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DYNAMIC ALIGNMENT AND SELECTIVE DECAY IN MHD

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DECAY PROCESSES IN MHD TURBULENCE: DYNAMIC ALIGNMENT AND SELECTIVE DECAY

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ABSTRACT

Under some circumstances, incompressible magnetohydrodynamic turbulence will evolve toward a state in which the velocity fields and magnetic fields are aligned or anti-aligned. We propose a mechanism for this effect and illustrate with numerical computations. Under some other circumstances, the energy appears to decay selectively toward a minimum energy state in which the kinetic energy has disappeared. It has not been possible so far to identify a boundary in the phase space which divides the two regimes.

INTRODUCTION

Turbulent, dissipative media obey a statistical mechanics whose central features are very far from clear. If the dissipative terms in the equations of motion are dropped, a truncated discretized representation of the remaining ideal equations seems to exhibit behavior which is well described by classical statistical mechanics: time averages are well represented by phase space averages calculated by Gibbsian techniques. But as soon as the dissipation (viscous or ohmic, for example) is turned on with even a very small coefficient, the picture changes drastically.

Typical dissipation amounts to a perfect or nearly perfect absorption in wavenumber space, if the discretization is accomplished by Fourier decomposition. The high-$\underset{\sim}{k}$ Fourier coefficients are systematically drained, and phase points no longer spend times in phase volumes which are proportional to those volumes. Even though this fractional depletion of the time spent in volumes of phase space corresponding to the excitations at high $\underset{\sim}{k}$ is not sharply characterized, it may not be the only thing happening in phase space which is non-standard, according to the perspectives of classical conservative statistical mechanics. There may also be preferences for the phase points of systems to spend excessive fractions of their time on exotic sets in the phase space: strange attractors, fractals, and the like. Most discussion of these possibilities has occurred for model systems with very low numbers of degrees of freedom. Efforts to draw conclusions, from these low order "dynamical systems," about realistic turbulent systems with many degrees of freedom have not always been very convincing. We have no candidates, for example, for a description of a set in the (Fourier) phase space that would lead to the Kolmogoroff spectrum.

An alternative to the "mappings" game, which characterizes increasingly remote-looking model systems, is to continue to deal with realistic turbulent systems, and to try for the present to give a physical characterization of the tendencies in their turbulent dynamics which appear to be general. This is also an inadequate characterization of their phase space behavior, but at least it may be regarded as physically useful information.

An example of such a physical effect which we wish to remark upon here is the dynamic alignment of velocity fields and magnetic fields in magnetohydrodynamic (MHD) turbulence. It was observed some time ago by Belcher and Davis (1971), in comparing solar-wind magnetometer measurements with velocity fields inferred from particle energy analyzers, that it often occurred that solar wind turbulence near 1 A.U. found itself in a highly aligned ($\underset{\sim}{v} = \underset{\sim}{B}$) or anti-aligned ($\underset{\sim}{v} = -\underset{\sim}{B}$) state.

[Here $\underset{\sim}{v}$ is the velocity field in a zero-momentum frame, expressed in units of an rms Alfvén speed, and $\underset{\sim}{B}$ is the turbulent magnetic field.] These alignment properties have been frequently observed since, though we attempt no survey of the literature here, and are often discussed rather inconclusively in terms of the properties of small-amplitude Alfvén waves. They have been seen also in a turbulence closure calculation by Grappin et al (1982), and have been computed dynamically by Pouquet et al (1983) and by Matthaeus et al (1982). An early important theoretical initiative on their role in the solar wind was due to Dobrowolny et al (1980). Our purpose here is to describe in print for the first time the computations of Matthaeus et al (1982), and to suggest a qualitative explanation for the observed alignment which differs from the one suggested by Pouquet et al (1983).

THE DYNAMICAL PROBLEM

We begin with the equations of incompressible MHD in a familiar dimensionless form:

$$\frac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v}\cdot\nabla\underset{\sim}{v} = -\nabla p + \underset{\sim}{B}\cdot\nabla\underset{\sim}{B} + \nu\nabla^2\underset{\sim}{v} \, , \tag{1}$$

$$\frac{\partial \underset{\sim}{B}}{\partial t} + \underset{\sim}{v}\cdot\nabla\underset{\sim}{B} = \underset{\sim}{B}\cdot\nabla\underset{\sim}{v} + \mu\nabla^2\underset{\sim}{B} \, , \tag{2}$$

where p is the total pressure (mechanical plus magnetic) and where $\nabla\cdot\underset{\sim}{v} = 0 = \nabla\cdot\underset{\sim}{B}$. The constants μ^{-1}, ν^{-1} are the magnetic and mechanical Reynolds numbers, respectively. We work in rectangular, periodic, two-dimensional geometry, with all variables independent of the Z coordinate: $\partial/\partial z \equiv 0$. We solve Eqs. (1) and (2) numerically, using a spectral-method code of the Orszag type (Orszag 1971, Patterson and Orszag 1971, Gottlieb and Orszag 1977) used previously (Fyfe et al 1976, 1977 a,b; Matthaeus and Montgomery 1980, 1981; Shebalin et al 1983).

We solve the initial value problem starting from random initial conditions. These are specified by choosing a set of random initial values for the Fourier coefficients in the expressions

$$\begin{aligned} \underset{\sim}{v} &= \Sigma_{\underset{\sim}{k}}\ \underset{\sim}{v}(\underset{\sim}{k},t)\ \exp(i\underset{\sim}{k}\cdot\underset{\sim}{x}) \\ \underset{\sim}{B} &= \Sigma_{\underset{\sim}{k}}\ \underset{\sim}{B}(\underset{\sim}{k},t)\ \exp(i\underset{\sim}{k}\cdot\underset{\sim}{x}) \end{aligned} \tag{3}$$

and then following the evolution of the Fourier-amplitudes by solving the Fourier transformed version of Eqs. (1) and (2).

The degree of alignment may be measured by the ratio of two ideal invariants, the cross helicity, H_c, and the energy, E:

$$\frac{H_c}{E} = \frac{\frac{1}{2}\int \underset{\sim}{v}\cdot\underset{\sim}{B}d^2x}{\frac{1}{2}\int(\underset{\sim}{v}^2+\underset{\sim}{B}^2)d^2x} = \frac{\Sigma_{\underset{\sim}{k}}\underset{\sim}{v}^*(\underset{\sim}{k},t)\cdot\underset{\sim}{B}(\underset{\sim}{k},t)}{\Sigma_{\underset{\sim}{k}}(|\underset{\sim}{v}(\underset{\sim}{k},t)|^2+|\underset{\sim}{B}(\underset{\sim}{k},t)|^2)} \quad . \tag{4}$$

The ratio H_c/E must lie within the range $-1/2 \leq H_c/E \leq +1/2$, the limits indicating perfect anti-alignment and perfect alignment, respectively. H_c and E are both conserved by Eqs. (1) and (2) if we set $\mu = \nu = 0$, but decay in absolute value if $\mu \neq 0$, $\nu \neq 0$. The decay of $|H_c|$ need not be monotonic, but often is; the decay of E is monotonic.

THE COMPUTATION

The simulation results described here consist of four runs, designated A, B, C, and D. All four were obtained using a $(128)^2$ code; that is, wave vectors with integer components ranging from $k_{min} = 1$ to $k_{max} = 64$ and their negative counterparts are included. Initial nonzero $|\underset{\sim}{B}(\underset{\sim}{k},0)|^2$ and $|\underset{\sim}{v}(\underset{\sim}{k},0)|^2$ were confined to a ring in $\underset{\sim}{k}$ space between $k^2 = 9$ and $k^2 = 25$. For runs A, B and C there was twice as much magnetic energy as kinetic energy in each mode, but for run D, kinetic and magnetic energies were equal. In all runs the spectrum was independent of $\underset{\sim}{k}$ within the ring and phases of magnetic modes were randomly chosen. The amount of cross helicity initially present is determined by selecting phase angles for kinetic Fourier amplitudes to make a specified angle θ with respect to the magnetic amplitude at the same $\underset{\sim}{k}$. This controls both the modal cross helicity, $H_c(\underset{\sim}{k}) = \mathrm{Re}\, \underset{\sim}{v}^*(\underset{\sim}{k})\cdot\underset{\sim}{B}(\underset{\sim}{k})$ and the total cross helicity. Runs A and D are initially highly aligned, with $\cos^{-1}(.95) = \theta$ while runs B and C are only moderately correlated with $\cos^{-1}(.3) = \theta$. In runs A, B and D, the parameters $\mu = \nu = 1/400$, and in run C, $\mu = \nu = 1/1000$. Times are measured in units of Alfvén transit times of unit distance, with timesteps of 1/256 such units. Other runs, not described here, have shown results similar to these four.

PHYSICAL RESULTS

Typical behavior is exhibited in Fig. 1: a monotonic <u>increase</u> of $|H_c/E|$. If we start with H_c/E negative, it decreases with time.

A simple explanation is suggested by writing Eqs. (1) and (2) in the Elsässer variables $\underset{\sim}{Z}^{\pm} = \underset{\sim}{v} \pm \underset{\sim}{B}$. We assume for the moment that interest lies in the low-$\underset{\sim}{k}$ region in the Fourier space, where the dissipative terms may be neglected. Eqs. (1) and (2) may be combined to give

$$\frac{\partial}{\partial t} \underset{\sim}{Z}^+ = -\underset{\sim}{Z}^-\cdot\nabla\underset{\sim}{Z}^+ - \nabla p \qquad (5)$$

$$\frac{\partial}{\partial t} \underset{\sim}{Z}^- = -\underset{\sim}{Z}^+\cdot\nabla\underset{\sim}{Z}^- - \nabla p \qquad (6)$$

The pressure p is, as usual, obtained by taking the divergence of either Eq. (5) or Eq. (6) and using $\nabla\cdot\underset{\sim}{Z}^{\pm} = 0$ to yield a Poisson equation.

Now suppose that in some mean square sense, $\underset{\sim}{Z}^+$ is large compared to $\underset{\sim}{Z}^-$: $|\underset{\sim}{Z}^+| >> |\underset{\sim}{Z}^-|$. It follows that $\underset{\sim}{Z}^+$ is nearly

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time independent, since its time derivative is of $0\ (\underset{\sim}{Z}^-)$:

$$\frac{\partial \underset{\sim}{Z}^+}{\partial t} \cong 0, \tag{5a}$$

[∇p is of the same order as $\underset{\sim}{Z}^- \cdot \nabla \underset{\sim}{Z}^+$]. However, the evolution of $\underset{\sim}{Z}^-$ is <u>not</u> slow, and is given to lowest significant order

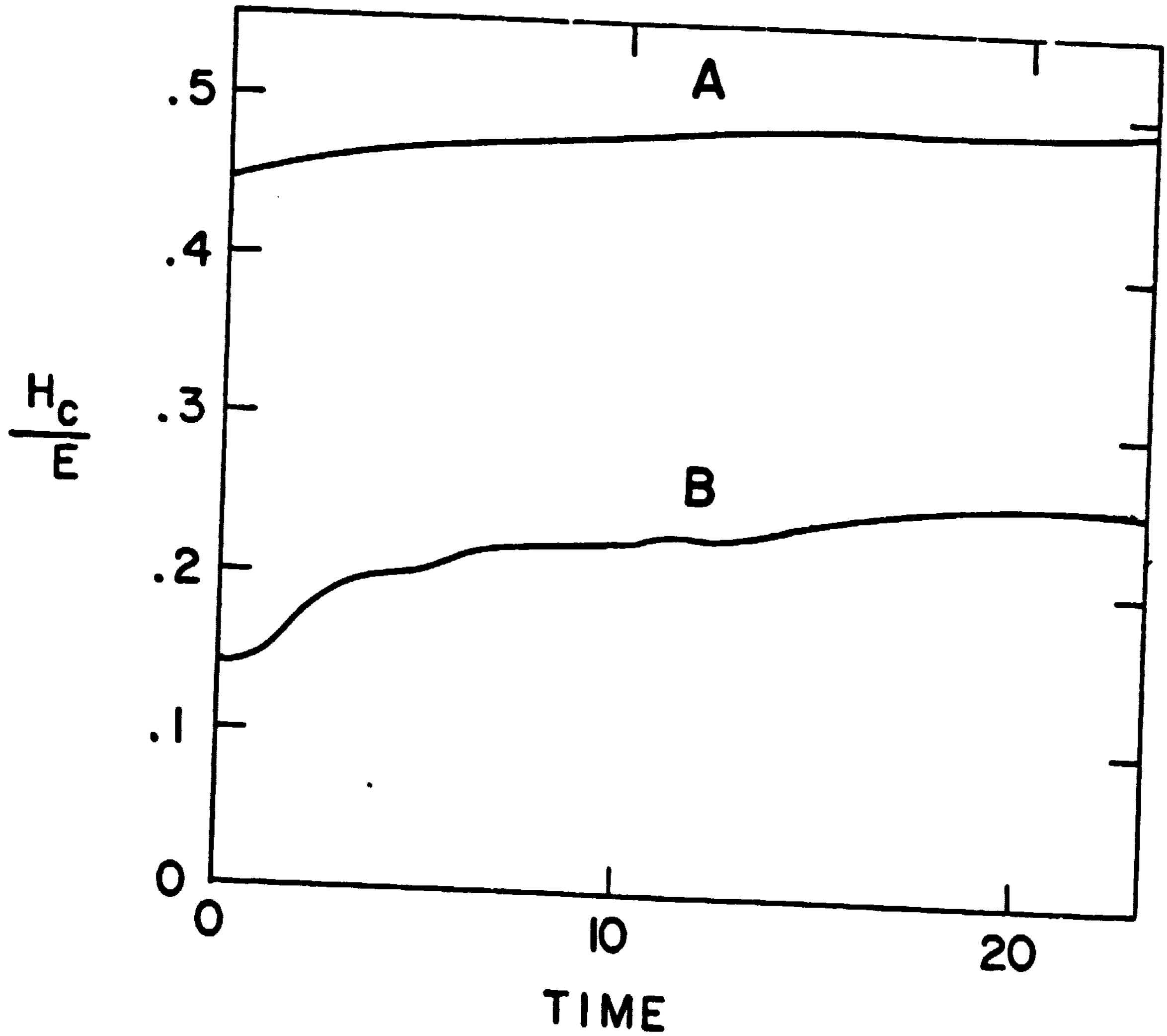
$$\frac{\partial}{\partial t} \underset{\sim}{Z}^- \cong -\underset{\sim}{Z}^+(t=0) \cdot \nabla \underset{\sim}{Z}^- - \nabla p, \tag{6a}$$

where $\underset{\sim}{Z}^+(0) \cdot \nabla \underset{\sim}{Z}^-$ is not negligible. Thus the fractional rate of change $|\underset{\sim}{Z}^+|^{-1} |\partial \underset{\sim}{Z}^+/\partial t|$ is small, but $|\underset{\sim}{Z}^-|^{-1} |\partial \underset{\sim}{Z}^-/\partial t|$ is not.

No spectral transfer in the $\underset{\sim}{Z}^+$ field is implied by Eq. (5a). Its spectrum is frozen at this order; if it is initially confined to wave numbers where dissipation is negligible, it will remain there. However, at every time step, Eq. (6a) will transfer the $\underset{\sim}{Z}^-$ spectrum to all additive combinations of wave numbers that can be made up of those present in $\underset{\sim}{Z}^+(0)$ and $\underset{\sim}{Z}^-$ at the previous time step. Clearly, the <u>transfer</u> of the $\underset{\sim}{Z}^-$ spectrum to higher wave numbers will continue at a fractional rate that does not go to zero as $\underset{\sim}{Z}^-$ becomes small. $\underset{\sim}{Z}^-$ will be dissipated as it reaches the higher $\underset{\sim}{k}$ values, thus enhancing the inequality $|\underset{\sim}{Z}^+| >> |\underset{\sim}{Z}^-|$. (A somewhat different result was obtained by Dobrowolyny et al, 1980).

We believe this mechanism, by which the majority species cannibalizes the minority one by sending it to high wave numbers to be dissipated, is the essential mechanism involved in dynamic alignment. [A pure $\underset{\sim}{Z}^+$ field is aligned and a pure $\underset{\sim}{Z}^-$ field is anti-aligned.] A more complicated dynamical calculation is required to make the effect precise, particularly for O(1) initial ratios of $|\underset{\sim}{Z}^+|/|\underset{\sim}{Z}^-|$.

Figure 2 shows the normalized cross helicity spectrum $2H_c(\underset{\sim}{k})/E(\underset{\sim}{k})$ vs. k at an early stage of the evolution of run A, $t \approx 0.19$. The initial value of this quantity was 0.895, near to its maximum possible value of 1.0, in the range of nonzero

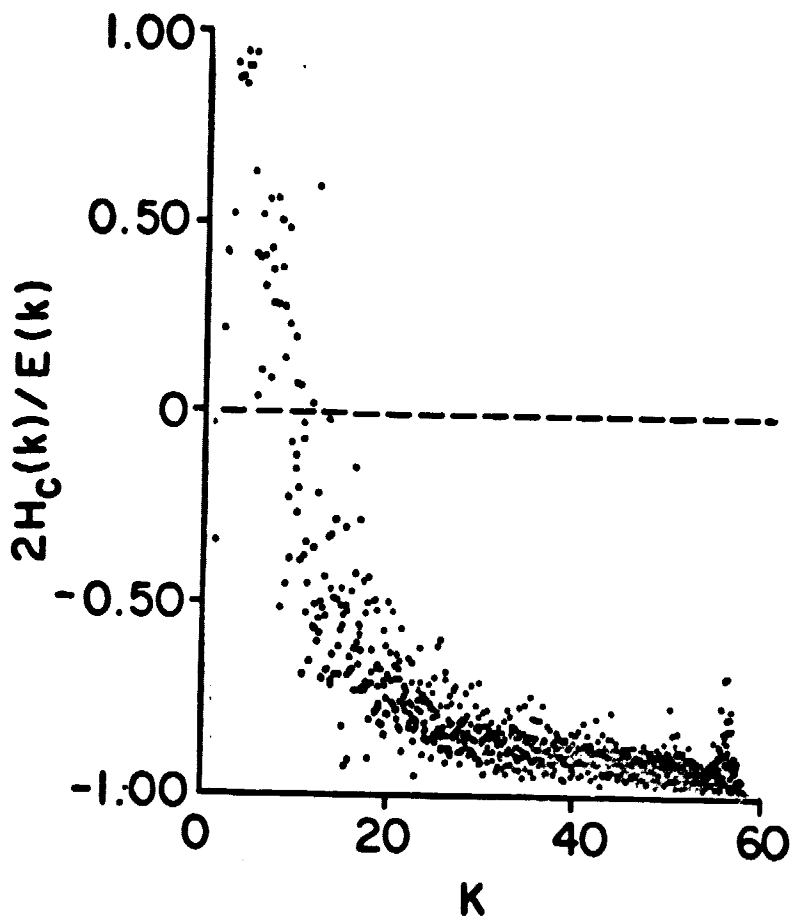

A
B
$\frac{H_c}{E}$
.5
.4
.3
.2
.1
0
0
10
20
TIME

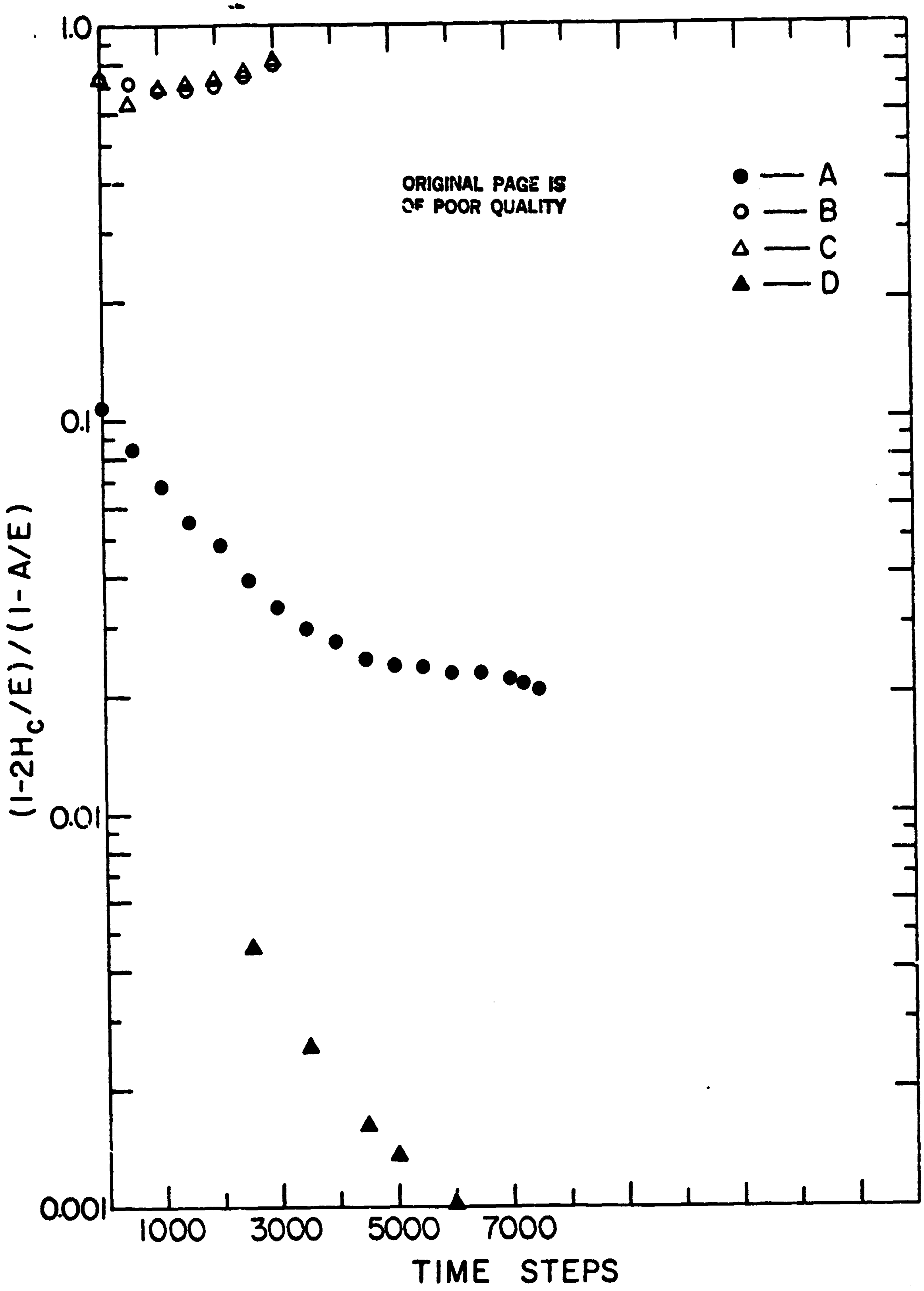
excitations, $3 \leq k \leq 5$. Figure 2 shows that the transfer of excitation to higher k's than those initially present consists almost entirely of negative cross helicity contributions, corresponding to highly anticorrelated small-scale magnetic and velocity field fluctuations. This typical behavior shows the transfer of $\underset{\sim}{Z}^-$ to higher $\underset{\sim}{k}$ where nothing was initially, and being dissipated there.

The picture seems, however, not to be as simple as the preceding argument would indicate. A competing process, the "selective decay" of energy relative to mean square vector potential $A \equiv \Sigma_{\underset{\sim}{k}} |\underset{\sim}{B}(\underset{\sim}{k},t)|^2/k^2$ also occurs under some circumstances, and at times competes with the increase of $|H_c/E|$ illustrated in Figs. 1 and 2. A is also an ideal invariant, and in several other computations (Matthaeus and Montgomery, 1980), the ratio E/A has been observed to decay monotonically, to close to its minimum value of $k^2_{min} = 1$.

The alignment process and selective decay cannot simultaneously proceed to their limiting states, since the limits $|2H_c/E| \to 1$ and $E/A \to k_{min}$ are incompatible. Figure 3 shows that for initially highly aligned runs, A and D, the ratio $(1 - 2H_c/E)/(1 - A/E)$ decreases indicating that the alignment process dominates (note that $k_{min} = 1$). However, the ratio increases for the less aligned runs B and C. At this writing, we do not know of any general theoretical considerations which determine which process "wins" in the long time high Reynolds number limits, for arbitrary initial conditions.

We may conjecture a (complicated or simple) boundary in the phase space of initial conditions on opposite sides of which the tendency is toward alignment and toward a minimum energy state. Similar uncertainties exist with regard to the three dimensional case, where magnetic helicity takes over the role of A, and the final selectively-decayed state is the


1.00
0.50
0
-0.50
-1.00
$2H_c(k)/E(k)$
0
20
40
60
K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A
B
C
D
1.0
0.1
0.01
0.001
(1-2H_C/E)/(1-A/E)
1000
3000
5000
7000
TIME STEPS

force-free "Taylor state." We have not succeeded, at this point, in identifying the boundary.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

Fig. 1. H_c/E for runs A and B vs. time in Alfvén transit times of unit distance. The increase of H_c/E with time shows that dynamic alignment of $\underset{\sim}{v}$ and $\underset{\sim}{B}$ is proceeding.

Fig. 2. Normalized cross-helicity spectrum $2H_c(k)/E(k)$ for run A at 5 = .19. $H_c(k)$ and $E(k)$ are modal spectra, i.e. average values over all $\underset{\sim}{k}$ with length k. Initially $H_c(k) = .895$ for $3 \leq k \leq 5$ and is zero elsewhere. Spectral transfer to higher k is shown to be almost entirely negative cross-helicity excitation (that is, the $\underset{\sim}{Z}^-$ field).

Fig. 3. $(1 - 2H_c/E)/(1 - A/E)$ for runs A, B, C, D. Increase of this ratio indicates dominance of selective decay; decrease signals dynamic alignment. The suggestion is that initially highly aligned initial conditions leads to ultimate dominance of alignment, but no threshold has been established.