# User's Manual for Axisymmetric Diffuser Duct (ADD) Code 

## Volume III-ADD Code Coordinate Generator

O. L. Anderson, G. B. Hankins, Jr., and D. E. Edwards
United Technologies Research Center

February 1982

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Lewis Research Center

Under Contract DEN 3-235
UIBRARY EOPY
for

## U.S. DEPARTMENT OF ENERGY Conservation and Renewable Energy Office of Vehicle and Engine R\&D

$\therefore 1983$

## NOTICE

This report was prepared to document work sponsored by the United States Government Nether the United States nor its agent, the United States Depanment of Energy, nor any Federal employees, nor any of their contractors. subcontractors or their employees, makes any warranty. express or implied, or assumes any legal liability or responsibility for the accuracy, completeness. or usefulness of any information. apparatus product or process disclosed. or represents that its use would not infringe privately owned rights

```
                0
            0, 0,*2
            0}01*
            0.02*5
            0 0 2*3
            0 02*4
    7 1 1 RN/NASA-CR-165598-4OL-3
            mpaynyoprea
```




```
                            OTPC81-65-VOL-3 CNT#: DUM-235 DE-HTOL-77CS-51040 82/02/00 3 vOLS
                            OO PAGES UNCLASSIFIED DOCUMENT
    UTTL: l!ge's marual for Axisvmmetric Difuser Duct (ADO code. Yolume 3: ADD
            code coormmete generator TLSP: Final Foport
```



```
    CORP: United Techmologies Comp. Eget Hartford, Conm. AvAlL. WTIS SAP: HC
        n05/MF A01
```



```
        programS
```



```
    ABA: Authon
    ABG: The Ueer's Manual contains a comelete deggiption of the compter codes
```



```
    references which descuibe the formulatim ot the nov rode and comparimone
    of calculation with experimental flowe, The imputroutput and genopal use
    of the code is descmbed in the firet volume. The second volumemtalne a
    detalled degoription of the coce moludme the globel stuctura of the
    code, list of FontRAM variables and desciptions of the subroutines. The
    third volume contains a detelled descmptim of the mouct code mhint
    generates condmate syetems for zrbitrarv axisymmetric oucts,
```


# User's Manual for Axisymmetric Diffuser Duct (ADD) Code 

## Volume III-ADD Code Coordinate Generator

O. L. Anderson, G. B. Hankins, Jr., and D. E. Edwards United Technologies Research Center
East Hartford, Connecticut 06108

February 1982

Prepared for
National Aeronautics and Space Administration Lewis Research Center
Cleveland, Ohio 44135
Under Contract DEN 3-235
for
U.S. DEPARTMENT OF ENERGY

Conservation and Renewable Energy
Office of Vehicle and Engine R\&D
Washington, D.C. 20545
Under Interagency Agreement DE-AIO1-77CS51040

USER'S MANUAL FOR
AXISYMMETRIC DIFFUSER DUCT
(ADD) CODE

TABLE OF CONTENTS

VOL. III ADD CODE COORDINATE GENERATOR

## Page

10.0 OPERATION OF CØDUCT CODE. . . . . . . . . . . . . . . . . . . . . III-1
10.1 Runstream . . . . . . . . . . . . . . . . . . . . . . . . . III-1
10.2 Input Format. . . . . . . . . . . . . . . . . . . . . . . III-2
10.3 Output Format . . . . . . . . . . . . . . . . . . . . . . III-10
10.4 Diagnostics and Failure Modes . . . . . . . . . . . . . . . III-21
11.0 GLOBAL STRUCTURE OF CØDUCT CODE . . . . . . . . . . . . . . . . . III-23
11.1 Main Program CめDUCT . . . . . . . . . . . . . . . . . . . . III-24
11.2 Global Tree Structure by Task . . . . . . . . . . . . . . . III-25
11.3 List of Labeled CøMMめN Blocks . . . . . . . . . . . . . . . III-27
12.0 DETAILED DESCRIPTION OF CQDUCT CODE . . . . . . . . . . . . . . . III-36
12.1 List of Subroutines . . . . . . . . . . . . . . . . . . . . III-37
12.2 Description of Subroutines. . . . . . . . . . . . . . . . . III-39

### 10.0 OPERATION OF CØDUCT CODE

### 10.1 Runstream

The following runstream for a UNIVAC 1100 operating system is used to assign input/output disc files and to execute the CØDUCT coordinate generator code.
@ASG, A
@USE
FILE9., D/0/TRK/300000
9., FILE9.
@ASG, A
FILE10.,D/0/TRK/300000
@USE
@XQT
CøDUCT
-
-
-
data cards
-
-
@FIN

FILE9 contains the coordinate data for a uniform mesh and FILE10 contains the data for a nonuniform mesh.

### 10.2 Input Format

The input format for the C $\emptyset D U C T$ code is described on the input data coding forms which follow. With the exception of the first card (Title card) and the duct geometry cards, the input data cards follow in sets of three cards. The first of three is a blank separator card. The second of three is the input variable name and the third of three is the value of the input variable. In general the input data is read as follows:

Card 1 Title Card
Cards 2-4 Program Control Parameters
Cards 5-7 Program Control Parameters
Cards 8-10 Coordinate Generator Parameters
Cards 11-13 Coordinate Generator Parameters
Card 14 Number of duct geometry coordinates
Cards $15+\quad$ Duct geometry coordinates

## CØDUCT CODE INPUT

Card 1 Title Card (18A4)


CøDUCT CODE INPUT

Cards 2-4 Program Control Parameters (T6, I3, T20, I1, T32, I1, T43, I2, T53, F5.2, T64, E8.3)


MAXIT IPRINT

Maximum number of iterations for conformal mapping solution Print option:
$=0$ Print only input and final mapping
$=1$ Print iteration results
$=2$ Print approximate solution

Coordinate generation option:
$=0$ Calculate conformal mapping only
$=1$ Calculate coordinates

Approximate solution option
$=0$ Program determines number of approximate potential lines
> 0 User specifies number of approximate potential lines

Step control in approximate solution calculations

Convergence criterion on conformal mapping iteration

## CめDUCT CODE INPUT

Cards 5-7 Program Control Parameters (T8, I1, T18, I3, T30, I3)
$=0$ Calculate approximate solution only
= 1 Calculate conformal mapping
$\underset{\substack{\text { H } \\ \underset{\sim}{H} \\ \multirow{2}{*}{}}}{ } \quad$ KN

Number of streamlines to generate on uniform grid, number of integration steps
Potential line integration step control:
$=0 \mathrm{KN}$ integration steps are used
$>0$ There will be NSD additional steps taken for each of the KN steps yielding NNS $=(K N-1) * N S D+K N$ total steps

## CØDUCT CODE INPUT

Cards 8-10 Coordinate Generation Control Parameters (T6, I3, T18, I3, T30, I3, T44, I1, T51, E8.3, T68, I1)


```
Cards 11-13 Coordinate Generation Control Parameters (T7, I2, T19, I2, T32, I1, T43, I2, T55, I1)
```



| IUUNIT | Output unit number for uniform mesh |
| :--- | :--- |
| INUNIT | Output unit number for nonuniform mesh |$\quad$| Smoothing option |
| :--- |
| ISMOOT |
| JXK |
| IXFG |

$=0$ Conformal mapping procedure
$=1$ IUUNIT data is read and interpolated to new grid in INUNIT

CøDUCT CODE INPUT
Card 14 Number of Duct Geometry Coordinates (I3)


NLF
Number of coordinate pairs defining the upper/lower walls
$\underset{\infty}{\underset{\sim}{B}}$


Upper wall $x$-coordinates

Upper wall $y$-coordinates

Lower wall x-coordinates
Lower wall y-coordinates
Where the data is read as follows

READ (IRUNIT,40) (XU(J), J=1,NLF)
READ (IRUNIT, 40) (YU(J), J=1,NLF)
READ (IRUNIT, 40) (XL(J), $\mathrm{J}=1, \mathrm{NLF}$ )
READ (IRUNIT, 40) (Y.L(J), J=1,NLF)

40 FORMAT (8F10.6)

### 10.3 Output Format

The printed output from the CØDUCT code is given on the following pages and is largely self explanatory. These pages contain the names of the subroutines which calculate the data as well as any print options which may be involved.

Input Data Echo Page (1)
Printed by RCNTRL Calculated by RCNTRL
Options None

## Description

Subroutine RCNTRL reads the input control data (cards 1-13) and prints the input control data.

Duct Coordinates Echo Page (2)
Printed by CORINP
Calculated by N/A
Options
None

## Description

Subroutine CORINP reads the duct wall coordinates as described in Section 10.2 and prints the wall coordinates.

| 1-2 |  |  |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |


| Heading | Variable | Description |
| :--- | :---: | :--- |
|  |  |  |
| $\mathrm{PT} \#$ | J | Wall coordinate number |
| XU | $\mathrm{X}_{\mathrm{U}}$ | , |
| YU | $\mathrm{Y}_{\mathrm{U}}$ | Upper wall input X coordinate |
| XL | $\mathrm{X}_{\mathrm{L}}$ | Upper wall input Y coordinate |
| YL | $\mathrm{Y}_{\mathrm{L}}$ | Lower wall input X coordinate |
|  |  | Lower wall input Y coordinate |


| Printed by | CORINP |
| :--- | :--- |
| Calculated by | SMARCL, SMOOTH |
| Options | ISMOOT |
|  | ISMOOT $=0$, no smoothing, no printout |
|  | ISMOOT $=1$ smoothed, printed |

## Description

The input coordinates are read in CORINP and subroutine SMARCL is called to cubic-spline smooth the wall data.
Pages $1-2$

| Printed by | RDAVIS, ROTATM |
| :--- | :--- |
| Calculated by | RDAVTS, ROTAIM |
| Options | None |

Description

Subroutine MDAVIS determines the orientation of the duct and forces the inlet walls parallel. The rotational and scaling constant $M$ and the exit wall angle are calculated and output by subroutine ROTATM.

| Heading | Variable | Description |
| :---: | :---: | :---: |
| None | $\theta_{1}$ | Angle of inlet lower wall to horizontal |
| None | h | True height of duct - perpendicular distance from lower wall to upper wall |
| None | M | Rotational and scaling constant used in Schwartz-Cristoffel mapping |
| None | $\alpha_{E}$ | Relative angle at exit of duct $+\pi$ |

## Approximate Potential Flow Page (6)

| Printed by | WALLV, ESTIMP |  |
| :--- | :--- | :--- |
| Calculated by | ESTIMP, ESTCOR, WALLV |  |
| Options | IPRINT |  |
|  | IPRINT $<2$ | no output |
|  | IPRINT $=2$ | output |

## Description

Subroutine ESTIMP solves for a geometric approximate potential solution in the duct using subroutine ESTCOR to determine the positions of a set of approximate potential lines. The wall velocities and curvatures at the end points of these lines are calculated and printed by subroutine WALLV. The solution along each wall is interpolated to the set of wall coordinates and an estimate of the Schwartz Cristoffel mapping is determined. This estimate is calculated and printed by subroutine ESTIMP.

| Heading | Variable | Description |
| :---: | :---: | :---: |
| PT \# | NP | Approximate potential line number |
| ARCL | S | Arc Length |
| VMEAN | V | Duct midline velocity |
| VWALL | $\mathrm{V}_{\mathrm{L}}$ | Lower wall velocity |
| KMEAN | $\overline{\mathrm{K}}$ | Duct midine curvature |

Approximate Potential Flow Pages (Cont'd)

| Heading | Variable | Description |
| :---: | :---: | :---: |
| PT\# | NP | Approximate potential line number |
| ARCL | S | Arc length |
| VMEAN | $\overline{\mathrm{v}}$ | Duct midline velocity |
| VWALL | $\mathrm{V}_{\mathrm{U}}$ | Upper wall velocity |
| KMEAN | $\overline{\mathrm{K}}$ | Duct midline curvature |
|  |  |  |
| Heading | Variable | Description |
| PT\# | J | Wall coordinate number |
| VWALL | $\mathrm{V}_{\mathrm{L}}$ | Lower wall velocity |
| KWALL | $\mathrm{K}_{\mathrm{L}}$ | Lower wall curvature |
| TT |  | t-plane pole estimates |
| B | b | $\zeta$-plane pole estimates |
|  |  |  |
| Heading | Variable | Description |
| PT非 | J | Wall coordinate number |
| VWALL | $\mathrm{V}_{\mathrm{u}}$ | Upper wall velocity |
| KWALL | $\mathrm{K}_{\mathrm{u}}$ | Upper wall curvature |
| TT | t | t-plane pole estimate |
| B | b | ち-plane pole estimate |

Iteration History Pages

| Printed by | MDAVIS |  |
| :--- | :--- | :--- |
| Calculated by | MDAVIS, STEP |  |
| Options | IPRINT |  |
|  | IPRINT $=0$ | not printed |
|  | IPRINT $>0$ | history printed |

## Description

The current estimate of $\mathrm{dz} / \mathrm{dt}$ is integrated along the duct walls in subroutine MDAVIS by steps $t_{j+1}-t_{j}$ in the $t-p l a n e$. The results of the integration and the error of the current iteration are printed.

Page_1 ITERATION HISTORY

| Heading | Variable | Description |
| :---: | :---: | :---: |
| PT非 | J | Wall coordinate number - j |
| TX | $\mathrm{t}_{\mathrm{x}}$ | Real component of $t_{j}$ |
| TY | $\mathrm{t}_{\mathrm{y}}$ | Imaginary component of $\mathrm{t}_{\mathrm{j}}$ |
| X | $\mathrm{z}_{\mathrm{x}}$ | Real component of $\mathrm{z}_{\mathrm{j}}$ |
| Y | $\mathrm{z}_{\mathrm{y}}$ | Imaginary component of $\mathrm{z}_{\mathrm{j}}$ |
| Ratio | R | Arc length ratio |
| ERROR |  | Absolute error $\mathrm{Zc}_{\mathbf{j}}-\mathrm{Z}_{\mathrm{j}}$ |
| DZDT | $\frac{d z}{d t}$ | Complex derivative ot mapping |

Iteration Summary Page (1)

| Printed by | MDAVIS, CLOSUR |
| :--- | :--- |
| Calculated by | MDAVIS, CLOSUR |
| Options | None |

## Description

Subroutine MDAVIS calculates the maximum relative error in the coordinate calculation for each iteration and tests for convergence of the mapping solution. Subroutine CLOSUR determines the integrated closure error of the solution.

Page 1_ _

| Heading | Variable |  | Description |
| :---: | :---: | :---: | :---: |
| ITERATION | $\nu$ | , | Iteration count |
| SCALED MAXIMUM ERROR | $\varepsilon$ | , | Maximum scaled error for the iteration |
| None | ZCL | , | Closure error $\left\|\bar{Z}_{1}-\overline{\mathrm{Z}}_{2}\right\|$ |
| None | $\bar{z}_{1}$ | , | Integration path \#1 endpoint |
| None | $\overline{\mathrm{Z}_{2}}$ | , | Integration path \#2 endpoint |

Mapped Duct Coordinate Page (2)

| Printed by | MDAVIS |
| :--- | :--- |
| Calculated by | MDAVIS |
| Options | None |

Description
Once the mapping iteration has terminated, the final solution and errors are printed for each wa!l coordinate.

| Heading | Variable | Description |
| :---: | :---: | :---: |
| TX | $\mathrm{t}_{\mathrm{x}}$ | Real part of t-plane pole location |
| TY | $\mathrm{t}_{\mathrm{y}}$ | Imaginary part of t-plane pole location |
| X | X | Image of T under mappping |
| Y | Y |  |
| XC | $\mathrm{X}_{\mathrm{C}}$ | Input coordinates |
| YC | $\mathrm{Y}_{\mathrm{C}}$ |  |
| EX | $\varepsilon_{\mathrm{X}}$ | Error $\left\|\mathrm{X}-\mathrm{X}_{\mathrm{C}}\right\|$ |
| EY | $\varepsilon_{\text {y }}$ | Error $\left\|Y-Y_{c}\right\|$ |
| S | S | Arc length |

Mesh Generation Page (1)

| Printed by | COORD |
| :--- | :--- |
| Calculated by | COORD |
| Options | None |

## Description

The $t-p l a n e$ uniform mesh that is used for the coordinate generation is described.


Heading
Variable
Description

None JL
None DSTEP
None
KN
Number of streamwise steps
Streamwise step size Number of uniform normal steps

### 10.4 Diagnostics and Failure Modes

Numerous checks are performed during the course of the calculation. If a non-fatal or correctable error occurs a DIAGNOSTIC message is printed and the calculation continues. If a fatal error occurs a FAILURE llode error is printed and the calculation stops. A DIAGNOSTIC message is printed of the form:
** DIANGOSTIC NO. XX for 2-D COORDINATE OPERATOR and a FAILURE Mode message is of the form:
** FAILURE iNO. XX for $\sum-D$ COORDINATE GENERATOR where XX refers to one of the conditions listed below.

## DIAGNOSTICS

1) NUMERICAL SOLUTION OF SCHWARTZ-CRISTOFFEL TRANSFORMATION FAILED TO CONVERGE

This error is detected in subroutine MDAVIS. It indicates that the scaled maximum error in the computed wall coordinates is greater than the input value ECONV after MAXIT iterations have been completed. By examining the ITERATION SUMMARY printed above the diagnostic message, one of three courses of action may be determined.
a) The Scaled Maximum Error (SME) appears to be converging. Reset MAXIT and rerun the case.
b) The SME has converged to a value different than zero. This can often be remedied by increasing the number of sub-steps (NSD), employed in the normal direction integration. If this does not solve the problem, more wall definition coordinates may be needed.
c) The SME is not converging. This often indicates that a poor initial potential flow solution was generated.
2) UPPER AND LOWER WALLS NOT PARALLEL AT INLET. UPPER WALL FORCED PARALLEL. TO LOWER WALL

This error is detected in subroutine ROTATM. It implies that the inlet upper and lower wall angles with respect to the horizontal differed by less than ten degrees but greater than $1.0-10$. The upper wall endpoint is moved to force the walls parallel.

FAILURE MODES

1) MESH DISTORTION PARAMETER EQUALS $=$ XXXXX

This error is detected in subroutine ROBRTS.
2) MESH DISTORTION PARAMETER EQUALS = XXXXX

This error is detected in subroutine DROBRT.
3) LOWER WALL ANGLE = XXXXX DEGREES

UPPER WALL ANGLE $=$ XXXXX DEGREES
WILL NOT FORCE PARALLEL IF DIFFERENCE IS > 10 DEG.

This error is detected in subroutine ROTATM. It indicates that the inlet walls are not sufficiently parallel, and the program will not force the walls parallel to avoid drastically changing the geometry.
4) DEGENERATE DERIVATIVE MAPPING FOR I = XXXXX

This error is detected in subroutine MDAVIS. It implies that $\left|\frac{d z}{d t}\right|<1 . D-8$ at wall point XXXXX.
5) INCONSISTENT OR INVALID INPUT

This error is detected in main program CODUCT. Check input data set.
6) READ ERROR ENCOUNTERED IN SUBROUTINE CORINP

This error, detected while reading duct wall coordinates, indicates an error in the input data set.
7) NUMBER OF INPUT POINTS EXCEEDS MAXIMUM (XXX)

This error, detected in subroutine CORINP, indicates that too many wall values are defined.
8) UNABLE TO COMPLETE APPROXIMATE SOLUTION

This error is detected in subroutine ESTCOR. It implies that more than ten attempts have been made to compute a single potential line and is usually due to very large wall curvatures.
9) INDEPENDENT STEP SIZE TOO SMALL

This error is detected in subroutine DERIV3. It indicates that two consecutively numbered wall coordinates are equal.
10) IMSL LIBRARY ICSVKU FAILURE NO. XXX

This error is detected in subroutine SMOOTH. It implies that the IMSL program ICSVKU cannot solve the spline problem. See IMSL manual to determine remedy.

### 11.0 GLOBAL STRUCTURE OF CØDUCT CODE

This section of the manual is intended for the special user who wishes to modify the CøDUCT code or adapt to a different computer. The section provides a global overview of the code in terms of the principal tasks. These tasks are clearly labeled in the main program C $\emptyset D U C T$ and agrees with the tasks listed on the Global Tree Structure Chart in Section 11.2. The global variables in labeled C $\varnothing M M$ N blocks are given in Section 11.3. Only the variables unique to the CめDUCT code are listed. Variables that are used by both the CøDUCT code and ADD code are listed in Section 6.0. Special problems associated with machine specific code are similar to those in the ADD code and are treated in Section 5.0.

### 11.1 Main Program CODUCT

Object
Main program for coordinate generator.
Options

| FG | $=0$ | Full solution |
| :---: | :---: | :---: |
|  |  | Interpolate only |
| IESTIM | $=0$ | Approximate solution only |
|  |  | Schwartz-Cristoffel transformation also |
| ICORD | $=0$ | No coordinate output |
|  |  | Coordinate output to disk files. |

## Theory

The control program CODUCT first calls subroutine RCNTRL (See Table 1) to read the user-input control parameters and options. Subroutine CORINP is then called to read the duct wall coordinates and, if requested, will smooth the wall coordinate using a cubic-spline fitting algorithm. Subroutine ESTCOR is called to geometrically determine the approximate potential flow solution necessary to start the conformed mapping iteration procedure. Then Davis' algorithm to compute the Schwartz-Cristoffel transformation is invoked by calling subroutine MDAVIS. Finally, subroutine COORD is called to generate and output the coordinate mesh parameters to disk file(s).

### 11.2 Global Tree Structure By Task

| Read Control Input |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| CODUCT | RCNTRL |  |  |  |
| Read and Smooth Duct Wall Data |  |  |  |  |
| CODUCT | CORINP | SMARCL | ARCLI <br> SMOOTH | ICSVKU |
| Calculate Approximate Potential Flow Solution |  |  |  |  |
| CODUCT | ESTIMP | ARCL1 <br> KURVTR <br> ESTCOR <br> WALLV | DER1V3 <br> INSECT <br> CROSS1 <br> DER1V3 <br> SUNBAR <br> UNBAR | CROSS1 |
| Calculate Srhwartz-Cristoffel Transformation |  |  |  |  |
| CODUCT | MDAVIS | ROTATM <br> INTNOR <br> STEP <br> TTUP <br> CLOSUR | $\begin{aligned} & \text { STEP } \\ & \text { STEP } \\ & \text { INTNOR } \\ & \text { INTSTR } \end{aligned}$ | $\begin{aligned} & \text { STEP } \\ & \text { STEP } \end{aligned}$ |


| Calculate Coordinates and Metrics |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CODUCT | COORD | CONSTR | CDS |  |  |  |
|  |  |  | ROBRTS |  |  |  |
|  |  |  | DROBRT |  |  |  |
|  |  | COOR1D | NORLIN | STEP |  |  |
|  |  |  | STRSTP | STEP |  |  |
|  |  |  | CDVDN |  |  |  |
|  |  |  | CORSTR |  |  |  |
|  |  |  | Q2INTD |  |  |  |
|  |  |  | BLKWRT | NTRAN8 |  |  |
|  |  |  | QPSTOR | QPCURV | INSECT | CROSS 1 |
|  |  | COORMD | STRSTP | STEP |  |  |
|  |  |  | CDVDN |  |  |  |
|  |  |  | CORSTR |  |  |  |
|  |  |  | QPSTOR |  |  |  |
|  |  |  | Q2INTP |  |  |  |
|  |  |  | BLKWRT | NTRAN\$ |  |  |
|  |  |  | QPSTOR | QPCURV | INSECT | CROSS 1 |
| Transform from Uniform to Non-uniform mesh |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| CODUCT | XfGRID | INITQ | BLKRED | NTRAN\$ |  |  |
|  |  | CONSTR |  |  |  |  |
|  |  | BLKRED | NTRAN\$ |  |  |  |
|  |  | Q2INTP |  |  |  |  |
|  |  | QPSTOR | QPCURV | InSECT | CROSS1 |  |
|  |  | BLKWRT | NTRAN\$ |  |  |  |

11.3 List of Labeled C $\varnothing M M \phi$ N Blocks

Name
BSMøTH
CESTP
СøøRC $\varnothing$
cøøRT
IPDAVS
NSDAVS
ØPDAVS
TITLE

Object
Variables for spline smoothing
Variables for approximate solution
Control options and parameters
Complex coordinates and derivative
Complex solution variables
Constants and parameters for mapping Intermediate Schwartz-Christoffel variables Run Title

List of Variables in C $\varnothing$ MM $\varnothing \mathrm{N} / \mathrm{BSM} \varnothing \mathrm{TH} /$ Variables for Spline Fitting

| Name | Symbol | Length | Type |
| :--- | :--- | :--- | :--- |
| A |  | Description |  |
| B | NXK | $\mathrm{R} * 4$ | Integration constant |
| CK | NXK | $\mathrm{R} * 4$ | Integration constant |
| WK | ICK,3 | $\mathrm{R} * 4$ | Spline coefficients |
| XK | IWK | $\mathrm{R} * 4$ | Work area |
| YPP | NXK | $\mathrm{R} * 4$ | Knot locations |
|  |  | NXNTD2 | $\mathrm{R} * 4$ |

List of Variables in C $\varnothing M$ M $\varnothing \mathrm{N} / \mathrm{CESTP} /$
Variables for Approximate Solution

| Name | Symbo 1 | Length | Type | Description |
| :---: | :---: | :---: | :---: | :---: |
| HT | $\mathrm{h}_{\mathrm{t}}$ | NXNTD2 | $\mathrm{R} * 4$ | Approximate duct height |
| IL, IU |  | NXNTD2 |  | Index lower/upper walls |
| KKL, KKU | $\mathrm{K}_{\mathrm{L}}, \mathrm{K}_{\mathrm{U}}$ | NXNTD2 | $R * 4$ | Curvature lower/upper walls |
| KMEAN | $\overline{\mathrm{K}}$ | NXNTD2 | R*4 | Curvature of mean line |
| SL, SU | $\mathrm{S}_{\mathrm{L}}, \mathrm{S}_{\mathrm{U}}$ | NXNTD2 | R*4 | Arc length lower/upper walls |
| SLI, SUI | $\mathrm{S}_{\mathrm{LI}}, \mathrm{S}_{\mathrm{UI}}$ | NXNTD2 | $R * 4$ | Arc length lower/upper walls |
| SMID | $\overline{\mathrm{S}}$ | NXNTD2 | $R * 4$ | Arc length mean line |
| TH | $\theta$ | NXNTD2 | R*4 | Angle of mean line with $x$ axis |
| VL, vu | $\mathrm{V}_{\mathrm{L}}, \mathrm{V}_{\mathrm{U}}$ | NXNTD2 | R*4 | Velocity lower/upper walls |
| VLI, VUI | $\mathrm{V}_{\mathrm{LI}}, \mathrm{V}_{\mathrm{UI}}$ | NXNTD2 | R*4 | Velocity lower/upper walls |
| VMEAN | $\overline{\mathrm{v}}$ | NXNTD2 | $R * 4$ | Velocity on mean line |
| XL, YL | $\mathrm{X}_{\mathrm{L}}, \mathrm{Y}_{\mathrm{L}}$ | NXNTD2 | R*4 | Input coordinate lower wall |
| XU, YU | $X_{U}, Y_{U}$ | NXNTD2 | R*4 | Input coordinate upper wall |
| XLI, YLI | $\mathrm{X}_{\mathrm{LI}}, \mathrm{Y}_{\text {LI }}$ | NXNTD2 | R*4 | Coordinates lower wall |
| XUI, YUI | $\mathrm{X}_{\mathrm{UI}}, \mathrm{Y}_{\mathrm{UI}}$ | NXNTD2 | $\mathrm{R} * / 4$ | Coordinates upper wall |

Note: subscript I denotes intersection of approximate potential line with wall

## List of Variables in CøMM $\mathrm{N}_{\mathrm{N}} / \mathrm{C} \varnothing \varnothing \mathrm{RC}$ (/

Control Options and Farameters

| Name | Symbol | Length | Type | Description |
| :---: | :---: | :---: | :---: | :---: |
| DDS | $(\Delta n / \Delta n)_{1}$ |  | R*4 | Ratio uniform/nonuniform grid at wall |
| IGRID |  |  | I*4 | Grid option |
| INUNIT |  |  | I*4 | Output unit for nonuniform grid |
| IRUNIT |  |  | I*4 | Read unit |
| ISMめФT |  |  | I*4 | Smoothing option |
| IUUNIT |  |  | I*4 | Output unit for uniform grid |
| IWUNIT |  |  | I*4 | Print unit |
| IXFG |  |  | I*4 | Transfer grid option |
| JL |  |  | I*4 | Number of output streamwise stations |
| JLPTS |  |  | $\mathrm{I} * 4$ | Number of input stations |
| JXK |  |  | I*4 | Number of knots for spline |
| KL |  |  | I*4 | Number of nonuniform streamlines |
| KN |  |  | I*4 | Number uniform streamlines |
| LøP |  |  | I*4 | Mesh distortion option |
| RADR | ${ }^{r}$ |  | $\mathrm{R} * 4$ | Reference length |
| TTL | ${ }^{\text {L }}$ L |  | $R * 4$ | Maximum t-plane coordinate lower wall |
| TTU | $\mathrm{t}_{\underline{u}}$ |  | R*4 | Maximum t-plane coordinate upper wall |

# List of Variables in CøMM $/ \mathrm{N} / \mathrm{C} \varnothing$ RT/ Complex Coordinates and Derivatives 

| Name | Symbol | Length | Type | Description |
| :--- | :--- | :--- | :--- | :--- |
| DZDTJ | $\left(\frac{d z}{d t}\right)^{J}$ | KLL | $C * 16$ | Mapping derivative |
| DZDTJP | $\left(\frac{d z}{d t}\right)^{J+1}$ | KLL | $C * 16$ | Mapping derivative |
| DZDTP2 | $\left(\frac{d z}{d t}\right)_{1}$ | KLL | $C * 16$ | Mapping derivative |
| DZDT1 | $\left(\frac{d z}{d t}\right)^{\prime}$ | KLL | $C * 16$ | Mapping derivative |
| ZJ | $Z^{J}$ | KLL | $C * 16$ | Coordinate in duct plane |
| ZJP | $Z^{J+1}$ | KLL | $C * 16$ | Coordinate in duct plane |
| ZP2 | $Z^{J+2}$ | KLL | $C * 16$ | Coordinate in duct plane |
| Z1 | $Z^{1}$ | KLL | $C * 16$ | Coordinate in duct plane |

```
List of Variables in C\emptysetMMMN/IPDAVS/
    Complex Solution Variables
```

| Name | Symbol | Length | Type | Description |
| :---: | :---: | :---: | :---: | :---: |
| B | $b^{\nu-1}$ | NXNT | $\mathrm{R} * 8$ | Location of poles |
| BNEW | $b^{v}$ | NXNT | $R * 8$ | Location of new poles |
| TG | $t^{v-1}$ | NXNT | C*16 | t-plane wall coordinates |
| TT | $t^{\nu}$ | NXNT | C*16 | New t-plane wall coordinates |
| Z | Z | NXNT | C*16 | Z-plane calculated wall coordinates |
| ZC | $\mathrm{Z}_{\mathrm{c}}$ | NXNT | $C * 16$ | Z-plane input wall coordinates |

## List of Variables in C $\emptyset M M \not \subset N / N S D A V S /$ <br> Constants and Parameters for Mapping

| Name | Symbol | Length | Type | Description |
| :---: | :---: | :---: | :---: | :---: |
| DM | $\mathrm{d}_{\mathrm{m}}$ |  | $R * 8$ | Automatic step size in approximate solution |
| ECøVV | $\varepsilon_{c}$ |  | $R * 8$ | Convergence criteria |
| IC | $0 .+1 . i$ |  | I*4 | Complex number $\sqrt{-1}$ |
| $I C \emptyset R D$ |  |  | I*4 | Coordinate generator option |
| IESTIM |  |  | I*4 | Approximate potential flow option |
| IPRINT |  |  | $I * 4$ | Print option |
| MAXIT |  |  | $I * 4$ | Maximum number of iterations |
| N |  |  | I*4 | Number of wall points (NLF+2) |
| NBE |  |  | I*4 | Number of non-trivial angle changes |
| NCL, NCU |  |  | I*4 | Number of lower/upper wall elements |
| NLF |  |  | I*4 | Number of lower wall points |
| NM1 |  |  | I*4 | $\mathrm{N}-1$ |
| NNS |  |  | $I * 4$ | Number of additional steps in integration |
| NUI |  |  | I*4 | NLF+1 |
| NIPゆT |  |  | I*4 | Approximate potential flow option |
| $\emptyset \mathrm{NE}$ | 1. $+0 . i$ |  | $C * 16$ | Complex 1.0 |
| XM | M |  | $\mathrm{C} * 16$ | Scale constant |
| ZER | 0. + 0i |  | C*16 | Complex 0.0 |

# List of Variables in C $\varnothing \mathrm{MM}$ ПN/TITLE/ <br> Run Title 

| Name | Symbol | Length | Type |
| :--- | :--- | :--- | :--- | Description

III-34

$$
\text { List of Variables in C } \emptyset \mathrm{MM} \mathrm{~N}_{\mathrm{N}} / \emptyset \mathrm{PDAVS/}
$$

| Name | Symbol | Length | Type | Description |
| :---: | :---: | :---: | :---: | :---: |
| ALPHA | $\alpha_{J}$ | NXNT | $R * 3$ | $\operatorname{Re}\left(\beta_{J}\right)+1$. |
| BETAM | $\beta_{J}$ | NXNT | C*16 | Change in wall angle |
| DELR | $\mathrm{b}_{\mathrm{J}+1^{-b}}{ }_{\mathrm{J}}$ | NXNT | $R * 8$ | Difference in pole locations |
| EXITA | $\alpha_{E}$ | 1 | $R * 8$ | Duct exit divergence angle $+\pi$ |
| HEIGHT | h | 1 | $R * 8$ | Duct inlet height |
| RATIO | $\mathrm{R}_{\mathrm{J}}$ | NXNT | $R * 8$ | Ratio of actual to calculated length |
| THETA1 | $\theta_{1}$ | 1 | $R * 8$ | Rotation of duct from real axis |
| ZET | $\zeta_{\mathrm{J}}$ | NXNT | C*16 | Estimated pole location |

### 12.0 DETAILED DESCRIPTION OF CØDUCT CODE

This section contains an alphabetic list of subroutines and a detailed description of each subroutine. Subroutines that are used by both the ADiv code and the CODUCT code are listed and described in Section 7.0. The description of the subroutines which follow have the same format. This format consists of the object or purpose of the subroutine, any options used by the subroutine, and a list of variables not in a $C \emptyset M M \emptyset N$ block which are used by the subroutine. Variables in $C \emptyset M M \emptyset N$ blocks are listed in Section 6.1 or Section 11.3 . Following the list of variables is a brief description of the analysis performed by the subroutine.

|  | 12．0 DETAIL DESCRIPTION OF CめDUCT CODE <br> 12．1 List of Subroutines |
| :---: | :---: |
| Name | Object |
| ARCL1 | Calculate arc length of input curve |
| BLKRED | See Section 7.2 |
| CDVDN | Calculate streamline curvature |
| ¢ $\varnothing$ BLK | Block data（IBM version） |
| CøDUCT | I＇ain program（see Section 11．1） |
| CLDSUR | Calculate closure error in mapping |
| CøNSTR | Store fixed data in Q1，Q2 arrays |
| Cø $\emptyset$ RD | Calculate coordinates and metrics |
| CDØRMD | Calculate coordinates $\mathrm{J}=2$ ， JL |
| CめØR1D | Calculate coordinates $\mathrm{J}=1$ |
| CめRINP | Read coordinate data |
| CøRSTR | Store coordinates in Q1 array |
| CRøSS1 | See Section 7.2 |
| DERIV3 | Calculate 3 point central difference derivative |
| DRØBRT | See Section 7.2 |
| ESTC＠R | Determine locationof approximate potential line |
| ESTIMP | Calculate approximate pole locations |
| INSECT | Determine intersection of potential line and wall |
| INTNØR | Calculate end point of potential line |
| INTSTR | Calculate end point of streamline |

12.1 List of Subroutines (Cont'd)

| Name | Object |
| :---: | :---: |
| KURVTR | Calculate curvature of input curve |
| MDAVIS | Solve Schwartz-Christoffel mapping |
| NØRLIN | Calculate single potential line |
| QPKURV | Interpolates curvatures at output location |
| QPSTøR | Store $Q$ parameters in Q1, Q2 arrays |
| Q2INTP | Interpolate from uniform to non-uniform mesh |
| RCNTRL | Reads user input control parameters |
| RめTATM | Calculate duct rotation and scaling |
| SMARCL | Cubic spline smoothing on arc length |
| SMめ ${ }^{\text {d }}$ | See Section 7.2 |
| STEP | Davis' integration formula |
| STRSTP | Integrate each streamline one step |
| SUNBAR | Store X, Y data into interpolation table T |
| TIUP | Update upper wall upstream point |
| UNBAR | Lagrange table interpolation |
| WALLV | Approximate potential flow wall velocity |
| XFGRID | Interpolate uniform to nonuniform grid |

### 12.2 Description of Subroutines

## Subroutine ARCLI (X,Y,NPT,S)

## Object

Calculate arc length of input curve

## Options

None

## Symbols

| NPT |  | Number of input points |
| :--- | :--- | :--- |
| $S(I)$ | $S_{I}$ | Arc length |
| $X(I), Y(I)$ | $X_{I}, Y_{I}$ | Coordinates of input curve |

## Theory

The arc length of a curve is given by

$$
\begin{equation*}
S_{J}=\sum_{I=2}^{J}\left\{\left(x_{I}-x_{I-1}\right)^{2}+\left(y_{I}-y_{I-1}\right)^{2}\right\}^{1 / 2} \tag{1}
\end{equation*}
$$

Object
Calculate streamline curvature

Options

None

List of Symbols
DZDT $\mathrm{d} Z / \mathrm{dt}$, Complex derivative of mapping

NPT , Number of points in DZDT
Q1 (7,K) $\partial v / \partial n$, Streamline curvature

Theory
The magnitude of the potential flow velocity is

$$
\begin{equation*}
V=\left|\frac{d t}{d Z}\right| \tag{1}
\end{equation*}
$$

Then the streamline curvature is given by

$$
\begin{equation*}
\kappa=-\partial V / \partial n \tag{2}
\end{equation*}
$$

The curvature is obtained by numerical differentiation using subroutine DERIV3 for $K=2$, $K L-1$. At the wall the streamline curvature is given by the wall curvature obtained from the input data.

Object
Calculate closure error in mapping
Options
None
List of Symbols

| NLF |  | Number of wall points <br> $Z$ |
| :--- | :--- | :--- |
| ZCL | $\varepsilon_{\text {CL }}$ | Duct plane coordinates |
| TT | $t$ | Closure error |

Theory
The solution is integrated from $t_{1}$ to $t_{\text {NLF }}+i$ by two paths to close the polygon. The closure error is defined by

$$
\begin{equation*}
\epsilon_{C L}=\left|z\left(\dagger_{N L F}+i\right)_{1}-z\left(\dagger_{N L F}+i\right)_{2}\right| \tag{1}
\end{equation*}
$$

## Subroutine CøBLK

## BLOCK DATA

Object
Defines default values for program control
List of Symbols
NXNT Maximum number of wall definition points total
NXNTD2 Maximum number of wall definition points for each wall, also maximum number of potential lines permissible

IST
Maximum number of streamlines
NVK Maximum number of knots to use in cubic-spline fit to wall data

IPOINT Logical unit number to read from
IWUNIT Logical unit number to write to

Object

Store fixed data in Q1, Q2 arrays

Options
IGRID $=0 \quad$ Uniform mesh
$=1$ Nonuniform mesh
$=2$ Both meshes
Input Symbols

| DDS | $(\Delta n / \Delta n)_{1}$ | Mesh distortion parameter |
| :--- | :--- | :--- |
| DETA | $\Delta \eta$ | Uniform transverse step size |
| JL | Number of streamwise stations |  |
| KL | Number of nonuniform streamlines |  |
| KN | Number of uniform streamlines |  |
| LøP | Mesh distortion option |  |
| SAVG |  | Average length of duct |

Output Symbo1s

Q1
Uniform mesh block data
Q2
Nonuniform mesh block data

Theory

CØNSTR is a general setup program that stores information into the Q1 and Q2 data blocks. These stored variables are ones that do not change for $J=1$, JL.

$$
\left.\begin{array}{l}
\mathrm{Q} 1(5, \mathrm{~K})=1 . \\
\mathrm{Q} 1(16, \mathrm{~K})=0 . \\
\mathrm{Q} 1(17, \mathrm{~K})=0 . \\
\mathrm{Q} 1(18, \mathrm{~K})=\eta \\
\mathrm{Q} 2(5, \mathrm{~K})=1 .  \tag{2}\\
\mathrm{Q} 2(16, \mathrm{~K})=\mathrm{d} \eta / \mathrm{dn} \\
\mathrm{Q} 2(17, \mathrm{~K})=\mathrm{n} \\
\mathrm{Q} 2(18, \mathrm{~K})=\eta
\end{array}\right\} \quad \mathrm{K}=1, \mathrm{KN}
$$

Object

Calculate coordinates and metrics

Options

$$
\begin{aligned}
\text { IGRID } & =0 & & \text { Uniform grid output to IUUNIT } \\
& =1 & & \text { Nonuniform grid output to INUNIT } \\
& =2 & & \text { Output both grids }
\end{aligned}
$$

Input Signals

| DDS | $\Delta \eta / \Delta n$ | Mesh distortion parameter |
| :--- | :--- | :--- |
| DETA | $\Delta \eta$ | Uniform step size |
| JL |  | Number of streamwise stations |
| JLPTS |  | Number of points on wall |
| KL | Number of nonuniform streamlines |  |
| KN |  | Number of uniform streamlines |
| L $\varnothing$ P |  | Mesh distortion option |
| NNS |  | Number of steps on potential line |

Output Symbols

Q1(I,K) Coordinate data uniform grid
Q2(I,K) Coordinate data nonuniform grid

Theory

Once the mapping solution has converged, the location of the poles are known and the solution can be obtained for any interior point by integrating the SchwartzChristoffel transformation. The streamwise integration step $\Delta S$ is defined in subroutine CめNSTR by

$$
\begin{equation*}
\Delta S=\left(\max \left(t_{U}, t_{L}\right)-\min \left(t_{U}, t_{L}\right)\right) /(J L-1) \tag{1}
\end{equation*}
$$

and the normal integration step is defined by

$$
\begin{equation*}
\Delta n=1 /(K N-1) \tag{2}
\end{equation*}
$$

Then integration of $\mathrm{d} Z / \mathrm{dt}$ with n constant produces a streamline and integration with $S$ constant produces a potential line.

To start the coordinate calculation, $C \varnothing \emptyset R 1 D$ is called to integrate the first potential line at the duct inlet. We note that dz/dt calculated by subroutine STEP is evaluated at the point

$$
\begin{equation*}
n_{k+1 / 2}=\left(n_{k+1}+n_{k}\right) / 2 \tag{3}
\end{equation*}
$$

such that the metric is given by

$$
\begin{equation*}
\left(\frac{d Z}{d t}\right)_{k}=\frac{1}{2}\left[\left(\frac{d Z}{d \dagger}\right)_{k+1 / 2}+\left(\frac{d Z}{d t}\right)_{k-1 / 2}\right] \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
V_{k}=1 /\left|\frac{d Z}{d t}\right|_{k} \tag{5}
\end{equation*}
$$

The remainder of the computation grid is constructed by integrating all the streamlines in the streamwise direction one step using subroutine $C \varnothing \varnothing R M D$. Again we note that the derivative is evaluated at the mid point so that

$$
\begin{gather*}
\left(\frac{d Z}{d t}\right)^{J}=\frac{1}{2}\left[\left(\frac{d Z}{d t}\right)^{J+1 / 2}-\left(\frac{d Z}{d t}\right)^{J-1 / 2}\right]  \tag{6}\\
V^{J}=1 /\left|\frac{d Z}{d t}\right|^{J} \tag{7}
\end{gather*}
$$

The integration is continued to $\mathrm{J}=\mathrm{JL}+1 / 2$.

## Subroutine CめØRMD (J)

## Object

Calculate coordinates $\mathrm{J}=2$, JL

Options

$$
\begin{aligned}
\text { IGRID } & =0 & & \text { Uniform grid output to IUUNIT } \\
& =1 & & \text { Nonuniform grid output to INUNIT } \\
& =2 & & \text { Output both grids }
\end{aligned}
$$

Input Symbols

| DSTEP | $\Delta S$ | Streamwise step size |
| :--- | :--- | :--- |
| DZDTJ | $\left(\frac{d Z}{d t}\right)^{J}$ | Derivative at J |
| DZDTJI | $\left(\frac{d Z}{d t}\right)^{J-1}$ | Derivative at J-1 |
| J |  | Streamwise index |
| KL |  | Number of nonuniform streamlines |
| KN |  | Number of uniform streamlines |
| ZJ | $Z^{J}$ | Coordinate at J |
| ZJI | $Z^{J-1}$ | Coordinate at $J-1$ |

Output Symbols
Q1 Coordinate data uniform grid
Q2 Coordinate data nonuniform grid

## Theory

The derivatives of the metrics ( $\partial V / \partial n, \partial V / \partial S$ ) are obtained by the three point difference formula. Thus we have

$$
\begin{equation*}
\left(\frac{d Z}{d t}\right)_{k}^{J}=\frac{1}{2}\left[\left(\frac{d Z}{d t}\right)_{k}^{J+1 / 2}+\left(\frac{d Z}{d t}\right)_{k}^{J-1 / 2}\right] \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{k}^{J}=1 /\left|\frac{d z}{d t}\right|_{k}^{J} \tag{2}
\end{equation*}
$$

## Subroutine CøøRMD (Cont'd)

Since $\mathrm{d} / \mathrm{d}$ t are known at the mid points we have

$$
\begin{equation*}
\frac{d v_{k}^{J}}{d s}=\frac{1}{2 \Delta s}\left\{\frac{1}{2}\left[v_{k}^{J+1 / 2}+v_{k}^{J+3 / 2}\right]-v_{k}^{J-1}\right\} \tag{3}
\end{equation*}
$$

The derivative $\partial V / \partial n$ is obtained using subroutine CDVDN and the remaining variables are defined and calculated for the KN points on the uniform mesh by calling subroutine C $\emptyset$ RSTR. Subroutine Q2INTP interpolates from the KN uniform mesh points to the KL nonuniform mesh points.

Ojbect
Calculate coordinates at $J=1$

## Options

$$
\begin{aligned}
\text { IGRID } & =0 & & \text { Uniform grid output to IUUNIT } \\
& =1 & & \text { Nonuniform grid output to INUNIT } \\
& =2 & & \text { Output both grids }
\end{aligned}
$$

## Input Symbols

| DSTEP | $\Delta S$ |
| :--- | :--- |
| KL | Streamwise step size |
| KN |  |
| Number of nonuniform streamlines |  |
| NNS |  |
|  |  |
|  | Number of uniform streamlines |

## Qutput Symbols

| DZDT2 | Derivative at $J=2$ |
| :--- | :--- |
| DZDT3 | Derivative at $J=1$ |
| Q1 | Coordinate data for uniform grid |
| Q2 | Coordinate data for nonuniform grid |
| Z2 | Coordinate at $J=2$ |
| Z3 | Coordinate at $J=3$ |

Theory
The first potential line is calculated by calling subroutine NøRLIN. Then derivatives of the metrics ( $\partial V / \partial n, \partial V / \partial S$ ) are obtained by 3 point difference formula. Thus we have

$$
\begin{gather*}
\left(\frac{d z}{d t}\right)_{k}^{J}=\frac{1}{2}\left[\left(\frac{d z}{d t}\right)_{k}^{J+1 / 2}+\left(\frac{d z}{d t}\right)_{k}^{J-1 / 2}\right]  \tag{1}\\
V_{k}^{J}=1 /\left|\frac{d Z}{d t}\right|_{k}^{J} \tag{2}
\end{gather*}
$$

The streamlines are integrated to $\mathrm{J}=4$ using subroutine STRSTP. Then we have

$$
\begin{equation*}
\left(\frac{\partial V}{\partial S}\right)_{k}^{\prime}=\frac{1}{\Delta S}\left\{-3 v_{k}^{\prime}+4\left[\frac{v_{k}^{3 / 2}+v_{k}^{5 / 2}}{2}\right]-\left[v_{k}^{5 / 2}+v_{k}^{7 / 2}\right]\right\} \tag{3}
\end{equation*}
$$

## Subroutine CøØR1D (Cont'd)

The derivative $\partial V / \partial n$ is calculated using subroutine CDVDN and the remaining variables are defined and calculated for the KN points on the uniform grid by calling subroutine C $\emptyset$ RSTR. Subroutine 22 INTP interpolates from the KN uniform grid to the KL nonuniform grid.

Object
Read coordinate data

## Options

ISMøФT $=0$ Do not smooth wall data $=1$ Smooth wall data

## Input Symbols

| JLPTS | Number of smoothed wall data points |
| :--- | :--- |
| JXK | Number of knots in spline smoothing |
| NLF | Number of input upper/lower wall data points |

## Output Symbols

| $\mathrm{XL}, \mathrm{YL}$ | $\mathrm{X}_{\mathrm{L}}, \mathrm{Y}_{\mathrm{L}}$ | Lower wall data points |
| :--- | :--- | :--- |
| $\mathrm{XU}, \mathrm{YU}$ | $\mathrm{X}_{\mathrm{U}}, \mathrm{Y}_{\mathrm{U}}$ | Upper wall data points |
| ZC | $\mathrm{X}_{\mathrm{C}}$ | Complex coordinates of wall data points |
| RADR | $\mathrm{r}_{\mathrm{r}}$ | Reference radius $\mathrm{XU}(1)$ |

## Theory

The subroutine reads the wall data in card image form. If ISMøøT = 1, a cubic spline smoothing routine SMARCL will produce a set of JLPTS data points for each wall. This subroutine also prints the smoothed and unsmoothed wall data.

Object
Store coordinates in Q1 array
Options
None
Input Symbols

| J |  | Streamwise station number |
| :--- | :---: | :--- |
| DZDT | $\left(\frac{\mathrm{dZ}}{\mathrm{dt}}\right)^{\mathrm{J}}$ | Derivative at J |
| ZI | $\mathrm{Z}^{\mathrm{J}}$ | Coordinates at J |

Output Sybmols
Q1 Coordinate data block
Theory
The following coordinate data are calculated at J.

$$
\begin{align*}
& \text { Q1 }(1, \mathrm{~K})=\operatorname{Im}\left(\mathrm{Z}_{\mathrm{K}}^{\mathrm{J}}\right) \quad=\mathrm{R}  \tag{1}\\
& \text { Q1 }(2, \mathrm{~K})=\operatorname{Re}\left(Z_{\mathrm{K}}^{\mathrm{J}}\right) \quad=\mathrm{Z}  \tag{2}\\
& \text { Q1 }(3, \mathrm{~K})=\operatorname{Re}(\mathrm{dZ} / \mathrm{dt})_{\mathrm{K}}^{\mathrm{J}} \quad=\partial \mathrm{R} / \partial \mathrm{n}  \tag{3}\\
& \text { QI }(4, \mathrm{~K})=\operatorname{Im}(\mathrm{dZ} / \mathrm{dt})_{\mathrm{K}}^{\mathrm{J}} \quad=\partial \mathrm{R} / \partial \mathrm{S}  \tag{4}\\
& \text { Q1 }(6, \mathrm{~K})=1 /|\mathrm{dZ} / \mathrm{dt}|_{\mathrm{K}}^{\mathrm{J}} \quad=\mathrm{V}  \tag{5}\\
& \text { Q1 }(9, K)=\int_{0}^{\mathrm{S}} \frac{\mathrm{dS}}{\mathrm{~V}}=\mathrm{X}  \tag{6}\\
& \text { Q1 }(10, K)=\int_{0}^{\mathrm{n}} \mathrm{~K} \frac{\mathrm{dn}}{\mathrm{~V}} \quad=\mathrm{Y}  \tag{7}\\
& \mathrm{Q} 1(11, \mathrm{~K})=\mathrm{Q}(10, \mathrm{~K}) / \mathrm{Q} 1(10, \mathrm{KN})  \tag{8}\\
& \text { Q1 }(12, K)=2 \pi \int_{0}^{n_{K}} \frac{R d n}{V}=A  \tag{10}\\
& \text { Q1 }(13, K)=2 \pi R  \tag{11}\\
& Q 1(14, K)=2 \pi \partial R / \partial n  \tag{12}\\
& Q 1(15, K)=2 \pi \partial R / \partial S \tag{13}
\end{align*}
$$

Object

Calculate 3 point central difference derivative

Options
None

## Input Symbols

| NPT | Point at which to evaluate derivative |
| :--- | :--- |
| NX | Number of data points for $X$ and $Y$ |
| $X, Y$ | Table of $N X$ independent/dependent variables |

Output Symbols

| DYDX | $d y / d x$ | First derivative |
| :--- | :--- | :--- |
| D2YDX2 | $d^{2} y / d x^{2}$ | Second derivative |

Theory

The finite difference formula are given by:

$$
\begin{align*}
& \left(\frac{d Y}{d x}\right)^{I}=\frac{Y^{I+1}-\left(1-r^{2}\right) Y^{I}-r^{2} Y^{I-1}}{x^{I+1}-x^{I}+r^{2}\left(x^{I}-x^{I-I}\right)}  \tag{1}\\
& \left(\frac{d^{2} y}{d x}\right)^{I}=\frac{Y^{I+1}-(1+r) Y^{I}+r y^{I-1}}{\left(x^{I+1}-x^{I}\right)^{2}+r\left(x^{I}-x^{I-I}\right)^{2}} \tag{2}
\end{align*}
$$

$$
\begin{equation*}
r=\frac{x^{I+1}-x^{I}}{x^{I}-x^{I-1}} \tag{3}
\end{equation*}
$$

If $I=1$ or NPT a diagnostic is printed
"INPUT PøINT XX OUT OF RANGE"
and both derivatives are set to 1.0 .

If $\left|x^{I+1}-x^{I}\right|$ or $\left|x^{I}-x^{I-1}\right|<10^{-15}$ a diagnostic is printed
"INDEPENDENT VARIABLE STEP SIZE LT 1.E-15"
and both derivatives are set to 1.0 .

Subroutine ESTCøR (SAVG, NPOT)

## Object

Determine location of approximate potential line.
Option
NIPDT $\quad=0 \quad$ Program determines number of lines > 1 NIPøT lines are calculated

Input Symbo1s

| NPøT | $\mathrm{N}_{\mathrm{p}}$ | Number of potential lines |
| :--- | :--- | :--- |
| SAVG | $\overline{\mathrm{S}}$ | Average duct length |
| $\mathrm{XL}, \mathrm{YL}$ | $\mathrm{X}_{\mathrm{L}}, \mathrm{Y}_{\mathrm{L}}$ | Lower wall coordinates |
| $\mathrm{XU}, \mathrm{YU}$ | $\mathrm{X}_{\mathrm{U}}, \mathrm{Y}_{\mathrm{U}}$ | Upper wall coordinates |

Output Symbols

| HT | h | Height of duct |
| :--- | :--- | :--- |
| IU, IL |  |  |
| TH | $\theta$ | Angle of mean line |
| XLI,YLI | $X_{\text {LI }}, Y_{\text {LI }}$ | Coordinates potential line lower wall |
| XUI,YUI | $X_{U I}, Y_{U I}$ | Coordinates potential line upper wall |

Theory
The object is to determine NPDT approximate potential lines in the duct where $N P \emptyset T=N L F / 3$ initially. The first potential line intersects the duct at $Z_{U 1}$ and $\Sigma_{\text {LI }}$ where the complex notation is used.

$$
\begin{equation*}
z=x+i y \tag{1}
\end{equation*}
$$

```
Subroutine ESTC\emptysetR (Cont'd)
```

We then construct a mean line Zm , (See Fig. 1), which satisfies the following conconditions:

$$
\begin{gather*}
\left|z_{m, J}-z_{m, J-1}\right|=\Delta s  \tag{2}\\
\left|z_{u 1, v}-z_{m, J}\right|=\left|z_{m, J}-z_{L I, J}\right|  \tag{3}\\
\left(z_{u I, J}-z_{L I, J}\right) \cdot\left(z_{m, J}-z_{m, J-1}\right)=0 \tag{4}
\end{gather*}
$$

It was found that the set of equations, Eq. (2) through (4) do not have a unique solution. Therefore Eq. (3) was replaced by a minimum condition on $D$ where:

$$
\begin{equation*}
D=1-\frac{\left|Z_{U I, J}-Z_{m, J}\right|}{\left|Z_{m, J}-Z_{L I, J}\right|} \tag{5}
\end{equation*}
$$

The algorithm consists of finding an angle $\theta_{J}$ which minimizes $D$. Thus we have from Eq. (1),

$$
\begin{equation*}
z_{m, J}^{\nu}=z_{m, j-1}+\Delta s \cdot\left[\cos \theta_{J}^{\nu}+i \sin \theta_{J}^{\nu}\right] \tag{6}
\end{equation*}
$$

A straight line normal to the mean line, from Eq. (4), is defined by the point $Z^{\nu} \mathrm{m}, \mathrm{J}$ and the point,

$$
\begin{equation*}
\tilde{Z}=Z_{m, j}+\Delta S \cdot\left[\cos \left(\theta_{j}^{\nu}+\pi / 2\right)+i \sin \left(\theta_{j}^{\nu}+\pi / 2\right)\right] \tag{7}
\end{equation*}
$$

The intersections of the line $\left(\underset{\sim}{Z}, Z_{m}, J\right)$ with the duct wall $\left(Z_{U I}^{\nu}, J, Z_{L I}^{\nu}, J\right)$ is determined using subroutine INSECT. Then $D$ is calculated and checked for a minimum. An iteration procedure determines the $\theta_{J}^{\nu}$ which minimizes $D$.

## Subroutine ESTCøR (Cont'd)

When the iteration has converged, a check is made to determine if the $Z_{m}$, $J$ potential line crosses the $Z_{m, J}$ potential line inside the duct. If it does, the distance along the mean line $\Delta S$ is increased

$$
\begin{equation*}
\Delta S=\Delta S \cdot 1.2 \tag{8}
\end{equation*}
$$

and the algorithm is repeated starting with Eq. (6). A maximum step $\Delta \mathrm{S}$ is fixed by some fraction of the duct height $d_{m}$. Thus

$$
\begin{equation*}
\Delta S=\min \left[d_{m}\left|Z_{U I, J}-Z_{L I, J}\right|, \Delta S\right] \tag{9}
\end{equation*}
$$

$$
Z=X+i Y
$$



Fig. 1. Geometric Construction of Potential Flow

Object
Calculate approximate pole locations
Options
None
Input Symbols

| N |  | Total number of poles $(2 * \mathrm{NLF})$ |
| :--- | :--- | :--- |
| NLF |  | Number of poles on each wall |
| ZC | $\mathrm{Z}_{\mathrm{C}}$ | Duct coordinates |

Output Symbols
B
TT

$$
\mathrm{b}_{\mathrm{I}}^{\mathrm{t}_{\mathrm{I}}}
$$

Pole locations in plane

$$
\text { Pole locations in } \mathrm{t} \text { plane }
$$

## Theory

The arc lengths $S_{U}$, $S_{L}$ to each pole in the duct ( $Z$ ) plane is determined using subroutine ARCL1. Subroutine ESTC $\varnothing$ R calculates the location of the approximate potential line and subroutine WALLV calculates the approximate potential flow velocities (metrics) at the pole locations in the 2 plane. Then the pole locations are given by:

$$
\begin{align*}
& t_{U I}=\int_{0}^{s_{U I}} V_{U I} d S  \tag{1}\\
& t_{L I}=\int_{0}^{s}{ }^{s_{L I}} V_{L I} d S  \tag{2}\\
& b_{U I}=\exp \left(-\pi t_{U I}\right)  \tag{3}\\
& b_{L I}=-\exp \left(-\pi t_{L I}\right) \tag{4}
\end{align*}
$$

## Object

Determine intermine intersection of potential line and wall.

Options

$$
\begin{array}{rlr}
\operatorname{IEXTRP} & =1 & \text { Extend last line segment for intersection } \\
& \neq 1 & \text { Do not extend last line segment. }
\end{array}
$$

Argument List
$(\mathrm{X} 1, \mathrm{Y} 1),(\mathrm{X} 2, \mathrm{Y} 2)$
$X, Y \quad Z$

NPT
(XI, YI)
$I \emptyset$

IERR

Points defining potential line

Points defining wall curve

Number of ( $X, Y$ ) points

Intersection point
Lower index of intersection point
Error flag $=0$ intersection found
$=-1$ no intersection found

Theory

The input coordinates (X,Y) are searched for an intersection with (X1,Y1), (X2,Y2) using subroutine CROSS1 which determines if an intersection occurs between

$$
\begin{equation*}
Z(I \varnothing) \leq Z_{I} \leq Z(I \varnothing+1) \tag{1}
\end{equation*}
$$

## Subroutine INTNØR (Arg, List)

Object

Calculate end point of potential line

Options

None

Argument List

| ZETO | $\zeta_{0}$ | Starting location in $\zeta$ plane |
| :--- | :---: | :--- |
| ZO | $Z_{0}$ | Starting location in $Z$ plane |
| ZETU | $\zeta_{u}$ | Final location in $\zeta$ plane |
| ZU | $Z_{U}$ | Final location in $Z$ plane |
| NNS |  | Number of steps |

Theory

The starting location in the $t$ plane is given by

$$
\begin{equation*}
t_{0}=i-\ln \left(\zeta_{0}\right) / \pi \tag{1}
\end{equation*}
$$

and the step size is given by

$$
\begin{equation*}
\Delta t=i \cdot 1 . /(N N S-1) \tag{2}
\end{equation*}
$$

Then

$$
\begin{equation*}
Z_{u}=Z_{0}+\sum_{j=1}^{N N S-1}\left\{\int_{t_{0}+\Delta t(J-1)}^{\dagger_{0}+\Delta t \cdot J} \frac{d Z}{d t} d t\right\} \tag{3}
\end{equation*}
$$

where the term in the bracket is evaluated using subroutine STEP.

```
Subroutine INTSTR (Arg. List)
```

Object
Calculate end point of streamline

Options
None

Argument List
ZETO $\zeta \quad$ Starting location in $\zeta$ plane
ZO $\mathrm{Z}_{0} \quad$ Starting location in Z plane

ZETU

ZU
Final location in $\zeta$ plane

Final location in $Z$ plane

List of Symbols
NLF Number of lower wall points

TT
$t_{I}$
Pole locations in t plane

Theory
The starting point in the $t$ plane is given by

$$
\begin{equation*}
t_{0}=i-\ln \left(\zeta_{0}\right) / \pi \tag{1}
\end{equation*}
$$

Define the streamline by

$$
\begin{equation*}
t_{n}=\operatorname{Im}\left(t_{0}\right) \tag{2}
\end{equation*}
$$

Then

$$
\begin{equation*}
Z_{U}=Z_{0}+\sum_{I=2}^{N L F}\left\{\int_{I-I+t_{n} \cdot i}^{t_{I-i}+t_{n} \cdot i} \frac{d Z}{d t} d t\right\} \tag{3}
\end{equation*}
$$

where the bracket is evaluated using subroutine STEP.

## Subroutine KURVTR (Arg. List)

Object

Calculate curvature of input curve

## Options

None

Argument List

| X,Y | X,Y | Coordinates of input curve |
| :--- | :--- | :--- |
| S | S | Arc length of input curve |
| NPT |  | Number of points on curve |
| KURV | K | Curvature of input curve |

Theory

The principal curvature of a curve is given by

$$
\begin{equation*}
\kappa=\frac{d X}{d S} \frac{d^{2} Y}{d S}-\frac{d Y}{d S} \frac{d^{2} x}{d S^{2}} \tag{1}
\end{equation*}
$$

Eq. (1) is evaluated by 3 point finite difference formula

## Object

Solve Schwartz-Christoffel Mapping

## Options

None

Input Symbols
EC $\quad \varepsilon_{c} \quad$ Convergence criteria
NLF Number of points on wall

TT $t$ Initial pole location $t$ plane
ZC $Z_{c} \quad$ Duct wall coordinates
Output Symbols

| TT | t | Final pole location in $t$ plane |
| :--- | :--- | :--- |
| Z | Z | Final pole location in Z plane |

Theory

The flow chart for this subroutine is shown on Fig. 2 and takes place in the following steps:

Step_1_Initialization
a) Calculate rotation constant $M$
b) Calculate duct exit divergence angle $\alpha_{e}$
c) Calculate Schwartz-Christoffel pole angle $\alpha_{i}$

## Step_2_Integrate Transformation

a) Integrate Schwartz-Christoffel transformation along each wall with a guess for the $b_{i}$ 's in the $\zeta$ plane using subroutine STEP.
b) Integrate Schwartz-Christoffel transformation along far upstream potential line with a guess for $b_{i}$ 's in $\zeta$ plane using subroutine NめRLIN.

## Subroutine MDAVIS (Cont'd)

## Step_3_Update Poles

a) Update poles on lower wall in $t$ plane by ratio of arc lengths.
b) Update first pole on upper wall using Step 2 b .
c) Update poles on upper wall in $t$ plane by ratio of arc lengths.
d) Calculate pole location $b_{i}$ 's in $\zeta$ plane.

Step_4_Check_Convergence
a) Calculate absolute error for all poles

$$
\varepsilon_{i}=\left|z_{i}-z_{C i}\right|
$$

b) Check convergence

$$
\max \left(\varepsilon_{i}\right)<\varepsilon_{c}
$$

c) Calculate closure error using subroutine CLøSUR
d) If not converged repeat Steps 2, 3, 4 If converged return


Fig. 2. Flow Chart for Subsonic MDAVIS

## Subroutine Nonim ( $\mathrm{J}, \mathrm{Z}$ )

Object
Calculate single potential line
Options
None
Input Symbols
$J \quad$ Calculate Jth potential line

KN Number of output stations
NNS Number of integration steps

| TT | $t_{i}$ | Pole locations in t plane |
| :--- | :---: | :--- |
| ZO | $Z_{o}$ | Initial Z location |

Output Symbols
$\begin{array}{lll}\text { Z1 } & Z_{K} & \text { Coordinates of potential line } \\ \text { DZDT1 } & (\mathrm{dZ} / \mathrm{d} t)_{K} & \text { Derivative }\end{array}$
Theory
After a converged solution for the pole locations is obtained, this subroutine integrates the potential line at the Jth station in NNS steps and outputs the coordinates and derivatives of KN stations. Then let us choose

$$
\begin{equation*}
N N S=N S D *(K N-1)+K N \tag{1}
\end{equation*}
$$

where NSD is the number of integration steps per output station. The integration starts at $Z_{o}$ in the $Z$ plane and $t_{o}$ given by

$$
\begin{equation*}
t_{0}=\operatorname{DSTEP}(J-1)-t_{L} / 2+0+i \tag{2}
\end{equation*}
$$

in the $t$ plane. The parameter $t_{L}$ is chosen in the approximate coordinate calculation to center the pole distributions about plus and minus values.

## Subroutine N $\emptyset$ RLIi (Cont'd)

The integration step is then given by

$$
\begin{equation*}
\Delta t=1 /(N N S-1) * i \tag{3}
\end{equation*}
$$

Then we have the recursion formula

$$
\begin{align*}
& Z_{1}=Z_{0}  \tag{4}\\
& Z_{K}=Z_{K-1}+\int_{t_{K-1}+\Delta t \cdot(L-1)}^{\dagger_{K-1}+\Delta t \cdot L} \frac{d Z}{d t} d t  \tag{5}\\
& \left(\frac{d Z}{d t}\right)_{t_{K}}=\frac{1}{2}\left[\left(\frac{d Z}{d t}\right)_{t_{K}-\Delta t}+\left(\frac{d Z}{d t}\right)_{t_{K}}+\Delta t\right] \tag{6}
\end{align*}
$$

## Object

Interpolates wall curvature at output location

## Options

None

Input Symbols

| KKL, KKU | $\mathrm{K}_{\mathrm{L}}, \mathrm{K}_{\mathrm{U}}$ | Curvature of lower/upper wall |
| :--- | :--- | :--- |
| $\mathrm{XL}, \mathrm{YL}$ | $\mathrm{X}_{\mathrm{L}}, \mathrm{Y}_{\mathrm{L}}$ | Input coordinates lower wall |
| $\mathrm{XU}, \mathrm{YU}$ | $\mathrm{X}_{\mathrm{U}}, \mathrm{Y}_{\mathrm{U}}$ | Input coordinates upper wall |
| $\mathrm{SL}, \mathrm{SU}$ | $\mathrm{S}_{\mathrm{L}}, \mathrm{S}_{\mathrm{U}}$ | Arc length lower/upper wall |
| Q1 |  | Coordinate data |

Output Symbols

| RHSI (3) | $\mathrm{K}_{\mathrm{L}}(\mathrm{J})$ | Curvature lower wall |
| :--- | :--- | :--- |
| RTSI (3) | $\mathrm{K}_{\mathrm{U}}(\mathrm{J})$ | Curvature upper wall |

Theory
The streamline curvature KKL and KKU is known at the input data points (XL,YL) and (XU,YU) respectively $\dot{r}_{\dot{\sim}}$ The coordinates are known at station $J$ for equal streamwise steps DSTEP. Let $\left(\mathrm{X}_{\mathrm{L}}, \tilde{Y}_{L}\right)$ and ( $\mathrm{X}_{\mathrm{U}}, \mathrm{Y}_{\mathrm{U}}$ ) be the lower and upper wall coordinates at station J obtained from the Ql array. A straight line is passed through these points and a serarch of the input coordinates is made using subroutine INSECT to determine the intersection on each wall. Subroutine INSECT returns an interpolation parameter which is used to calculate $K_{L}(J), K_{U}(J)$

Subroutine QPSTøR(J)
Object
Store Q parameters in Q1, Q2 arrays

## Sptions

| IGRID | $=0$ |  | Uniform grid |
| ---: | :--- | ---: | :--- |
|  | $=1$ |  | Nonuniform grid |
|  | $=2$ |  | Both grids |
| IXFG | $=0$ |  | Uniform grid |
|  | $=1$ |  | Interpolate only |

Input Symbols

| DSTEP | $\triangle$ S | Streamwise step size |
| :--- | :--- | :--- |
| JL |  | Number of potential lines |
| KL |  | Number of streamlines |
| RADR | $r_{r}$ | Reference radius |

Output Symbols
RHS1,RMS1,RTS1 Wall coordinate data uniform grid
RHS2,RMS2, RTS2 Wall coordinate data nonuniform grid
QPARM1 Parameters for uniform grid
QPARM2 Parameters for nonuniform grid
Theory
The wall coordinate data and gria parameters are calculated by this subroutine.

## Object

Interpolate from uniform grid to nonuniform grid

## Options

None

## Input Sybmols

KL
KN
Q1
Output Symbols
Q2
Coordinate data for nonuniform grid

Theory
The normal coordinate for a uniform grid is $\mathrm{Q} 1(19, \mathrm{~K}) \mathrm{K}=1, \mathrm{KN}$ and the normal coordinate for the nonuniform grid is $\mathrm{Q} 2(18, \mathrm{~K}) \mathrm{K}=1, \mathrm{KL}$. The Q 2 variables are obtained from the Q1 variables by linear interpolation using the normal coordinate as the independent variable.

## Subroutine RCNTRL

## Object

Reads user input control parameters

## Options

None

## Input Signals

See input data Section 10.2

## Theory

This subroutine reads the input control parameters, checks for inconsistencies and prints the input data.

## Object

Calculate duct rotation and scaling

## Options

None

## Input Symbols

ZC
$Z_{c}$
Input duct coordinates

Output Symbols
XM M Rotational constant
Theory
Far upstream of the duct inlet $\zeta \rightarrow \infty$ and the Schwartz-Christoffel transformation reduces to

$$
\begin{equation*}
\frac{d Z}{d \zeta}=\frac{M}{\zeta} \tag{1}
\end{equation*}
$$

Integrating Eq. (1) we have

$$
\begin{equation*}
Z=m \ln \zeta+Z_{0} \tag{2}
\end{equation*}
$$

The tranformation to the $t$ plane is given by

$$
\begin{equation*}
\ln \zeta=\pi(i-1) \tag{3}
\end{equation*}
$$

and Eq. (2) becomes a duct with parallel walls.

$$
\begin{equation*}
z=M \pi(i-t)+z_{0} \tag{4}
\end{equation*}
$$

Subtracting the lower wall from the upper wall we have

$$
\begin{equation*}
Z_{U}-Z_{L}=-M \pi i \tag{5}
\end{equation*}
$$

## Subroutine ROTATM (Cont'd)

The height of the duct is given by

$$
\begin{equation*}
H=\left|Z_{U}-z_{L}\right| \tag{6}
\end{equation*}
$$

Hence

$$
\begin{equation*}
Z_{U}-Z_{L}=H e^{i(\theta+\pi / 2)} \tag{7}
\end{equation*}
$$

where $\theta$ is the angle of the duct with respect to the real axis.

Then solving for M using Eq. (5) and Eq. (7) we have

$$
\begin{equation*}
M=\frac{-H}{\pi} e^{i \theta} \tag{8}
\end{equation*}
$$

Since this solution requires parallel walls, this subroutine will modify the Nth point of the data set to insure parallel walls at the inlet so that the correct duct height H can be determined at the inlet.

Object
Cubic spline smoothing on arc length
Options
None

## Argument List

| JX | Number of input coordinate points |  |
| :--- | :--- | :--- |
| JXB | Number of ouptut coordinate points |  |
| JXK | Number of knots in spline fit |  |
| $\mathrm{X}, \mathrm{Y}$ | $\mathrm{X}, \mathrm{Y}$ | Input coordinates |
| S | X | Arc length along input curve |
| SB | $\overline{\mathrm{S}}$ | Arc length 'along output curve |
| $\mathrm{XB}, \mathrm{YB}$ | $\overline{\mathrm{X}, \overline{\mathrm{Y}}}$ | Output smoothed coordinates |

Theory
The arc length along the input curve is calculated using subroutine ARCLI. Then an increment of arc length is defined by

$$
\begin{equation*}
\Delta \bar{S}=(S(J X)-s(1)) /(J X B-1) \tag{1}
\end{equation*}
$$

The curves $X(S)$ and $Y(S)$ are smoothed using subroutine SMøゆTH which returns $\bar{X} \cdot(\bar{S})$ and $\bar{Y}(\bar{S})$ for $J X B$ points spaced $\Delta \bar{S}$ in length

Object

Calculate Integration Step For Schwartz-Christoffel Transformation

## Options

None

Variables

| $B(K)$ | $=$ | $\mathrm{b}_{\mathrm{K}}$ | , | Location of pole in $\zeta$ plane |
| :---: | :---: | :---: | :---: | :---: |
| BETM (K) | = | $-\alpha_{K} / \pi$ | , | Turning angle in Z plane |
| DZ | = | $\Delta \mathrm{Z}_{\mathrm{m}}$ | , | Step size in Z plane |
| DZETD | $=$ | $\begin{aligned} & \Delta \zeta_{\mathrm{m}} \\ & \Delta \mathrm{t}_{\mathrm{m}} \end{aligned}$ | , | Step size in $\zeta$ plane Step size in $t$ plane |
| XM | $=$ | M | , | Scale factor |
| ZETD1 | $=$ | $\zeta_{\mathrm{m}}$ | , | Initial $\zeta$ |
| ZETD2 | $=$ | $\zeta_{m+1}$ | , | Final $\zeta$ |
| NBE | $=$ | N | , | Number of poles |
| GAMA | = | $\gamma_{m}$ | , | Exit divergence angle |

Theory

The second order integration formula evaluated at the mid point is given by Davis Ref. 1 as

$$
\begin{equation*}
\left(\frac{d Z}{d \zeta}\right)_{M+1 / 2}=\frac{M}{\zeta_{M+1 / 2}} \zeta_{M+1 / 2}^{\alpha M / \pi} \prod_{I=1}^{N}\left\{\frac{\left(\zeta_{M+1}-b_{I}\right)^{-a_{I} / \pi+1}-\left(\zeta_{M}-b_{I}\right)^{-\alpha_{I} / \pi+1}}{\Delta \zeta_{M}\left(-\alpha_{I} / \pi+1\right)}\right\} \tag{1}
\end{equation*}
$$

where

$$
\begin{align*}
\Delta \zeta_{M} & =\zeta_{M+1}-\zeta_{M}  \tag{2}\\
\zeta_{M+1 / 2} & =\frac{1}{2}\left(\zeta_{M+1}+\zeta_{M}\right) \tag{3}
\end{align*}
$$

## Subroutine STEP (Cont'd)

The transformation to the $t$ plane is given by

$$
\begin{equation*}
\left(\frac{d t}{d \zeta}\right)_{M+1 / 2}=-\frac{1}{\pi \zeta_{M+1 / 2}} \tag{4}
\end{equation*}
$$

Then we have

$$
\begin{equation*}
\Delta Z_{M}=\left(\frac{d Z}{d \zeta}\right)_{M+1 / 2} /\left(\frac{d t}{d \zeta}\right)_{M+1 / 2} \Delta \dagger_{M} \tag{5}
\end{equation*}
$$

where $\Delta t_{m}$ is chosen by the input $\zeta^{\prime} s$.

$$
\begin{equation*}
\Delta t=-\left(\ln \zeta_{M+1}-\ln \zeta_{M}\right) / \pi \tag{6}
\end{equation*}
$$

## References

1. Davis, R. T.: Numerical Methods for Coordinate Generation Based on SchwartzChristoffel Transformations.

## Subroutine STRSTP

## Object

Integrate each streamline one step

Options

None

Argument List

| J |  | Streamwise station |
| :--- | :--- | :--- |
| ZI | $\mathrm{Z}^{\mathrm{J}}$ | Coordinates at J |
| ZO | $\mathrm{Z}^{\mathrm{J}+1}$ | Coordinates at $\mathrm{J}+1$ |
| DZDTO | $(\mathrm{dZ} / \mathrm{dt})^{\mathrm{J}+1 / 2}$ | Derivative at mid point |

Theory

This subroutine integrates $K=1, K N$ streamlines one step

$$
\begin{equation*}
Z_{k}^{J+1}=Z_{k}^{J}+\left(\frac{d Z}{d t}\right)_{k}^{J+1 / 2} \Delta t \tag{1}
\end{equation*}
$$

using subroutine STEP.

Subroutine SUNBAR (X,Y,T,NPT;NORDER)
Object
Stores $X, Y$ data into interpolating Table $T$
Options
None

## Input Symbols

$X, Y \quad X, y$ Independent variable arrays
NPT Number of data pairs (X,Y)
NORDER Interpolation order (= 1, 2 or 3 )
Output Symbols
T Output interpolation table

## Theory

The data is stored into $T$ as follows

```
T(1) = 1.
T(2) = NORDER
T(3) = NPT
T(4) = 0.
T(J + 4) = X(J), J = 1,NPT
T(J + NPT + 4) = Y(J), J = 1,NPT
```

Object

Update upper wall upstream point

## Options

None

## Input Symbols

| ITER | $\nu$ | Iteration number |
| :--- | :--- | :--- |
| THETAI | $\theta_{1}$ | Angle of duct rotation from real axis |
| TT | $\mathrm{t}_{i}^{\nu}$ | Location of poles in $t$ plane |
| ZC | $\mathrm{Z}_{\mathrm{ci}}$ | Input wall coordinates |
| ZU | $\mathrm{Z}_{\mathrm{U}}$ | End point of potential line integration |

Output Symbols
$\operatorname{TT}(N) \quad t_{N}^{v+1} \quad$ Updated $t$ plane coordinate at point $N$
Theory

This subroutine updates the corner point $Z_{N}$ to close the polygon by jumping from the lower wall to the upper wall as shown in Fig. 3. With known $t_{i}$, the point $Z_{N}^{\nu}$ is determined by integrating along the path $\left(Z_{c 1}, A, Z_{N}^{\nu}\right)$. The error in closing the polygon is given by

$$
\begin{equation*}
\varepsilon=\left|Z_{N}^{\nu}-Z_{C N}\right| \tag{I}
\end{equation*}
$$

The update $t_{N}^{\nu+1}$, is determined in the following manner. Let us define upstream points $t_{1}$ and $t_{N}^{\prime}$ given by

$$
\begin{gather*}
t_{1}^{\prime}=t_{1}-\sigma\left|t_{1}^{\nu}\right|  \tag{2}\\
t_{N}^{\prime}=f_{1}+i \tag{3}
\end{gather*}
$$

## Subroutine TTUP (Cont'd)

where $\sigma$ is a parameter chosen to move $t_{1}^{\prime}$ sufficiently upstream to approximate the limiting case $t \rightarrow-\infty$

$$
\begin{equation*}
Z=\pi M(i-t)+Z_{0} \tag{4}
\end{equation*}
$$

Then

$$
\begin{equation*}
z_{N}^{\prime}-Z_{1}^{\prime}=-\pi M i \tag{5}
\end{equation*}
$$

The point $Z_{1}^{\prime}$ is determined by integrating along the path ( $Z_{c 1}, Z_{i}^{\prime}$ ) and $Z_{N}^{\prime}$ is determined from Eq. (5). A ratio of wall lengths is defined

$$
\begin{equation*}
R=\frac{\left|Z_{C N}-Z_{N}^{\prime}\right|}{\left|Z_{N}{ }^{2}-Z_{N}^{\prime}\right|} \tag{6}
\end{equation*}
$$

and the update for $t_{N}^{\nu}$ is given by

$$
\begin{equation*}
t_{N}^{\nu+1}=t_{N}^{\prime}+R\left(t_{N}^{\nu}-t_{N}^{\prime}\right) \tag{7}
\end{equation*}
$$



Fig. 3. Update for Corner Point

Subroutine UNBAR (T,IK,XIN,YIN,ZZ,KK)

Object

Interpolate a univariate or bivariate table.

## Input Symbols

| T | $=$ Name of the array which contains the table values. |
| ---: | :--- |
| IK $\quad=\quad$Element of the array at which the table starts. If <br> you have only one table in the array, IK=ONE. |  |
| XIN $\quad=\quad$ Independent variable in the X-sense. |  |
| YIN $\quad=\quad$Independent variable in the Y-sense. If the table <br> is a univariate, then YIN is zero. |  |

Output Symbols

| Z2 | = | Dependent variable |
| :---: | :---: | :---: |
| KK | = | Off Table indicator |
|  | = | 0 Normal evaluation |
|  | = | 1 Off On X Min. |
|  | $=$ | 2 Off On X Max. |
|  | = | 3 Off On Y Min. |
|  | $=$ | 4 Off On X Min. and Y Min. |
|  | $=$ | 5 Off On X Max, and Y Min. |
|  | $=$ | 6 Off On Y Max. |
|  | = | 7 Off On X Min. and Y Max. |
|  | = | 8 Off On X Max. and Y Max. |
|  | = | Less Than 0, Table set up |

Theory

If either variable is off the table, UNBAR will return the corner value. This implies that UNBAR will not extrapolate and does not recognize any discontinuities. The table must be set up as follows-all numbers are in floating point mode.

```
T(IK) = Curve No.
T(IK+1) = Degree of Interpolating (1, 2, 3)
T(IK+2) = NX. No. of X values
T(IK+3) = NY. No. of Y values. (in univariate make zero)
T(IK++) = X values in ascending order.
T(IK++) = Y values in ascending order.
T(IK++) = Z values. Put them in following order- (Z(1.1),Z(1,2),
        Z(1,3)---Z(1,NY),Z(2,1),Z(2,2)---Z(2,NY)---Z(NX,1),
        Z(NX,2)---Z(NX,NY). For bivariate only.
```


## Subroutine UNBAR (Cont'd)

A Lagrongian interpolation polynomial of degree 1 , 2 or 3 will be used for the interpolation depending upon $T(I K+1)$.

## Subroutine WALLV

## Object

Approximate potential flow velocity

## Options

None

Input Symbols

| HT | , Approximate duct height |
| :--- | :--- |
| IL,IU | Indices for lower/upper wall potential |

NPめT $\quad N_{p} \quad$ Number of potential lines

SAVG
$\bar{S}$
, Average duct length

SL, SU

TH
$S_{L}, S_{U}$
, Arc length lower/upper wall

XL, YL
$\theta$
, Mean line angle
$\mathrm{XU}, \mathrm{YU}$
$X_{L}, Y_{L}$
, Coordinates lower wall
$\mathrm{XU}, \mathrm{YU}$
$X_{U}, Y_{U}$
, Coordinates upper wall

XLI,YLI
$X_{L I}, Y_{L I}$
, ESTCめR coordinates lower wall
XUI, YUI
$X_{U I}, Y_{U I}$
ESTC $\emptyset \mathrm{R}$ coordinates upper wall
Output Symbols

VL,VU $\quad V_{L}, V_{U} \quad$ Velocity lower/upper wall
Theory

For each computed potential line, the wall velocity can be estimated as follows

$$
\begin{equation*}
\bar{v}=1 / n \tag{1}
\end{equation*}
$$

and the curvature of the mean line by

$$
\begin{equation*}
\overline{\mathbf{K}}=\frac{d \theta}{d \bar{S}} \tag{2}
\end{equation*}
$$

## Subroutine WALLV (Cont'd)

Then define

$$
\begin{equation*}
\phi=\frac{1-\bar{K} /(2 \bar{V})}{1+\bar{K} /(2 \bar{V})} \tag{3}
\end{equation*}
$$

and the approximate velocity at the wall is given by

$$
\begin{equation*}
v_{U I}=\frac{2}{1+\phi} \bar{v} \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
V_{L I}=\frac{2 \phi}{1+\phi} \bar{V} \tag{5}
\end{equation*}
$$

Arc lengths $S_{\text {LI }}$ and $S_{\text {UI }}$ may be calculated using subroutine ARCL for the lower and uppwer walls defined by ( $\mathrm{X}_{\mathrm{LI}}, \mathrm{Y}_{\mathrm{LI}}$ ) and ( $\mathrm{X}_{\mathrm{UI}}, \mathrm{Y}_{\mathrm{UI}}$ ). The linear interpolation is used with arc length as an independent variable to interpolate $V_{L}$ and $V_{U}$ from the table $\mathrm{V}_{\mathrm{LI}}, \mathrm{V}_{\mathrm{UI}}$.

## Subroutine XFGRID

Object
Interpolate uniform to nonuniform grid

## Options

None

Input Symbols

DDS Mesh distortion parameter
KL Number of output streamlines
Q1 Input coordinate data array

## Output Symbols

Q2
Output coordinate data array

Theory

If IXFG option is turned on, only this subroutine is called by the main program CØDUCT. This subroutine reads in input coordinate file from unit IUUNIT, interpolates the Q1 array with KN uniformly spaced streamlines to obtain the Q2 array with KL nonuniformly spaced streamlines and stores the output on unit INUNIT.

|  | Report No. <br> NASA CR-165598 | 2. Government Accession No. |  | 3. Recipient's Catalog No. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4. Title and Subtitle <br> User's Manual for Axisymmetric Diffuser Duct (ADD) Code Volume III - ADD Code Coordinate Generator |  |  |  | 5. Report Date February 1982 <br> 6. Performing Organization Code 778-32-01 |  |
|  |  |  |  |  |  |
| 7. Author(s) <br> O. L. Anderson, G. B. Hankins, Jr., and D. E. Edwards |  |  |  | 8. Performing Organization Report No. UTRC81-65 |  |
|  |  |  |  | 10. Work Unit No. |  |
|  |  |  |  |  |  |
|  |  |  |  | 11. Contract or Grant No. DEN 3-235 |  |
|  |  |  |  | 13. Type of Report and Period Covered Contractor Report |  |
|  | 12. Sponsoring Agency Name and Address <br> U. S. Department of Energy Office of Vehicle and Engine R\&D Washington, D. C. 20545 |  |  |  |  |
|  |  |  |  | 14. Sponsoring Agency Gode Report No. DOE/NASA/0235-2 |  |
|  | 15. Supplementary Notes <br> Final Report. ' Prepared under Interagency Agreement DE-AI01-77CS51040. Project Manager, K. L. McLallin, Aerothermodynamics and Fuels Division, NASA Lewis Research Center, Cleveland, Ohio 44135. |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  | 16. Abstract <br> This User's Manual contains a complete description of the computer codes known as the Axisymmetric Diffuser Duct ( $A D D$ ) code. It includes a list of references which describe the formulation of the ADD code and comparisons of calculation with experimental flows. The input/output and general use of the code is described in the first volume. The second volume contains a detailed description of the code including the global structure of the code, list of FORTRAN variables, and descriptions of the subroutines. The third volume contains a detailed description of the CODUCT code which generates coordinate systems for arbitrary axisymmetric ducts. |  |  |  |  |
|  |  |  |  |  |  |  |  |

*For sale by the National Technical Information Service, Springfield, Virginia 22161


