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A STUDY INTO THE LOSS OF LOCK OF THE SPACE TELESCOPE FINE GUIDANCE SENSOR

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April 1983

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George C. Marshall Space Flight Center
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This report documents the results of a study into the loss of lock phenomenon associated with the Space Telescope (ST) Fine Guidance Sensor (FGS). The primary cause of loss of lock has been found to be a combination of cosmic ray spikes and photon noise due to a 14.5 Mv star. The probability of maintaining lock versus time is estimated both for the baseline FGS design and with parameter changes in the FGS firmware which will improve the probability of maintaining lock. The parameters varied are changeable in-flight from the ground and hence do not impact the design of the FGS hardware.

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LIST OF SYMBOLS

A	Photoelectron counts in tube A
B	Photoelectron counts in tube B
D	Parameter in Kolmogorov-Smirnov test
F(x)	Distribution function (d.f.) for the random variable X
f(x)	Probability density function (p.d.f.) for the random variable X
K_{0G}	Optical lever arm, dimensionless
K_o	Parameter in linearization algorithm, arc-sec
K_1	Parameter in linearization algorithm, arc-sec
K_3	Parameter in linearization algorithm, dimensionless
K_4	Parameter in Q computation, photoelectron counts
K_{13}	Proportional gain in P-I control law, (arc-sec/sec)/(arc-sec)
K_{14}	Integral gain in P-I control law, (arc-sec/sec)/(arc-sec sec)
N	Sample size
n	Total number of runs in each case
R(x)	Reliability function (r.f.) for the random variable X
R	Output of the linearization algorithm, arc-sec
r	Number of runs in each case which lost lock
Q	Normalized fine error signal, dimensionless
Q_L	Output of Q limiter, dimensionless
Q_{LIM}	Limit on Q, dimensionless
$S_N(x)$	Distribution function from the random sample
X	Random variable
x	A specific value for the random variable X
x_i	Number of cosmic spikes until loss of lock for the ith run
β_T	Input to the interferometer, arc-sec
σ	Standard deviation of the distribution
μ	Mean of the distribution
$\hat{\mu}$	Estimated value for μ

TECHNICAL MEMORANDUM

A STUDY INTO THE LOSS OF LOCK OF THE SPACE TELESCOPE FINE GUIDANCE SENSOR

SUMMARY

This report documents the results of a study into the loss of lock phenomenon associated with the Space Telescope (ST) Fine Guidance Sensor (FGS). The primary cause of loss of lock has been found to be a combination of cosmic ray spikes and photon noise due to a 14.5 Mv star. The probability of maintaining lock versus time is estimated both for the baseline FGS design and with parameter changes in the FGS firmware which will improve the probability of maintaining lock. The parameters varied are changeable in-flight from the ground and hence do not impact the design of the FGS hardware.

I. INTRODUCTION

The Space Telescope (ST) is an unmanned, Earth orbiting astronomical observatory to be inserted into a 600 km circular orbit by the Space Shuttle in the mid-1980's. The ST will have the capability of pointing at celestial targets with an accuracy of 0.01 arc-sec and a stability of 0.007 arc-sec RMS for observation times up to 10 hours [1]. This is accomplished by the onboard pointing control system (PCS) which includes a system of rate gyros, reaction wheels, an onboard digital computer, and a fine guidance sensor (FGS) [2]. The fine guidance sensor is basically a system of three star trackers located at the focal plane of the telescope. Two out of the three star trackers are normally employed to generate vehicle attitude information which is used to periodically update that derived from the rate gyro outputs. Each star tracker acquires and tracks known guide stars as dim as 14.5 Mv. Tracking is accomplished by means of a fine lock loop, internal to each star tracker, which consists of a two axis Koesters prism interferometer, four photomultiplier tubes, digital control logic in the fine guidance electronics (FGE), and two star selector servos (gimbal rate servos). Optical elements are mounted on the star selector servo (SSS) shafts which deflect the guide star image in such a way so as to compensate for vehicle line of sight motions in order to keep the guide star at null in the field of view of the interferometer. Outputs from the photomultiplier tubes and outputs from the gimbal encoders mounted on the SSS shafts are routed to the PCS digital computer every 0.025 sec and used to update the computed vehicle attitude derived by integrating the rate gyro outputs. These updates take place once per second. However, disturbances to the FGS fine lock loop can cause the FGS to lose lock on the guide star it is tracking. To date, the principal cause of loss of lock has been found to be a combination of cosmic ray spikes striking the PMT photocathode and photon noise due to a 14.5 Mv star.

This report presents the results of a study to:

a) Analyze the loss of lock phenomenon associated with the FGS fine lock loop.

b) Predict the probability of maintaining lock with the FGS, calculated based on loss of lock results from a single axis digital simulation of the ST PCS/FGS.

c) Offer potential solutions for improving the probability of maintaining lock which do not require redesign of the FGS.

II. DESCRIPTION OF THE FGS FINE LOCK LOOP

The primary purpose of the FGS fine lock loop is to keep the guide star image at null in the field of view of the two axis Koesters prism interferometer. A simplified functional block diagram of one axis of the fine lock loop is shown in Figure 1. The outputs of the interferometer are the photons from the guide star which, in turn, strike the photocathodes of the PMTs, two per axis. The photons generate photoelectron counts in the PMTs and these are integrated over 0.025 sec intervals and read out by the FGE. These are inputs into the calculation of Q, the normalized fine error signal. The transfer characteristic for the interferometer from the input to the interferometer, denoted by β_T, to the normalized error signal, Q, is shown in Figure 2, both for the ideal theoretical curve and the simplified model used in the simulation. Because of the uncertainty that exists as to the location of the tails of the actual manufactured interferometers, the simplified model used in the simulation goes to zero at ±0.047 arc-sec.

Returning to Figure 1, the output of the Q computation is limited to ±1 and this result is input into the linearization algorithm which:

a) Transforms the normalized error signal into object space.

b) Helps to linearize the interferometer transfer characteristic in the region between ±0.01 arc-sec for β_T.

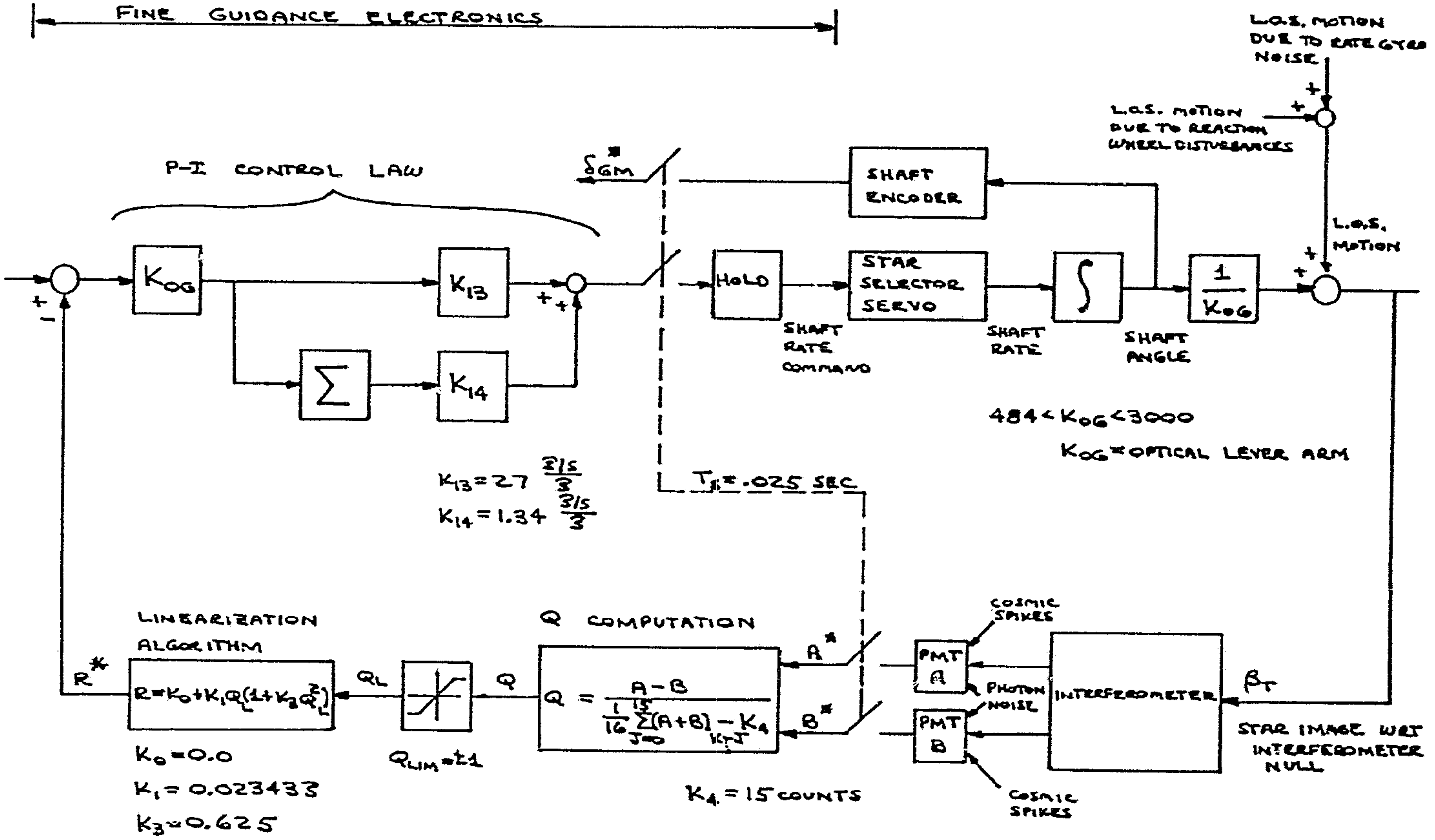
Nominal values for K_0, K_1, and K_3 in the linearization algorithm are given in Figure 1. The output R of the linearization algorithm is then transformed into gimbal space by the optical lever arm K_{0G} after a sign inversion, and then this result is input into the fine lock loop proportional-integral (P-I) control law which generates gimbal rate commands to the star selector servo (SSS) every 0.025 sec. The gimbal rate commands to the SSS result in a gimbal rate and hence a change in shaft angle which, in turn, moves the star image by virtue of the optical elements on the SSS shaft in the optical train on the FGS. The action of the fine lock loop compensates for line of sight motions, such as those generated by rate gyro noise and reaction wheel induced oscillations, to try to keep the star image at interferometer null and hence in the field of view of the interferometer. However, because of the finite bandwidth of the fine lock loop (gain crossover frequency is about 4 Hz), it is possible for disturbances to move the star image outside the field of view of the interferometer. If there are no immediate subsequent disturbances to the fine lock loop which drive the star image back into the interferometer field of view, then loss of lock occurs and telescope pointing is affected.

III. AN ANALYSIS OF THE LOSS OF LOCK PHENOMENON

As stated previously, loss of lock of the FGS on a guide star it is tracking occurs when disturbances to the fine lock loop cause the star image to move outside the field of view of the interferometer (±0.047 arc-sec in the simulation model) and there are no immediate subsequent disturbances which drive the star image back into the interferometer field of view. Four disturbances to the fine lock loop have been simulated to date:

a) Line of sight motion due to rate gyro noise

b) Line of sight motion due to reaction wheel induced oscillations



* DATA SENT TO SSM COMPUTER EVERY .025 SEC FOR PCS ATTITUDE UPDATE CALCULATIONS

Figure 1. FGS fine lock loop functional block diagram.

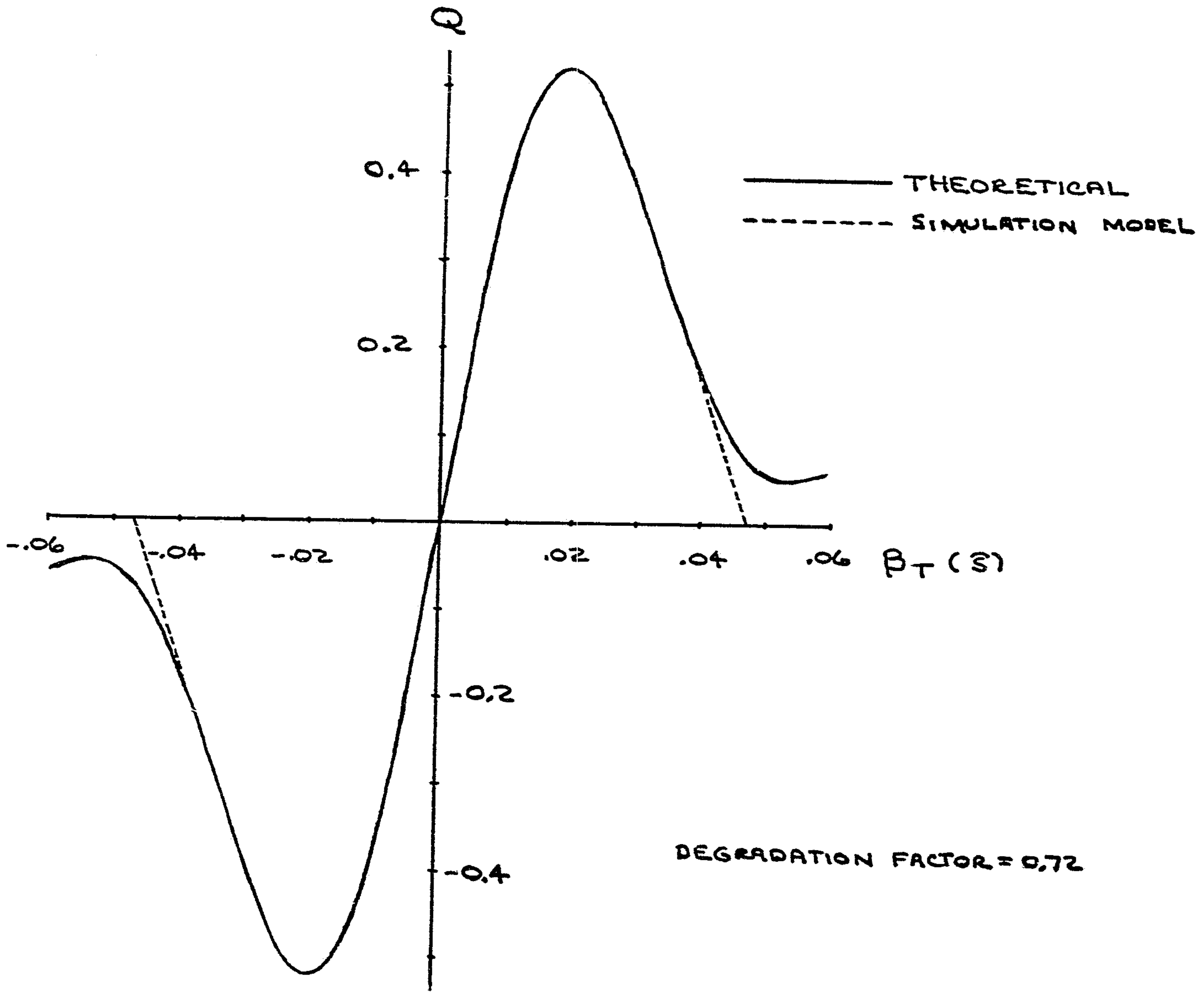


Figure 2. Interferometer transfer characteristic.

c) Photon noise

d) Cosmic ray spikes.

It was found that rate gyro noise has little effect on loss of lock. Simulation runs were made with line of sight motion due to reaction wheel induced oscillations at a frequency of 50 Hz (these disturbances have been predicted to be predominantly in the frequency range of 50 Hz to 60 Hz) and a zero-to-peak amplitude equal to 0.01 arc-sec (it is predicted that the requirement of 0.003 arc-sec RMS due to reaction wheel induced oscillations will be satisfied), and it was found the oscillations actually alleviated the loss of lock problem because their effect was to extend the field of view of the interferometer. It turns out that loss of lock is caused primarily by the combination of cosmic ray spikes and photon noise due to a 14.5 Mv star. At a 13 Mv star, loss of lock does not occur with or without cosmic ray spikes. Also, the FGS has no requirement to track stars dimmer than 14.5 Mv for PCS control.

To understand what combinations of cosmic ray spikes and photon noise from a 14.5 Mv star cause loss of lock, a simplified analysis was performed with a single axis simulation of the fine lock loop. Initially, all line of sight motion, as well as photon noise, was set equal to zero so that the only source of disturbance to the fine lock loop was a single cosmic ray spike, affecting one 0.025 sec sample from the output of PMT A. The amplitude of the cosmic ray spike was such that it produced 300 photoelectron counts in PMT A as shown in Figure 3. Also shown in Figure 3 is the response of the fine lock loop, in terms of β_T versus time, to this disturbance. It can be seen from the fine lock loop response that:

a) $|\beta_T|_{max}$ is 0.025 arc-sec, which is well within the field of view of the interferometer (± 0.047 arc-sec) and hence loss of lock did not occur

b) $|\beta_T|_{max}$ occurred between 0.05 sec and 0.1 sec

c) The response settled out well within 1 sec of time.

Next the amplitude of the cosmic ray spike was varied and $|\beta_T|_{max}$ was recorded. This was done for $K_3 = 0.625$ (nominal) and $K_3 = 0$. The results were plotted and are shown in Figure 4. Several conclusions can be drawn from Figure 4:

a) The value of $|\beta_T|_{max}$ increases with increasing amplitude for the cosmic ray spike until the amplitude is in the neighborhood of 60 photoelectron counts. This is because with a 14.5 Mv star, Q becomes 1 when the amplitude of the cosmic spike is about 60 counts and the Q limiter saturates at ±1. Hence, the output of the Q limiter is ±1 for amplitudes of cosmic spikes greater than about 60 photoelectron counts.

b) Consequently, $|\beta_T|_{max}$ peaks out and at a value of 0.025 arc-sec for $K_3 = 0.625$ and 0.014 arc-sec for $K_3 = 0$. Since the field of view of the interferometer is ±0.047 arc-sec, one cosmic spike (or one photon noise spike for that matter) will not cause loss of lock. In fact, one would predict that two spikes in the same tube large enough to saturate the Q limiter would be needed for loss of lock to occur with $K_3 = 0.625$, while four spikes would be needed with $K_3 = 0$. Hence, one would expect loss of lock to be improved by reducing K_3 below its nominal value of 0.625.

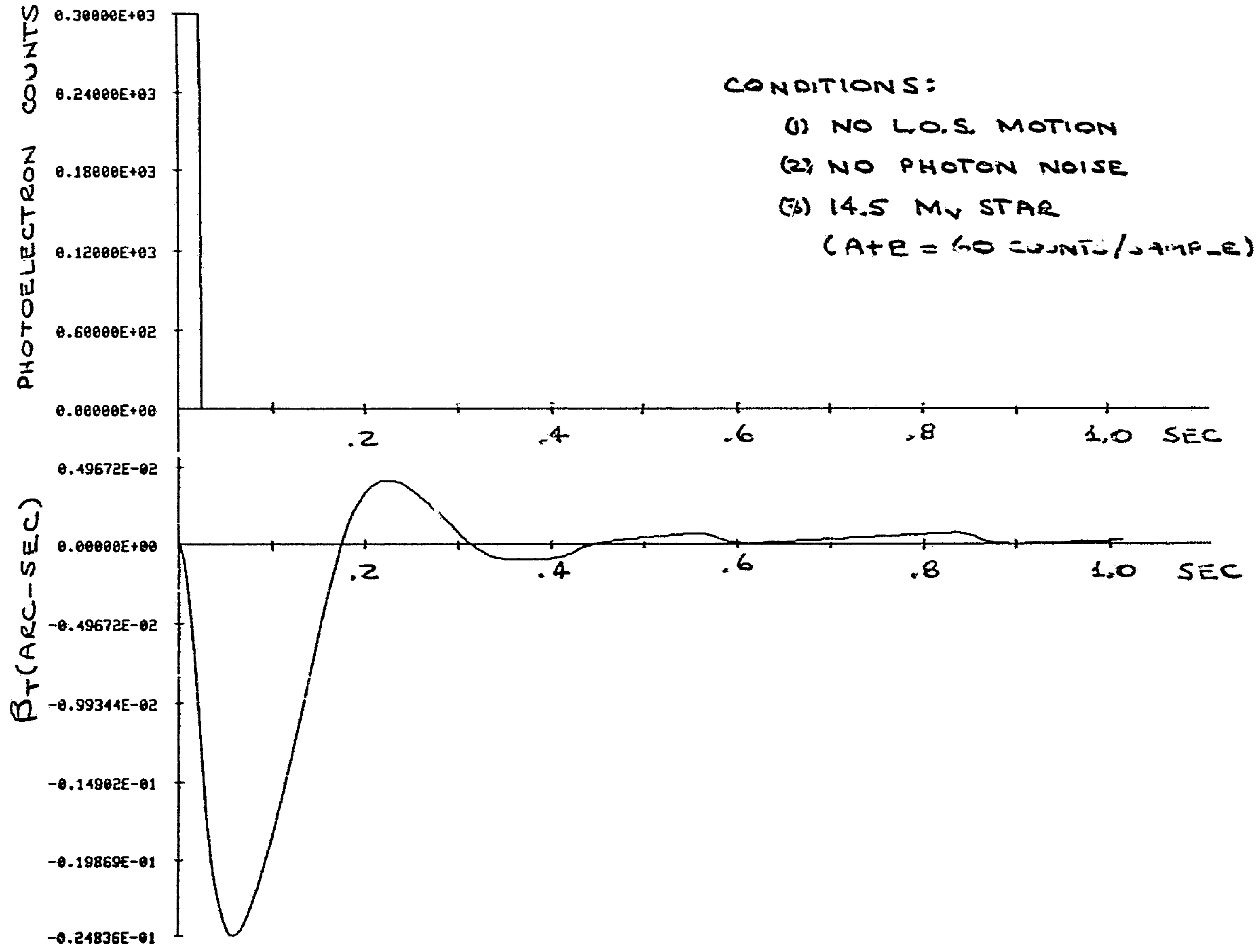


Figure 3. FGS fine lock loop response to a cosmic ray spike.

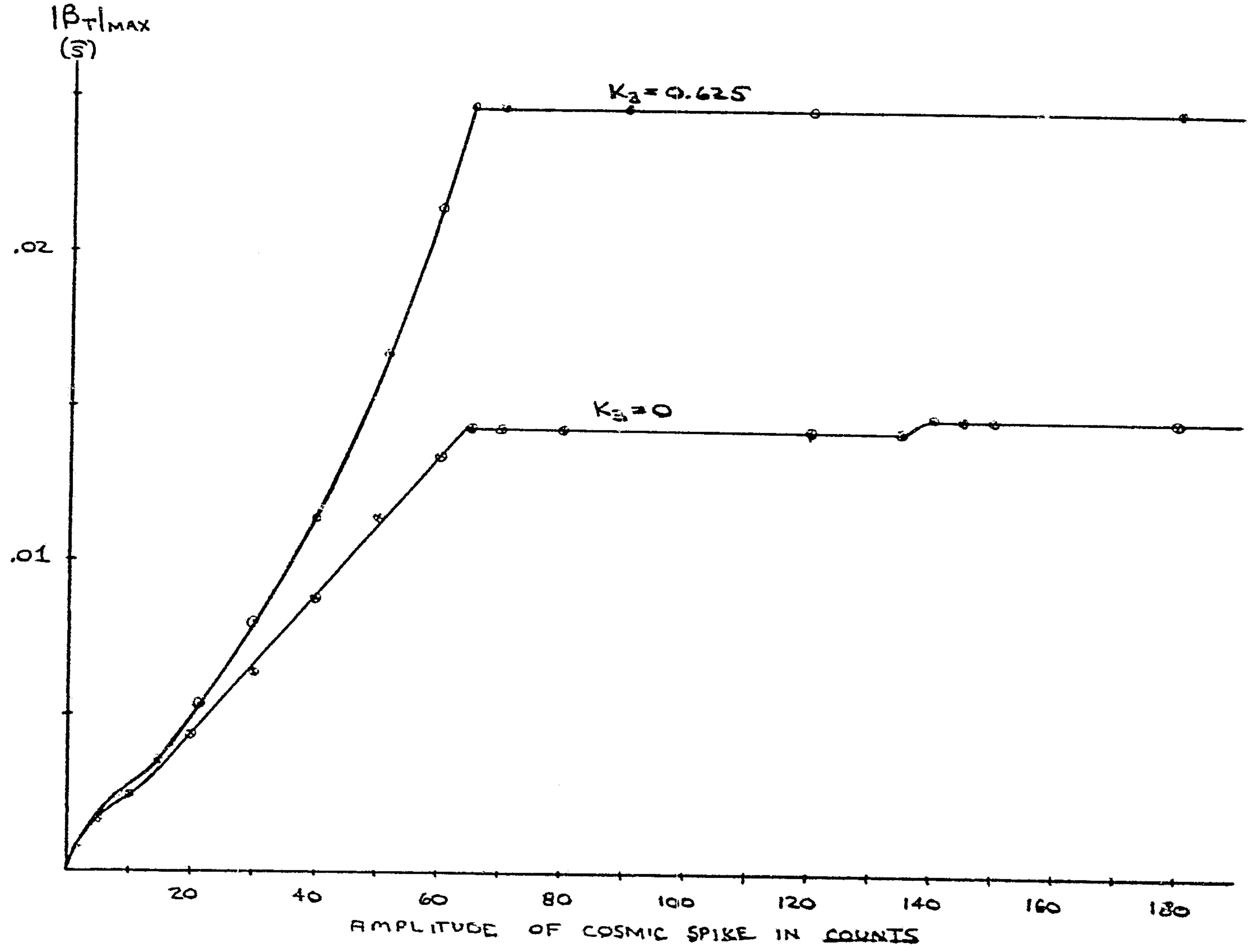


Figure 4. $|\beta_T|_{\max}$ versus amplitude of cosmic ray spike.

To test the theory that two cosmic ray spikes could cause loss of lock, the response of the fine lock loop to two cosmic ray spikes was simulated. The results are shown in Figure 5. The cosmic spikes were large enough to saturate the Q limiter and had the same polarity (were inserted in the same PMT). When they occurred back to back, or else one 0.025 sec sample apart, loss of lock occurred. When they were two 0.025 sec samples (or 0.05 sec) apart, loss of lock did not occur. It can also be seen in the loss of lock cases that once loss of lock occurs, the star image is driven away from null at a constant rate. This is due to the fact that the integral path in the fine lock loop control law sums the two cosmic spikes so that its output is a constant resulting in a constant rate command driving the star image away from null. One would suspect that reducing the value of the gain K_{14} in the integral path would improve the loss of lock problem, also.

It should be pointed out that it should be possible to find combinations of three or more smaller amplitude noise spikes which will cause loss of lock. This was not investigated, however. What has been established is that one noise spike (cosmic ray or photon) will not cause loss of lock regardless of amplitude because of the Q limiter in the FGE. Two noise spikes of the same polarity (in the same tube) large enough to saturate the Q limiter and close together will cause loss of lock.

To further understand the nature of the loss of lock problem, simulation runs were made with a single axis simulation of the PCS/FGS under more realistic conditions with cosmic spikes of various amplitudes. In these simulation runs, the PCS/FGS was configured in what is known as Configuration 2 wherein the FGS outputs (PMT counts and gimbal encoder readouts) every 0.025 sec are used to update once per second the vehicle attitude computed onboard from the rate gyro outputs. This is the normal mode for fine pointing. It was assumed that one rate gyro had failed and of the remaining five, the best four were chosen so as to minimize rate gyro noise. There was no reaction wheel induced jitter. The FGS was tracking a 14.5 Mv star and the two axis photon noise equivalent angle was 0.0046 arc-sec. Cosmic spikes were inserted in PMT A every 2 sec. 2 sec was chosen based on the 1 sec settling time of the fine lock loop. Hence, the effects of one cosmic spike settled out before the next arrived, which is what happens in the actual situation as shall be seen later. Finally, the simulation run times were 5400 sec long. The amplitudes of the cosmic spikes in a given run were held constant, but varied from run to run. It was observed whether loss of lock occurred in 5400 sec. The results are shown in Table 1. The results indicate that the FGS maintains lock until the amplitude of the cosmic spikes are 52 photoelectron counts or more. Hence, it is seen that loss of lock does not occur unless cosmic spikes are present and their amplitude exceeds approximately 50 counts. Should cosmic spikes greater than 50 counts exist in the actual situation, one would intuitively expect the probability of maintaining lock to be related to the number of these events that occur.

In a separate study, summarized in Table 2, it was found that the mean time between cosmic spikes greater than 50 photoelectron counts, hitting any one of eight PMT's (four PMT's per star tracker with two normally used for control) was 673 sec with a pulse amplitude discriminator (PAD) setting of 80 photoelectron counts in the FGS and 1293 sec with a PAD setting of 40 photoelectron counts.[1] The PAD is a setting in the electronics at the output of each PMT which discriminates against some of the cosmic spikes or photon noise spikes which exceed the setting. Unfortunately, the PAD does not discriminate against all the cosmic spikes which exceed the threshold setting and its ability to do so depends on the direction of the cosmic rays relative to the PMT. It has been predicted that 40 is a conservative value for the PAD setting at the beginning of life and 80 at the end of life, the change caused by degradations in the electronics with time. These results will be used in Section IV of this study to estimate the probability of maintaining lock with the FGS.

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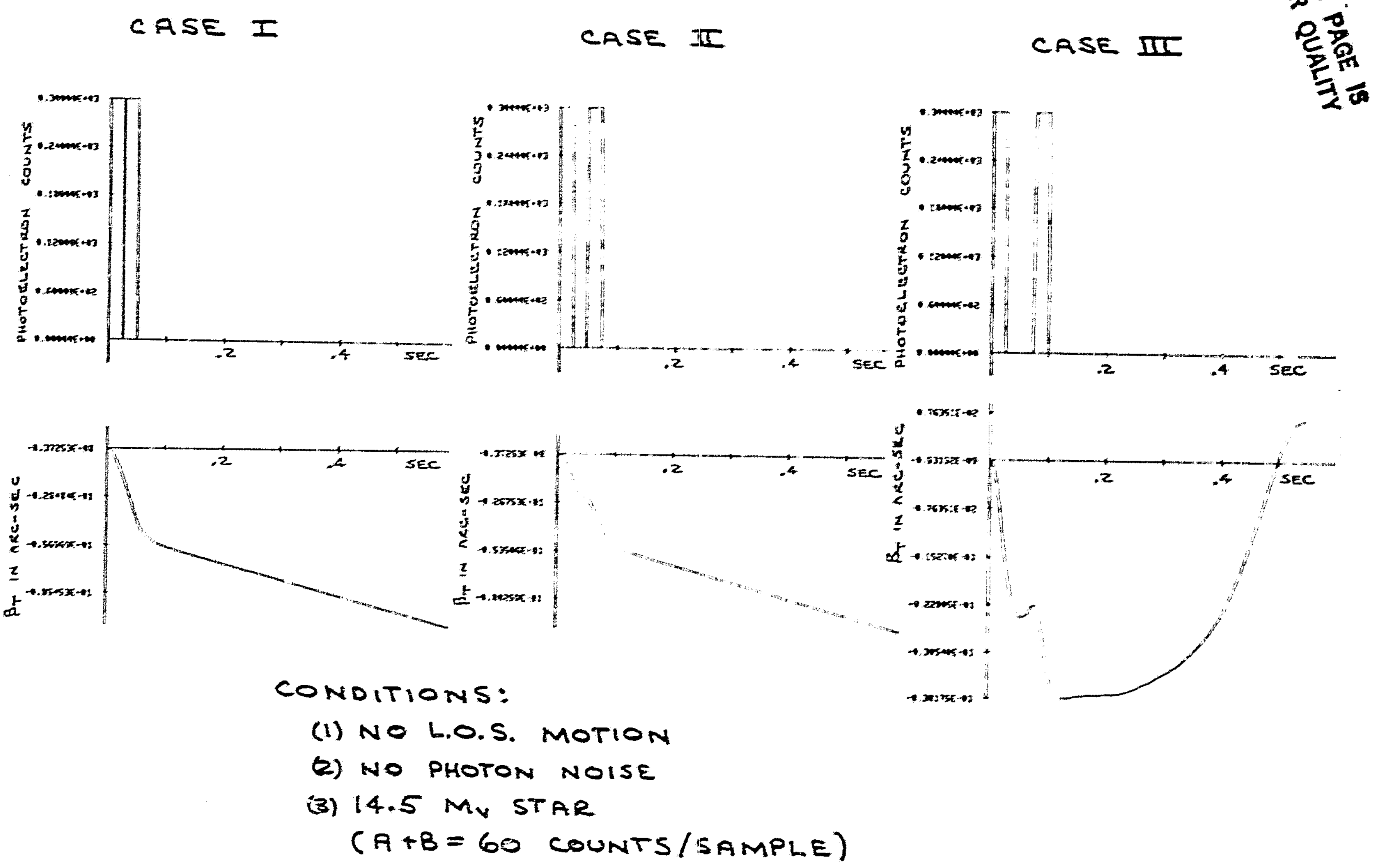
[1] This is the result of an in-house study by William B. Chubb of NASA-Marshall Space Flight Center in February, 1983.



CONDITIONS:

(1) NO L.O.S. MOTION
(2) NO PHOTON NOISE
(3) 14.5 M_V STAR
(A+B = 60 COUNTS/SAMPLE)

Figure 5. Fine lock loop response to two cosmic ray spikes.

TABLE 1. SIMULATION RESULTS VARYING AMPLITUDE OF COSMIC RAY SPIKES

SIMULATION CONDITIONS:

(1) PCS CONFIGURATION 2
(2) BEST 4 OUT OF 5 RATE GYROS
(3) NO REACTION WHEEL INDUCED JITTER
(4) 14.5 m_v STAR, 2 AXIS PHOTON NOISE ($1\sigma = 12$ COUNTS $\equiv .00463$ NEA)
(5) COSMIC SPIKE EVERY 2 SEC IN TUBE A
(6) 5400 SEC RUN

AMPLITUDE OF COSMIC SPIKE (COUNTS)	Q	SIMULATION RESULT
≤ 50	≤ 0.84	HELD LOCK
51	0.86	HELD LOCK
52	0.87	LOST LOCK AFTER 2518 SPIKES
53	0.89	—— ” —— 1912 ”
55	0.92	—— ” —— 1912 ”
57	0.96	—— ” —— 1123 ”
59	0.99	—— ” —— 1120 ”
61	1.02	—— ” —— 1120 ”
63	1.06	—— ” —— 277 ”
65	1.09	—— ” —— 277 ”

TABLE 2. RATE OF OCCURRENCE OF COSMIC RAY NUCLEI

ORGANIZATION		NAME
EL51	**MARSHALL SPACE FLIGHT CENTER** RATE OF OCCURRENCE OF COSMIC RAY NUCLEI	W. B. CHUBB DATE FEBRUARY 1983

Z	Particles Per Sec Per (.25 cm^2) Tube X 10^{-3} [illegible]	Lp	Orbital Correction Factor	8-Tube Correction Factor	Discriminator Setting	Blanking Factor Downward	Blanking Factor Upward	"Q" Factor <.8 No. [illegible] (Actually <.87) Downward	"Q" Factor Upward	Particles Per Sec X 10^{-3}	Seconds Between Particles
1	60.976	60.976	.2155	8	80						
					40						
2	9.259	9.259			80	Q <.8 for Average Fluorescence +2σ, Therefore, no effect.					
					40						
3	.041	.041			80						
					40						
4	.0195	.0195			80						
					40						
5	.067	.067			80	1	1	.472	.472	.1090	9,171
					40	.0818	1	0	.472	.0545	18,341
6	.238	.238			80	.826	1	.8207*	1	.74705	1,339
					40	0	.7412	0	1	.3041	3,288
7	.0563	.0563			80	0	1	1	1	.09706	10,302
					40	0	.5774	0	1	.0560	17,843
8	.2137	.2137			80	0	.255	1	1	.2966	3,372
					40	0	.515	0	1	.1897	5,270
9	.0043	.0043			80	0	.6703	1	1	.0074	134,894
					40	0	.4554	0	1	.0034	296,211
≥10	.2315	.2315			80	0	.5774	1	1	.2304	4,340
					40	0	.4158	0	1	.1659	6,026
				Total Effect	80					1.487	673
					40					.773	1,293

Z			1	2	3	4	5	6	7	8	9	10
Fluorescence		Max	4	16	36	64	100	144	196	256	324	400
		Avg	2	8	18	32	50	72	98	128	162	200
Cherenkov Yield	Downward	Max	5.5	22	49.5	88	137	198	269.5	352	445.5	550
		Avg	2	8	18	32	50	72	98	128	162	200
	Upward	Avg -2.					35.9	55.0	72.2	105.4	136.5	171.7

* [illegible] Both
Discriminator
and "Z" Effect

IV. THE PROBABILITY OF MAINTAINING LOCK WITH THE BASELINE SYSTEM

It has been established that loss of lock occurs when the fine lock loop tracks a 14.5 Mv star and cosmic ray spikes exceeding 50 photoelectron counts are present. However, time to loss of lock, as it turns out, is not deterministic, but depends on the probability of getting a large cosmic ray spike surrounded by the right photon noise profile. To determine the probability of maintaining lock as a function of time, the preferred approach would be to simulate the cosmic spikes at the frequency they are expected to occur in the actual situation, which is once every 673 sec or 1293 sec, depending on the PAD setting. The simulation would be run until loss of lock and then this procedure would be repeated a number of times varying the photon noise profile from run to run to get a random sample of the time to loss of lock. From this, the probability of maintaining lock versus time could be estimated. However, simulation run times would be intolerably long to do this. Consequently, the scheme employed was:

a) Insert cosmic spikes every 2 sec in PMT A in the simulation. (2 sec was chosen to be as small as possible to save computer run time but larger than the 1 sec settling time of the fine lock loop to a single cosmic ray spike. Hence, the effect of one spike will have settled out before the next arrives which is what happens in the actual situation.) Run until loss of lock and record the number of cosmic spikes until loss of lock. (Since loss of lock does not occur unless cosmic spikes exceeding 50 photoelectron counts are present, then intuitively one would figure that the probability of maintaining lock is related to the number of these cosmic spike events which occur.)

b) Make 25 simulation runs varying the photon noise profile from run to run and record the number of cosmic ray spikes until loss of lock for each run. From this random sample, estimate the probability of maintaining lock versus the number of cosmic spikes until loss of lock.

c) Multiply the number of cosmic spikes until loss of lock by the mean time between cosmic spikes to determine probability versus time.

The idea of subjecting the FGS to cosmic spikes at a higher rate than it will experience in normal operation is analogous to what is known as accelerated life testing, whereby different types of components are tested in a far more severe environment than they will normally be expected to operate in so as to significantly shorten the time to failure and hence the time required for life testing. The reliability of one versus another can be quickly ascertained. Sometimes, preliminary experimentation is carried out to determine the relationship between the proportion of failures that can be expected under nominal conditions and under various levels of accelerated environmental conditions [3].

The simulation conditions used in the 25 runs are similar to those used previously:

a) PCS configuration 2

b) Best four out of five gyros

c) No reaction wheel induced jitter

d) 14.5 Mv star with two axis photon noise

e) Cosmic spikes every 2 sec in tube A, large enough to saturate the Q limiter

f) 5400 sec simulation run times.

The simulation results in terms of the number of cosmic spikes until loss of lock are shown in Table 3 where the data has been ordered such that $x_1 < x_2 < \ldots < x_{25}$ where x_i is the number of cosmic spikes until loss of lock for the ith run. It should be observed that the mean and standard deviation for the random sample are 63 and 59, respectively. A histogram was constructed from the data to show the number of times loss of lock occurred (class frequency) for various intervals of the random variable (the number of cosmic spikes until loss of lock). The result is shown in Figure 6. This was then

TABLE 3. SIMULATION RESULTS FOR THE BASELINE SYSTEM DESIGN

RUN #	# OF COSMIC SPIKES TIL LOSS OF LOCK
1	8
2	8
3	11
4	13
5	16
6	16
7	18
8	23
9	26
10	26
11	28
12	38
13	41
14	46
15	51
16	68
17	71
18	73
19	76
20	83
21	91
22	148
23	168
24	171
25	226
MEAN	63
ST.DEV.	59

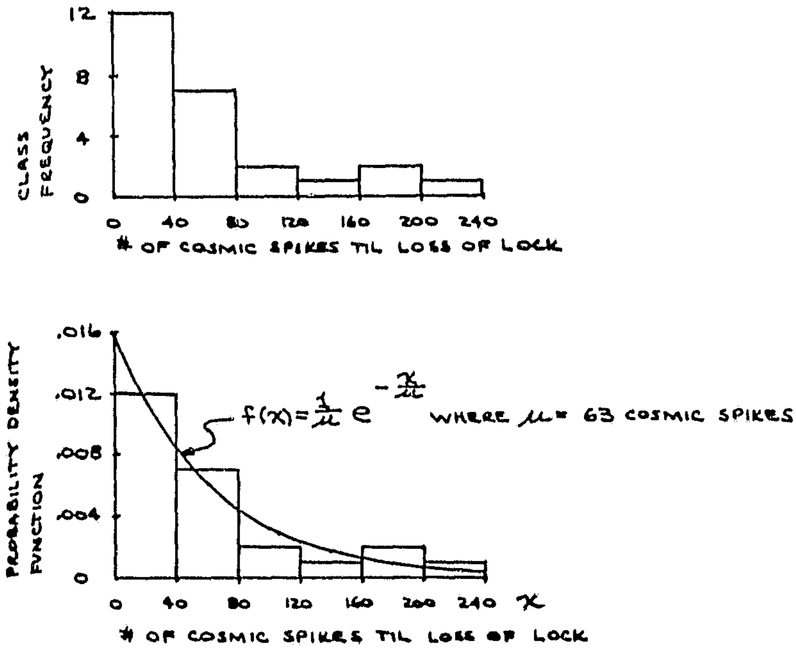


Figure 6. Histograms based on the simulation results for the baseline system design.

put into the form of a probability density function (p.d.f.) by dividing the class frequencies by 25 (the total number of samples) and then dividing this by 40 (the width of each class interval). This result is also shown in Figure 6. However, the p.d.f. derived from the random sample looks suspiciously like the p.d.f. of an exponential distribution which has the form

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}} \tag{1}$$

where x is a specific value for the random variable X and μ is the mean of the distribution of X [4]. If μ equals the mean of the random sample or

$$\mu = 63 \quad , \tag{2}$$

the result is shown in Figure 6. It can be seen that the p.d.f. derived from the random sample matches the theoretical curve quite well. Furthermore, for the exponential distribution, σ, the standard deviation of the distribution of X, is equal to the mean of the distribution or

$$\sigma = \mu \quad . \tag{3}$$

From Table 3, it can be seen that the standard deviation of the random sample is close to the mean of the random sample. This then is a second piece of information which infers that the number of cosmic spikes until loss of lock has an exponential probability distribution. A formal test of goodness of fit of the random data of Table 3 to an exponential distribution was performed using the Kolmogorov-Smirnov test.[1] According to this test, the distribution function from a random sample is compared with that for a specified theoretical distribution [5]. The distribution function (d.f.) F(X), where

$$F(X) = \Pr(X \leqslant x) \quad , \tag{4}$$

obtained from the random data of Table 3, was plotted along with that for the exponential distribution given by

$$F(x) = 1 - e^{-\frac{x}{\mu}} \tag{5}$$

with

$$\mu = 63 \quad . \tag{6}$$

[1] The author wishes to express his appreciation to Mario Rheinfurth and Frank Pizzano, both of Marshall Space Flight Center, for their assistance in this area.

The results are shown in Figure 7. When the two curves are compared at values in the random sample, the absolute values for the differences in the two curves was observed. The maximum was found to be 0.081 as seen in Figure 7. This then was compared with a critical value found in Table 4. In general, the Kolmogrov-Smirnov test does not apply where parameters in the theoretical distribution are estimated from the random sample being tested. However, the test has been worked out for the exponential distribution where the mean of the distribution is unknown and must be estimated from the random sample at hand. Table 4 is the result of that effort [6]. A general rule of thumb is to use a level of significance of 0.05 (i.e., the probability of false rejection is 0.05). At that level of significance and with a sample size

$$N = 25 \quad , \tag{7}$$

it is seen that the critical value for D in the table is

$$D = 0.210 \tag{8}$$

Since the value for D from Figure 7, which is

$$D = 0.081 \quad , \tag{9}$$

is less than the critical value found in Table 4, the hypothesis that the random data of Table 3 comes from an exponential distribution cannot be rejected. To even further argue that the number of cosmic spikes until loss of lock has an exponential distribution, the exponential distribution is characteristic of the probability distribution for the time to failure of a system or component which has a constant failure rate [7]. Furthermore, a system or component with a constant failure rate does not wear out with time but is "as good as new" while it still functions. For the system at hand as modeled, there is nothing in it which wears out with time and indeed is "as good as new" while it maintains lock. Hence, by this reasoning one would expect that the time (or number of cosmic spikes) until loss of lock would follow an exponential distribution.

Once the fact is accepted that the number of cosmic spikes until loss of lock has an exponential distribution, the real function of interest is not the p.d.f. or the d.f. but the reliability function R(X), which is the probability that loss of lock will occur after x number of cosmic spikes or

$$R(x) = \Pr(X > x) \tag{10}$$

and is given by

$$R(x) = 1 - F(x) = e^{-\frac{x}{\mu}} \quad . \tag{11}$$

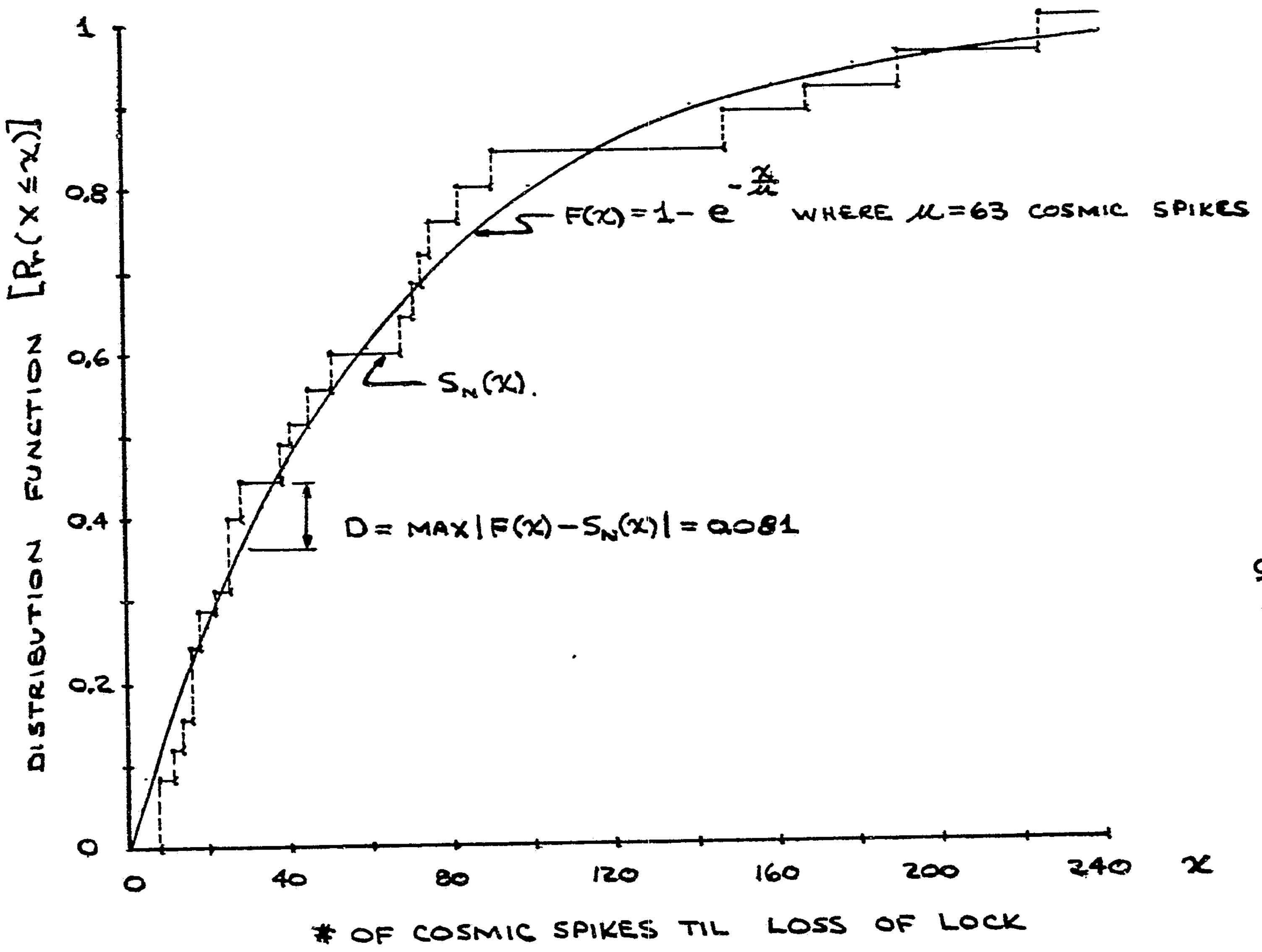


Figure 7. Distribution functions based on the simulation results for the baseline system design.

TABLE 4. KOLMOGOROV-SMIRNOV TEST FOR THE EXPONENTIAL DISTRIBUTION WITH MEAN UNKNOWN

Sample Size N	Level of Significance for $D = \text{Max}\lvert F^*(X) - S_N(X)\rvert$				
	.20	.15	.10	.05	.01
3	.451	.479	.511	.551	.600
4	.396	.422	.449	.487	.548
5	.359	.382	.406	.442	.504
6	.331	.351	.375	.408	.470
7	.309	.327	.350	.382	.442
8	.291	.308	.329	.360	.419
9	.277	.291	.311	.341	.399
10	.263	.277	.295	.325	.380
11	.251	.264	.283	.311	.365
12	.241	.254	.271	.298	.351
13	.232	.245	.261	.287	.338
14	.224	.237	.252	.277	.326
15	.217	.229	.244	.269	.315
16	.211	.222	.236	.261	.306
17	.204	.215	.229	.253	.297
18	.199	.210	.223	.246	.289
19	.193	.204	.218	.239	.283
20	.188	.199	.212	.234	.278
25	.170	.180	.191	.210	.247
30	.155	.164	.174	.192	.226
Over 30*	$\frac{.86}{\sqrt{N}}$	$\frac{.91}{\sqrt{N}}$	$\frac{.96}{\sqrt{N}}$	$\frac{1.06}{\sqrt{N}}$	$\frac{1.25}{\sqrt{N}}$

REFERENCE: LILLIEFORS, H., "ON THE KOLMOGOROV-SMIRNOV TEST FOR THE EXPONENTIAL DISTRIBUTION WITH MEAN UNKNOWN," JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION, VOL. 64, NUMBER 325, MARCH 1969, PP. 387-389.

For

$$\mu = 63 \quad , \tag{12}$$

the reliability function, or equivalently, the probability of maintaining lock, is as shown in Figure 8.

However, the probability of maintaining lock versus time is preferred. This is arrived at, then, by multiplying the number of cosmic spikes until loss of lock in Figure 8 by the mean time between cosmic spikes given in Table 2. The result is shown in Figure 9. It indicates that the probability of maintaining lock for 10 hr (maximum observation time) under FGS control with two FGSs tracking 14.5 Mv stars is about 0.6 at the beginning of the mission (PAD setting = 40 photoelectron counts) and about 0.4 at the end of the mission (PAD setting = 80 photoelectron counts). For 24 hr under FGS control with two FGSs tracking 14.5 Mv stars, the probability of maintaining lock is about 0.3 at the beginning of the mission and about 0.1 at the end of the mission.

V. IMPROVING THE PROBABILITY OF MAINTAINING LOCK

The next logical question which arises is: What can be done to improve the probability of maintaining lock shown in Figure 9? Simulation runs were made varying two parameters in the fine lock loop, K_3 and K_{14}, both of which are firmware parameters in the FGE, changeable in-flight from the ground. The various combinations of K_3 and K_{14} considered are shown in Table 5. Case I corresponds to the nominal values for K_3 and K_{14} and hence is a reference which the others are compared against. The simulation conditions are the same as those used in the analysis in Section IV and are shown in Table 5. In each case, 25 simulation runs were made varying the photon noise profile from run to run, the simulation run times being limited to 5400 sec in all cases. Because of the limited run times, loss of lock did not occur in some of the runs. However, this can be accommodated, with the problem at hand, in calculating $\hat{\mu}$, the estimated value for the mean number of cosmic spikes until loss of lock. Since the random data is drawn from an exponential distribution, a good estimator for μ is given by the expression in Table 5, which accounts for the fact that some of the observations may be censored [8]. The variable r represents the number of runs in each case which did lose lock, (n - r) being the number which did not lose lock. The resulting estimates for μ in each case are presented in the table.

To see the sensitivity of $\hat{\mu}$ to K_3, $\hat{\mu}$ was plotted against K_3 for a nominal value for K_{14} as shown in Figure 10. It indicates that below about 0.4 for K_3, $\hat{\mu}$ is very sensitive to K_3 and, there, a considerable improvement in $\hat{\mu}$ can be realized by a small reduction in K_3.

Given values for $\hat{\mu}$ in each case and using the procedure followed in Section IV, the probability of maintaining lock versus time was determined for each case and the results are shown in Figure 11.

The results indicate that a considerable improvement can be realized by reducing K_3 and/or K_{14}. Indeed, if K_3 is set to zero, the probability of maintaining lock is 1 regardless of the time interval.

The effects that reducing K_3 and K_{14} have on other aspects of system performance need to be considered, but will not be addressed in this study.

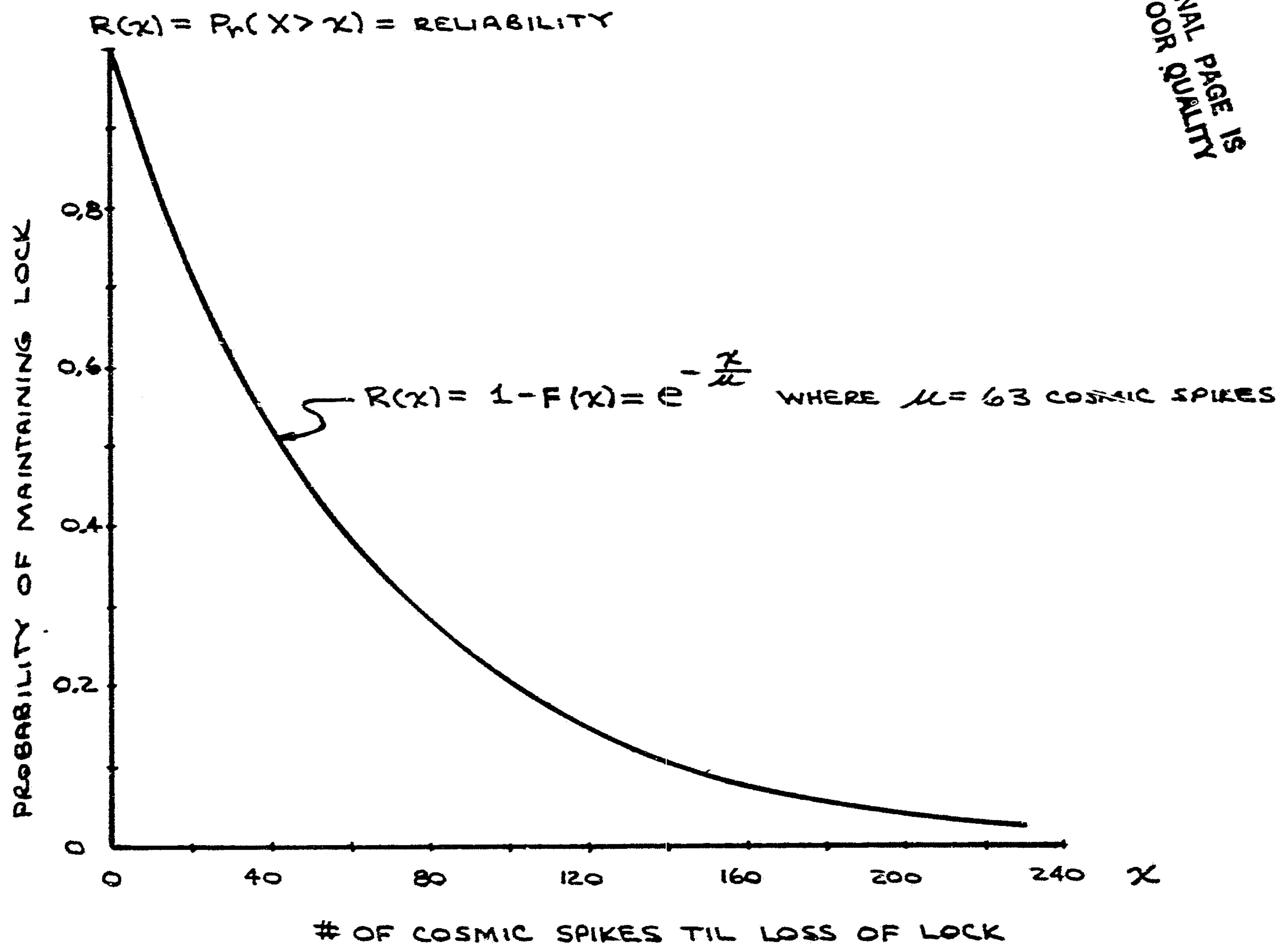


Figure 8. Probability of maintaining lock versus the number of cosmic spikes until loss of lock for the baseline system design.

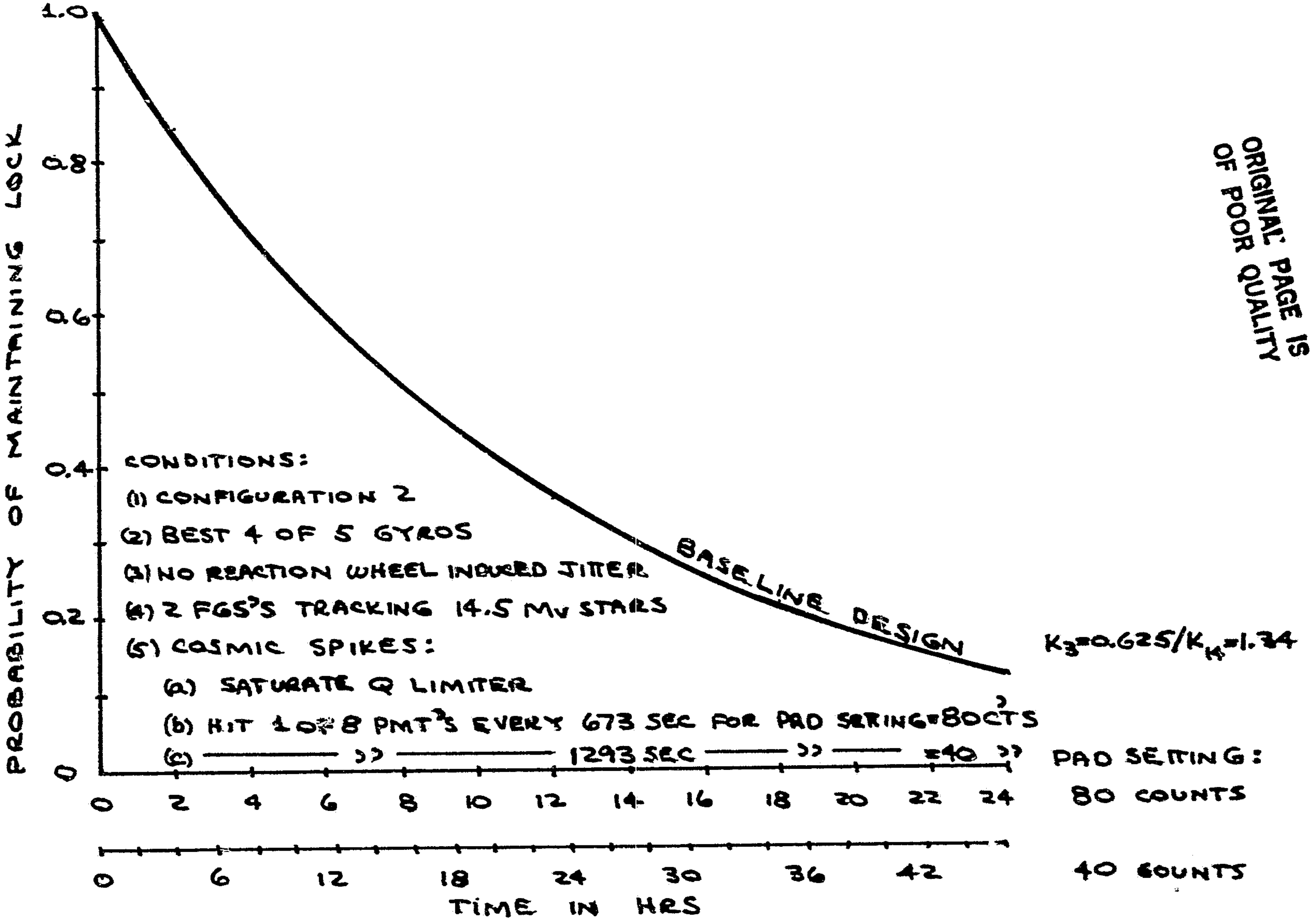


Figure 9. Probability of maintaining lock versus time for the baseline design.

TABLE 5. RESULTS OF SIMULATION VARYING K_3 AND K_{14}.

SIMULATION CONDITIONS:

(1) PCS CONFIGURATION 2
(2) BEST 4 OF 5 GYROS
(3) NO REACTION WHEEL INDUCED JITTER
(4) COSMIC SPIKE EVERY 2 SEC IN TUBE A, LARGE ENOUGH TO SATURATE Q LIMITER
(5) 25 5400 SEC SIMULATION RUNS FOR EACH CASE

METHOD FOR ESTIMATING μ:

(1) ASSUME RANDOM SAMPLES ARE DRAWN FROM AN EXPONENTIAL DISTRIBUTION $\left[f(x;\mu)=\frac{1}{\mu}e^{-\frac{x}{\mu}}\right]$
(2) LET ESTIMATOR BE:

$$\hat{\mu}=\frac{\sum_{i=1}^{r}x_i-(n-r)x_r}{r}$$

WHERE

$\hat{\mu}$ = ESTIMATED VALUE FOR MEAN # OF COSMIC SPIKES TIL LOSS OF LOCK
n = # OF RUNS = 25
r = # OF RUNS WHICH LOST LOCK IN 5400 SEC SIMULATION RUN TIME
x_i = # OF COSMIC SPIKES TIL LOSS OF LOCK FOR i^{th} RUN WHICH LOST LOCK ($1 \leq i \leq r$)

(REF: EPSTEIN, B., "ESTIMATION FROM LIFE TEST DATA," TEHHOMETRICS, VOL. 2, NO. 4, NOV 1960, PP. 447-454.)

CASE	K_3	K_{14}	r	$\hat{\mu}$
I	0.625	1.34	25	63
II	0.5	1.34	25	242
III	0.5	0.67	25	1305
IV	0.3125	1.34	19	2183
V	0	1.34	0	∞

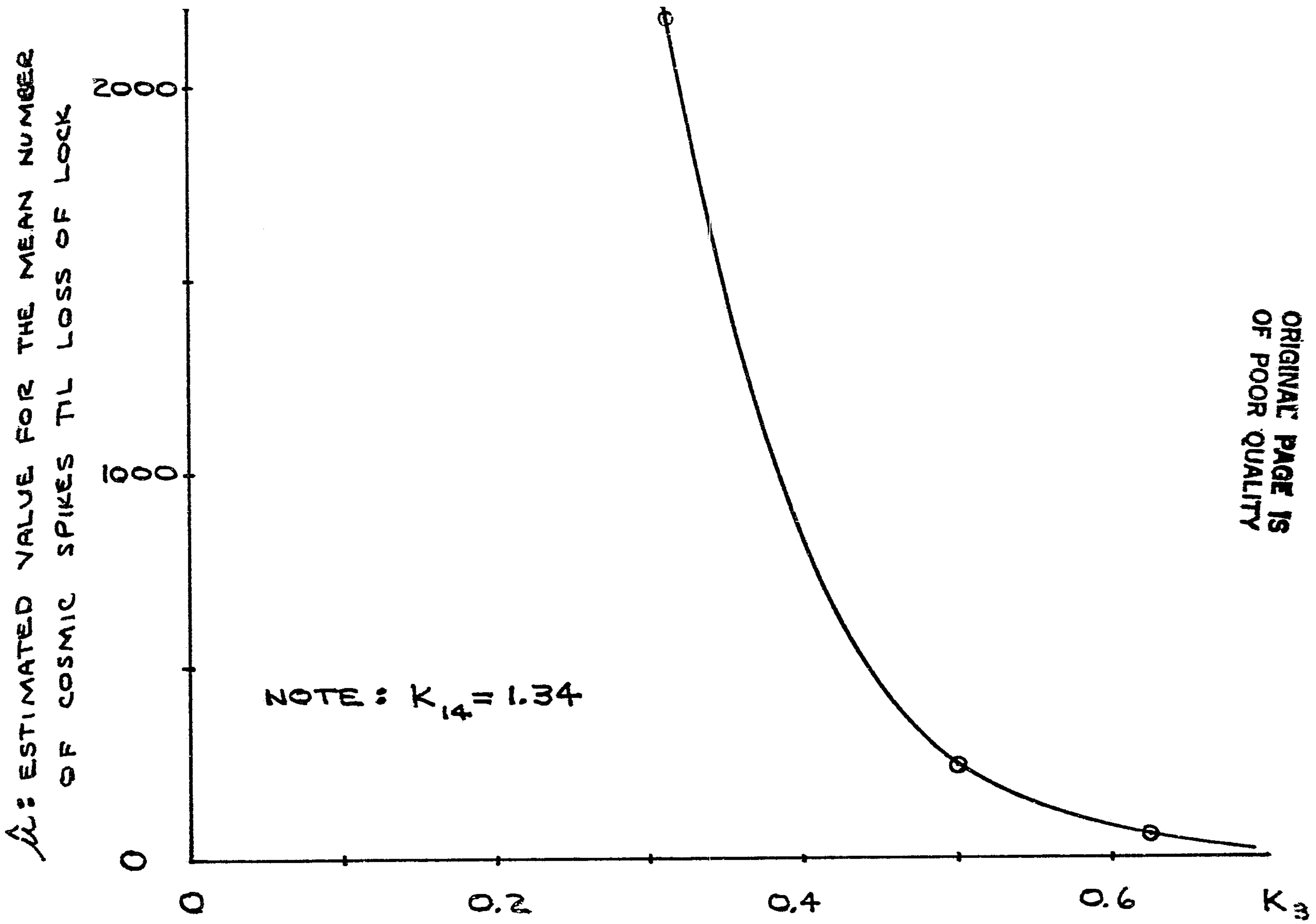


Figure 10. $\hat{\mu}$ versus K_3 for a nominal K_{14}.

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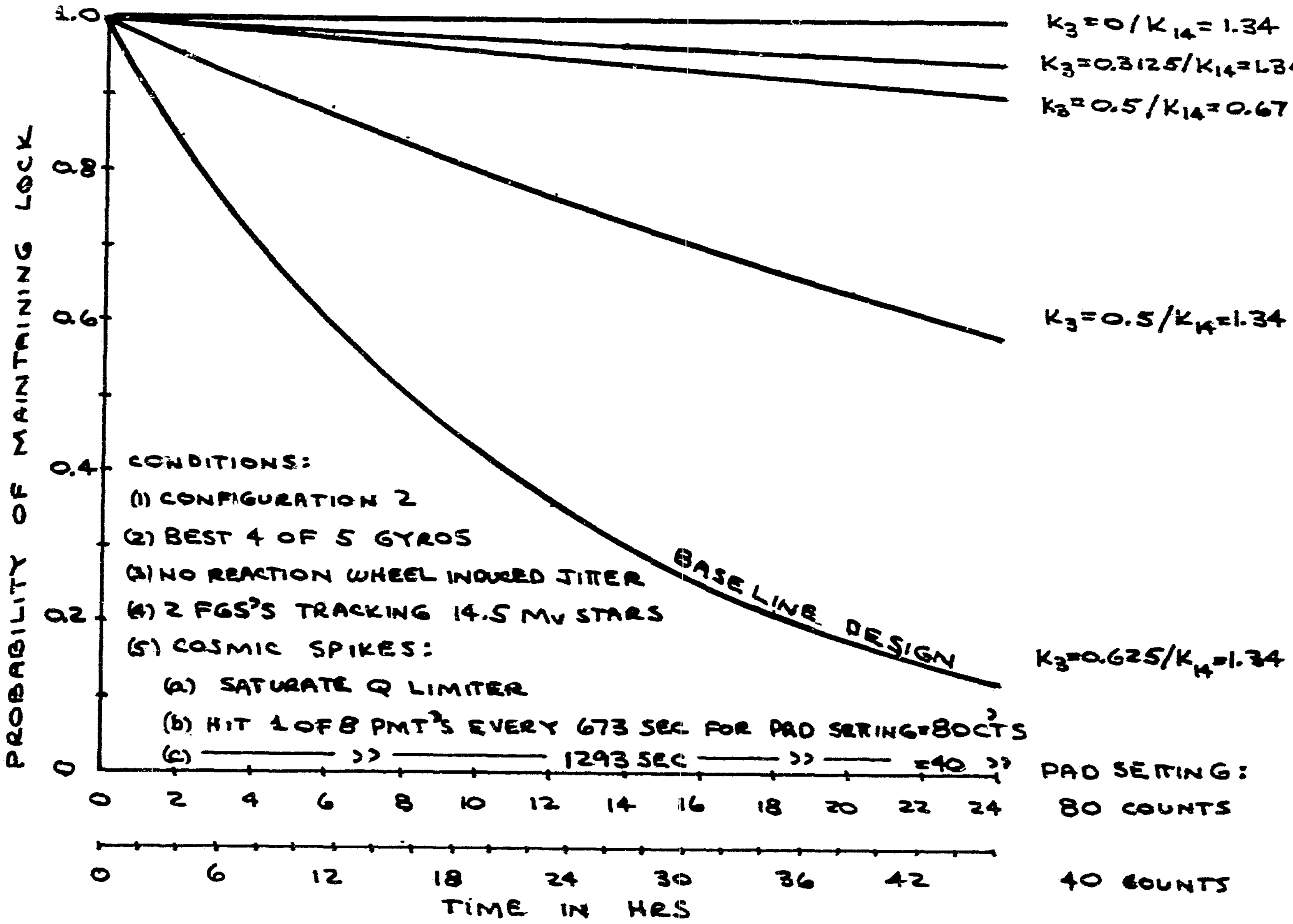

Figure 11. Improving the probability of maintaining lock.

VI. CONCLUSIONS

To summarize the overall conclusions of this study:

1) It was found that the time to loss of lock has an exponential probability distribution which is characteristic of a system which has a constant failure rate. A system with a constant failure rate does not wear out with time but is "as good as new" while it still functions. The FGS as modeled has nothing in it which wears out with time and, consequently, should exhibit a constant failure rate. Hence, one would expect the time until loss of lock to have an exponential probability distribution.

2) In the baseline design, with 2 FGSs tracking 14.5 Mv stars, the probability of maintaining lock:

a) For 10 hr is about 0.6 at the beginning of the mission (PAD setting = 40 photoelectron counts) and about 0.4 at the end of the mission (PAD setting = 80 photoelectron counts).

b) For 24 hr is about 0.3 at the beginning of the mission (PAD setting = 40 photoelectron counts) and about 0.1 at the end of the mission (PAD setting = 80 photoelectron counts).

3) The probability of maintaining lock can be improved by reducing the values for the FGS fine lock loop parameters K_3 and K_{14}. These are firmware parameters in the FGE, but are changeable in-flight from the ground. Indeed, setting K_3 equal to zero eliminates loss of lock altogether and the probability of maintaining lock becomes 1 for all time. The effects that reducing K_3 and K_{14} have on other aspects of system performance need to be considered, but were not a part of this study.

REFERENCES

1. Bahcall, J. and O'Dell, C.: The Space Telescope Observatory. NASA TM-78301, June 1979.

2. Nurre, G., Daugherty, H., and Tompetrini, K.: Space Telescope Pointing Control System. IEEE Eascon Conference, Washington, D.C., November 18, 1981.

3. Miller, I. and Freud, J.: Probability and Statistics for Engineers. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965, p. 373.

4. Hogg, R., and Craig, A.: Introduction to Mathematical Statistics. MacMillion Publishing Company, New York, 1978, pp. 105-106.

5. Miller, I. and Freud, J.: Probability of Statistics for Engineers. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965, pp. 222-223.

6. Lilliefors, H.: On the Kolmogorov-Smirnov Test for the Exponential Distribution with Mean Unknown. Journal of the American Statistical Association, Vol. 64, No. 325, March 1969, pp. 387-389.

7. Meyer, P.: Introductory Probability and Statistical Applications. Addison-Wesley Publishing Co., Reading, Mass., 1970, pp. 229-231.

8. Epstein, B.: Estimation From Life Data. Technometrics, Vol. 2, No. 4, November 1960, pp. 447-454.

APPROVAL

A STUDY INTO THE LOSS OF LOCK OF THE SPACE TELESCOPE FINE GUIDANCE SENSOR

By Michael E. Polites

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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