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SIMULATION OF LARGE TURBULENT VORTEX STRUCTURES WITH THE PARABOLIC NAVIER-STOKES EQUATIONS

J. V. Rakich,[1] R. T. Davis,[2] and M. Barnett[3]

ABSTRACT

The theoretical basis for well-posed marching of a Parabolic Navier-Stokes (PNS) computational technique for supersonic flow is discussed and examples given to verify the analysis. It is demonstrated that stable computations can be made even with very small steps in the marching direction. The method is applied to cones at large angles of attack in high Reynolds-number, supersonic flow. Streamline trajectories generated from the numerical solutions demonstrate the development of vortex structures on the lee side of the cone.

INTRODUCTION

The computation of steady supersonic viscous flows with one set of equations, valid in both the boundary layer and the inviscid supersonic region, has long been attempted to avoid the difficulties with matching conditions for separate inviscid and boundary-layer equations. Use of the unsteady form of the NS equations has been successful but inefficient, in terms of computer time and memory requirements. Therefore, many authors [1-8] have used steady marching or iterative methods to solve numerically the time-invariant NS equations. Single-pass methods have been used for supersonic flows, where the inviscid region is a well-posed initial-value problem, and where the viscous region is a boundary layer, which is itself amenable to a marching solution. The difficulty with solving the complete inviscid/viscous domain at once lies in the thin subsonic layer near the wall, where the pressure is determined by the solution. The absence of a downstream boundary condition makes the problem ill-posed. This combined problem is basically an interaction of a boundary layer with an inviscid, supersonic stream. Lighthill [9] analyzed this problem and found solutions for the boundary-layer displacement thickness of the form $\delta = a\exp(kx)$. One can interpret this exponential growth as the onset of streamwise expansion or compression, which may lead to separation; these solutions have become known as "departure" solutions corresponding to some undetermined downstream boundary condition. This paper describes an approach for avoiding the exponentially growing solution, and presents results for turbulent flow over a cone at large angles of attack.

THE PNS EQUATIONS

The governing equations are written in general curvilinear conservation-law form as follows [8]:

$$\frac{\partial}{\partial\xi}\{J^{-1}[\xi_x E^* + \xi_y F^* + \xi_z G^*]\} + \frac{\partial}{\partial\eta}\{J^{-1}[\eta_x(E - E_v) + \eta_y(F - F_v) + \eta_z(G - G_z)]\}$$
$$+ \frac{\partial}{\partial\zeta}\{J^{-1}[\zeta_x(E - E_v) + \zeta_y(F - F_v) + \zeta_z(G - G_v)]\} = \frac{\partial}{\partial\xi}\{-J^{-1}[\xi_x P_1 + \xi_y P_2 + \xi_z P_3]\} \quad . \qquad (1)$$

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Here J is the Jacobian of the coordinate transormation; E, F, and G are the inviscid flux vectors; and F_v and G_v are the viscous flux vectors--all written in terms of Cartesian velocity components. Note that the viscous terms are omitted from the ξ derivative term; this is the fundamental parabolizing approximation. The ( )* terms are central to the discussion of parabolic marching, and are given by the column vectors

$$\left.\begin{aligned} E^* &= \{\rho u,\ \rho u^2 + \omega p,\ \rho uv,\ \rho uw,\ (\rho e_t + p)u\}^T \ , \\ F^* &= \{\rho v,\ \rho uv,\ \rho v^2 + \omega p,\ \rho vw,\ (\rho e_t + p)v\}^T \ , \\ G^* &= \{\rho w,\ \rho uw,\ \rho vw,\ \rho w^2 + \omega p\ ,\ (\rho e_t + p)w\}^T \ , \end{aligned}\right\} \tag{2}$$

where

$$\rho e_t = p/(\gamma - 1) + 0.5\rho(u^2 + v^2 + w^2) \ .$$

Here u, v, and w are Cartesian velocity components, p is the pressure, ρ the density, and γ the ratio of specific heats. The P terms on the right side are given by

$$\left.\begin{aligned} P_1 &= \{0,\ (1 - \omega)p,\ 0,\ 0,\ 0\}^T \ , \\ P_2 &= \{0,\ 0,\ (1 - \omega)p,\ 0,\ 0\}^T \ , \\ P_3 &= \{0,\ 0,\ 0,\ (1 - \omega)p,\ 0\}^T \ . \end{aligned}\right\} \tag{3}$$

Following Vigneron et al. [4], the parameter ω is included with the pressure term in the momentum equations. It was shown that the equations can be made formally parabolic by utilizing an appropriate functional form for ω. Considering the two-dimensional viscous subset of equations (1), the mathematical character of the equations is governed by the eigenvalues of the differential system, leading to the condition

$$\omega \leq \gamma M_x^{\ 2}/[1 + (\gamma - 1)M_x^{\ 2}] \ . \tag{4}$$

If this condition is satisfied, then the equations remain parabolic even in the viscous region near the wall where the Mach number is less than unity; a similar form also governs the inviscid region, but is not pertinent to the present work. The purpose of this paper is to further demonstrate the effectiveness of this approach for interacting supersonic flows where the problem can be solved in a single sweep of the PNS equations.

In previous work with the PNS method, a "sublayer" approach was used, in which the pressure gradient term was completely removed in the subsonic region of flow. This is equivalent to a step function for ω, and this places greater significance on the P terms on the right side (RHS) of the equation. Usually, backward-difference approximations have been used for the pressure gradient term on the RHS. However, it can be shown that a backward difference leads to numerical instability for sufficiently small marching steps (which may be a manifestation of the departure behavior). When the ω function is used, the RHS can be set to zero, to a good approximation, making the numerical computation stable, even for small steps. Thus, the ω function and a zero RHS together permit the investigation of departure behavior, which requires very small streamwise steps.

NUMERICAL METHODS

The present computational results are obtained with the Beam-Warming factored algorithm in delta form. Details can be found in reference [8]. Full second-order accuracy can be maintained by use of central differences in η and ζ, and with a three-point backward difference in ξ. However, for the present results, the Euler implicit method is used for the marching direction, making the method first-order accurate in the ξ direction.

TRIPLE-DECK ANALYSIS

For subsonic flows, that is, elliptic equations, a downstream boundary condition must be specified. When attempting to march such a system without specification of the downstream condition, there are an infinity of possible solutions that correspond to various attached or separated flows. When the viscous layer is thin, the departure solution is governed by the interaction of the viscous and inviscid regions near the wall [9]. Stewartson [10] further studied such flows using a triple-deck analysis consisting of a Lighthill sublayer, an inviscid shear region, and the inviscid external flow. This analysis establishes the order of magnitude of the interaction region.

The pressure is constant across both the lower and middle decks, and the velocity in the middle deck corresponds with the attached velocity profile upstream of the interaction, but displaced by δ. Utilizing linear supersonic theory for the pressure at the top edge of the middle deck, one obtains the following momentum equation for the lower deck:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \rho_e u_e{}^2 / [\rho_w (M_e{}^2 - 1)^{\frac{1}{2}}] \frac{d^2 \delta}{dx^2} \quad , \tag{5}$$

where the subscript w refers to wall, and e refers to the top edge of the middle deck. Introducing the stream function in (5), and utilizing the Lighthill form for the displacement thickness δ, yields an Airy equation. It is found from the solution that the triple-deck scale is given by

$$\Delta\tilde{x} = 0.8272 (M^2 - 1)^{3/4} (T_\infty / T_w)^{1/2} C_f{}^{5/4} Re^{3/8} \ \Delta x = k \ \Delta x \quad , \tag{6}$$

where x is the nondimensional physical distance, Re is the Reynolds number, C_f is the skin friction coefficient, T is the temperature, and M is the Mach number. The pressure gradient that results is

$$dp/dx = C \exp(kx) \quad . \tag{7}$$

The importance of this analysis to computations with the PNS equations is that it provides a test of the effectiveness of methods for damping the unwanted departure solutions. To illustrate, we consider a flat-plate boundary layer. The solution shown in Figure 1 at $x/L = 0.5$ is obtained with the ω parameter turned on, and then ω is set to 1.0, so that the full PNS equations are marched for a short distance. The solution immediately starts to "depart" until at $x = 0.5055$ the ω factor is again turned on, and the solution decays toward the well-known boundary-layer solution. If the triple-deck scale had not been resolved by the step size

Δx, the departure would have appeared as an oscillatory divergence. We note here that other investigators report their methods are unable to suppress the departure behavior for small marching steps, while the present approach is quite effective.

Figure 2 demonstrates that the computed departure solutions ($\omega = 1$) agree with Lighthill's linear analysis. This figure shows the magnitude of the computed pressure gradient as a function of the triple-deck distance scale $\tilde{x} = kx$. The initial slope of the computed departure agrees with the linear analysis until nonlinear effects take over.

COMPUTATION OF TURBULENT FLOWS

Turbulent computations are performed with the Reynolds averaged form of the PNS equations and an algebraic, or zero-equation, eddy viscosity model. Details are given in reference [5] and will not be repeated here. We note, however, that no special modifications were needed for the present calculations, even with a large crossflow separation on the leeward side.

Figures 3 and 4 compare the circumferential variation of pressure and surface stream angle from the present computations with the experimental results of Rainbird [11]. Generally, good agreement is obtained with experiment. The main differences are caused by an incorrect prediction of the location of separation, believed to be due to the inadequacy of the simple turbulence model used. These results were obtained on a CDC 7600 computer with 50 grid points between the body and the shock, and with 47 unequally spaced meridian planes.

FLOW-FIELD SIMULATION

The simulation of turbulent vortex structures is achieved by tracing particle paths using the velocity field obtained from the PNS solution. Initial positions for the tracer particles are specified at field locations near the cone surface, and $x/L = 0.1$. The particle paths are then determined from a simple Euler predictor-corrector, finite-difference scheme. The velocity field is taken at $x = L$ and is assumed invariant with x for the purposes of this simulation.

The flow simulation is shown in Figures 5a and 5b. Three distinct vortices are observed--one emanating from the primary separation, the second from the secondary separation of the flow coming down the leeward plane of symmetry, and a third below the primary vortex and having the same rotation as the primary vortex. This flow structure is suggestive of a global type of mixing in large turbulent structures, which is significant to the development of turbulence models.

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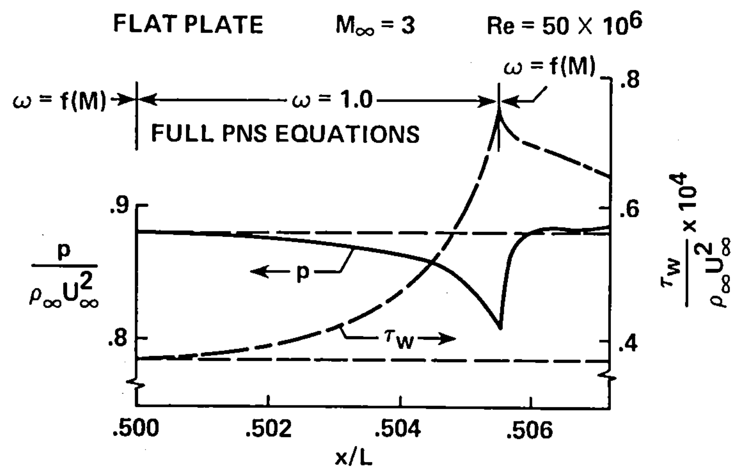


Fig. 1. Influence of "Omega" parameter on departure and recovery.

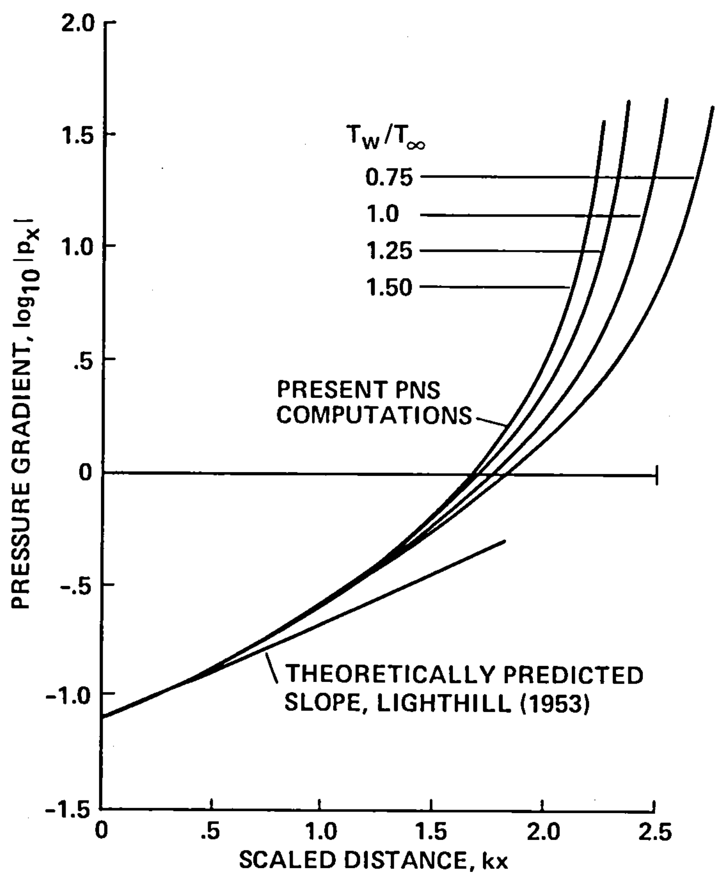


Fig. 2. Departure solution for a flat plate; $M_\infty = 3$, $Re = 50\times10^6$, $T_W = 100$ K.

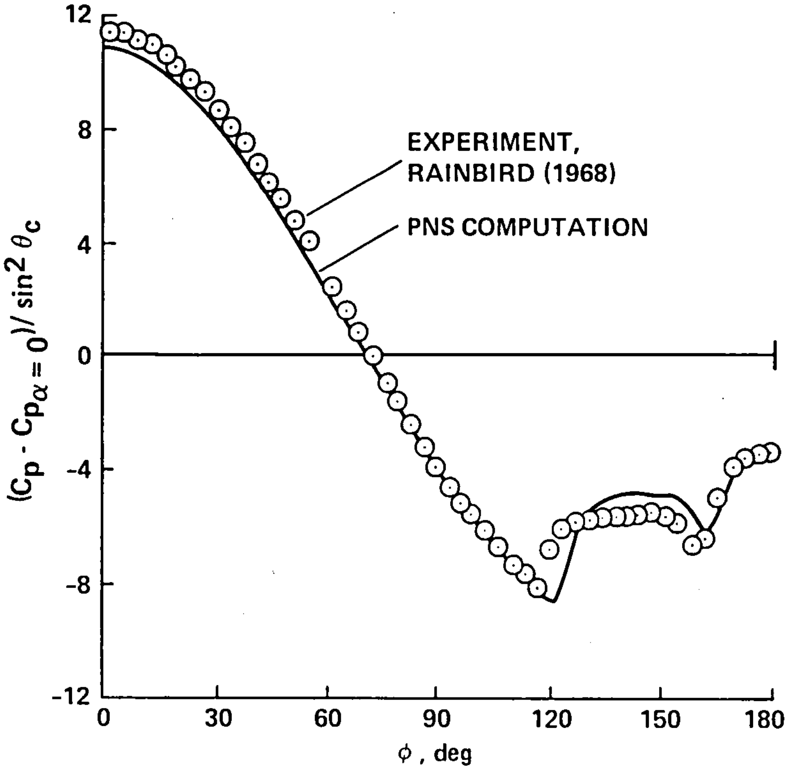


Fig. 3. Circumferential pressure distribution for a 12.5° cone; $M_\infty = 1.8$, $\alpha = 22.7°$, $Re = 25 \times 10^6$.

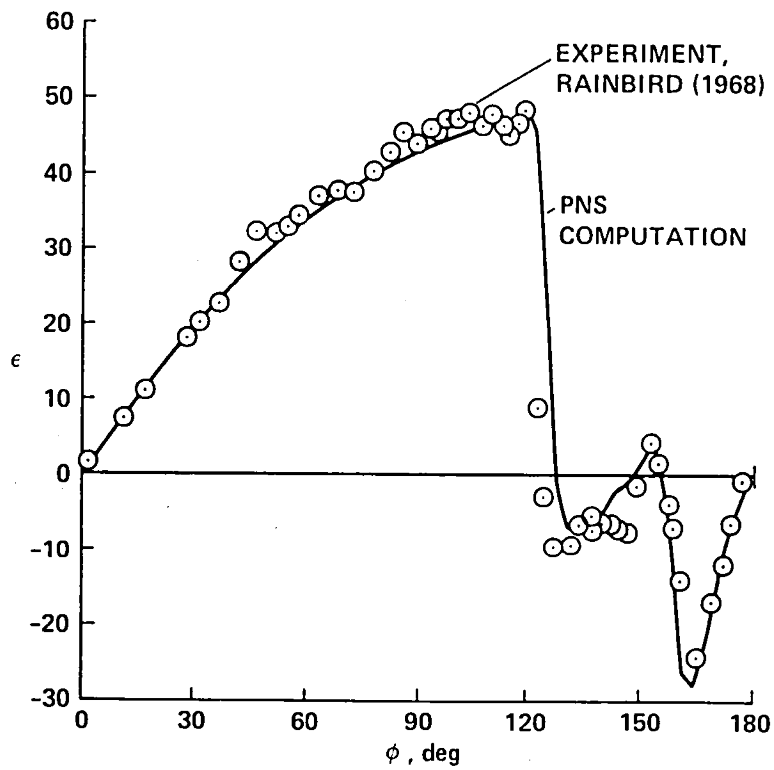
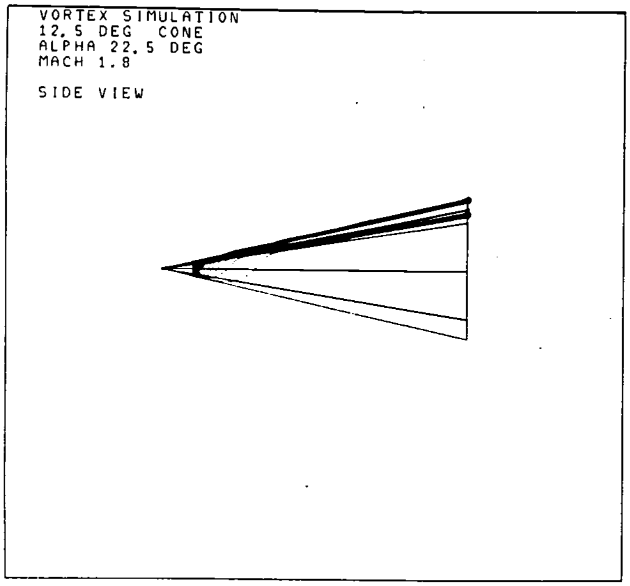
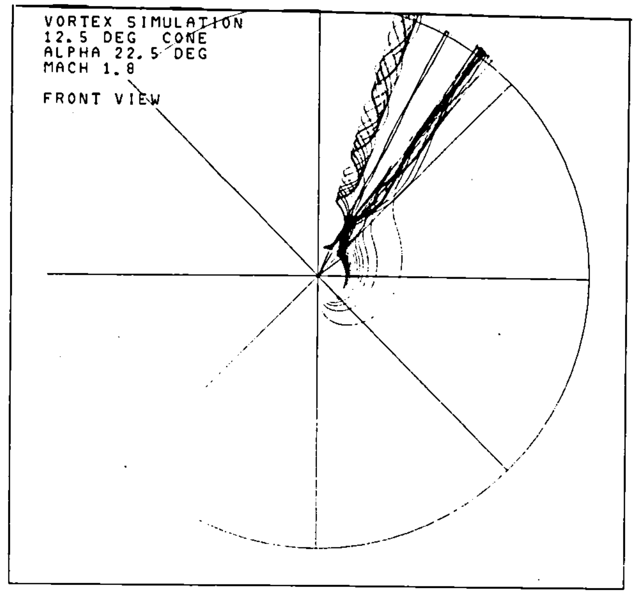


Fig. 4. Surface stream angle relative to the generators; $\theta_c = 12.5°$, $M = 1.8$, $\alpha = 22.75°$, $Re = 25 \times 10^6$.



(a) Side view.



(b) Front view.

Fig. 5. Streamlines around a 12.5° cone; $M = 1.8$, $\alpha = 22.75°$, $Re = 25 \times 10^6$.

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16. Abstract

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