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# ELECTRICAL ENGINEERING

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UNIVERSITY OF NOTRE DAME, NOTRE DAME, INDIANA

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to the  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
on  
NASA Grant NSG-3048  
ALTERNATIVES FOR JET ENGINE CONTROL\*  
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\*This work has been carried out under the direction of

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## ABSTRACT

This report deals with progress made on the Grant NSG-3048 during the six month period beginning October 1, 1982 and ending March 31, 1983. The NASA Technical Officer for this period was Dr. Kurt Seldner of Lewis Research Center. The director of the research at the University of Notre Dame was Dr. Michael K. Sain, who has been assisted by Mr. Daniel Bugajski and Mr. Joseph O'Sullivan, graduate research assistants in the Department of Electrical Engineering. Mr. Thomas A. Klingler has received the degree of Master of Science based upon his studies under this Grant, for his thesis entitled "Nonlinear Modeling of a Turbofan Jet Engine: A Tensor Method Application". Mr. Klingler is presently with the Pontiac Division of General Motors Corporation, where he is a principal in the modernization of the Division's automobile engine simulation and test facilities.

Researches during this period have stressed tensor model order reduction, recursive tensor model identification, input design for tensor model identification, software development for nonlinear feedback control laws based upon tensors, and development of the CATNAP software package for tensor modeling, identification, and simulation. This report is focused upon the last of these. The other projects are scheduled for future reports.

## ACKNOWLEDGEMENTS

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Special thanks go to Mr. Thomas A. Klingler, whose M.S. Thesis forms the core of this report, and to Mr. Daniel Bugajski, who assisted Mr. Klingler with the simulations reported.

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## SECTION I

### INTRODUCTION

Models have always been an important aspect of applications engineering in the area of multivariable control system design. See for example [1]. Practical industrial examples of the use of models are provided by gas turbine engine control systems, which commonly use models to generate control and response trajectories for various power demands. These models, when scheduled over the operating envelope, can reduce the compensation normally required of the controller, and thus provide the feedback loop with an opportunity to achieve better accuracy in the presence of noise and parametric uncertainties.

The scheduling of feedforward models and feedback compensation typically produces nonlinearities, even if the local models are linear. Accordingly, there is a basic interest in fundamental approaches which incorporate nonlinearity at the outset. Such approaches should (1) reduce to the earlier schemes for variables with very small excursions, (2) be amenable to scheduling, and (3) offer opportunities for determination of parameters from simulation data.

One such approach uses the notions of power series and algebraic tensors [2] to generate a class of nonlinear models [3]. The important feature of the algebraic tensor is that it provides an organized way of describing the power series expansion formula, lending itself with relative ease to programming on a digital computer.

This report summarizes an investigation focused on the use of the

above technique for the identification and testing of the algebraic nonlinear model structure which utilizes the tensor approach [4]. The basic strategy involves initial condition and control signal design in order that the nonlinear model not only outperforms a standard linear model locally about an expansion point, but also establishes a larger region of model validation. Emphasis throughout the study is placed on the application of these ideas in nonlinear model generation using a turbofan jet engine simulation.

The remainder of the report is outlined as follows. In Section II the background material necessary for an understanding of the modeling methodology is presented. Section III describes the modeling software which has evolved into a computer-aided design package capable of handling the overall process of model formulation, identification, and validation for any particular nonlinear modeling problem developed within the tensor framework. Next, Section IV presents an overview of the QCSE engine along with its corresponding digital simulation. QCSE is the name of the turbofan jet engine used in the modeling exercises of this investigation. A detailed identification with full validation studies, as well as a summary of the modeling technique as applied to QCSE, is included in Section V. Finally, Section VI offers a few pertinent closing remarks.



## SECTION 1.1 NONLINEAR MODEL STRUCTURE

The purpose of this section is to relate some of the background material which is fundamental to the work described in the succeeding sections. The first portion summarizes important ideas developed in the previous work [ 3 ] which serves as a basis for this study. Since a rigorous development of these topics is not the purpose of this report, the presentation here is limited. However, the discussion should supply an adequate foundation for the material to follow. Finally, the section closes with the introduction of a technique used to normalize the bulk data required in the identification phase.

### 2.1 ALGEBRAIC TENSORS IN MODELING

Current research indicates that the use of tensor algebra [ 2 ], in modeling nonlinear dynamical systems, is quite advantageous. Furthermore, these studies maintain that the use of algebraic tensors gives a unified view of the nonlinear modeling problem. The important feature of the algebraic tensor is that it provides an organized way of describing the power series expansion formula. In addition, its use allows for the implementation of linear parameter identification techniques.

Due to the brevity of the following treatment, the reader is advised to consult the works [ 3 , 5 , 6 ] for detailed expositions of the topics discussed. With this in mind, consider the following nonlinear ordinary differential equation

$$\dot{x} = f(x,u) . \tag{2-1}$$

Here  $x \in X$  is a  $n$ -tuple of states and  $u \in U$  is a  $m$ -tuple of inputs, where  $X$  and  $U$  are real vector spaces. The function  $f : X \times U \rightarrow X$ , under certain assumptions, may be expressed in a power series expansion of two variables. Moreover, if we employ the universal bilinear tensor product function  $\otimes : X \times U \rightarrow X \otimes U$  [7], as a powerful tool to organize the higher order mixed partial derivatives from the expansion, then the right side of (2-1) can be expressed in the form

$$f(x,u) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \tilde{L}_{ij} \underbrace{(x \otimes x \otimes \dots \otimes x)}_{i \text{ times}} \otimes \underbrace{(u \otimes u \otimes \dots \otimes u)}_{j \text{ times}}. \quad (2-2)$$

Therefore, a nonlinear model of (2-1) can be obtained by a truncation of (2-2). Substituting (2-2) into (2-1), and rewriting the result in a different form, yields the model structure

$$\dot{x} = [\tilde{L}_{10} \tilde{L}_{01} \tilde{L}_{20} \tilde{L}_{11} \tilde{L}_{02} \tilde{L}_{30} \dots] x_T \quad (2-3)$$

where  $x_T$  is a vector stacked with ordered tensor product terms given by the expression

$$x_T = (x, u, x \otimes x, x \otimes u, u \otimes u, x \otimes x \otimes x, \dots)^T. \quad (2-4)$$

It should be noted that  $x_T$  contains a number of redundant terms due to the commutativity of scalar multiplication in the field; however, through use of the symmetric tensor algebra [7] these redundancies are eliminated and a reduced-size version of (2-3) is constructed.

To identify the  $\tilde{L}_{ij}$  parameters, (2-1) is perturbed from  $(x_0, u_0)$  and excited with cosinusoidal inputs. The state solutions and control inputs are sampled at  $h$  selected time points and are loaded into the  $pxh$  matrix  $X_T$ . The sampled values of  $x$  and  $u$  comprise the first  $n+m$  rows of  $X_T$ , whereas the remaining  $p-(n+m)$  rows contain monomial

terms which are products of various combinations of the first  $n+m$  rows. These monomial terms are generated via an efficient ordering algorithm [3] which minimizes the required number of multiplications by re-using any products previously computed instead of repeating the operations. The  $n \times h$  matrix  $\dot{X}$  is formed by loading derivative estimates for  $\dot{x}$  at the  $h$  points in time. As an illustration, a truncation approximation which retains tensor product terms through second degree is given by the equation

$$\dot{X} \approx [\hat{L}_{10} \hat{L}_{01} \hat{L}_{20} \hat{L}_{11} \hat{L}_{02}] X_T . \quad (2-5)$$

Finally, to arrive at the desired  $\hat{L}_{ij}$  operators, a singular value decomposition is employed to solve the minimal least-squares problem.

## 2.2 DATA NORMALIZATION

Recall from the previous section that sampled data, in the form of derivative estimates, state solutions, and control inputs, is used to construct a least-squares minimization. Frequently though, when physical systems are being considered, this raw data spans a wide range of units and therefore taxes the abilities of the coefficient matrix software. Consequently the need for a practical, yet easy to implement, normalization technique is essential. The rest of this section is devoted to outlining one such scheme.

To begin, let

$$L = [\hat{Y}_{10} \hat{Y}_{01} \hat{Y}_{20} \hat{Y}_{11} \hat{Y}_{02} \hat{Y}_{30} \dots] ; \quad (2-6)$$

then by substitution equation (2-3) becomes

$$\dot{X} = LX_T . \quad (2-7)$$

Now, let  $\dot{Z} = VX$ , where  $V$  is a normalization matrix given by

$$\left[ \begin{array}{ccccccc} \frac{1}{\gamma_1} & & & & & & \\ & \frac{1}{\gamma_2} & & & & & \\ & & \frac{1}{\gamma_3} & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & & & \frac{1}{\gamma_n} \end{array} \right] ;$$

nxn

$$\gamma_i; i = 1, \dots, n \triangleq \max_j |X(i,j)|; j = 1, \dots, h .$$

Similarly let  $Z_T = WX_T$ . In this situation the normalization matrix  $W$  takes on a rather complicated form due to the nature of  $X_T$ , but through use of the Kronecker product [ 8 ] of two matrices,  $W$  can be defined as the partitioned matrix

$$\left[ \begin{array}{ccccccc} A & & & & & & \\ \hline & B & & & & & \\ \hline & & A \otimes A & & & & \\ \hline & & & A \otimes B & & & \\ \hline & & & & B \otimes B & & \\ \hline & & & & & A \otimes A \otimes A & \\ \hline & & & & & & \ddots \end{array} \right]$$

pxp



Applying this transformation to (2-7) yields the normalized matrix equation

$$\dot{Z} = \bar{L} Z_T \quad (2-8)$$

which can then be solved for  $\bar{L}$ . The data in  $\dot{Z}$  and  $Z_T$  now takes on real values between -1.0 and 1.0.

Work backwards now to reconstruct the original system. Equation (2-8) can be written as  $\dot{VX} = \bar{L}WX_T$  and manipulated to equal (2-7) when  $L = V^{-1}\bar{L}W$ .

This approach will prove to be quite valuable when the turbofan jet engine problem is tackled in Section V.

In closing, this section has addressed certain issues of theoretical interest which serve as a prerequisite to the material contained in the pages that follow. The remainder of the report centers around the application of these ideas.

## SECTION III

### CATNAP

The intent of this section is to present a detailed discussion of the Computer-Aided Tensor Nonlinear Modeling Applications Package (CATNAP) currently used as a development tool in the formulation, identification, and validation of nonlinear models of the type mentioned in Section II.

#### 3.1 OVERVIEW

The structure of CATNAP is based upon ideas from distributed processing and local networking [9] in which computations are spread over multiple machines. The package [10], developed using the extensive capabilities of the IBM and DEC Command Procedure Languages with the strengths of FORTRAN and SPEAKEASY, is tailored to utilize existing computer hardware effectively, as well as to provide the fastest possible user turnaround time. Development of a package such as CATNAP is an absolute necessity to apply the ideas of [3].

Specifically, CATNAP is divided into two segments, each of which is implemented on an independent computing system. These segments are entitled GENERATE and SIMULATE. GENERATE is implemented on a Time Sharing Option [11] of the IBM 370-168 mainframe computer system, where the memory dependent and highly computational routines of the package can benefit from use of the virtual memory and floating point hardware. Once a structured nonlinear model is identified, it is then transferred to the DEC PDP 11/60, where, using SIMULATE, it can be analyzed and compared to a linear model as well as the true system. In this manner, the user has

at his disposal both the graphics capabilities of a video terminal and an electrostatic printer/plotter for the immediate display of data and comparative trajectories. Furthermore, both of these segments are highly interactive and contain straightforward input prompts as well as informative error messages.

### 3.2 GENERATE

The GENERATE segment of CATNAP is primarily made up of three routines governed by a higher level supervisor. This supervisory level is written using the command procedure language CLIST [12] and performs the following main functions in sequence:

- (1) prompts the user for the name of the desired loader routine to be executed;
- (2) passes control to the loader routine defined in (1);
- (3) passes control to IDENTIFY; and,
- (4) upon user request, passes control to TRANSFER.

In addition to these main functions, certain maintenance roles such as file creation, allocation and deletion are handled by this supervisor.

Figure 3.1 contains a block diagram depicting the structure of GENERATE.

Associated with each nonlinear system to be modeled, there exists a loader routine which performs the model formulation task. These routines are stored in a library and are typically written in double precision FORTRAN. The purpose of any loader routine is to excite the given nonlinear system via initial condition and control input perturbations and to sample the states, inputs and derivative estimates over  $h$  selected points in time. The system is then represented by the matrix equation



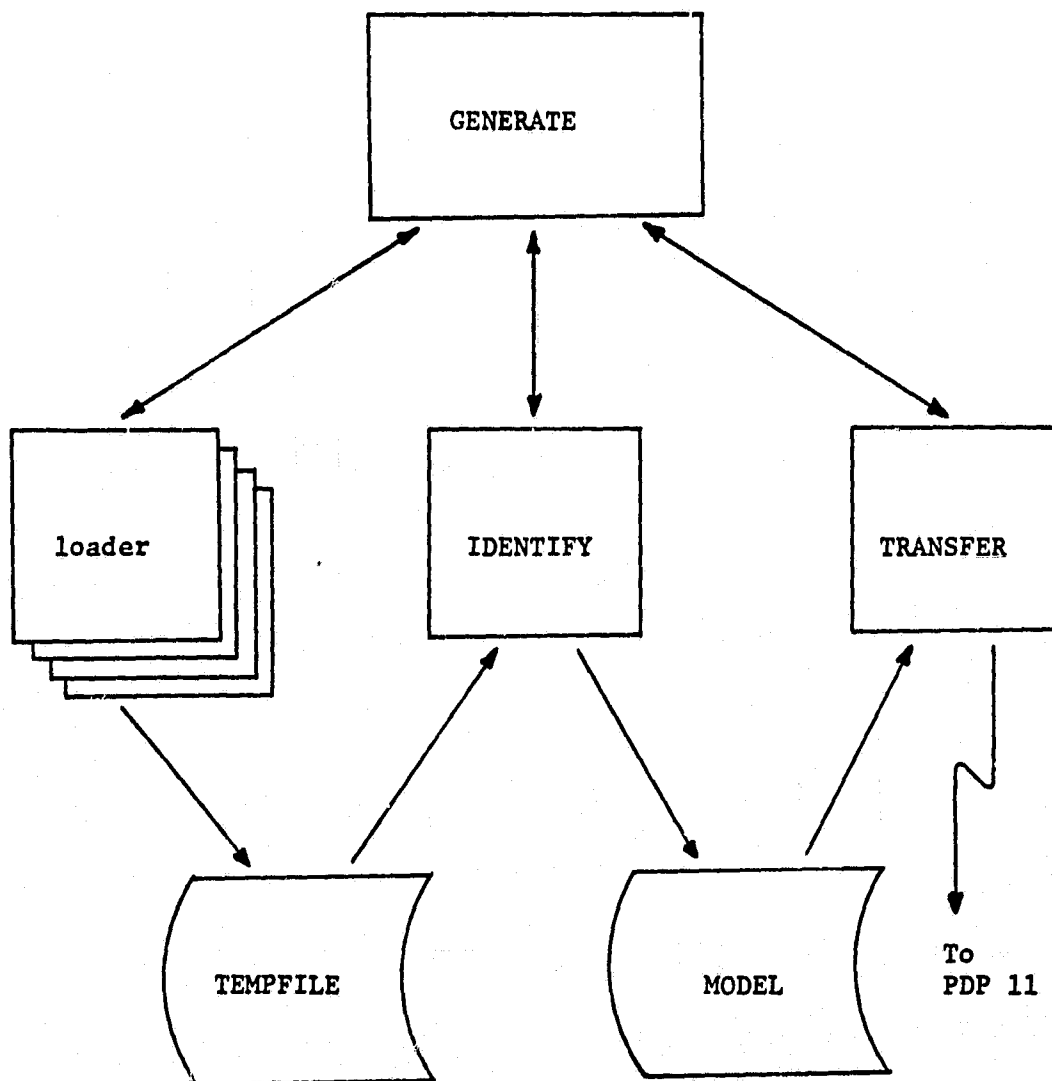


Figure 3.1. Block diagram for the CATNAP segment GENERATE.

$$\dot{X} = \begin{matrix} \tilde{L}_{10} & \tilde{L}_{01} & \tilde{L}_{20} & \tilde{L}_{11} & \tilde{L}_{02} & \tilde{L}_{30} & \dots \\ \downarrow & & & \downarrow & & & \\ nxh & & & nxp & & & pxh \end{matrix} X_T .$$

The first  $n+m$  rows of the matrix  $X_T$  are formed from the sampled values of  $x$  and  $u$  while the remaining  $p-(n+m)$  rows are formed from combinatorial products of those first  $n+m$  rows.  $\dot{X}$  is formed by loading derivative estimates for  $\dot{x}_1, \dot{x}_2, \dots, \dot{x}_n$  at the  $h$  time points. The number  $p$  depends on  $n$  and  $m$ , and the degree of the truncation approximation. All this data is then stored in TEMPFIL for later use.

The ordering algorithm which is used to construct  $X_T$  utilizes the important characteristic that higher degree tensor terms are "built up" from lower degree terms. Incorporating this idea into the methodology reduces the computational aspects of the scheme significantly. In fact, this improved algorithm operates nearly 3.5 times faster for second degree approximations than the previously used method. Even a greater improvement is expected for higher degree approximations.

Using the above formulation technique, CATNAP can accommodate any particular nonlinear modeling problem since the problem specifics are transparent to the remainder of the package. The only requirement is that TEMPFIL contains the appropriate data.

After the completion of any chosen loader routine, the program IDENTIFY is executed. IDENTIFY reads the interim data from TEMPFIL and forms a least squares minimization problem which is solved for the partitioned matrix containing the desired  $\tilde{L}_{ij}$  parameters. These  $\tilde{L}_{ij}$  parameters are recorded at the terminal as well as entered into the

MODEL data file. It should be noted here that IDENTIFY is written in the high level language SPEAKEASY [13], which is based on the concepts of arrays and matrices and processes these as entities. This results in the elimination of the many loops necessary in other programming languages. The main reason for employing SPEAKEASY here is that the highly efficient routine SIMEQUAT can be easily used to solve the least squares problem via singular value decomposition, thus reducing the apparent complexity of the problem to a minimum.

Once the identified data is entered into the MODEL data file, the user may elect to execute the program TRANSFER. Upon a yes response to a supervisory prompt, the program is submitted batch to the IBM 370-168. TRANSFER is merely a Job Control Language (JCL) deck which sends a copy of the file MODEL, containing the  $\tilde{L}_{ij}$  parameters, to the DEC PDP 11/60 computing system by the way of a Remote Job Entry port, and stores it in the nonlinear model library. An excellent account of JCL can be found in [14].

### 3.3 SIMULATE

Shifting our concern away from the discussion of GENERATE, we now focus our attention on the SIMULATE segment of CATNAP. Basically, two routines plus a supervisory comprise the structure of SIMULATE. Figure 3.2 offers an illustration of this structure to supplement the following presentation. Written in the form of an Indirect Command File [15] this supervisor allows the user to:

- (1) create new simulator routines;

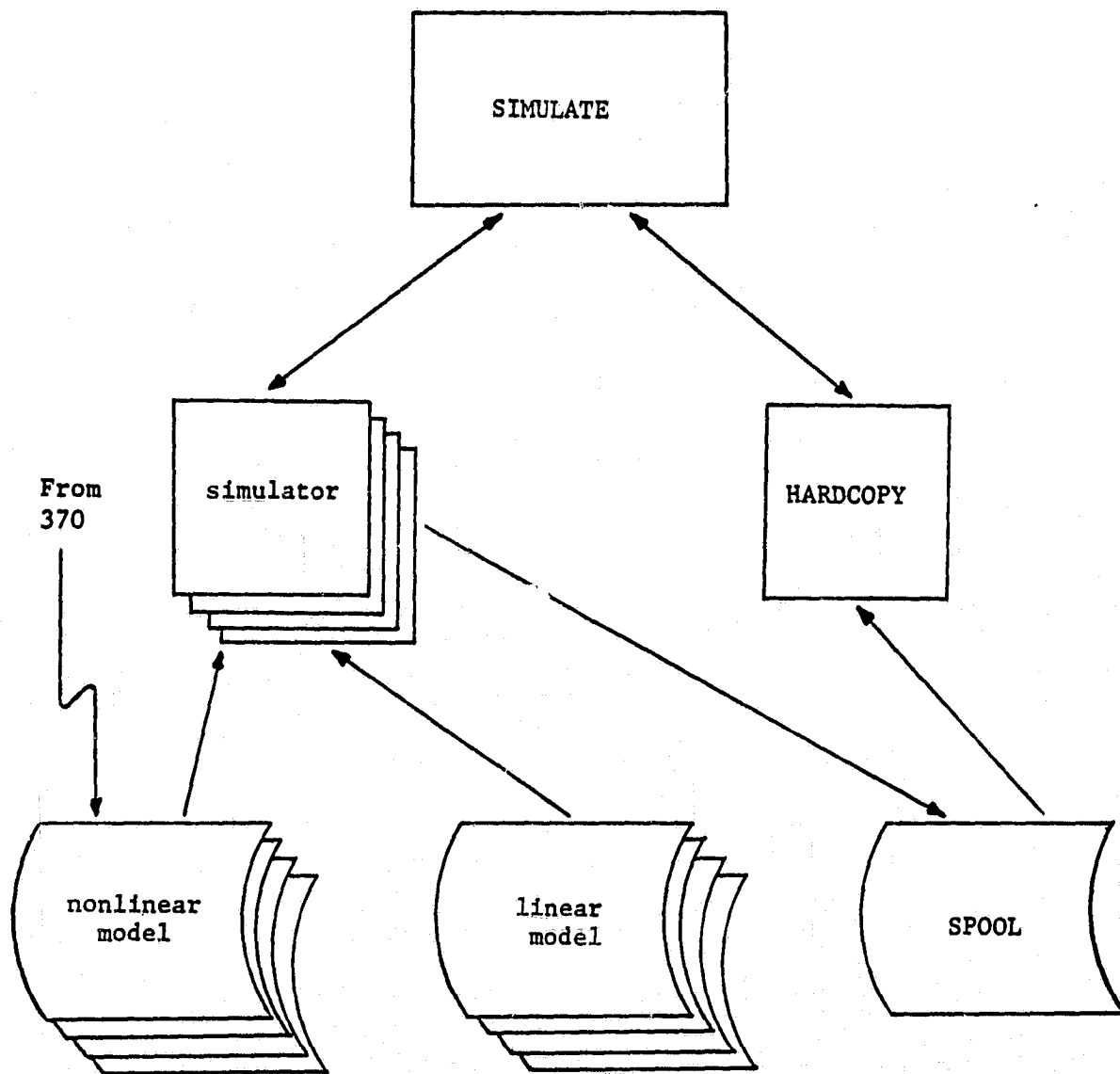


Figure 3.2. Block diagram for the CATNAP segment SIMULATE.

- (2) execute existing simulator routines; and,
- (3) execute VERSATEC which produces hardcopy plots.

As earlier, this supervisor performs a number of file maintenance duties in addition to the above functions.

Fundamentally, the heart of this segment is contained within any given simulator routine. Typically a simulator routine, written in FORTRAN, exists for each nonlinear modeling problem studied; however, only one subroutine in that program is altered among versions, and that is the application subroutine TRUES. The remainder of the program stays unchanged. TRUES contains the true system representation of the nonlinear system being modeled, and is used extensively in comparison studies. Because of the number of TRUES subroutines that exist, a library has been created to store the various simulator routines.

The execution of a particular simulator routine can be divided into three steps; (1) problem configuration; (2) systems integration; and (3) solution display. The first of these steps requires the user to decide which of the available systems, true solution, linear model and/or nonlinear model, should be included in the session configuration. When a model is chosen, the user is asked to enter the name of the desired model. That model is then read into the simulator from the appropriate library. The linear models which are used in CATNAP are generally identified by standard techniques and are available for use in comparison studies.

Next, the user is prompted for various integration parameters such as stepsize and upper time limit as well as initial conditions, input amplitudes and frequencies. The configured systems are then integrated

and the solutions are sampled at 100 points, evenly spaced in time. Finally, to assist in the data analysis, a number of options are available to the user. They include:

- (1) printing the solutions on the Versatec;
- (2) displaying the comparative trajectories on the Tektronix graphics terminal;
- (3) writing the trajectory solutions to SPOOL for hard-copy plotting at a later time; and,
- (4) solving the configuration for another set of initial conditions and control inputs.

The use of these options provides a powerful yet flexible capability for the study of model performance and validity. Furthermore, when all three systems are included in the configuration, an additional error criterion is generated and used in testing the performance of the nonlinear model versus that of the linear model (See [3]).

Let  $\epsilon_i^N$  denote the absolute maximum error in the nonlinear model solution, as compared to the true simulation solution, over the time range of simulation for the  $i$ th state variable. Similarly, define  $\epsilon_i^L$  for the linear model error. Then  $\epsilon_i$  is the difference  $\epsilon_i^N - \epsilon_i^L$ . Thus, if  $\epsilon_i$  is negative, the nonlinear model has exhibited a smaller maximum absolute error in the  $i$ th state, and in that sense has outperformed the linear model.

At the completion of any particular simulation session, the user has the option to plot the selected curves that he spooled earlier. The routine HARDCOPY, again written in FORTRAN, reads the trajectory solutions from SPOOL and records at the Versatec printer/plotter, a data sheet corresponding to each plot set which follows. The comparative trajectories

themselves are then plotted. In this way, permanent records of results are made available which are suitable for inclusion in reports such as this one.

This section has attempted to summarize in an easy-to-read format the functions and capabilities of the CATNAP software package. The computing hardware situation at the University of Notre Dame has, however, at this time a rather significant gradient. Within the next calendar year, progress is expected in (1) replacing the IBM 370-168 with a newer generation system, (2) establishing a new system in the College of Engineering, and (3) studying the possibilities for networking machines in the Department of Electrical Engineering. Accordingly, the specifics of CATNAP will have to evolve continually. Appendix B gives some details for students who use the package this year. While it is illustrative, it can certainly be skipped on an initial reading.

## SECTION IV

### QCSEE

Since the primary goal of the report is to apply the modeling methodology described in Section II on a practical example of a turbofan jet engine, it is necessary that the reader be somewhat familiar with the operation of the jet engine. Therefore, the intent of this section is to present an overview of the jet engine and its corresponding real-time digital simulator. More specifically, attention will center around NASA's Quiet, Clean, Shorthaul Experimental Engine (QCSEE).

#### 4.1 TURBOFAN JET ENGINE

The major purpose of the QCSEE ("Quixie") program [16,17] is to develop the technology required for quiet, clean, economical propulsion systems to be used in commercial short takeoff and landing (STOL) aircraft. As a result, QCSEE will greatly expand airline capabilities in densely populated metropolitan areas where low noise and atmospheric pollution requirements are critical.

The QCSE engine, shown in Figure 4.1, will pioneer with several concepts not all currently used on existing turbofans. The engine features:

- (1) an extremely high-bypass ratio;
- (2) a gear-driven reversible-pitch fan with composite fan blades;
- (3) a variable geometry fan exhaust nozzle; and,
- (4) a digital electronic control system combined with a hydromechanical fuel control.

The following will be an attempt to present a brief description pertinent



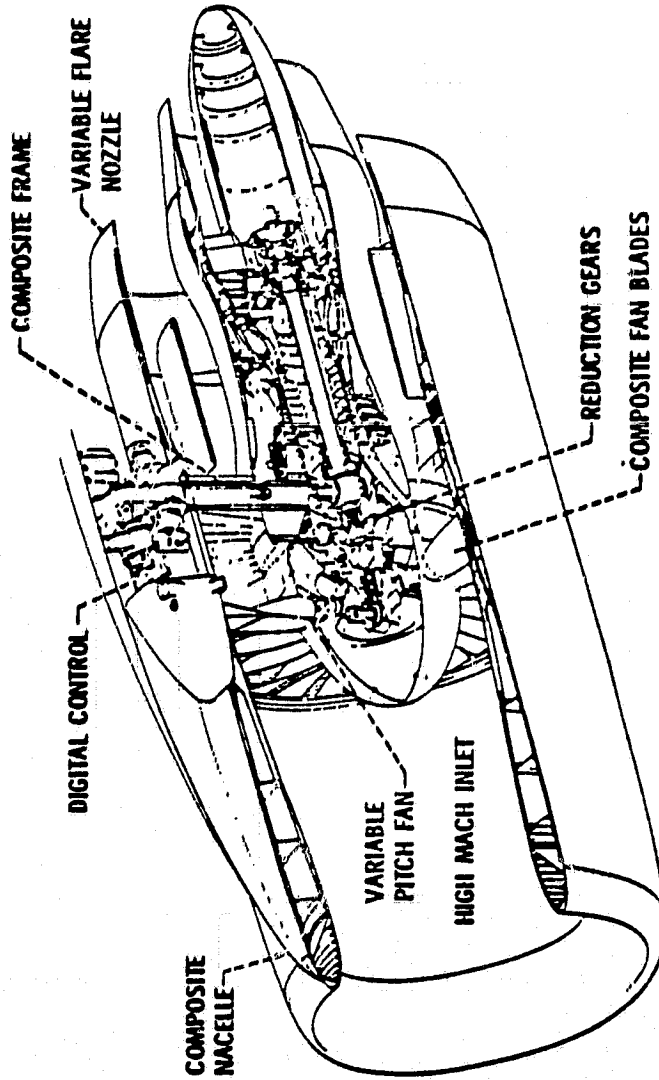


Figure 4.1. QCSEE Propulsion System.

to each of these features.

First, by doubling the ratio of cool, dense bypass air to high-temperature high-pressure air flowing through the core, QCSEE is able to reduce the airflow velocity 40% while maintaining engine thrust, thus improving lift. Equally as important, the noise level of the jet engine is reduced significantly. Current estimates indicate that QCSEE will have a takeoff and landing "footprint" of approximately .8 square miles, whereas current transport aircraft have noise footprints ranging anywhere from 6 to 30 square miles.

Another advantage of the extremely high-bypass system is that it causes the engine exhaust temperature to be relatively low when compared to temperatures of other turbofan engines. This cooler air reduces acoustic and thermal fatigue in the flap systems, and therefore permits manufacturers the use of conventional airframe materials. On the other hand, QCSEE employs the use of exotic composite materials, whenever feasible, to reduce weight. Examples indicating where composite structural materials are used include the inlet duct and fan blades.

The second innovative feature of the QCSE engine is that it uses variable-pitch fan blades. Variable pitch will eliminate the need for large, complex, heavy thrust-reversal systems which are typically used on turbofans. The almost instantaneous response to engine thrust settings that variable pitch fan blades provide will give pilots improved control in the critical landing and takeoff portions of the flight envelope. Currently, a 3 to 5 second time lag exists in engine acceleration from approach thrust to full thrust. This time lag can be reduced to less than

one second with the QCSE engine. This is achieved by maintaining high fan speed, with a low thrust pitch angle. To achieve full thrust the fan pitch angle is merely changed accordingly.

The next important feature of QCSEE is its fan exhaust nozzle. The fan exhaust nozzle is a variable-area, four-flap design capable of area change from takeoff to cruise, as well as opening to a flared position to form an inlet in the reverse thrust mode.

Finally, QCSEE utilizes an on-board system to control various steady-state and transient characteristics. The system designed to accomplish this task incorporates two basic control components, a modified hydromechanical fuel control and an engine-mounted, 4000 word core memory, digital electronic control designed specifically for QCSEE (See [18]).

An F101 fuel pump is used in the system for supplying fuel for engine operation, and for operating servomechanisms in the hydromechanical control. A constant pressure hydraulic pump supplies fluid for operation of the actuators which position the fan nozzle and the hydraulic motor which drives the variable-pitch actuators.

An important estimation in any jet engine is the thrust. Here, the controller uses the ratio of compressor discharge pressure to free stream total pressure as a measure of engine thrust. Incidentally, the QCSE engine is designed to provide 17,400 pounds of installed thrust at takeoff on a 90° fahrenheit day.

The ultimate goal of propulsion engineers is to equip aircraft with fly-by-wire systems where essentially the entire flight is managed and

controlled by a sophisticated computer. In this arrangement, the pilot would merely serve as a backup. Advanced versions of QCSEE are expected to contain fly-by-wire systems where flight plans from one airport to another would be relayed from the ground-based center to the on-board computer. Then, during takeoff, cruise, and approach the appropriate plan would be initiated with the computer controlling the necessary parameters such as thrust, fan-blade setting, and exhaust nozzle area.

It can be determined from the above discussion that QCSEE is a rather large engine made up of several bulky component systems. Consequently on an experimental basis, it is not very practical, let alone affordable, to use a real turbofan. A useful alternative, though, is to use a digital simulation of the engine. In this spirit, the remainder of the section outlines the QCSEE simulation deck which is used in the modeling exercises to follow.

#### 4.2 DIGITAL SIMULATION

With the advent of faster, more reliable computers, real-time digital simulation has come to play an important role in the development of advanced aircraft and propulsion systems. Simulations are advantageous since, at a meager cost, they can enhance product development by eliminating the need to build expensive prototypes. Furthermore, simulation provides a safe means of studying various failure modes such as engine stall, which might otherwise be impossible to perform due to the hazardous conditions involved. Accordingly, the QCSEE simulator program [19,20] was established to achieve detailed real-time propulsion simulation to be used in manned flight simulators designed specifically for STOL aircraft.

For reference, Appendix A contains a flowchart of the QCSEE simulator as well as a list of important engine and control variables used throughout the simulation.

The analytical model, developed from component performance maps of the various engine dynamics, incorporates multivariable functional representations of two or three variables. Since 40 percent of the total calculation time is consumed in function generation, a number of more efficient generating schemes are employed. Linear interpolation, polynomial and geometric curve fitting, and table search are among the techniques used to generate these functions. The eight engine dynamics chosen as state variables for the system include two fan speeds, four pressures, and two temperatures. In addition, there are four sensor states and four control actuator states.

A digital controller is incorporated into the overall simulation, which uses absolute error signals to achieve feedback in the control loop. The error signals proceed through a segment which contains a lag compensator for stability and transient anticipation, logic for switching to other control modes, selectors through which limits are applied, and elements which generate the output signals. Overrides are applied to each of the manipulated variables under certain conditions for safety or operational reasons. A block diagram of the QCSEE simulator control structure can be found in Figure 4.2.

The computer program is written in double precision ANSI FORTRAN V, where great care has been taken to minimize unnecessary computations.

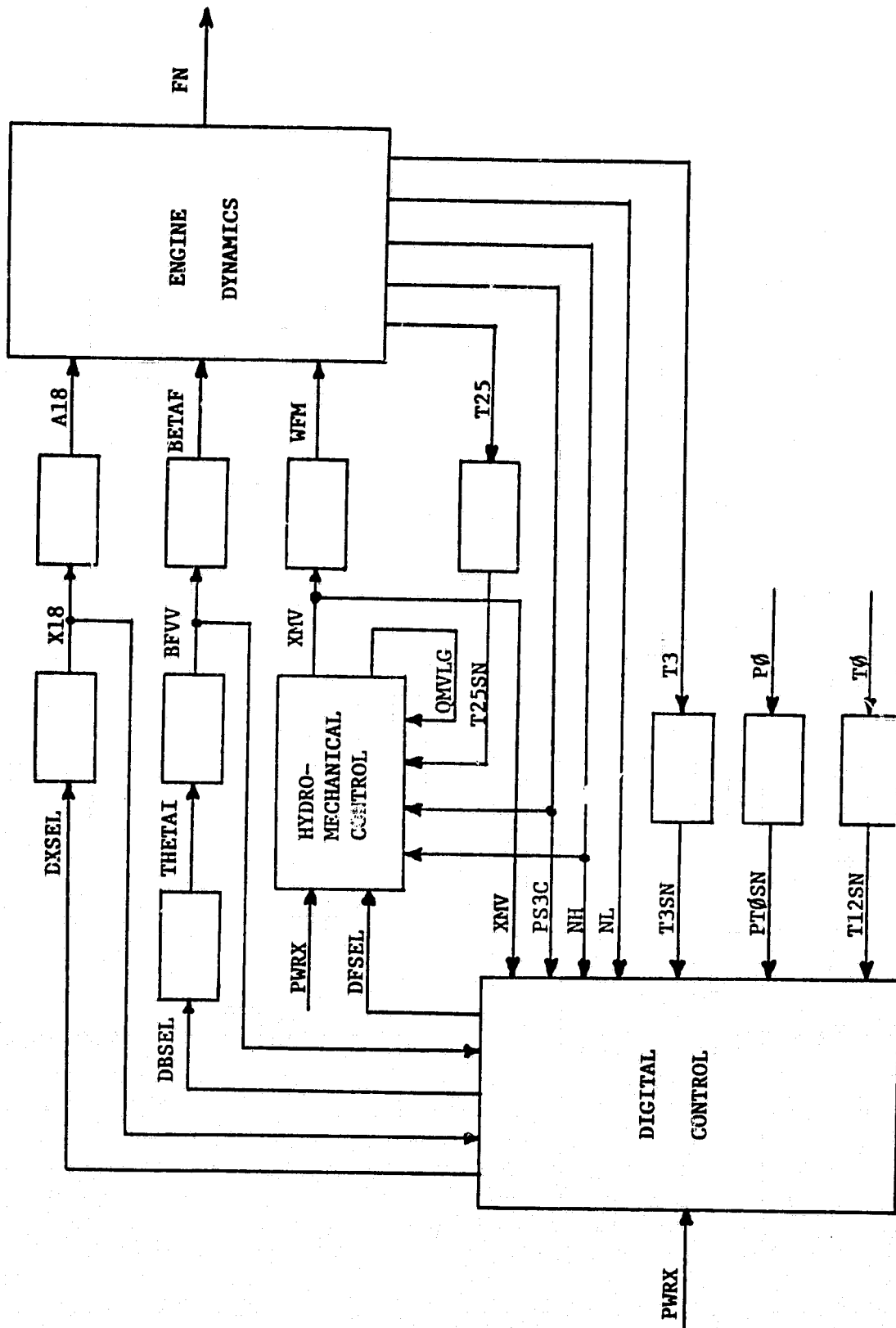


Figure 4.2. QCSEE Control Structure.

Functions such as exponential are avoided, and whenever possible, divide computations are replaced by multiplications. Surprisingly, conditional branches are substituted for subroutine calls throughout the code. The program contains provisions for monitoring and displaying ninety engine and control variables, for detecting certain malfunctions, and for taking corrective action in the event of some critical malfunction such as fan drive gear failure, loss of fan speed signal, and certain control computation faults. The integration stage of the routine utilizes a second-order modified Euler algorithm [21] which was found to be computationally faster than a standard fourth-order Runge-Kutta method. The concept of point averaging, rather than slope averaging, is used in this modified Euler algorithm, where it has been proved stable over a wide range of transients.

The engine states used in the modeling discussions in the next section are the combustor discharge pressure (P4GS), the core nozzle total pressure (P8GS), and the rotor dynamics in the form of fan speed (NL), and compressor speed (NH). As a comparison, the fan attains a speed of 3000 rpm, at cruise, whereas the compressor reaches a speed near 12500 rpm. The four remaining states are the total pressures from the fan inlet and fan discharge, and the exit temperatures from the compressor and combustor. All three engine control inputs are employed in the modeling, namely, the main burner fuel flow (WFM), the exhaust nozzle area (A18) and the fan pitch angle (BETAF). To illustrate QCSEE's fuel consumption, the main burner fuel flow attains a value of 4000 lb/hr at cruise, and a value as high as 6600 lb/hr during takeoff.

QCSEE has one primary input variable for manipulation, the percent power demand (PWRX). In reality, this power setting is a digital electronic input supplied from the control room, or in an aircraft application, from the pilot or aircraft flight control computer. In the simulator, though, power demand is just a numerical percentage with idle speed occurring at 62.5%. Cruise speed is considered to be about 85% PWRX, while full throttle occurs at 100% PWRX. A sample operation might be to run QCSEE into the steady state (approximately ten seconds) at idle speed, and then switch to a desired PWRX to perform particular transient response studies. Finally, the program permits modification of several environmental parameters, however, the simulator is normally operated at sea level and zero velocity.

The complex, advanced design of the QCSE engine lends itself as a practical yet challenging nonlinear modeling problem worth studying. In the next section the QCSEE simulator is incorporated into CATNAP and used to formulate nonlinear models of the type mentioned thus far.



## SECTION V

### QCSEE MODELING

A useful way to test the feasibility of the modeling technique under investigation is to apply the ideas presented thus far using a nonlinear digital simulation. Accordingly, this section summarizes a research effort focused on developing nonlinear models of tensor type for the QCSE engine by using the CATNAP software discussed in Section III. Specifically, the aim of the section is two-fold. First, the problem is formulated in reference to the jet engine digital simulation. This involves the creation of an appropriate CATNAP loader routine which utilizes the QCSEE deck. Then, an example four-state, three-control nonlinear model of the QCSE engine is presented. The majority of this section consists of simulation results in the form of solution plots and tabulated error criteria. These test results are provided to illustrate a comparison of the true, linear, and nonlinear solutions, and to establish an acceptable region of model validity.

#### 5.1 FORMULATION

Attention in this section will center around the formulation of a reduced order four-state, three-control analytical model. As mentioned in the previous section, the chosen states are the combustor discharge pressure ( $P4GS$ ), the core nozzle total pressure ( $P8GS$ ), the fan speed ( $NL$ ), and the compressor speed ( $NH$ ). Recall also that the main burner fuel flow ( $WFM$ ), the exhaust nozzle area ( $A13$ ), and the fan pitch angle ( $BETA_F$ ) are the control inputs employed. Since the region of interest for identification is the operating envelope, an expansion is made about

a particular non-zero point,  $(\bar{x}_0, \bar{u}_0)$ , on the envelope. If the four-tuple  $x$  is given by the expression

$$x(t) = \bar{x}(t) - \bar{x}_0 \quad (5-1)$$

where

$$\begin{aligned} \bar{x}_1 &= \text{P4GS (psi) ,} \\ \bar{x}_2 &= \text{P8GS (psi) ,} \\ \bar{x}_3 &= \text{NL (rpm) ,} \\ \bar{x}_4 &= \text{NH (rpm) ,} \end{aligned}$$

and the three-tuple  $u$  is taken to be

$$u(t) = \bar{u}(t) - \bar{u}_0 \quad (5-2)$$

where

$$\begin{aligned} \bar{u}_1 &= \text{WFM (lb}_m\text{/hr) ,} \\ \bar{u}_2 &= \text{A18 (in}^2\text{) ,} \\ \bar{u}_3 &= \text{BETAF (deg.) ,} \end{aligned}$$

then from Section 2.1 the differential equation representation, with tensor notation applied, has the following form:

$$\begin{aligned} \dot{x} &= \hat{L}_{10} x + \hat{L}_{01} u \\ &+ \hat{L}_{20} (x \otimes x) + \hat{L}_{11} (x \otimes u) \\ &+ \hat{L}_{02} (u \otimes u) + \dots \end{aligned} \quad (5-3)$$

It should be noted that in the off-simulator examples (See [ 3 ]) the equilibrium point (point of expansion) was taken to be zero so that a multivariable equivalent of a Maclaurin series expansion was used. In this study, however, the point of expansion is not zero and a Taylor series of several variables is used.

The model presented in this section is of degree two; that is, second degree tensor terms

$$(x \otimes x), (x \otimes u), (u \otimes u)$$

are the highest degree terms retained. Therefore, the truncation approximation of equation (5-3) takes on the structure

$$\dot{x} \approx [\hat{L}_{10} \hat{L}_{01} \hat{L}_{20} \hat{L}_{11} \hat{L}_{02}] x_T . \quad (5-4)$$

In this particular situation  $x_T$  contains thirty-five monomials, seven of degree one, and twenty-eight of degree two. These are

$$\begin{aligned} x_T = & (x_1, x_2, x_3, x_4, u_1, u_2, u_3, x_1^2, x_1x_2, x_1x_3, x_1x_4, x_2^2, x_2x_3, x_2x_4, \\ & x_3^2, x_3x_4, x_4^2, x_1u_1, x_1u_2, x_1u_3, x_2u_1, x_2u_2, x_2u_3, x_3u_1, x_3u_2, \\ & x_3u_3, x_4u_1, x_4u_2, x_4u_3, u_1^2, u_1u_2, u_1u_3, u_2^2, u_2u_3, u_3^2)^T . \end{aligned} \quad (5-5)$$

Here, the elements of (5-5) are listed according to the established ordering convention.

Appropriate engine operation for the model identification involves opening the loop by deactivating the controller and independently inserting the individual control inputs (See [22]). In this strategy, nonlinearities of the plant exist—which might otherwise be less noticeable had the controller been present in the loop. This scheme is portrayed in Figure 5.1 where a constant desired power percentage (DPP) is applied and the controller is effectively "turned-off" by equating the controller state derivatives with zero. In this way, an input vector of the form

$$(\bar{u})_i = (\bar{u}_0)_i + \alpha_i \cos(2\pi \phi_i t), \quad i = 1, 2, 3 \quad (5-6)$$

can be substituted, where  $\alpha$  and  $\phi$  are three-tuples containing amplitudes and frequencies (in hertz) respectively. An important point to

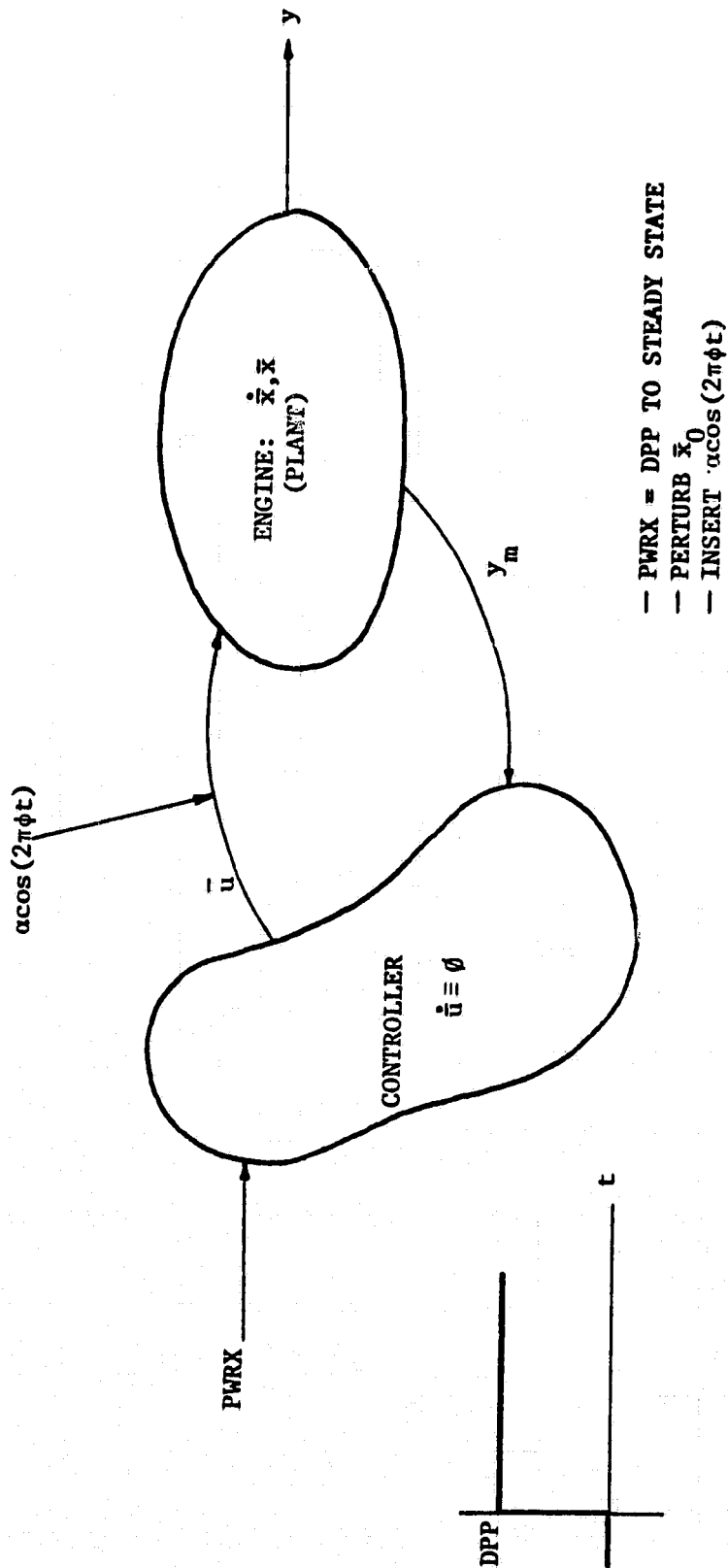


Figure 5.1. Open Loop Identification Scheme.

note is that, in the open loop situation, the choice of input signals is critical. This is due to the fact the engine itself has certain physical limits, which in turn have been placed in the simulator. In reality, exceeding these limits could cause severe damage to the engine, an example of which is turbine melt down.

Using the QCSEE digital simulation, and implementing the above ideas, a CATNAF loader routine (QCSELOAD) is constructed to set up the problem for the least squares identification. Moreover, if  $h$  time points are used in the sampling procedure, the matrix equation

$$\dot{X} \approx [L_{10} \ L_{01} \ L_{20} \ L_{11} \ L_{02}] X_T \quad (5-7)$$

$\begin{matrix} 4 & \downarrow & & & & \\ x & \times & h & & & \\ & & & 4 & \downarrow & \\ & & & x & \times & 35 \\ & & & & & & 35 & \downarrow & \\ & & & & & & x & \times & h \end{matrix}$

is formulated, with matrix dimensions as shown, for fourth order models keeping second degree tensor terms in the approximation. In addition, provisions are made available in the code which permit normalization of the sampled data according to the scheme outlined in Section 2.2.

In order to perform comparative model simulations, the engine program must be incorporated into a CATNAF simulator routine. Consequently, program QIXSIM is built using QCSEE as the true system in the application subroutine TRUES. QIXSIM is used extensively in the next section where results for various simulations of the example model are presented.

## 5.2 EXAMPLE MODEL

To aid in the selection of input signals, a family of parametric plots have been constructed using QCSEE steady state data from idle (62.5% PWRX) to maximum power (100% PWRX). Figure 5.2 contains an example of one such steady state plot. From these plots a set of accept-

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STEADY STATE PLOTS:  
○ - WFM; △ - A18; ★ - BETA

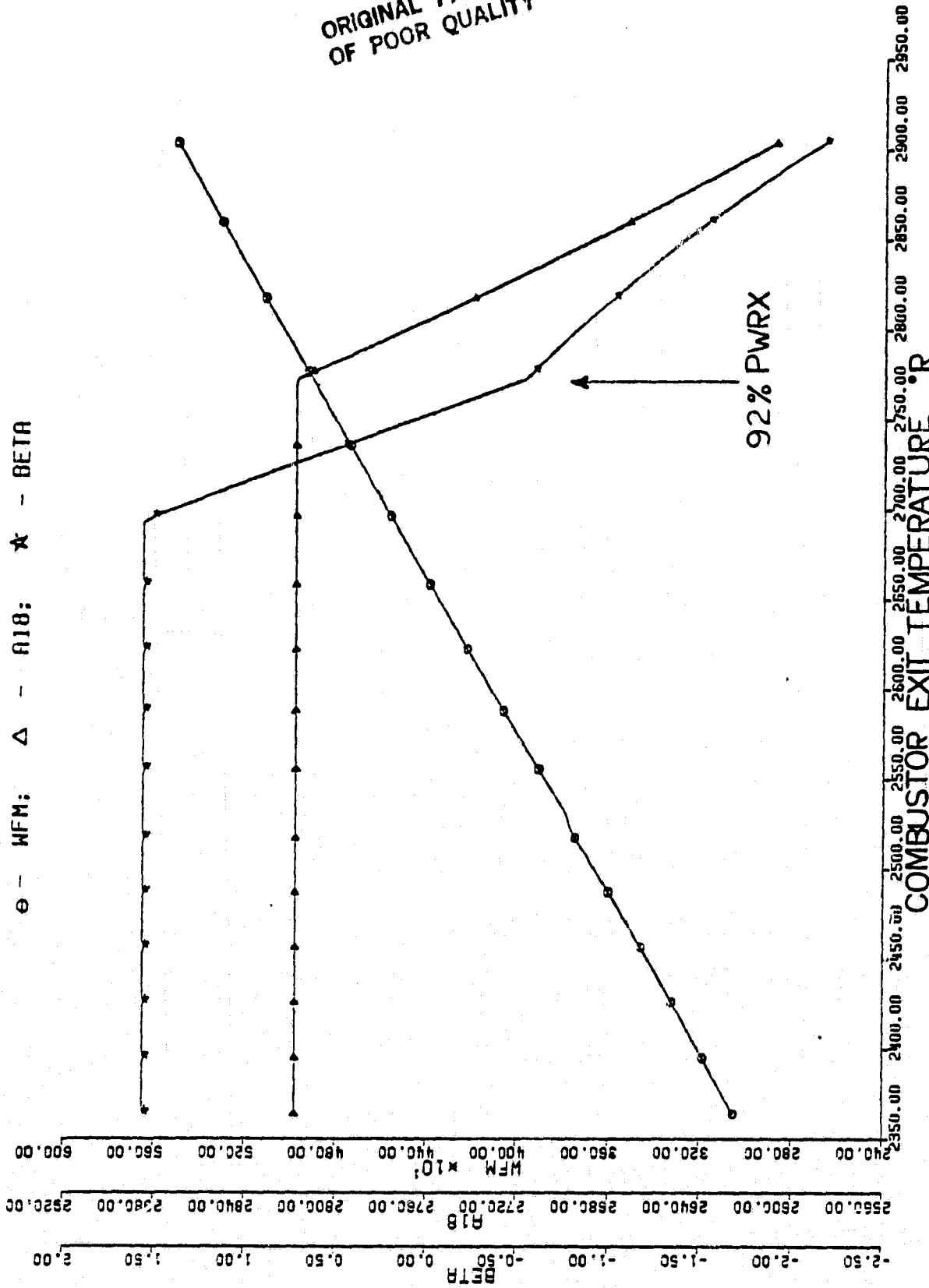


Figure 5.2. Example Steady State Response: Parameterization with PWRX.

able input signals can be selected. Acceptable state perturbations can be selected in a similar fashion. Another important feature of these steady state plots is that they suggest regions of nonlinearity. From Figure 5.2 it appears that in the locality of 92% power demand the engine is nonlinear due to the abrupt changes in exhaust nozzle area and fan pitch angle, with this in mind, 92% shall be established as the operating point of the current example.

The following is an overview of the particular model formulation using QCSELOAD. The QCSEE simulator is run, closed loop, with a 92% power demand for ten seconds to settle all transients. This produces the equilibrium value  $(\bar{x}_0, \bar{u}_0)$ . The initial conditions thus generated form the point of expansion for the series truncation approximation in the model formulation. Within the digital simulation program the controller is disconnected by setting the control derivatives to zero. From the steady state plots a point  $(\hat{x}, \hat{u})$  is chosen on the engine operating line at 93% power demand. The state variables are perturbed  $\tilde{x}$  from their equilibrium values where

$$\tilde{x} = \hat{x} - \bar{x}_0 . \quad (5-8)$$

Furthermore, the control variables are manipulated so that the input vector (eq. 5-6) is inserted, where  $\alpha$  is given by the expression

$$\alpha = \hat{u} - \bar{u}_0 , \quad (5-9)$$

and

$$\phi = (2.0, 1.0, 1.3) .$$

The observed states and inputs are sampled over a six second interval; 100 samples are evenly spaced at 0.06 seconds, and the normalized dif-

ference between these values and the corresponding equilibrium values, together with the normalized ordered tensor product terms and normalized state derivative values comprise the data necessary for the identification. Once this is complete, the model parameters contained in matrix representations of the  $\hat{L}_{ij}$  are easily computed from IDENTIFY and then sent to the PDP 11/60 via TRANSFER.

Using the GENERATE segment of CATNAP, two models have been identified: a second-degree nonlinear model, and a first-degree linear model. The linear model has been identified for use in comparison studies. Accordingly, such a nonlinear model is expected to out perform the linear model in a region about the point of expansion. Complete listings of the identified model operators for both the linear and nonlinear models are contained in Appendix C.

Verification of the nonlinear model involves conducting numerous simulations for various combinations of the parameters  $x_0$ ,  $\alpha$ ,  $\phi$ , and comparing results against those of the true system. Frequently though, as is the case here, a third system is included in the comparison studies; in the form of a linear approximation. This provides the engineer with a gauge to determine how well a new nonlinear model performs as compared to a traditional representation. Utilizing the CATNAP capabilities of QIXSIM and HARDCOPY, several validation studies have been completed to date, all yielding satisfactory results.

Four such tests will be presented here, the first of which consists of twenty-five simulations along the line of engine operation with



different choices of  $x_0$ , and  $\alpha$ , and one triplet  $(\phi_1, \phi_2, \phi_3)$ . Results of this test are tabulated in Table 5.1, where  $\phi = (2.1, 1.0, 1.5)$  for all simulations. By observing the table, one can see that the error criterion  $(\epsilon_1)$  discussed in Section 3.3 is negative for each state  $(x_1)$  throughout all individual simulations. This indicates that the model has outperformed the linear approximation in a region about the expansion point. To illustrate the tracking ability of the model, consider a representative comparison plot of these solutions against the true solution. Take as an example the simulation with conditions from Table 5.1, line 18 which is depicted in Figures 5.3 and 5.4. Figure 5.3 is presented merely to give the reader an opportunity to see the format of a CATNAP generated data sheet, whereas the actual trajectory comparisons are presented in Figure 5.4. Clearly, the model solution tracks the true solution well in all four states throughout the time interval.

The second test represents on the line studies of model performance for a variety of input frequencies and zero initial conditions. Table 5.2 shows the results of these error analyses. As in the previous test, the  $\epsilon_1$  are negative for all simulations. A comparison plot of one of these tests, Table 5.2 line 8, is given in Figure 5.5a for variable P4GS and in Figure 5.5b for variable NL. The next figure shows the ability of the model to track a step response. Figures 5.6a and 5.6b offer comparison plots with conditions from Table 5.2 line 7 for variables P4GS and P8GS respectively. In both Figures 5.5 and 5.6 the model simulation follows the true solution well while the linear approximation is less accurate.

Table 5.1: Comparison Studies on the Operating Line;  $\phi = (2.1, 1.0, 1.5)$ 

Initial State Conditions				Input Amplitudes			Error Criteria			
$x_1$	$x_2$	$x_3$	$x_4$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\epsilon_1$	$\epsilon_2$	$\epsilon_3$	$\epsilon_4$
0.010	0.001	0.010	0.500	-18.800	0.000	0.115	-0.224	-0.004	-1.71	-3.46
0.010	0.001	0.010	0.500	37.350	-6.830	-0.159	-0.203	-0.004	-2.56	-3.98
0.010	0.001	0.010	0.500	55.780	-14.030	-0.198	-0.246	-0.004	-3.63	-5.39
0.010	0.001	0.010	0.500	74.290	-21.200	-0.239	-0.296	-0.006	-5.18	-6.99
0.010	-0.001	0.010	-0.750	-18.800	0.000	0.115	-0.307	-0.005	-1.77	-4.89
0.010	-0.001	0.010	-0.750	18.920	0.000	-0.111	-0.297	-0.006	-1.97	-5.55
0.010	-0.001	0.010	-0.750	55.780	-14.030	-0.198	-0.367	-0.007	-2.05	-8.00
-0.010	0.001	-0.010	0.750	-37.460	0.000	0.228	-0.318	-0.006	-1.48	-5.37
-0.010	0.001	-0.010	0.750	-18.800	0.000	0.115	-0.281	-0.005	-1.91	-4.38
-0.010	0.001	-0.010	0.750	37.350	-6.830	-0.159	-0.260	-0.005	-3.01	-4.86
-0.010	0.001	-0.010	0.750	55.780	-14.030	-0.198	-0.289	-0.005	-4.20	-6.24
-0.010	0.001	-0.010	0.750	74.290	-21.200	-0.239	-0.341	-0.006	-5.60	-7.80
-0.010	-0.001	-0.010	-0.750	-18.800	0.000	0.115	-0.210	-0.004	-1.43	-3.47
-0.010	-0.001	-0.010	-0.750	18.920	0.000	-0.111	-0.249	-0.004	-1.49	-3.57
-0.010	-0.001	-0.010	-0.750	55.780	-14.030	-0.198	-0.396	-0.007	-2.01	-6.15
-0.010	-0.001	-0.010	-0.750	74.290	-21.200	-0.239	-0.466	-0.008	-4.87	-7.48
-0.075	0.001	-0.250	0.500	55.780	-14.030	-0.198	-0.491	-0.009	-5.76	-9.44
-0.075	0.001	-0.250	0.500	74.290	-21.200	-0.239	-0.533	-0.009	-6.98	-11.10
0.050	0.002	2.500	5.000	74.290	-21.200	-0.239	-0.243	-0.004	-4.88	-7.59
0.075	0.002	5.000	2.500	74.290	-21.200	-0.239	-0.239	-0.004	-5.19	-7.54
0.100	0.002	2.000	1.000	18.920	0.000	-0.111	-0.124	-0.002	-0.85	-2.11
0.100	0.002	2.000	1.000	37.350	-6.830	-0.159	-0.145	-0.002	-1.30	-2.98
0.100	0.002	2.000	1.000	55.780	-14.030	-0.198	-0.190	-0.003	-2.72	-4.54
0.100	0.002	2.000	1.000	74.290	-21.200	-0.239	-0.239	-0.004	-5.04	-7.19
0.100	0.002	2.500	5.000	74.290	-21.200	-0.239	-0.135	-0.003	-3.95	-6.13

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\*\*\*\*\* PROBLEM SUMMARY \*\*\*\*\*  
 CONFIGURATION: TRUE, LINEAR & NONLINEAR  
 NUMBER OF STATES: 4  
 NUMBER OF CONTROLS: 3  
 LENGTH OF TENSOR TERM VECTOR: 35  
 DEGREE OF APPROXIMATION: 2  
 \*\*\*\*\*

SOLUTION PARAMETERS:  
 \*\*\*\*\*

INTEGRATION STEP SIZE: 0.010  
 UPPER TIME LIMIT OF INTEGRATION: 2.000  
 NUMBER OF PLOT POINTS: 100  
 SPACING BETWEEN PLOT POINTS: 0.020

STATE NUMBER	INITIAL CONDITION	ERROR CRITERION
1	-0.075	-0.533E+00
2	0.001	-0.915E-02
3	-0.250	-0.698E+01
4	0.500	-0.111E+02

CONTROL NUMBER	AMPLITUDE	FREQUENCY (CYCLES/SEC)
1	74.290	2.100
2	-21.200	1.000
3	-0.239	1.500

Figure 5.3. Data Sheet for Figure 5.4.

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COMPARATIVE SOLUTION PLOTS: STATE # 1  
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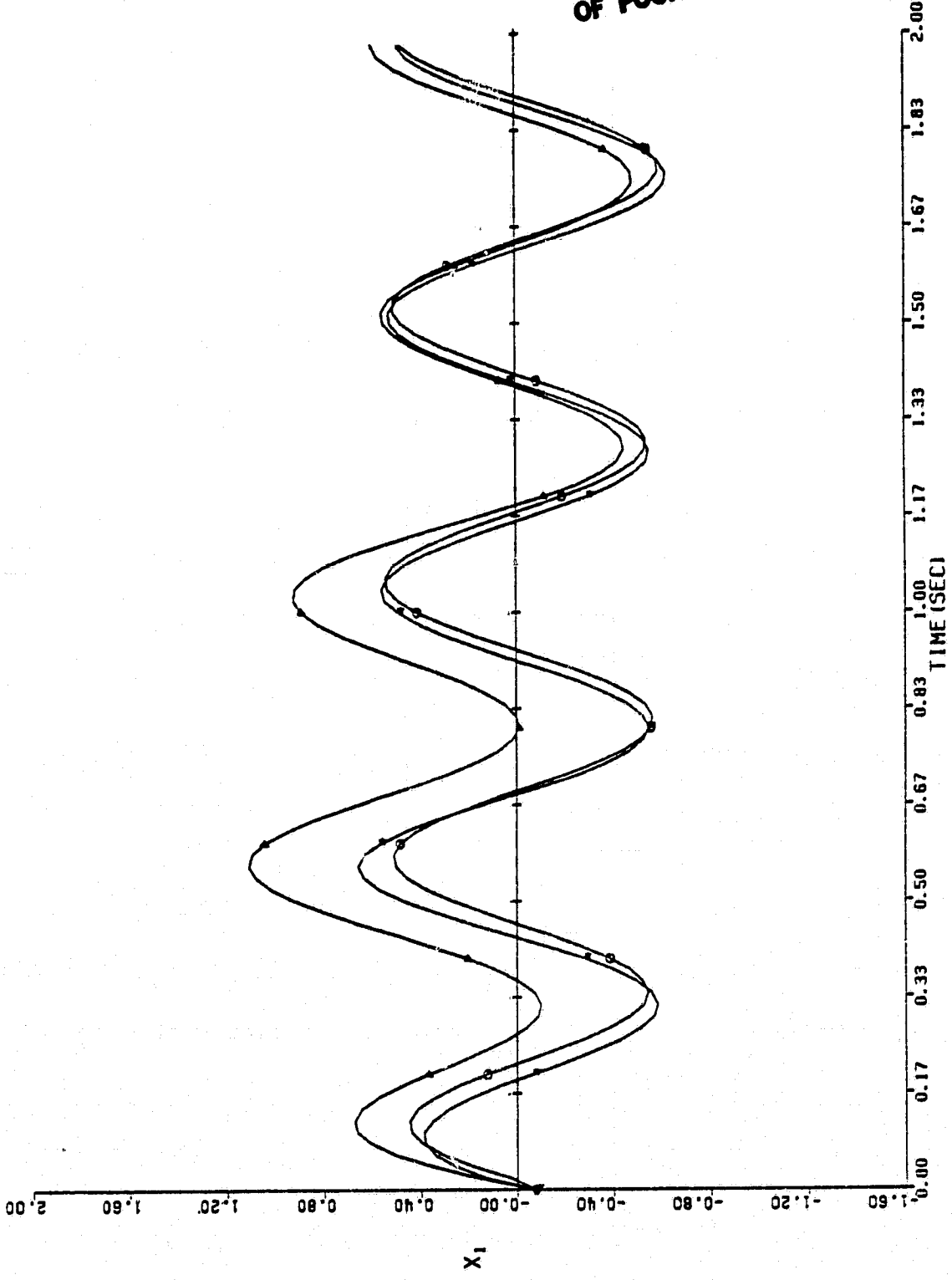


Figure 5.4a. Table 5.1, Line 18.

COMPARATIVE SOLUTION PLOTS: STATE # 2  
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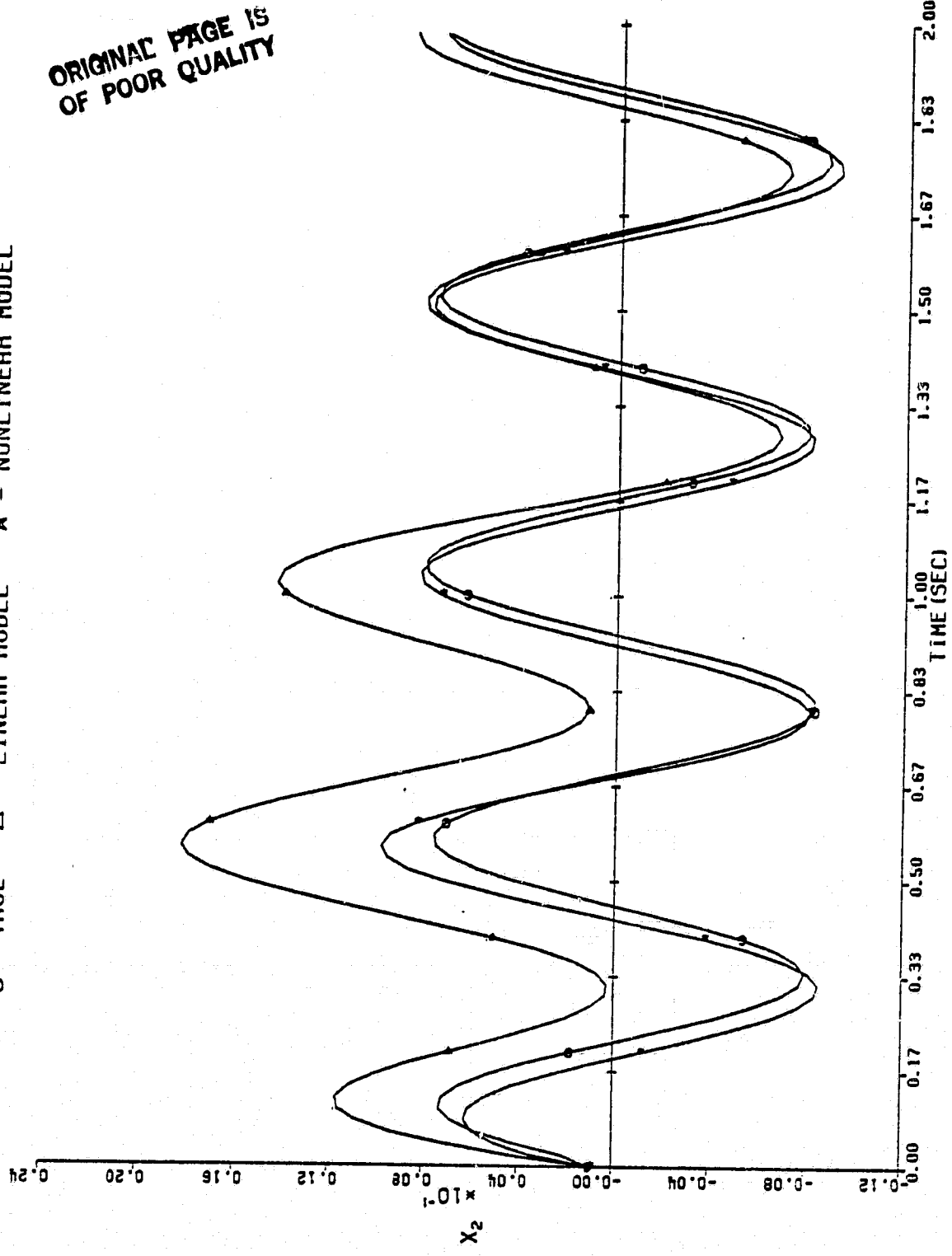


Figure 5.4b. Table 5.1, Line 18.

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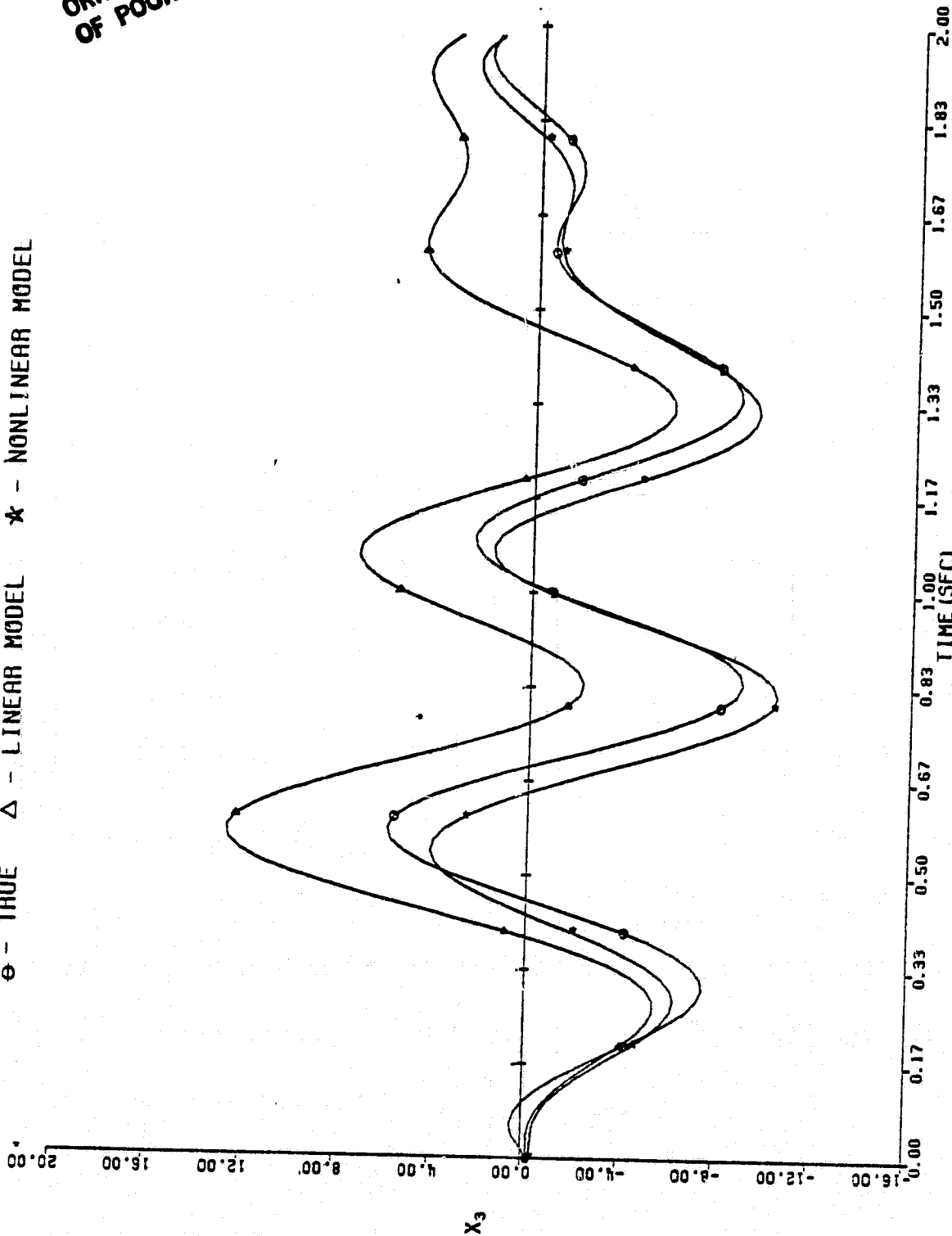


Figure 5.4c. Table 5.1.1, Line 18.

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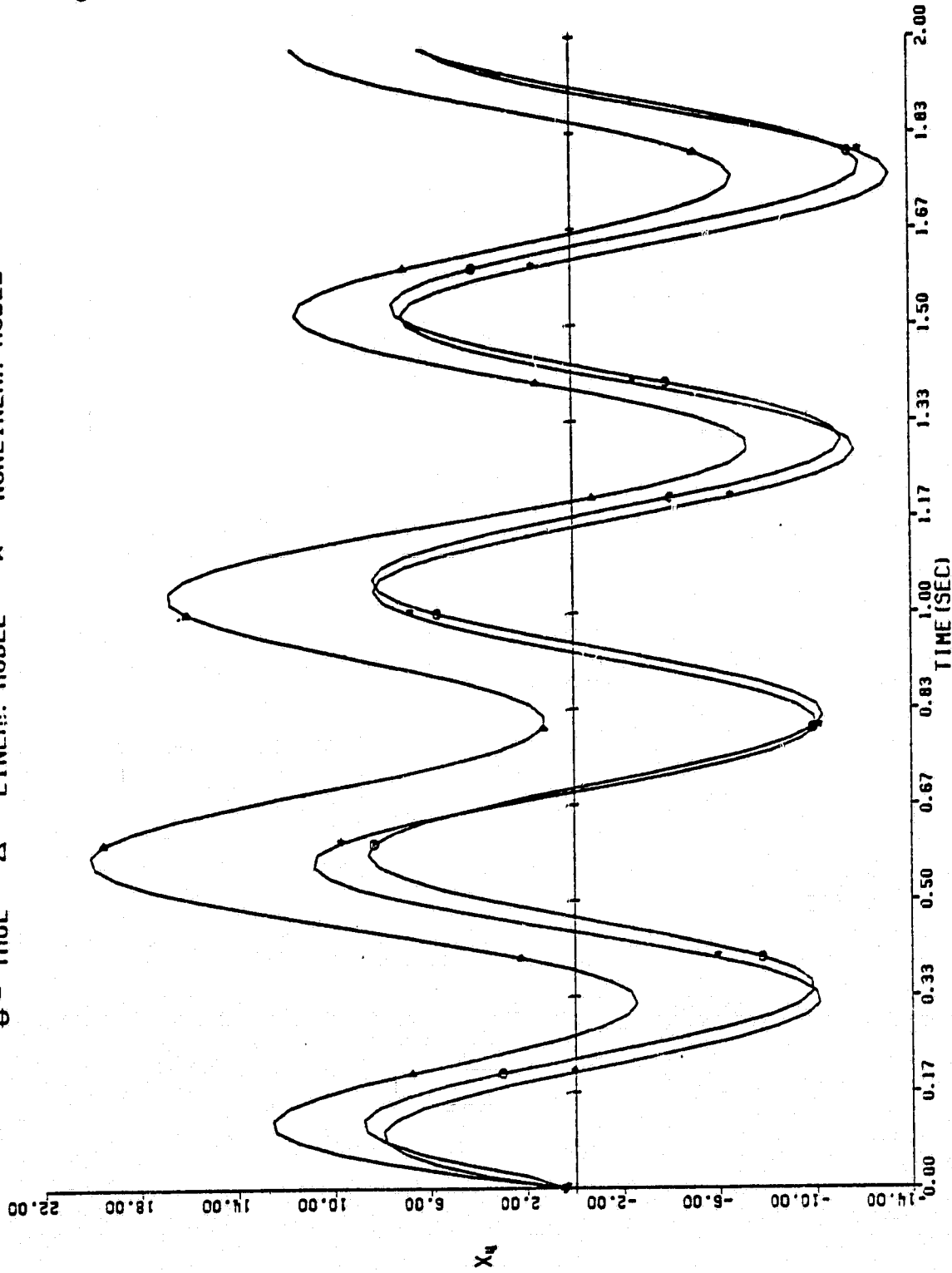


Figure 5.4d. Table 5.1, Line 18.

Table 5.2: Comparison Studies on the Operating Line;  $x_0 = (0.0, 0.0, 0.0, 0.0, 0.0)$ 

Input Amplitudes			Input Freq. (Hz)			Error Criteria			
$\alpha_1$	$\alpha_2$	$\alpha_3$	$\phi_1$	$\phi_2$	$\phi_3$	$\epsilon_1$	$\epsilon_2$	$\epsilon_3$	$\epsilon_4$
-18.800	0.000	0.115	0.25	0.00	1.00	-0.824	-0.015	-4.46	-12.70
-18.800	0.000	0.115	0.25	0.00	0.75	-0.789	-0.014	-4.95	-12.90
-18.800	0.000	0.115	0.25	0.00	1.25	-0.830	-0.015	-4.91	-13.50
-18.800	0.000	0.115	0.25	0.00	1.50	-0.821	-0.015	-5.07	-13.70
-18.800	0.000	0.115	0.35	0.00	0.50	-0.552	-0.010	-4.22	-8.51
-18.800	0.000	0.115	0.75	0.00	0.50	-0.196	-0.004	-1.39	-3.39
18.920	0.000	-0.111	0.00	0.00	0.00	-1.040	-0.018	-10.40	-14.00
18.920	0.000	-0.111	0.25	0.00	0.50	-0.696	-0.013	-7.46	-8.75
18.920	0.000	-0.111	0.25	0.00	0.75	-0.781	-0.014	-5.01	-13.20
18.920	0.000	-0.111	0.25	0.00	1.00	-0.816	-0.014	-4.42	-14.00
18.920	0.000	-0.111	0.25	0.00	1.25	-0.838	-0.014	-4.98	-13.70
18.920	0.000	-0.111	0.25	0.00	1.50	-0.826	-0.015	-4.74	-13.80
18.920	0.000	-0.111	0.35	0.00	0.50	-0.526	-0.009	-5.01	-9.94
18.920	0.000	-0.111	0.75	0.00	0.50	-0.211	-0.004	-1.63	-3.13
18.920	0.000	-0.111	1.00	0.00	0.50	-0.164	-0.002	-0.88	-1.81
37.350	-6.830	-0.159	2.00	1.00	1.30	-0.057	-0.001	-0.68	-1.37
37.350	-6.830	-0.159	2.00	1.50	1.00	-0.057	-0.001	-0.77	-1.47
55.780	-14.030	-0.198	2.00	1.00	1.30	-0.100	-0.002	-2.50	-2.79
55.780	-14.030	-0.198	2.00	1.50	1.00	-0.097	-0.002	-2.57	-3.01
55.780	-14.030	-0.198	2.00	1.75	1.50	-0.079	-0.001	-1.73	-1.95
74.290	-21.200	-0.239	3.00	1.00	1.50	-0.111	-0.002	-1.71	-3.41
74.290	-21.200	-0.239	2.00	1.00	1.30	-0.148	-0.003	-4.63	-5.40
74.290	-21.200	-0.239	2.00	1.50	1.00	-0.146	-0.002	-5.26	-5.55
74.290	-21.200	-0.239	2.00	1.75	1.50	-0.112	-0.002	-4.17	-4.12
74.290	-21.200	-0.239	3.00	1.00	1.50	-0.178	-0.003	-3.92	-5.75



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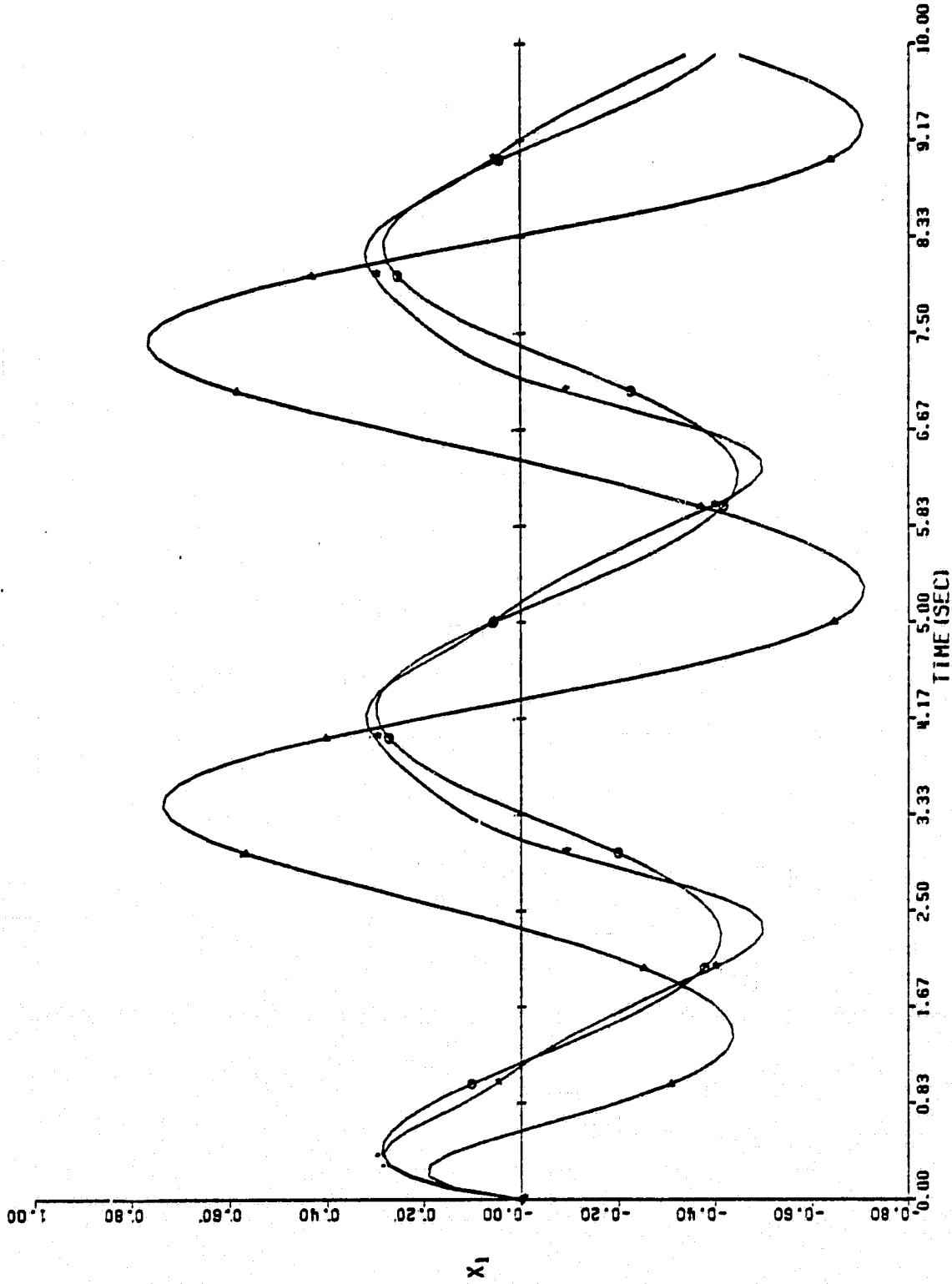


Figure 5.5a. Table 5.2, Line 8.

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COMPARATIVE SOLUTION PLOTS: STATE # 3  
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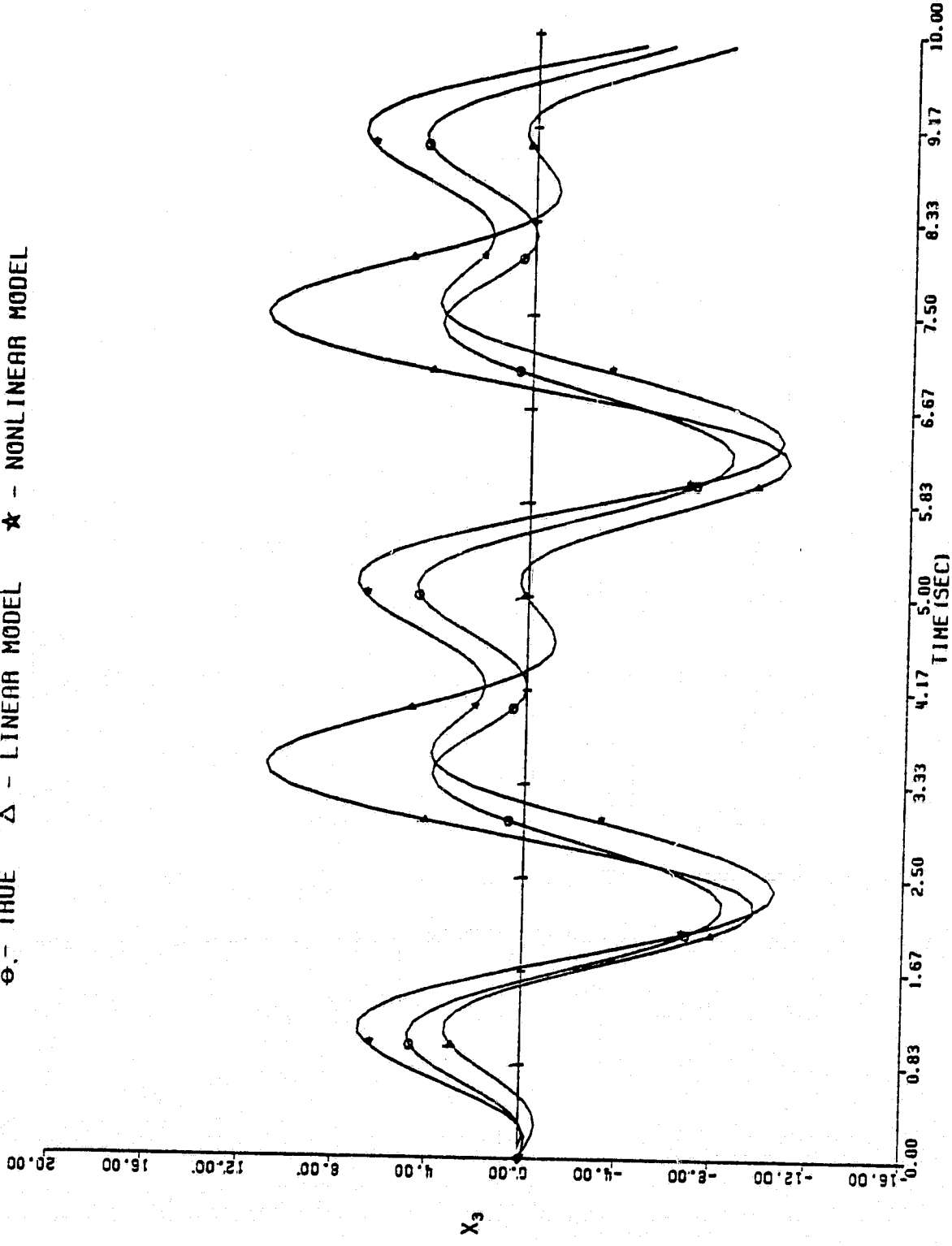


Figure 5.5b. Table 5.2, Line 8.

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COMPARATIVE SOLUTION PLOTS: STATE # 1  
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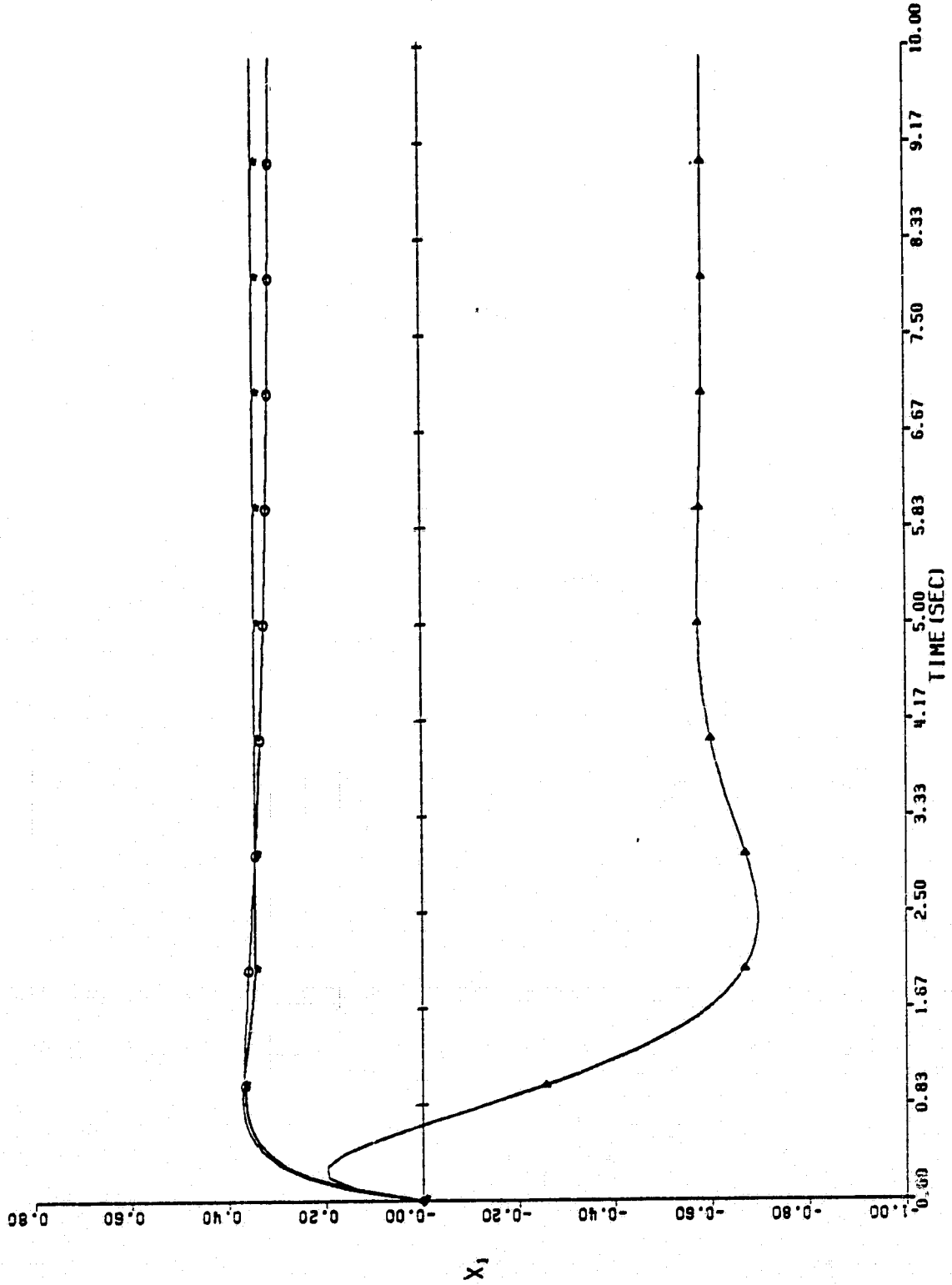
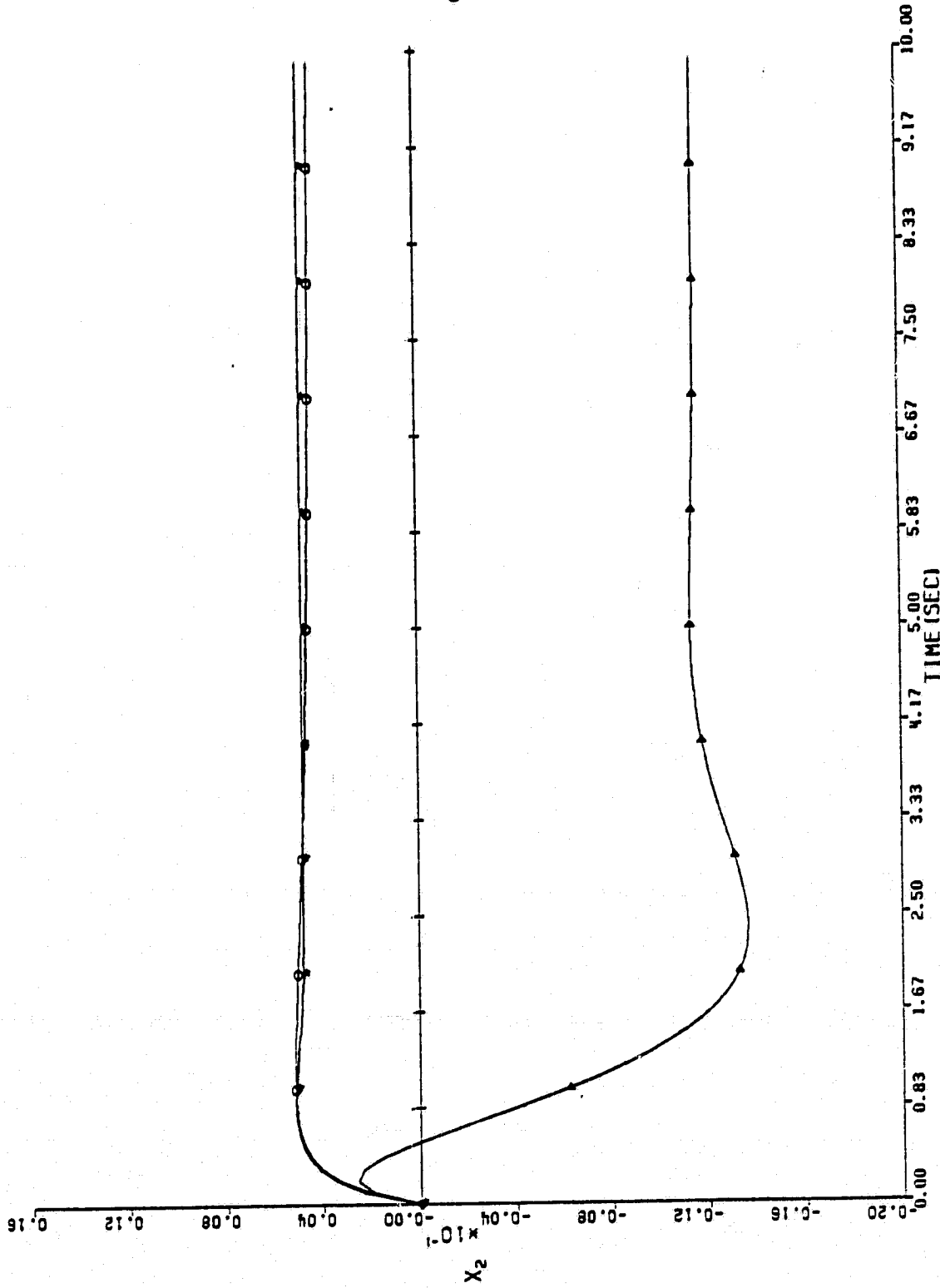


Figure 5.6a. Table 5.2, Line 7.

COMPARATIVE SOLUTION PLOTS: STATE # 2  
○ - TRUE    △ - LINEAR MODEL    ★ - NONLINEAR MODEL



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Figure 5.6b. Table 5.2, Line 7.

The third test considers several possible combinations of  $x_0$ , and  $\phi$ , and fixes  $\alpha = (55.78, -14.03, -0.198)$ . These results are recorded in Table 5.3. Again, the error analyses indicate that the model outperforms the linear approximation for the various test conditions. Figure 5.7 offers an illustration which uses the conditions from line 20 of Table 5.3.

The final test represents analyses for various amplitudes and frequencies less initial conditions. The unique feature of this test is that the excursions are made away from the typical engine line of operation. This is done because in practice such a model may be required to perform within a tubular region, with the operating line at its center, instead of just along the line itself. The error analyses associated with these simulations are contained in Table 5.4. The final plots to be presented here are for a simulation with conditions from line 6 of Table 5.4. Figure 5.8a offers the comparative plot for variable P8GS while Figure 5.8b illustrates variable NH.

Table 5.3: Comparison Studies on the Operating Line;  $\alpha = (55.78, -14.03, -0.198)$ 

Initial State Conditions				Input Freq. (Hz)			Error Criteria			
$x_1$	$x_2$	$x_3$	$x_4$	$\phi_1$	$\phi_2$	$\phi_3$	$\epsilon_1$	$\epsilon_2$	$\epsilon_3$	$\epsilon_4$
0.010	0.000	0.01	-1.00	1.80	0.80	1.35	-0.183	-0.003	-2.66	-4.52
0.010	0.000	0.01	-1.00	1.80	0.90	1.35	-0.173	-0.003	-3.39	-3.41
0.010	0.000	0.01	-1.00	1.80	1.00	1.35	-0.167	-0.003	-2.61	-3.83
0.010	0.000	0.01	-1.00	1.80	1.10	1.35	-0.162	-0.003	-2.58	-3.76
0.010	0.000	0.01	-1.00	1.80	1.20	1.35	-0.157	-0.003	-2.04	-3.45
0.010	0.000	0.01	-1.00	1.80	1.30	1.35	-0.153	-0.003	-2.78	-3.68
0.010	0.000	0.01	-1.00	1.80	1.40	1.35	-0.149	-0.003	-2.84	-3.15
0.010	0.000	0.01	-1.00	1.80	1.50	1.35	-0.146	-0.003	-3.40	-2.44
-0.025	0.002	4.00	2.00	1.80	1.10	0.75	-0.565	-0.010	-6.47	-10.80
-0.025	0.002	4.00	2.00	1.80	1.10	0.85	-0.563	-0.010	-6.22	-11.40
-0.025	0.002	4.00	2.00	1.80	1.10	0.95	-0.565	-0.010	-5.98	-11.90
-0.025	0.002	4.00	2.00	1.80	1.10	1.15	-0.589	-0.010	-5.55	-12.40
-0.025	0.002	4.00	2.00	1.80	1.10	1.25	-0.599	-0.011	-5.39	-12.60
-0.025	0.002	4.00	2.00	1.80	1.10	1.35	-0.605	-0.011	-5.24	-12.70
-0.025	0.002	4.00	2.00	1.80	1.10	1.45	-0.611	-0.011	-5.14	-12.80
-0.025	0.002	4.00	2.00	1.80	1.10	1.55	-0.617	-0.011	-5.05	-12.80
-0.025	0.002	4.00	2.00	1.80	1.10	1.65	-0.621	-0.011	-4.95	-12.80
0.110	0.001	-20.00	12.00	1.60	1.10	1.35	-0.472	-0.009	-5.03	-6.66
0.110	0.001	-20.00	12.00	1.70	1.10	1.35	-0.460	-0.008	-4.94	-5.89
0.110	0.001	-20.00	12.00	1.80	1.10	1.35	-0.447	-0.008	-4.98	-5.51
0.110	0.001	-20.00	12.00	1.90	1.10	1.35	-0.422	-0.007	-4.98	-5.68
0.110	0.001	-20.00	12.00	2.00	1.10	1.35	-0.416	-0.007	-4.98	-5.79
0.110	0.001	-20.00	12.00	2.10	1.10	1.35	-0.425	-0.008	-5.07	-5.86
0.110	0.001	-20.00	12.00	2.22	1.10	1.35	-0.425	-0.008	-5.09	-5.88
0.110	0.001	-20.00	12.00	2.30	1.10	1.35	-0.421	-0.008	-5.15	-5.90

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COMPARATIVE SOLUTION PLOTS: STATE # 1  
○ - TRUE    △ - LINEAR MODEL    ★ - NONLINEAR MODEL

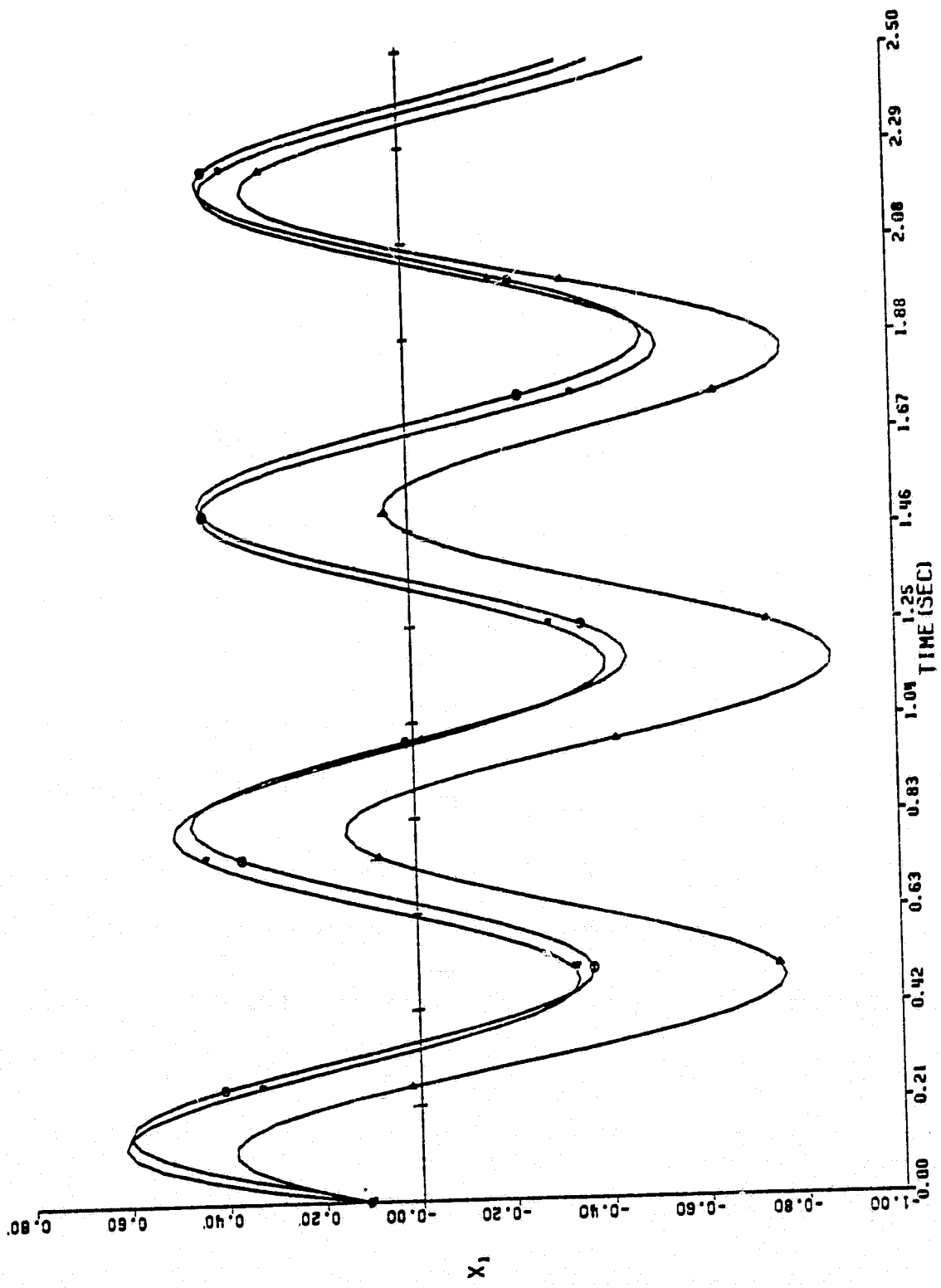


Figure 5.7a. Table 5.3, Line 20.

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COMPARATIVE SOLUTION PLOTS: STATE # 2  
e - TRUE    Δ - LINEAR MODEL    ★ - NONLINEAR MODEL

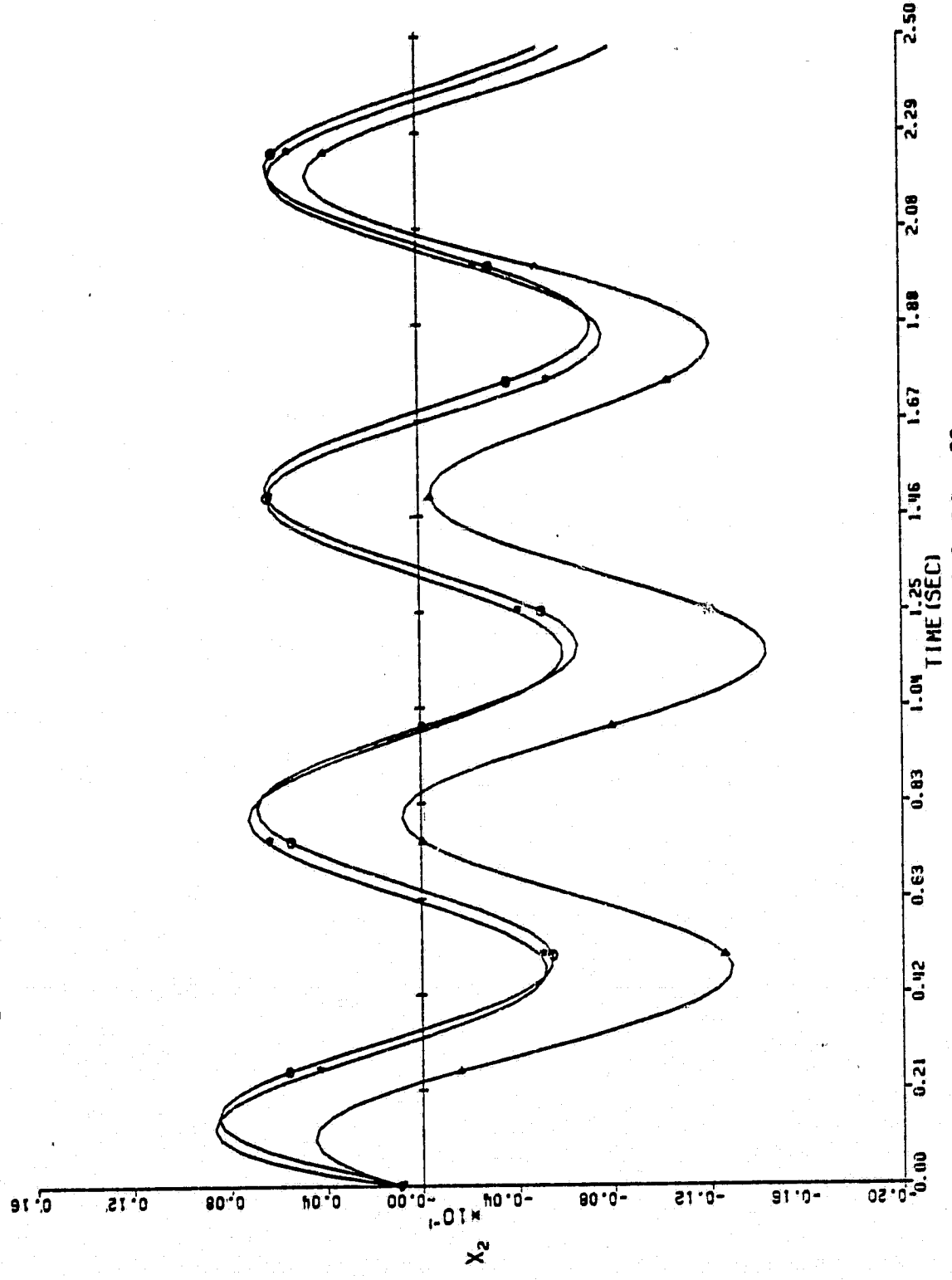


Figure 5.7b. Table 5.3, Line 20.



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COMPARATIVE SOLUTION PLOTS: STATE # 3  
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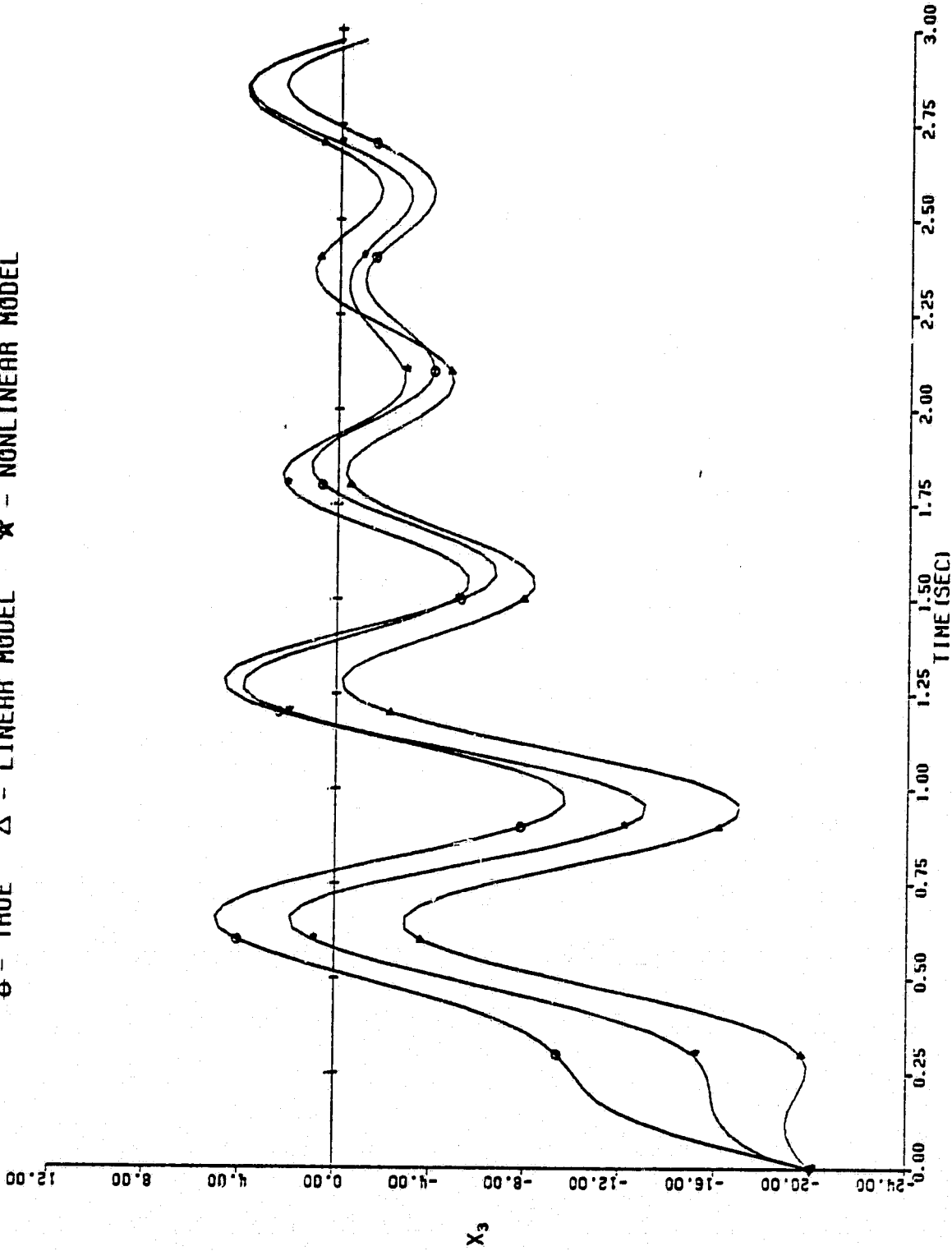


Figure 5.7c. Table 5.3, Line 20.

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COMPARATIVE SOLUTION PLOTS: STATE # 4  
o - TRUE    Δ - LINEAR MODEL    ★ - NONLINEAR MODEL

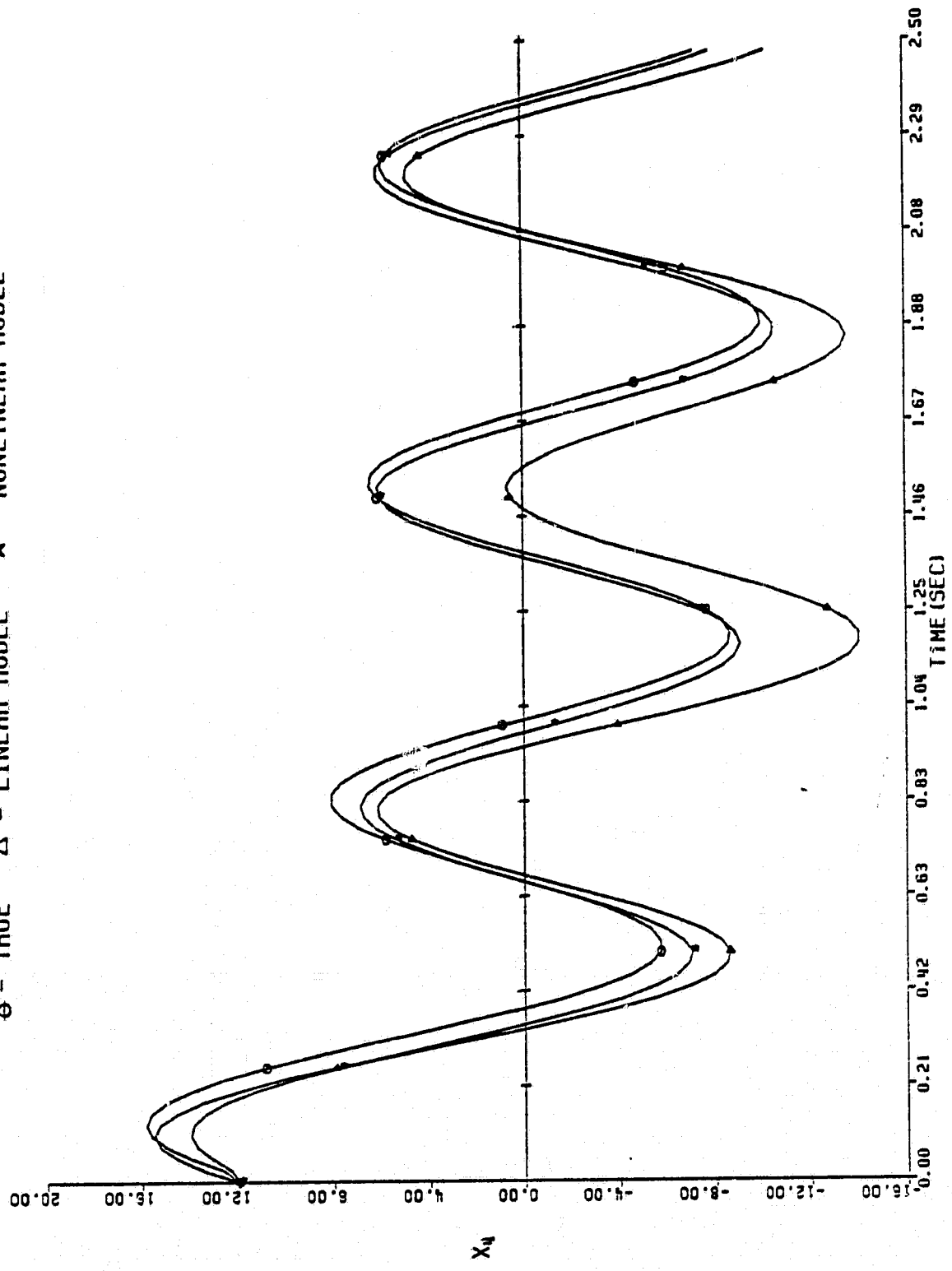


Figure 5.7d. Table 5.3, Line 20.

Table 5.4: Comparison Studies off the Operating Line;  $x_0 = (0.0, 0.0, 0.0, 0.0, 0.0)$ 

Input Amplitudes			Input Freq. (Hz)			Error Criteria			
$\alpha_1$	$\alpha_2$	$\alpha_3$	$\phi_1$	$\phi_2$	$\phi_3$	$\epsilon_1$	$\epsilon_2$	$\epsilon_3$	$\epsilon_4$
50.000	-21.200	- 0.239	1.50	1.00	1.30	-0.179	-0.003	- 5.09	- 4.47
55.000	-21.200	- 0.239	1.50	1.00	1.30	-0.187	-0.003	- 5.36	- 4.85
60.000	-21.200	- 0.239	1.50	1.00	1.30	-0.195	-0.004	- 5.69	- 5.31
68.000	-21.200	- 0.239	1.50	1.00	1.30	-0.207	-0.004	- 6.47	- 5.96
70.000	-21.200	- 0.239	1.50	1.00	1.30	-0.210	-0.004	- 6.64	- 5.83
73.000	-21.200	- 0.239	1.50	1.00	1.30	-0.218	-0.004	- 6.22	- 5.60
75.000	-21.200	- 0.239	1.50	1.00	1.30	-0.224	-0.004	- 5.91	- 6.16
77.000	-21.200	- 0.239	1.50	1.00	1.30	-0.239	-0.004	- 5.58	- 6.70
80.000	-21.200	- 0.239	1.50	1.00	1.30	-0.242	-0.004	- 5.28	- 4.86
74.290	-16.000	- 0.239	1.75	0.75	1.50	-0.247	-0.004	- 3.09	- 6.48
74.290	-17.000	- 0.239	1.75	0.75	1.50	-0.255	-0.004	- 3.58	- 6.68
74.290	-18.000	- 0.239	1.75	0.75	1.50	-0.263	-0.004	- 4.20	- 6.86
74.290	-19.000	- 0.239	1.75	0.75	1.50	-0.271	-0.005	- 4.65	- 7.02
74.290	-20.000	- 0.239	1.75	0.75	1.50	-0.278	-0.005	- 5.34	- 7.32
74.290	-21.000	- 0.239	1.75	0.75	1.50	-0.283	-0.005	- 5.87	- 7.50
74.290	-22.000	- 0.239	1.75	0.75	1.50	-0.287	-0.005	- 6.60	- 7.78
74.290	-23.000	- 0.239	1.75	0.75	1.50	-0.291	-0.005	- 7.19	- 7.84
74.290	-21.200	- 0.250	2.00	1.25	1.00	-0.186	-0.003	- 5.04	- 6.91
74.290	-21.200	- 0.275	2.00	1.25	1.00	-0.178	-0.003	- 5.18	- 6.65
74.290	-21.200	- 0.300	2.00	1.25	1.00	-0.171	-0.003	- 5.33	- 6.23
74.290	-21.200	- 0.325	2.00	1.25	1.00	-0.168	-0.003	- 5.37	- 5.89
74.290	-21.200	- 0.350	2.00	1.25	1.00	-0.166	-0.003	- 5.53	- 5.60
74.290	-21.200	- 0.375	2.00	1.25	1.00	-0.165	-0.003	- 5.40	- 5.36
74.290	-21.200	- 0.400	2.00	1.25	1.00	-0.165	-0.003	- 5.27	- 5.16
74.290	-21.200	- 0.450	2.00	1.25	1.00	-0.166	-0.003	- 5.45	- 4.88

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COMPARATIVE SOLUTION PLOTS: STATE # 2  
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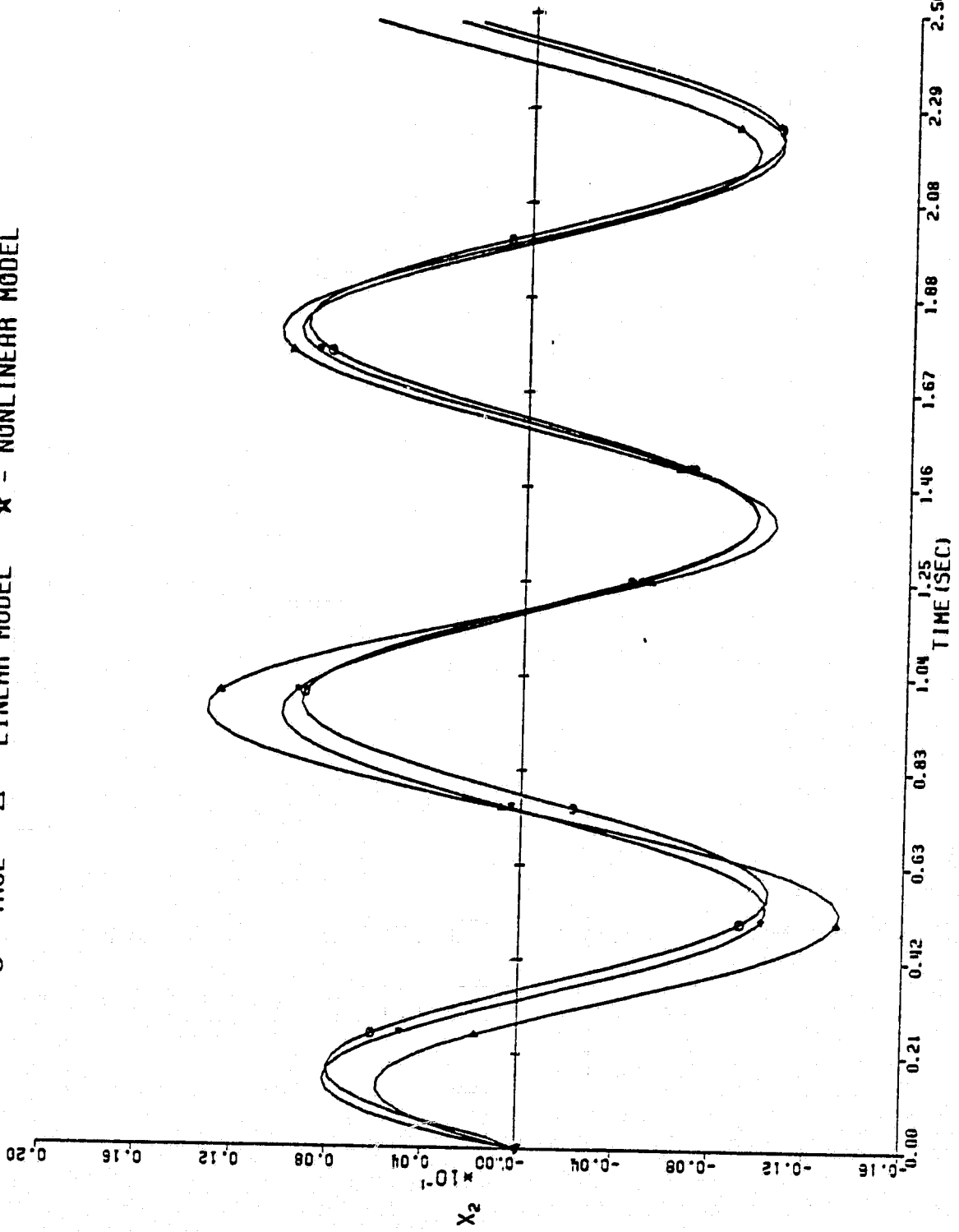


Figure 5.8a. Table 5.4, Line 6.

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COMPARATIVE SOLUTION PLOTS: STATE # 4  
○ - TRUE    △ - LINEAR MODEL    ★ - NONLINEAR MODEL

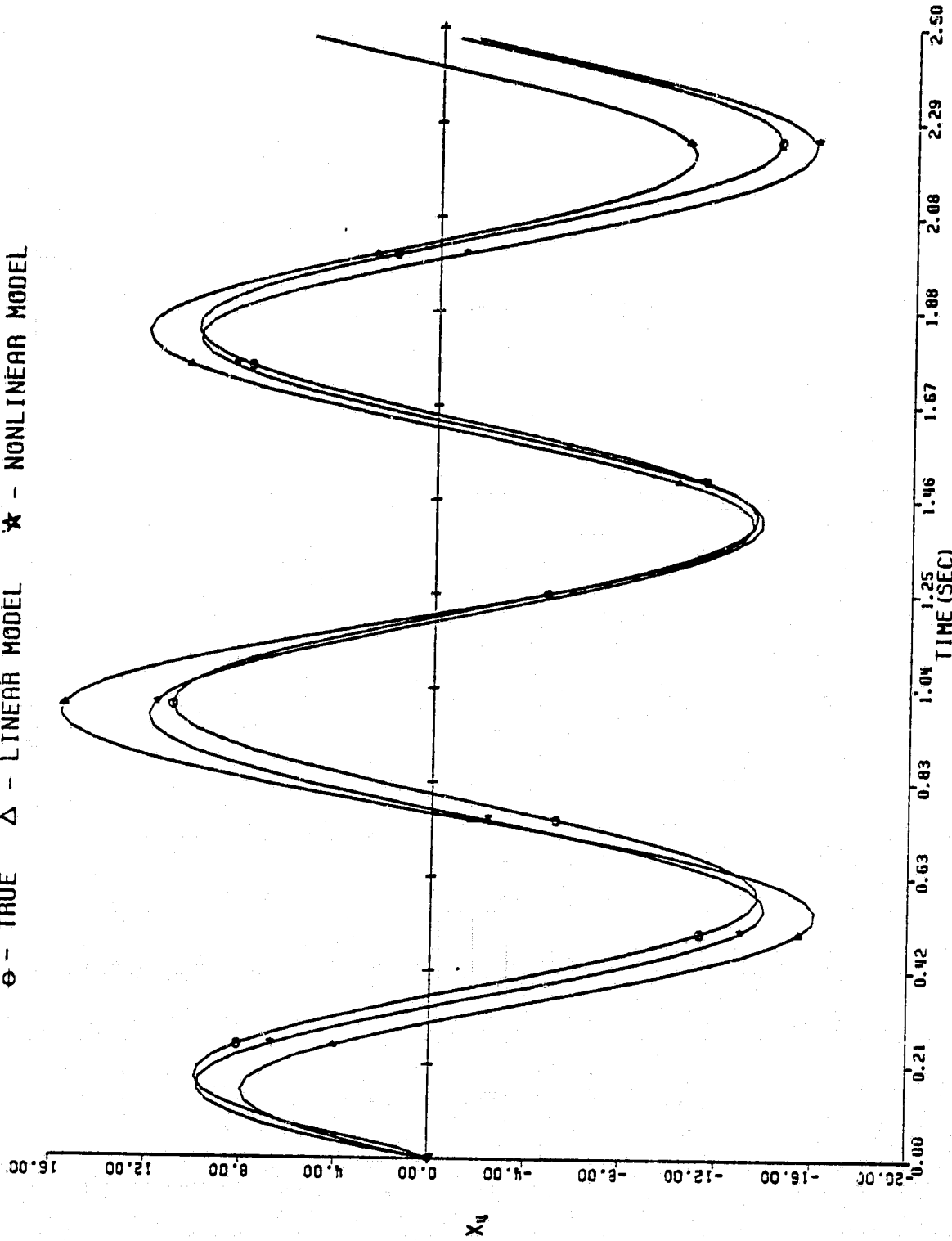


Figure 5.8b. Table 5.4, Line 6.

## SECTION VI

### CONCLUSIONS

Based upon the results included in this report it can be said that the tensor approach, used to model the QCSEE digital simulation, has tested out well. This is true in the sense that (1) initial conditions and input signals have been chosen so that the nonlinear model outperforms the linear approximation locally and also expands the region of desirable model behavior; (2) suitable nonlinear model performances have been observed over a wide variety of initial conditions, input amplitudes, and input frequencies; and (3) acceptable nonlinear model trajectories have been viewed in a qualitative fashion. Besides the QCSEE modeling effort, important milestones have been reached, as a result of this study, in the area of specialized computer software development. With the inception of the CATNAP package, tensor nonlinear modeling software capability has been substantially advanced.

A potentially useful idea which has stemmed from this work is the concept of nonlinear model identification using a "layered" tensor approach. Because of the possibilities, it is perhaps worth introducing this approach here.

Basically, the idea requires that the model structure (equation 2-3) be split into several smaller identification problems. Initially, a first-degree approximation is formulated, using the original technique, and identified. This is called the first layer. Next, a second-degree model is formulated; however, this time the response from the

first-degree model is constructed and subtracted from the derivative estimate matrix of the second-degree formulation. The second-degree formulation uses the identified first-layer parameters and the appropriate first degree monomials. This results in a second-layer formulation which contains only second degree tensor terms. Once identified, these parameters (second-layer) are combined with the first-layer parameters to form a model structure containing both first and second degree tensor terms. It should be noted that the magnitude of the initial condition and input perturbations used for excitation usually increase as the layer number does. This is done to assure that the derivative estimates maintain sufficient rank.

To add third degree tensor terms, the second-degree model response is constructed, and subtracted from the derivative estimate matrix of the third-degree formulation. A third-layer formulation containing only third-degree tensor terms is the result. When the third layer parameters are identified they are combined with the first and second layer ones to form a model structure. Clearly, any number of layers could be added using this scheme.

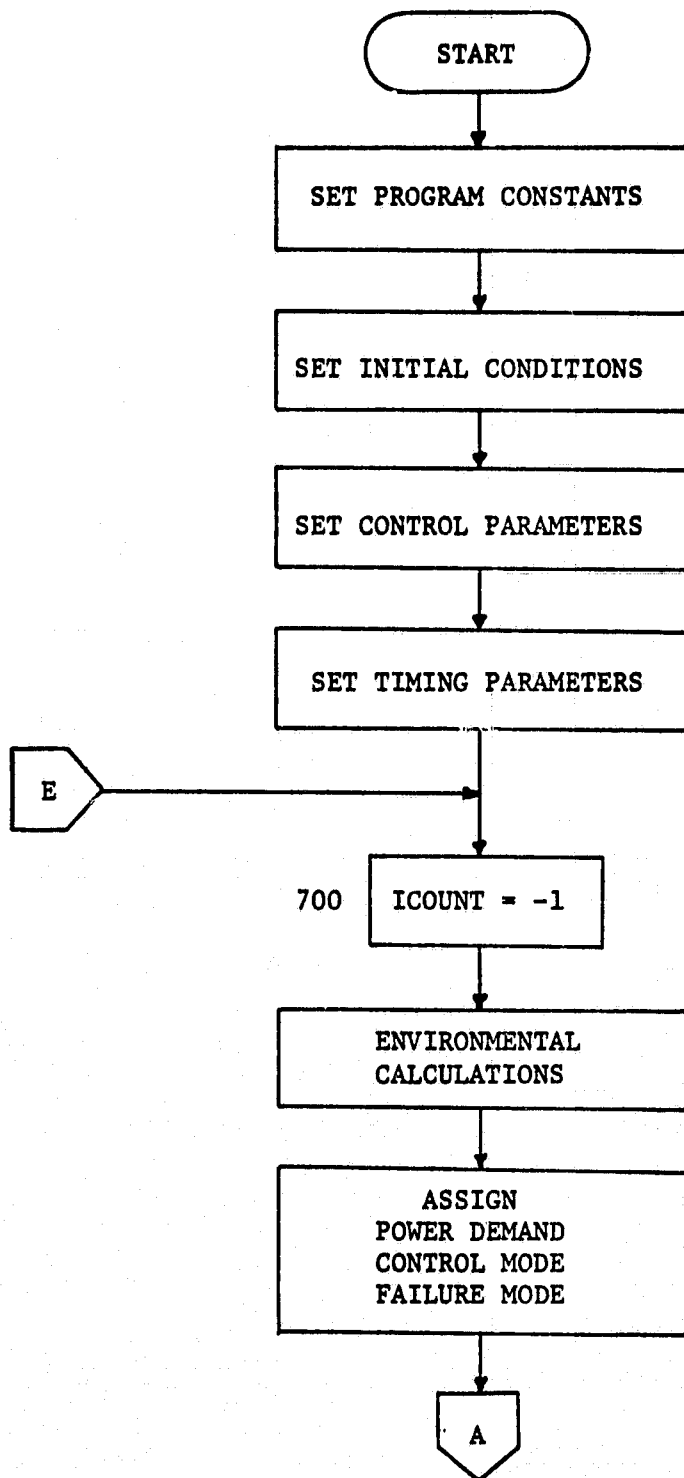
Intuitively, one would expect this approach to reduce the size of the nonlinear model identification problem significantly, as well as reduce the chance of obtaining dependent data. In addition, this approach should enlarge the region of acceptable model behavior. In this regard, it is believed that further studies are warranted. In particular, a thorough investigation of the layered tensor approach is recommended. Some preliminary efforts have already begun in this area.

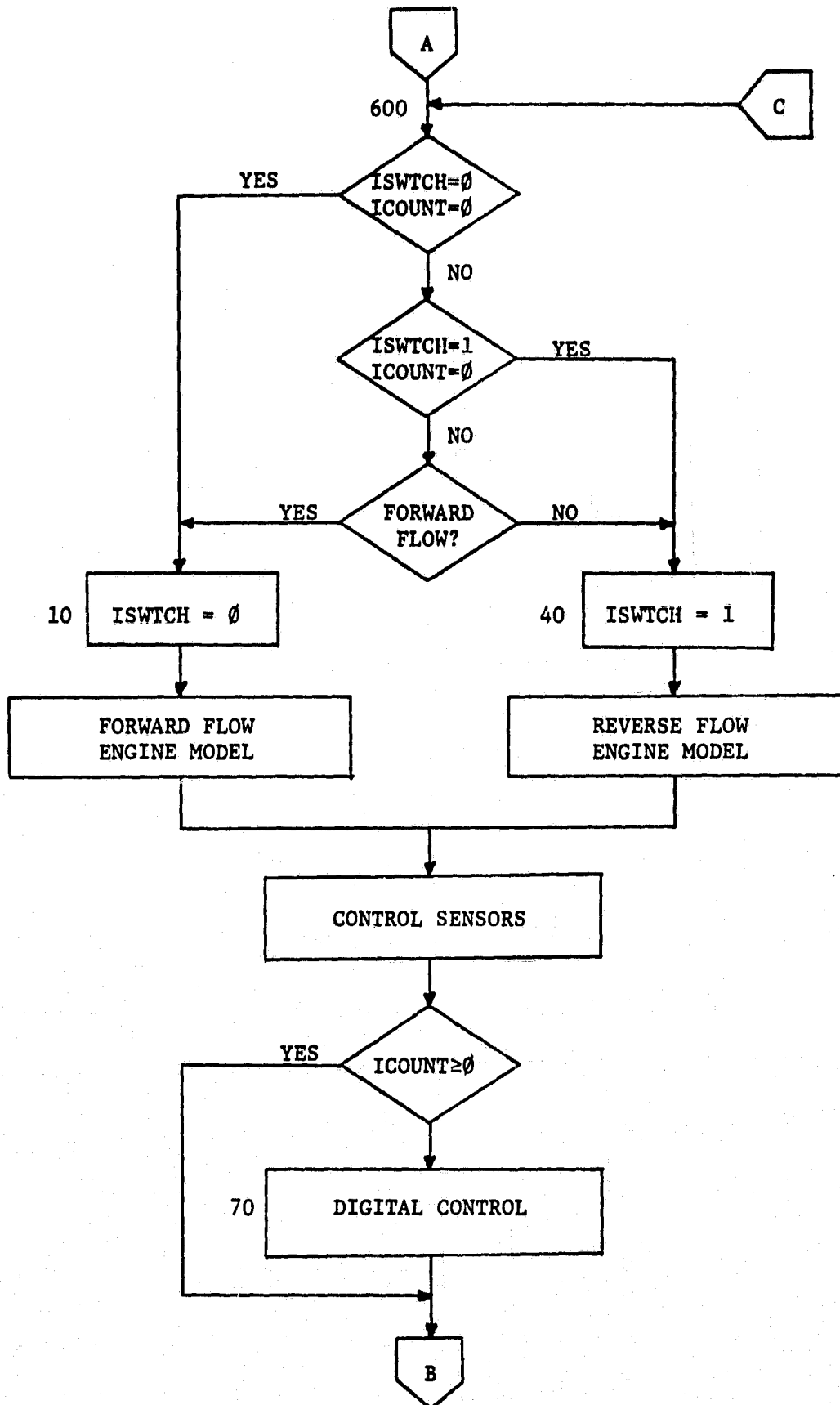
**APPENDIX A**

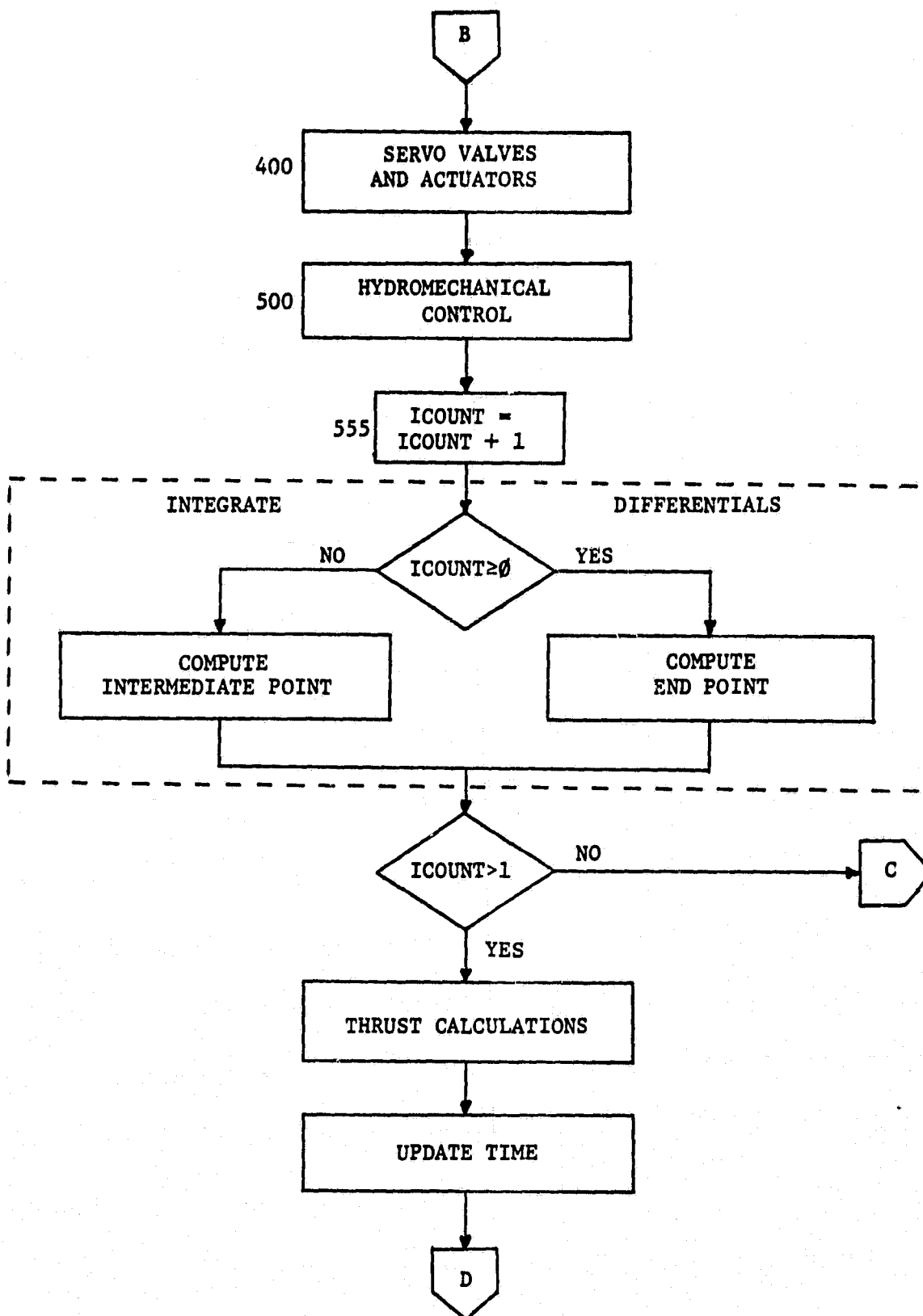
**QCSEE SIMULATION FLOWCHART AND VARIABLE LIST**

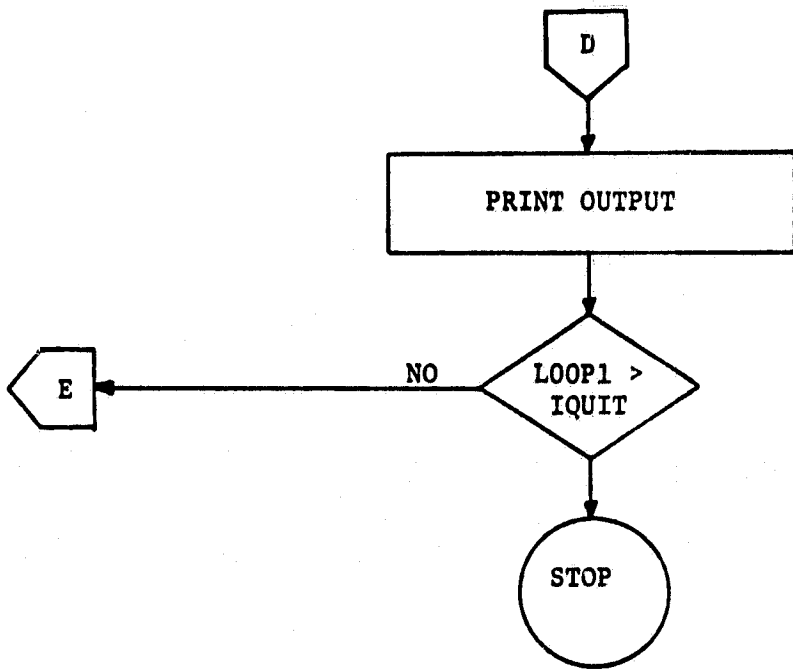


## FLOWCHART FOR QCSEE DIGITAL SIMULATION









## VARIABLE LIST FOR OCSEE DIGITAL SIMULATION

ALT	- Altitude - ft.
MNO	- Mach Number ~ dim.
PO	- Ambient Pressure - psia
TO	- Ambient Temperature - °R
VO	- Freestream Velocity
FN	- Engine Net Thrust - $lb_f$
FGROSS	- Engine Gross Thrust - $lb_f$
FRAM	- Engine Ram Thrust ~ $lb_f$
FG8	- Core Engine Gross Thrust ~ $lb_f$
FG11	- Fan Reverse Gross Thrust ~ $lb_f$
FG18	- Fan Forward Gross Thrust ~ $lb_f$
P11	- Engine Inlet Total Pressure - psia
PS11	- Engine Inlet Static Pressure - psia
XM11	- Inlet Mach Number dim.
P12Q11	- Inlet Total Pressure Ratio, $P_{12}/P_{11}$ ~ dim.
P12GDT	- Fan Inlet Duct Total Pressure Iteration Potential ~ psia/sec
P12GS	- Fan Inlet Duct Total Pressure Updated Iteration Value ~ psia
P13GDT	- Fan Discharge Total Pressure Iteration Potential ~ psia/sec
P13GS	- Fan Discharge Total Pressure Updated Iteration Value ~ psia
P13Q12	- Fan Pressure Ratio - $P_{13}/P_{12}$
P18	- Fan Nozzle Total Pressure - psia
P25	- Compressor Inlet Total Pressure - psia
P3	- Compressor Discharge Total - psia
P3Q25	- Compressor Pressure Ratio - $P_3/P_{2.5}$

PS3C - Compressor Discharge Pressure at Sensor Location ~ psia  
 EPRSN - Sensed Engine Pressure Ratio ~ dim.  
 P4GDT - Combustor Discharge Pressure Iteration Potential ~ psia/sec  
 P4GS - Combustor Discharge Pressure Updated Iteration Value ~ psia  
 P42 - Low Pressure Turbine Discharge Total Pressure - psia  
 P8GDT - Core Nozzle Total Pressure Iteration Potential ~ psia/sec  
 P8GS - Core Nozzle Total Pressure Updated Iteration Value ~ psia  
 T12 - Fan Inlet Temperature - °R  
 T13 - Fan Discharge Temperature - °R  
 T25 - Compressor Inlet Temperature - °R  
 T3 - Compressor Discharge Temperature - °R  
 WQW12 - Fan Flow Function Ratio ~ dim.  
 W12RO - Forward Fan Map Flow Function ~ dim.  
 W12 - Fan Airflow - lb<sub>m</sub>/sec  
 W18 - Fan Nozzle Airflow - lb<sub>m</sub>/sec  
 W25 - Compressor Inlet Airflow - lb<sub>m</sub>/sec  
 W3 - Compressor Discharge Airflow - lb<sub>m</sub>/sec  
 W4 - Combustor Airflow - lb<sub>m</sub>/sec  
 W8 - Core Nozzle Airflow - lb<sub>m</sub>/sec  
 T4 - Combustor Discharge Temperature - °R  
 T41C - Calculated Turbine Inlet Temperature - °R  
 T42 - High Pressure Turbine Discharge Temperature - °R  
 T8 - Low Pressure Turbine Discharge Temperature - °R  
 HPF - Fan Horsepower ~ hp.  
 HPLT - Low Pressure Turbine Horsepower ~ hp.

NLDT - Fan Speed Derivative - rpm/sec  
 NL - Fan Rotor Speed - rpm  
 HPC - High Pressure Compressor Horsepower ~ hp.  
 HPHT - High Pressure Turbine Horsepower ~ hp.  
 NHDT - Compressor Speed Derivative - rpm/sec  
 NH - Compressor Speed - rpm  
 PCNLR - Percent Corrected Fan Speed  
 PCNHR - Percent Corrected Core Speed  
 PCNHRD - Percent Corrected Compressor Speed Demand  
 PCEPRD - Percent Demanded Engine Pressure Ratio  
 PWRX - Power Demand - Percent  
 ICTRMO - Control Mode Logic Variable, 0 = Forward, 1 = Reverse  
 IB1F - Inadvertent Reverse and Out-Reverse Interlock Logic Signal  
 NLK - Scheduled Fan Speed Demand ~ rpm  
 NLMXR - Fan Speed Selector Value ~ rpm  
 XMV - Fuel Metering Value Position - in.  
 WFM - Main Burner Fuel Flow - lb<sub>m</sub>/hr  
 X18 - Fan Nozzle Area Actuator Position - in.  
 A18 - Fan Nozzle Area - in<sup>2</sup>  
 EPRD - Demanded Engine Pressure Ratio ~ dim.  
 DFSEL - Digital Fuel Control Selector ~ in/sec  
 EPWR - Digital Engine Pressure Ratio Control Error ~ in/sec  
 ETWR - Digital Maximum Turbine Inlet Temperature Control Error ~ in/sec  
 ENWRH - Digital Core Speed Control Error ~ in/sec  
 ENWRL - Digital Fan Speed Control Error ~ in/sec

ENWRI - Digital Core Idle Speed Control Error ~ in/sec

QMV - Hydromechanical Power Piston Flow ~ in<sup>3</sup>/sec

QEOR - Equivalent Electronic Override Power Piston Flow ~ in<sup>3</sup>/sec

QNH - Hydromechanical Core Speed Control Power Piston Flow ~ in<sup>3</sup>/sec

QDEC - Hydromechanical Deceleration Limit Control Power Piston Flow  
~ in<sup>3</sup>/sec

QACC - Hydromechanical Acceleration Limit Control Power Piston Flow  
~ in<sup>3</sup>/sec

BETA F - Fan Pitch Angle - Degree

DXSEL - Digital Fan Nozzle Position Control Selector ~ in/sec

EX18 - Digital Inlet Mach Number Control Error ~ in/sec

EX18MX - Digital Maximum Fan Nozzle Position Control Error ~ in/sec

EX18MN - Digital Minimum Fan Nozzle Position Control Error ~ in/sec

EX18RV - Digital Fan Nozzle Reverse Position Control Error ~ in/sec

DBSEL - Digital Fan Pitch Control Selector ~ in/sec

EBNBR - Digital Fan Pitch Angle Floor Schedule Control Error ~ V/V/sec

EBMR - Digital Maximum Fan Pitch Angle Control Error ~ V/V/sec

EBREV - Digital Reverse Fan Pitch Angle Control Error ~ V/V/sec

THETA I - Fan Pitch Mechanism Drive Motor Position ~ deg.

BFVV - Fan Pitch Position Sensor Signal ~ V/V



**APPENDIX B**  
**CATNAP USER'S GUIDE**

APPENDIX B  
CATNAP USER'S GUIDE

This appendix takes the user step-by-step through the operation of CATNAP to identify and simulate a nonlinear model. The process is divided into three main sections and covers everything from logging-on to logging-off.

The first section is devoted to identifying a nonlinear model and transferring it to the Radiation Laboratory using the GENERATE segment of CATNAP which is resident on the IBM 370-168 computer system. The process of retrieving the nonlinear model from the Radiation Laboratory and entering it into the model library on the electrical engineering department PDP 11/60 computer is the concern of the second section. The third and final section uses the CATNAP segment SIMULATE, which is on the PDP 11/60, and leads the user through a sample simulation session.

**B.1 MODEL IDENTIFICATION PROCEDURE**

This section takes the user through the process of identifying a nonlinear model. This step-by-step process begins with logging-on the IBM 370-168, checking the user library, entering the GENERATE supervisory, executing a desired loader routine, executing the SPEAKEASY program IDENTIFY, sending the model to the Radiation Laboratory and finally logging-off the system.

To log on, the user must initiate a connection between the terminal and the computer, and then identify himself to the system as an authorized user by entering a valid TSO identification number. The discussion to

follow will be directed to users of DECwriter terminals only, since the procedure for making the connection is terminal dependent.

To log on at a DECwriter, the user must turn the power switch on the keyboard to ON, and check to make sure that the two switches on the back of the telephone coupler are in the power on and half duplex mode. On the keyboard, the CAPS LOCK key should be in the "down" position. In addition, the panel to the left of the keyboard should have all keys in the "up" position except for the HDX (half-duplex) key and the 300 BAUD key, which should be down. Lift the telephone receiver, dial the remote access telephone number and listen for a tone. Upon hearing the tone, place the receiver in the coupler with the cord toward the back of the coupler. When the carrier light on the front of the coupler becomes illuminated strike the capital letter "X" a couple of times and then hit the carriage return key (referred to from now on as CR) on the right side of the keyboard. (Note: Throughout this appendix, information printed by the computer will be underlined; all other information is user input.) When the proper connection is made, the terminal will print a log on message.

IKJ54012A ENTER LOGON-

Respond by typing

LOGON XXXXXX SIZE(1000) (CR)

where XXXXXX is the user's TSO ID and the SIZE keyword requests additional memory for the session. If the system accepts the ID it will print an acknowledgement.

XXXXXX LOGON IN PROGRESS

The computer will then print any broadcast messages and the word READY.

READY

It is usually a good idea to list the user library to see what is stored in it. The user should have on hand the following files in order to identify models: CLIST, CNTL, SPEAKEZ.DATA and MODEL.DATA. Also, the user should have on file a FORTRAN application loader routine for the particular problem being studied. The SPACELST command is used to list the files stored.

## SPACELST (CR)

The computer responds by listing the names of all files stored on disk under the user's TSO ID.

```

      .
      .
      OTHER FILES
      .
      .
      .
      CLIST
      CNTL
      MODEL.DATA
      QCSELOAD.DATA
      SPEAKEZ.DATA

```

READY

Seeing that the required files are present, the user can now access the GENERATE program. Incidentally, QCSELOAD.FORT is the loader routine used to formulate nonlinear models of the QCSEE simulator.

To enter the supervisory the user must execute the proper command list program.

## EX (GENERATE) (CR)

The program will begin to run and respond by printing an introduction.

WELCOME TO THE NONLINEAR MODEL GENERATION SEGMENT OF THE COMPUTER  
AIDED TENSOR NONLINEAR MODELING APPLICATIONS PACKAGE (CATNAP)!

ENTER NAME OF DESIRED LOADER ROUTINE:

The user can then enter the name of the loader routine to be executed. As an example, say the user wishes to run QCSELOAD.FORT. The user would then merely enter QCSELOAD as follows:

ENTER NAME OF DESIRED LOADER ROUTINE: QCSELOAD (CR)

Control would then pass from the supervisor to QCSELOAD and execution would begin. (Note: Since the purpose of this appendix is to outline the CATNAP package in general and not focus on a particular application routine, QCSELOAD will not be addressed here; however, Appendix C contains listings of sample model formulations and identifications which utilize QCSELOAD.)

At the termination of the loader routine control is passed to the SPEAKEASY processor and the message

TSO SPEAKEASY III PI

is printed at the terminal followed by the prompt symbol (:\_). The user then types the string

:\_ SIZE=500;GET IDENTIFY:IDENTIFY:QUIT (CR)

to begin program IDENTIFY which prints the model coefficients at the terminal and in file MODEL.DATA. Once the model coefficients are printed control is returned to the supervisor where a couple of questions are asked.

DO YOU WISH TO SEND THIS MODEL TO THE RADIATION LABORATORY?[Y/N]:

A "Y" response to this question would initiate execution of program

TRANSFER which would send a copy of MODEL.DATA to the PDP 11/55 computer located in the Radiation Laboratory. An "N" response would merely cause the supervisor to print the next question.

DO YOU WISH TO IDENTIFY ANOTHER MODEL?[Y/N]:

A "Y" response here would cause the program to branch back to the question

ENTER NAME OF DESIRED LOADER ROUTINE:

whereas an "N" response would cause the supervisor to exit.

READY

Once in this mode, the user is ready to disengage connection with the IBM 370-168. This is achieved by entering the NDLOGOFF command.

NDLOGOFF (CR)

The computer will respond with

XXXXXX LOGGED OFF TSO

and the carrier light will turn off. The user should then hang up the telephone and turn the power switches on the coupler and DECwriter off.

## B.2 NONLINEAR MODEL RETRIEVAL

The procedure to retrieve nonlinear models from the Radiation Laboratory is the topic presented in this section.

Before beginning, the user must select the correct RK05 disk pack from the disk cabinet in the EE department PDP 11/60 laboratory. This disk pack is labeled CNTL1. It should be placed in a plastic bag (also located in the cabinet) in order to prevent the disk from collecting dust or getting wet while in transit. The disk is now ready to be carried to the Radiation Laboratory.

Upon entering the PDP 11/55 computer laboratory in the basement of the Radiation Laboratory the user should remove the disk pack from the bag and place it in one of the two RK05 disk drives available (either DK0: or DK1:). Press the RUN/LOAD switch in front and note that the RDY (ready) light should turn on. Also, if the WTPROT (write protect) lamp is lit, the WTPROT switch should be depressed so that the light goes out. Now the disk is loaded and the user is free to log on.

When one of the available terminals is turned on the computer sends a prompt (>) and waits for the user to supply a command. To log on the user types HELLO.

```
> HELLO (CR)
```

Next the computer prompts for the user's account number (UIC) or last name. For this discussion the correct UIC is [45,1].

```
>ACCOUNT OR NAME: [45,1] (CR)
```

To complete the log on procedure the user's password is entered. Notice that in order to maintain privacy the password itself is not echoed back to the terminal.

```
>PASSWORD: (password) (CR)  
not echoed
```

Following any broadcast messages the computer will respond with the prompt symbol.

It is now necessary to allocate and mount the disk for use. To achieve this the user would enter

```
>ALL DKn: (CR)  
>MOU DKn: CNTL1 (CR)
```

where  $DKn$ : is either  $DK0$ : or  $DK1$ : depending upon which  $RK05$  drive is being used. From this point onward let us assume that  $DK1$ : is being used.

Once this is complete the user has access to his disk and is ready to begin the copy procedure. Prior to performing the copy, the user must list his directory to find out the name of the model file. This involves using the PIP command.

≥ PIP /LI (CR)

The computer will respond by listing all the names of files stored on the default disk under the user's UIC.

.  
.  
OTHER FILES  
.  
.  
.

RADLAB.437  
777.437

The two files of interest are RADLAB and 777. RADLAB contains the JCL code used to send the model over and is no longer of any use, whereas 777 contains the nonlinear model coefficients and is the file we wish to retrieve.

It is important to mention here that the .437 which is appended to the file name indicates that RADLAB and 777 were contained in the 437th job sent over this particular remote job entry port on a given day. This is useful to know. For example, say three models were consecutively sent to the Radiation Laboratory during an identification



session. Then, three RADLAB files and three 777 files would be present on the PDP 11/55. Distinguishing them from each other might be difficult without the job number; however, with the appended number the order of arrival becomes obvious.

Once the name of the desired model file is known, the copy can be initiated via a set of PIP commands. An example illustrating this for file 777.437 might be as follows:

```
>PIP DK1:[45,1]*.*;*/DE (CR)
>PIP DK1:[45,1]Q92S4MA.DAT;1=777.437 (CR)
>PIP RADLAB.437;1,777.437;1/DE (CR)
```

The first line deletes any files on DK1:[45,1] that are present from a previous transfer session. Line two performs the actual copy, and the last line deletes the necessary files on the default disk. It should be noted that the name Q92S4MA is chosen to be unique of other model names in the library, and furthermore is the name that will be requested later in the SIMULATE segment. (Comment: Q92S4MA is the name of the QCSEE model discussed in Chapter V.)

When the user has completed these steps he logs off the computer by typing BYE:

```
>BYE (CR)
```

The terminal is now ready to be turned off.

Moving over to the disk drive, the user should depress the RUN/LOAD switch. Upon doing this the RDY light will turn out. The user MUST wait approximately two minutes until the LOAD light comes on. Only then can he remove the disk pack from the drive and put it back into the

plastic bag. (Warning: Removing the disk prematurely will severely damage the disk drive and disk pack.)

Upon returning to the PDP 11/60 laboratory in the engineering building the disk should be loaded in the drive according to the same procedure as before. The user now logs on one of the Tektronix terminals.

>HELLO (CR)

This time however, the UIC is different but the password is the same.

```
>ACCOUNT OR NAME: [17,1] (CR)
>PASSWORD: (password) (CR)
           not echoed
```

When the user is logged on, a short indirect command file (ICF) is automatically executed. Its purpose is to allocate and mount the disk as well as program the various special function keys on the Tektronix terminal. After it is finished the prompt symbol (>) will appear.

To complete the model retrieval process (for the example presented), the user types the command

```
>PIP DK2:[17,7]Q92S4MA.DAT;1=DK2:[45,1]Q92S4MA.DAT (CR)
```

which enters the file Q92S4MA into the model library located in directory DK2:[17,7].

The user is now free to log off or proceed to the next section.

### B.3 MODEL SIMULATION PROCEDURE

This final section takes the user step-by-step through the process of simulating a nonlinear model on the PDP 11/60. If the user is not already logged-on at a Tektronix terminal he should do so by entering the following commands.

```

>HELLO (CR)
>ACCOUNT OR NAME: [17,1] (CR)
>PASSWORD: (password) (CR)
not echoed

```

When the log on is complete, that includes the automatic execution of the LOGIN ICF, and the user sees the prompt symbol (>) he should strike and PT (point) key to the right of the keyboard followed by a CR. @SIMULATE will appear on the screen and execution of the CATNAP segment SIMULATE will begin. Figure B.1 contains a diagram indicating the prompt sequences available in the supervisory program. To illustrate the use of SIMULATE, consider the following sample simulation session.

```

> @SIMULATE (CR)

```

The screen is cleared and an introduction is displayed.

```

>: WELCOME TO THE NOTRE DAME DEPARTMENT OF ELECTRICAL ENGINEERING
>: COMPUTER-AIDED TENSOR NONLINEAR MODELING APPLICATIONS PACKAGE!
>:
>* DO YOU WISH TO EXECUTE AN EXISTING SIMULATION ROUTINE?[Y/N]: Y (CR)
>* ENTER NAME OF DESIRED SIMULATION ROUTINE: QIXSIM (CR)

```

(Note: QIXSIM is the QCSE engine simulation routine which incorporates the QCSEE digital deck in subroutine TRUES.) The screen is cleared again and program QIXSIM starts executing.

```

WELCOME TO THE DYNAMICAL SYSTEM SIMULATION ROUTINE OF THE COMPUTER
AIDED TENSOR NONLINEAR MODELING APPLICATIONS PACKAGE (CATNAP)!

```

```

ENTER # OF STATES & # OF CONTROLS TO BE USED THROUGHOUT THIS SESSION:

```

The user responds by typing:

```

4 3 (CR)

```

The program then asks a series of questions which, of course, the user answers.

ORIGINAL PAGE IS OF POOR QUALITY

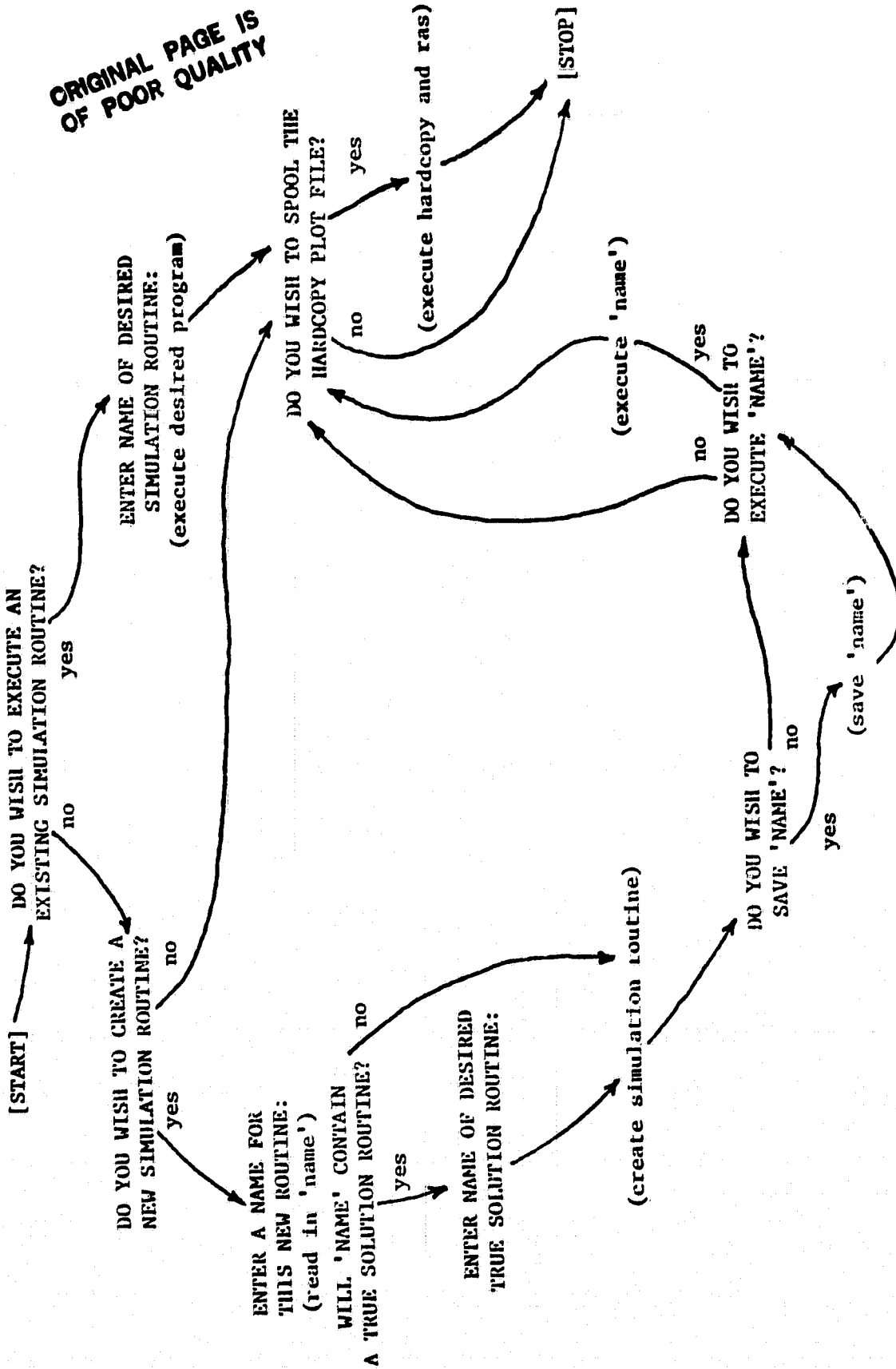


Figure B.1. Prompt sequence diagram for the SIMULATE supervisory program.

SHOULD THIS SESSION INCLUDE THE USE OF A TRUE SOLUTION ROUTINE?[Y/N]:  
Y (CR)

SHOULD THIS SESSION INCLUDE THE USE OF A LINEAR MODEL?[Y/N]:  
Y (CR)

ENTER NAME OF DESIRED LINEAR MODEL FILE:

Q92L4MA (CR) (QCSEE 4 state linear model at 92% power demand)

USING LINEAR MODEL FILE: DK2:[17,7]Q92L4MA.DAT

SHOULD THIS SESSION INCLUDE THE USE OF A NONLINEAR MODEL?[Y/N]:  
Y (CR)

ENTER NAME OF DESIRED NONLINEAR MODEL FILE:

Q92S4MA (CR) (QCSEE 4 state nonlinear model at 92% power demand)

USING NONLINEAR MODEL FILE: DK2:[17,7]Q92S4MA.DAT

The above set of questions is used to establish the session configuration as well as isolate the necessary model files.

Following this, the screen is cleared once more and a problem summary is displayed.

```
*****PROBLEM SUMMARY*****
      CONFIGURATION: TRUE, LINEAR & NONLINEAR
      NUMBER OF STATES: 4
      NUMBER OF CONTROLS: 3
      LENGTH OF TENSOR TERM VECTOR: 35
      DEGREE OF APPROXIMATION: 2
*****
```

Next the user is prompted for various simulation parameters.

ENTER INTEGRATION STEPSIZE, & UPPER TIME LIMIT OF INTEGRATION:  
(answers) (CR)

ENTER THE 4 INITIAL CONDITIONS:  
(answers) (CR)

ENTER AMPLITUDE, & FREQUENCY...OF FORCING FUNCTION FOR CONTROL #1:  
(answers) (CR)

ENTER AMPLITUDE, & FREQUENCY...OF FORCING FUNCTION FOR CONTROL #2:  
(answers) (CR)

ENTER AMPLITUDE, & FREQUENCY...OF FORCING FUNCTION FOR CONTROL #3:  
(answers) (CR)

ENTER DESIRED POWER SETTING:  
92.0 (CR)

Once these questions are answered the simulator begins to integrate the three systems. It should be mentioned that the last question regarding power setting is specific to this problem and is contained in the appli-

cation subroutine TRUES instead of the mainline as are the other questions.

After the solutions are sampled and the error criteria are computed and displayed the user is asked:

DO YOU WISH TO HAVE THE SOLUTIONS PRINTED AT THE VERSATEC?[Y/N]:

For most simulations, an "N" response here is appropriate since the raw data is not really very useful. A more useful output form is the comparative trajectory plots which are displayed at the Tektronix terminal when the user answers "Y" to the next question.

DO YOU WISH TO HAVE THE SOLUTIONS PLOTTED AT THE TEKTRONIX 4025?[Y/N]:

If the user wants to make hardcopy versions of these plots he may wish to answer "Y" to the following question.

DO YOU WISH TO HAVE THE SOLUTIONS FILE WRITTEN FOR HARDCOPY PLOTTING?[Y/N]:

An "N" response would cause the program just to skip to the next question.

DO YOU WISH TO RERUN SIMULATE WITH THE CURRENT CONFIGURATION?[Y/N]:

A "Y" response to this prompt would cause the program to jump back to

ENTER INTEGRATION STEPSIZE, & UPPER TIME LIMIT OF INTEGRATION:

whereas an "N" response would cause the program to end; and control would be returned to the supervisor.

When the supervisor receives control it initiates yet another prompt.

>\* DO YOU WISH TO SPOOL THE HARDCOPY PLOT FILE?[Y/N]:

If the user answered "Y" to the earlier question regarding hardcopy plotting, then he should answer "Y" here. An "N" response at this time terminates CATNAP. When a "Y" answer is detected, the program HARDCOPY is executed which prints corresponding data sheets along with the desired plots at the Versatec.

WELCOME TO THE HARDCOPY PLOTTING ROUTINE OF THE COMPUTER-AIDED  
TENSOR NONLINEAR MODELING APPLICATIONS PACKAGE (CATNAP)

SHOULD DATA SHEETS BE SEPARATED BY A BLANK PAGE?[Y/N]:

A "Y" response to this question is only necessary when report quality data sheets are required; otherwise, "N" should be entered to reduce paper consumption.

When more than one curve is present on a plot the user is given the choice of having single vertical axis scaling or multiple vertical axis scaling.

SHOULD THE PLOTS HAVE SEPARATE VERTICAL SCALES FOR EACH CURVE?[Y/N]:

Here, an "N" response is generally preferred since it implements the more desirable single vertical axis scaling.

Finally, the data sheets and plots are printed at the Versatec. At the completion of program HARDCOPY, CATNAP terminates and the user is free to log off.

> BYE (CR)

**APPENDIX C**  
**QCSEE MODEL IDENTIFICATIONS**



READY  
 EX (GENERATE)  
 THERE ARE 17 USERS ON TSD

WELCOME TO THE NONLINEAR MODEL GENERATION SEGMENT OF THE COMPUTER  
 AIDED TENSOR NONLINEAR MODELING APPLICATIONS PACKAGE (CATNAP)!

ENTER NAME OF DESIRED LOADER ROUTINE:QCSELOAD  
 G1 COMPILER ENTERED  
 SOURCE ANALYZED  
 PROGRAM NAME = MAIN  
 \* NO DIAGNOSTICS GENERATED  
 SOURCE ANALYZED  
 PROGRAM NAME = VECT  
 \* NO DIAGNOSTICS GENERATED  
 SOURCE ANALYZED  
 PROGRAM NAME = INDEX  
 \* NO DIAGNOSTICS GENERATED  
 SOURCE ANALYZED  
 PROGRAM NAME = QCSEE  
 \* NO DIAGNOSTICS GENERATED  
 \*STATISTICS\* NO DIAGNOSTICS THIS STEP

ENTER THE DESIRED POWER SETTING:  
 ?  
 92.0 92.0000

STEADY STATE ENGINE DATA  
 P8GS= 184.045 P8GS= 16.093 NL= 3066.130 NH= 12804.447  
 WFN= 4885.806 A18= 2817.710 RETAF= -0.413

ENTER INITIAL CONDITIONS FOR THE 4 STATES:  
 ?  
 185.63 16.12 3065.98 12835.12  
 185.6300 16.1200 3065.9800 12835.1200

ENTER INITIAL CONDITIONS FOR THE 3 CONTROLS:  
 ?  
 4960.16 2796.51 -0.654  
 4960.1600 2796.5100 -0.6540

ENTER FREQUENCIES FOR PERTURBING THE 3 CONTROLS:  
 ?  
 2.0 1.0 1.3  
 2.0000 1.0000 1.3000

ENTER SAMPLING PARAMETERS:  
 APPROXIMATION DEGREE  
 # SAMPLE POINTS  
 INTERVAL BETWEEN SAMPLES  
 QCSEE INTEGRATION STEPSIZE

1 100 .06 .01  
1 100 .0.0600 0.0100

DO YOU WISH TO NORMALIZE THE DATA?CY/NJ:  
Y

THE MATRIX OF SAMPLED MONOMIAL TERMS  
HAS 7 ROWS AND 100 COLUMNS.

TSO SPEAKEASY III FI 5:44 PM AUGUST 1, 1982  
!\_SIZE=500;GET IDENTIFY;IDENTIFY;QUIT

EXECUTION STARTED

PARTITION NUMBER 1.

FRITITION (A 4 BY 4 MATRIX)

-37.884 1954.5 -.0083009 .17971  
-.58412 30.144 -1.0833E-4 .0025439  
-57.622 4702.9 -1.651 -.77243  
-539.68 28922 .24067 1.4807

...WITH EIGENVALUES:

VALUES (A VECTOR WITH 4 COMPONENTS)

-3.3876 -2.8127 -.85501+1.13551i -.85501-1.13551i

PARTITION NUMBER 2.

FRITITION (A 4 BY 3 MATRIX)

.1019 -.0021608 .32914  
.0014992 5.2707E-5 .0074535  
.76391 1.2122 159.62  
1.8011 .24183 .74896

S (A 7 COMPONENT ARRAY)

7.6749 7.0603 5.6368 4.2097 2.5819 .10972 .47781

THE MAXIMUM SINGULAR VALUE:

MAX = 7.6749

THE MINIMUM NONZERO SINGULAR VALUE:

MIN = .10972

...AND THEIR RATIO:

RTIO = 69.949

MANUAL MODE

SPACE USED 10 K NOW, 43 K PEAK, SIZE 500 K

DO YOU WISH TO SEND THIS MODEL TO THE RADIATION LABORATORY?CY/NJ:N

DO YOU WISH TO IDENTIFY ANOTHER MODEL?CY/NJ:Y

ENTER NAME OF DESIRED LOADER ROUTINE:QCSELOAD

ENTER THE DESIRED POWER SETTING:

? 92.0  
92.0000

STEADY STATE ENGINE DATA  
P405= 184.045 P0GS= 16.093 NL= 3066.130 NH= 12004.447  
WFH= 4885.806 A18= 2817.710 RETAF= -0.413

ENTER INITIAL CONDITIONS FOR THE 4 STATES:

? 185.63 16.12 3065.98 12835.12  
185.6300 16.1200 3065.9800 12835.1200

ENTER INITIAL CONDITIONS FOR THE 3 CONTROLS:

? 4960.16 2796.51 -0.654  
4960.1600 2796.5100 -0.6540

ENTER FREQUENCIES FOR PERTURBING THE 3 CONTROLS:

? 2.0 1.0 1.3  
2.0000 1.0000 1.3000

ENTER SAMPLING PARAMETERS:

APPROXIMATION DEGREE  
# SAMPLE POINTS  
INTERVAL BETWEEN SAMPLES  
QCSEE INTEGRATION STEP SIZE

? 2 100 .06 .01  
2 100 0.0600 0.0100

DO YOU WISH TO NORMALIZE THE DATA?Y/NJ:

Y

THE MATRIX OF SAMPLED MONOMIAL TERMS  
HAS 35 ROWS AND 100 COLUMNS.

TSO SPEAKEASY III FI 5:37 PM AUGUST 1, 1982  
?-SIZE=5001GET IDENTIFY;IDENTIFY;QUIT

EXECUTION STARTED

ORIGINAL PAGE IS  
OF POOR QUALITY

PARTITION NUMBER 1.

PRITION (A 4 BY 4 MATRIX)  
 -21.041 616.67 -0.002663 .31963  
 -.18723 -.21128 -3.126E-5 .0048976  
 214.82 -12477 -1.6317 -2.2455  
 -399.67 19012 .065554 1.9287

...WITH EIGENVALUES:

VALUES (A VECTOR WITH 4 COMPONENTS)  
 -8.3536+39261 -8.3536-.39261 -2.2157 -2.0321

PARTITION NUMBER 2.

PRITION (A 4 BY 3 MATRIX)  
 .11091 .0023403 .56467  
 .0016872 3.5111E-5 .0088131  
 .83946 .81252 162.48  
 1.8416 .058423 1.5405

PARTITION NUMBER 3.

PRITION (A 4 BY 10 MATRIX)  
 ROW 1 539.51 -49661 -8.7168 -17.701 918917 592.33 1120.3 -.0028601 .013409 .041562  
 ROW 2 8.0387 -751.87 -1.12934 -.2586 14524 8.7823 16.529 -4.3155E-5 2.0514E-4 6.2388E-4  
 ROW 3 -3601.6 221321 -6.251 209.87 1203595 545.47 -13178 -.003049 -.084325 -.55865  
 ROW 4 7741.7 -626936 -69.488 -332.29 6414384 4677.1 22620 -.018613 .13546 .19139

PARTITION NUMBER 4.

PRITION (A 4 BY 12 MATRIX)  
 ROW 1 .028089 -1.513 -87.161 3.9051 95.693 5602.1 -.0028633 7.5705E-4 .050727 -.0040343 .0070703  
 .3557  
 ROW 2 5.9437E-4 -.027876 -1.3895 .041057 1.7866 99.502 -4.2393E-5 1.1138E-5 7.5398E-4 -5.7672E-5 1.1339E-4  
 .0055366  
 ROW 3 4.5414 53.174 -826.34 -387.52 -3571.2 57636 -.0038821 .0047397 .30542 .047048 -.093217  
 -.039728  
 ROW 4 -2.4326 -40.664 -1250.9 317.7 2664.4 81590 -.022073 .0020903 .14073 -.10462 .12241  
 4.2035

PARTITION NUMBER 5.

PRITION (A 4 BY 6 MATRIX)  
 1.9052E-6 -2.672E-4 -.023965 .001057 .017131 -1.9480  
 1.672E-7 -5.6709E-6 -3.8536E-4 1.6124E-5 2.7262E-4 -.029457  
 .0021967 .012615 -.30127 -.043636 -.084512 -7.8143  
 -.0021547 -.010365 -.35059 .0098707 .23752 -9.2130

S (A 35 COMPONENT ARRAY)

9.9931 8.4386 7.6725 5.8088 5.1673 4.4472 3.7611 3.5515 2.9605 2.6686 2.2088  
 2.2643 2.0418 1.7111 1.4305 1.4836 1.2193 .73954 .41134 .34883 .20952 .21625  
 .15943 .1137 .080232 .053959 .049209 .033613 .0075497 .005474 .0027162 .0013924 7.6795E-4  
 1.9031E-4 4.9696E-5

THE MAXIMUM SINGULAR VALUE:

MAX = 9.9931

THE MINIMUM NONZERO SINGULAR VALUE:

MIN = 4.9496E-5

...AND THEIR RATIO:

RTIO = 201086

MANUAL MODE  
SPACE USED 13 K NOW, 132 K PEAK, SIZE 500 K  
DO YOU WISH TO SEND THIS MODEL TO THE RADIATION LABORATORY/NJ:Y  
DO YOU WISH TO IDENTIFY ANOTHER MODEL/NJ:N  
READY

**APPENDIX D**

## GRANT BIBLIOGRAPHY, INCEPTION TO PRESENT

1. R.J. Leake, J.L. Melsa and M.K. Sain, "Preliminary Proposal on Modern Approaches to Jet Engine Control", November 1, 1974.
2. R.J. Leake, M.K. Sain and J.L. Melsa, "Proposal to NASA for Support of a Work Entitled 'Alternatives for Jet Engine Control'", November 27, 1974.
3. R.J. Leake and M.K. Sain, "Semi-Annual Status Report on NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1975-August 31, 1975.
4. R.J. Leake, M.K. Sain and J.L. Melsa, "Proposal for Continuation of NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 1, 1975.
5. J.C. Shearer, "An IBM 370/158 Installation and User's Guide for the DYNGEN Jet Engine Simulator", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, November 1975.
6. T.C. Brennan, "Simplified Simulation Models for Control Studies of Turbo-jet Engines", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, January 1976; also published as Technical Report 757, November 1975.
7. R.J. Leake, M.K. Sain and J.L. Melsa, "Final Technical Report on NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1975 - February 29, 1976.
8. R.R. Gejji, "Polynomial Techniques Applied to Multivariable Control", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, May 1976; also published as Technical Report EE 761, March 1976.
9. V. Seshadri, "Compensation of Multivariable Control Systems", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana; May 1976; also published as Technical Report EE 763, May 1976.
10. A.J. Maloney, III, "Graphics Analysis of Dominance in Jet Engine Control Models", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, August 1976; also published as Technical Report EE 765, June 24, 1976.
11. M.K. Sain, R.J. Leake, R. Basso, R. Gejji, A. Maloney and V. Seshadri, "Alternative Methods for the Design of Jet Engine Control Systems", Proceedings Fifteenth Joint Automatic Control Conference, pp. 133-142, July 1976.
12. J.C. Shearer, W.E. Longenbaker, Jr. and R.J. Leake, "An IBM 370/158 Installation and User's Guide to the DYNGEN Jet Engine Simulator", Technical Report EE 766, July 1976.

13. R.R. Gejji and M.K. Sain, "A Jet Engine Control Problem for Evaluating Minimal Design Software", Proceedings Midwest Symposium on Circuits and Systems, pp. 238-243, August 1976.
14. V. Seshadri and M.K. Sain, "Interaction Studies on a Jet Engine Model by Characteristic Methodologies", Proceedings Midwest Symposium on Circuits and Systems, pp. 232-237, August 1976.
15. R. Basso and R.J. Leake, "Computational Alternatives to Obtain Time-Optimal Jet Engine Control", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, August 1976; also published as Technical Report EE 767, September 1976.
16. R.J. Leake and M.K. Sain, "Semi-Annual Status Report on NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1976-August 31, 1976.
17. R. Basso and R.J. Leake, "Computational Methods to Obtain Time-Optimal Jet Engine Control", Proceedings Fourteenth Allerton Conference on Circuit and System Theory, pp. 652-661, September 1976.
18. M.K. Sain and V. Seshadri, "A Result on Pole Assignment by Exterior Algebra", Proceedings Fourteenth Allerton Conference on Circuit and System Theory, pp. 399-407, September 1976.
19. R.R. Gejji, "A Computer Program to Find the Kernel of a Polynomial Operator", Proceedings Fourteenth Allerton Conference on Circuit and System Theory, pp. 1091-1100, September 1976.
20. R.J. Leake and M.K. Sain, "Proposal for Continuation of NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 25, 1976.
21. R.M. Schafer, W.E. Longenbaker and M.K. Sain, "System Dominance: A Preliminary Report on an Alternate Approach", Technical Report, University of Notre Dame, November 25, 1976.
22. R.J. Leake and M.K. Sain, "Final Technical Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1976-February 28, 1977.
23. R.M. Schafer, "A Graphical Approach to System Dominance", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, May 1977; also published as Technical Report EE 772, April 1, 1977.
24. R.J. Leake, J.G. Allen and R.S. Schlunt, "Optimal Regulators and Follow-Up Systems Determined from Input-Output Signal Monitoring", Proceedings IEEE International Symposium on Circuits and Systems, April 1977.
25. W.E. Longenbaker and R.J. Leake, "Hierarchy of Simulation Models for a Turbofan Gas Engine", Proceedings Eighth Annual Pittsburgh Conference on Modeling and Simulation, April 1977.



26. P.W. Hoppner, "The Direct Approach to Compensation of Multivariable Jet Engine Models", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, May 1977; also published as Technical Report EE 774, May 1977.
27. R.R. Gejji and R.J. Leake, "Time-Optimal Control of a Single Spool Turbojet Engine Using a Linear Affine Model", Technical Report EE 7711, University of Notre Dame, June 1977.
28. R.M. Schafer, R.R. Gejji, P.W. Hoppner, W.E. Longenbaker and M.K. Sain, "Frequency Domain Compensation of a DYNGEN Turbofan Engine Model", Proceedings Sixteenth Joint Automatic Control Conference, pp. 1013-1018, June 1977.
29. R.R. Gejji and M.K. Sain, "Application of Polynomial Techniques to Multivariable Control of Jet Engines", Proceedings Fourth IFAC Symposium on Multivariable Technological Systems, pp. 421-429, July 1977.
30. R.R. Gejji, "Use of DYGABCD Program at Off-Design Points", Technical Report EE 7703, University of Notre Dame, July 1977.
31. E.A. Sheridan and R.J. Leake, "Non-Interactive State Request Jet Engine Control with Non-Singular B Matrix", Proceedings Twentieth Midwest Symposium on Circuits and Systems, pp. 539-543, August 1977.
32. R. Gejji, R.M. Schafer, M.K. Sain and P. Hoppner, "A Comparison of Frequency Domain Techniques for Jet Engine Control System Design", Proceedings Twentieth Midwest Symposium on Circuits and Systems, pp. 680-685, August 1977.
33. R.J. Leake and M.K. Sain, "Semi-Annual Status Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1977 -August 31, 1977.
34. W.E. Longenbaker, "Time Optimal Control of a Two-Spool Turbofan Jet Engine", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, January 1978; also published as Technical Report 7714, September 1977.
35. R.J. Leake and J.G. Comiskey, "A Direct Method for Obtaining Nonlinear Analytical Methods of a Jet Engine", Proceedings International Forum on Alternatives for Linear Multivariable Control, National Electronics Conference, Chicago, pp. 203-212, October 1977.
36. M.K. Sain and R.J. Leake, "Proposal for Continuation of NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 25, 1977.
37. J.A. Ortega and R.J. Leake, "Discrete Maximum Principle with State Constrained Control", SIAM Journal on Control and Optimization, Vol. 15, No. 6, pp. 109-115, November 1977.
38. Michael K. Sain and V. Seshadri, "Pole Assignment and a Theorem from Exterior Algebra", Proceedings IEEE Conference on Decision and Control, pp. 291-295, December 1977.

39. R. Michael Schafer and Michael K. Sain, "Some Features of CARDIAD Plots for System Dominance", Proceedings IEEE Conference on Decision and Control, pp. 801-806, December 1977.
40. M.K. Sain, "The Theme Problem", in Alternatives for Linear Multivariable Control, M.K. Sain, J.L. Peczkowski and J.L. Melsa, Editors. Chicago: National Engineering Consortium, 1978, pp. 20-30.
41. R.M. Schafer and M.K. Sain, "Input Compensation for Dominance of Turbofan Models", in Alternatives for Linear Multivariable Control, M.K. Sain, J.L. Peczkowski, and J.L. Melsa, Editors. Chicago: National Engineering Consortium, 1978, pp. 156-169.
42. J.L. Peczkowski and M.K. Sain, "Linear Multivariable Synthesis with Transfer Functions", in Alternatives for Linear Multivariable Control, M.K. Sain, J.L. Peczkowski, and J.L. Melsa, Editors. Chicago: National Engineering Consortium, 1978, pp. 71-87.
43. R.J. Leake and M.K. Sain, "Final Technical Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1977 -February 28, 1978.
44. M.K. Sain and R.J. Leake, "Semi-Annual Status Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1978 -August 31, 1978.
45. R.J. Leake, J.L. Peczkowski and M.K. Sain, "Step Trackable Linear Multivariable Plants", Technical Report EE 789, September 1978.
46. R.M. Schafer and M.K. Sain, "CARDIAD Design: Progress in the Four Input/Output Case", Proceedings Sixteenth Allerton Conference on Communication, Control, and Computing, p. 567, October 1978.
47. M.K. Sain, "Proposal for Continuation of NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", November 1, 1978.
48. M.K. Sain, "On Exosubsets and Internal Models", Proceedings IEEE Conference on Decision and Control, pp. 1069-1073, January 1979.
49. M.K. Sain, "Annual Technical Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1978 -February 28, 1979.
50. V. Seshadri and M.K. Sain, "An Application of Exterior Algebra to Multivariable Feedback Loops", Proceedings Conference on Information Sciences and Systems, Johns Hopkins University, pp. 337-342, 1979.
51. M.K. Sain and R.M. Schafer, "Alternatives for Jet Engine Control", Propulsion Controls Symposium, NASA Lewis Research Center, Preprints, p. III-15, May 1979.
52. J.G. Comiskey, "Time Optimal Control of a Jet Engine Using a Quasi-Hermite Interpolation Model", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, May 1979; also published as Technical Report EE 791, May 1979.

53. R.M. Schafer and M.K. Sain, "Frequency Dependent Precompensation for Dominance in a Four Input/Output Theme Problem Model", Proceedings Eighteenth Joint Automatic Control Conference, pp. 348-353, June 1979.
54. M.K. Sain, "Semi-Annual Status Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1979-August 31, 1979.
55. V. Seshadri and M.K. Sain, "Loop Closures and the Induced Exterior Map", Proceedings Seventeenth Allerton Conference on Communications, Control, and Computing, pp. 753-761, October 1979.
56. R. Michael Schafer and Michael K. Sain, "CARDIAD Approach to System Dominance with Application to Turbofan Engine Models", Proceedings Thirteenth Asilomar Conference on Circuits, Systems, and Computers, pp. 78-82, November 1979.
57. R.J. Leake, J.L. Peczkowski and M.K. Sain, "Step Trackable Linear Multivariable Plants", International Journal of Control, Vol. 30, pp. 1013-1022, December 1979.
58. R.R. Gejji, "On the Total Synthesis Problem of Linear Multivariable Control", Ph.D. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, May 1980; also published as Technical Report EE 801, January 1980.
59. V. Seshadri, "Multivariable Feedback Loop Closures: An Approach by Exterior Algebra", Ph.D. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, January 1980; also published as Technical Report EE 800, January 1980.
60. M.K. Sain, "Semi-Annual Status Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", September 1, 1979-February 29, 1980.
61. R.M. Schafer, "On the Design of Dynamical Compensation for Diagonal Dominance", Ph.D. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, May 1980.
62. M.K. Sain, A. Ma and D. Perkins, "Sensitivity Issues in Decoupled Control Systems Design", Proceedings Twelfth Southeastern Symposium on System Theory, pp. 25-29, May 1980.
63. S. Yurkovich and M. Sain, "Generating Nonlinear Models from Digital Simulations: A Tensor Approach", Proceedings Eleventh Pittsburgh Conference on Modeling and Simulation, pp. 797-801, May 2, 1980.
64. M.K. Sain, "Proposal for Continuation of NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", June 19, 1980.
65. R.R. Gejji, "Reliable Floating Point Computation of Minimal Bases", Proceedings Nineteenth Joint Automatic Control Conference, Paper WA-8B, August 1980.
66. M.K. Sain, "Nineteen Month Technical Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", March 1, 1979 -September 30, 1980.

67. S. Yurkovich and M. Sain, "A Tensor Approach to Modeling of Nonhomogeneous Nonlinear Systems", Proceedings Eighteenth Allerton Conference on Communications, Control, and Computing, pp. 604-613, October 1980.
68. M.K. Sain, "Quotient Signal Flowgraphs: New Insights", Proceedings IEEE International Conference on Circuits and Computers, p. 417, October 1980.
69. Stephen Yurkovich, "Application of Tensor Ideas to Nonlinear Modeling and Control", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, January 1981.
70. M.K. Sain, "Semi-Annual Status Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 1, 1980 -March 31, 1981.
71. V. Seshadri, "Exterior Algebra and Simultaneous Pole-Zero Placement", Proceedings Conference on Information Sciences and Systems, Johns Hopkins University, 1981.
72. J.L. Peczkowski and M.K. Sain, "Scheduled Nonlinear Control Design for a Turbojet Engine", Proceedings IEEE International Symposium on Circuits and Systems, pp. 248-251, April 1981.
73. M.K. Sain, "Status of Computer-Aided Control System Feedback Design: Algebraic Methods", GE-RPI-NSF Workshop on Control Design, May 20, 1981.
74. Stephen Yurkovich, Thomas A. Klingler and Michael K. Sain, "Tensor Ideas for Nonlinear Modeling of a Turbofan Jet Engine: Preliminary Studies", Proceedings Twelfth Annual Pittsburgh Conference on Modeling and Simulation, pp. 1423-1427, May 1981.
75. M.K. Sain, "Proposal for Continuation of NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", June 11, 1981.
76. M.K. Sain and J.L. Peczkowski, "An Approach to Robust Nonlinear Control Design", Proceedings Joint Automatic Control Conference, FA-3D, June 1981.
77. M.K. Sain, "Twelve Month Technical Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 1, 1980 -September 30, 1981.
78. M.K. Sain, "Semi-Annual Status Report on NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 1, 1981-March 31, 1982.
79. T.A. Klingler, S. Yurkovich and M.K. Sain, "An Application of Tensor Ideas to Nonlinear Modeling of a Turbofan Jet Engine", Proceedings Thirteenth Pittsburgh Conference on Modeling and Simulation, April 1982.
80. J.P. Hill, "Solution of Nonlinear Optimal Control Problems Using the Algebraic Tensor: An Example", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, June 1982.

81. M.K. Sain and R.M. Schafer, "A Computer-Assisted Approach to Total Feedback Synthesis", Proceedings American Control Conference, pp. 195-196, June 1982.
82. M.K. Sain and J.L. Peczkowski, "Nonlinear Multivariable Design by Total Synthesis", Proceedings American Control Conference, pp. 252-260, June 1982.
83. M. Sain and S. Yurkovich, "Controller Scheduling: A Possible Algebraic Viewpoint", Proceedings American Control Conference, pp. 261-269, June 1982.
84. T.A. Klingler, "Nonlinear Modeling of a Turbofan Jet Engine: A Tensor Method Application", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, August 1982.
85. T.A. Klingler, S. Yurkovich and M.K. Sain, "A Computer-Aided Design Package for Nonlinear Model Applications", Preprints Second IFAC Symposium on Computer Aided Design of Multivariable Technological Systems, pp 345-353, September 1982.
86. M.K. Sain, "Twelve Month Technical Report, NASA Grant NSG-3048, 'Alternatives for Jet Engine Control'", October 1, 1981 -September 30, 1982.

## REFERENCES

1. E. Kreindler and D. Rothchild, "Model Following in Linear Quadratic Optimization", Journal of the American Institute of Aeronautics and Astronautics, Volume 14, July 1976, pp. 835-842.
2. M.K. Sain, "The Growing Algebraic Presence in Systems Engineering: An Introduction", Proceedings of the IEEE, Volume 64, No. 1, January 1976, pp. 96-111.
3. S. Yurkovich, "Application of Tensor Ideas to Nonlinear Modeling and Control", M.S. Thesis, Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, January 1981.
4. S. Yurkovich and M.K. Sain, "A Tensor Approach to Modeling of Nonhomogeneous Nonlinear Systems", Proceedings Eighteenth Allerton Conference on Communication, Control, and Computing, October 1980.
5. T.A. Klingler, S. Yurkovich, and M.K. Sain, "An Application of Tensor Ideas to Nonlinear Modeling of a Turbofan Jet Engine", Proceedings Thirteenth Pittsburgh Conference on Modeling and Simulation, April 1982, to appear.
6. M. Sain and S. Yurkovich, "Controller Scheduling: A Possible Algebraic Viewpoint", Proceedings 1982 American Control Conference, June 1982, pp. 261-269.
7. W.H. Greub, Multilinear Algebra. New York: Springer-Verlag, 1967.
8. M.K. Sain, "Applications of Modern Algebra in Engineering Introductory Lecture Notes", Department of Electrical Engineering, Texas Tech University, Lubbock, Texas, 1975.
9. A.S. Tanenbaum, Computer Networks. Englewood Cliffs: Prentice-Hall, Inc., 1981.
10. T.A. Klingler, S. Yurkovich, and M.K. Sain, "A Computer-Aided Design Package for Nonlinear Model Applications", Proceedings IFAC Symposium on Computer-Aided Design of Multivariable Technological Systems, September 1982, to appear.
11. "OS/VS2 TSO Terminal User's Guide", IBM GC28-0645-2, IBM Corporation, Poughkeepsie, New York, 1975.
12. "OS/VS2 TSO Command Language Reference", IBM GC28-0646-3, IBM Corporation, Poughkeepsie, New York, 1976.
13. S. Cohen, and S. Pieper, "The Speakeasy III Reference Manual", Speakeasy Computing Corporation, Chicago, Illinois, 1979.

14. G.D. Brown, System 370 Job Control Language. New York: John Wiley and Sons, 1977.
15. "RSX-11M/M-Plus MCR Operations Manual", DEC AA-H263A-TC, Digital Equipment Corporation, Maynard, MA, 1979, Chapter 5.
16. C.E. Wise, "Turbofan of the Future", Machine Design, August 22, 1974, pp. 20-25.
17. J.R. Mihaloew and C.E. Hart, "Real Time Digital Propulsion System Simulation for Manned Flight Simulators", NASA TM-78958, Lewis Research Center, Cleveland, Ohio, July 1978.
18. "Quiet Clean Short-haul Experimental Engine (QCSEE) Under-the-Wing (UTW) Engine Digital Control System Design Report", NASA CR-134920, Lewis Research Center, Cleveland, Ohio, January 1978.
19. "Quiet Clean Short-haul Experimental Engine (QCSEE) Under-the-Wing (UTW) Simulation Report", NASA CR-134914, Lewis Research Center, Cleveland, Ohio, July 1977.
20. J.R. Mihaloew, "A Nonlinear Propulsion System Simulation Technique for Piloted Simulators", Proceedings Twelfth Pittsburgh Conference on Modeling and Simulation, April 1981, pp. 1407-1417.
21. D.D. McCracken and W.S. Dorn, Numerical Methods and FORTRAN Programming. New York: Wiley, 1964, p. 322.
22. S. Yurkovich, T.A. Klingler, and M.K. Sain, "Tensor Ideas for Non-linear Modeling of a Turbofan Jet Engine: Preliminary Studies", Proceedings Twelfth Pittsburgh Conference on Modeling and Simulation, April 1981, pp. 1423-1427.
23. B. Carnahan, H.A. Luther, and J.O. Wilkes, Applied Numerical Methods. New York: Wiley, 1979.
24. J.A. Turek and E.F. Phelps, "A Beginners Guide to Drawing Graphics on the Tektronix 4025 Terminal", Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN, May 1979.