Research on the Control of Large Space Structures*

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*Paper not presented at conference. This work was partially supported by NASA Langley Research Center under grant NSG-1603.

INTRODUCTION

The research effort on the control of large space structures at the University of Houston has concentrated on the mathematical theory of finite-element models; identification of the mass, damping, and stiffness matrix; assignment of damping to structures; and decoupling of structure dynamics. The objective of the work has been and will continue to be the development of efficient numerical algorithms for analysis, control, and identification of large space structures. The major consideration in the development of the algorithms has been the large number of equations that must be handled by the algorithm as well as sensitivity of the algorithms to numerical errors.

The finite-element model that has been used in the linear second-order matrix differential equation

$$\frac{Md^2x(t)}{dt^2} + \frac{\bar{C}dx(t)}{dt} + \bar{K}x(t) = f(t) \tag{1}$$

where $M \epsilon R^{m\times m}$ is the mass matrix, $\bar{C}\epsilon R^{m\times m}$ is the damping matrix, $\bar{K}\epsilon R^{m\times m}$ is the stiffness matrix, $x(t)\epsilon R^{m\times 1}$ is the node displacement vector, and $f(t)\epsilon R^{m\times 1}$ is the forcing function vector.

The Laplace transform of equation (1) gives the matrix equation

$$[Ms^2+\bar{C}s+\bar{K}]x(s) = B(s) \tag{2}$$

where B(s) contains the initial condition information as well as the forcing function. If s is replaced in equation (2) by λ, equation (2) then takes the form of a lambda matrix:

$$\bar{A}(\lambda)X(\lambda) = B(\lambda) \tag{3}$$

If it is assumed that the initial conditions are zero and no forcing function is present, then

$$\bar{A}(\lambda)X(\lambda) = [M\lambda^2+\bar{C}\lambda+\bar{K}]X(\lambda) \tag{4}$$

is the homogeneous equation that will be of interest. The latent roots λ_i of $A(\lambda)$ are given by

$$\det \bar{A}(\lambda) = \det[M\lambda^2+\bar{C}\lambda+\bar{K}] = 0 \tag{5}$$

and the latent vectors y_i are obtained from

$$\bar{A}(\lambda_i)y_i = [M\lambda_i^2+\bar{C}\lambda_i+\bar{K}]y_i = 0$$

provided that all latent roots are distinct, of multiplicity one.

Lancaster (ref. 1) and Dennis et al. (ref. 2), as well as others, have published material on lambda matrices. One of the tasks during the past research period has been to consider their work as well as extensions of the algebraic theory of lambda matrices for the control of structures.

ALGEBRAIC THEORY OF LAMBDA MATRICES

A comprehensive treatment of the algebraic theory of lambda matrices cannot be presented in this short paper. Only the essentials necessary to understand the damping assignment problem will be given.

Consider the lambda matrix

$$\bar{A}(\lambda) = Q[I\lambda^2+C\lambda+K]Q^T = QA(\lambda)Q^T \tag{6}$$

where Q is the Cholesky matrix of the decomposition of $M = QQ^T$. The normalization process in equation (6) will retain the symmetry of C and K. The lambda matrix $A(\lambda)$ will be considered in the following work and it will be assumed that the latent roots of $A(\lambda)$ appear in conjugate pairs as well as being distinct. This restriction is not necessary but is made to simplify the analysis.

It can be shown that the matrix

$$A_c = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \tag{7}$$

has eigenvalues λ_i that are equal to the latent roots of $A(\lambda)$. The right and left eigenvectors of A_c contain the right and left latent vectors of $A(\lambda)$ with

$$y_{ci} = \begin{bmatrix} y_i \\ \lambda_i y_i \end{bmatrix} \tag{8}$$

and

$$z_{ci} = \begin{bmatrix} (\lambda_i I + C) z_i \\ z_i \end{bmatrix} \tag{9}$$

where y_{ci} and z_{ci} are the right and left eigenvectors, respectively. An eigenprojector, P_i, will be defined as the matrix

$$P_i = \frac{y_{ci} z_{ci}^T}{z_{ci}^T y_{ci}} \tag{10}$$

where the matrix A_{ci} defined by

$$A_{ci} = P_i \ A_c = A_c P_i \tag{11}$$

will have the same eigenvectors as A_c but all eigenvalues will be zero except for λ_i, which will be the same as in A_c. The eigenprojectors are the matrix residues of the partial fraction expansion of $[A_c(\lambda)]^{-1}$ with

$$[A_c(\lambda)]^{-1} = \sum_{i=1}^{2m} \frac{P_i}{\lambda - \lambda_i} \tag{12}$$

where

$$P_i = \lim_{\lambda \to \lambda_i} (\lambda - \lambda_i)[A_c(\lambda_i)]^{-1} \tag{13}$$

A latent projector will be defined as the matrix residues of the partial fraction expansion of $[A(\lambda)]^{-1}$ where

$$[A(\lambda)]^{-1} = \sum_{i=1}^{2m} \frac{\hat{P}_i}{(\lambda-\lambda_i)} \tag{14}$$

with

$$\hat{P}_i = \lim_{\lambda\to\lambda_i} (\lambda-\lambda_i)[A(\lambda)]^{-1} \tag{15}$$

The latent projectors are also given by

$$\hat{P}_i = \frac{y_i z_i^T}{z_i^T \frac{dA(\lambda_i)}{d\lambda} y_i} \tag{16}$$

where y_i and z_i are the latent vectors of $A(\lambda)$.

To complete the limited discussion of the algebraic theory, the relationship that exists between the eigenprojectors and latent projectors must be given. By inverting $A_c(\lambda)$ it can be shown that

$$P_i = \hat{P}_i \begin{bmatrix} \lambda_i I+C & I \\ -K & \lambda_i I \end{bmatrix} \tag{17}$$

with the past definitions holding.

ASSIGNMENT OF DAMPING

An algorithm to assign damping to a particular undamped mode has been developed, although the theory has not been fully explored at this time. Considerable work remains to be carried out to develop and test a comprehensive algorithm. Therefore the material in this section is only preliminary.

Consider the undamped structure with the associated lambda matrix

$$A(\lambda) = I\lambda^2+K \tag{18}$$

If it is assumed that K is symmetric and positive definite then all latent roots of $A(\lambda)$ lie along the $j\omega$ axis. Since K is positive definite all of its eigenvalues will be real with $\bar{\lambda}_i = \omega_{ni}^2$, and the eigenprojectors of K are given by

$$(\bar{\lambda}I-K)^{-1} = \sum_{i=1}^{m} \frac{\bar{P}_i}{(\bar{\lambda}-\bar{\lambda}_i)} \tag{19}$$

where it can be shown that P_i is given by

$$\bar{P}_i = \hat{P}_i \quad \lambda_i + \hat{P}_i^* \quad \lambda_i^* \tag{20}$$

which will be defined as the augmented latent projector. It is not difficult to show that the undamped A_c is

$$A_{cu} = \begin{bmatrix} \sum_{i=1}^{m} (\hat{P}_i\lambda_i^2+\hat{P}_i^*\lambda_i^{*2}) & \sum_{i=1}^{m} (\hat{P}_i\lambda_i+\hat{P}_i^*\lambda_i^*) \\ -\sum_{i=1}^{m} (\hat{P}_i\lambda_i+\hat{P}_i^*\lambda_i^*)K & \sum_{i=1}^{m} (\hat{P}_i\lambda_i^2+\hat{P}_i^*\lambda_i^{*2}) \end{bmatrix} \tag{21}$$

but since the undamped matrix is

$$A_{cu} = \begin{bmatrix} 0 & I \\ -K & 0 \end{bmatrix} \tag{22}$$

then

$$\sum_{i=1}^{m} (\hat{P}_i\lambda_i^2+\hat{P}_i^*\lambda_i^{*2}) = 0 \tag{23}$$

$$\sum_{i=1}^{m} (\hat{P}_i\lambda_i+\hat{P}_i^*\lambda_i^*) = I \tag{24}$$

The damped case has a similar form with the exception that the upper diagonal block is modified with

$$A_{cd} = \begin{bmatrix} \sum_{i=1}^{m} (\hat{P}_i \lambda_i^2 + \hat{P}_i^* \lambda_i^{*2} + C\lambda_i + C\lambda_i^*) & \sum_{i=1}^{m} (\hat{P}_i \lambda_i + \hat{P}_i^* \lambda_i^*) \\ -\sum_{i=1}^{m} (\hat{P}_i \lambda_i + \hat{P}_i^* \lambda_i^*) K & \sum_{i=1}^{m} (\hat{P}_i \lambda_i^2 + \hat{P}_i^* \lambda_i^{*2}) \end{bmatrix} \tag{25}$$

$$= \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}$$

Suppose now that the undamped structure is to have damping added to the jth mode (i.e., λ_j and λ_j^*) and all other modes are to remain undamped. Using equations (21) and (22), A_c becomes

$$A_{cd} = \begin{bmatrix} \sum_{\substack{i=1 \\ i \neq j}}^{m} (\hat{P}_i \lambda_i^2 + \hat{P}_i^* \lambda_i^{*2}) & \sum_{\substack{i=1 \\ i \neq j}}^{m} (\hat{P}_i \lambda_i + \hat{P}_i^* \lambda_i^*) \\ -\sum_{\substack{i=1 \\ i \neq j}}^{m} (\hat{P}_i \lambda_i + \hat{P}_i^* \lambda_i^*) K & \sum_{\substack{i=1 \\ i \neq j}}^{m} (\hat{P}_i \lambda_i^2 + \hat{P}_i^* \lambda_i^{*2}) \end{bmatrix} +$$

$$\begin{bmatrix} (\hat{P}_j \lambda_j^2 + \hat{P}_j^* \lambda_j^{*2} + C_j \lambda_j + C_j \lambda_j^*) & \hat{P}_j \lambda_j + \hat{P}_j^* \lambda_j^* \\ -(\hat{P}_j \lambda_j + \hat{P}_j^* \lambda_j^*) K & \hat{P}_j \lambda_j^2 + \hat{P}_j^* \lambda_j^{*2} \end{bmatrix} \tag{26}$$

which can be rewritten as

$$A_{cd} = \begin{bmatrix} 0 & \sum_{\substack{i=1 \\ i \neq j}}^{m} \bar{P}_i \\ -\sum_{\substack{i=1 \\ i \neq j}}^{m} \bar{P}_i K & 0 \end{bmatrix} + \begin{bmatrix} 0 & \bar{P}_j \\ -\bar{P}_j K & -C_j \end{bmatrix} \tag{27}$$

and also in the form

$$A_{cd} = A_{cd} \sum_{\substack{i=1 \\ i \neq j}}^{2m} P_i + A_{cd} P_j = A^{+} + A_j \qquad (28)$$

The matrix A^{+} denotes the matrix constructed from the summation and A_j is the complement to A^{+}.

The last step in the algorithm is to recognize that A^{+} is orthogonal to A_j; thus

$$\begin{bmatrix} 0 & \sum_{\substack{i=1 \\ i \neq j}}^{m} \overline{P}_i \\ -\sum_{\substack{i=1 \\ i \neq j}}^{m} \overline{P}_i K & 0 \end{bmatrix} \begin{bmatrix} 0 & \overline{P}_j \\ -\overline{P}_j K & -C_j \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (29)$$

Finally, C_j is the matrix that is orthogonal to $\sum_{\substack{i=1 \\ i \neq j}}^{m} \overline{P}_i$. The eigenprojector $\overline{P}_j$ must be orthogonal to all $\overline{P}_i$ as the set of eigenprojectors have the orthogonal property

$$\overline{P}_i \overline{P}_j = 0 \qquad i \neq j \qquad (30)$$

The C_j matrices are therefore nothing more than $\alpha_j \overline{P}_j$ where α_j is a scaling constant. The trace of C_j is

$$\text{trace } (C_j) = \sum_{i=1}^{m} c_{ii,j} = 2\sigma_j \qquad (31)$$

where trace $(\bar{P}_j) = 1$. The scaling constants are then used to place the latent roots of $A(\lambda)$ with

$$\sigma_j = \frac{\alpha_j}{2} \tag{32}$$

$$\omega_j^2 = \omega_{nj}^2 - \sigma_j^2 \tag{33}$$

An example will now be given to illustrate the algorithm. Let $C = 0$ and select K as

$$K = \begin{bmatrix} 300 & -200 \\ -200 & 350 \end{bmatrix}$$

which has eigenvalues $\pm j11.1105$ and $\pm j22.9468$. The augmented projectors are

$$\bar{P}_1 = \begin{bmatrix} 0.562017 & 0.496139 \\ 0.496139 & 0.437983 \end{bmatrix}$$

and

$$\bar{P}_2 = \begin{bmatrix} 0.437983 & -0.496139 \\ -0.496139 & 0.562017 \end{bmatrix}$$

The matrices

$$C_1 = \begin{bmatrix} 1 & 0.8827824 \\ 0.8827824 & 0.7793048 \end{bmatrix}$$

and

$$C_2 = \begin{bmatrix} 1 & -1.1327785 \\ -1.1327785 & 1.283187 \end{bmatrix}$$

are orthogonal to $\bar{P}_1$ and $\bar{P}_2$. Note that the columns in C_1 and C_2 can be scaled by constants and remain orthogonal to $\bar{P}_1$ and $\bar{P}_2$.

The eigenvalues of

$$A_c = \begin{bmatrix} 0 & I \\ -K & -C_j \end{bmatrix}$$

are given in the computed results below.

A(I,J) MATRIX

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -300 & 200 & 0 & 0 \\ 200 & -350 & 0 & 0 \end{bmatrix}$$

EIGENVALUES	REAL	IMAGINARY
1	0	22.9468
2	0	-22.9468
3	3.04403E-22	11.1105
4	3.04403E-22	-11.1105

No damping

A(I,J) MATRIX

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -300 & 200 & -1 & -.882783 \\ 200 & -350 & -.882783 & -.779305 \end{bmatrix}$$

EIGENVALUES	REAL	IMAGINARY
1	3.99893E-08	22.9468
2	3.99893E-08	-22.9468
3	- .889652	11.0748
4	- .889652	-11.0748

Damping in lowest mode

A(I,J) MATRIX

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -300 & 200 & -1 & 1.13278 \\ 200 & -350 & 1.13278 & -1.28319 \end{bmatrix}$$

EIGENVALUES	REAL	IMAGINARY
1	-1.14159	22.9184
2	-1.14159	-22.9184
3	2.84795E-08	11.1105
4	2.84795E-08	-11.1105

Damping in highest mode

A(I,J) MATRIX

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -300 & 200 & -1 & .125 \\ 200 & -350 & .125 & -1.03125 \end{bmatrix}$$

EIGENVALUES	REAL	IMAGINARY
1	-.570798	22.9399
2	-.570798	-22.9399
3	-.444827	11.1019
4	-.444827	-11.1019

Damping in both modes

The undamped system matrix and its eigenvalues are given first. The lowest mode was then damped with the system matrix and its eigenvalue given. The third test was to include damping for the highest mode where $C = C_2$. The last test was to combine the damping for the two modes with

$$C = \frac{1}{2}(C_1+C_2) = \alpha_1\overline{P}_1+\alpha_2\overline{P}_2 = 0.889655\overline{P}_1 + 1.141595\overline{P}_2$$

The system matrix and its eigenvalues are then given. It should be noted that the new values of σ_j of the two modes are

$$\sigma_1 = \frac{0.889652}{2} = 0.444827$$

and

$$\sigma_2 = \frac{1.141595}{2} = .570797$$

The algorithm allows damping to be assigned to all modes and to the prediction of the location of the system eigenvalues.

The major problem that remains to be resolved is that of constructing the $\bar{C}$ matrix, which is symmetric and positive definite as well as realizable. It will be assumed that $\bar{C}$ and $\bar{K}$ are tridiagonal with the off-diagonal elements having a smaller magnitude than the diagonal elements. The off-diagonal elements must be negative with positive diagonal terms.

IDENTIFICATION OF M, C, AND K

The quadrature algorithm for identifying the mass, damping, and stiffness matrices of a structure is still under study. The algorithm performs well with simulated data, but attempts to utilized data from the beam experiment at NASA Langley Research Center have not produced useable results. It is believed that the conditions on the beam during the data collection may not have satisfied the requirements of the algorithm.

The quadrature identification, as well as other algorithms, will receive major attention in the future. One of the major problems in developing an efficient algorithm is the availability of test data, either by computer simulation or from a test bed. The Langley Research Center experimental beam is a suitable test bed, but planning and running test data are costly and time consuming. Plans for the future include construction of an electronic analog test bed that will be low cost and will provide flexibility in the types of structures that can be simulated. The test bed will be used primarily in the development of the identification algorithm, but it can also be used for developing control algorithms.

SIMULATION OF STRUCTURES

Some of the preliminary work on designing a structures test bed at the University of Houston has been completed. The type of simulator considered is an electronic analog that will be constructed from low-cost operational amplifiers, resistors, and capacitors. Data acquisition will be handled by a PDP 11/70 computer for signal processing. The PDP 11/70 is equipped with analog-to-digital and digital-to-analog converters and has adequate storage to perform on-line data collection and processing.

CONCLUSIONS

Some of the work that has been supported under grant from NASA Langley Research Center has been described in this report. Preliminary details on the assignment of damping are covered, as well as some information on identification and simulation.

The identification work has not been described since the algorithm is given in reference 3. The simulation facility is still in the planning stage and a decision to build such a test bed has not been made. Further descriptions of this research are given in references 4 and 5.

REFERENCES

1. Lancaster, P.: Lambda Matrices and Vibrating Systems. Pergamon Press, N.Y., 1966.

2. Dennis, J. E., Jr.; Traub, J. F.; and Weber, R. P.: The Algebraic Theory of Matrix Polynomials. SIAM J. Numer. Analy., vol. 13, no. 6, Dec. 1976, pp. 831-845.

3. AIAA Guidance and Control Conference. A Collection of Technical Papers. Albuquerque, N.M., August 19-21, 1981.

4. Denman, E. D.: Roots of Real Matrices. Linear Alg. & Appl., vol. 36, 1981, pp. 133-139.

5. Denman, E. D.; and Leyva-Ramos, J.: Spectral Decomposition of a Matrix Using the Generalized Sign Matrix. Appl. Math. & Comp., vol. 8, no. 4, June 1981, pp. 237-250.