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ENGINEERING CALCULATIONS FOR COMMUNICATIONS SYSTEMS PLANNING

C.A. Levis
C.H. Martin
C.W. Wang
D. Gonsalvez

The Ohio State University

ElectroScience Laboratory

Department of Electrical Engineering
Columbus, Ohio 43212

First Summary Report 713533-2
(1/20/81-7/15/82)
NASA Grant No. DAG 3-159

March 1983

NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 43212

NOTICES

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO.	2.	3. Recipient's Accession No.

4. Title and Subtitle
ENGINEERING CALCULATIONS FOR COMMUNICATIONS SYSTEMS PLANNING

5. Report Date
March 1983

6.

7. Author(s)
C.A. Levis, C.H. Martin, C.W. Wang, and D. Gonsalvez

8. Performing Organization Rept. No.
ESL 713533-2

9. Performing Organization Name and Address
The Ohio State University ElectroScience Laboratory
Department of Electrical Engineering
Columbus, Ohio 43212

10. Project/Task/Work Unit No.

11. Contract(C) or Grant(G) No.
(C)
(G) NAG 3-159

12. Sponsoring Organization Name and Address
NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135

13. Type of Report & Period Covered
First Summary Report
1/20/81-7/15/82

14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

The single-entry interference problem is treated for frequency-sharing between the Broadcasting Satellite and Inter-satellite services near 23 GHz. It is recommended that very long (more than 120° longitude difference) intersatellite hops be relegated to the unshared portion of the band. When this is done, it is found that suitable orbit assignments can be determined easily with the aid of a set of universal curves.

An attempt to develop synthesis procedures for optimally assigning frequencies and orbital slots for the Broadcasting Satellite Service in Region 2 has been initiated. In this report several discrete (0-1) programming and continuous optimization techniques are discussed which appear relevant to the problem.

17. Document Analysis a. Descriptors

b. Identifiers/Open-Ended Terms

Satellite Communications
Broadcasting Satellite
Intersatellite
Orbit Assignment
Frequency Assignment
Spectrum Allocation
Satellite System Planning

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
	Unclassified	63
	20. Security Class (This Page)	22. Price
	Unclassified	

(See ANSI-Z39.18) *See Instructions on Reverse* OPTIONAL FORM 272 (4-77) (Formerly NTIS-35) Department of Commerce

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I. INTRODUCTION AND OVERVIEW

Radio telecommunications, and the satellite services in particular, are governed by international agreements. The technical analyses on which such agreements may be based are performed by the International Consultative Committee on Radio (CCIR), using as input the work of study groups established for that purpose. NASA is actively supporting the work of these study groups in regard to satellite communications systems. The purpose of this grant is to develop computational methods and to perform engineering calculations which will assist in this task. The grant was initiated in January 1981 and augmented in July, 1981; this is the first summary report.

Two specific tasks were undertaken during the period. The first deals with frequency-sharing between the Inter-satellite Service and the Broadcasting-satellite Service in the band near 23 GHz. The 1979 World Administrative Radio Conference (WARC-79) promulgated an Inter-satellite Services (ISS) band from 22.55 to 23.55 GHz. This band is to be shared with other services, including the Broadcasting-satellite Service (BSS) in the 22.55 to 23.00 GHz band [1]. Because of potential interference, the detailed implementation was not spelled out by WARC-79. Under the Grant, this Laboratory has assisted NASA in evaluating the potential interference between the BSS and ISS users, and in defining usable orbit geometries. The results will be found in Section II, which is intended as a preliminary draft for a paper to be submitted to an appropriate technical journal. Additional information, including computer program documentation is contained in a technical report prepared under this Grant [2]. In its initial stages, this work also made significant contributions to a CCIR Study Group Report [3].

Since July 1981, the problem of synthesizing optimal and near-optimal spectrum and orbit assignments for broadcast satellites has been a second topic of investigation. The goal of broadcasting-satellite service system synthesis is to specify for each service area under consideration a set of channel assignments, polarizations, and an orbital slot in a manner which minimizes the amount of bandwidth required. Implicit in this is the requirement that a specified number of channels be supplied to each service area and that these channels be protected from interference. Protection ratios of 35 dB single-entry cochannel, 19 dB single-entry adjacent channel, 30 dB aggregate are typical. Other constraints, such as eclipse protection and minimum elevation angle, limit the flexibility of assignments.

Our aim is to devise computerized spectrum/orbit synthesis techniques which will be useful at two important international conferences. The first, to be held in 1983, deals with broadcast satellite services (BSS), the distribution of information (especially television) to many users simultaneously, either as individual users or on a community or regional basis. The second, to be held in 1985, deals with fixed satellite services (FSS), i.e., point-to-point communications by means of satellites between specific fixed stations on the Earth. Since in both the FSS and BSS cases up- and down-link calculations are involved, the techniques to be developed for the 1983 and 1985 conferences exhibit a certain amount of commonality. However, there are also significant differences such as the number of up- and down-links to be considered, the coverage area (which affects the antenna patterns strongly), and the modulation methods to be employed.

The spectrum/orbit assignment problem is still far from a definitive solution, but some progress has been made and is reported in Section III. It is our aim to have significant improvements over present techniques available for the 1983 conference, and a quasi-optimal computer code for the 1985 conference.

Conclusions and recommendations are summarized in Section IV.

II. SINGLE-ENTRY INTERFERENCE BETWEEN BROADCAST SATELLITE AND INTERSATELLITE SYSTEMS SHARING FREQUENCIES NEAR 23 GHz

A. INTRODUCTION

The 1979 World Administrative Radio Conference (WARC-79) authorized the use of the 22.55-23.55 GHz band by the Intersatellite Service (ISS) and the use of the 22.55-23.0 GHz band by the Broadcasting-Satellite Service (BSS) in region II, the Western Hemisphere [1]. Full use of these bands requires frequency-sharing between the two services in the 22.55-23.0 GHz frequency range. The purpose of this paper is to define acceptable satellite orbital assignments for this purpose. Calculations for some specific systems have been made previously by CCIR study groups [3-5]. Since the systems are still being defined, the more general problem is addressed here. Only the co-polarized case is considered;

additional discrimination is, of course, possible by the use of orthogonal polarizations, and the extension to this case is straightforward. Also we do not consider isolation which might be obtained by special modulation schemes, generally at the sacrifice of efficient use of the spectrum by at least one of the services.

Under these conditions, the only available means for preventing interference between the two services is by the discrimination available from the antenna patterns and station geometries. Figure 1 shows the geometric parameters of interest, with the Earth radius exaggerated for clarity; the actual ratio between the Earth radius and the radius of the geosynchronous orbit is 1:6.6. The receiving (RX) ISS satellite is located at A, the transmitting (TX) ISS satellite is shown at C, the BSS satellite transmitter is at B, and a BSS Earth station receiver is shown at E. It is assumed that the ISS(TX) antenna is pointed at the ISS(RX) antenna, and vice versa, as required by good system design. Similarly, the Earth station antenna is assumed pointed at the BSS transmitter. We also assume for simplicity that the BSS transmitter happens to be pointed directly at the Earth station for which interference is being computed; while this need not be precisely true, it can be shown to have no significant effect on the conclusions to be drawn.

In Figure 1, ψ_1 and ψ_4 denote the angles between the axes of the receiving antennas and their respective potential interference sources; ψ_2 and ψ_3 describe the angles between the transmitter antenna axes and the receivers with which they might potentially interfere. Protection against interference results from the fact that, in general, receiving

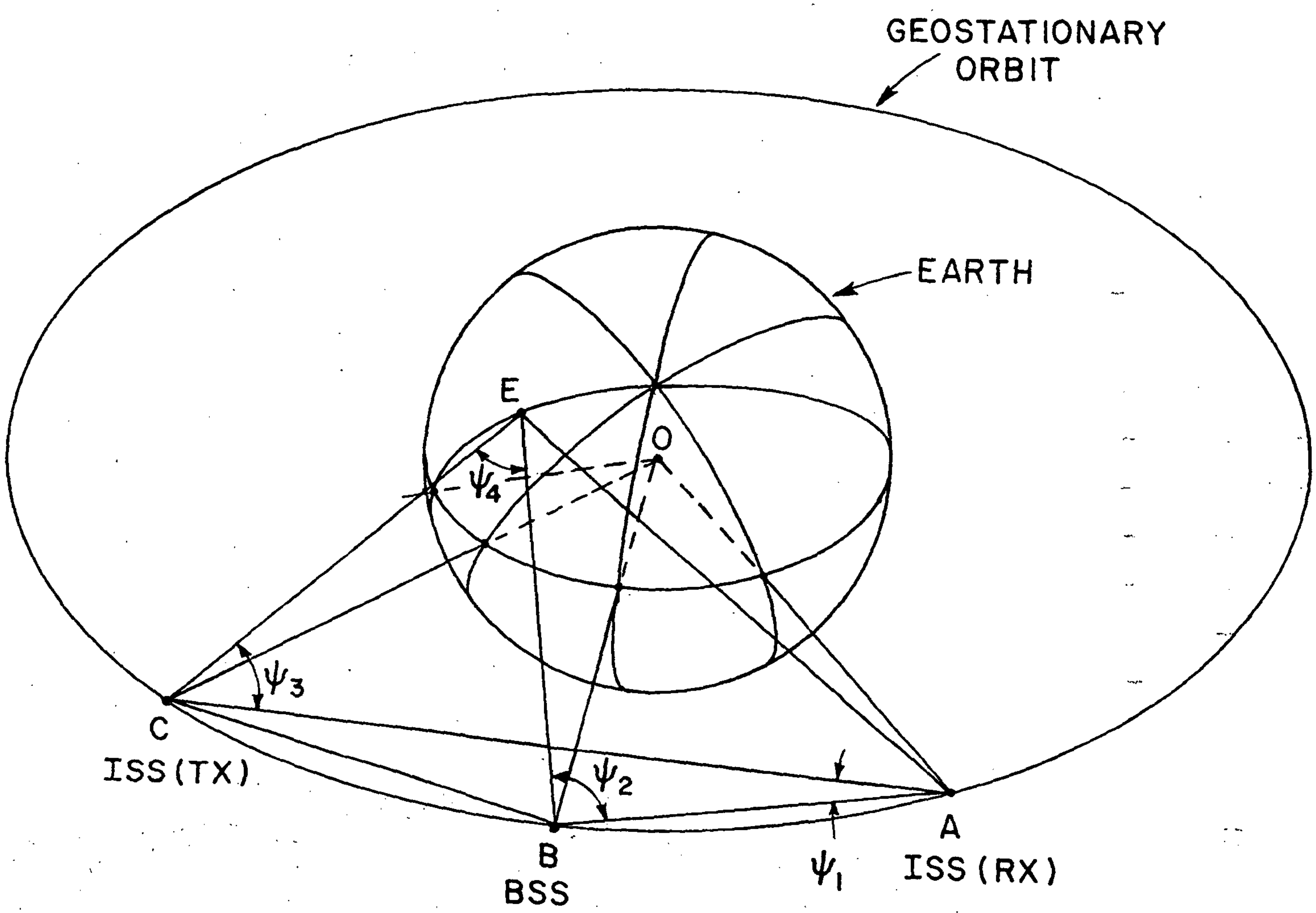


Figure 1. Simplified geometry for analysis of ISS/BSS systems. (Earth radius exaggerated for clarity).

antennas are not pointed at the interfering transmitters and transmitting antennas are not pointed at receivers where they would cause interference, i.e., the angles ψ_1 to ψ_4 in Figure 1 are not too small. The task addressed here is to define "not too small" quantitatively in terms of the systems parameters and orbital assignments.

Before looking at quantitative interference calculations, it is useful to consider the approximate geometries which make the various ψ_i angles small. The more complete coordinate system of Figure 2 is useful for this purpose. The symbols θ_1, θ_2, θ_3 denote central equatorial angles with respect to the Earth. The center of the Earth is indicated by O; the North Pole by P. Clockwise central angles are taken as positive, counterclockwise as negative, while with the ψ_i angles only the magnitude is of interest. In the figure, the θ_1 central angle happens to be negative, and all other central angles positive, but this is not necessarily true. All the θ angles can be defined over any 360° interval. The location of the BSS satellite is used as reference for all the central angles. The central angle from the BSS satellite to the ISS(RX) satellite is denoted by θ_1, that from the BSS satellite to the ISS(TX) satellite by θ_2, and the equatorial angle from the BSS satellite to the Earth station longitude by θ_3. The Earth station latitude is denoted by ℓ. The letters u,v,w,x,y,z denote distances.

The specific equations relating these variables which are used to calculate the ψ_i are given in the Appendix. From these equations, or alternatively from consideration of Figure 2, the following conditions are evident.

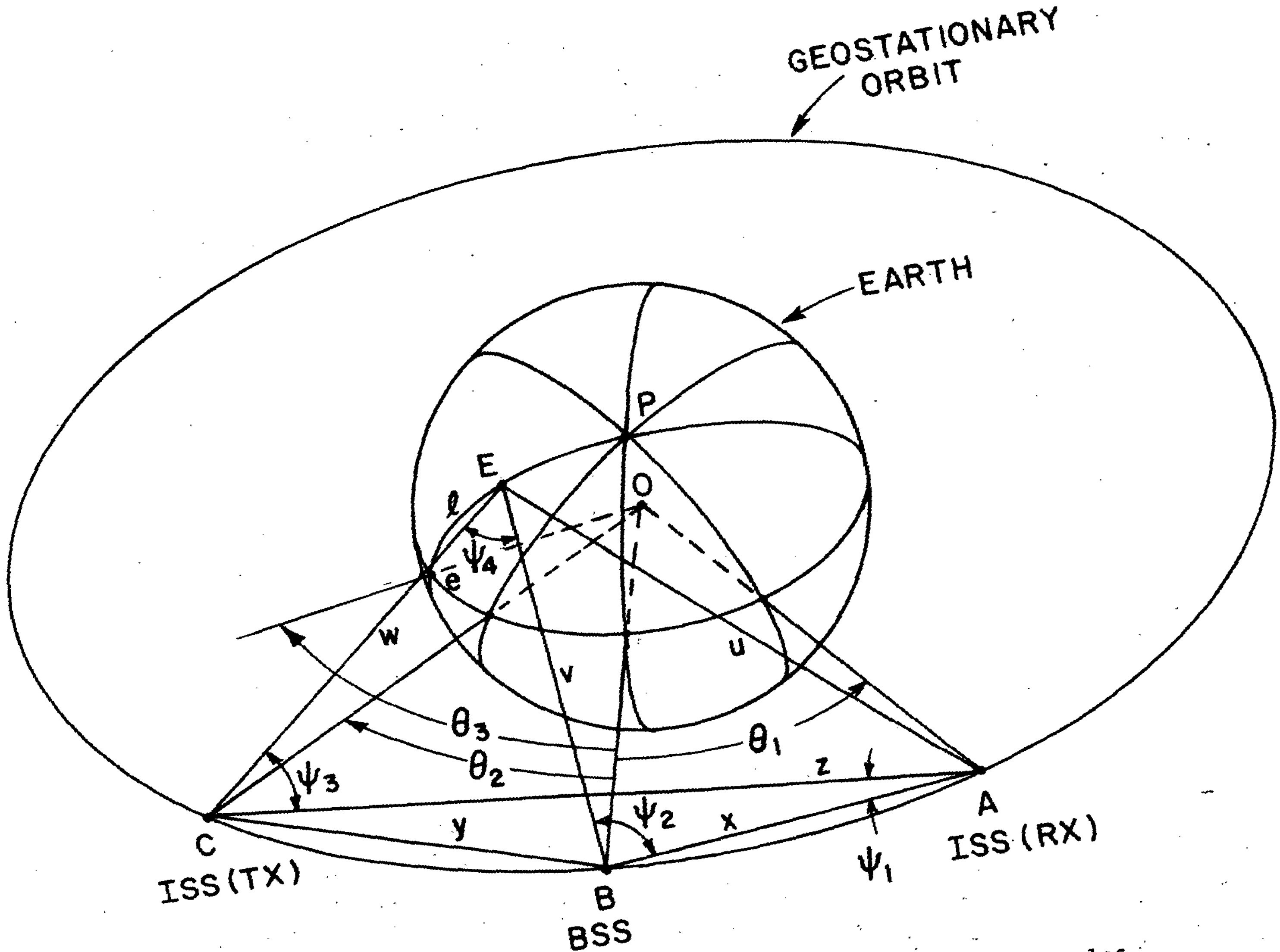


Figure 2. System geometry, showing all geometrical variables.

To make ψ_3 small, we require $\theta_1-\theta_2$ near ±160° and (θ_3,ℓ) near $[(\theta_1+\theta_2)/2, 0°]$. The geometry for the + sign is shown in Figure 3. In practice, the Earth station coordinates cannot approach (±90°,0°) too closely because of the requirement of a minimum elevation angle for the BSS as seen from the Earth station receiver.

To make ψ_2 small, we require θ_1 near ±160° and (θ_3,ℓ) near (±80°,0°) with the signs coordinated to be both positive or both negative. The corresponding geometry, for negative signs, is shown in Figure 4.

To make ψ_1 and ψ_4 small, the requirement is that θ_2 be very small in magnitude. It should be noted that ψ_1 decreases more rapidly than ψ_4 as the magnitude of θ_2 approaches zero, as shown in Figure 5.

A condition which would result in maximum interference to the ISS system exists when ψ_1 and ψ_2 are both small simultaneously. This requires θ_2 small in absolute value and θ_1 near ±160°, and consequently $|\theta_2-\theta_1|$ will also be near 160°. The geometry is shown in Figure 6; it is seen that all four of the ψ angles become small under this condition, which represents a "pathological" situation where all possible interference contributions become large. In order to avoid its occurrence, it may be wise not to allow very long frequency-shared inter-satellite paths, e.g., paths longer than about 120° might be restricted to the part of the spectrum not shared with the broadcast-satellite service.

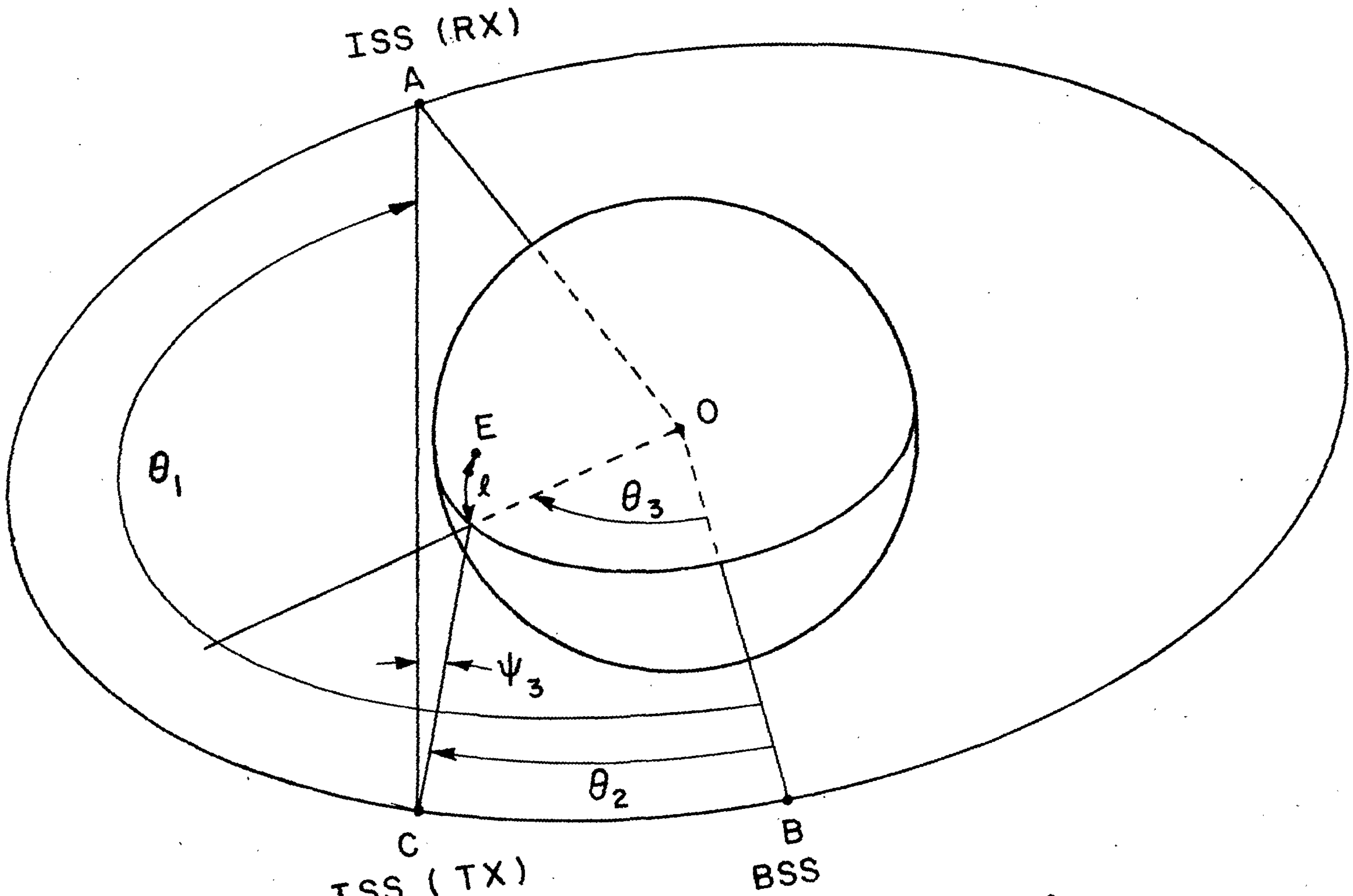


Figure 3. A geometry for small ψ_3: $\theta_1 = \theta_2 \approx 160°$, $(\theta_3, \ell) \approx ([\theta_1+\theta_2]/2, \ell)$.

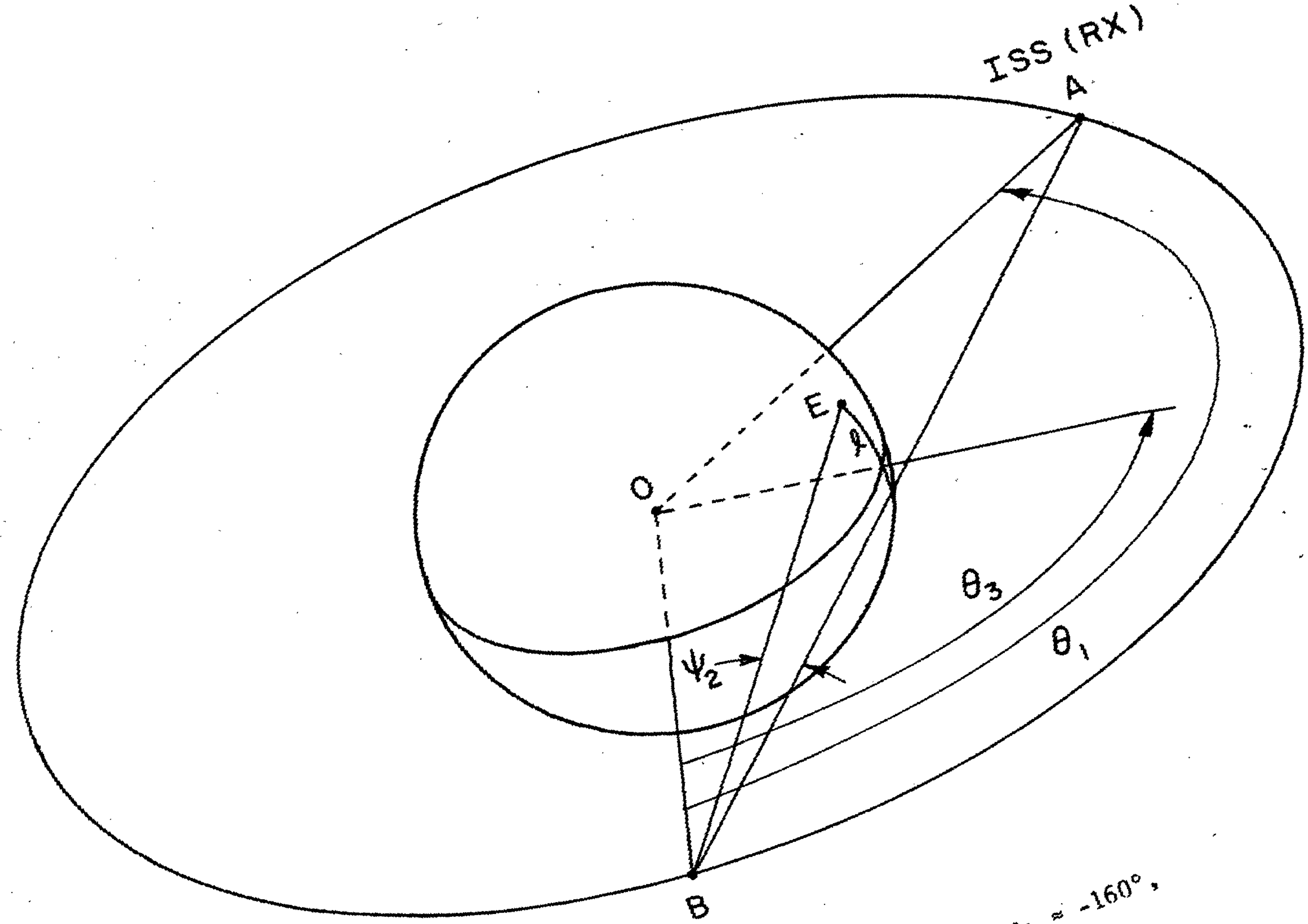


Figure 4. A geometry for small ψ_2: $\theta_1 \approx -160°$, $(\theta_3, \ell) \approx (-80°, 0°)$.

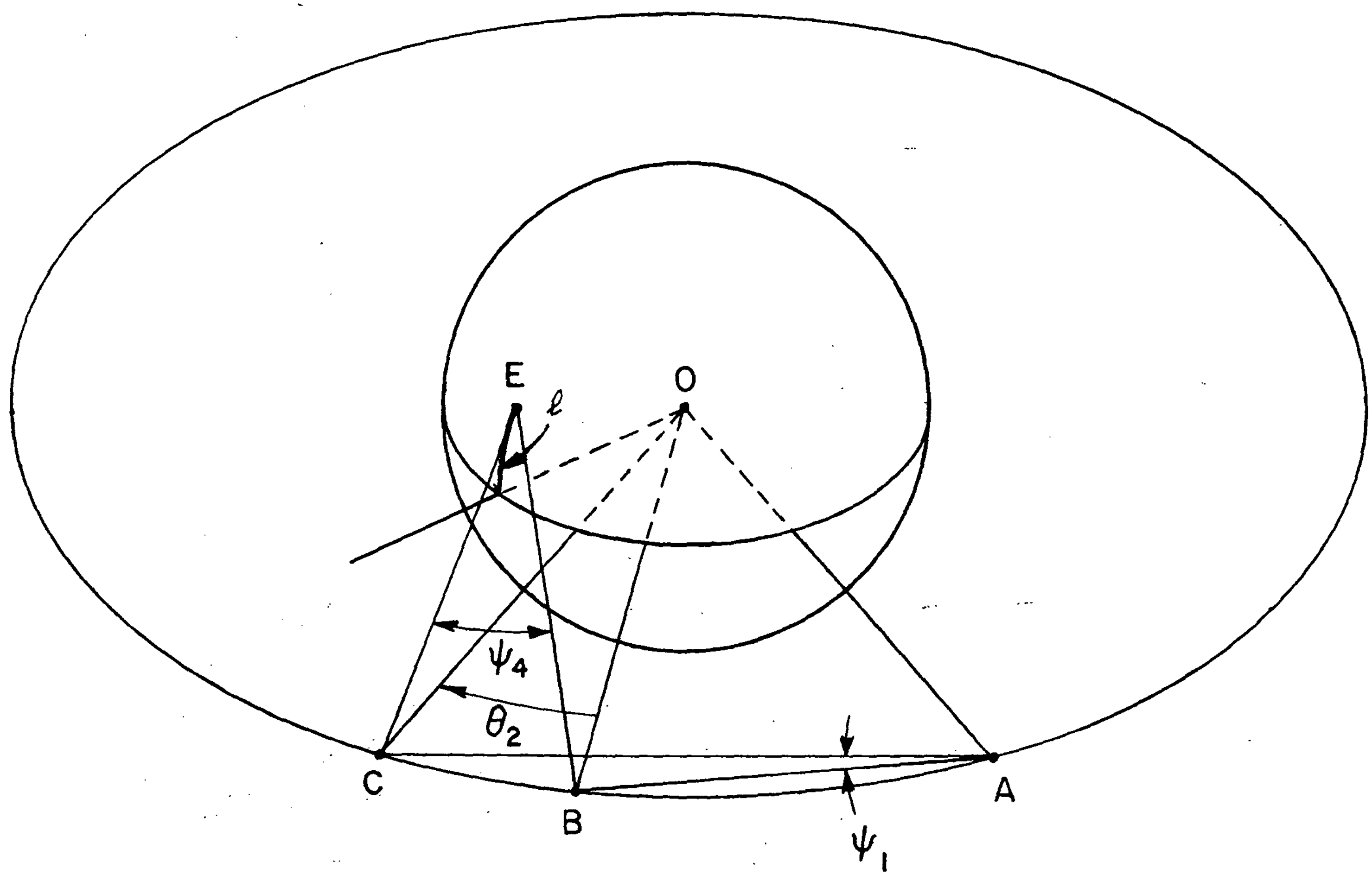


Figure 5. Geometry for small ψ_1, ψ_4: small $|\theta_2|$.

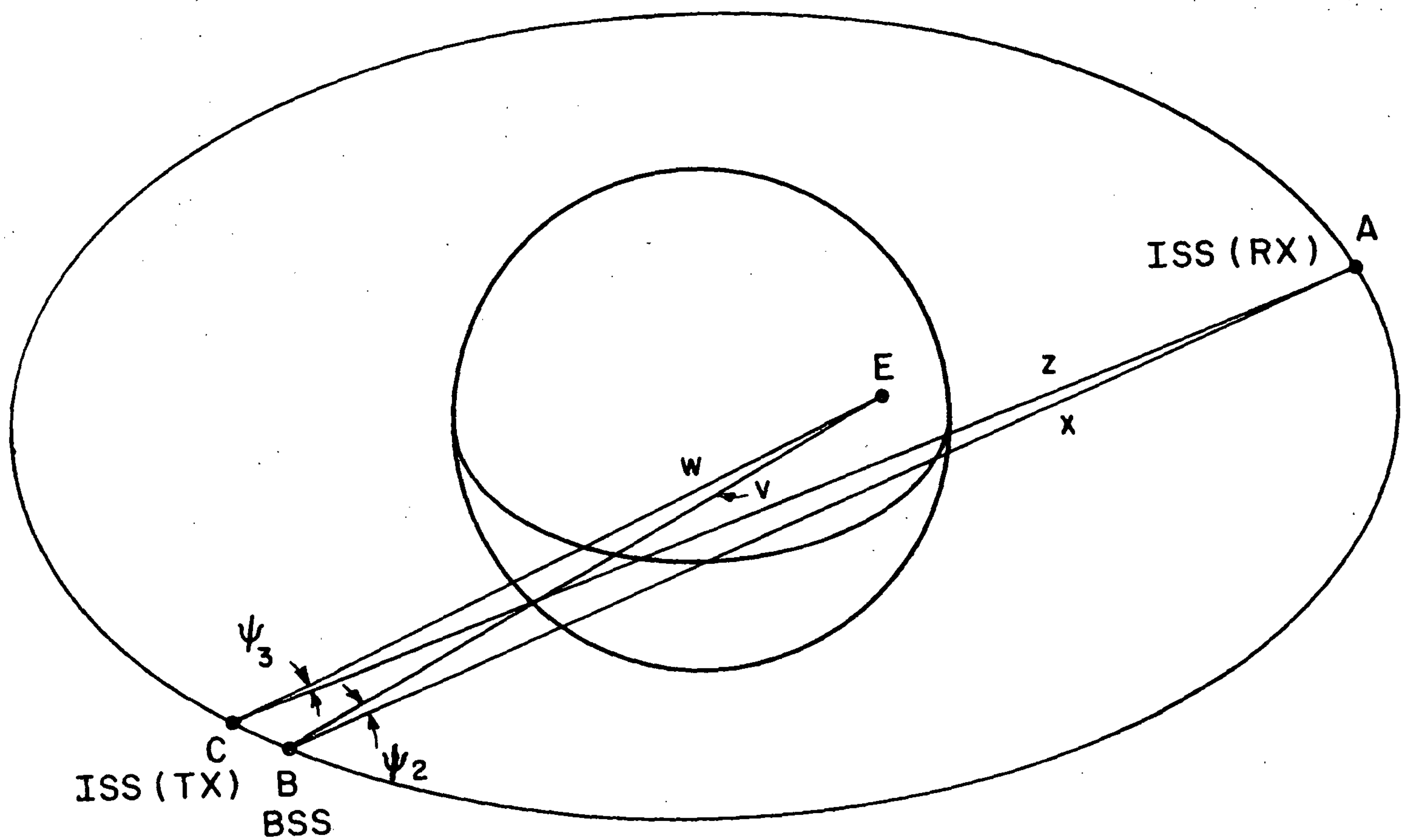


Figure 6. "Pathological" geometry: all ψ small.

B. TYPICAL SYSTEM PARAMETERS

The aim of this paper is to characterize acceptable geometries, from the point of view of mutual interference, for a considerable range of systems parameters, such as transmitted power, antenna gains, receiver noise temperature, etc. However, if the parameters are changed too drastically, the very nature of the problem changes. The "typical" system values about which variations might be made are shown in Table 1, together with an explanation of the symbols which will be used throughout this paper. The values for the BSS system are taken from CCIR Report 215-4[6,7], those for the ISS system from a proposed design [3]. The antenna discrimination of the ISS system is adopted from CCIR Report 558-1 [8] and is shown in Figure 7; those of the BSS system are taken from CCIR Report 810 [9] for community reception and WARC-BS-77-Annex 8 [10] for transmission. They are given in Figures 8 and 9, respectively. Pointing and station-keeping errors are ignored in this study. Halfpower beamwidths are calculated from gain by the relationship

$$\psi_0 = \sqrt{27,000/G} \quad \text{(degrees)} \quad . \qquad (1)$$

It should be noted that over most of their angular range, at angles well removed from the main lobe, the discrimination patterns are constant. This property turns out very useful because the number of variables is reduced; this facilitates the construction of families of universal curves for displaying the results in a particularly useful form.

TABLE 1

TYPICAL SYSTEMS PARAMETER VALUES

f - frequency - 22.75 GHz

P^T_{BSS} - BSS transmitter power - 34 dBW

G^T_{BSS} - BSS transmit antenna gain - 36 dB

G^R_{BSS} - BSS receive antenna gain - 43 dB

T_{BSS} - BSS receiver noise temperature - 1100 K.

B_{BSS} - BSS bandwidth - 40 MHz.

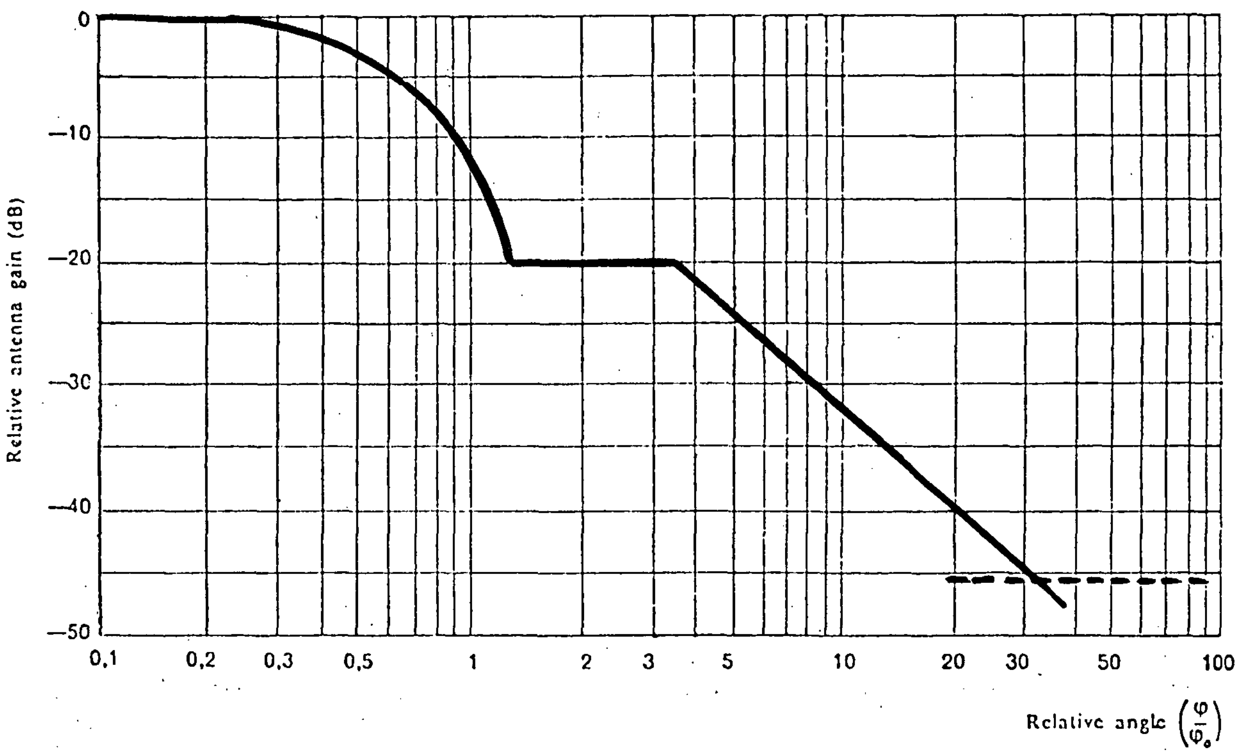
P_{ISS} - ISS transmitter power - 10 dBW.

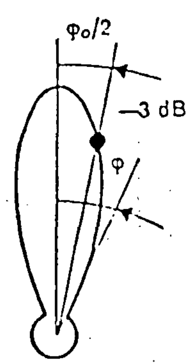
G^T_{ISS} - ISS transmit antenna gain - 52 dB.

G^R_{ISS} - ISS receive antenna gain - 52 dB.

T_{ISS} - ISS receiver noise temperature - 1000 K.

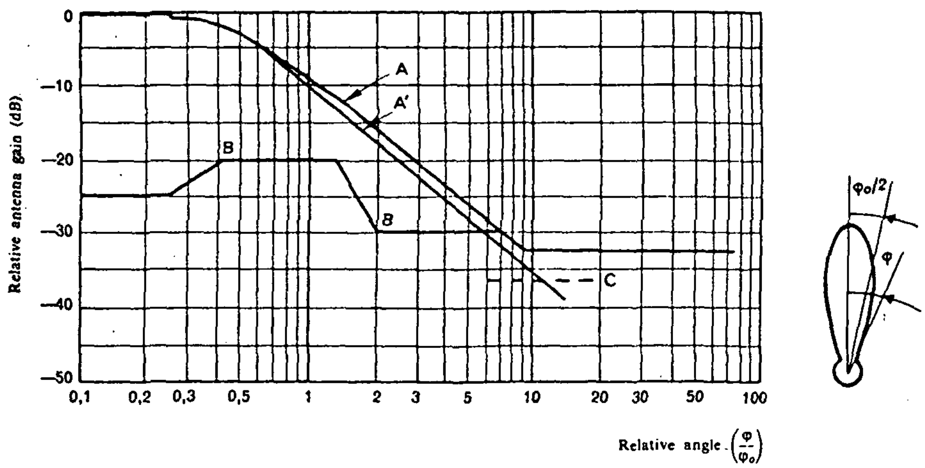
B_{ISS} - ISS bandwidth - 125 MHz





$-12\left(\frac{\psi}{\psi_o}\right)$	for $0<\psi\leqslant 1.3\ \psi_o$
-20	for $1.3\ \psi_o<\psi\leqslant 3.15\ \psi_o$
$-7.5-25\log_{10}\left(\frac{\psi}{\psi_o}\right)$	for $3.15\ \psi_o<\psi\leqslant\psi_i$
$-G-10$	for $\psi_i<\psi$

Figure 7. ISS antenna discrimination patterns. From references [8,10].



Co-polar and cross-polar reference patterns for receiving antenna

Curve A: Co-polar component for individual reception without side-lobe suppression

0	for $0 \leqslant \varphi \leqslant 0.25\,\varphi_o$
$-12\left(\frac{\varphi}{\varphi_o}\right)^2$	for $0.25\,\varphi_o < \varphi \leqslant 0.707\,\varphi_o$
$-\left[9.0 + 20\log_{10}\left(\frac{\varphi}{\varphi_o}\right)\right]$	for $0.707\,\varphi_o < \varphi \leqslant 1.26\,\varphi_o$
$-\left[8.5 + 25\log_{10}\left(\frac{\varphi}{\varphi_o}\right)\right]$	for $1.26\,\varphi_o < \varphi \leqslant 9.55\,\varphi_o$
-33	for $9.55\,\varphi_o < \varphi$

Curve A': Co-polar component for community reception without side-lobe suppression

0	for $0 \leqslant \varphi \leqslant 0.25\,\varphi_o$
$-12\left(\frac{\varphi}{\varphi_o}\right)^2$	for $0.25\,\varphi_o < \varphi \leqslant 0.86\,\varphi_o$
$-\left[10.5 + 25\log_{10}\left(\frac{\varphi}{\varphi_o}\right)\right]$	for $0.86\,\varphi_o < \varphi$ up to intersection with Curve C (then Curve C)

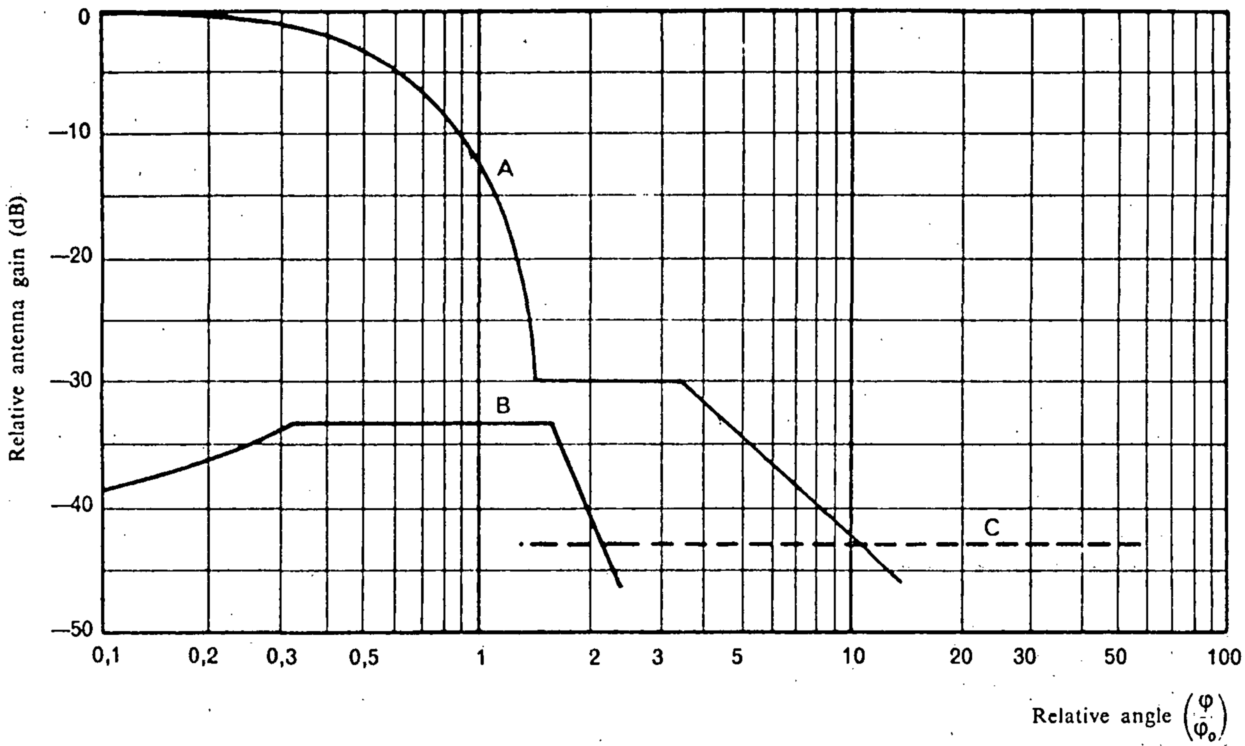
Curve B: Cross-polar component for both types of reception

-25	for $0 \leqslant \varphi \leqslant 0.25\,\varphi_o$
$-\left(30 + 40\log_{10}\left\lvert\frac{\varphi}{\varphi_o} - 1\right\rvert\right)$	for $0.25\,\varphi_o < \varphi \leqslant 0.44\,\varphi_o$
-20	for $0.44\,\varphi_o < \varphi \leqslant 1.4\,\varphi_o$
$-\left(30 + 25\log_{10}\left\lvert\frac{\varphi}{\varphi_o} - 1\right\rvert\right)$	for $1.4\,\varphi_o < \varphi \leqslant 2\,\varphi_o$

-30 until intersection with co-polar component curve; then as for co-polar component

Curve C: Minus the on-axis gain

Figure 8. BSS earth station (receive) antenna discrimination pattern. From references [9,10]. Curve A' is used in the calculations of this report.



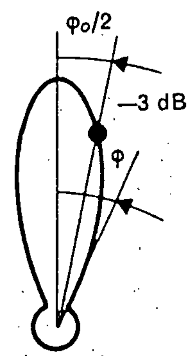
Curve A: Co-polar component

$$-12\left(\frac{\varphi}{\varphi_o}\right)^2 \quad \text{for } 0 \leqslant \varphi \leqslant 1.58\,\varphi_o$$

$$-30 \quad \text{for } 1.58\,\varphi_o < \varphi \leqslant 3.16\,\varphi_o$$

$$-\left[17.5 + 25\log_{10}\left(\frac{\varphi}{\varphi_o}\right)\right] \quad \text{for } 3.16\,\varphi_o < \varphi$$

after intersection with curve C: as curve C



Curve B: Cross-polar component

$$-\left(40 + 40\log_{10}\left|\frac{\varphi}{\varphi_o} - 1\right|\right) \quad \text{for } 0 \leqslant \varphi \leqslant 0.33\,\varphi_o$$

$$-33 \quad \text{for } 0.33\,\varphi_o < \varphi \leqslant 1.67\,\varphi_o$$

$$-\left(40 + 40\log_{10}\left|\frac{\varphi}{\varphi_o} - 1\right|\right) \quad \text{for } 1.67\,\varphi_o < \varphi$$

after intersection with curve C: as curve C

Curve C: Minus the on-axis gain.

Figure 9. BSS satellite (transmit) antenna discrimination pattern. From reference [10].

C. INTERFERENCE CRITERIA

Two commonly used criteria for interference calculations are the interference-to-noise ratio I/N and the carrier-to-interference ratio C/I. The interference-to-noise ratio is useful in predicting the degradation of system performance due to interference directly by the relation

$$\frac{C}{N+I} = \frac{C}{N}\left(1 + \frac{I}{N}\right)^{-1} \quad , \tag{2}$$

where C denotes carrier power, I interference power, and N receiver noise power.

The ratio I/N is also useful to describe system acceptability in terms of an allowed noise margin N_m by the inequality

$$N_m > 1 + \frac{I}{N} \quad . \tag{3}$$

On the assumption that the number of interferers to any one receiver will not be large, systems might be designed for I/N ratios in the range -5 dB to -10 dB.

The carrier-to-interference criterion implies the notion that excess interference can be overcome by boosting carrier power, although at the penalty of possibly increased interference to other users. System performance can be calculated from C/I by

$$\left(\frac{C}{N+I}\right)^{-1} = \left(\frac{C}{N}\right)^{-1} + \left(\frac{C}{I}\right)^{-1} \quad . \tag{4}$$

In terms of noise margin, the acceptability relation can be shown to be

$$\frac{N}{C}(N_m - 1) > \left(\frac{C}{I}\right)^{-1} \quad . \qquad (5)$$

A more complete discussion will be found in the thesis by Wang [11].

D. INTERFERENCE TO THE ISS SYSTEM

1. The General Case

Equations for the single-entry interference-to-noise ratio and carrier-to-interference ratio can be obtained by application of the Friis transmission equation [12,13] as

$$\frac{I}{N} = \frac{P_{BSS}\, G^T_{BSS}\, D^T_{BSS}(G^T_{BSS}, \psi_2)\, G^R_{ISS}\, D^R_{ISS}(G^R_{ISS}, \psi_1)\, c^2}{f^2\, (4\pi)^2\, x^2\, k\, T_{ISS}\, B_{ISS}} \quad , \qquad (6)$$

$$\frac{C}{I} = \frac{P_{ISS}\, G^T_{ISS}}{P_{BSS}\, G^T_{BSS}} \cdot \frac{1}{D^T_{BSS}(G^T_{BSS}, \psi_2) D^R_{ISS}(G^R_{ISS}, \psi_1)} \cdot \frac{x^2}{z^2} \quad . \qquad (7)$$

Since the goal here is to present the results in a form which is helpful for system design and specifically, wherever possible, by the use of universal curves, it is useful to eliminate as many variables as possible by combining them into universal factors which are easily calculated from the given system parameters. We define two factors R_1 and R_2 by

$$\frac{I}{N} \cdot f^2 \cdot \frac{B_{ISS}\, T_{ISS}}{P_{BSS}} \equiv R_1 = \frac{c^2}{(4\pi)^2\, k} \cdot \frac{G^T_{BSS} D^T_{BSS}(G^T_{BSS}, \psi_2) G^R_{ISS} D^R_{ISS}(G^R_{ISS}, \psi_1)}{x^2} \quad , \qquad (8)$$

$$\frac{C}{I} \cdot \frac{P_{BSS}\ G^T_{BSS}}{P_{ISS}\ G^T_{ISS}} \equiv R_2 = \frac{1}{D^T_{BSS}(G^T_{BSS}, \psi_2) D^R_{ISS}(G^R_{ISS}, \psi_1)} \cdot \frac{x^2}{z^2} , \qquad (9)$$

where the dependence on central angles arises through $\psi_1(\theta_1, \theta_2)$, $\psi_2(\theta_1, \theta_3)$, $x(\theta_1)$ and $z(\theta_1, \theta_2)$. In application, the left sides of these equations, which define the universal factors R_1, R_2, are evaluated by the user from the system parameters, while the right side has been evaluated by computer and is presented in the form of graphs. In order to minimize the hand-computation labor, the numerical constants have been included on the right (computer-generated) side; in the case of the I/N calculation by Equation (8) this makes the equation dependent on the units employed, and the units Hertz, Watts, Kelvins are implied. While there are still too many variables involved in these equations to permit plotting universal curves, it is useful at this time to look at the shape of the allowed regions and relate them to the antenna discrimination angle (ψ_i) conditions discussed previously. A computer code implementing the right side of Equation (8) has been written [14] and a plot appears in Figure 10. The forbidden region of unacceptable interference appears near, and is symmetric with respect to, the diagonal line $\theta_2 = 0$, which slants from the lower right to the upper left of the figure. This is the condition for $\psi_1 = 0$. The broadening of the unallowed region near the center of the figure occurs because x, the separation distance between the BSS and ISS(RX) satellites becomes small so that unacceptable interference can be received even via the ISS receiving antenna sidelobes. The broadening of the region near the top

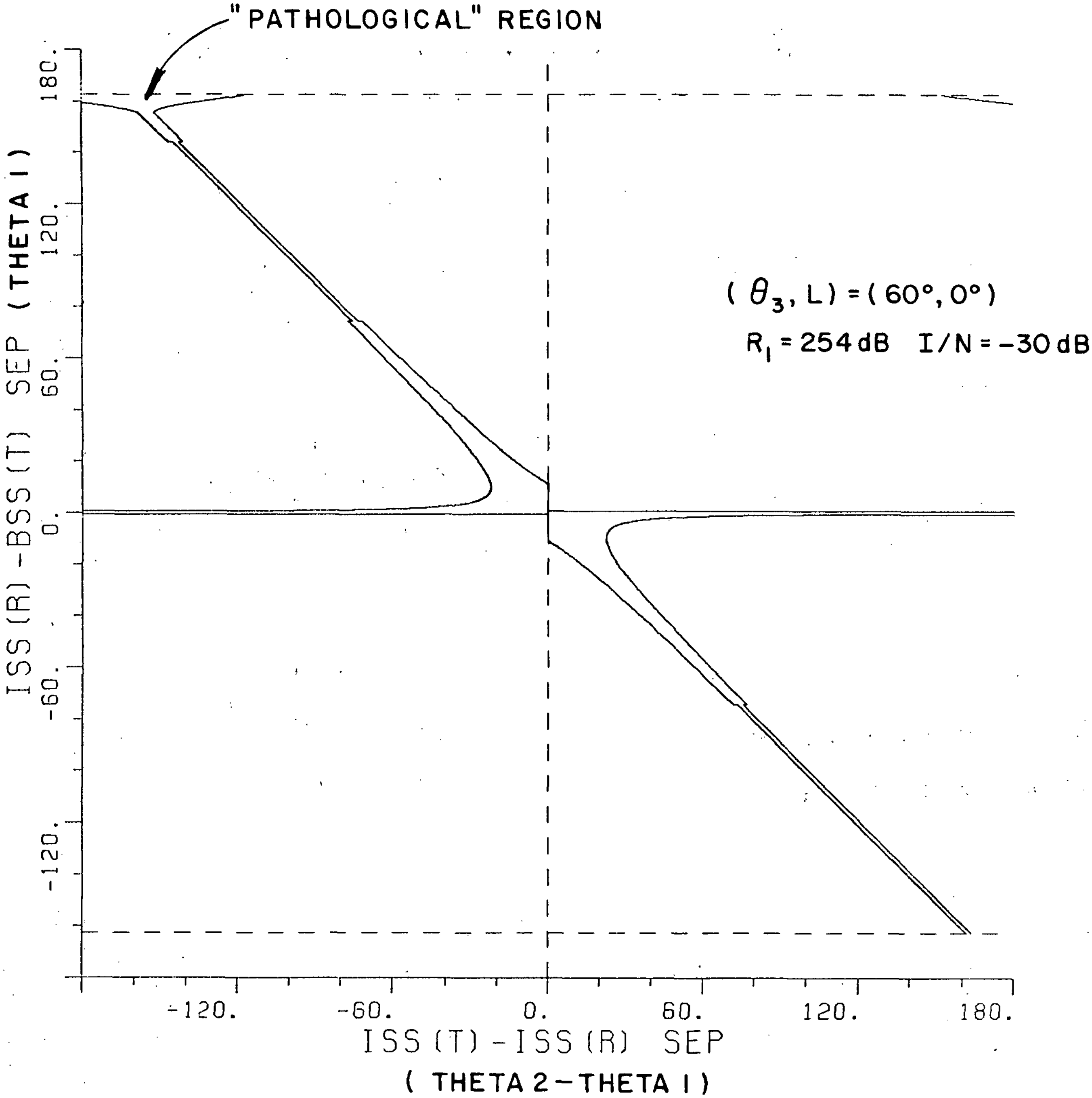


Figure 10. Sample solution of interference to an ISS system by a BSS system, using the unrestricted computer code IING [14]. I/N is used as criterion. The narrow regions indicate unacceptable geometries. THETA 1, THETA 2 refers to θ_1, θ_2 in Figure 2. See Table 1 for system parameter values.

left ($\theta_2-\theta_1=-160°$, $\theta_1=160°$) occurs due to ψ_2 also being small, so that the BSS transmitter points at the ISS receiver. The combination of the two effects, ψ_1 small, ψ_2 small represents the pathological condition alluded to previously, which can be avoided by restricting the maximum length of frequency-shared intersatellite links to central equatorial angles on the order of 120°.

The computer code used for producing Figure 10 is useful not only to give an intuitive feel for system behavior, but also as a definitive tool for evaluating allowable system geometries for proposed specific systems. It deals with all possible geometries. A similar presentation for the C/I criterion, based on Equation (9), appears in Figure 11. This computer code is also documented in the technical report [15].

2. Universal Curves

When the ISS receiver is not illuminated by the main or near side lobes of the BSS transmitter, e.g., for the geometry of Figure 12, Equations (8) and (9) can be simplified by setting

$$G^T_{BSS} \cdot D^T_{BSS} \left(G^T_{BSS}, \psi_2(\theta_1, \theta_3, \ell)\right) = 1 \quad . \tag{10}$$

The reduction in the number of variables then allows the plotting of families of universal curves with the universal factors R_1 or R_2 as parameter. One such family is required for each value of ISS receiver antenna gain. Figure 13 shows an example of such a universal curve for the case of I/N used as the criterion; note that for the systems of Table 1 with I/N = -10 dB, R_1 has the value 274.1 dB. Figure 14 shows

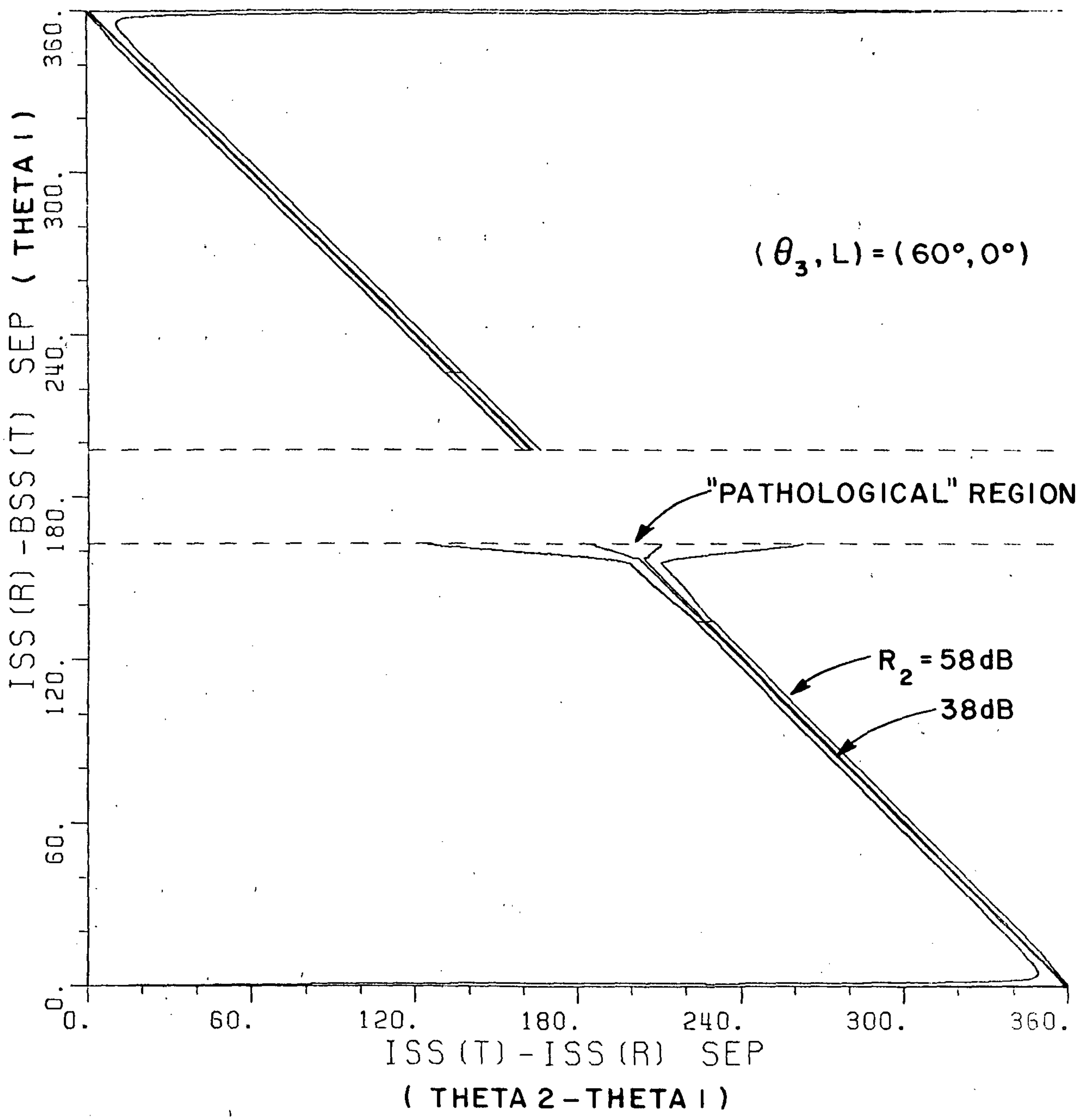


Figure 11. Sample solution of interference to an ISS system by a BSS system, using the unrestricted computer code ICIG [15]. C/I is used as criterion. See Table 1 for system parameter values.

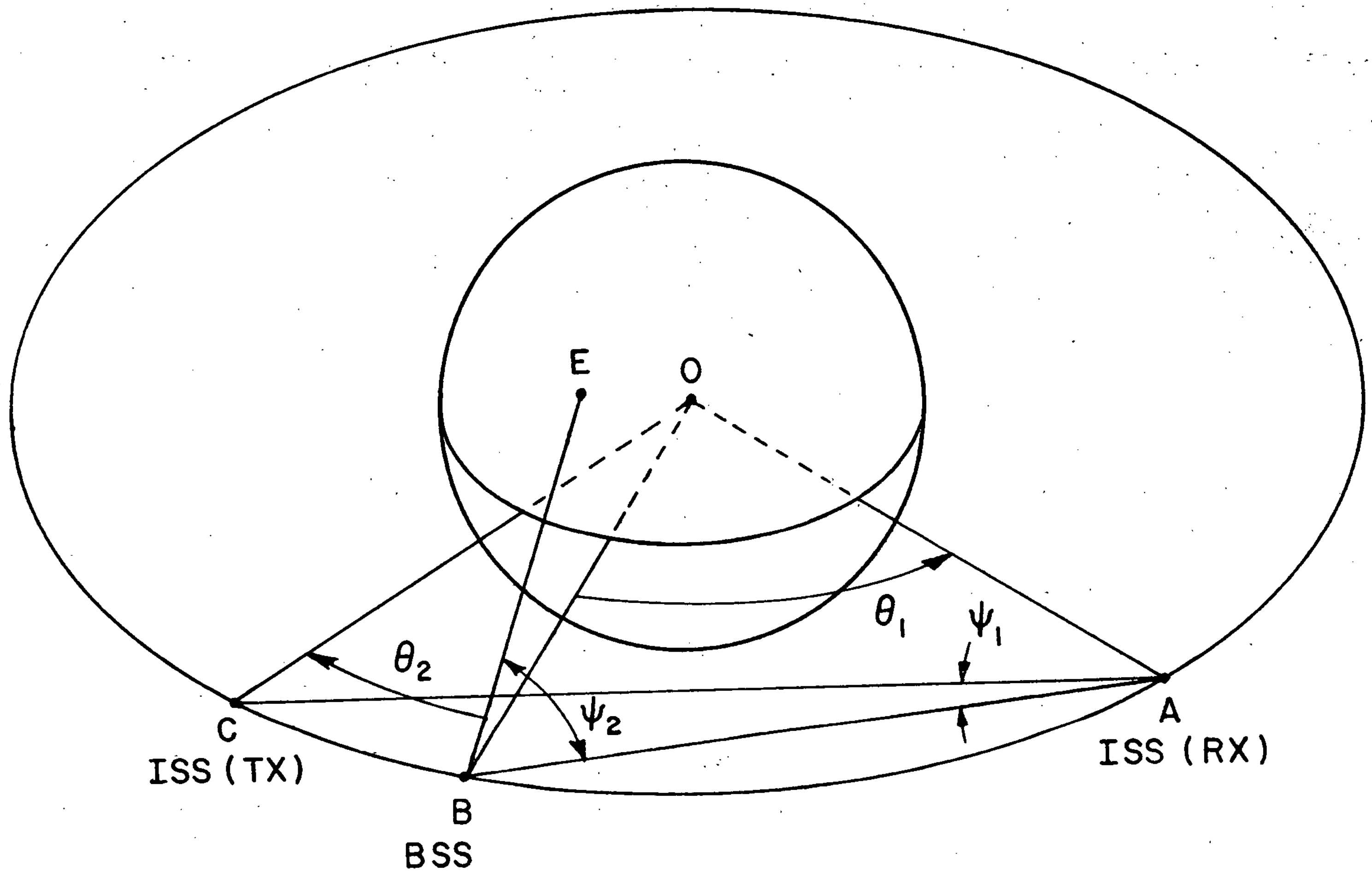


Figure 12. Example of a geometry for which the universal curves of Figures 13, 14, 15 are applicable (ψ_2 large).

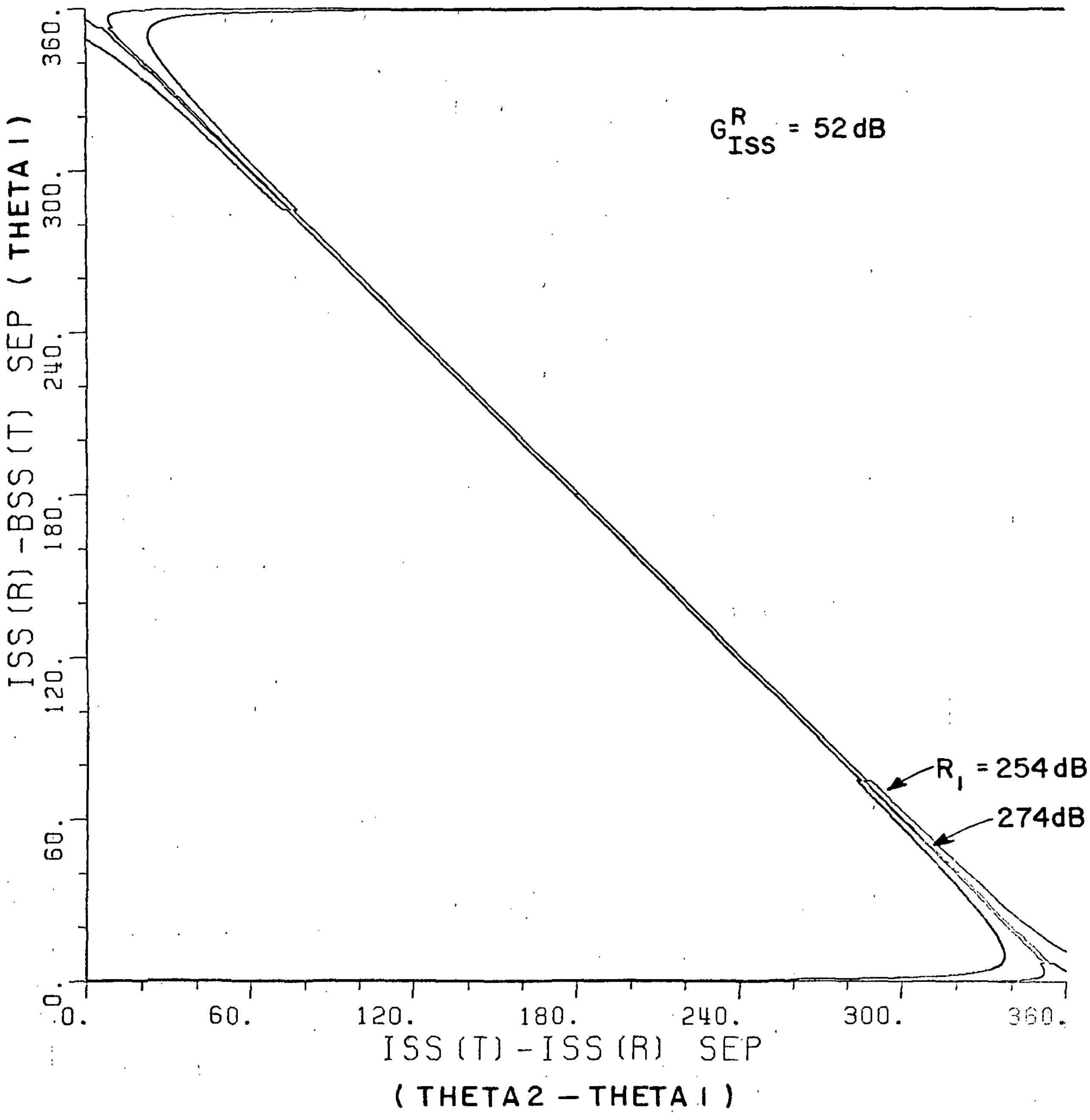


Figure 13. Universal curve for interference to an ISS system by a BSS system when the ISS receiver is illuminated by the back or far side lobes of the BSS transmitter. I/N criterion.

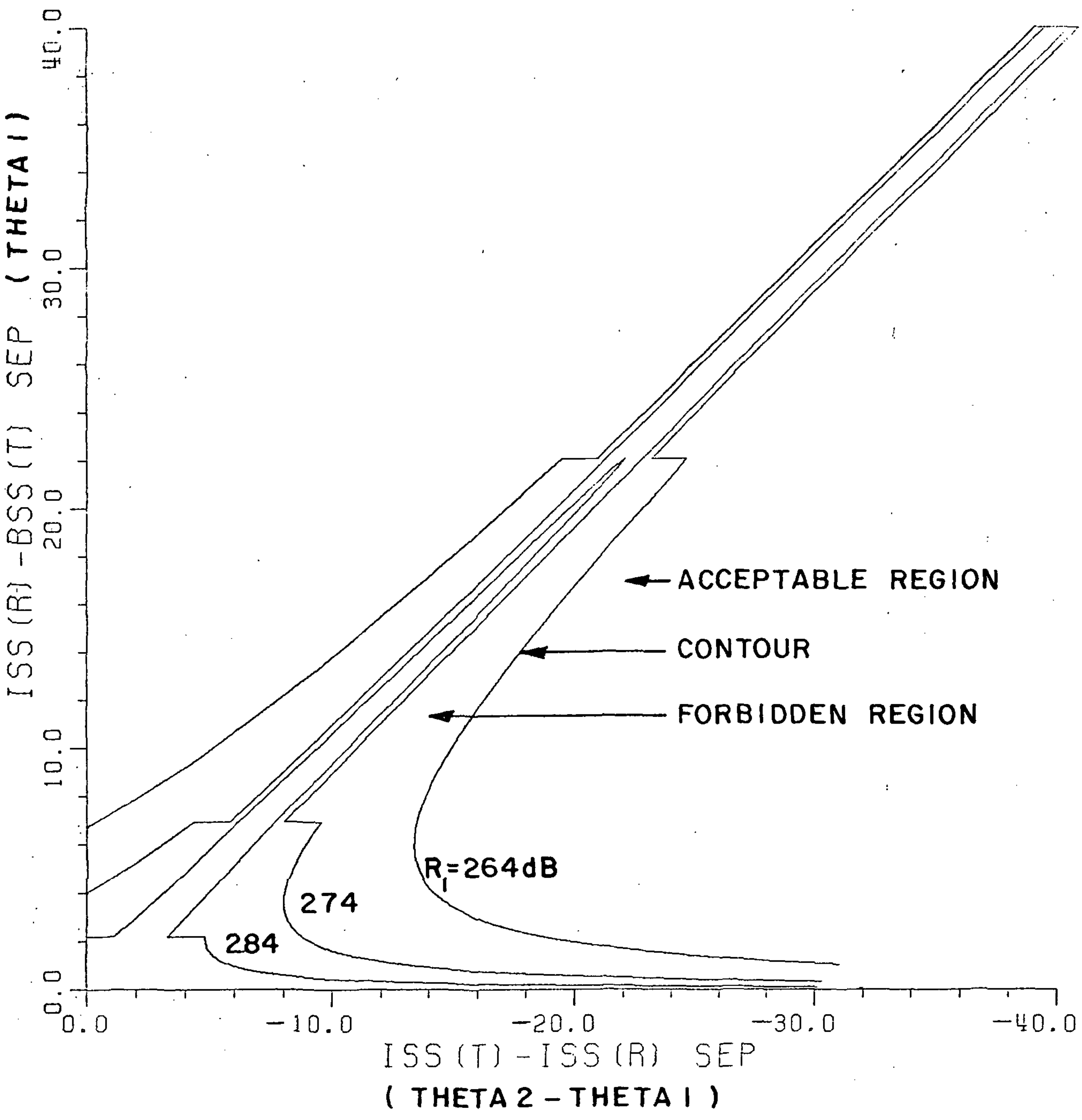


Figure 14. Enlargement of part of Figure 13 for numerical application. (The abscissa is reversed compared to Figure 13).

an enlargement of the most significant region of this plot (with the abscissa reversed). The set of curves corresponding to Figure 12 but with C/I used as criterion is shown in Figure 15. For the system of Table 1 with C/I = 30 dB, the value of R_2 is 38 dB. The region of applicability of the universal curves of Figures 13, 14, and 15, as defined by Equation (10), is shown in Figure 16.

In principle, universal curves can also be constructed for the case when the ISS(RX) antenna receives interference only via its far side lobes, as illustrated in Figure 17, i.e., when ψ_1 is large and ψ_2 is small. Then analogously to Equation (10) we have

$$G_{ISS}^{R}\, D_{ISS}^{R}\, (G_{ISS}^{R}, \psi_1(\theta_2)) = 0.1 \quad , \tag{11}$$

which may be used to simplify the right side of Equations (8) and (9) to involve only the central angles, θ_1, θ_3, and ℓ. A computer code for this geometry for the I/N criterion has been written, but it turns out the there are no forbidden regions unless the value of R_1 is on the order of 230 dB, some 40 dB below the values associated with the typical system of Table 1 [16]. The physical reasons are a) the ISS(RX) antenna gain typically is much higher than that of the BSS(TX) antenna, which must illuminate a substantial Earth region for broadcast purposes, b) the far side-lobe region for the BSS(TX) antenna is taken as 0 dBi [6,7], while that for the ISS(RX) antenna is taken as -10 dBi [8], c) for the interference geometry of Figure 12 the distance between the interfering transmitter at B and the affected receiver at A can become small, but this is not possible for the geometry of Figure 17, and

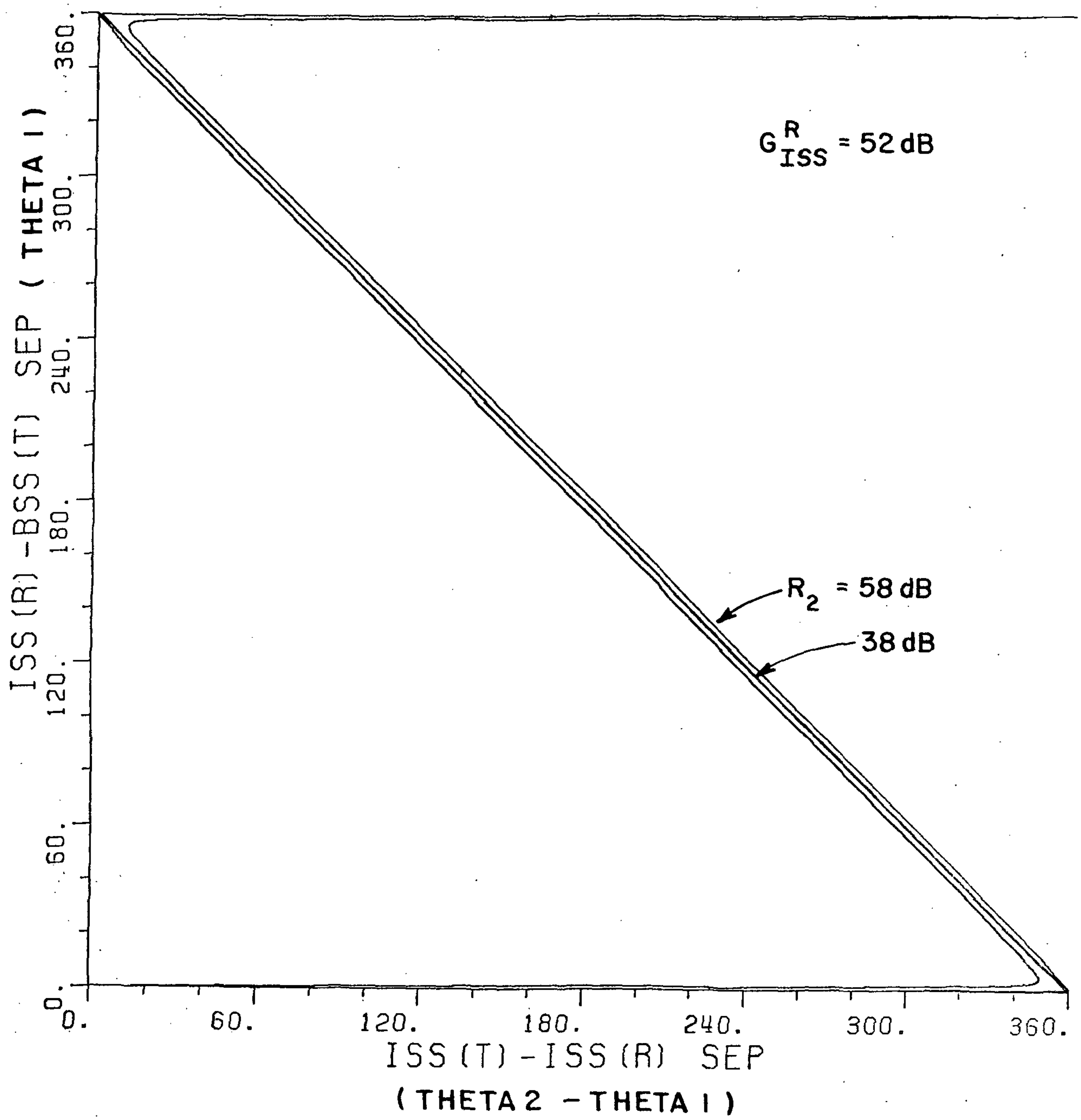


Figure 15. Universal curve for interference to an ISS system by a BSS system when the ISS receiver is illuminated by the back or far side lobes of the BSS transmitter. C/I criterion.

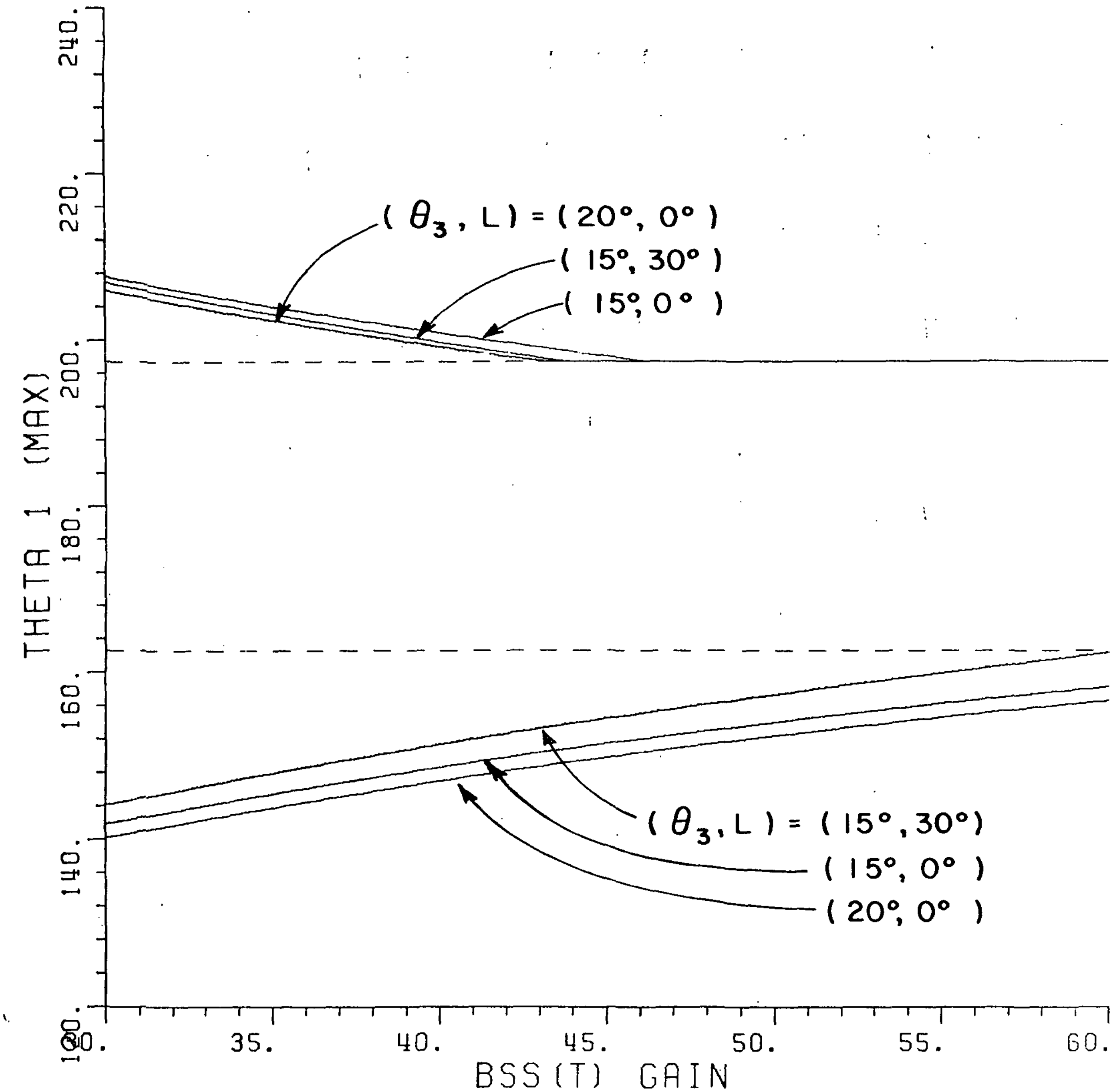


Figure 16. Regions of validity for the universal curves of Figures 13 to 15. The universal curves are not valid for points between the respective contours showing earth station locations as parameter; however, for the region between the dashed lines the BSS transmitter is screened from the ISS receiver by the Earth, so that interference is impossible.

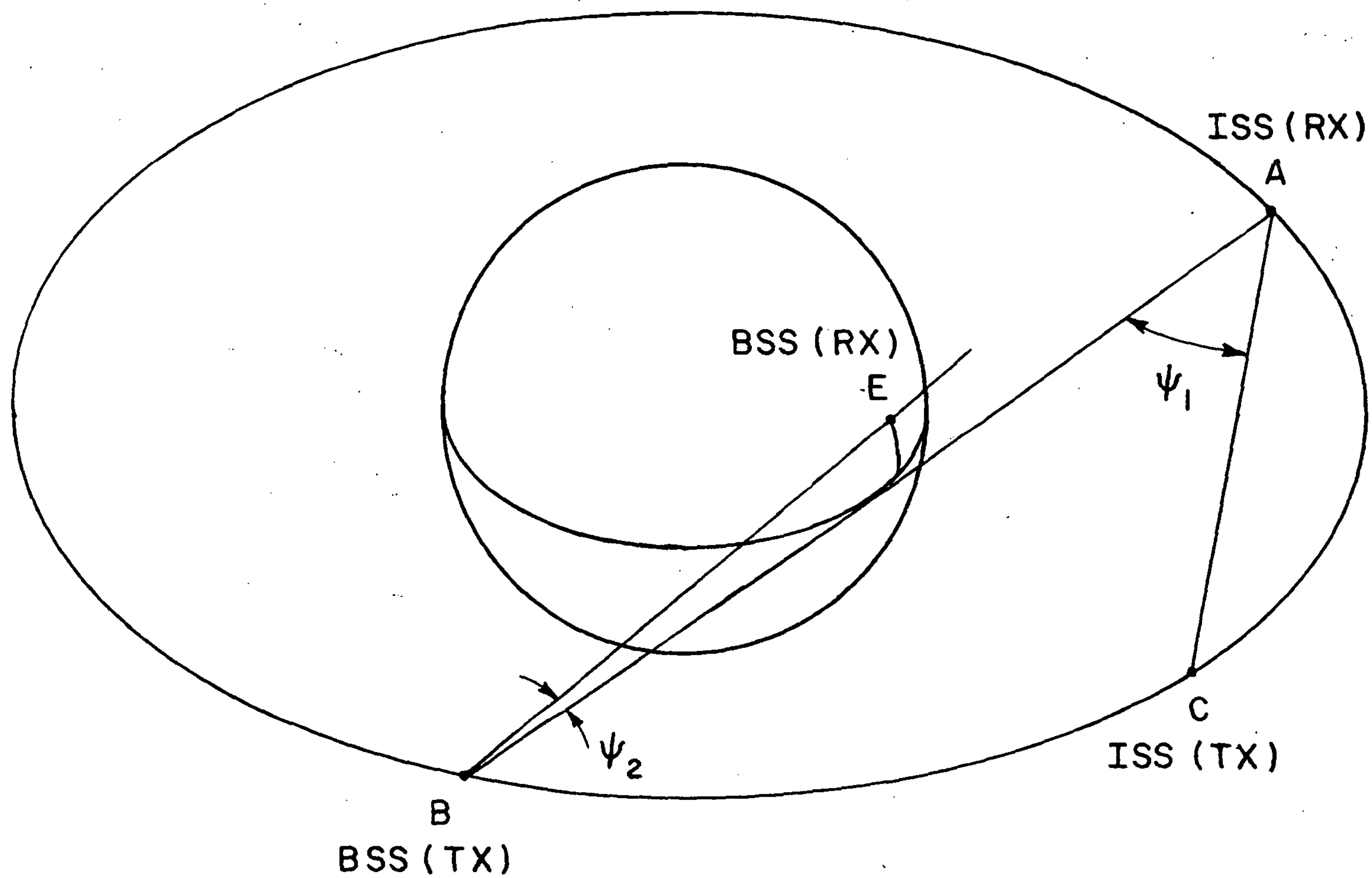


Figure 17. Example of a second geometry for which universal curves may be constructed (ψ_1 large).

d) the smallness of ψ_1 is unrestricted, while that of ψ_2 is restricted by the requirement of a minimum elevation of the BSS transmitter at B as seen from the Earth station at E. Thus the geometry of Figure 17 does not lead to interference for systems parameter values even moderately close to those currently considered, and the curves are not needed. The same is true for the C/I criterion.

E. INTERFERENCE TO THE BSS SYSTEM

In analogy with Equations (8) and (9), the following relations can be obtained for the interference criteria when the ISS transmission causes interference in the Earth-station BSS receiver

$$\frac{I}{N} f^2 \frac{T_E B_E}{P_{ISS}} \equiv R_3 = \frac{c^2}{(4\pi)^2 k} \frac{G^T_{ISS} D^T_{ISS} (G^T_{ISS}, \psi_3) G^R_E D^R_E (G^R_E, \psi_4)}{w^2} , \quad (12)$$

$$\frac{C}{I} \frac{P_{ISS} G^T_{ISS}}{P_{BSS} G^T_{BSS}} \equiv R_4 = \frac{1}{D^T_{ISS} (G^T_{ISS}, \psi_3) D^R_E (G^R_E, \psi_4)} \frac{w^2}{v^2} , \quad (13)$$

where the central-angle dependence occurs through $\psi_3(\theta_1, \theta_2, \theta_3, \ell)$, $\psi_4(\theta_2, \theta_3, \ell)$, $v(\theta_3, \ell)$, and $w(\theta_2, \theta_3, \ell)$.

Computer codes have been written to implement these equations, and the results for an example are shown in Figures 18 and 19, respectively [17]. In these graphs, the region between the vertical dashed lines is excluded because the Earth would block the ISS link; the region above the top dashed line and that below the bottom one are excluded because

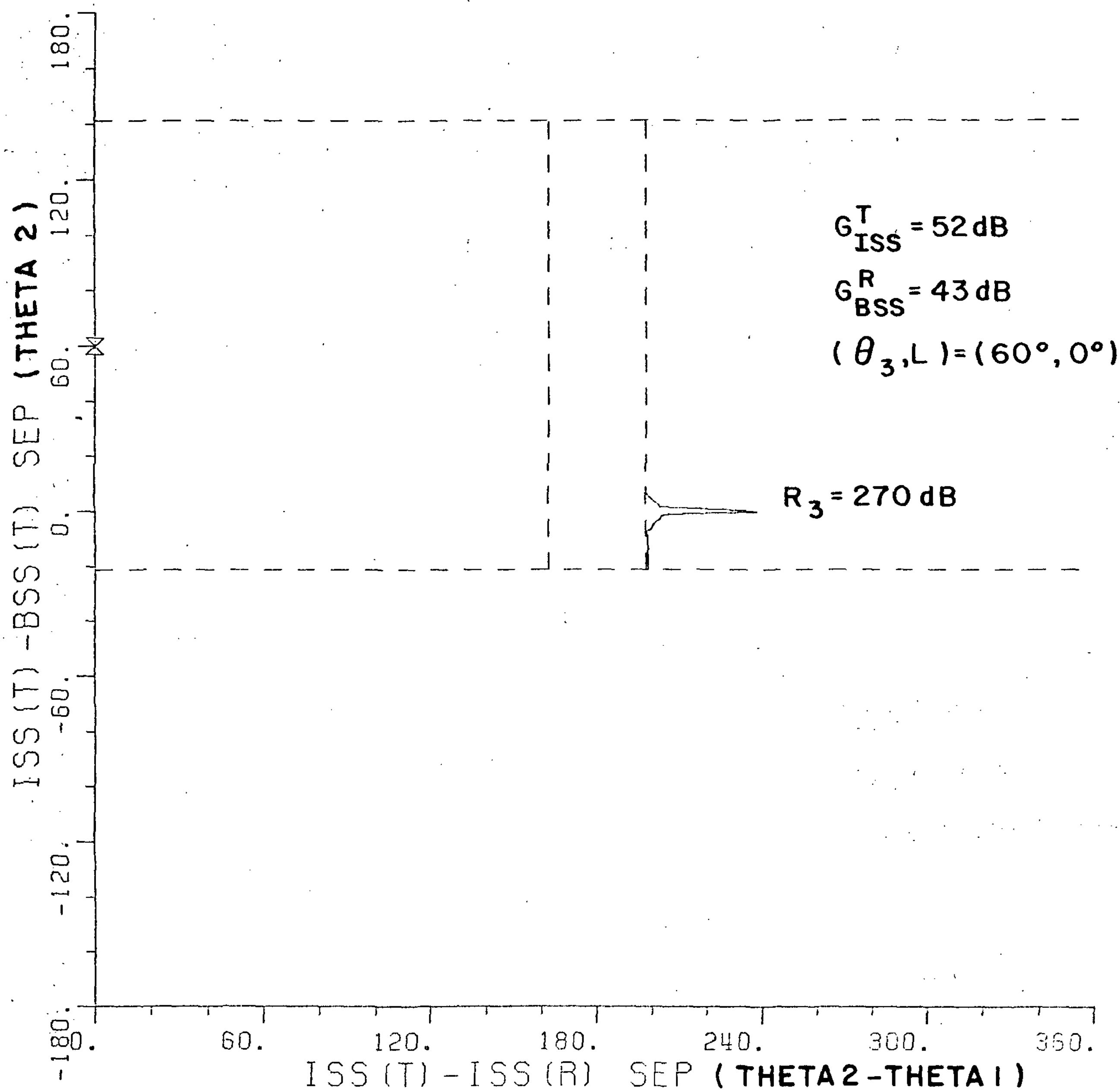


Figure 18. Sample solution of interference to a BSS earth station by an ISS system, using the unrestricted computer code EING [17]. I/N is used as criterion. The small "spike" defines the prohibited region. The narrow vertical region between dashed lines defines geometries which are unrealistic because the ISS system is obstructed by the Earth. For regions above the top and below the bottom horizontal dashed lines interference cannot occur because the BSS earth station cannot "see" the ISS transmitter.

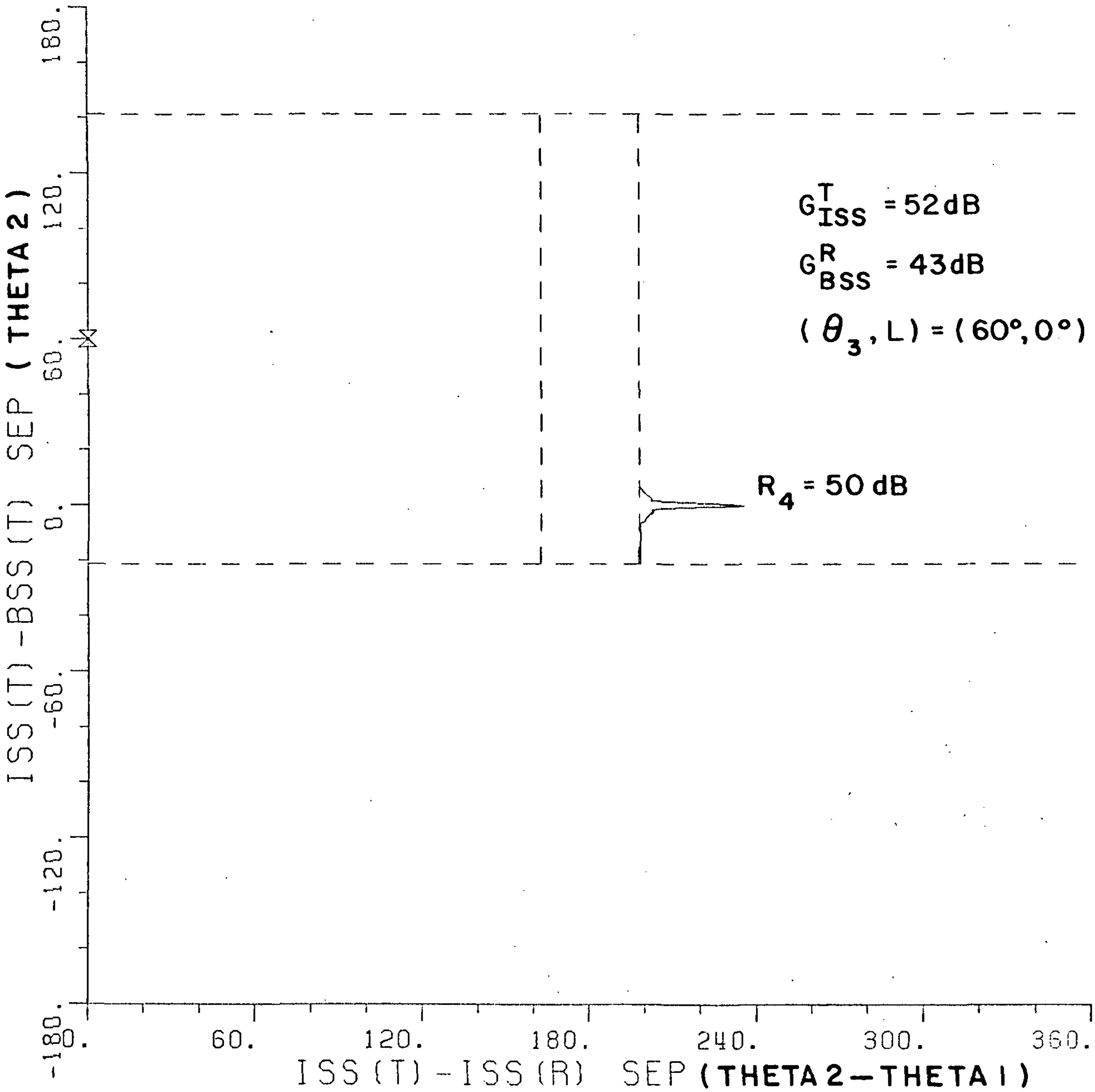


Figure 19. Sample solution of interference to a BSS earth station by an ISS system, using the unrestricted computer code ECIG [17]. C/I is used as criterion. See Figure 18 for details of the presentation.

the BSS transmitter would be below the horizon as seen from the Earth station.

If the systems parameters for the typical system of Table 1 had been used for these calculations, no unacceptable region would have been shown in the graphs. To obtain the graphs, R_3 was lowered by 23 dB and R_4 was raised by 28 dB from their Table 1 values; this is equivalent to requiring I/N of -33 dB and C/I of 58 dB instead of the conventional -10 dB, +30 dB values, with other parameters unchanged. Even so, the region of unacceptable interference lies in the quasi-pathological region; if satellite paths over 120° in central angle length were assigned to the unshared portion of the spectrum, the unacceptable regions in these graphs would be covered by these exclusions.

It should be noted that when ψ_3 and ψ_4 are both small in Equations (12) and (13), ψ_1 and ψ_2 will also be small (see Section II, above), therefore a pathological geometry from the point of view of interference to the BSS system is also pathological with respect to the ISS system. From these calculations it appears that the interference to the ISS system is the more restrictive constraint, so that for a broad range of system parameters interference to the BSS system will automatically be within allowed bounds if pathological geometries are excluded and interference to the ISS system is kept within allowed bounds. Defining quantitatively the range of system variables for which this statement is true remains as a task still to be completed.

When either ψ_3 or ψ_4 is sufficiently large so that the interference involves only the far side-lobe region of the Earth station or the

ISS(TX) antenna, the problem can again be reduced to one amenable to universal curve presentation. Computer codes for these cases have been written and universal curves plots constructed [18], but the R_3 and R_4 values had to be degraded even more unrealistically from the typical values of Table 1 before any unacceptable region appeared.

F. CONCLUSIONS

The most serious interference between proposed ISS and BSS systems near 23 GHz arises when the interference is transmitted from the main lobe or near sidelobes of the interfering transmitter and is received through the main lobe or near side lobes of the receiver. In this report this situation is termed "pathological". It can occur for both interference to the BSS system and to the ISS system. A necessary condition for this situation is a long ISS path. It is therefore recommended that long ISS paths (e.g., those subtending an equatorial central angle greater than 120°) be restricted to the 23.00-23.55 GHz range, which is not frequency-shared with the BSS.

When the very long ISS paths are excluded from frequency-sharing, simple sets of universal curves can be constructed to define acceptable satellite and Earth station locations, based on either I/N or C/I as the acceptability criterion. To use these curves, the designer performs a simple multiplication of various system parameters to find a universal factor; contours of the universal factor then allow the range of acceptable locations to be read directly from the charts.

In principle, one set of charts is required to define interference to the ISS system, and a second set is required for interference to the BSS system; the acceptable region is then the intersection of the individually acceptable regions. In practice, interference to the ISS system turns out to be the more restrictive condition for a wide range of reasonable system parameters. In this case only the ISS charts need be consulted. The precise definition of the parameter range for which this statement is true is yet to be determined, but it appears to include most, if not all, practical systems.

Programs have been written which allow a proposed solution to be tested by calculating the resulting interference criterion (I/N or C/I) explicitly. A proposed design procedure would include use of the charts to select a suitable geometry and then to verify it by the direct calculation of the resulting interference.

III. BROADCASTING-SATELLITE SERVICE SPECTRUM/ORBIT ASSIGNMENT SYNTHESIS

A. INTRODUCTION

The goal of broadcasting-satellite service assignment synthesis is to specify for each service area under consideration a set of channel, polarization, and orbital slot assignments in a manner which minimizes the amount of bandwidth required. Implicit in the problem is the requirement that a specified number of channels be available to each service area and that these channels be protected from interference. Protection ratios of 35 dB single-entry co-channel, 19 dB single-entry adjacent channel, and 30 dB aggregate are typical. Other factors, such as eclipse protection and minimum elevation angle, limit the flexibility of solutions.

The BSS assignment synthesis problem is a discrete-continuous nonlinear optimization problem. The discrete design variables are the choice of channel assignments and polarizations; the continuous design variables are the orbital slot assignments. The problem is nonlinear in several respects, e.g., antenna patterns, angular calculations, and pre-detection interference-to-signal ratios. Finally, it is an optimization problem in the sense that the objective is to minimize the required bandwidth, subject to design restrictions such as eclipse protection, etc.

Mathematically, this problem is extremely difficult if the stated objective is to find the optimum solution and supply proof of its optimality. Optimization theory can at most give an indication that a solution is a local optimum with respect to the continuous design variables. Some form of enumeration would be required to provide similar statements including the discrete design variables. Simpler but related combinatorial optimization problems that result from very strong assumptions are (in the language of computational complexity theory) NP-complete. This means they are among the hardest problems known.

However, this pessimistic assessment refers to the possibility of obtaining a proven optimum. This is, of course, not necessary from a practical point of view. Indeed, the objective of this research is to develop methods that will give good (or acceptable) system synthesis with a high degree of reliability and a minimum of computational expense. In this regard, efforts have been concentrated in four areas that seem to hold the keys to a successful approach. They are the following:

1. Incorporation of the SOUP program,
2. Development of user insights,
3. Exact algorithms,
4. Heuristic procedures.

An illustration of how these four elements might be arranged in a synthesis process is given in Figure 20. It should be noted that we are still in the early stages of research in this area, and the approach may need to be changed as we progress.

B. INCORPORATION OF THE SOUP PROGRAM

The initial effort in this research program was to assess the appropriate role for the SOUP program in a synthesis method. It was assumed from the outset that SOUP would be the final judge as to the acceptability of a synthesis plan. However, it was not clear that an iterative method could afford to make extensive use of the entire SOUP calculation process due to limits on computation time. Therefore, after installing SOUP on our VAX 11/780 system, timing tests were run with 90 test points. Run times on the order of 10 seconds confirmed our suspicions that any practical synthesis method could make only limited use of SOUP.

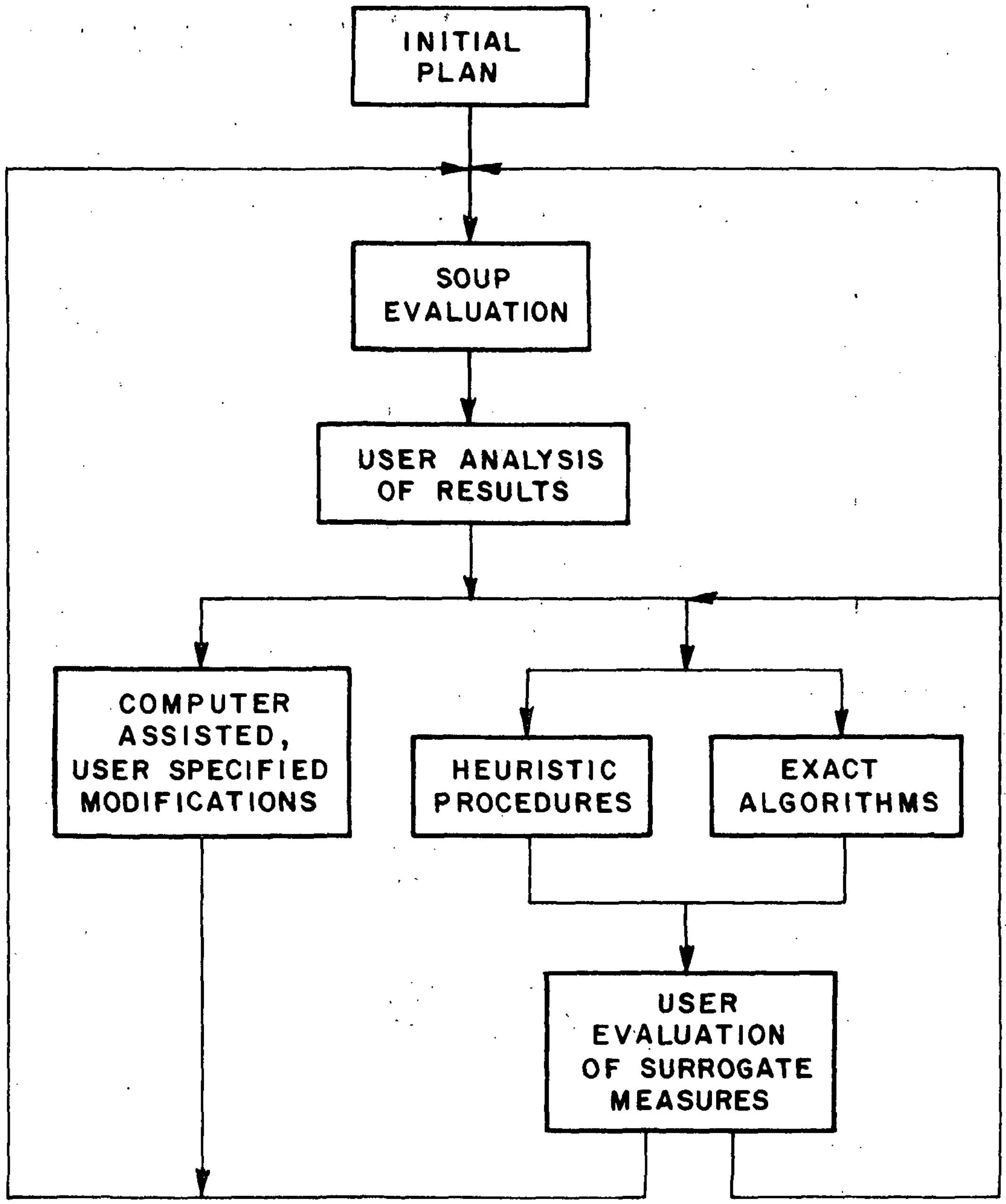


Figure 20. Synthesis process flow chart.

Attention then turned to examining the logical structure of the SOUP calculations in order to determine what subset could be included in a "stripped down" version to be employed to provide surrogate measures of the quality of the synthesis plan. This surrogate measure would be used to direct the iterative search procedures and therefore must be implemented as efficiently as possible. The heart of the SOUP calculation was found to be the calculation of certain antenna pointing separation angles. It proved possible to streamline the calculation of these angles significantly compared to their calculation in SOUP.

For example, Figure 21 on the following page illustrates the calculation of the angle α between the vector from the satellite to the main (i.e., intended) receiver (M) and the vector from the satellite to a receiver suffering interference (I). If we let ϕ refer to the satellite longitude, ϕ_I and θ_I the longitude and latitude of I, ϕ_M and θ_M the longitude and latitude of M, and the orbital radius R = 6.6134 earth radii then the modified calculation of α is as follows:

$$\|e\| = [2\{1-\cos(\phi_M-\phi_I)\cos(\theta_M)\cos(\theta_I) - \sin(\theta_M)\sin(\theta_I)\}]^{\frac{1}{2}} \quad (14)$$

$$\|a\| = [R^2 + 1 - 2R\cos(\theta_M)\cos(\phi_M - \phi)]^{\frac{1}{2}} \quad (15)$$

$$\|b\| = [R^2 + 1 - 2R\cos(\theta_I)\cos(\phi_I - \phi)]^{\frac{1}{2}} \quad (16)$$

$$s = (\|e\| + \|a\| + \|b\|)/2 \quad (17)$$

$$\alpha = 2\arccos[\{s(s-\|e\|)/(\|a\|\ \|b\|)\}^{\frac{1}{2}}] \quad . \quad (18)$$

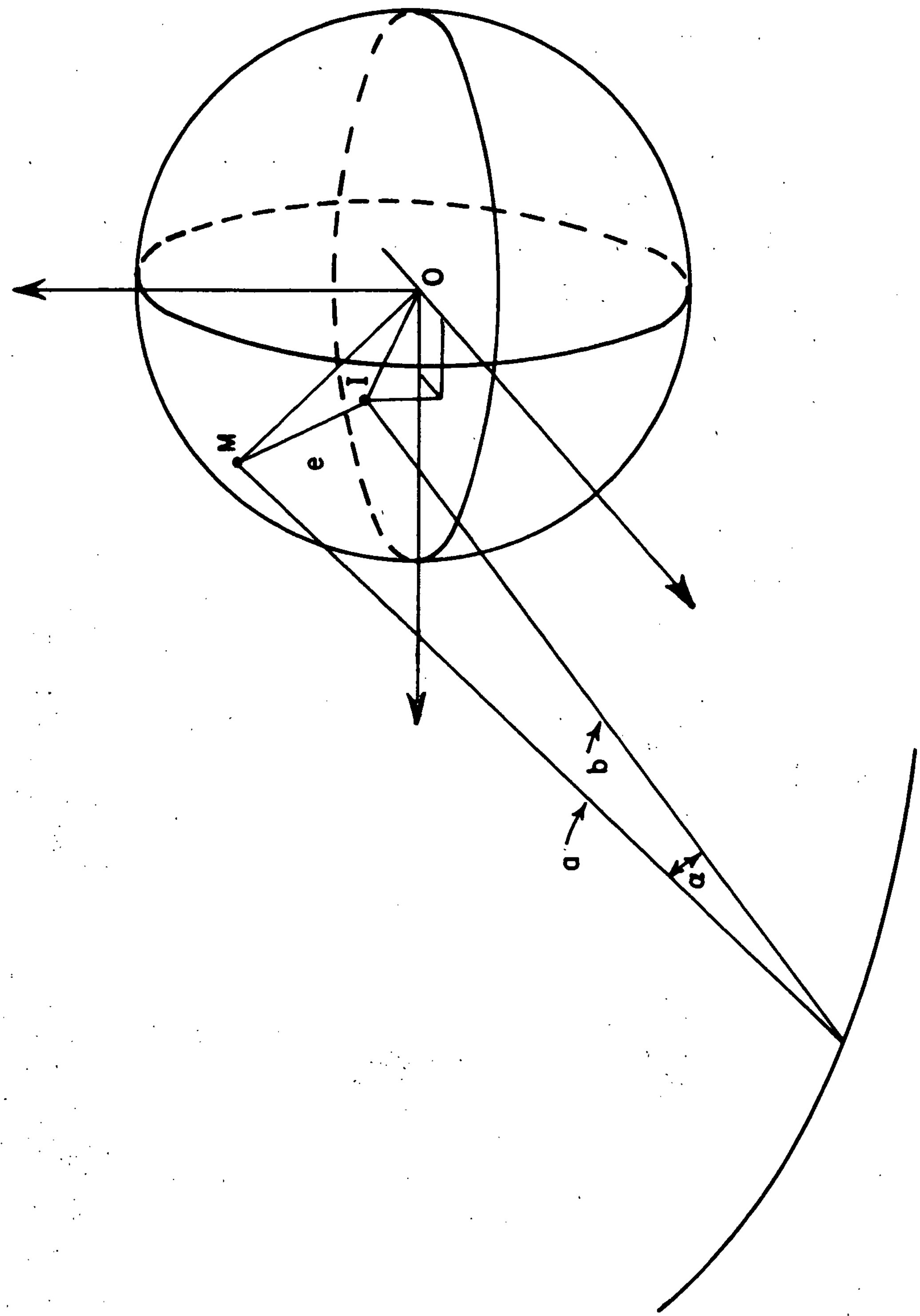


Figure 21. Illustration of angle calculation.

This calculation requires 14 function evaluations and 27 arithmetic operations as compared with 18 and 39, respectively, in the SOUP equivalent. Furthermore, it is possible to reduce these calculations even more by storing a few intermediate results, such as $\|e\|$, which do not change with satellite position. It should also be mentioned that this calculation deals with distances in multiples of earth radii while SOUP uses kilometers, which introduces a noticeable numerical error.

Another aspect of the implementation of the SOUP program at OSU that required attention was discrepancies in the outputs for runs made with identical data at NASA Lewis Research Center and OSU. The differences showed up in the margin calculations with differences up to 0.5 dB. In an effort to pinpoint the source of the error, runs were made on the VAX in both single-precision and double-precision modes and on the Amdahl 470 of the Instruction and Research Computing Center of OSU. None of the four sets of runs (NASA, VAX-single precision, VAX-double precision, and Amdahl) were in complete agreement. The tentative conclusion was reached that the differences are the result of trigonometric calculations with small angles.

C. DEVELOPMENT OF USER INSIGHTS

The second area of effort in this project was the development of user insights, i.e., a base of knowledge developed through experience with potential system designs. The rationale for this effort was twofold. First, to be able to design an iterative procedure for this problem, one must possess some feel for how the system reacts to design variables, with respect to both direction and rate of change. Second, due to the complexity of the problem it may not be possible to develop a method that can proceed with the synthesis of an acceptable plan without iteration with a user possessing substantial insight into the problem. For example, it is hard to imagine how a classical optimization approach could uncover the potential benefit of selective use of crossed-path geometry without user intervention.

One of the approaches we have taken to develop this basis of experience is to implement a computer-assisted version of a manual synthesis method which is currently in use at NASA Lewis Research Center. At its present level of implementation, our program attempts to specify an assignment of satellite orbit locations that satisfies the discrimination criteria for every pair of service areas. The user inputs satellite orbit positions and the program calculates the discrimination between all pairs and provides this information in the form of a discrimination matrix. The user can then make changes in orbit positions and improve the matrix. Pairwise comparison is also provided as an option.

The program does not, and is not expected to, provide optimum orbit assignments. However, it is intended to provide good starting solutions. It has already demonstrated the capability to provide important insights, such as highly unacceptable assignments and cliques of areas with little or no interaction. It also has provided a basis for testing the sensitivity of approximations in the angle calculations, which consume most of the computation time.

An anticipated extension to the program is to include frequency assignments and polarizations and to reduce the demands on user input.

D. EXACT ALGORITHMS

1. General Considerations

Although it is clear that no standard optimization algorithm can be applied directly to this problem in its entirety, standard techniques may have a role to play when some of the design variables have been fixed at trial values and the others are to be optimized. For example, suppose that each service area has been assigned a set of channels and polarizations and that now it is desired to determine the corresponding orbital slots that minimize total system interference.

This problem can be attacked by classical gradient-search methods for continuous variable problems. This is, of course, precisely the problem form encountered under block allocation schemes.

There is, however, a potentially large computational burden associated with this approach. A gradient-search algorithm requires estimates of the partial derivations of the objective function with respect to each of the design variables, in this case the orbit locations for each service area. Obtaining these estimates will require a number of system interference calculations equal to the number of service areas. Furthermore, due to the strong interactions that may be anticipated between the variables, modified gradient procedures may be required to give reasonable rates of convergence. These will require an estimate of the Hessian matrix, i.e., the matrix of second partial derivatives, and the computations associated with this grow as the square of the number of service areas. Clearly, the use of SOUP with up to 90 service areas is out of the question since these calculations must be repeated for every trial solution.

A promising approach to overcoming the computational cost of a full-blown gradient search procedure is based on two considerations. First, it is reasonable to expect that most of the system interference will be isolated in relatively few, perhaps 7 to 12, of the service areas. Therefore, only the orbit positions of these services areas and those of the service areas causing the interference need to be included in the optimization. Second, this is another example where a surrogate measure of interference, one that can be calculated much more quickly than an exact calculation in SOUP, can be useful.

As another example of the potential role of exact algorithms there are aspects of the overall problem that lead to combinatorial optimization problems. While it has not been determined precisely how the solutions of these problems fit into the total synthesis of the satellite communication system, it is clear that they may play on important part.

Consider the situation in which each pair of regions has been specified as either interfering or non-interfering with respect to co-channel assignment. A pair of service areas are co-channel interferers if the elliptical beam assigned to one service area reaches the other without sufficient angular antenna discrimination. This may be true in both directions, but need not be bidirectional for the pair to be co-channel interferers. Clearly, service areas sharing a common border will be interferers, but geographic contiguity is not a necessary condition.

We limit our discussion initially to the case of assigning a single channel to each service area so as to avoid co-channel interference. However, this is not as limited a scenario as one might suppose at first. The case of assigning a fixed number of channels to each service area does not change the nature of the problem since this may be viewed as assigning groups consisting of this fixed number of adjacent channels so as to avoid co-channel interference between groups. The consideration of adjacent-channel interference may be handled by selecting the channels for each group so that adjacent channel interference does not occur within a group. For example, group 1 could consist of channels 1,3,5,7 and group 2 of channels 9,11,13,15 in the case of four channels per service area. Then groups 1 and 2 could be assigned to a pair of interfering service areas without causing adjacent-channel interference.

The problem under discussion may be formulated as a graph-coloring in which the nodes of the graph represent service areas and a pair of nodes are connected by an arc if the corresponding pair of service areas are interferers. We begin by describing a set-covering approach suggested by Cameron [19] and then suggest alternatives.

2. Cameron's Approach

Cameron considered the problem of assigning a single channel to each service area so as to minimize the number of channels required [19]. The restriction on the assignment pattern was that no pair of interfering service areas could be assigned the same channel, in order to prevent co-channel interference. He suggested solving a sequence of minimum-cardinality set-covering problems to determine the minimum number of channels required and the corresponding channel assignment pattern.

Cameron observed that the problem could be formulated as a graph-coloring problem. However, since service areas need not be geographically contiguous to interfere, the graph need not be planar and hence the famous four-color theorem does not apply. Graph coloring is a notoriously hard combinatorial problem (even for planar graphs). In fact, finding a coloring that requires no more than twice the minimum number of colors is among the hardest combinatorial optimization problems.

Although the minimum-cardinality set-covering problem used in Cameron's approach is also a very difficult combinatorial optimization problem, practical experience with problems of this type has been relatively good and several computer codes exist with promise of solving problems of the size encountered in the current context.

To describe Cameron's formulation (which was not expressed mathematically), suppose we have a set of n service areas. We propose using p channels for assignment to the service areas. We wish to determine 1) is p a sufficient number of channels to provide a feasible assignment? and 2) if so, what is the assignment pattern?

We introduce E which is defined as the set of all interfering pairs of service areas, i.e.,

$$E = \{(q,r) \mid \text{service areas } q \text{ and } r \text{ interfere}\}. \tag{19}$$

Now define the following two sets of decision variables

$$x_{ij} = \begin{cases} 1 \text{ if channel } i \text{ \underline{is not used} for service area } j \\ 0 \text{ otherwise} \end{cases} \tag{20}$$

$$y_{ij} = \begin{cases} 1 \text{ if channel } i \text{ \underline{is used} for service area } j \\ 0 \text{ otherwise} \end{cases} \tag{21}$$

for i=1, ..., p; j=1, ..., n. The channel assignment set-covering problem for p channels is then denoted CASC(P):

$$\text{minimize} \quad \sum_{i=1}^{p} \sum_{j=1}^{n} x_{ij} + \sum_{i=1}^{p} \sum_{j=1}^{n} y_{ij} \tag{22}$$

$$\text{subject to} \quad \sum_{i=1}^{p} y_{ij} \geq 1, \quad j = 1, \ldots, n \ , \tag{23}$$

$$x_{iq} + x_{ir} \geq 1, \; i = 1, \ldots, p; \; (q,r) \varepsilon E \ , \tag{24}$$

$$x_{ij} + y_{ij} \geq 1, \; i = 1, \ldots, p; \; j=1, \ldots, n \ , \tag{25}$$

$$x_{ij}, y_{ij} = 0 \text{ or } 1. \tag{26}$$

The first set of constraints insures that each service area j has at least one channel assigned to it as defined by the y variables; the objective function will insure that the optimum solution will have exactly one channel assignment. The second set of constraints insures that, for each channel, any potentially interfering pairs of service areas must not assign that frequency to at least one of the two regions as defined by the x variables. The final set of constraints requires that for each (i,j) either the corresponding y variable or the corresponding x variable or both equal 1. Of course, if both equal one, they contradict each other and this is the essence of the determination of the sufficiency of p.

Clearly from the second set of constraints, the objective function is at least np. If a solution to CASC(P) can be found with an objective function value equal to np, this demonstrates that p channels are sufficient since for any (i,j)

$$x_{ij} + y_{ij} = 1. \qquad (27)$$

Thus the x and y variables are then consistent and the y variables specify a feasible channel assignment. However, if the optimum value of the objective function exceeds np then all feasible solutions have

$$x_{ij} = y_{ij} = 1 \qquad (28)$$

for at least one (i,j) and hence all solutions are contradictory. In such a case, one concludes p channels are not sufficient.

To determine the minimum number of channels required amounts to determining the smallest value of p for which CASC(P) has an optimum objective function value of np. Although the relative computability of set-covering problems is an attractive feature, there are some drawbacks to this approach. These are the following:

1. Several problems (for different values of p) may have to be solved.
2. The problem is somewhat larger in the number of variables and in the number of constraints than in other possible formulations.
3. In practice, many codes stop short of finding an optimum solution but can deliver good approximately optimal solutions. However, they are of no use in this approach.

3. <u>0-1 Programming Formulation</u>

Assume as before that there are n service areas. Let m be a number of channels known to be sufficient. Trivially m=n will work; however, smaller values may be obtained by heuristically generating a reasonable (although probably not optimal) channel assignment. For any region j define C_j as the set of service areas that interfere with service area j and decision variables

$$x_i = \begin{cases} 1 \text{ if channel } i \text{ is used} \\ 0 \text{ otherwise} \end{cases} , \qquad (29)$$

$$y_{ij} = \begin{cases} 1 \text{ if channel } i \text{ is assigned to service area } j \\ 0 \text{ otherwise} \end{cases} \quad . \tag{30}$$

The problem can now be stated as

$$\text{minimize} \quad \sum_{i=1}^{m} x_i \tag{31}$$

$$\text{subject to} \quad \sum_{i=1}^{m} y_{ij} = 1, \quad j = 1, \ldots, n \quad , \tag{32}$$

$$\sum_{j=1}^{m} y_{ij} - nx_i \leq 0, \quad i = 1, \ldots, m \quad , \tag{33}$$

$$\sum_{K \varepsilon C_j} y_{ik} + |C_j|\, y_{ij} \leq |C_j|, \quad i=1, \ldots, m;\ j=1, \ldots, n, \tag{34}$$

where

$$x_i,\ y_{ij} = 0 \text{ or } 1 \quad , \tag{35}$$

and $|C_j|$ represents the number of service areas in the set C_j.

The first set of constraints insures that for each service area exactly one channel is assigned. The second set of constraints insures that for each channel, if that channel is assigned to one or more regions, this fact is reflected by the corresponding x variable being equal to one. Finally, the last set of constraints insures, for each channel and service area, that assigning the channel to a given service area eliminates the possibility of assigning it also to any interfering service areas. The objective function minimizes the number of channels used.

Note that any solution to this problem, whether optimal or not, yields a feasible assignment of channels (unlike the Cameron formulation where for some p even optimal solutions may not yield a feasible assignment). Hence, approximately optimal solutions may be found, and these can be quite useful. Also observe the following comparison of problem size given in Table 2, where $|E|$ is the number of pairs of interfering regions. Since m and p are essentially equivalent and $|E|$ is potentially very large, the 0-1 program is considerably smaller then Cameron's.

Table 2

Problem Size Comparison

	Cameron	0-1 program
number of variables	2 np	nm + m
number of constraints	$np + n + p\|E\|$	mn + m + n

4. An Alternative Set-Covering Formulation

Consider again a problem with n service areas. Generate a set of subsets $P_1, \ldots, P_q$ of these areas such that within any given subset no pair of service areas interfere. Insure that every region is contained in one or more of the subsets. Then define the parameters

$$a_{ij} = \begin{cases} 1 \text{ if service area j is in the subset } P_j \\ 0 \text{ otherwise} \end{cases} \qquad (36)$$

and the decision variable

$$x_i = \begin{cases} 1 \text{ if subset } P_i \text{ is included in the cover} \\ 0 \text{ otherwise} \end{cases} \quad . \tag{37}$$

Consider now a solution to the following set-covering problem:

$$\text{minimize} \quad \sum_{i=1}^{q} x_i \tag{38}$$

$$\text{subject to} \quad \sum_{i=1}^{q} a_{ij}\, x_i \geq 1, \quad j = 1, \ldots, n \quad , \tag{39}$$

where

$$x_i = 0 \text{ or } 1 \quad . \tag{40}$$

The solution selects a minimum number of subsets such that every service area is contained in at least one subset. This provides a channel assignment pattern in the sense that every subset selected for the cover is assigned its own channel. Service areas belonging to more than one subsets selected in the cover may be assigned a channel chosen arbitrarily from the several subsets indicated.

The quality of the channel assignment (i.e., the number of channels used) depends, of course, on the selection of the set of subsets. If, however, the subsets are chosen as maximal-cardinality non-interfering subsets (i.e., no pair of service areas in the subset interfere and no further service areas can be added without destroying this property) and all such subsets are selected, the channel assignment given by the solution to the set-covering problem is guaranteed to use the minimum number of channels.

Unfortunately, the problem of determining all maximal-cardinality non-interfering subsets (maximal cardinality independent sets is the graph theory equivalent) is itself a difficult combinational problem. In practice, one would use a heuristic to generate a reasonable set of subsets in the hope of finding good and possibly optimum channel assignments.

5. Multiple Channel Adjacent Frequency Model

Although we believe that the previously considered single-channel, (co-channel) interference model addresses the multiple-channel adjacent-channel interference cases indirectly, we consider now a 0-1 model for addressing these issues directly. To this end we define C_j - the set of service areas that are co-channel interferers to j, and C'_j - the set of service areas that are adjacent-channel interferers of j.

For simplicty of notation, assume that the service areas have been ordered so that the first $n'<n$ service areas possess adjacent channel interferers. Also let r_j be the number of channels to be assigned to service area j. Using the same set of variables as in the previous 0-1 programming formulation the model is

$$\text{minimize} \quad \sum_{i=1}^{m} 2^i x_i \tag{41}$$

$$\text{subject to} \quad \sum_{i=1}^{m} y_{ij} = r_j, \quad j = 1, \ldots, n \;, \tag{42}$$

$$\sum_{j=1}^{n} y_{ij} - nx_i < 0, \quad i = 1, \ldots, m \ , \tag{43}$$

$$\sum_{K \epsilon C_j} y_{ik} + |C_j| \ y_{ij} < |C_j|, \quad i=1, \ldots, m; \ j=1, \ldots, n, \tag{44}$$

$$\sum_{K \epsilon C'_j} (y_{i-1,j} + y_{i+1,j}) + 2|C'_j| y_{ij} \leq 2|C'_j|$$

$$i = 1, \ldots, m; \ j = 1, \ldots, n'; \tag{45}$$

where

$$x_i, \ y_{ij} = 0, 1 \ . \tag{46}$$

The differences between this 0-1 model and the previous model are a more complex objective function and an additional set of constraints. The change in the objective function is required to insure that bandwidth, and not simply the number of channels used, is minimized. This was not necessary in the absence of adjacent-channel restrictions since whichever channels were used could be reordered to eliminate unused channels from the required bandwidth. This, of course, is generally not possible with adjacent-channel restrictions. Consequently the objective must insure explicitly that bandwidth is minimized. The additional set of constraints is needed to insure adjacent-channel restrictions are observed.

It must be noted that these models are likely to be difficult to solve for a proven optimum solution for a 90 service area and 40 channel problem. However, obtaining good solutions may be relatively easy, especially when only a subset of the service areas are considered, e.g., only service areas with unresolved interference problems. Our own computational experience and that of Baybars [20] confirms this.

E. HEURISTIC PROCEDURES

The review of three Canadian papers was an important part of the process of attempting to conceptualize the features of heuristic approaches likely to provide a good starting solution.

Chovinard and Vachon [21] present a method based on exhaustive (but implicit) enumeration of all possible channel assignments and polarizations to service areas, given a preassignment of orbital position to service areas. This brute-force approach seems unlikely to be of much help in realistically sized problems and does not offer any insights into the problem. Nedzela and Sidney [22] offer an approach that is a heuristic based on matrices indicating the freedom of choice for the service areas remaining to be considered later when assigning a channel, polarization, and orbital slot for a service area under consideration currently. By making the current assignment on the basis of maximizing the resulting freedom for later choices, a sequence of assignments is made that either results in a successful plan based on a prespecified number of channels or a procedure failure. At each step, the selection of service area to be considered next is made by choosing the one with minimum remaining freedom. Christensen's procedure [23] is an interactive system that offers the user a menu of routines, some of which are automatic and some of which require manual input, to aid in synthesizing a plan. Both this paper and the Nedzela and Sidney paper offer some well-conceived heuristic approaches which are part of our current thinking on the subject.

The drawbacks of the Canadian procedures are twofold: 1) some of the approximations that have been made may not be valid. For example, in the method of Nedzela and Sidney, assignments based on the minimum freedom matrix may not be feasible when tested by SOUP and, of course, there is no guarantee of optimality; and 2) a combination of all three methods was proposed with a supervisory iterative process. However, there seem to be certain difficulties encountered in trying to combine the methods and, so far, integration of the three procedures has not been achieved. Consequently, though these procedures are certainly useful, they fall short of solving the assignment synthesis problem.

IV. CONCLUSIONS AND RECOMMENDATIONS

The problem of allocating satellite orbital locations to the Broadcasting-Satellite Service and the Inter-satellite Service near 23 GHz has been solved with respect to single-entry interference, subject to the constraint that very long inter-satellite links (e.g., those separated by more than 120° of equatorial arc) be allocated frequencies in the unshared portion of the band. The allocation procedure involves the use of universal charts. To use these charts, the designer calculates a universal factor by simple multiplication of certain system parameters; acceptable geometries are then found from contours of that parameter value on charts whose coordinates are satellite separation angles. Programs for checking the validity of the resulting assignments have also been prepared.

It is recommended that an atlas of such charts be prepared and published, together with instructions for its use. Certain minor points remain still to be resolved, e.g., whether the assignment on the basis of interference to the ISS always assures protection also to the BSS for practical system parameter values, or whether two sets of charts may be required in certain cases.

Considerable progress has been made in formalizing the concepts of broadcasting-satellite service assignment synthesis. An important feature of our suggested approach is the combination of exact algorithms, heuristic procedures and user insights. We are confident that the formulations and solution procedures described in this report are effective methods for the corresponding subproblems of the overall system synthesis problem. However, the most challenging part of the problem remains to be formally addressed. This is the design and testing of a software package that combines and effectively interfaces the several subprocedures already developed. This effort is ready to be initiated. In the process, additional procedures and approaches may emerge.

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APPENDIX

EQUATIONS FOR CALCULATING DISTANCES
AND ANTENNA ANGLES FROM CENTRAL ANGLES

The equations below may be used to calculate the distances and angles shown in Figure 2. The symbols β and r denote the radius of the geostationary orbit and of the earth, respectively.

$$u^2 = \beta^2 + r^2 - 2r\beta \cos\ell \cos(\theta_3-\theta_1) \quad (A.1)$$

$$v^2 = \beta^2 + r^2 - 2r\beta \cos\ell \cos\theta_3 \quad (A.2)$$

$$w^2 = \beta^2 + r^2 - 2r\beta \cos\ell \cos(\theta_3-\theta_2) \quad (A.3)$$

$$x = 2\beta \sin \left|\frac{\theta_1}{2}\right| \quad (A.4)$$

$$y = 2\beta \sin \left|\frac{\theta_2}{2}\right| \quad (A.5)$$

$$z = 2\beta \sin \left|\frac{\theta_2-\theta_1}{2}\right| \quad (A.6)$$

$$\psi_1 = \cos^{-1}[(x^2+z^2-y^2)/(2xz)] \quad (A.7)$$

$$\psi_2 = \cos^{-1}[(x^2+v^2-u^2)/(2xv)] \quad (A.8)$$

$$\psi_3 = \cos^{-1}[(w^2+z^2-u^2)/(2wz)] \quad (A.9)$$

$$\psi_4 = \cos^{-1}[(w^2+v^2-y^2)/(2wv)] \quad (A.10)$$

$$\psi_1 = \frac{\theta_2}{2}, \quad \theta_1 > \theta_2 \quad (A.11)$$

$$\psi_1 = 180° - \frac{\theta_2}{2}, \quad \theta_1 < \theta_2 \quad (A.12)$$

Note that in Equations (A.11) and (A.12) the ranges of θ_1, θ_2 are from 0° to 360°.