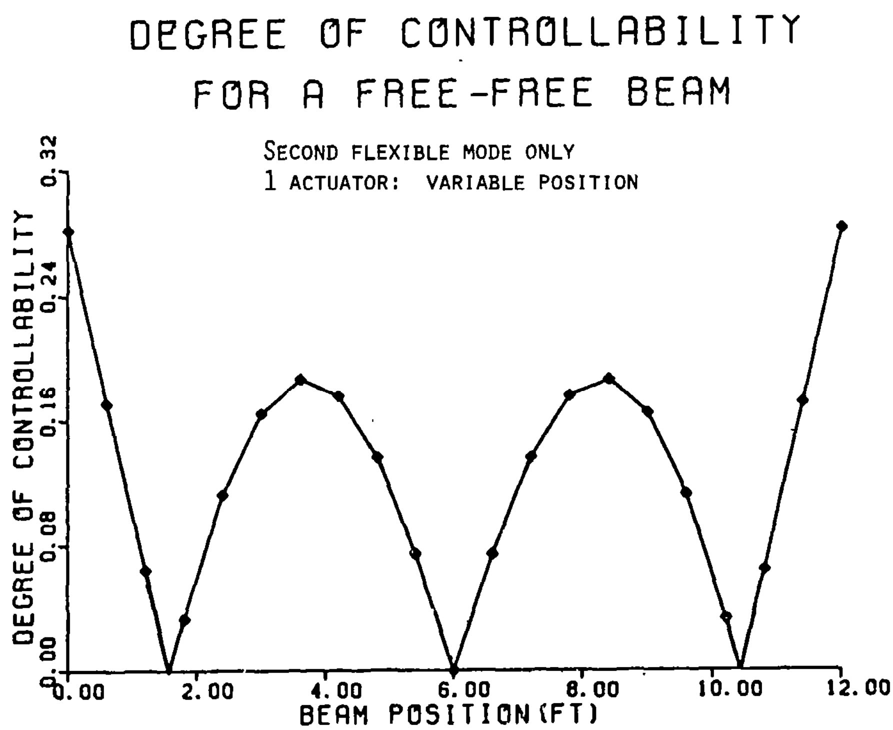
$$V_R \approx \prod_i (\nu_i)^{1/2}$$

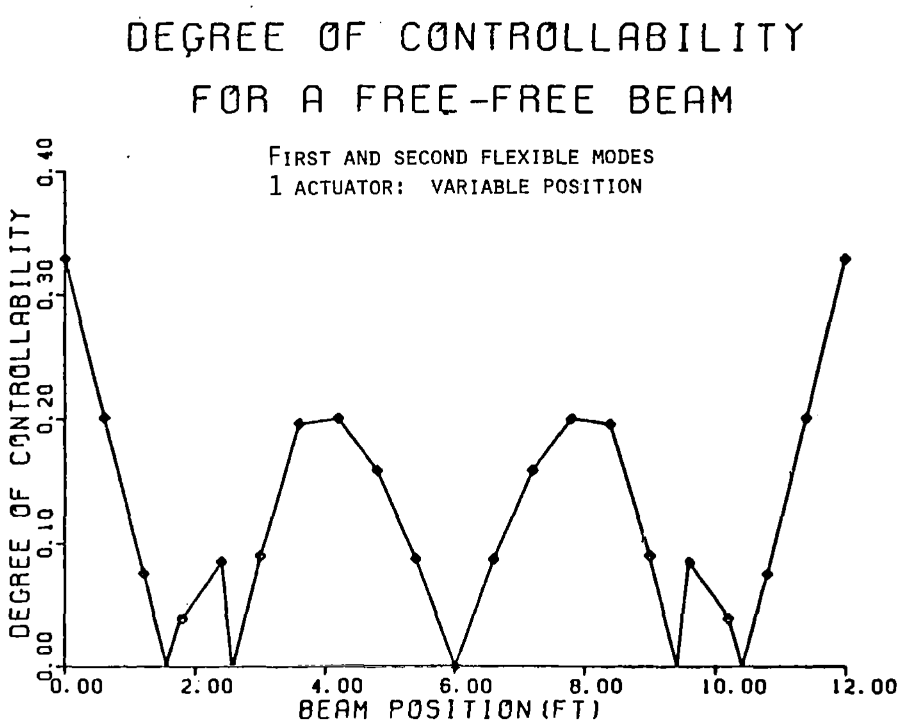
where the ν_i are the eigenvalues of DV(0)D.

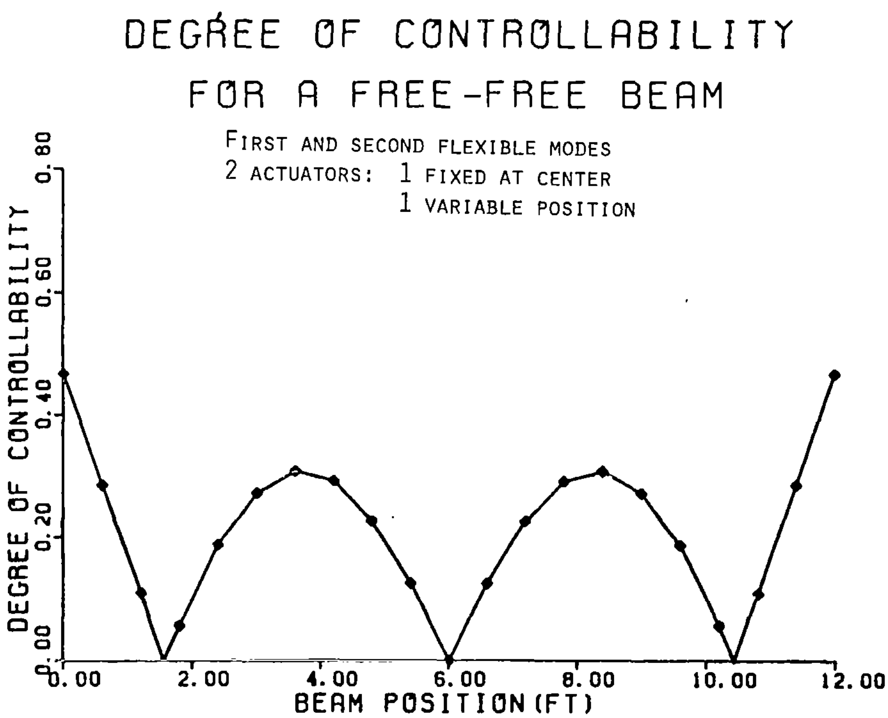
Then $$V_S \approx (\nu_{i_{min}})^{n/2}$$

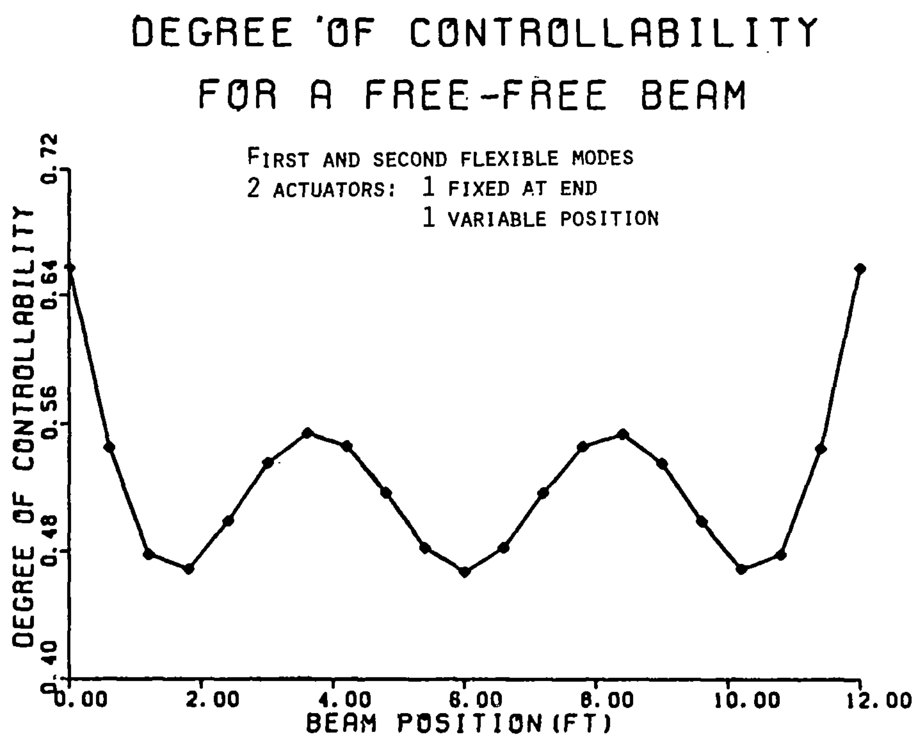
An analytic solution is available for V(0).


DEGREE OF CONTROLLABILITY
FOR A FREE-FREE BEAM
FIRST FLEXIBLE MODE ONLY
1 ACTUATOR: VARIABLE POSITION
DEGREE OF CONTROLLABILITY
0.00
0.20
0.40
0.60
0.80
0.00
2.00
4.00
6.00
8.00
10.00
12.00
BEAM POSITION (FT)


DEGREE OF CONTROLLABILITY
FOR A FREE-FREE BEAM
SECOND FLEXIBLE MODE ONLY
1 ACTUATOR: VARIABLE POSITION
DEGREE OF CONTROLLABILITY
0.00
0.08
0.16
0.24
0.32
0.00
2.00
4.00
6.00
8.00
10.00
12.00
BEAM POSITION (FT)


DEGREE OF CONTROLLABILITY
FOR A FREE-FREE BEAM
FIRST AND SECOND FLEXIBLE MODES
1 ACTUATOR: VARIABLE POSITION
DEGREE OF CONTROLLABILITY
0.00
0.10
0.20
0.30
0.40
0.00
2.00
4.00
6.00
8.00
10.00
12.00
BEAM POSITION (FT)


DEGREE OF CONTROLLABILITY
FOR A FREE-FREE BEAM
FIRST AND SECOND FLEXIBLE MODES
2 ACTUATORS: 1 FIXED AT CENTER
1 VARIABLE POSITION
DEGREE OF CONTROLLABILITY
0.00
0.20
0.40
0.60
0.80
0.00
2.00
4.00
6.00
8.00
10.00
12.00
BEAM POSITION (FT)



ONE MORE CONSIDERATION

WE WANT THE MEASURE OF CONTROLLABILITY TO REFLECT THE FACT THAT A SYSTEM WITH MORE ACTUATORS OF EQUAL EFFECTIVENESS HAS GREATER CONTROL CAPABILITY THAN ONE WITH FEWER ACTUATORS.

THE DC JUST DEFINED IS MADE PROPORTIONAL TO THE NUMBER OF ACTUATORS PLACED AT THE SAME LOCATIONS IF THE ELEMENTS OF R ARE SCALED INVERSELY WITH THE NUMBER OF ACTUATORS.

FOR DIAGONAL R, CHOOSE $R_{o_{ii}}$ TO REFLECT THE RELATIVE COST OF THE DIFFERENT CONTROLS. THEN

$$R_{ii} = R_{o_{ii}}/m$$

m = total number of actuators

A MEASURE OF THE DEGREE OF OBSERVABILITY

1. IN TERMS OF AN INFORMATION MATRIX, DETERMINE HOW MUCH INFORMATION CAN BE DERIVED ABOUT THE SYSTEM STATE IN TIME T, STARTING FROM ZERO INFORMATION, USING THE GIVEN SET OF SENSORS.

2. DEFINE THE DEGREE OF OBSERVABILITY TO BE A MEASURE OF THE SIZE OF THIS INFORMATION MATRIX.

STEP 1. THE INFORMATION MATRIX

WE WANT THE DEGREE OF OBSERVABILITY TO BE A PROPERTY OF THE SYSTEM, NOT OF THE ENVIRONMENT IN WHICH IT OPERATES. SO DO NOT CONSIDER STATE DRIVING NOISE.

THEN

$$\dot{J} = - JA - A^T J + C^T N^{-1} C$$

$$J(0) = 0$$

CORRESPONDING TO THE SYSTEM MODEL

$$\dot{\underline{x}} = A\underline{x} + B\underline{u}$$

$$\underline{y} = C\underline{x} + \underline{n}$$

$$\overline{\underline{n}(t_1)\underline{n}(t_2)^T} = N\,\delta(t_2 - t_1)$$

AN ANALYTIC SOLUTION IS AVAILABLE FOR J(T).

STEP 2. THE SIZE OF THE MATRIX

ONE WAY TO MEASURE THE SIZE OF J(T) IS TO INDICATE THE VOLUME CONTAINED WITHIN THE SURFACE

$$\underline{v}^T J(T)^{-1} \underline{v} = 1$$

BUT THE VARIABLES SHOULD BE SCALED TO REFLECT THE RELATIVE IMPORTANCE OF ERRORS IN THE DIFFERENT STATE VARIABLES.

$$\underline{w} = F \, \underline{v}$$

$$F = \begin{bmatrix} e_{1_{max}} & & & \bigcirc \\ & e_{2_{max}} & & \\ & & \ddots & \\ \bigcirc & & & e_{n_{max}} \end{bmatrix}$$

WHERE $e_{i_{max}}$ IS THE MAXIMUM TOLERABLE ERROR IN THE ESTIMATE OF x_i.

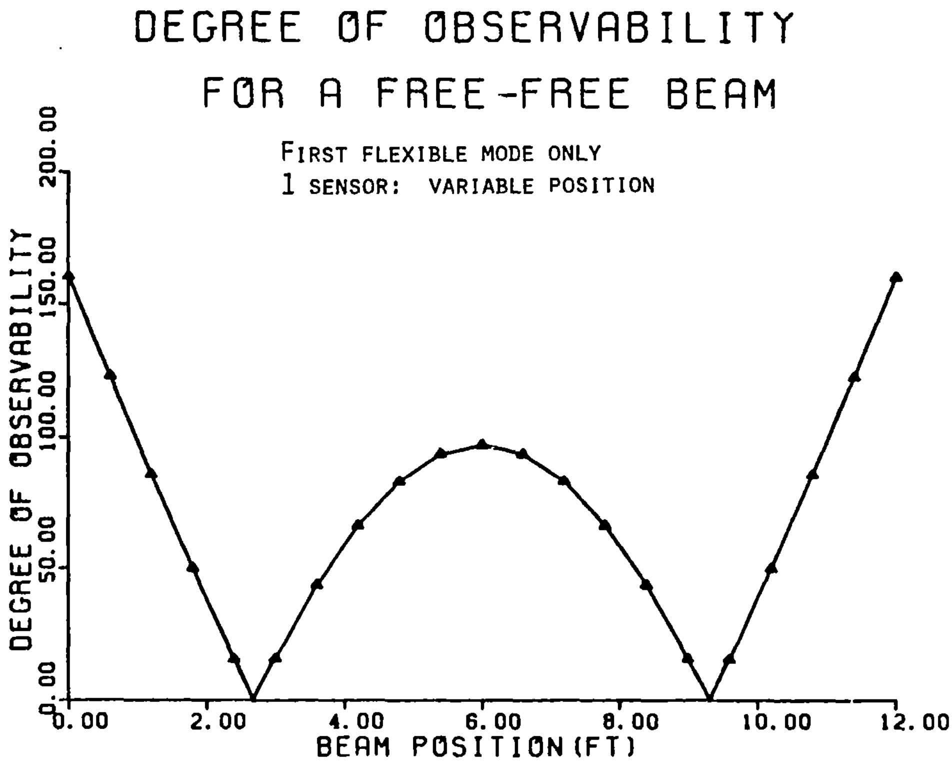
THE DEGREE OF OBSERVABILITY IS DEFINED WITH RESPECT TO THIS VOLUME IN THE SPACE OF EQUALLY IMPORTANT ERRORS ($\underline{w}$) JUST AS THE DEGREE OF CONTROLLABILITY WAS DEFINED FOR THE VOLUME OF THE RECOVERY REGION IN THE SPACE OF EQUALLY IMPORTANT CONTROL CHARACTERISTICS.

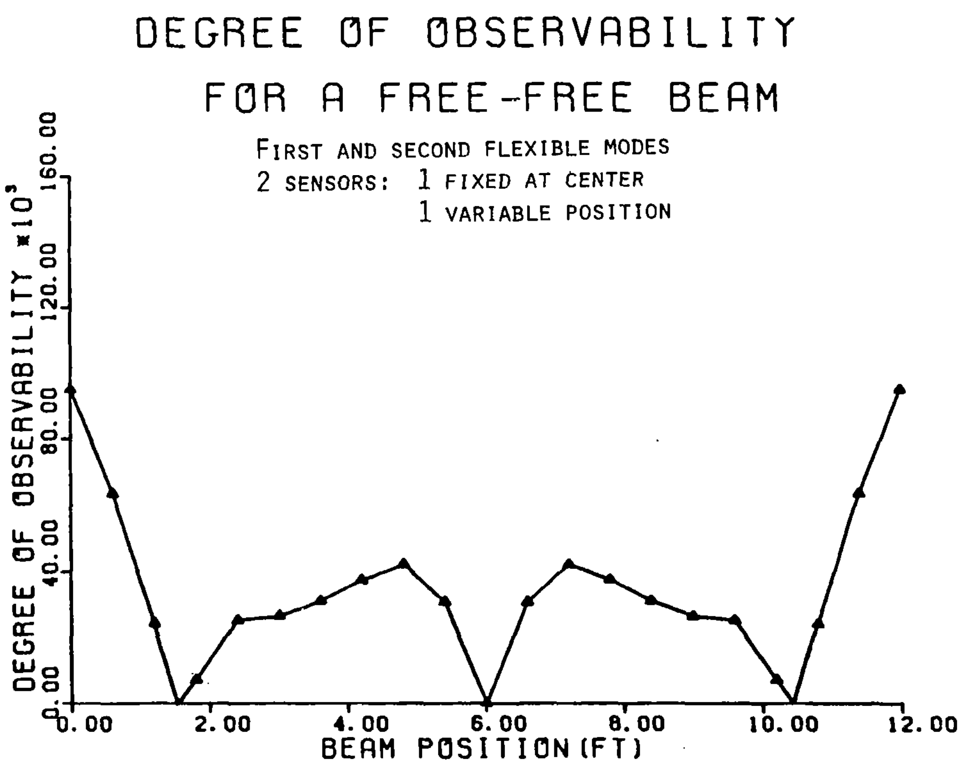
$$DO = \sqrt[n]{V_S + \frac{V_S}{V_R}(V_R - V_S)}$$

$$V_R = \prod_i (\nu_i)^{1/2}$$

$$V_S = (\nu_{i_{min}})^{n/2}$$

WHERE THE ν_i ARE THE EIGENVALUES OF $FJ(T)F$.


DEGREE OF OBSERVABILITY
FOR A FREE-FREE BEAM
FIRST FLEXIBLE MODE ONLY
1 SENSOR: VARIABLE POSITION
DEGREE OF OBSERVABILITY
0.00
50.00
100.00
150.00
200.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
BEAM POSITION (FT)


DEGREE OF OBSERVABILITY
FOR A FREE-FREE BEAM
FIRST AND SECOND FLEXIBLE MODES
2 SENSORS: 1 FIXED AT CENTER
1 VARIABLE POSITION
DEGREE OF OBSERVABILITY *10³
0.00
40.00
80.00
120.00
160.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
BEAM POSITION (FT)

RECOGNITION OF COMPONENT FAILURES IN THESE MEASURES

LET f INDICATE THE STATE OF FAILURES AMONG THE ACTUATORS OR SENSORS. THEN FOR EVERY f, $DC(f)$ OR $DO(f)$ CAN BE COMPUTED AS JUST DESCRIBED.

FROM THE STATISTICAL MODEL FOR FAILURES OF THE DIFFERENT COMPONENTS ONE CAN EXPRESS $P[f(t) = f_i]$.

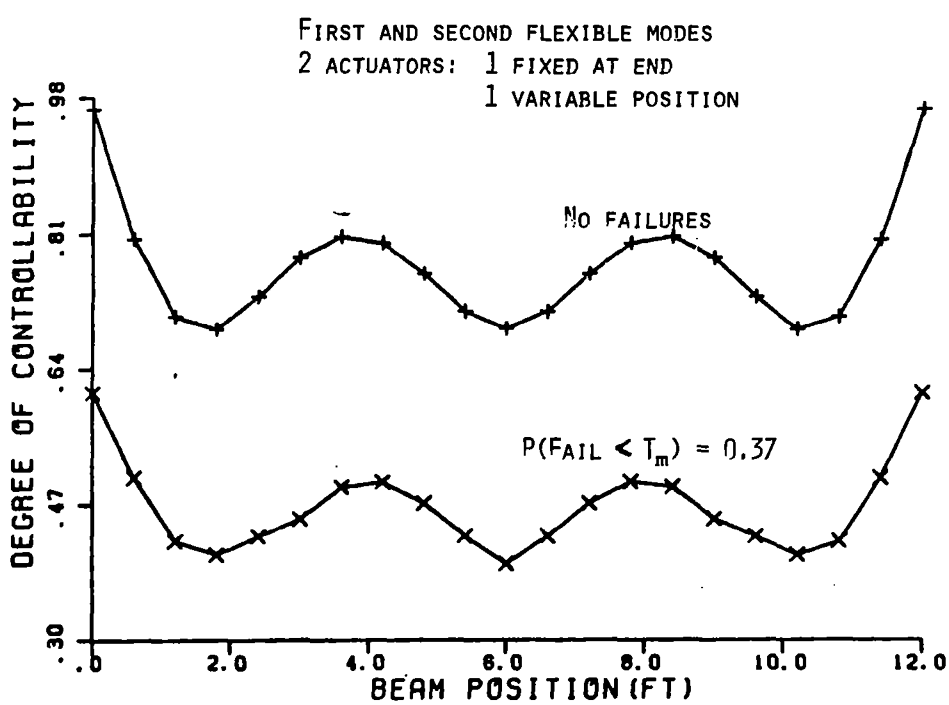
THE AVERAGE, OVER THE MISSION PERIOD, OF THE EXPECTED DEGREE OF CONTROLLABILITY OR OBSERVABILITY IS TAKEN AS THE FINAL MEASURE.

$$ADC = \frac{1}{T_m}\int_0^{T_m} \overline{DC(t)}\, dt$$

$$= \frac{1}{T_m}\int_0^{T_m} \sum_i DC(f_i)\, P[f(t) = f_i]\, dt$$

$$= \sum_i DC(f_i)\, \frac{1}{T_m}\int_0^{T_m} P[f(t) = f_i]\, dt$$

THE DEGREE OF OBSERVABILITY IS COMPUTED IN THE SAME WAY.



OPTIMIZING COMPONENT LOCATIONS

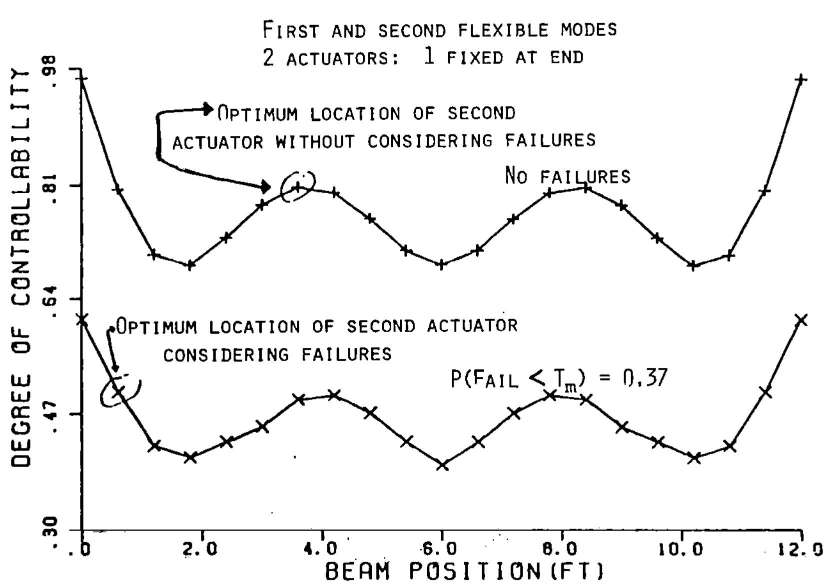
THE AVERAGE DEGREE OF CONTROLLABILITY IS A FUNCTION OF THE CHOICE OF ACTUATOR LOCATIONS; LET ℓ INDICATE THE VARIOUS ADMISSIB LE CHOICES.

$$ADC \Rightarrow ADC(\ell)$$

AN OPTIMIZATION ROUTINE IS REQUIRED TO FIND THE OPTIMUM LOCATIONS.

$$ADC^* = \max_{\ell \in L} ADC(\ell)$$

THE OPTIMUM SENSOR LOCATIONS ARE DEFINED IN THE SAME WAY USING THE AVERAGE DEGREE OF OBSERVABILITY.



CHOICE OF THE NUMBER OF COMPONENTS

THE OPTIMUM AVERAGE DEGREE OF CONTROLLABILITY IS A FUNCTION OF THE NUMBER OF ACTUATORS IN THE SYSTEM.

$$ADC^* \Rightarrow ADC^*(m)$$

WITH THE LIKELY CONSTRAINT THAT MULTIPLE ACTUATORS CANNOT BE PLACED IN THE SAME LOCATION, $ADC^*(m)$ SHOWS DIMINISHING RETURNS WITH INCREASING m.

THE SAME IS TRUE FOR THE OPTIMUM AVERAGE DEGREE OF OBSERVABILITY.