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SELECTING BAND COMBINATIONS WITH THEMATIC MAPPER DATA.

by

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Background. Since the human eye employs three primary colors, and the Thematic Mapper returns seven bands of data, one obvious problem that arises in making color composite images is the choice of bands. The choice in non-trivial, since three bands can be selected from seven in 35 ways. Also, any band can be assigned any color. This gives a total of 210 different possible color presentations of TM three-band images. In this note we present a way of reducing that 210 to a single choice, decided uniquely by the statistics of a scene or subscene, and taking full account of any correlations that exist between different bands.

We should remark here that one well-known and widely used approach to this problem of choice is through the use of principal component images. However, such methods offer a new problem as great as the one that they solve. For although the first three principal components contain in a statistical sense as much information as can be presented using three colors, the resulting scene is completely data dependent. It is thus difficult for an interpreter to apply any previous experience of color-surface relationships to the analysis of a principal components image.

Definition of the method. Consider the 7 x 7 variance-covariance matrix M for the scene or subscene, ignoring for the moment the fact that the thermal band is of inherently lower resolution than the rest. Any triplet of bands will be represented within this 7 x 7 matrix by a 3 x 3 submatrix.

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Considering now the 3-dimensional subspace spanned by any particular band triplet, the associated variance-covariance matrix defines an ellipsoid within the subspace. Further, the sum of the squared principal axes of this ellipsoid represents the total variance accounted for by these three bands (see Figure 1). One could plausibly (but as we shall see, wrongly) argue that the best three bands are those with the largest sum of squared principal axes, and hence accounting for the largest total variance. This is, after all, exactly the argument applied in employing principal component images. Since the trace of a matrix is invariant under rotational transformations, and since the sum of squared principal axes is equal to that trace, the band triplet that accounts for the most possible variance can be found from the original variance-covariance matrix simply by selecting the three bands with the largest diagonal elements. There is no need to examine all 35 band combinations.

To see what is wrong with this approach, consider an extreme case where there happens to be perfect correlation between a pair of bands. For convenience, suppose that those bands are 1 and 2, and suppose that the variance of band 1 (and therefore of 2) is larger than that of any other band. The 7 x 7 matrix M then has the form:

$$\left(\begin{array}{cc|cccc} a & a & \cdots & & & \\ a & a & \cdots & & & \\ \hline \vdots & \vdots & b & & & \\ & & & c & & \\ & & & & \ddots & \\ & & & & & \ddots \end{array}\right)$$

where a>b,c,...

The rotation matrix that will diagonalize the upper left 2 x 2 submatrix then has the form:

$$\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \vert & \\ -1/\sqrt{2} & 1/\sqrt{2} & \vert & 0 \\ \hline & 0 & \vert & I \end{pmatrix}$$

and thus _after_ rotation the upper left 2 x 2 submatrix will have the form:

$$\begin{pmatrix} 2a & 0 \\ 0 & 0 \end{pmatrix}$$

As expected, one eigenvalue is zero; but the other is the sum of the variances from the original bands 1 and 2. Since a is assumed to be large, both bands 1 and 2 will be included in the triplet that accounts for maximum variance -- despite the fact that if either one of them is used, adding the other contributes no new information.

The problem lies in the use of _total variance_ as the measure for the information content of the band triplets. This is equivalent to use of the sum of squares of ellipsoid principal axes, and there is no penalty associated with a very small principal axis provided that it occurs in association with a large axis (see Figures 2 and 3), as was the case for the above example.

We propose the use of a new measure for the information content of the triplet, and one that avoids the undesirable property demonstrated above. We will select the ellipsoid of _maximum volume_. This discourages selection of pairs of bands with high correlation, since in such cases one eigenvalue will be close to zero and the corresponding ellipsoid volume will be small.

Since the ellipsoid volume is simply $4/3\pi abc$, where a, b, and c are the principal axes of the ellipsoid, the volume of the ellipsoid associated with a particular band triplet is a constant multiple of the square root of the product of the eigenvalues for the 3 x 3 variance-covariance matrix of that triplet. However, under rotational transformation the product of the eigenvalues is equal to the determinant of the original 3 x 3 submatrix. Thus we can select the band triplet that provides the ellipsoid of maximum volume simply by computing and ranking in order the determinants of each 3 x 3 principal submatrix of the original matrix M. The band triplets associated with these determinants will then be ranked in order of decreasing overall information content. Given the original matrix M, the total computation to achieve this ranking is trivial. It requires a few hundred multiplications, followed by a sort of a list of 35 items. A BASIC program to perform this is given as an Appendix to this note.

This procedure gives the best triplet, but the assigment of colors is still to be made. Now we can make use of the actual variances (the diagonal elements of M). Since the eye is most sensitive to green, next to red, and least to blue, we will assign green to the band triplet member of maximum variance (i.e. most variation within the image), red to the triplet member of second largest variance, and blue to the triplet member of smallest variance. The definition of bands for production of a color image is now complete.

Examples and comments. The procedure has been applied to a number of scenes of very different ground cover, including Washington D.C., Death Valley, and Cement, Oklahoma. The results for Washington and for Death Valley are given in Tables 1 and 2, together with the associated variance-covariance matrices. The following comments apply to all scenes studied to date.

1) The band combination 1,4,5 (in the order blue, red, green) is usually, but not always, the selected triplet. In cases where it does not rank first,

it ranks second or third.

2) The natural color combination 1,2,3 and the standard false color combination 2,3,4, both place far down in the rankings. In the case of Washington, the natural color combination is 29th (lower than anything except some thermal band combinations, which are low for another reason to be discussed shortly); the 2,3,4 combination was ranked in 16th place. For Death Valley, the 1,2,3 natural color combination ranked 32nd, and the 2,3,4 combination just above it, at 31st. This is presumably a consequence of the very high correlations between the first four bands.

3) Triplets that rank high always include either band 5 or band 6 (note: the bands here are ordered by increasing wavelength, so the thermal band is band 7). This emphasizes the great importance of these new bands on general information-bearing grounds.

4) The triplet selected is not always or even usually the triplet with the greatest individual variances, though large variances are naturally preferred somewhat in the selection process.

Other considerations and comments.

1) The statistical analysis performed here used P tapes (all that we had available) in which the original histograms had already been modified by the gains and offsets. It would be preferable to work with data that have had no gains or offsets applied, i.e., with A tapes prior to any radiometric correction. If band selection of this type becomes common, it would be nice to have A tapes generally available from the EROS Data Center.

2) The thermal band is of lower resolution than the rest, thus it would not be appropriate to give it the same weight in the selection process. How should one therefore de-weight it? One argument runs as follows: The maximum information that a scene can contain is given by the number of pixels, since

In the ultimate case there would be no correlation between pixels, and each would carry independent information about some feature of the surface. In such a case, the amount of information that the thermal band can contribute is only 1/16th that of the other bands, because there are 16 times fewer pixels in that band. Therefore one should deweight the thermal channel by a factor of 16. Such deweighting was performed in the experiments reported here. However, we should also note that this made no difference at all to the preferred band triplets, since even without deweighting we found no case where a triplet involving the thermal channel was in the top five.

3) It is obvious when one looks at images created from the triplet 1,4,5 that for some applications this combination will be much inferior to others, such as natural color and standard false color. This restates the old truth, one man's noise is another man's signal. However, the preferred triplets have another advantage: they provide images of unusual clarity, with far less residual striping than is seen in, for example, the natural color images.

4) Although combinations such as 1,4,5 produce images that are at first sight unfamiliar and unusual, the assigned colors are not scene-dependent. Thus in contrast to the scene dependent colors of principal component or ratio images, the interpreter quickly learns to associate colors with particular ground condition. We therefore believe that there are definite advantages to seeking color composites from the original bands, rather than through band ratios or band combinations.

Figure 1: The variance-covariance ellipsoid, principal axes $\sqrt{\lambda_1}$, $\sqrt{\lambda_2}$, $\sqrt{\lambda_3}$.

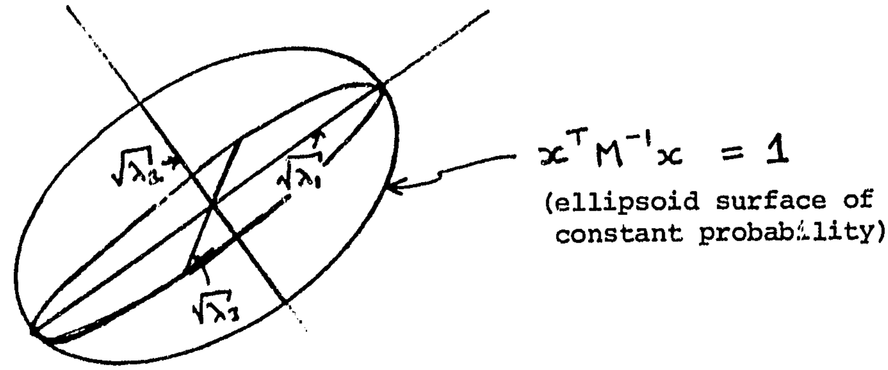
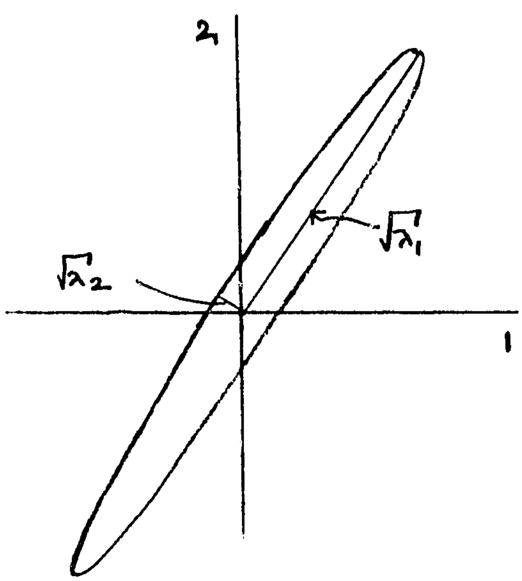


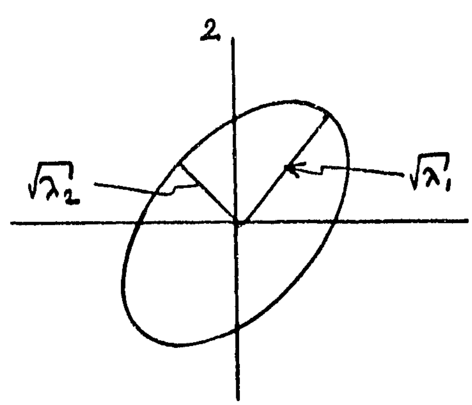
Figure 2. High correlation, bands 1 and 2.



$\lambda_1 + \lambda_2$ large,

$\lambda_1 \cdot \lambda_2$ small

Figure 3. Low correlation, bands 1 and 2, but lower individual variances.



$\lambda_1 + \lambda_2$ smaller than in Fig.2.

$\lambda_1 \cdot \lambda_2$ larger than in Fig. 2.

Based on ellipsoid volumes, Fig. 2 case is preferred over Fig. 1 case, although the former accounts for a greater total variance.

APPENDIX: THE BEST-BAND PROGRAM.


```
20  PRINT "SELECTION OF BEST THREE BANDS BASED ON ELLIPSOID VOLUME"
30  PRINT "DEATH VALLEY WITH REDUCED THERMAL VARIANCE"
40  DIM R(36),Q(36)
50  DIM U(36),V(36)
60  DIM M(8,8)
```

70 REMARK: M is the variance-covariance matrix for the scene or subscene.
80 REMARK: The arrays R,Q,U and V are storage arrays used in the program.
90 REMARK: Note that the program assumes that band 7 is the thermal data, and band 6 is the 2.2 micrometre data.
100 REMARK: The instructions 190 to 230 (except for 220, which sets a count) reduce the variance of the thermal channel to allow for the lower spatial resolution of the thermal channel pixel.

```
190  FOR I = 1 TO 6
200 M(I,7) = M(I,7) / 4
210  NEXT
220 C = 1
230 M(7,7) = M(7,7) / 16
240  PRINT "RANK        DETERMINANT     COMBINATION"
250  FOR I = 1 TO 5
260  FOR J = I + 1 TO 6
270  FOR K = J + 1 TO 7
280 D1 = M(I,I) * (M(J,J) * M(K,K) - M(J,K) ~ 2)
290 D2 = M(I,J) * (M(J,K) * M(I,K) - M(I,J) * M(K,K))
300 D3 = M(I,K) * (M(I,J) * M(J,K) - M(I,K) * M(J,J))
310 DT = D1 + D2 + D3
```

315 REMARK: The next instruction makes the determinant an integer; this is not necessary, it is done for convenience of output only.

```
320 DT =  INT (DT)
330 N = 100 * I + 10 * J + K
340 R(C) = DT:Q(C) = N
350 C = C + 1
360  NEXT
370  NEXT
380  NEXT
```

385 REMARK: The next piece of code sorts the determinant into descending order.

```
390  FOR I = 1 TO 35
400 N = 0
410  FOR J = 1 TO 35
420  IF R(I) | R(J) THEN 440
430 N = N + 1
440  NEXT
450 U(N) = R(I):V(N) = Q(I)
460  NEXT
470  FOR I = 1 TO 35
480  PRINT I,U(I),V(I)
490  NEXT
500  PR# 0
510  END
```

SELECTION OF BEST THREE BANDS BASED ON ELLIPSOID VOLUME
WASHINGTON SCENE WITH REDUCED VARIANCE ON THE THERMAL CHANNEL

VARIANCE-COVARIANCE MATRIX, THERMAL BAND IS BAND 7

53.32	27.41	35.74	5.86	36.04	33.56	7.77
27.41	17.01	21.35	11.36	29.35	21.29	4.13
35.74	21.35	31.66	20.01	46.56	31.03	6.69
5.86	11.36	20.01	131.71	131.64	38.14	8.26
36.04	29.35	46.56	131.64	210.83	86.25	19.1
33.56	21.29	31.03	38.14	86.25	50.01	11.51
7.77	4.13	6.69	8.26	19.1	11.51	9.8

}

Table 1.a Variance-covariance matrix for the Washington D.C. scene.

SELECTION OF BEST THREE BANDS BASED ON ELLIPSOID VOLUME
THIS IS THE WASHINGTON SCENE WITH REDUCED VARIANCE ON THE THERMAL CHANNEL

RANK	DETERMINANT	COMBINATION
1	433858	145
2	205811	345
3	138551	146
4	124784	245
5	101638	456
6	71723	156
7	62960	346
8	49759	135
9	39992	134
10	39609	246
11	36060	356
12	22847	125
13	21953	256
14	16732	124
15	11646	235
16	9709	234
17	7967	136
18	5094	457
19	4752	157
20	3634	126
21	3606	147
22	2294	467
23	2194	357
24	1945	347
25	1616	236
26	1386	567
27	1348	257
28	1130	247
29	727	123
30	688	167
31	276	367
32	215	137
33	175	267
34	84	127
35	43	237

TABLE 1.b Ranked results for Washington D.C. scene.

SELECTION OF BEST THREE BANDS BASED ON ELLIPSOID VOLUME
DEATH VALLEY WITH REDUCED THERMAL VARIANCE

VARIANCE-COVARIANCE MATRIX, THERMAL BAND IS BAND 7

251.64	146.31	198.55	176.41	246.36	144.63	5.3
146.31	90.4	125.4	112.95	178.63	105.16	10.33
198.55	125.4	181.12	163.27	276.44	162.93	22.5
176.41	112.95	163.27	159.7	262.74	152.99	14.79
246.36	178.63	276.44	262.74	627.47	366.9	75.38
144.63	105.16	162.93	152.99	366.9	223.38	48.73
5.3	10.33	22.5	14.79	75.38	48.73	69.89

}

Table 2.a Variance-covariance matrix for the Death Valley scene.

SELECTION OF BEST THREE BANDS BASED ON ELLIPSOID VOLUME
DEATH VALLEY WITH REDUCED THERMAL VARIANCE

RANK	DETERMINANT	COMBINATION
1	1462581	145
2	859695	156
3	684248	135
4	601687	146
5	432952	345
6	346425	157
7	328331	356
8	319827	245
9	275534	456
10	263989	136
11	219239	256
12	204146	125
13	167450	346
14	137060	357
15	127643	246
16	121117	167
17	107494	457
18	103781	235
19	89506	257
20	76827	126
21	75913	134
22	49163	367
23	40621	467
24	39230	236
25	37614	147
26	31621	267
27	21579	137
28	21322	124
29	20256	567
30	9168	347
31	8118	234
32	7895	123
33	7197	247
34	5037	127
35	2407	237

Table 2.b Ranked results for Death Valley scene.