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Acoustic Modal Analysis of the Pressure Field in the Tailpipe of a Turbofan Engine



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ACOUSTIC MODAL ANALYSIS OF THE PRESSURE FIELD IN THE TAILPIPE OF A TURBOFAN ENGINE

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SUMMARY

The results of a modal analysis of the pressure field in the tailpipe of a turbofan engine are presented. Modal amplitudes, at the tailpipe inlet and exit, are presented, as a function of frequency, for several operating conditions. The modal amplitudes were obtained using an optimization routine to obtain a best fit between measured cross-spectra and an analytical expression for the cross-spectra between pressures at circumferentially spaced locations. The measured pressure field was decomposed into a set of five modal amplitudes corresponding to the (0,0), (1,0), (2,0), (3,0), and (4,0) modes. The analysis was limited to frequencies below 1500 Hz where higher order modes are cutoff. The results of the analysis showed that at low frequencies, up to the cut-on frequency of the (1,0) mode, the (0,0) mode (plane wave) dominated the pressure field. The frequency range from the cut-on of the (1,0) mode to the cut-on of the (2,0) mode was dominated by the (1,0) mode. The (2,0) mode dominated from its cut-on frequency to the upper limit of the analysis, i.e., 1500 Hz. The contribution of modes other than the dominant mode was usually small.

INTRODUCTION

The modal content of the sound field in a duct is an important property of that sound field. Numerous analyses of the propagation in, and radiation from ducts exist in the literature. These analyses almost always describe the propagation and radiation of the sound field in terms of its modal content. In this paper, the modal content of the sound field in the tailpipe of the AiResearch QCGAT (Quiet Clean General Aviation Turbofan) engine is determined from measured pressure fluctuations in the tailpipe. Fluctuating pressure measurements were made at five circumferentially spaced locations at both the tailpipe inlet and exit. The lack of measurements at varying radial locations limited the analysis to frequencies below 1500 Hz where higher order radial modes are cutoff and not expected to be present. This is not a serious limitation since combustion noise, which propagates through the tailpipe, makes its most significant contribution to the total far-field noise of a turbofan engine at frequencies below 1000 Hz.

The modal analysis procedure used in the current work is the same as that used in reference 1 to determine the modal content of the pressure field in a full scale annular combustor. In this paper, modal amplitudes at the tailpipe inlet and exit are presented for several engine speeds over a frequency range of 0 to 1500 Hz.

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ENGINE DESCRIPTION

The engine used for these tests was the AiResearch QCGAT engine. This engine is a modified version of the AiResearch TFE 731-3 engine. The modifications were made as part of the NASA QCGAT program to demonstrate the applicability of large turbofan engine noise and pollution reduction technology to small general aviation turbofan engines. The engine is a front-fan, twin-spool configuration with a design point bypass ratio of 3.7 and a sea level static thrust of 17 300 N. (3900 lb.). For the data presented in this report, the separate flow exhaust version of the engine was used. More details of the engine can be found in reference 2.

INSTRUMENTATION AND DATA PROCESSING

The data and results reported in this paper were obtained from ten dynamic pressure probes located in the engine tailpipe. Five of these pressure probes were located near the tailpipe inlet, 0.66 m from the exit, and five were located near the tailpipe exit, 0.05 m from the exit. These probes were spaced circumferentially at 32° , 47° , 77° , 137° , and 212° from the top of the engine looking toward the engine inlet. The probe locations are shown schematically in figure 1.

The probes used to measure the fluctuating pressures in the tailpipe were cooled "semi-infinite" waveguide tubes using conventional condenser microphones as the actual transducer. A detailed description of their design and response characteristics may be found in reference 3. These probes were installed so as to measure the fluctuating pressure at the tailpipe wall.

The signals from the pressure probes were FM recorded simultaneously on magnetic tape for later processing. The record lengths at each operating condition were approximately two minutes in duration to provide adequate statistical confidence in the results. Pressure auto- and cross-spectra were obtained by off-line processing of the tape recorded data on a two channel fast Fourier transform signal processor with built-in analog to digital converters and 120 dB/octave anti-aliasing filters. The processor was capable of direct computation of up to a 4096 member ensemble average of a 1024 point forward or inverse Fourier transform to yield either frequency domain (coherence, auto- and cross-spectra, and transfer functions) or time domain (correlation) information. The digital auto- and cross-spectra were then transmitted from the processor to a central computer to perform the computations required for modal decomposition.

ANALYSIS

As previously mentioned, the modal analysis technique used in this study is identical to that used in reference 1. The technique involves writing an analytically derived expression for the cross-spectra between pressures at circumferentially spaced locations in terms of circumferential modal amplitudes. A least squares optimization routine is then used to obtain a set of modal amplitudes that yield a best fit to measured pressure cross-spectra. In reference 1, several assumptions were made in obtaining the analytical expression for the cross-spectra. These assumptions were:

1. Radial modes above the lowest order are not important for the frequency range of interest;

2. The amplitudes of the clockwise and counterclockwise rotating circumferential modes of the same order are equal; and

3. Circumferential modes of different orders are independent and do not correlate.

The applicability of these assumptions to the present data will now be discussed.

The assumption that higher order radial modes are not important and therefore can be neglected is based on the fact that the first higher order radial mode, i.e., the (0,1) mode, is cutoff at frequencies below about 1450 Hz at the tailpipe inlet and is cutoff at frequencies below 1640 to 1940 Hz at the tailpipe exit, depending on engine operating conditions (see table I). At frequencies where a mode is cutoff, the amplitude of the mode decays exponentially with distance from its source. Thus even if there were a source of higher order radial modes in the tailpipe (the tailpipe is not usually considered to contain significant sources of core noise), the presence of these modes would be felt over only a small extent of the tailpipe. Thus it is felt that over the frequency range of 0 to 1500 Hz only the lowest order radial modes need to be considered.

The equality of the amplitudes of the clockwise and counterclockwise rotating modes of the same order results in a predicted cross-spectral phase of either 0° or 180°. Assessment of the appropriateness of this assumption will be delayed until the measured cross-spectra are discussed.

The assumption that modes of different order are independent will be discussed next. The degree of dependence between the modes depends to a large extent on the nature of the source. If a source is well organized, i.e., correlated spatially, then the modal structure of the resulting pressure field will probably consist of modes that are highly correlated with each other. However, if the spatial characteristics of the source are more random in nature than the resulting modal structure will be less organized and consist of uncorrelated modes. For the frequency range of this analysis, 0 to 1500 Hz, the major source of noise propagating through the tailpipe of a turbofan engine is combustion noise. Combustion noise is generally considered to result from random fluctuations in heat release in the combustor. These fluctuations are random both in time and space. Thus the assumption of uncorrelated modes is consistent with the nature of the dominant core noise source over the frequency range of interest.

With these assumptions, the analytical expression for the cross-spectrum, G_{00} , of two pressure measurements, one at a reference angle of 0 degrees and the other at an angle 0 from the reference, can be shown to be (eq. (7) of ref. 1)

$$G_{0\theta}(f) = 4 \sum_{m=0}^{M-1} A_m^2 \cos m\theta \qquad (1)$$

where m is the mode number, M is the highest order mode considered, and A_m is the amplitude of the mth mode, and f is frequency.

The phase angle, $ullet_{O_{\Theta}}$, of $G_{O_{\Theta}}$ is given by

$$\bullet_{0\theta}(f) = \begin{cases} 0^{\circ} \text{ for } G_{0r}(f) \ge 0\\ 180 \text{ for } G_{0\theta}(f) < 0 \end{cases}$$
(2)

Equations (1) and (2), together with the measured pressure cross-spectra from the tailpipe, form the basis for the modal analysis. Equation (1) can be written for each of the five measured cross-spectra. If it is further assumed that only the first five circumferential modes make significant contributions to the pressure field in the tailpipe over the frequency range of interest, five equations in five unknowns result. (The cutoff frequencies for modes of order higher than (4,0) are well beyond the frequency range of interest for this study, thus these modes, like the higher order radial modes should not make a significant contribution to the pressure field.) The resulting equations were solved using a least squares search routine to determine the modal amplitudes. This approach avoids potential matrix inversion problems that might result from a direct solution to the five equations. The least squares approach also allows for experimental error and statistical variance that might create a situation where no algebraic solution to the five equations exists and also allows for extension of the method to the case when the equations are overdetermined, i.e., more equations than unknowns.

The search routine used in this study is identical to that used in reference 1. The search routine finds a set of coefficients, A_m , to minimize the function S, where

$$S = \sum_{i=1}^{5} \left[\left| G_{0\theta_{i}} \right|_{e^{-4}} \right| \sum_{m=0}^{4} A_{m}^{2} \cos m\theta \right]^{2} W_{i}$$
(3)

Here, $|G_{00,j}|_{e}$ is the magnitude of the measured cross-spectrum between the pressure at the reference position and the pressure at the angle θ_{j} . The term W_{i} in equation (3) is a weighting factor which is equal to unity when the sign of the reconstructed cross-spectrum (which is real) is the same as the real part of the measured cross-spectra. If the signs are different, W_{i} is set equal to ten. This value of W_{i} was based on a subjective comparison of the reconstructed and measured cross-spectra. The value chosen is one which effectively requires the search routine to select a set of coefficients which accurately reconstructs both the amplitude and phase of the measured data. The value of the weighting function can vary from experiment to experiment. In fact, a weighting of five was used in reference 1.

RESULTS

Measured Cross-Spectra

The approach used herein to perform the modal analysis at each of the tailpipe axial locations entails the use of five cross-spectra from the five

pressure signals measured at each axial location. The cross-spectrum is defined for a pair of random pressure signals, at a give frequency f, as:

$$\overline{G_{0\theta}(f)} = \overline{P_0(f) \cdot P_{\theta}^{\star}(f)}$$
(4)

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The overbar indicates an ensemble average, and $P_0(f)$ and $P_{\theta}(f)$ are the Fourtier transforms of the measured pressure signals at the reference angle, and at the angle θ from the reference, respectively. The asterisk denotes the complex conjugate.

In general, $G_{00}(f)$ is a complex quantity containing both amplitude and phase information. Measured cross-spectra amplitudes for an engine speed of 19 200 rpm are shown figures 2 and 3 for the five angular separations of 0°, 75°, 135°, 165°, and 180°. These cross-spectra were determined using the 212° probe as the reference. The 0° case is, of course, the auto-spectrum of the reference microphone signal. The amplitudes at the tailpipe inlet are shown in figure 2 and those at the tailpipe exit are shown in figure 3. The cross-spectra amplitudes at the tailpipe inlet are characterized by several peaks with sharp dips between the peaks. The amplitudes at the tailpipe exit are somewhat smoother.

Measured cross-spectra phase are shown in figures 4 to 7 for engine speeds of 10 200 rpm and 19 200 rpm. Additional phase measurements are shown because of the relevance of the measured phase to the assumption that the amplitudes of clockwise and counterclockwise rotating modes of the same order are equal. This assumption resulted in a cross-spectrum phase of either O or 180. As can be seen from the data, there appears to be a strong tendency for the measured phase to be near one of these two angles. In some cases, such as 75° separation, for all four conditions shown, there are ranges of frequencies where the phase exhibits significant scatter. These regions correspond to regions of small cross-spectra magnitude resulting in low signal-to-noise ratio and therefore more random variation in phase. For example, figure 6(a) from 900 Hz to 1200 Hz, corresponds to a frequency range where the amplitude of the cross-spectrum, figure 2(b), is about 7 dB below that of the auto-spectrum, figure 2(a). Even over these frequency ranges the data tend to scatter about 0° or 180°. Although there are a few exceptions, the bulk of the measured phase data support the assumption of equal amplitude of clockwise and counterclockwise rotating modes of the same order.

Modal Content

The output of the modal analysis, described previously, is a set of five modal amplitudes at each frequency. Results of the modal analysis are presented in figures 8 to 13. Each figure consists of a series of plots showing the modal amplitudes compared to the total measured auto-spectrum i.e., each plot shows what portion of the total pressure signal is associated with a given mode. It should be noted, however, that when a modal amplitude is more than 10 dB below the total, the actual amplitude is probably not very accurate.

In figure 8, the modal amplitudes at the tailpipe inlet are shown for an engine speed of 19 200 rpm. In figure 8(a) the (0,0) mode (plane wave) is compared with the total auto-spectrum. The amplitude of the plane wave is

within about 2 dB of the auto-spectrum up to about 600 Hz, beyond which it is typically 10 dB or more below the total, and at many frequencies being below 100 dB and thus not plotted. In figure 8(b), the (1,0) circumferential mode is plotted. This mode is not a significant contributor to the tocal below 700 Hz but dominates from about 700 Hz to about 1100 Hz. From table I, it can be seen that at the tailpipe inlet at 19 200 rpm, the (1,0) mode cuts on at 696 Hz. It is at this frequency that the (1,0) mode starts to dominate the total auto-spectrum. Also from table I, the (2,0) circumferential mode cuts on at 1155 Hz. In figure 8(c), it can be seen that the (2,0) mode begins to make a significant contribution to the total at approximately this frequency. The (2,0) mode also makes some contribution to the total at frequencies below 600 Hz. In this frequency range, the plan wave, figure 8(a), does not equal the total and the difference between the plane wave and the total is made up by contributions from other modes. Except for a few isolated frequencies, the (3,0) and (4,0) modes do not make a significant contribution to the auto-spectrum.

Plots similar to those of figure 8 are presented for the tailpipe exit in figure 9. The results are nearly the same as for the tailpipe inlet. The (0,0) mode (plane wave), figure 9(a), dominates up to about 800 Hz which is the cut-on frequency for the (1,0) circumferential mode. The (1,0) mode, figure 9(b), then dominates until the (2,0) mode, figure 9(c), cuts on at 1327 Hz. Again the (3,0) and (4,0) modes make no significant contributions over the frequency range of this study. The major difference between the results at the exit and those at the inlet is that the plane wave nearly equals the total at low frequencies and the contributions from other modes at these low frequencies are negligible.

In figures 10 to 13, plots similar to those of figures 8 and 9 are presented for engine speeds of 10 200 and 15 150 rpm. The plots of the (3,0) and (4,0) modes are not presented since they do not make significant contributions. The results at these speeds are similar to those at 19 200 rpm.

The lack of a contribution to the auto-spectrum from cutoff modes, such as the (1,0) at low frequencies and the (3,0) and (4,0) modes is not surprising since these modes, being cutoff, decay exponentially with distance from a source. The contribution, at low frequencies, of the (2,0) mode to the auto-spectrum at the tailpipe inlet could imply that the measurement location was near a source for this mode.

Reconstruction of Measured Data

An indicator of the degree to which the modal analysis procedure yields an acceptable set of modal amplitudes is how well the modal amplitudes can be used to reconstruct the original measured auto- and cross-spectra. In figures 14 to 17, reconstructed auto- and cross-spectra are compared with measured auto- and cross-spectra. In figures 14 and 15, the amplitudes of the spectra are compared for 19 200 rpm at the tailpipe inlet and exit. The phase comparisons are made in figures 16 and 17. It is quite clear from figures 14 to 17, which are typical of all speeds, that the search routine used is very effective in determining a set of modal amplitudes which accurately reconstructs the measured auto- and cross-spectra.

CONCLUSIONS

The modal content of the sound field in the tailpipe of the separate flow exhaust version of the AiResearch QCGAT engine was determined. The modal analysis neglected higher order radial modes and assumed that circumferential modes of different orders were independent. The analysis also assumed that the magnitudes of counter rotating modes of the same order were equal. A least squares optimization routine was used to decompose measured auto- and crossspectra into a set of five modal amplitudes for the (0,0), (1,0), (2,0), (3,0), and (4,0) modes. The analysis was limited to frequencies below 1500 Hz. Results were presented at three engine speeds for both the tailpipe inlet and the tailpipe exit.

The resultant modal decomposition indicated that at low frequencies, up to the cut-on frequency of the (1,0) mode, the (0,0) mode, plane wave, dominated the pressure field. The frequency range from the cut-on of the (1,0) mode to the cut-on of the (2,0) mode was dominated by the (1,0) mode. The (2,0) mode dominated from its cut-on to the upper limit of the analysis, i.e., 1500 Hz. A more concise way of stating the above is that as frequency increases, the most recent cut-on mode dominated the pressure field. The contribution of modes other than the dominant mode was usually small, with the total contribution of the other modes being 2 dB or less. These results indicate that an analysis of the propagation in, or the radiation from, the tail-pipe based on the assumption of a single dominant mode, is adequate. The modal amplitudes determined in this study could be used as the input for such an analysis.

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Engine speed, rpm	Tailpipe inlet*					Tailpipe exit*				
	f1,0	f2,0	f3,0	f4,0	f0,1	f1,0	f2,0	f3,0	f4,0	f0,1
10 200	694	1151	1584	2004	1444	912	1513	2081	2634	1898
15 150	701	1162	1600	2025	1459	885	1470	2019	2556	1842
19 200	696	1155	1588	2010	1449	800	1327	1825	2310	1665

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TABLE I. - MODE CUT-ON FREQUENCIES

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*f is the cut-on frequency of the acoustic mode of circumferential order m and radial order n.



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Figure 2, - Concluded,



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Figure 5. - Measured cross-spectrum phase at tailpipe exit between reference microphone and microphone at angle 6 from reference, 10 200 rpm engine speed.



Figure 6. - Measured cross-spectrum phase at tailpipe inlet between reference microphone and microphone at angle 0 from reference, 19 200 rpm engine speed.



Figure 7. - Measured cross-spectrum phase at tailpipe exit between reference microphone and microphone at angle A from reference, 19 200 rpm engine speed.



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Figure 9. - Modal content of auto-spectrum at the tailpipe exit, engine speed 19 200 rpm.



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Figure 9. - Continued.







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Figure 10. - Conclusion,



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Figure 14. - Concluded,



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Figure 16, - Comparison of measured and reconstructed cross-spectra phase at the tailpipe inlet engine speed 19 200 rpm,



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16. Abstract									
The results of a modal anal fan engine are presented. presented, as a function of modal amplitudes were obtai between measured cross-spec between pressures at circum field was decomposed into a (0,0), (1,0), (2,0), (3,0), frequencies below 1500 Hz w analysis showed that at low mode, the (0,0) mode (plane range from the cut-on of th nated by the (1,0) mode. T the upper limit of the anal than the dominant mode was	ysis of the pre Modal amplitude frequency, for ned using an op tra and an anal ferentially spa set of five mo and (4,0) mode here higher ord frequencies, u wave) dominate e (1,0) mode to he (2,0) mode d ysis, i.e., 150 usually small.	ssure field in t s, at the tailpi several operati timization routi ytical expression ced locations. dal amplitudes of s. The analysis er modes are cut p to the cut-on d the pressure f the cut-on of t ominated from it 0 Hz. The contr	the tailpipe of ipe inlet and ing conditions ine to obtain a on for the cross The measured corresponding was limited to off. The ress frequency of field. The from the (2,0) mode ts cut-on freq ribution of mode	f a turbo- exit, are . The a best fit ss-spectra pressure to the to sults of the the (1,0) equency was domi- uency to des other					
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