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The study of reconnection in the context of one fluid, two dimensional magnetohydrodynamics (MHD), with spatially uniform constant density, viscosity and resistivity is thought to retain most of the physics important in reconnection. Much of the existing reconnection literature makes use of this approach. This discussion focuses on attempts to determine the properties of reconnection solutions to MHD as precisely as possible without regard to the intrinsic limitations of the model.

Models of Reconnection

The basic properties of the MHD reconnection process were elucidated in the model of Parker (1958) and Sweet (1957). Parker noticed that when a magnetic field of strength B_o reverses sign across a neutral sheet of thickness ℓ, pressure imbalance draws fluid towards the reconnection zone from the strong field sides. The (incompressible) flow emerges from the weak field sides carrying "reconnected" flux with it. In this model the rates of reconnection are determined by requiring that the magnetic flux entering the diffusion dominated region is annihilated by ohmic dissipation as fast as it is brought in. The result obtained depends crucially on estimating the dissipation rate per unit length as proportional to $\mu(B_o/\ell)^2$ where μ is the resistivity. The electric current density has been assumed uniform in the diffusion dominated region, so we may say that the reconnection zone in the Parker-Sweet model is "structureless". Although the qualitative features of the Parker-Sweet model have withstood the test of time, there are reasons to doubt that realistic reconnection zones are without internal structure. For example, Dungey (1958) noticed that at the position of a magnetic x-point there is the possibility of a very intense electric discharge. This suggests that the concentration of electric current within the reconnection zone may be strongly peaked near the center, so that the dissipation rate assumed in the Parker-Sweet model is a lower bound.

The Petschek (1964) model incorporates modifications to the Parker-Sweet geometry which predicts faster reconnection. In Petschek reconnection the diffusion dominated region is narrower, the electric current more intense and field lines can be annihilated more quickly. Parker (1979) has noted that any mechanism which results in a narrower diffusion region can elevate reconnection rates to the Petschek level. Petschek's original discussion establishes the geometry leading to rapid reconnection by introducing a flow field supported by an external agency. Thus, the model has traditionally been associated with forced rather than spontaneous reconnection.

Another approach to reconnection originates with the "tearing mode" theory of Furth, Killeen and Rosenbluth (1963). Tearing mode analysis involves solving the equations which result from linearizing the MHD equations about an equilibrium (or quasi-equilibrium) magnetic field profile. As with any linearized treatment of nonlinear equations the usefulness of tearing mode theory depends on two assumptions: that the equilibrium profile adequately represents a configuration which actually appears in nature, and that infinitesimal amplitude perturbation results are good predictors of finite amplitude situations.

Turbulence and the Role of Simulation in Reconnection Studies

Much of the detailed knowledge of reconnection phenomena is based onn computer simulation of the MHD equations. The task of accurately simulating solutions to fluid-like equations such as MHD in the nonlinear regime is not an easy one, mainly due to the appearance of turbulence in the solutions. MHD turbulence in two dimensions has been extensively studied (Kraichnan and Montgomery, 1980) and is usually discussed in terms of Fourier wavenumber decomposition of the magnetic and velocity fields. Perhaps the single most important feature of MHD turbulence is the appearance of dynamically important excitations over a wide range of spatial scales. While the standard reconnection geometries do not lend themselves to a description in terms of homogeneous turbulence, for which the Fourier description is most relevant (Batchelor, 1982), a conservative approach to studying reconnection would be to allow for the possibility that the same sort of complex behavior my develop in reconnection solutions. The typical fate of MHD turbulence viewed as an initial value problem runs something like this: Initially there are no fine scales present, all of the energy is in large scale magnetic (B) and/or velocity field (V) (Fourier) modes. As time advances, magnetic and mechanical excitations spread to a wide range of spatial scales. The characteristic time scales for evolution of the system are the eddy-turnover times T_B = L/B and T_V = L/V where L is the scale of the energy containing fluctuations, V is the r.m.s. velocity, and B the r.m.s. magnetic field strength in Alfvén speed units (Fyfe and Montgomery, 1976). The excited scales can usually be roughly divided into three regimes: the energy containing large scales, the smallest excited scales which constitute a dissipation range and an intermediate range in which energy spectra exhibit a near powerlaw dependence on wavenumber. The system is also characterized by dimensionless mechanical and magnetic Reynolds numbers, R and R_m which have reciprocal relationship to the viscosity and resistivity, respectively. At large values of R and R_m most of the dissipation occurs at scales very much smaller than the energy containing range. Accordingly, estimates of dissipation wavenumbers (Batchelor, 1982; Fyfe et al., 1977) increase with decreasing viscosity and resistivity. The spatial resolution required for accurate simulation is in no way related to the energy containing scale. It has been conjectured (Matthaeus and Montgomery, 1980) and to some extent numerically verified, that in the limit of very large R and R_m energy is dissipated at a nonzero rate independent of R and R_m. This is similar to the conjecture of Kolmogorov (1941) and Oubukhoff (1941) which is widely accepted in the hydrodynamic turbulence community (e.g., chapter 6 of Batchelor, 1982).

It is well outside the scope of this commentary to attempt a complete summary of investigations of MHD turbulence but the moral with regard to simulations is rather simple. Turbulence at high Reynolds numbers involves a large number of degrees of freedom (Fourier modes) and this number increases along with the Reynolds number. Accurate simulations must retain an adequate number of Fourier modes (or their equivalents) and must freely allow among them the transfers of excitation implied by the MHD equations. There are a number of ways that simulation design can run afoul of these guidelines.

"Numerical diffusion" (Potter, 1977) afflicts many finite difference schemes. Put simply, a numerical scheme attempts to convect a fluid element but adds a "bit of Laplacian" in the process. Use of such schemes may dissipate energy (or some other quantity) by both numerical <u>and</u> physical processes, making it difficult to determine how processes such as reconnection depend on the physical dissipation coefficients.

Some techniques (e.g., Sato and Hayashi 1979) make use of neighboring cell filtering methods to smooth numerically calculated time derivatives, ostensibly to stabilize the method. This is a questionable approach when simulating a process which delicately depends on the formation of small scale structures such as a diffusion region in reconnection, since filtering suppresses the transfer of excitation to high wavenumber in ways that are likely to be inconsistent with MHD. Large scale dynamical features may be qualitatively reproduced, but information about rates and dependence on Reynolds numbers is probably lost.

The use of artificial dissipation terms explicitly added to numerical MHD methods is also not uncommon. These are usually of "superdissipation" type (Meneguzzi et al., 1981) wherein the extra dissipation terms are effective only at very high wavenumber, or the "anomalous" dissipation type which for example (Sato and Hayashi, 1979) may increase dissipation when the electric current exceeds specified thresholds. The justifications for these approaches are quite different and both must be interpreted with care. Superdissipation (Basdevant, 1981) techniques attempt to model the unretained very high k modes as a sink of energy and are of limited utility since a fairly large unmodified intermediate wavenumber range must be present to preserve accuracy (Lamkin et al., 1983). Anomalous dissipation usually is motivated by considerations outside the scope of MHD.

Another important issue in MHD simulations is that of numerical reconnection and the preservation of ideal MHD topological (pointwise) invariants in codes when resistivities are set to zero (Matthaeus and Montgomery, 1980). It is computationally impossible to preserve pointwise conservation laws for arbitrary initial conditions and arbitrarily long times. Codes treating nonlinearities in an internally consistent manner avoid numerical reconnection in cases of physical interest by restricting the values of Reynolds numbers used. When resistivity and viscosity are set to zero accurately modeled nonlinearities should cause spectral transfer to the limit of resolution in a few turnover times, leading inevitably (and appropriately!) to numerical reconnection. This phenomenon is in no way connected with dissipation of energy. Concern with maintaining pointwise conservation laws has sometimes led simulators to introduce some of the numerical artifices discussed above to delay or prevent numerical reconnection by supressing transfer to small scales.

Some numerical models include much finer spatial resolution in the direction across the initial current sheet than in the direction along it. There is a possibility that a subtle problem is inherent in these anisotropic representations, particulary for incompressible MHD. In incompressible MHD excitations at wavenumber $\underline{k}$ are directly coupled to those at wavenumbers $\underline{r}$ and $\underline{p}$ only when $\underline{r} + \underline{p} = \underline{k}$. The coefficients of the nonlinear couplings (Fyte and Montgomery, 1976) vanish when $\underline{p}$ and $\underline{r}$ are parallel. As a rule of thumb, the most efficient couplings occur when $\underline{r}$ and $\underline{p}$ differ in magnitude by a ratio of about $\sqrt{2}$ and are about 45° apart in angle. To achieve rapid reconnection the current sheet must become much thinner than its initial width. High k_y modes must be excited by nonlinear interactions, where the y direction is across the current sheet. If very much higher resolution is retained in y than in x (the direction perpendicular to y), a substantial number of the wavenumber triads which pump the high k_y modes are excluded. To efficiently excite high k_y modes, k_x modes must be retained which do not differ from k_y in order to magnitude. At this writing, we do not know of any systematic study which has been performed to evaluate this effect. Highly anisotropic representations may underestimate the thinning of the current sheet near x-points.

There are a larger number of pitfalls in numerical techniques which may tend to prevent small scale structures from appearing in MHD simulations. This may result in a qualitatively correct picture of the reconnection process which is nontheless "slower" than that which is strictly implied by the MHD model.

Spectral Method Reconnection Simulations

The simulation of the nonlinear evolution of the periodic sheet pinch (Matthaeus and Montgomery, 1981; Matthaeus, 1982) is a somewhat nonstandard approach to studying the reconnection process. The two dimensional MHD equations are solved numerically in a periodic box containing initially two sheet currents (to achieve periodicity) using the Orszag-Patterson Spectral Method (Orszag, 1971). The use of this simulation geometry prohibits identification with any particular physical system, but allows many of the features of reconnection to be accurately determined. The principal advantages of the numerical scheme are the absence of aliasing and numerical dissipation errors and direct control of spatial resolution.

These simulations have shown the development of a reconnection zone near magnetic x-points which form along the initially present current sheets. The dynamics are triggered by broad band low level noise rather than by symmetrized eigenmode perturbutions which have sometimes been used. Plasma flows into the reconnection zone from the strong field sides. Outflow along the current sheet is at a substantial fraction of the Alfven speed. Magnetic field lines are continuously reconnected, eventually forming large magnetic islands. Enhanced dissipation occurs at x-points due to the formation of electric current filaments within the reconnection zone. A quadrupole-like distribution of vorticity is self-consistently generated around current filaments augmenting the characteristic flow pattern. The vorticity is generated by the curl of the Lorentz force which becomes large near the x-point corners. This effect is absent in the Parker-Sweet-Type reconnection which is driven by magnetic pressure gradients. The vorticity sheets present in the Petschek model are not localized near the reconnection zone and no generation mechanism is identified.

The appearance of intense current filamentation and quadrupole-like vorticity distributions in these simulations indicates that MHD reconnection zones are highly structured. It is likely that these structures enhance the reconnection process to rates above those of Parker and Sweet by narrowing the reconnection zone in a way that is reminiscent of the Petschek model, but without the need for an externally supported flow field. The effect can be qualitatively understood in terms of the local dynamics. Current filamentation results from the characteristic inflow/outflow pattern. Current filaments interact with the magnetic field to produce the vorticity quadrupoles. The quadrupoles are associated with an augmentation of the characteristic flow pattern, more current filamentation, and so on. The principal difficulty in treating this effect quantitatively arises from advection of the vorticity, causing the localized quadrupole distributions to elongate. The delocalization of the vorticity slows the reconnection process until a new phase of vorticity generation begins, which accounts for the bursts of reconnection activity seen in these simulations.

A very simplified scaling discussion has been given (Matthaeus, 1982) which was intended to demonstrate that fine structures in the reconnection zone might plausibly result in rates of reconnection which do not vanish for large magnetic Reynolds numbers. For this to occur the volume of the reconnection zone must shrink as R_m increases, while the intensity of the current increases enough that the Ohmic dissipation rate remains constant. This is the same sort of limiting behavior required for "selective decay" (Matthaeus and Montgomery, 1980) to occur in MHD and is again analogous to the Kolmogoroff-Oubukhoff conjecture concerning hydrodynamic turbulence.

It is of interest to note that the selective decay process occurs through what might be termed "progressive reconnection" (see figure 8 of Matthaeus and Montgomery, 1980). If a dissipative MHD simulation begins with 10 or so randomly positioned magnetic islands, turbulence leads ultimately to a state with just two large magnetic islands. At every stage of the turbulent decay leading to this final state, reconnection zones form between adjacent magnetic islands, exhibiting the same type of current and vorticity structures found in sheet pinch simulations. There is an intriguing possibility that global turbulent MHD turbulence processes and reconnection rates are integrally related. In an interesting recent paper Frisch et al. (1983) have investigated the appearance of small scale structures near neutral points in 2 dimensional MHD turbulent flows. They use very high resolution codes (256^2 and 512^2) with zero dissipation and run for short times. They define a "smallest excited scale" which monotonically decreases in time, indicating that dissipation alone can set a lower limit to the scale size of structures appearing near x-points.

Very recently we have completed (Lamkin and Matthaeus, 1983) simulations of the MHD sheet pinch at several values of R and R_m up to 1000. The most important feature of the results is that the time required for growth of "reconnected" magnetic islands does not increase as R_m is increased. The reconnection zones decrease in size as R_m is increased but we have not yet conclusively shown that the scaling suggested above obtains.

Conclusions

The overall importance of reconnection depends on how rapidly it occurs compared to other plasma dynamical processes. Historically, reconnection time scales have been thought to be considerably slower than eddy turnover times or Alfvén transit times of unit characteristic distance. In the Parker-Sweet model and in tearing mode theory the characteristic reconnection times tend to zero algebraically as the magnetic Reynolds number goes to infinity. In many cases of astrophysical and fusion-related interest, the magnetic Reynolds numbers are so large that reconnection becomes slow in comparison with other dynamical processes. On the other hand, fine scale structures appearing near x-points may be able to reinforce the reconnection process, and increase rates of reconnection above the usual estimates. This would require a reassessment of the overall importance of reconnection.

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