NASA Contractor Report 166824

A PROCEDURE FOR COMBINING ACOUSTICALLY INDUCED AND MECHANICALLY INDUCED LOADS

FIRST PASSAGE FAILURE

DESIGN CRITERIA

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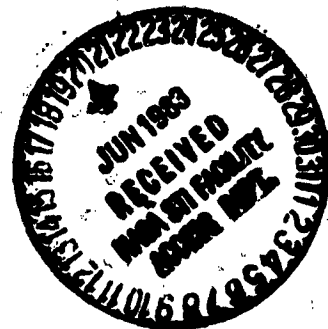Lockheed Way
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National Aeronautics and
Space Administration

Goddard Space Flight Center
Greenbelt, Maryland 20771

1. Report No. NASA CR-166824	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A Procedure For Combining Acoustically Induced And Mechanically Induced Loads (First Passage Failure Design Criterion)		5. Report Date April 1983
		6. Performing Organization Code
7. Author(s) Crowe/Honricks		8. Performing Organization Report No. LMSC/D885405 / SS-1633
9. Performing Organization Name and Address Lockheed Missiles and Space Company 1111 Lockheed Way Sunnyvale, California 94086		10. Work Unit No.
		11. Contract or Grant No.
		13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address Goddard Space Flight Center Greenbelt, Maryland 20771 Tech. Monitor F. On		FINAL
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

An analytical technique is developed for predicting the combined loads
during STS launch due to mechanical inputs and acoustically induced random
vibrations. The procedure assumes that the individual acoustic load and
mechanical load have been previously obtained. The procedure assumes that
the structure is linear and that the acoustic load can be taken as being
Gaussian and stationary. The mechanical load is treated as either a
known deterministic transient or a non-stationary random variable. The
procedure results in a predicted probability distribution function for the
combined load. The effect on the predicted combined loading of flight to
flight variations in the acoustic pressure field is discussed. An analytical
approximation to defining combined design loads for a first passage failure
criterion is presented and experimental verification provided.

17. Key Words (Selected by Author(s)) Space Shuttle Vibroacoustic Data, Combined Mechanical Transient Acoustic Induced Loads, First Passage Failure	18. Distribution Statement		
19. Security Classif. (of this report) UNCLASSIFIED	20. Security Classif. (of this page) UNCLASSIFIED	21. No. of Pages 81	22. Price*

PREFACE

An analytical technique for predicting the design requirements of the combined acoustic and mechanical loads incurred during the Space Shuttle launch phase due to the simultaneous occurrence of low frequency transient inputs and acoustically induced random vibrations has been developed. This document describes the underlying theoretical rationale leading to the proposed design method.

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1.0 INTRODUCTION

During the launch of a Space Shuttle Transportation System (STS) the simultaneous occurrence of mechanically transmitted low frequency transient excitation, generated by the ignition of the solid rocket boosters, and acoustically induced random vibration requires that the design loads specified for STS payload components include the effect of this combined excitation. Documented herein is a study that first develops a statistical description of how the transient-mechanical and acoustically induced loads are expected to combine, and then proceeds to describe and validate a procedure for setting design requirements for loads expected to combine in this manner. A basic assumption in this study is that the individual acoustic load and mechanical load have been previously obtained.

The combined load statistics are developed by taking the acoustically induced load to be a random population, assumed to be stationary. Each element of this ensemble of acoustically induced loads is assumed to have the same power spectral density (PSD), obtained previously from a random response analysis employing the given acoustic field in the STS cargo bay as a stationary random excitation. The mechanically induced load is treated as either (1) a known deterministic transient, or (2) a non-stationary random variable of known first and second statistical moments which vary with time. A method is then shown for determining the probability that the combined load would, at any time, have a value equal to or less than a certain level.

Having obtained a statistical representation of how the acoustic and mechanical loads are expected to combine, an analytical approximation for defining design levels for these loads is presented using the First-Passage failure criterion. Using this criterion the probability of the first occurrence of a certain level (the design value for the combined load) within a prescribed period (the desired service life of the STS payload component) can be determined. Empirical verification of this approach for establishing design loads is then provided.

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While the procedure for establishing design loads is developed assuming a stationary acoustic pressure field, this study also indicates how this procedure can be modified to account for non-stationary acoustic excitation. Finally, a method is presented for establishing design loads that account for random flight-to-flight variation in the acoustically induced load. The salient features of a random process (the acoustically induced load) with which the reader should be familiar and that are employed in the text of this document are presented in Appendix A.

2.0 A PROCEDURE FOR COMBINING ACOUSTICALLY AND MECHANICALLY INDUCED LOADS

A procedure for predicting combined acoustically and mechanically induced loads has been developed. The procedure assumes that the individual acoustic load and mechanical load have been previously obtained. Two cases are considered: (1) the mechanically induced load is taken to be a known deterministic transient; (2) the mechanically induced load is taken to be a non-stationary random variable.

2.1 Deterministic Transient Mechanical Load Plus Stationary Random Acoustic Load

In this case the mechanical load, $X(t)$ due to a staging event, is taken to be a known deterministic transient (e.g. Figure 1). This information is obtained from techniques (computer codes) currently employed to determine the mechanically induced transient load for staging events of expendable boosters, and entails a response analysis of the payload, as represented by a finite element model, to the known deterministic staging excitation. For any time, t_1, as the response due to staging is taken to be deterministic, we have

$$E\left[X(t_1)\right] = X(t_1)$$

$$E\left[X^2(t_1)\right] = X^2(t_1) \tag{1}$$

$$\sigma^2 X(t_1) = E\left[X^2(t_1)\right] - E^2\left[X(t_1)\right] = 0$$

TIME HISTORY – LIFTOFF

X (t)
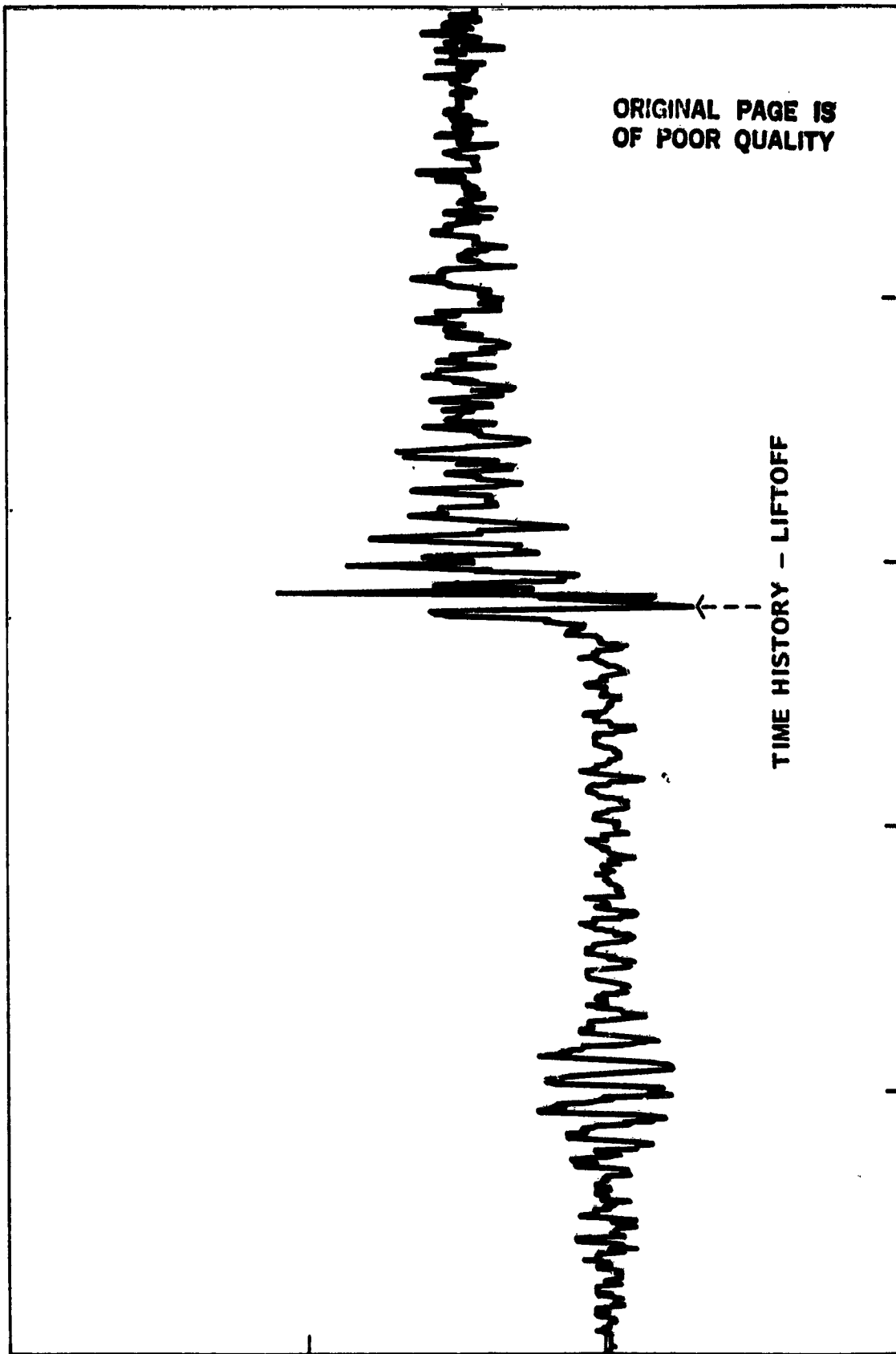
Fig. 1 Deterministic Transient Mechanical Load

where

$E [\ ]$ indicates expected value, and hence

$E \left[X(t_1) \right]$ is the mean value of $X(t_1)$

$E \left[X^2(t_1) \right]$ is the mean square value of $X(t_1)$

$\sigma^2 X(t_1)$ is the variance of $X(t_1)$

As shown in Appendix A, the acoustically induced load, $Y(t)$, is a random population assumed to be stationary and Gaussian with:

$$E \left[Y(t) \right] = 0 \qquad \text{(Equation A14)}$$

$$E \left[Y^2(t) \right] = \int_0^\infty S_y(W) \, dW \qquad \text{(Equation A18)} \qquad (2)$$

$$\sigma_Y^2 = E \left[Y^2(t) \right] - E^2 \left[Y(t) \right] = E \left[Y^2(t) \right] \qquad \text{(Equation A19)}$$

where

$E \left[Y(t) \right]$ is the mean value of $Y(t)$

$E \left[Y^2(t) \right]$ is the mean square of $Y(t)$

σ_Y^2 is the variance of $Y(t)$

$S_y(W)$ is the power spectral density of $Y(t)$ (e.g., Figure 2)

Explicit reference to time for the acoustic load statistical parameters will be eliminated in the sequel, i.e.,

$$E \left[Y(t) \right] = E \left[Y \right]$$

$$E \left[Y^2(t) \right] = E \left[Y^2 \right] \text{, etc.}$$

Assuming a linear system, the combined load, $Z(t)$, is a superposition (sum) of the loads due to the staging transient, $X(t)$, and the acoustics, $Y(t)$.

$$Z(t) = X(t) + Y(t) \qquad (3)$$

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$S_Y(\omega)$

ω
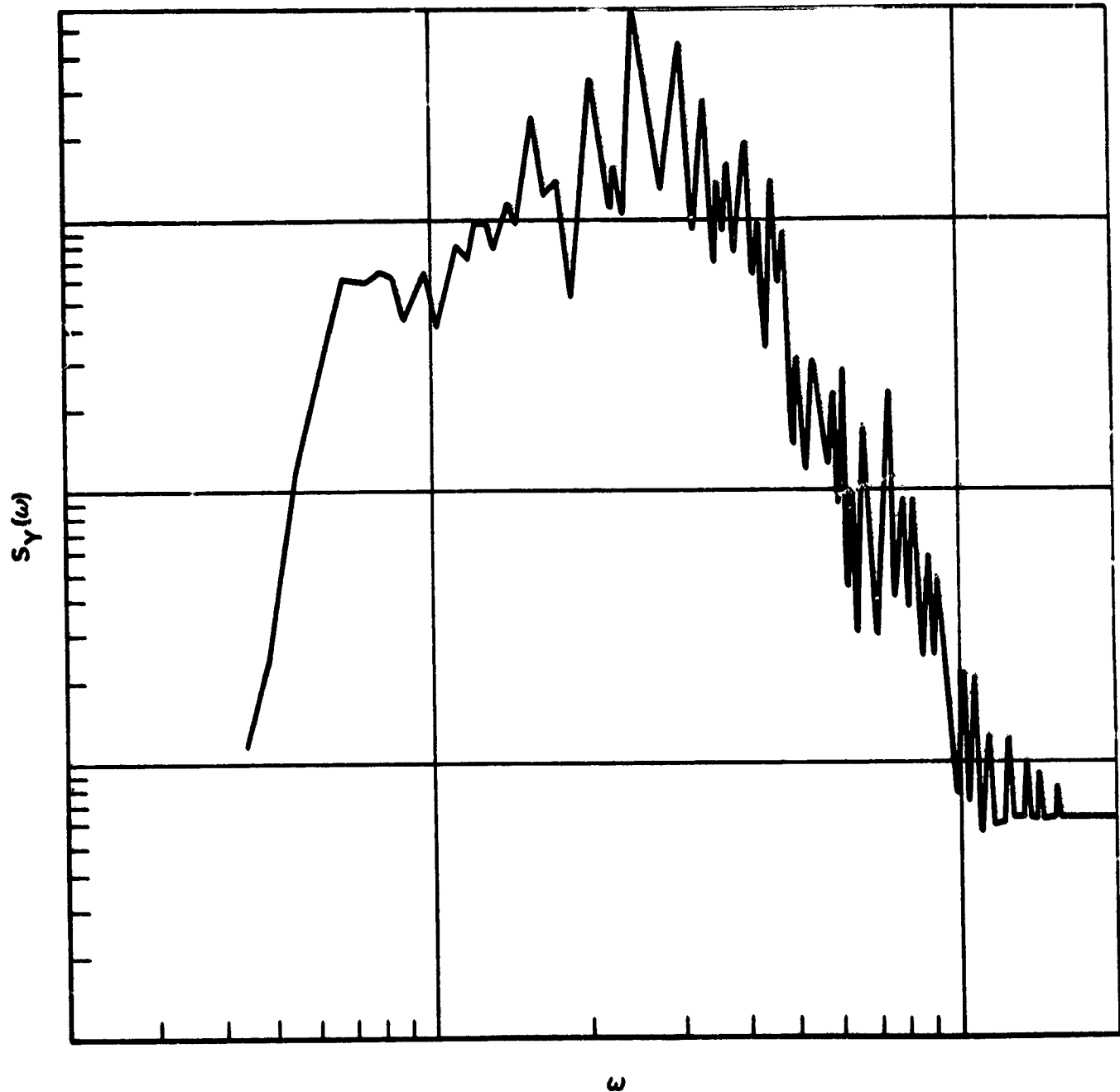
Fig. 2   Power Spectral Density of Stationary Random Acoustic Load

or at any given t , t_1

$$Z(t_1) \quad = \quad X(t_1) + Y(t_1) \tag{4}$$

taking expected values

$$E \quad [Z(t_1)] \quad = \quad E \quad [X(t_1)] \quad \cdot \quad E \quad [Y] \tag{}$$
$$E \quad [Z(t_1)] \quad = \quad X(t_1) \tag{5}$$

That is, the mean value of the total load at any time, t_1, is merely the load at that time due to the transient since the expected value of the load due to acoustics is zero.

Squaring the total load

$$Z^2(t_1) \quad = \quad X^2(t_1) + Y^2(t_1) + 2 X(t_1) \cdots t_1) \tag{6}$$

and taking expected values

$$E \quad \left[Z^2(t_1)\right] \quad = \quad E \left[X^2(t_1)\right] + E \left[Y^2\right] + 2E \quad \left[X(t_1) \; Y(t_1)\right] \tag{7}$$

where

$$E \quad \left[X(t_1) \; Y(t_1)\right] \quad = \quad COV(X(t_1) \; Y(t_1)) + E \left[X(t_1)\right] \; E \left[Y\right] \tag{8}$$

and COV $(X(t_1) \; Y(t_1))$ is the covariance of $X(t_1) \; Y(t_1)$ defined as:

$$COV(X(t_1) \; Y(t_1)) \quad = \quad E \quad \left\{[X(t_1) - E(X(t_1))] \quad [Y(t_1) - E(Y)]\right\} \tag{9}$$

hence $\quad COV(X(t_1) \; Y(t_1)) \quad = \quad 0$

and, from Equation 7 $\quad E \left[Z^2(t_1)\right] \quad = \quad X^2(t_1) + \sigma_Y^2 \tag{10}$

Now, the mean square value is the variance plus the square of the mean

$$\sigma_Z^2(t_1) + E^2 \left[Z(t_1)\right] \quad = \quad X^2(t_1) + \sigma_Y^2 \tag{11}$$

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or
$$\sigma_Z^2(t_1) + X^2(t_1) = X^2(t_1) + \sigma_Y^2 \tag{12}$$

$$\sigma_Z^2(t_1) = \sigma_Y^2 \tag{13}$$

and the standard deviation of the combined load is equal to the standard deviation of the acoustic load.

$$\sigma_{Z(t_1)} = \sigma_Y \tag{14}$$

Finally, as the acoustically induced load is normally distributed, we obtain the Pth percentile of the combined load from

$$P_{Z(t_1)}^{\%} = E\left[Z(t_1)\right] + K^{P\%}\sigma_{Z(t_1)} \tag{15}$$

or,

$$P_{Z(t_1)}^{\%} = X(t_1) + K^{P\%}\sigma_Y \tag{16}$$

where

$X(t_1)$ is the mechanically induced load at time, t_1, due to the staging event. σ_Y is the standard deviation of the stationary acoustically induced load and is obtained as the square root of the area under the acoustic load PSD determined in the random response analysis of the acoustic excitation. $K^{P\%}$ is the appropriate constant relating a multiple of the standard deviation to the Pth percentile for a normal distribution (e.g. for the 95th percentile, $K = 1.645$).

Thus, the predicted percentile combined load at any time, t_1, is obtained by adding to the deterministic load at time, t_1, due to the transient, $X(t_1)$, the appropriate constant multiplied by the standard deviation of the stationary acoustic load, σ_Y.

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As presented above, the Pth percentile level represents the probability
that an ensemble member will, at a given instant, have a value equal to or
less than this level, or equivalently were the phenomenon ergodic (which the
combined load is not), the proportion of time this member spends at or below
this level. It should be noted, however, that this information about an
ensemble member is not that desired for determining combined load design
values. Combined load design values are developed in section 3.0. The
reason for including the present probablistic description for combined loads
is to support the design load rationale given in section 3.0.

2.2 Non-Stationary Random Mechanical Load Plus Stationary Random Acoustic Load

In the previous section we considered the problem of superposing the load due
to acoustic excitation, which was taken to be a stationary random variable,
on the load due to a staging transient which was taken to be deterministic.
In this section we make the extension to the situation where the load due to
the staging transient is not deterministic but is itself a non-stationary
random variable (e.g. Figure 3). This situation arises, for example, when
the predicted transient load comes from an ensemble of transient excitations.

The load, $X(t)$, due to staging is taken to be a known, non-stationary random
variable. For any time, t_1, we know $E[X(t_1)]$ and $\sigma_{X(t_1)}$ by methods
currently employed in predicting the mechanically induced transient load for
staging events of expendable boosters. This entails performing a series of
deterministic response analyses employing, one at a time, the members of the
staging transient excitation ensemble, and subsequently forming the
statistical parameters of the resultant responses.

Note, as the transient load is non-stationary

$$E\left[X(t_1)\right] \neq E\left[X(t_2)\right]$$

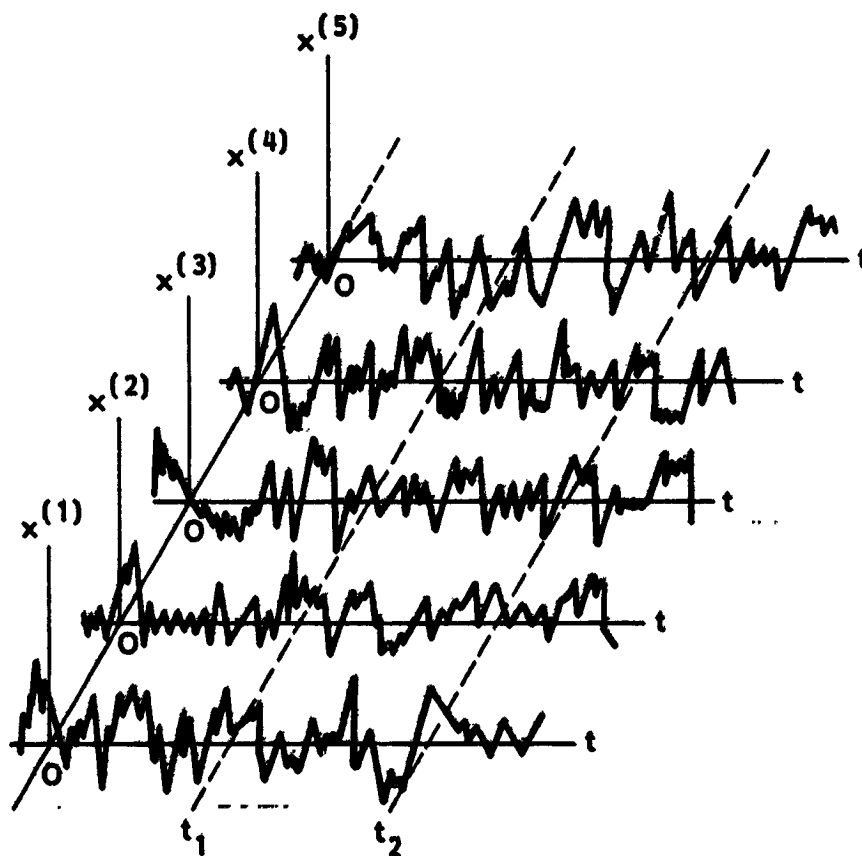$$\sigma_{X(t_1)} \neq \sigma_{X(t_2)}$$

(17)

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Fig. 3   Non-stationary Random Mechanical Load

The results obtained in Appendix A pertaining to the load due to acoustics, $Y(t)$, still hold – namely, for any time.

$$E\left[Y\right] = 0$$

$$E\left[Y^2\right] = \int_0^\infty S_y(W)dW \tag{18}$$

$$\sigma_Y = \left[\int_0^\infty S_y(W)dW\right]^{\frac{1}{2}}$$

As before, the total load, $Z(t)$, is a superposition (linearity assumption) of the loads due to the transient and the acoustics

$$Z(t) = X(t) + Y(t) \tag{19}$$

or, at any given time, t_i

$$Z(t_i) = X(t_i) + Y(t_i) \tag{20}$$

taking expected values

$$E\left[Z(t_i)\right] = E\left[X(t_i)\right] + E\left[\overset{0}{Y}\right]$$

$$\tag{21}$$

$$E\left[Z(t_i)\right] = E\left[X(t_i)\right]$$

That is, the mean value of the total load at any time, t_i, is equal to the mean value of the transient load at time, t_i, since the expected value of the load due to acoustics is zero.

Now, squaring the total load and taking expected values –

$$E\left[Z^2(t_i)\right] = E\left[X^2(t_i)\right] + E\left[Y^2\right] + 2E\left[X(t_i)\ Y(t_i)\right] \tag{22}$$

where

$$E\left[X(t_i)Y(t_i)\right] = COV\left(X(t_i)\ Y(t_i)\right) + E\left[X(t_i)\right]\ E\left[\overset{0}{Y}\right] \tag{23}$$

and the covariance is:

$$COV\left(X(t_i)\ Y(t_i)\right) = E\left\{\left[X(t_i) - E(X(t_i))\right]\left[Y(t_i) - E(Y)\right]\right\} \tag{24}$$

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which can be written in terms of the correlation coefficient, ρ x y as:

$$\text{COV } (X(t_i) \; Y(t_i)) = \rho_{X(t_i)Y(t_i)} \; \sigma_{X(t_i)} \; \sigma_Y \tag{25}$$

where - $1 \leq \rho_{X(t) \; Y(t)} \leq + 1$

hence, equation 22 becomes

$$E\left[Z^2(t_i)\right] = E\left[X^2(t_i)\right] + E\left[Y^2\right] + 2 \rho_{X(t_i)Y(t_i)} \sigma_{X(t_i)} \sigma_Y \tag{26}$$

again, the mean square value is the variance plus the square of the mean

$$\sigma_Z^2 {}_{(t_i)} + E^2\left[Z(t_i)\right] = \sigma_{X(t_i)}^2 + E^2\left[X(t_i)\right] + \sigma_Y^2 + E^2\left[Y\right] \tag{27}$$

$$\underbrace{\qquad\qquad\qquad}_{\text{cancels}}$$

$$+ 2 \rho_{X(t_i)Y(t_i)} \; \sigma_{X(t_i)} \; \sigma_Y$$

or

$$\sigma_{Z(t_i)}^2 = \sigma_{X(t_i)}^2 + \sigma_Y^2 + 2 \rho_{X(t_i) \; Y(t_i)} \; \sigma_{X(t_i)} \; \sigma_Y \tag{28}$$

and the standard deviation of the combined load is:

$$\sigma_{Z(t_i)} = \left[\sigma_{X(t_i)}^2 + \sigma_Y^2 + 2 \rho_{X(t_i) \; Y(t_i)} \; \sigma_{X(t_i)} \; \sigma_Y\right]^{\frac{1}{2}} \tag{29}$$

Finally, as before, assuming the total load is normally distributed, the pth percentile is given as:

$$P_{Z(t_i)}^{\%} = E\left[Z(t_i)\right] + K^{P\%} \sigma_{Z(t_i)} \tag{30}$$

or,

$$P_{Z(t_i)}^{\%} = E\left[X(t_i)\right] + K^{P\%}\left[\sigma_{X(t_i)}^2 + \sigma_Y^2 + 2 \rho_{X(t_i)Y(t_i)} \sigma_{X(t_i)} \sigma_Y\right]^{\frac{1}{2}} \tag{31}$$

To make further progress we must make some assumption regarding the degree of correlation between the random variables X(t) and Y(t). We consider three cases.

1. If the load due to the transient is totally uncorrelated with the load due to acoustics, i.e., $\rho_{X(t)Y(t)} = 0$

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then

$$P^{\%}_{Z(t_1)} = E\left[X(t_1)\right] + K^{P^{\%}}\left[\sigma^2_{X(t_1)} + \sigma^2_Y\right]^{\frac{1}{2}} \tag{32}$$

That is, the standard deviation of the total load is a
Root-Sum-Square (RSS) of the standard deviations of the
transient load and the acoustic load.

2. If the load due to the transient is perfectly correlated,
with positive correlation, with the load due to acoustics,
i.e., $\rho_{X(t)Y(t)} = +1$ (The loads are in-phase and
reinforce each other)

then

$$P^{\%}_{Z(t_1)} = E\left[X(t_1)\right] + K^{P^{\%}}(\sigma_{X(t_1)} + \sigma_Y) \tag{33}$$

That is, the standard deviation of the total load is the
sum of the standard deviations due to the transient and
the acoustic loads.

3. If the load due to the transient is perfectly correlated,
with negative correlation, with the load due to acoustics,
i.e., $\rho_{X(t)Y(t)} = -1$ (The loads are out of phase and
subtract from each other)

then,

$$P^{\%}_{Z(t_1)} = E\left[X(t_1)\right] + K^{P^{\%}}\left|\sigma_{X(t_1)} - \sigma_Y\right| \tag{34}$$

That is, the standard deviation of the total load is the
absolute value of the difference of the standard deviations
due to the transient and the acoustics.

As the acoustically and mechanically induced loads at Shuttle laun'h arise
from fundamentally two different excitation sources and arrive at the payload
by different and circuitous paths, it seems reasonable to take the first case
($\rho XY = 0$) as being representative of the load for that event.

3.0 COMBINED DESIGN LOADS FOR FIRST PASSAGE FAILURE

In the previous sections it has been demonstrated that, under the assumptions
made, the combined load can be represented as a non-stationary Gaussian random
variable with time-varying mean. As given in equation 5 the time-varying mean
of the combined load is merely the time-varying deterministic load due to the
mechanical transient. As given in equation 14 the standard deviation of the
combined load is the standard deviation of the stationary acoustically induced
load. The probability that a member of the combined load ensemble would, at a
given instant, have a value equal to or less than a certain level, was also
determined. It was noted that for an ergodic phenomenon this could be
considered as the proportion of time this member spent at or less than a
certain level. It might be expected that a combined load design level could
be considered adequate if set such that this proportion of time was large.
However, the proportion of time spent at or less than some level is not
particularly relevant since a failure could occur the first time the
magnitude of the combined loads equaled, or slightly exceeded, the design
level. Therefore, for design purposes, one is interested in the first time
that a certain level is reached and what the probability is that this will
occur during the service life of, for example, a payload. This time will be
different for each member of the combined load ensemble. The ensemble of
"first" times has some distribution which must be known to establish the
desired probability of occurrence of a given value for the combined load.
The above considerations fall under the category of "First-Passage" theory.

A classical problem in stochastic theory is to determine the probability $P(T)$
dT that the value of a random process surpasses a threshold for the first time
during the interval from T to T + dT. The resulting First-Passage probability
density $P(T)$ has considerable importance as a reliability measure in random

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vibration studies. However, an exact solution to this problem has not been found for even the simplest version of the problem - which is to consider the stationary response of a simple oscillator excited by white noise. However, a number of papers dealing with approximation methods which provide First-Passage probabilities have been written.

In the study of many random processes the mean is assumed to be zero. If the random process serves as input to a linear system, a non-zero or time-varying mean can be considered separately and handled as a deterministic process. Thus, as seen in the previous sections, if one is only interested in response statistics, such as mean and standard deviation, a time-varying mean presents no problem. However, the First-Passage problem for a system subjected to random excitation is not merely one of finding the response statistics.

In general, the first-passage probability density depends in a complicated manner, on the characteristics of the dynamic system involved, on the nature of the excitation, on the initial conditions imposed, as well as on the magnitude of the threshold.

The first-passage problem of interest at hand is that for a structure subjected to random excitation with time-varying mean. That is, it has been shown that the combined load can be taken as a deterministic process, the mean (i.e. the mechanically induced load), plus an additive random process with zero mean (the acoustically induced load). Having previously obtained the statistics for the combined load, a method for approximating the first-passage probability is now presented.

The time-varying mean is handled as a deterministic process and allows for formulation of the first-passage problem for a fixed barrier from two different outlooks. The first outlook is the one of considering the combined load as a random process with time-varying mean and studying first-passage statistics for a fixed barrier (e.g., Figure 4). The second view is to consider the combined load as a random process with zero mean and to investigate the first-passage problem for a time-varying barrier (e.g. Figure 5). In this case the barrier is made time-varying by subtracting the process mean. The
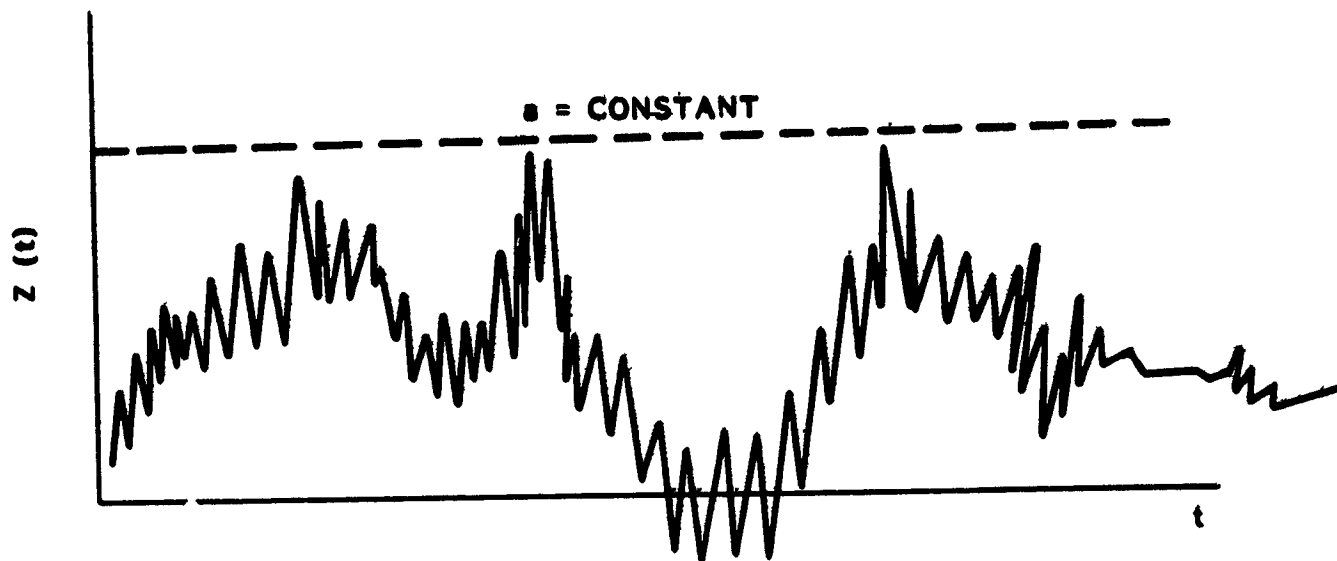
14

a = CONSTANT

z (t)

t

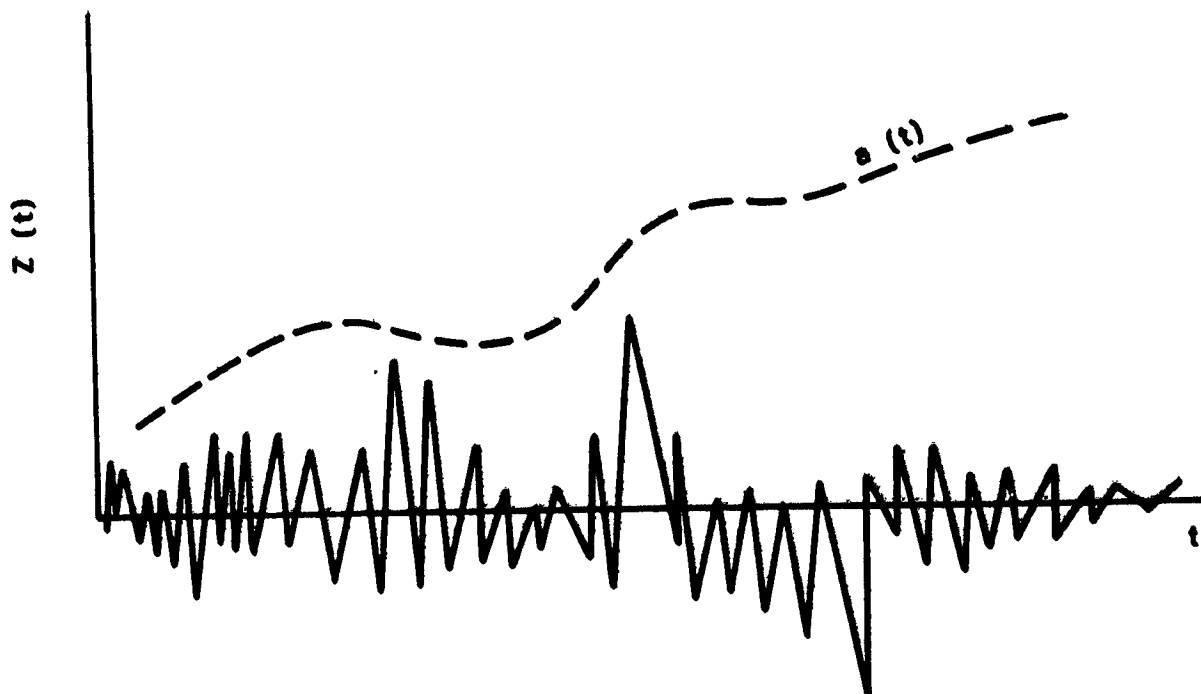Fig. 4   Random Process with Time Varying Mean (Fixed Barrier a)

a (t)

z (t)

t

Fig. 5   Random Process with Zero Mean (Variable Barrier a (t))

two outlooks are in essence equivalent but the latter is adopted here. Thus, the first-passage problem is considered in terms of a time-varying barrier for a zero mean stationary Gaussian random process.

The simplest approximation to the failure rate, called the Poisson approximation, assumes the barrier cro_ ings occur so rarely that they can be considered as statistically independent events. The assumption of independence of crossings has been attacked but it has been pointed out that any tendency of the crossings to cluster in groups or clumps (i.e., not be statistically independent) makes the Poisson assumption conservative, and hence the use of the Poisson assumption in design introduces an error on the conservative side.

The Poisson process is one whose properties have been thoroughly explored. In particular, it is known that for Poisson processes the First-Passage probability density function is an exponential function with a single parameter which is simply related to the expected rate of crossings, ν_a. That is:

$$P(T) = \nu_a \exp \left(- \int_o^T \nu_a dt\right) \tag{35}$$

Thus, on the basis of the assumption that the crossings of the level $Z(t) = A$ constitute a Poisson process we have a "solution" to the distribution to the time of failure. When the distribution density, $P(T)$, for the time to failure is known, the probability of failure in the interval $T_1 < T < T_2$ is:

$$P(T_2, T_1) = \int_{T_1}^{T_2} P(T) \, dT \tag{36}$$

or,

$$P(T_2, T_1) = \int_{T_1}^{T_2} \nu_a \exp \left(- \int_o^T \nu_a dt\right) dT \tag{37}$$

For a positive barrier, $\alpha(t) > 0$, the failure rate is the upcrossing rate, ν_a^+, of the level $\alpha(t)$. For a negative barrier $\alpha(t) < 0$, the failure rate is the downcrossing rate, ν_a^-, of the level $\alpha(t)$. In general the total expected rate of barrier crossings is the sum of the upcrossing rate for a positive barrier and the downcrossing rate for a negative barrier, i.e.:

$$\nu_a = \nu_a^+ + \nu_a^-$$

The barrier crossing rate, ν_a, for a stationary random process for a
__fixed__ barrier was first derived by Rice (Reference 1). This was extended to
the case of __general__ curve crossing of a non-stationary Gaussian process by
Cramer and Leadbetter (Reference 2). The upcrossing rate, $\nu^+(t,\alpha)$, for a
positive barrier level and the downcrossing rate, $\nu^-(t,\alpha)$ for a negative
barrier level are given by (Reference 2, page 288).

$$\nu^+(t,\alpha) = \frac{\dot{\sigma}(1-\mu^2)^{\frac{1}{2}}}{\sigma\sqrt{2\pi}} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right) \left[\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-\eta^2}{2}\right) + \eta \phi(\eta)\right]$$

(38)

$$\nu^-(t,\alpha) = \frac{\dot{\sigma}(1-\mu^2)^{\frac{1}{2}}}{\sigma\sqrt{2\pi}} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right) \left[\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\eta^2}{2}\right) - \eta(1-\phi(\eta))\right]$$

where (for the problem at hand):

$\sigma^2 = E\left[Y^2(t)\right]$;   the variance of the acoustically induced load

$\dot{\sigma}^2 = E\left[\dot{Y}^2(t)\right]$;   the variance of the time rate of change of the
acoustically induced load

$\mu = \dfrac{E\left[Y(t)\,\dot{Y}(t)\right]}{\sigma(t)\,\dot{\sigma}(t)}$; The normalized covariance or correlation
coefficient for the above two parameters

$\alpha(t)$ is the time-varying barrier, $\alpha(t) = A - X(t)$ (i.e. $\alpha(t)$ is
obtained by subtracting the mechanically induced load from the
fixed barrier, A).

Further, we define:

$$\eta = \frac{1}{(1-\mu)^{1/2}}\left(-\frac{\dot{\alpha}}{\dot{\sigma}} + \frac{\mu\alpha}{\sigma}\right)$$

$$\dot{\alpha} = \frac{d}{dt}\left(\alpha(t)\right) = -\dot{X}(t)$$

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and,

$\Phi(\eta)$ designates the normalized Gaussian distribution function:

$$\Phi(\eta) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\eta} e^{-t^2/2} \, dt$$

For the problem at hand, the acoustically induced load is taken as
stationary. It is known (Reference 3, page 33) that for a stationary
process, $Y(t)$ there is no correlation between $Y(t)$ and $\dot{Y}(t)$, i.e.,

$$E\left[Y(t) \quad \dot{Y}(t)\right] = 0$$

and hence,

$$\mu = \frac{E\left[Y(t) \quad \dot{Y}(t)\right]}{\sigma(t) \quad \dot{\sigma}(t)} = 0$$

Further, for a stationary process,

$$\dot{\sigma}^2 = \int W^2 Sy(W) dW$$

Thus, for the general curve crossing of a __stationary__ Gaussian process,
equations 38 become:

$$\nu^+(t,\alpha) = \frac{1}{2\pi} \frac{\dot{\sigma}}{\sigma} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2\dot{\sigma}^2}\right) - \frac{\dot{\alpha}}{\dot{\sigma}} \int_{-\infty}^{-\dot{\alpha}/\dot{\sigma}} e^{-t^2/2} dt\right]$$

$$\nu^-(t,\alpha) = \frac{1}{2\pi} \frac{\dot{\sigma}}{\sigma} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2\dot{\sigma}^2}\right) - \frac{\dot{\alpha}}{\dot{\sigma}} \int_{-\infty}^{-\dot{\alpha}/\dot{\sigma}} e^{-t^2/2} dt + \sqrt{2\pi} \frac{\dot{\alpha}}{\dot{\sigma}}\right] \quad (39)$$

Further, should the stationary Gaussian random process with zero mean also be
__narrow__ __band__ with center frequency W_o then it is known (Reference 3, page
44) that:

$$W_o = \frac{\dot{\sigma}}{\sigma}$$

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Hence, the curve crossing of a stationary, narrow band, Gaussian process is:

$$\nu^+(t,\alpha) = \frac{W_o}{2\pi} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2W_o^2\sigma^2}\right) - \frac{\dot{\alpha}}{W_o\sigma}\int_{-\infty}^{-\dot{\alpha}/W_o\sigma} e^{-t^2/2}\,dt\right]$$

$$\nu^-(t,\alpha) = \frac{W_o}{2\pi} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2W_o^2\sigma^2}\right) - \frac{\dot{\alpha}}{W_o\sigma}\int_{-\infty}^{-\dot{\alpha}/W_o\sigma} e^{-t^2/2}\,dt + \sqrt{2\pi}\frac{\dot{\alpha}}{\sigma W_o}\right] \tag{40}$$

Finally, should the barrier be constant,

$$\alpha(t) = \alpha; \quad \dot{\alpha} = 0$$

Then the barrier crossing of a __fixed__ barrier by a stationary, narrow band, Gaussian process is given by:

$$\nu^+(\alpha) = \nu^-(\alpha) = \frac{W_o}{2\pi} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right) \tag{41}$$

which is the result first given by Rice (Reference 1) or (Reference 3, page 107).

For the First-Passage problem of the combined load, under the assumptions stated, equations 39 are in general applicable. If in addition the acoustically induced load can be taken as narrow band, then equations 40 may be used. In either case, equations 39 or equations 40 are employed to calculate the expected rate of barrier crossing, $\nu_a = \nu_a^+ + \nu_a^-$, which is substituted into equation 37 to evaluate the relationship between the probability of failure $P(T_2, T_1)$ during the interval $T_1 < T < T_2$ and the barrier level A (where A is the combined design load).

To illustrate, consider the somewhat restrictive case in which the acoustically induced load can be taken as a narrow band process of center frequency W_o and standard deviation σ. This, and other, information on the acoustically induced load is obtained from a random response analysis with the

structure represented by a finite element model and employing the acoustic pressure field as a stationary stochastic excitation. Further, the mechanically induced deterministic transient load, $X(t)$, is previously obtained by a response analysis of the payload, as represented by a finite element model, to the known deterministic transient staging excitation. Assume the combined loading condition lasts for 10 seconds (i.e., $T_1 = 0$ and $T_2 = 10$). We seek the barrier level, A, such that the probability of failure, $P(T_o)$, during this interval is one percent (i.e., the probability of success is 99 percent, $P_{success} = 100 - P_{failure}$).

The relevant equations are equations 37 and 40:

$$P(T_2, T_1) = \int_{T_1=0}^{T_2=T_o} \nu_a \exp\left(-\int_0^T \nu_a \, dt\right) \, dT$$

with $\nu_a = \nu_a^+ + \nu_a^-$

$$\nu_a^+ = \frac{W_o}{2\pi} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2W_o^2\sigma^2}\right) - \frac{\dot{\alpha}}{W_o\sigma}\int_{-\infty}^{-\dot{\alpha}/W_o\sigma} e^{-t^2/2} \, dt\right] \; ; \alpha(t) > 0$$

$$\nu_a^- = \frac{W_o}{2\pi} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2W_o^2\sigma^2}\right) - \frac{\dot{\alpha}}{W_o\sigma}\int_{-\infty}^{-\dot{\alpha}/W_o\sigma} e^{-t^2/2} \, dt + \sqrt{2\pi} \frac{\dot{\alpha}}{\sigma W_o}\right] ; \alpha(t) < 0$$

where $T_o = 10$, $P(T_o) = 0.01$

$$\alpha(t) = A - X(t), \quad \dot{\alpha}(t) = -\dot{X}(t)$$

and W_o, $X(t)$, $\dot{X}(t)$, σ are known a priori and we solve for A (the combined design load).

An interpretation of the above is that out of 100 flights, on the average, only one will exceed the design level A when exposed to the combined load of ten-second duration.

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Note that intuition is served in that the above equations for predicting first-passage failure are functions of amplitude, duration, and frequency content of the combined loading condition. The longer the duration of the combined loading condition the greater the probability that a prescribed barrier level, A, will be exceeded. The greater the frequency content of the combined loading condition, the greater the number of excursions per unit time and hence the greater the probability that a prescribed barrier level will be exceeded.

Obviously, the solution of these equations must be carried out numerically employing a digital computer. The problem with any type of crossing rate approach for a rapidly moving barrier is that the statistics must be computed at frequent intervals. This can become quite expensive, especially when the frequency is high.

Extension of this technique for First-Passage failure to the case where the mechanical load is taken as a non-stationary random variable is made by taking the combined load mean as given in equation 21 and the combined load standard deviation as given in equation 29 and employing equations 38 for the crossing rates.

4.0 EXPERIMENTAL INVESTIGATION

To evaluate the proposed method of combining low frequency transient loading with acoustically induced loads, experimental methods were employed whereby a test specimen was separately and simultaneously exposed to acoustic excitation and portable shaker force transients.

The test specimen (Figure 6) was a 14-foot diameter right-circular cylinder, 10 feet long, with double wall wooden end caps. The cylinder was of .125 inch aluminum skin with circumferential and longitudinal stiffeners, and was tested with a simulated payload component that weighed 45 lbs and had an installation first mode at 27 Hz (Figure 7). The specimen was suspended by steel cables from the work platform in the large reverberant chamber of the LMSC acoustic test facility. A shaker was suspended using steel cables and the stinger attached to the mass simulator, as shown in Figures 6 and 7.

The test instrumentation was comprised of: (1) four cell microphones to monitor the acoustic field present in the reverberant chamber; (2) twelve Endevco 2220 accelerometers to monitor the mass simulator responses located as shown in Figure 8 and 9; and (3) four uniaxial strain gages (two pair of back-to-back gages) as shown in Figures 8 and 9. In addition, a load cell (Figure 7) was employed to monitor the shaker input force to the specimen.

It should be noted that the techniques discussed herein are methods for combining signals and subsequently setting "design levels" for those signals under a first-passage failure criterion, regardless of the physical significance of the signal. Thus, while the variable of immediate interest is design load, or stress, for the purpose of empirically verifying the proposed techniques more readily measureable variables such as acceleration or strain will serve.

Fig. 6 Test Configuration Overview
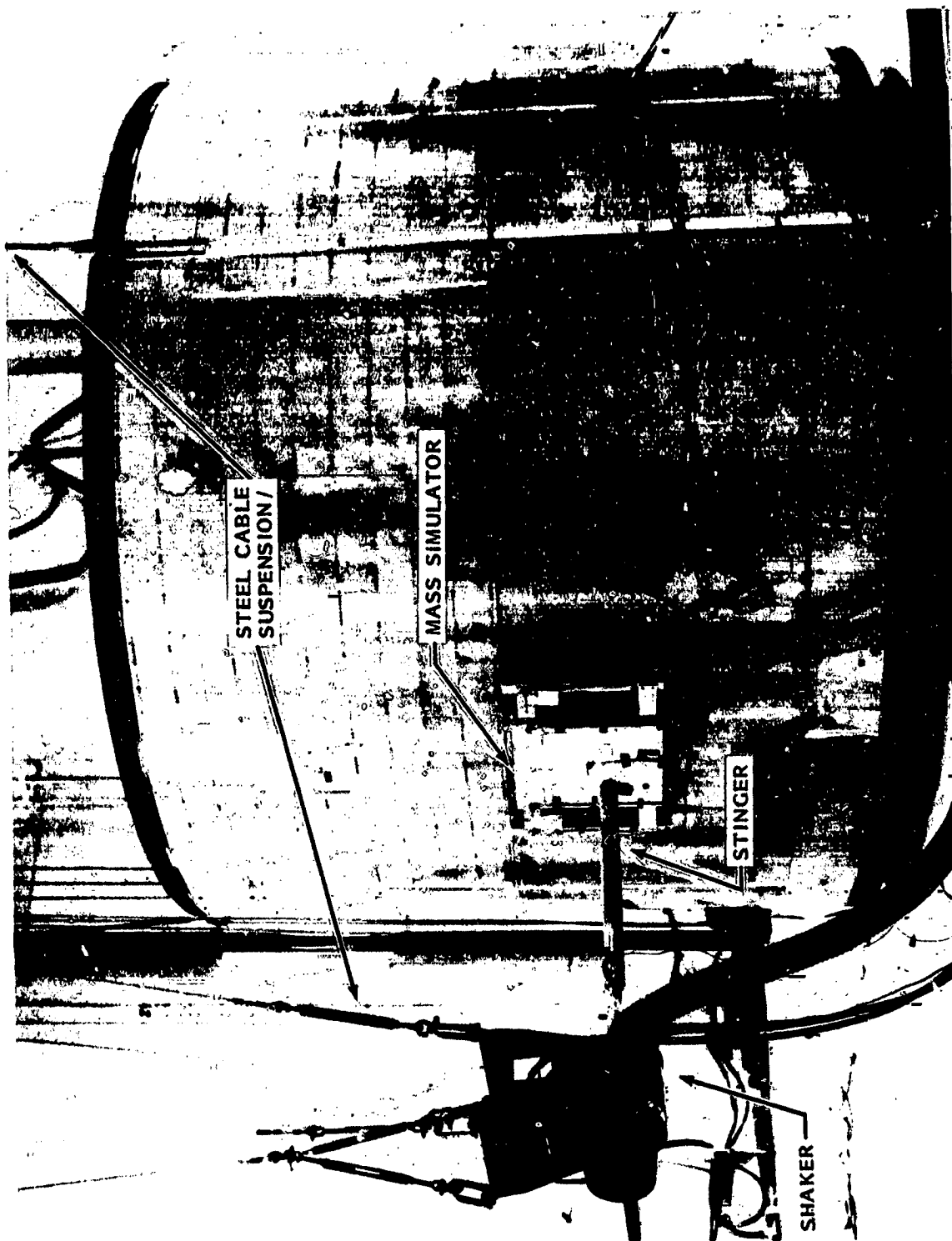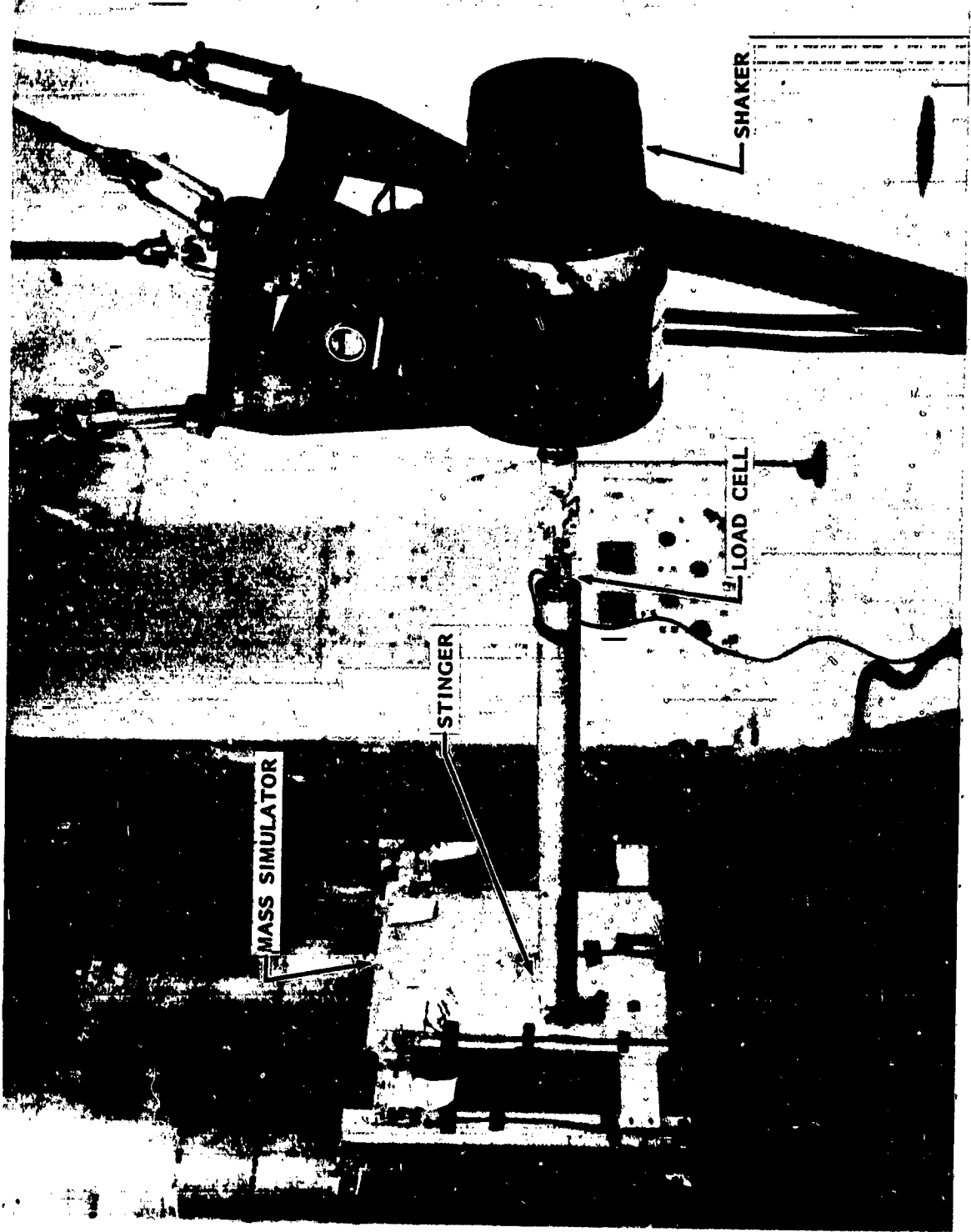
STEEL CABLE
SUSPENSION/

MASS SIMULATOR

STINGER

SHAKER

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Fig. 7 Test Configuration – Close-up

(6431a 12/07/82)

Fig. 8 Simulator Instrumentation Location

(6431a 12/07/82)
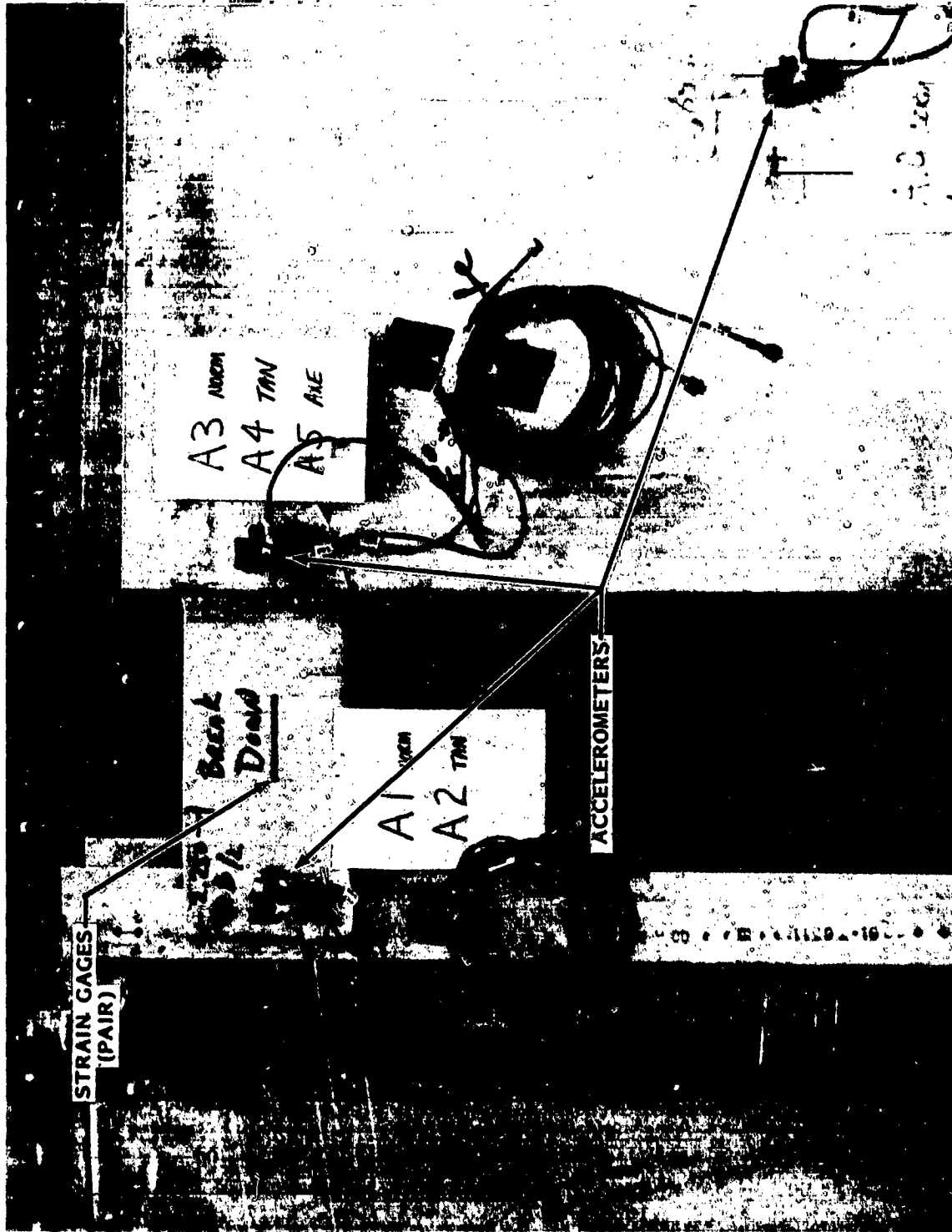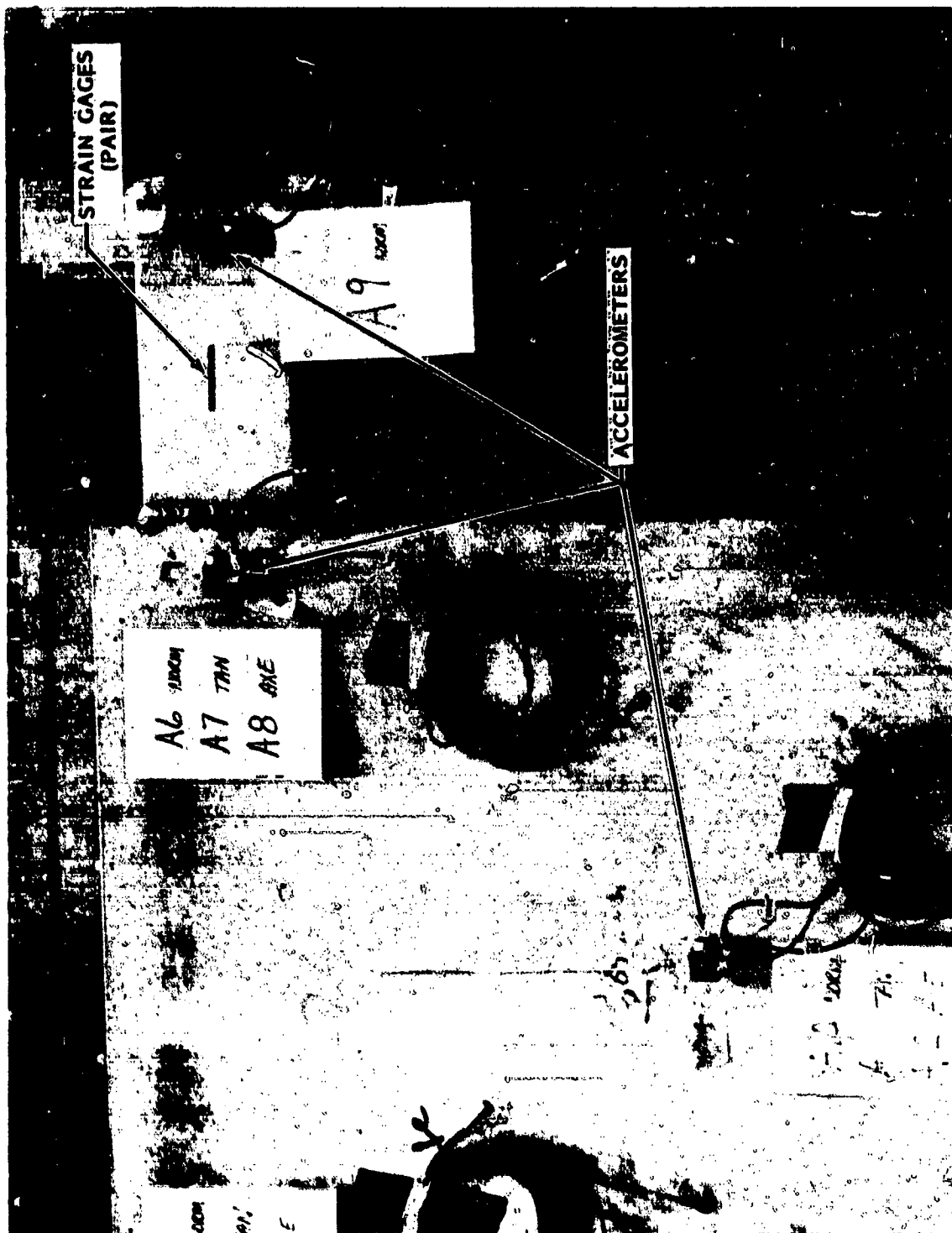
Fig. 9 Simulator Instrumentation Location

(6431a 12/07/82)

As the first-passage failure level is a function of the amplitude, duration and frequency content of the combined response, two acoustic spectra and three shaker transients were employed in the testing. The two acoustic spectra, differing in low frequency energy content, are shown in Table I. Based on the results of a shaker sine-sweep test, the shaker transients were selected that would engender strong mass simulator responses of magnitudes comparable to the responses generated by the acoustic spectra. The transient excitation applied by the shaker was controlled "open-loop" using a magnetic tape loop containing the desired transient. One tape loop was made containing the first transient which was repeated 100 times at 10-second intervals. The mathematically constructed transient was of 4-second duration with a sine envelope and frequency content near a resonance (27 Hz) of the mass simulator to ensure sufficient response amplitudes. Two additional tape loops were made containing the second and third transients, each also repeated 100 times. The second transient, repeated at 10-second intervals was of 4-second duration with a sine envelope and frequency of 50 Hz. The third transient, repeated at 2.5-second intervals, was of 1-second duration with a sine envelope and frequency at 270 Hz.

To confirm repeatability of the shaker transient excitation and to obtain the response, $X(t)$, due to the mechanical transient, each transient in turn was applied 100 times and the response of the twelve accelerometers and four strain gages recorded. To obtain the response $Y(t)$, due to the acoustic fields, each spectrum (Table I) was applied to the specimen in the reverberant chamber for a duration of three minutes and the accelerometer and strain gage data recorded. Finally, to obtain combined responses, $Z(t)$, three combinations of acoustic field and shaker transient, as listed below, were tested and the accelerometer and strain gage data recorded:

1. spectrum A plus transient 1 applied 100 times for a duration of 1000 seconds
2. spectrum B plus transient 2 applied 100 times for a duration of 1000 seconds
3. spectrum B plus transient 3 applied 100 times for a duration of 250 seconds

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TABLE I

ACOUSTIC TEST SPECTRA

1/3 OCTAVE	SPL	
CENTER FREQUENCY	A	B
25	128.3	-
31.5	134.0	-
40	133.9	-
50	134.5	134.5
63	133.7	133.7
80	135.6	135.6
100	134.9	134.9
125	135.6	135.6
160	135.4	135.4
200	133.7	133.7
250	132.2	132.2
315	132.0	132.0
400	131.2	131.2
500	130.7	130.7
630	130.7	130.7
800	130.0	130.0
1000	127.9	127.9
1250	128.2	128.2
1600	127.3	127.3
2000	124.6	124.6
O.A.	145.7	145.0

Preliminary data processing was completed for one accelerometer and one strain
gage for the test sequence. The accelerometer selected was accelerometer #6,
as it was a normal accelerometer located in the corner of the mass simulator
(see Figure 9), and hence had a strong response signal for both simulator
translational modes (e.g. 27 Hz) and rotational modes (e.g., 50 Hz). Due to
geometric symmetry, the strain gage selection was arbitrary and gage #2 was
chosen.

For the transient-only tests employing transient #1 (27 Hz) and transient #2
(50 Hz) both of which were of 4-second pulse duration, the response data X(t),
were digitized at a rate of 2,000 samples per second, or 8,000 samples per
pulse. For the transient only test employing transient #3 (270 Hz) which was
of 1-second pulse duration, the response data were digitized at 4,000 samples
per second (pulse). The time derivative of these response data were then
calculated using a simple "delta-delta" formulation, i.e.,

$$\dot{X} = \Delta X / \Delta t$$

For the short duration high frequency third transient, the strain gage data
were low in magnitude and of questionable quality. Hence, for the high
frequency (270 Hz) transient, only the accelerometer data was processed.

For the acoustic only tests, the power spectral densities of the responses,
$S_y(W)$, were determined. From these, the standard deviations of the
acoustically induced response, σ_y, and the time rate of change of the
acoustically induced response $\sigma_{\dot{y}}$, were calculated

$$\sigma_y = \left[\int S_y(W) dW \right]^{\frac{1}{2}}$$

$$\sigma_{\dot{y}} = \left[\int W^2 S_y(W) dW \right]^{\frac{1}{2}}$$

For the combined acoustic/transient tests employing transient #1 and transient
#2, response data, Z(t), were digitized at 2,000 samples per second (8,000
samples per pulse). For the combined acoustic/transient test employing trans-
ient #3, the accelerometer response data were digitized at 4,000 samples per
second (pulse).

For the acoustic-only tests and the combined acoustic/transient tests, cell microphone one-third octave SPLs were generated to ensure repeatability of the acoustic fields.

4.1 Comparison of Theory and Experiment

Comparison of the first-passage level theoretical prediction and experimental data for the combined acoustic/transient loading conditions are presented in Figures 10-14 and Tables II - VI as described below. Employing a digital computer, the theoretical prediction computations were performed following the general steps 2 through 5 of Table VII. The empirical curves were obtained by merely observing the percentage (number) of a response ensemble that did not exceed (even one time) each prescribed barrier level.

Figure	Table	Instrument	Condition
10	II	Accel #6	Spectrum A + Transient 1
11	III	Accel #6	Spectrum B + Transient 2
12	IV	Accel #6	Spectrum B + Transient 3
13	V	Gage #2	Spectrum A + Transient 1
14	VI	Gage #2	Spectrum B + Transient 2

The theoretical predictions are made employing equations 37 and 39:

$$P(T) = \int_0^T \nu_a \exp\left(-\int_0^t \nu_a \, d\tau\right) dt \tag{37}$$

where, $\nu_a = \nu_a^+ + \nu_a^-$

$$\nu_a^+ = \frac{1}{2\pi} \frac{\dot{\sigma}}{\sigma} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2\dot{\sigma}^2}\right) - \frac{\dot{\alpha}}{\dot{\sigma}} \int_{-\infty}^{-\dot{\alpha}/\dot{\sigma}} e^{-t^2/2} \, dt\right]$$

$$\tag{39}$$

$$\nu_a^{-1} = \frac{1}{2\pi} \frac{\dot{\sigma}}{\sigma} \exp\left(\frac{-\alpha^2}{2\sigma^2}\right)\left[\exp\left(\frac{-\dot{\alpha}^2}{2\dot{\sigma}^2}\right) - \frac{\dot{\alpha}}{\dot{\sigma}} \int_{-\infty}^{-\dot{\alpha}/\dot{\sigma}} e^{-t^2/2} \, dt + \sqrt{2\pi} \frac{\dot{\alpha}}{\dot{\sigma}}\right]$$

and

σ^2 is the variance of the acoustically induced response

$\dot{\sigma}^2$ is the variance of the time rate of change of the acoustically induced response

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α is the time-varying barrier, $\alpha(t) = A - X(t)$ (i.e.,
           $\alpha(t)$ is obtained by subtracting the mechanically
           induced response from the fixed barrier)

$\dot{\alpha}$ $=$ $d/dt\,(\alpha(t)) = -\dot{X}$

T $=$ 4 or 1 (i.e. the combined load is applied for either
           4 seconds or 1 second)

From the transient-only runs are obtained the transient response, $X(t)$, and
the time rate of change of the transient response, $\dot{X}(t)$. From the acoustic-
only runs are obtained σ, the standard deviation of the acoustically induced
response and $\dot{\sigma}$, the standard deviation of the time rate of change of the
acoustically induced response. These two cases permit the predicted first-
passage probability of failure to be made for various levels, A, via equations
37 and 39. The probability of not crossing a given level is the compliment
of the probability of failure given by equation 37, i.e.,

$$P_{SUCCESS} = 100 - P_{FAILURE}$$

To the extent that the proposed approach is valid, the combined responses
will follow the first-passage probability given by equations 37 and 39.

As seen in Figures 10-14 and Tables II-VI, the agreement between the first-
passage failure theoretical prediction and the empirical data is quite good.
With the exception of the strain gage data for the spectrum B/transient 2
combination, the predictions are slightly conservative showing typical posi-
tive margins of 1.0 to 1.5 dB. This result is expected and is attributed to
the previously stated conservative Poisson assumption that the barrier cross-
ings are statistically independent events. The strain gage data for the
spectrum B/transient 2 combination (Figure 14 and Table VI) shows excellent
agreement between theoretical prediction and test data but without margin,
the difference between prediction and test being at most 0.2 dB. Why no mar-
gin is perceived for this case has not been explained at this time, other
than to point out that the disparities between prediction and empirical data
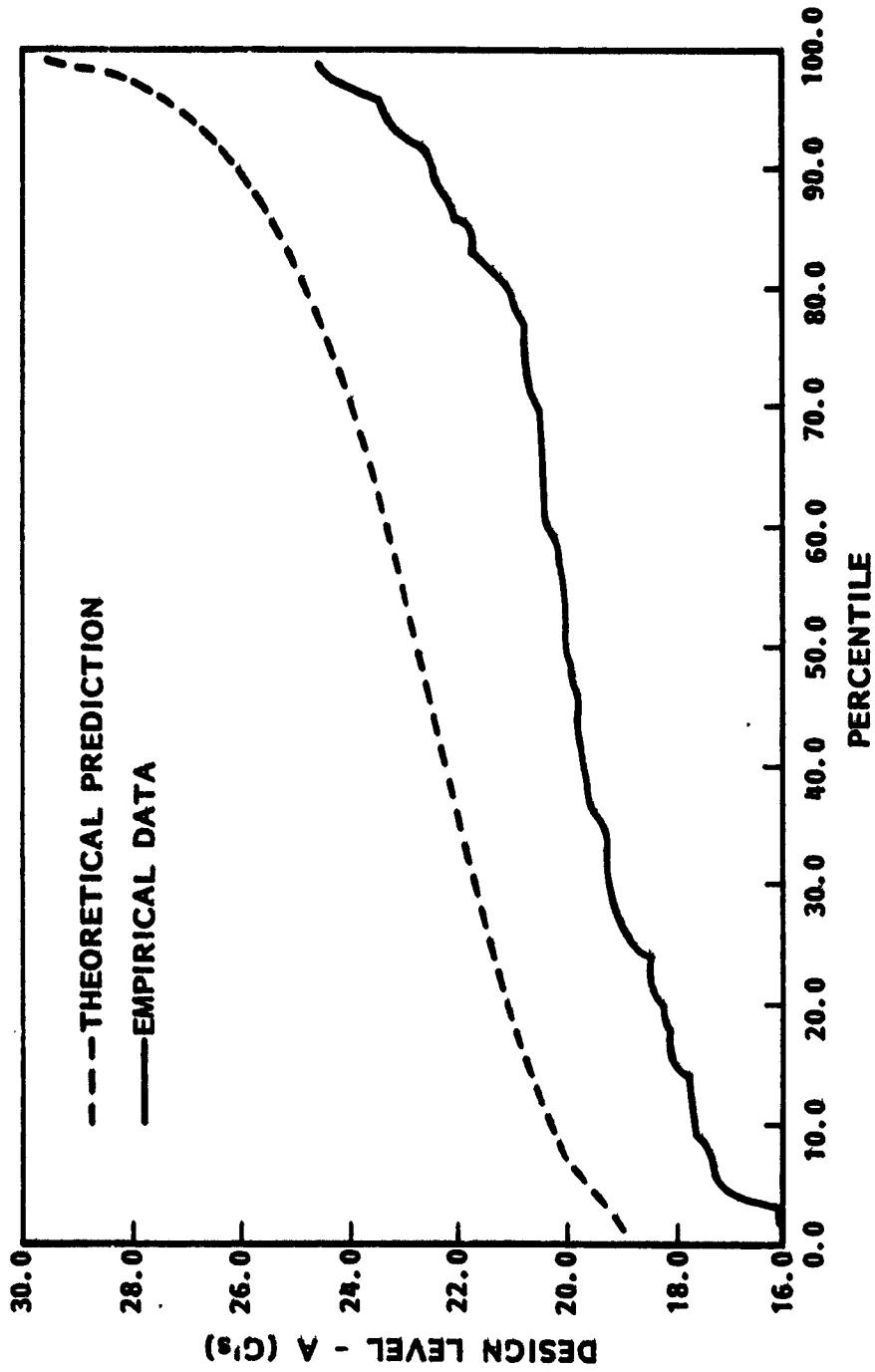are well within the historical uncertainties of typical vibro-acoustic data.

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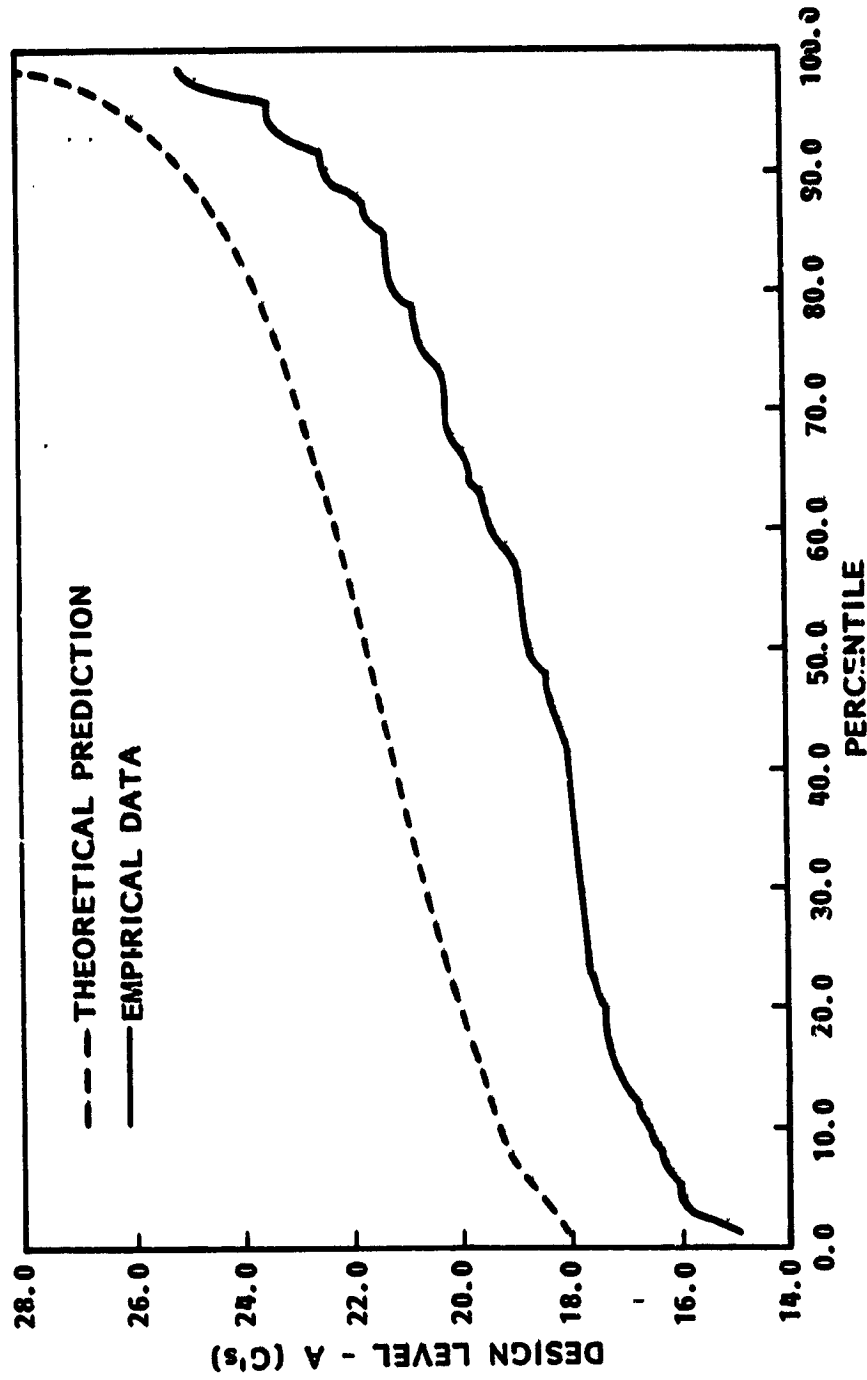Fig. 10 First Passage Level Accelerometer 6 -- Spectrum A/Transient 1

Fig. 11 First Passage Level Accelerometer 6 – Spectrum B/Transient 2
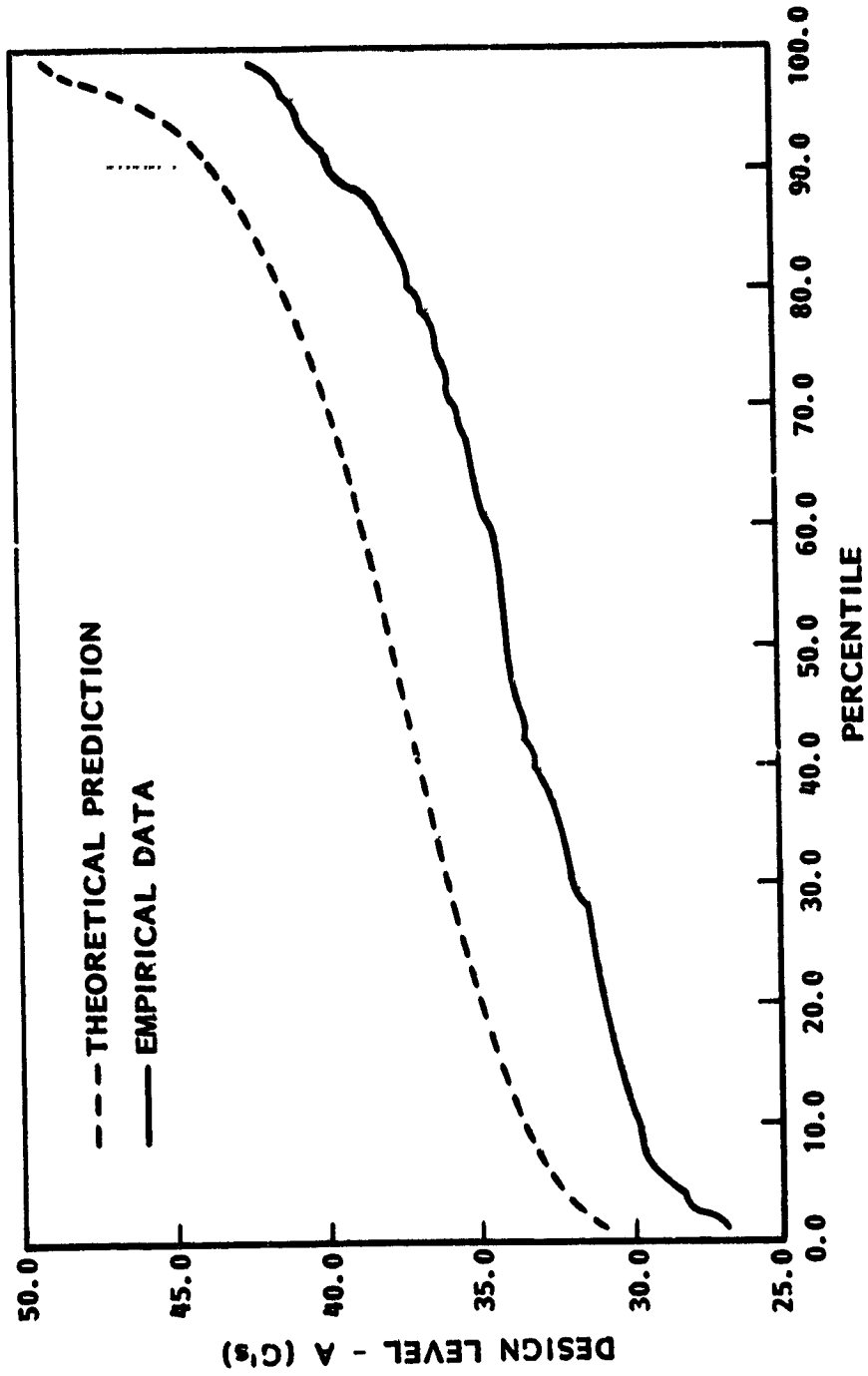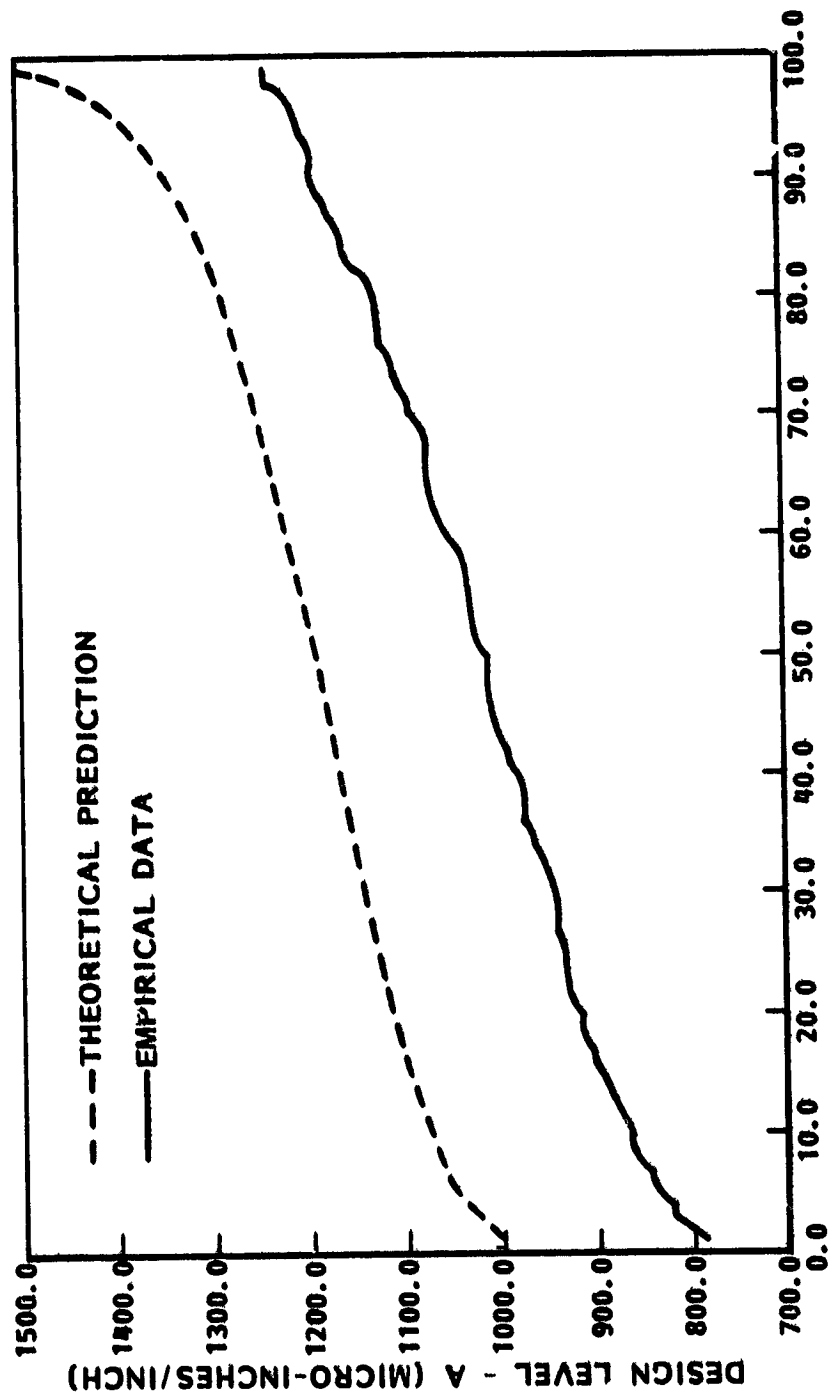
Fig. 12 First Passage Level Accelerometer – Spectrum B/Transient 3
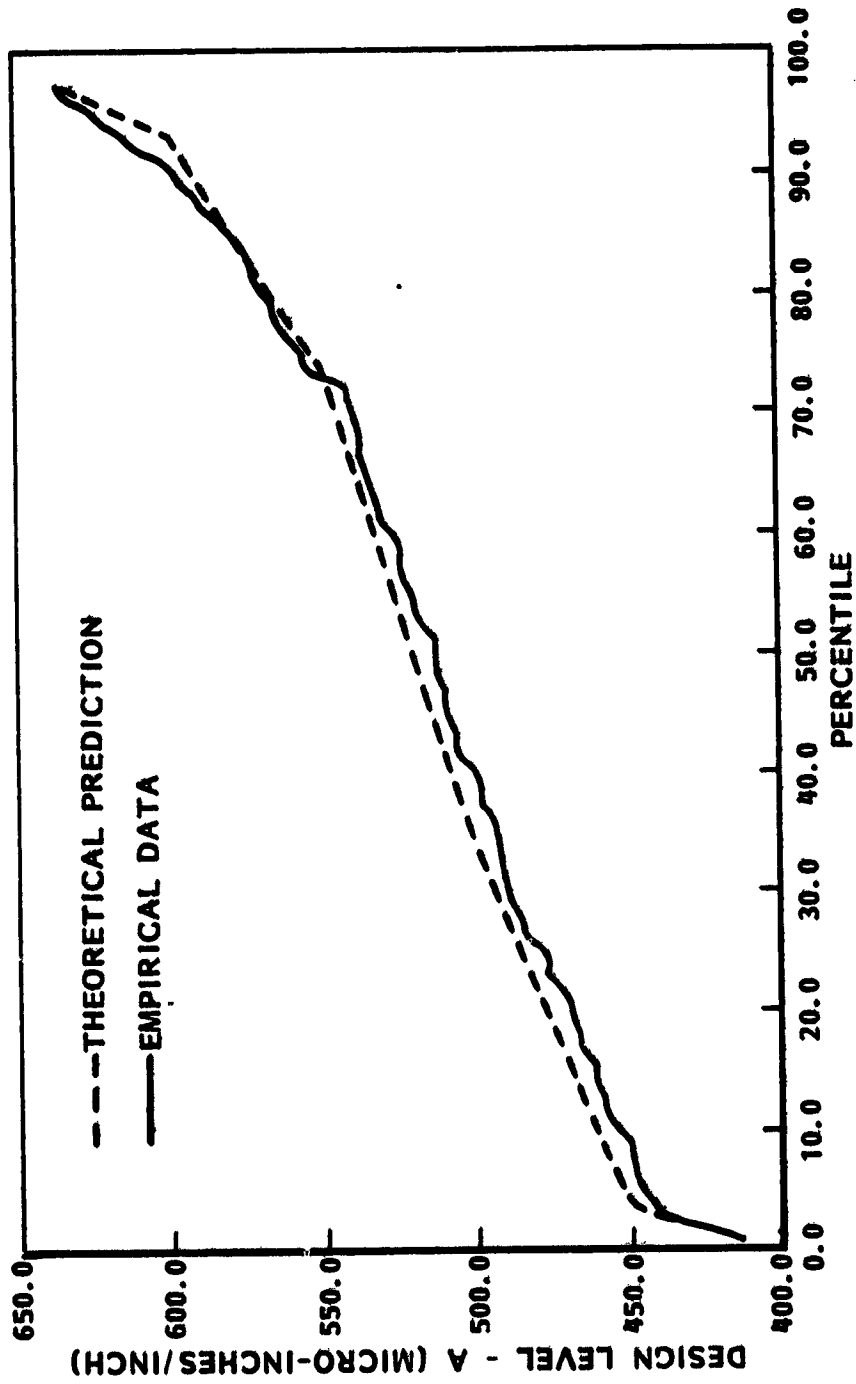
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Fig. 13 First Passage Level Strain Gage 2 – Spectrum A/Transient 1

Fig. 14 First Passage Level Strain Gage 2 - Spectrum B/Transient 2

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TABLE II

FIRST-PASSAGE LEVEL COMPARISON

Accelerometer #6

Spectrum A + Transient 1

Probability of not Exceeding (%)	Theoretical Prediction (g's)	Empirical Data (g's)	Margin Ratio (dB)
10	20.3	17.7	1.15 (1.2)
20	21.1	18.3	1.15 (1.2)
30	21.6	19.2	1.13 (1.0)
40	22.2	19.7	1.13 (1.0)
50	22.7	20.0	1.14 (1.1)
60	23.3	20.3	1.15 (1.2)
70	23.9	20.5	1.17 (1.3)
80	24.8	21.0	1.18 (1.4)
90	26.0	22.4	1.16 (1.3)
92	26.4	22.7	1.16 (1.3)
94	26.9	23.3	1.15 (1.2)
95	27.1	23.3	1.16 (1.3)
97	27.9	24.1	1.16 (1.3)
99	29.6	24.6	1.20 (1.6)

TABLE III

FIRST-PASSAGE LEVEL COMPARISON

Accelerometer #6

Spectrum B + Transient 2

Probability of not Exceeding (%)	Theoretical Prediction (g's)	Empirical Data (g's)	Margin Ratio (dB)
10	19.2	16.5	1.16 (1.3)
20	20.0	17.3	1.16 (1.3)
30	20.5	17.6	1.16 (1.3)
40	21.0	17.9	1.17 (1.4)
50	21.5	18.5	1.16 (1.3)
60	22.1	19.1	1.16 (1.3)
70	22.7	19.9	1.14 (1.1)
80	23.5	20.8	1.13 (1.1)
90	24.7	22.2	1.11 (0.9)
92	25.0	22.2	1.13 (1.0)
94	25.5	23.1	1.10 (0.9)
95	25.7	23.1	1.11 (0.9)
97	26.5	24.2	1.10 (0.8)
99	27.9	24.7	1.13 (1.1)

TABLE IV

FIRST-PASSAGE LEVEL COMPARISON

Accelerometer #6

Spectrum B + Transient 3

Probability of not Exceeding (%)	Theoretical Prediction (g's)	Empirical Data (g's)	Margin Ratio (dB)
10	33.6	29.6	1.13 (1.0)
20	34.9	30.8	1.13 (1.1)
30	35.9	31.0	1.16 (1.3)
40	36.9	33.1	1.11 (0.9)
50	37.8	33.9	1.12 (0.9)
60	38.7	34.3	1.13 (1.0)
70	39.8	35.5	1.12 (1.0)
80	41.2	37.0	1.11 (0.9)
90	43.3	39.5	1.10 (0.8)
92	44.0	40.0	1.10 (0.8)
94	44.7	40.6	1.10 (0.8)
95	45.3	40.6	1.12 (1.0)
97	47.5	41.1	1.16 (1.3)
99	49.0	42.2	1.16 (1.3)

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TABLE V

FIRST-PASSAGE LEVEL COMPARISON

Strain Gage #2

Spectrum A + Transient 1

Probability of not Exceeding (%)	Theoretical Prediction 10^{-6} inches/inch	Empirical Data 10^{-6} inches/inch	Margin Ratio (dB)
10	1071	865	1.24 (1.9)
20	1111	916	1.21 (1.7)
30	1140	946	1.21 (1.6)
40	1166	979	1.19 (1.5)
50	1191	1011	1.18 (1.4)
60	1219	1053	1.16 (1.3)
70	1248	1090	1.14 (1.2)
80	1287	1128	1.14 (1.1)
90	1343	1192	1.13 (1.0)
92	1361	1193	1.14 (1.1)
94	1383	1203	1.15 (1.2)
95	1394	1208	1.15 (1.2)
97	1431	1221	1.17 (1.4)
99	1498	1240	1.21 (1.6)

TABLE VI

FIRST PASSAGE LEVEL COMPARISON

Strain Gage #2

Spectrum B + Transient 2

Probability of Not Exceeding (%)	Theoretical Prediction 10^{-6} inches/inch	Empirical Data 10^{-6} inches/inch	Margin Ratio (dB)
10	461	454	1.02 (0.13)
20	478	469	1.02 (0.17)
30	495	490	1.01 (0.09)
40	509	500	1.02 (0.15)
50	521	513	1.02 (0.13)
60	533	528	1.01 (0.08)
70	546	540	1.01 (0.10)
80	567	569	1.00 (-0.03)
90	592	598	0.99 (-0.09)
92	597	611	0.98 (-0.20)
94	609	623	0.98 (-0.20)
95	618	624	0.99 (-0.08)
97	636	637	1.00 (-0.01)
99	645	648	1.00 (-0.04)

Having achieved rather good agreement between the theoretical first-passage prediction and the empirical data, it is not surprising that the assumptions upon which that prediction is based—were reasonably valid for the test configuration. A brief discussion of the empirical verification of the underlying assumptions is found in Appendix B.

5.0 AN APPROXIMATION TO A NON-STATIONARY ACOUSTIC PRESSURE FIELD

The previously developed analytical technique for predicting the combined loads during STS launch due to transient mechanical inputs and acoustically induced random vibrations is based upon three assumptions: (1) that the system (structure) is linear thereby permitting superposition of the two loads; (2) that the acoustic load is Gaussian; and (3) that the acoustic load is stationary. During STS launch the third assumption of the acoustic load being stationary is not physically justifiable, but as the true evolutionary nature of the acoustic pressure may not be known, and in fact the acoustic forcing function often is taken as a stationary envelope of the evolutionary acoustic SPL, the assumption of the acoustic load being stationary is made as a pragmatic analytical expedient.

Should the evolutionary nature of the acoustic pressure be known, this truly non-stationary forcing function could be approximated as being quasi-stationary in temporally contiguous intervals. The previously developed analytical techniques of combining transient mechanical inputs with acoustically induced random vibrations and subsequently predicting the probability of first-passage barrier crossing of a prescribed level, A, could then be affected for each individual interval. The probability of first-passage barrier crossing of a level, A, for the entire STS launch event is then obtained as the sum of the probabilities of first-passage barrier crossing of that level, A, in each of the sub-intervals comprising the launch event. This quasi-stationary approximation is illustrated in Figure 15.
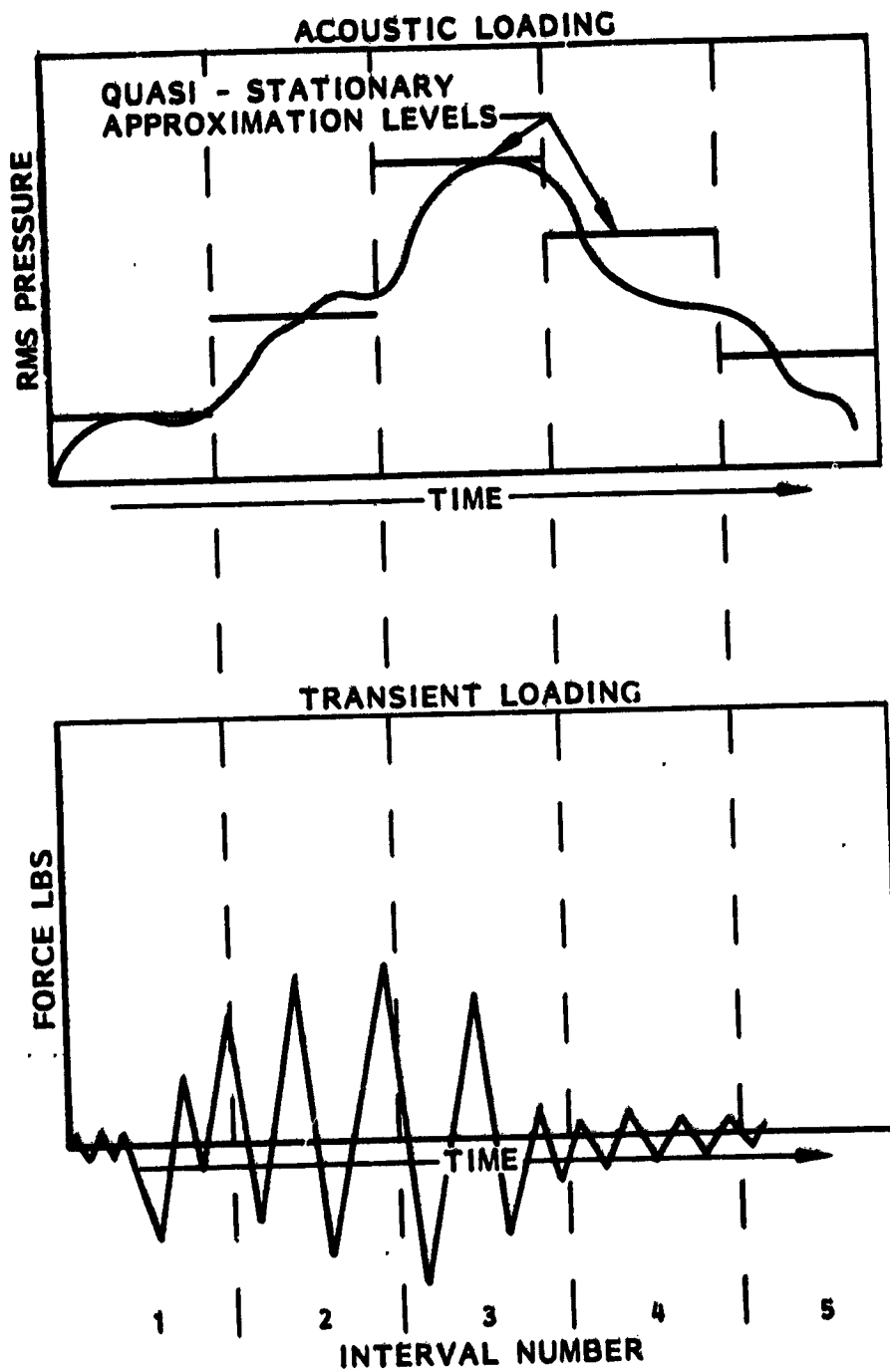
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Fig. 15   Quasi-Stationary Approximation to the Acoustic Field

6.0 THE EFFECT OF FLIGHT-TO-FLIGHT VARIATIONS IN THE ACOUSTIC PRESSURE FIELD PSD ON THE PREDICTED COMBINED LOADING

The question may arise as to how flight-to-flight variations in the acoustic field power spectral density, PSD, should be handled. To establish an acoustic load based on the average of the flight-to-flight variations appears to be a non-conservative approach but to base it on the maximum seems overly conservative. This section proposes an approach to handling this variation. Consider an ensemble of PSD's representing the flight acoustic pressure fields (e.g. Figure 16). For a spatially uncorrelated acoutic pressure field (i.e., neglecting the crosspower spectral density of the acoustic pressure), the response power spectral density is related to the acoustic pressure power spectral density by the transfer function, T_{ij}:

$$S_y(X_i,W) = T_{ij} S_p(X_j,W) \tag{42}$$

where, $\qquad T_{ij} = H(X_i,X_j,W) \ H^*(X_i, X_j,W)$

and,

$S_y(X_i,W)$ is the PSD of the response variable y, at frequency W and location X_i.

$H(X_i, X_j,W)$ is the frequency response function relating the response at location X_i to an acoustic pressure at location X_j at frequency , W.

$H^*(X_i, X_j,W)$ is the complex conjugate of $H(X_i, X_j, W)$

$S_p(X_j,W)$ is the PSD of the acoustic pressure at location X_j and frequency W.

Assuming the field is stationary (i.e. statistical parameters are independent of time) and taking expected values across the ensemble of flight acoustic pressure fields results in, from equation 42:

44

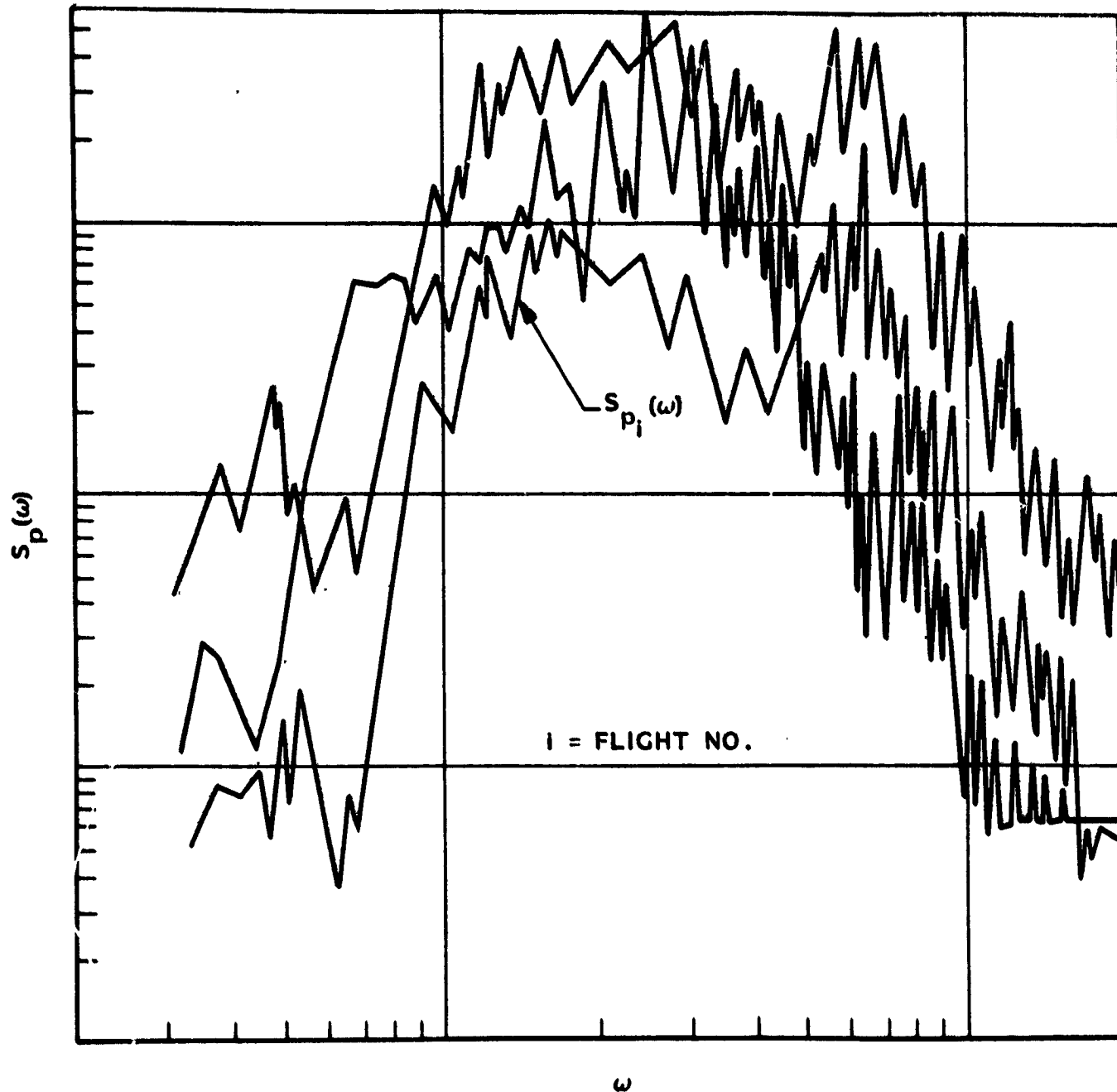Fig. 16   Flight-to-Flight Variation in Acoustic Pressure
Field Power Spectral Density

$$E\left[S_y(X_i,W)\right] = T_{ij}\ E\left[S_p(X_j,W)\right] \tag{43}$$

Squaring equation 42 and taking expected values across the ensemble results in:

$$E\left[S_y^2(X_i,W)\right] = T_{ij}^2\ E\left[S_p^2(X_j,W)\right] \tag{44}$$

As the mean-square of a random variable is equal to the variance plus the square of the mean, equation 44 may be rewritten as:

$$\sigma^2_{S_y(X_i,W)} + E^2\left[S_y(X_i,W)\right] = T_{ij}^2\ \sigma^2 S_p(X_j,W) + T_{ij}^2\ E^2\left[S_p(X_j,W)\right] \tag{45}$$

which by equation 43 reduces to

$$\sigma^2_{S_y(X_i,W)} = T_{ij}^2\ \sigma^2_{S_p(X_j,W)} \tag{46}$$

and taking the square-root of equation 46

$$\sigma_{S_y(X_i,W)} = T_{ij}\ \sigma_{S_p(X_j,W)} \tag{47}$$

where $\sigma_{S_y(X_i,W)}$ and $\sigma_{S_p(X_j,W)}$ are the flight-to-flight standard deviations of the PSDs of the response variable, Y, and the acoustic pressure, respectively.

Equations 43 and 47 state that the expected value (flight-to-flight mean) of the response PSD and the flight-to-flight standard deviation of the response PSD are obtained from mapping the expected value of the acoustic pressure PSD and the standard deviation of the acoustic pressure PSD, respectively, employing the transfer function,

$$T_{ij} = H(X_i,X_j,W)\ H^*(X_i,X_j,W)$$

Now assume that the population of acoustic pressure PSDs (from flight to flight) are normally distributed. If the acoustic pressure PSD is normally distributed, then so is the response PSD, for the latter is linearly related to the former (equation 42). With the ensemble of response PSDs normally distributed, the Mth percentile of the response PSD is given by:

$$P^{M\%}_{S_y(X_i,W)} = E\left[S_y(X_i,W)\right] + K^{M\%} \sigma_{S_y(X_i,W)} \tag{48}$$

where $K^{M\%}$ is the appropriate constant relating a multiple of the standard deviation to the Mth percentile for a normal distribution.

Substituting equations 43 and 47 into equation 48 results in:

$$P^{M\%}_{S_y(X_i,W)} = T_{ij} E\left[S_p(X_j,W)\right] + K^{M\%}\left[T_{ij}\, \sigma_{S_p(X_j,W)}\right]$$

or   (49)

$$P^{M\%}_{S_y(X_i,W)} = T_{ij}\left\{ E\left[S_p(X_j,W)\right] + K^{M\%} \sigma_{S_p(X_j,W)}\right\}$$

But, with the acoustic pressure PSDs normally distributed, the term in brackets is the Mth percentile of the acoustic pressure PSD. Thus,

$$P^{M\%}_{S_y(X_i,W)} = T_{ij}\, P^{M\%}_{S_p(X_j,W)} \tag{50}$$

Equation 50 states that the Mth percentile PSD of the response variable is obtained by mapping the Mth percentile PSD of the acoustic pressure field by the usual transfer function, $T_{ij} = H(X_i,X_j,W)\ H^*(X_i,X_j,W)$.

The standard deviation of the stationary random acoustically induced load, σ_Y, is obtained as the square root of the area under the acoustic load PSD determined in the random resonse analysis of the acoustic excitation,

$$\sigma_{Y(X_i)} = \left[\int_0^\infty S_y(X_i, W) \ dW \right]^{\frac{1}{2}} \tag{51}$$

When the standard deviation of the acoustically induced load, $\sigma_Y^M(X_i)$, is based on the Mth percentile of the acoustic pressure PSD, equation 51 becomes:

$$\sigma_{Y(X_i)}^M = \left[\int_0^\infty P_{S_y}^{M\%}(X_i, W) \ dW \right]^{\frac{1}{2}}$$

or from equation 50 (52)

$$\sigma_{Y(X_i)}^M = \left[\int_0^\infty T_{ij} \ P_{S_p}^{M\%}(X_j, W) \ dW \right]^{\frac{1}{2}}$$

The choice of which percentile, M, to use is a subjective, non-analytical decision.

Recall the case when the mechanically induced load, $X(t)$, was taken to be a known deterministic transient. For this case the Pth percentile of the total load, $Z(t)$, was given by;

$$P_{Z(t)}^\% = X(t) + K^{P\%} \sigma_Y \tag{53}$$

Now, when this P^{th} percentile of the total load is in turn based upon the Mth percentile PSD of the acoustic pressure, substituting σ_Y^M for σ_Y (with the location i dependence implied) equation 53 becomes:

$$P_{Z(t)}^\% = X(t) + K^{P\%} \left[\sigma_Y^M \right] \tag{54}$$

48

or

$$P^{\%}_{Z(t)} = X(t) + K^{P^{\%}} \left[\int_0^\infty T_{ij} \, P^{M^{\%}}_{S_p}(W) \, dW \right]^{\frac{1}{2}} \tag{54}$$

Similarly, for the case where the mechanically induced load is a non-stationary random variable, the Pth percentile of the total response, $Z(t_i)$, at any time, t_i, was given by:

$$P^{\%}_{Z(t_i)} = E\left[X(t_i)\right] + K^{P^{\%}} \left[\sigma^2_{X(t_i)} + \sigma^2_Y + 2\rho_{X(t_i) \, Y(t_i)} \, \sigma_{X(t_i)} \, \sigma_Y \right]^{\frac{1}{2}}$$

And, when this P^{th} percentile of the total response is in turn based upon the Mth percentile PSD of the acoustic pressure, (i.e., replacing σ_Y with σ_Y^M)

$$P^{\%}_{Z(t_i)} = E\left[X(t_i)\right] + K^{P^{\%}} \left[\sigma^2_{X(t_i)} + \sigma_Y^{M^2} + 2\rho_{X(t_i) \, Y(t_i)} \, \sigma_{X(t_i)} \, \sigma_Y^M \right]^{\frac{1}{2}} \tag{55}$$

where in equation 55, σ_Y^M is given by equation 52.

Referring to equations 52, 54 or 55, it is seen that the effect of flight to flight variations in the PSD of the acoustic pressure on the predicted combined load depends (1) on the relative magnitude of $P^{M^{\%}}_{S_p}(X_j,W)$ with respect to the excitation pressure $^P S_p(W)$ without flight-to-flight variations considered and (2) on the magnitude of the transient load considered by itself (i.e., if the transient load $X(t)$ is very high compared to the acoustically induced load, then consideration of flight-to-flight variations in the acoustic load may not cause an appreciable change in the combined load). Similarly, when the acoustically induced response is based upon the Mth percentile PSD of the acoustic pressure, σ_Y^M given by equation 52 is substituted for σ in the equations for calculating First-Passage probability.

7.0 CONCLUSION

An analytical technique has been developed for predicting the combined loads during STS launch due to transient mechanical inputs and acoustically induced random vibrations. The procedure assumes that the individual acoustic load and mechanical load have been previously obtained by currently employed techniques alluded to herein. The procedure is based upon three assumptions: (1) That the system (structure) is linear thereby permitting superposition of the two loads; (2) that the acoustic load is Gaussian; and (3) that the acoustic load is stationary. During STS launch the third assumption of the acoustic load being stationary is not physically justifiable, but as the true evolutionary nature of the acoustic pressure field may not be known, and in fact the acoustic forcing function often is taken as a stationary envelope of the evolutionary acoustic SPLs, the assumption of the acoustic load being stationary is made as a pragmatic analytical expedient. Should the evolutionary nature of the acoustic pressure be known, this non-stationary forcing function could be approximated as being quasi-stationary in temporally contiguous intervals. The procedure results in a predicted probability distribution function for the combined load; the combined load has been shown to be represented as a non-stationary Gaussian random variable with time-varying mean. Finally, a technique for obtaining combined design loads under a first-passage failure criterion has been developed and empirical verification of the proposed technique provided. A summary of the procedural steps leading to a First-Passage failure design level are listed in Table VII.

It is recommended that the following additional work be performed.

(1) Develop and document computer code "run streams" for performing first passage failure analyses on Univac, VAX, and CDC machines.

(2) Empirically verify analytical techniques for predicting acoustic induced loads and their resultant strains by employing the data obtained in the experimental investigation, Section 4.0.

TABLE VII

A SUMMARY OF PROCEDURAL STEPS LEADING TO A
FIRST-PASSAGE FAILURE DESIGN LEVEL

1. Identify acoustic pressure SPL

    a.   or SPLs if using quasi-stationary approximation

    b.   if accounting for flight-to-flight variations select desired M^{th}
         percentile SPL (subjective).

2. Using the selected SPL (or SPLs) calculate the standard deviations of the
   acoustically induced response, σ_y, and the time rate of change of the
   acoustically induced response, $\dot{\sigma}_y$, by a random reponse analysis of the
   structure.

3. Select desired transient and perform respon:e analysis to obtain
   mechanical transient response, $X(t)$, and time derivative of transient
   response $\dot{X}(t)$.

4. Select candidate barrier levels, A.

5. Perform First-Passage failure analysis (equation 37) to obtain the
   probability distribution function of first-passage failure.

6. Either:

    a.   select the desired probability (subjective) to which the structure
         is to be designed and obtain from the probability distribution
         function the corresponding design level, or

    b.   select the design level and obtain the corresponding probability.

REFERENCES

1. Rice, S. O., "Mathematical Analysis of Random Noise", in <u>Selected Papers on Noise and Stochastic Processes</u> (Editor, N. Wax), Dever, New York, 1954

2. Cramer, H. and Leadbetter, M. F., <u>Stationary and Related Stochastic Processes</u>, John Wiley, New York, 1967

3. Crandall, S. H. and Mark W. D., <u>Random Vibration in Mechanical Systems</u>, Academic Press, New York, 1963

APPENDIX A

PRELIMINARY CONSIDERATIONS ON
THE ACOUSTICALLY INDUCED LOAD

The purpose of this appendix is to develop some of the salient features of a random process (the acoustically induced load) with which the reader should be familiar and that are employed in the text of this document.

The acoustically induced load is taken as a random population, assumed to be stationary and Gaussian and of the form:

$$Y_j(t) = \Sigma A_n \, SIN \, (W_n t + \phi_{nj}) \qquad (A\text{-}1)$$

or, at a given time, t_1

$$Y_j(t_1) = \Sigma A_n \, SIN \, (W_n t_1 + \phi_{nj}) \qquad (A\text{-}2)$$

where:

A_n = Amplitude of the nth frequency component

W_n = Circular frequency of the nth frequency component

ϕ_{nj} = Phase angle of the nth frequency component for the jth member of the population

Each element, $Y_j(t)$, of the ensemble has the same power spectral density (PSD) obtained previously from a random response analysis employing the acoustic field in the STS cargo bay as a stationary random excitation. The elements of the ensemble differ and are deterministically unknown because the phase angles differ and are unknown. In this formulation the load due to acoustics is represented as a number of sinusoidal components with random phase angles uniformly distributed over the range $(-\pi, +\pi)$. As the phase angles are assumed to be uniformly distributed between $-\pi$ and $+\pi$, the probability density function of ϕ_n is flat at the level, $\frac{1}{2}\pi$, in that interval.

It is known that for some function of ϕ_n, say $G(\phi_n)$, the expected value of $G(\phi_n)$ is defined by:

$$E \, [G(\phi_n)] = \int_{-\infty}^{\infty} G(\phi_n) P(\phi_n) d\phi_n \qquad (A\text{-}3)$$

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where $P(\phi_n)$ is the probability density function of ϕ_n.

In particular the following will prove to be useful.

$$E\left|\cos\phi_n\right| = \int_{-\infty}^{\infty}\cos\phi_n \, P(\phi_n)d\phi_n = \int_{-\pi}^{\pi}\frac{\cos\phi}{2\pi}\, d\phi_n = \tfrac{1}{2}\pi\left[\sin\phi_n\right]_{\pi}^{\pi} = 0$$

$$E\left|\sin\phi_n\right| = \int_{-\infty}^{\infty}\sin\phi_n \, P(\phi_n)d\phi_n = \int_{-\pi}^{\pi}\frac{\sin\phi_n}{2\pi}\, d\phi_n = \tfrac{1}{2}\pi\left[-\cos\phi_n\right]_{\pi}^{\pi} = 0$$

$$E\left|\cos^2\phi_n\right| = \int_{-\infty}^{\infty}\cos^2\phi_n \, P(\phi_n)d\phi_n = \int_{-\pi}^{\pi}\frac{\cos^2\phi_n}{2\pi}\, d\phi_n = \tfrac{1}{2}\pi\left[\frac{\phi_n}{2} + \tfrac{1}{4}\sin 2\phi_n\right]_{-\pi}^{\pi}$$

$$= \tfrac{1}{2}\pi\left[\pi/2 + \pi/2\right] = \tfrac{1}{2} \qquad (A-4)$$

$$E\left|\sin^2\phi_n\right| = \int_{-\infty}^{\infty}\sin^2\phi_n \, P(\phi_n)d\phi_n = \int_{-\pi}^{\pi}\frac{\sin^2\phi_n}{2\pi}\, d\phi_n = \tfrac{1}{2}\pi\left[\frac{\phi_n}{2} - \tfrac{1}{4}\sin 2\phi_n\right]_{-\pi}^{\pi}$$

$$= \tfrac{1}{2}\pi\left[\pi/2 + \pi/2\right] = \tfrac{1}{2}$$

$$E\left|\sin\phi_n \cos\phi_n\right| = \int_{-\infty}^{\infty}\sin\phi_n\cos\phi_n \, P(\phi_n)d\phi_n = \int_{-\pi}^{\pi}\tfrac{1}{2}\pi(\sin\phi_n\cos\phi_n)d\phi_n$$

$$= \tfrac{1}{2}\pi\left[\tfrac{1}{2}\sin^2\phi_n\right]_{-\pi}^{\pi} = 0$$

In a similar fashion, we consider the second order probability density function between two phase angles, ϕ_n, and ϕ_m. As before the phase angles are assumed to be uniformly distributed between $-\pi$ and $+\pi$ (i.e. $-\pi \leq \phi_n \leq \pi$ and $-\pi \leq \phi_m \leq +\pi$, each with uniform distribution). Hence the second order probability density function between ϕ_n and ϕ_m is a flat surface of level $\frac{1}{4}\pi^2$. By definition, two random variables are independent if:

$$P_2 (\phi_n, \phi_m) = P_1 (\phi_n) P_1 (\phi_m) \tag{A-5}$$

which is the case at hand (i.e. $\frac{1}{4}\pi^2 = \frac{1}{2}\pi \times \frac{1}{2}\pi$). Thus, we have taken the phase angles to be not only uniformly distributed, but statistically independent.

It is known that for some function of (ϕ_n, ϕ_m), say $G(\phi_n, \phi_m)$, the expected value of $G(\phi_n, \phi_m)$, is defined by:

$$E \; [G(\phi_n, \phi_m)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\phi_n \phi_m) \; P_2 \; (\phi_n \phi_m) \; d\phi_n \; d\phi_m \tag{A-6}$$

In particular, the following will prove to be useful.

$$E \; [\cos\phi_n \cos\phi_m] = \int_{-\infty}^{\infty} \frac{\cos\phi_n}{2\pi} \; d\phi_n \int_{-\infty}^{\infty} \frac{\cos\phi_m}{2\pi} \; d\phi_m = 0$$

$$E \; [\sin\phi_n \sin\phi_m] = \int_{-\infty}^{\infty} \frac{\sin\phi_n}{2\pi} \; d\phi_n \int_{-\infty}^{\infty} \frac{\sin\phi_m}{2\pi} \; d\phi_m = 0 \tag{A-7}$$

$$E \; [\sin\phi_n \cos\phi_m] = \int_{-\infty}^{\infty} \frac{\sin\phi_n}{2\pi} \; d\phi_n \int_{-\infty}^{\infty} \frac{\cos\phi_m}{2\pi} \; d\phi_m = 0$$

We had the family of loads due to acoustic excitation to be of the form:

$$Y(t_1) = \Sigma A_n \; \sin(W_n t_1 + \phi_n) \tag{A-8}$$

where $Y(t_1)$ is a random variable because ϕ_n is a random variable. A_n is obtained from a frequency analysis of response to acoustic excitation, or, by an identity of trigonometry

$$Y(t_1) = \Sigma A_n \left[SIN(W_n t_1) \cos \phi_n + \cos(W_n t_1) \sin \phi_n \right] \tag{A-9}$$

taking expected values

$$E\left[Y(t_1)\right] = \Sigma A_n \left\{ E\left[SIN(W_n t_1)\cos\phi_n\right] + E\left[\cos(W_n t_1)\sin\phi_n\right] \right\} \tag{A-10}$$

and as $SIN\ W_n t_1$ and $\cos \phi_n$ are uncorrelated

and as $\cos W_n t_1$ and $SIN\ \phi_n$ are uncorrelated

$$E\ [Y(t_1)] = \Sigma A_n \left| E\ [SIN(W_n t_1)]\ E\ [\cos\phi_n] + E\ [\cos(W_n t_1)]\ E\ [SIN\ \phi_n] \right| \tag{A-11}$$

but as $W_n t_1$ is a deterministic quantity

$$E\ \left[SIN(W_n t_1)\right] = SIN(W_n t_1)$$

$$\tag{A-12}$$

$$E\ \left[\cos(W_n t_1)\right] = \cos(W_n t_1)$$

and we have already shown that

$$E\ \left[\cos\ \phi_n\right] = E\ \left[SIN\ \phi_n\right] = 0 \tag{A-13}$$

hence

$$E\left[Y(t_1)\right] = \Sigma A_n \left\{(SIN\ W_n t_1)(0) + (\cos W_n t_1)(0)\right\} = 0$$

$$\tag{A-14}$$

$$E\ \left[Y(t_1)\right] = 0$$

Thus, the expected value (mean) of the response due to acoustic loading is zero.

Now, squaring the response due to acoustic excitation gives

$$Y^2(t_1) = \sum_n A_n^2 \, [SIN(W_n t_1)COS\phi_n + COS(W_n t_1)SIN\phi_n]^2$$

$$+ \, 2 \sum_n \sum_m A_n A_m \, [SIN(W_n t_1)COS\phi_n + COS(W_n t_1)SIN\phi_n] \qquad (A\text{-}15)$$

$$x\,[SIN(W_m t_1)COS\phi_m + COS(W_m t_1)SIN\phi_m]$$

or expanding and taking expected values

$$E\,[Y^2(t_1)] = \sum_n A_n^2 \, [(SIN^2 W_n t_1)\; E(COS^2\phi_n)^{1/2} + (COS^2 W_n t_1)\; E(SIN^2\phi_n)^{1/2}$$

$$+\, 2\, SIN(W_n t_1)COS(W_n t_1)\; E(SIN\phi_n COS\phi_n)^{0}\,]$$

$$+2 \sum_n \sum_m A_n A_m \left[SIN(W_n t_1)SIN(W_m t_1)\; E(COS\phi_n COS\phi_m)^{0} \right.$$

$$+\, SIN(W_n t_1)COS(W_m t_1)\; E(COS\phi_n SIN\phi_m)^{0} \qquad (A\text{-}16)$$

$$+\, SIN(W_m t_1)COS(W_n t_1)\; E(SIN\phi_n COS\phi_m)^{0}$$

$$\left. +\, COS(W_n t_1)COS(W_m t_1)\; E(SIN\phi_n SIN\phi_m)^{0} \right]$$

and $E\left[Y^2(t_1)\right]$ reduced to

$$E\left[Y^2(t_1)\right] = \sum \frac{A_n^2}{2} \left[SIN^2 W_n t_1 + COS^2 W_n t_1 \right]$$

and

$$E\left[Y^2(t_1)\right] = \sum \frac{A_n^2}{2} \qquad (A\text{-}17)$$

which is Parseval's formula for a Fourier series (i.e. the mean square value equals 1/2 the sum of the squares of the component amplitudes).

In practice, the mean-square acoustically induced response, $E\left[Y^2(t)\right]$ is the area under the one-sided acoustically induced response PSD, $S_y(W)$, and is obtained by integrating the same over frequency, i.e.

$$E\left[Y^2(t)\right] = \int_0^\infty S_y(W)dW \qquad (A-18)$$

Further, as the expected value (mean) of the acoustically induced response is zero, the mean-square acoustically induced response equals the variance of that response.

$$E\left[Y^2(t)\right] = \sigma_Y^2 \qquad (A-19)$$

EMPIRICAL VERIFICATION OF
UNDERLYING ASSUMPTIONS

Having achieved rather good agreement between the theoretical First-Passage
prediction and the empirical data, it is not surprising that the assumptions
upon which that prediction is based were reasonably valid for the test con-
figuration. The prediction of the combined response is based upon three
assumptions: (1) that the structure is linear, thereby permitting superposi-
tion of the two loads; (2) that the acoustic load is Gaussian; and (3) that
the acoustic load is stationary. Hence, the combined response is represented
as a non-stationary Gaussian random variable with time-varying mean (the
time-varying mean being the response to the transient).

To confirm the validity of this representation, empirical ensemble statistics
for the combined loading responses (i.e. statistics across the ensemble of 100
samples) at various arbitrary time points were compared with the predicted
ensemble statistics based upon equation 16.

$$P^{\#}_{Z(t_i)} = X(t_i) + K^P \sigma^{\#}_Y$$

Figures B-1, B-2, and B-3 depict such comparisons for the accelerometer #6
data, spectrum A/transient 1 loading condition of 4-second duration, at the
times $t_i = 1$ second, $t_i = 2$ second, and $t_i = 3$ second, respectively.
The agreement, if not excellent, is considered adequate. Further, if a
temporal average of the ensemble statistics over the 4-second duration (i.e.,
over 8000 digitized time points) is made, excellent agreement is found as.
seen in Figure B-4. Similar results for the other combined loading condi-
tions for accelerometer #6 and strain gage #2 are presented in Figures B-5
through B-20. Taking the combined response as a non-stationary Gaussian
random variable with time-varying mean appears to be a reasonable assumption
that is essentially correct.

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GAUSSIAN DISTRIBUTION COMPARISON
ACCELEROMETER 6 / TRANSIENT 1 + SPECTRUM A / T-1 SECOND

EMPIRICAL DATA
GAUSSIAN DISTRIBUTION

LEVEL (G S)

PERCENTILE
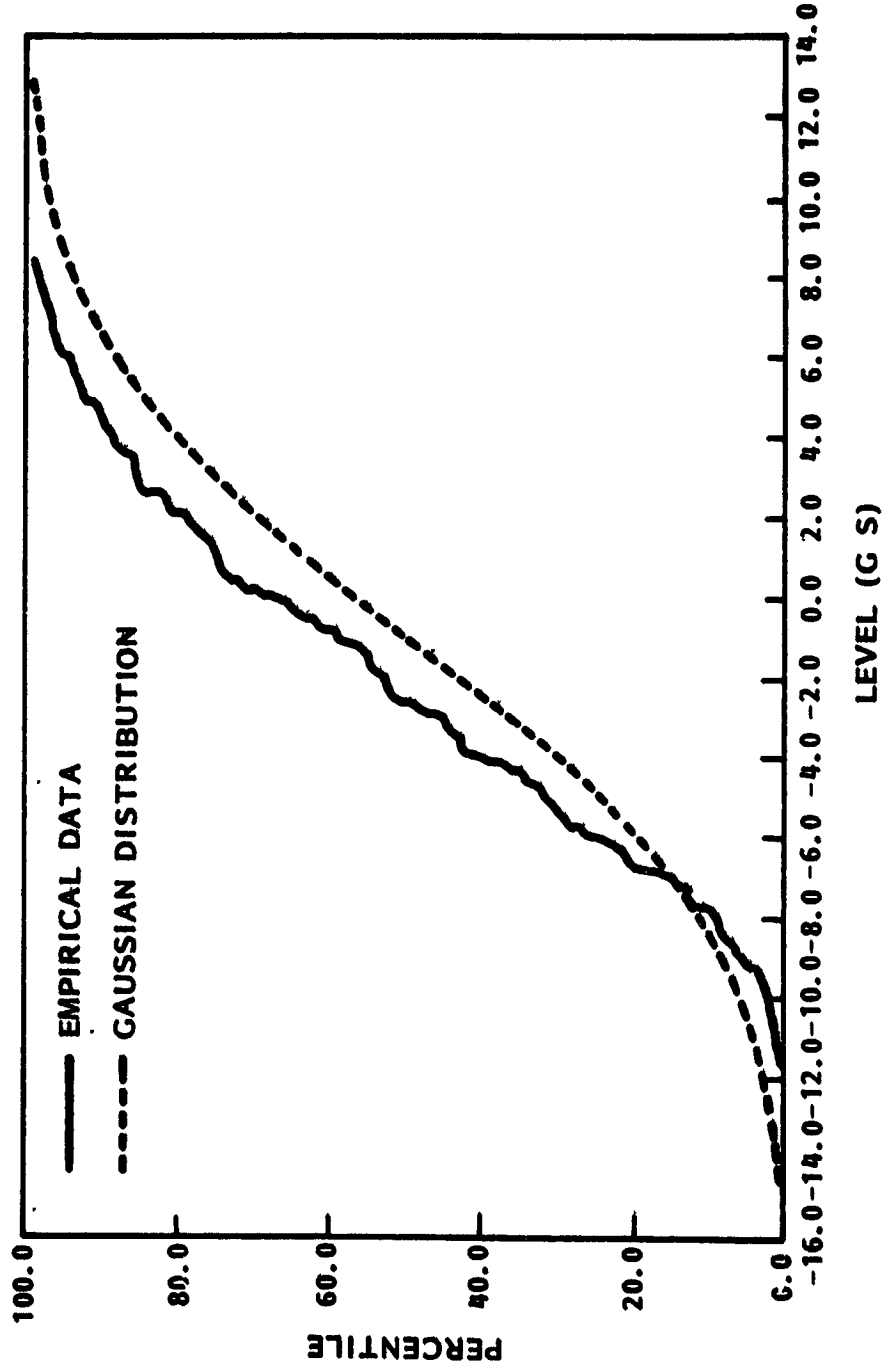
Fig. B-1   Gaussian Distribution Comparison
           Accelerometer 6/Transient 1 + Spectrum
           A/T-1 Second
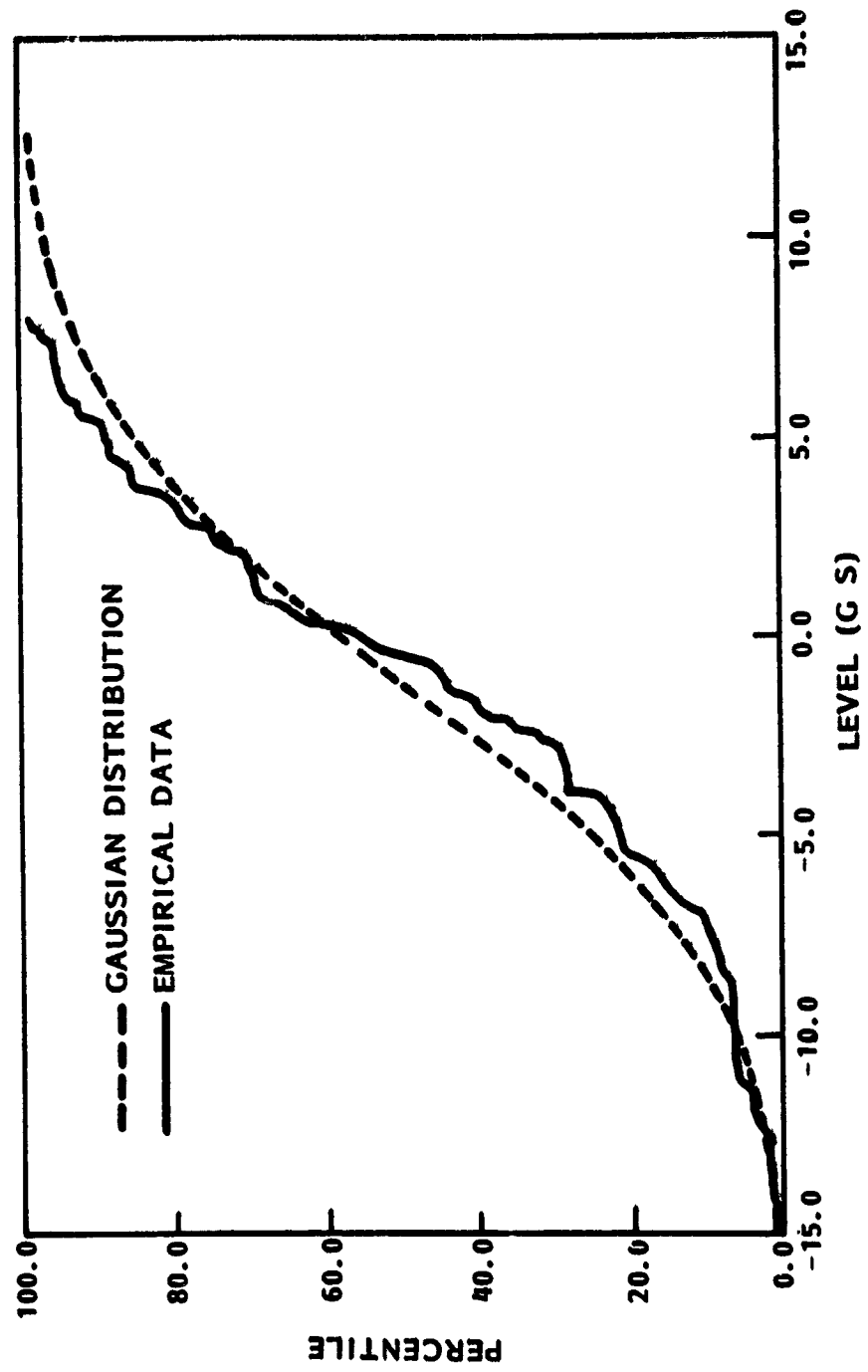
Fig. B-2 Gaussian Distribution Comparison
Accelerometer 6/Transient 1 + Spectrum
A/T-2 Second

GAUSSIAN DISTRIBUTION COMPARISON
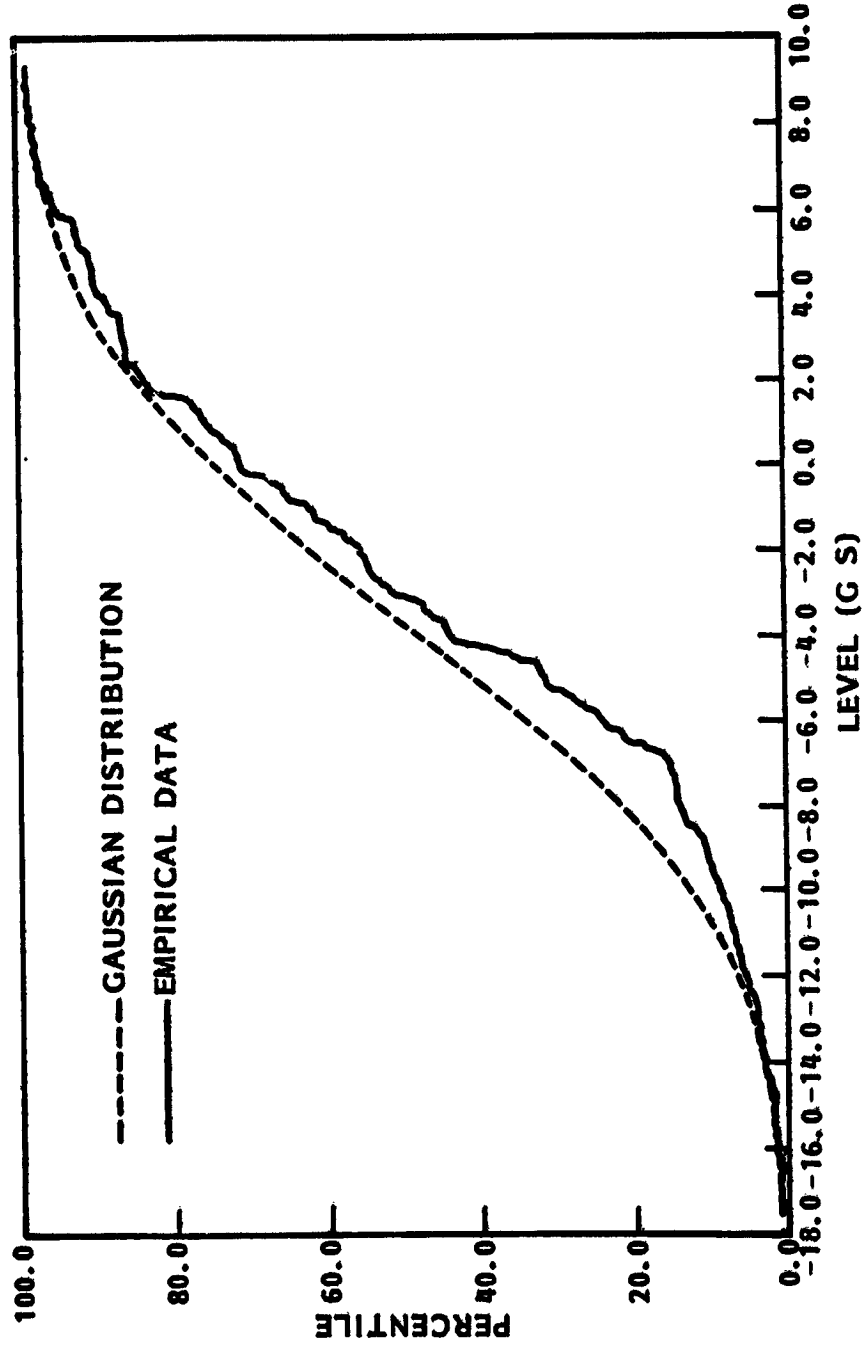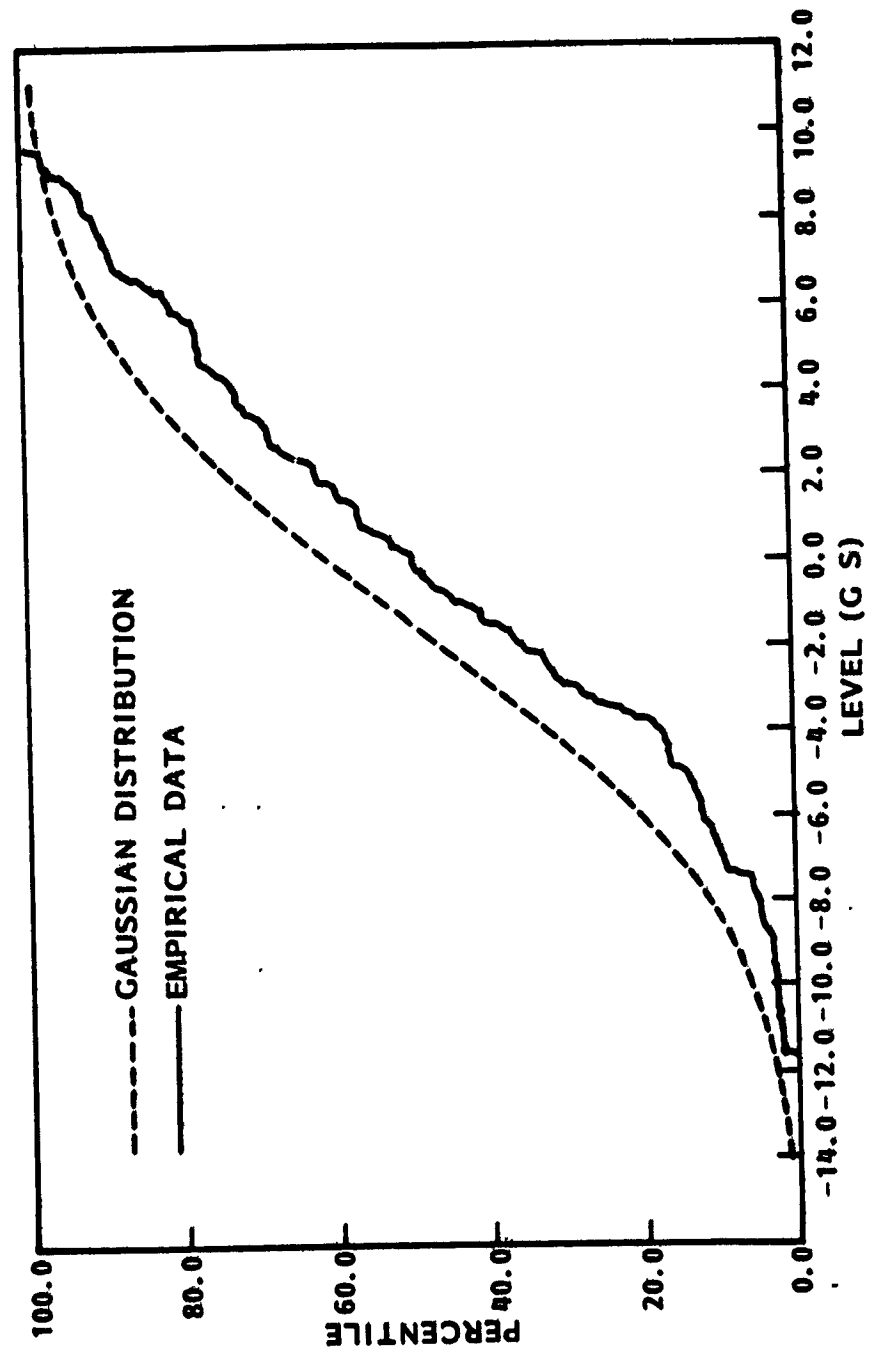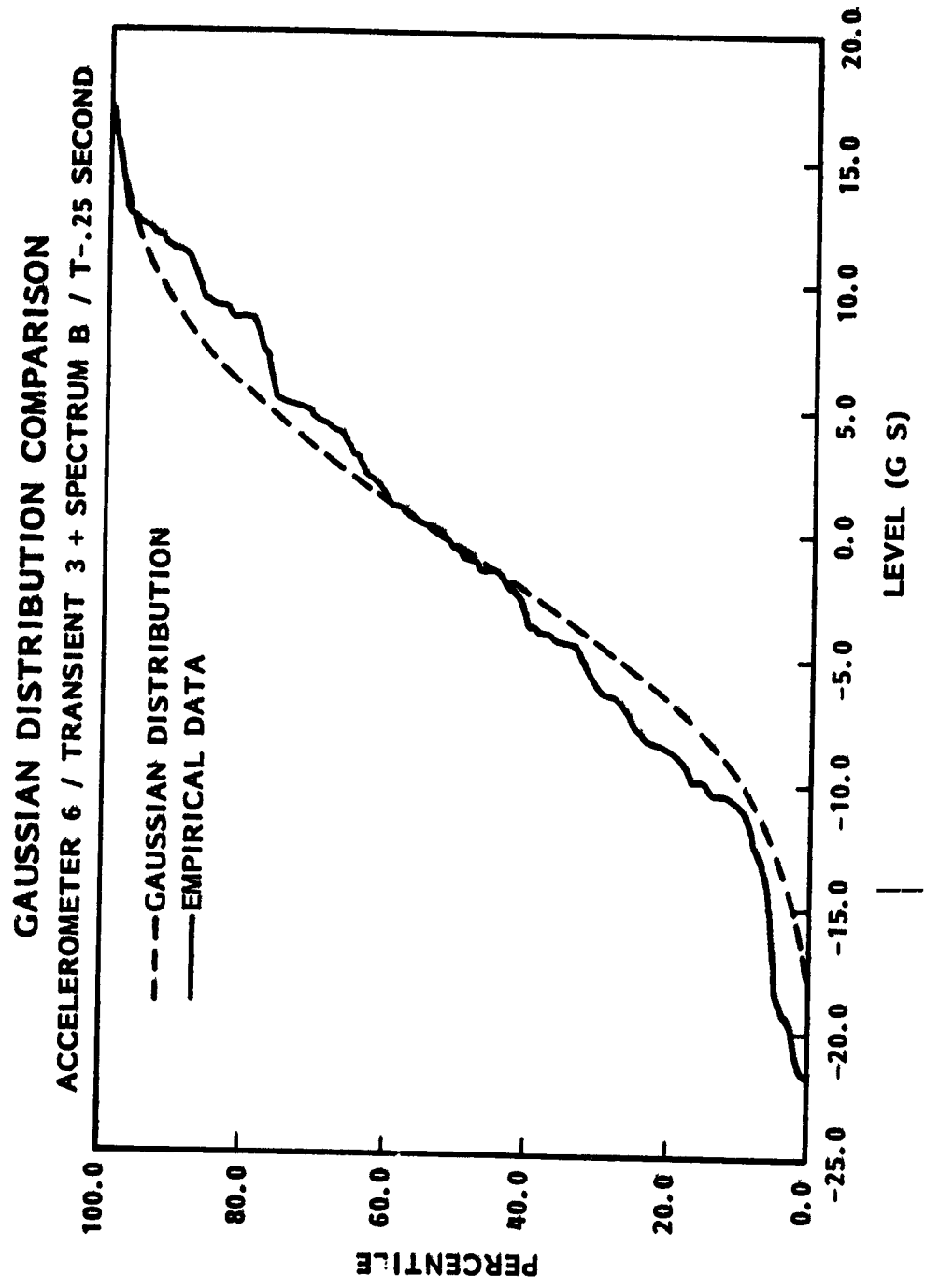ACCELEROMETER 6 / TRANSIENT 1 + SPECTRUM A / T-3 SECOND

GAUSSIAN DIRTRIBUTION -----
EMPIRICAL DATA _____

Fig. B-3   Gaussian Distribution Comparison
Accelerometer 6/Transient 1 + Spectrum
A/T-3 Second

GAUSSIAN DISTRIBUTION COMPARISON

ACCELEROMETER 6 / TRANSIENT 1 + SPECTRUM A / TIME AVERAGED

GAUSSIAN DISTRIBUTION -----

EMPIRICAL DATA ———

LEVEL (G S)

PERCENTILE
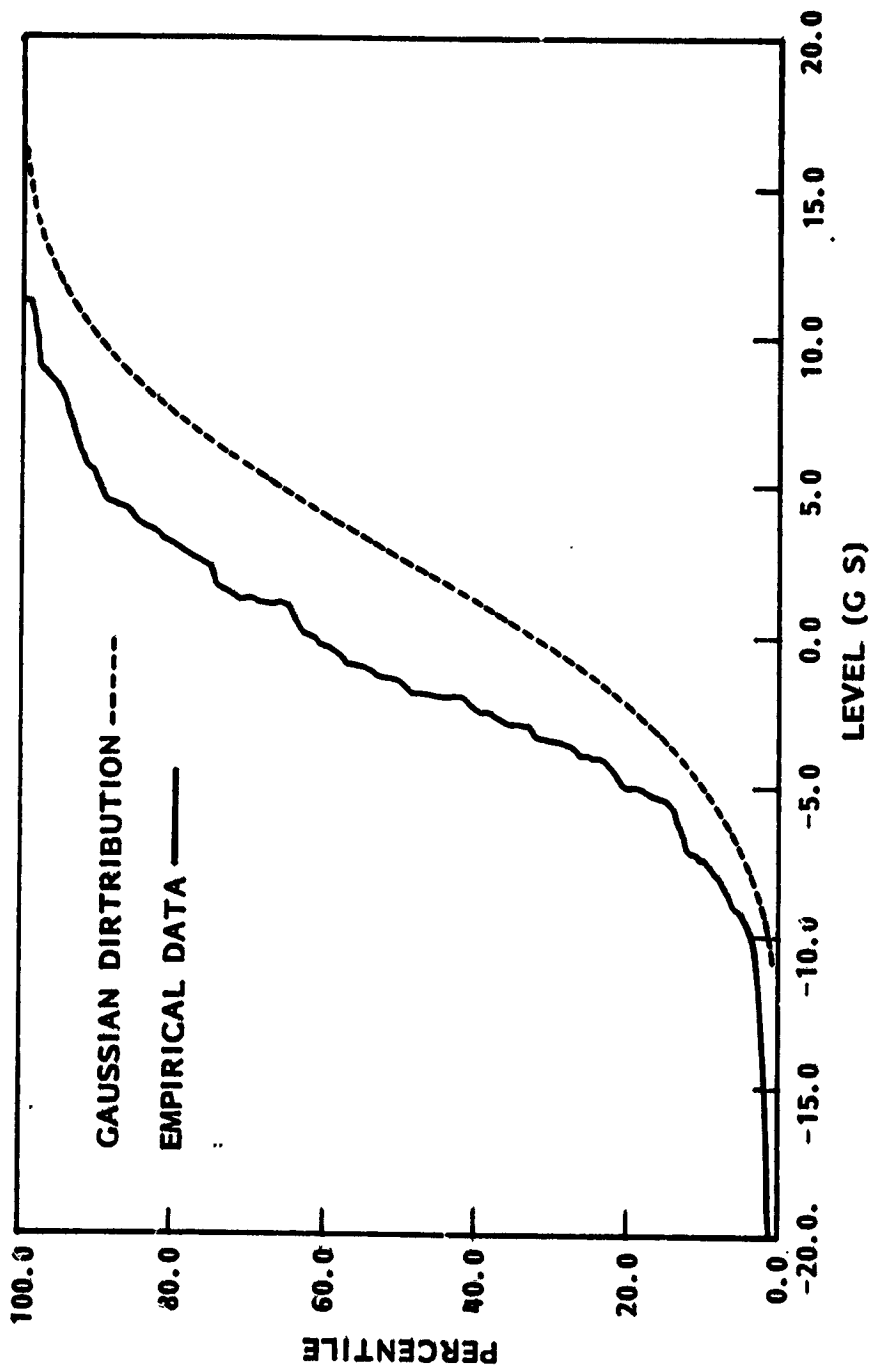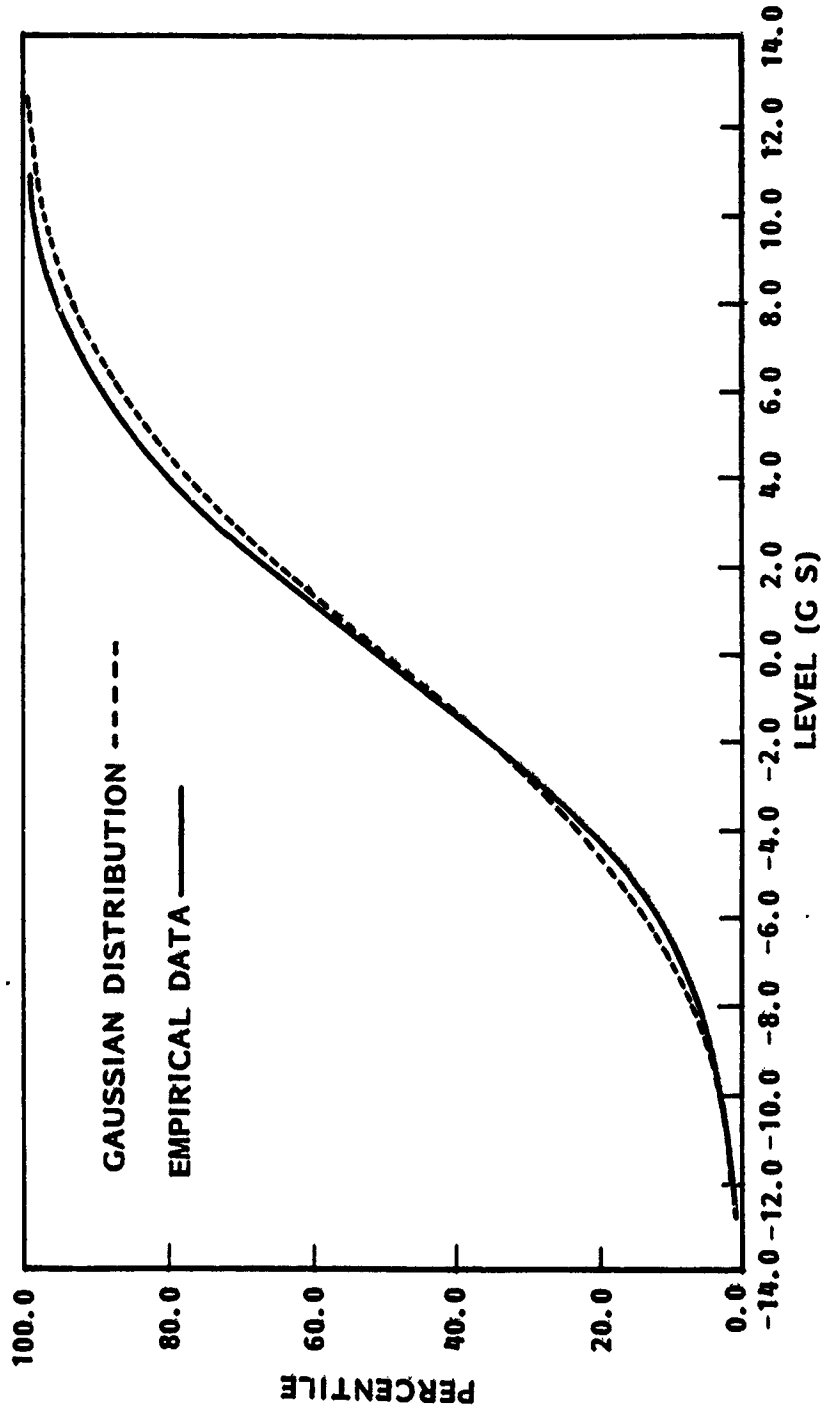
Fig. B—4   Gaussian Distribution Comparison
Accelerometer 6/Transient 1 + Spectrum
A/Time Averaged

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GAUSSIAN DISTRIBUTION COMPARISON

ACCELEROMETER 6 / TRANSIENT 2 + SPECTRUM B / T-1 SECOND

- - - - GAUSSIAN DISTRIBUTION
———— EMPIRICAL DATA

Fig. B-5   Gaussian Distribution Comparison
            Accelerometer 6/Transient 2 + Spectrum
            B/T-1 Second

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GAUSSIAN DISTRIBUTION COMPARISON
ACCELEROMETER 6 / TRANSIENT 2 + SPECTRUM B / T-2 SECOND

----- GAUSSIAN DISTRIBUTION
——— EMPIRICAL DATA

Fig. B-6   Gaussian Distribution Comparison
Accelerometer 6/Transient 2 + Spectrum
B/T-2 Second

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GAUSSIAN DISTRIBUTION COMPARISON
ACCELEROMETER 6 / TRANSIENT 2 + SPECTRUM B / T-3 SECOND

---- GAUSSIAN DISTRIBUTION
—— EMPIRICAL DATA

PERCENTILE

LEVEL (G S)
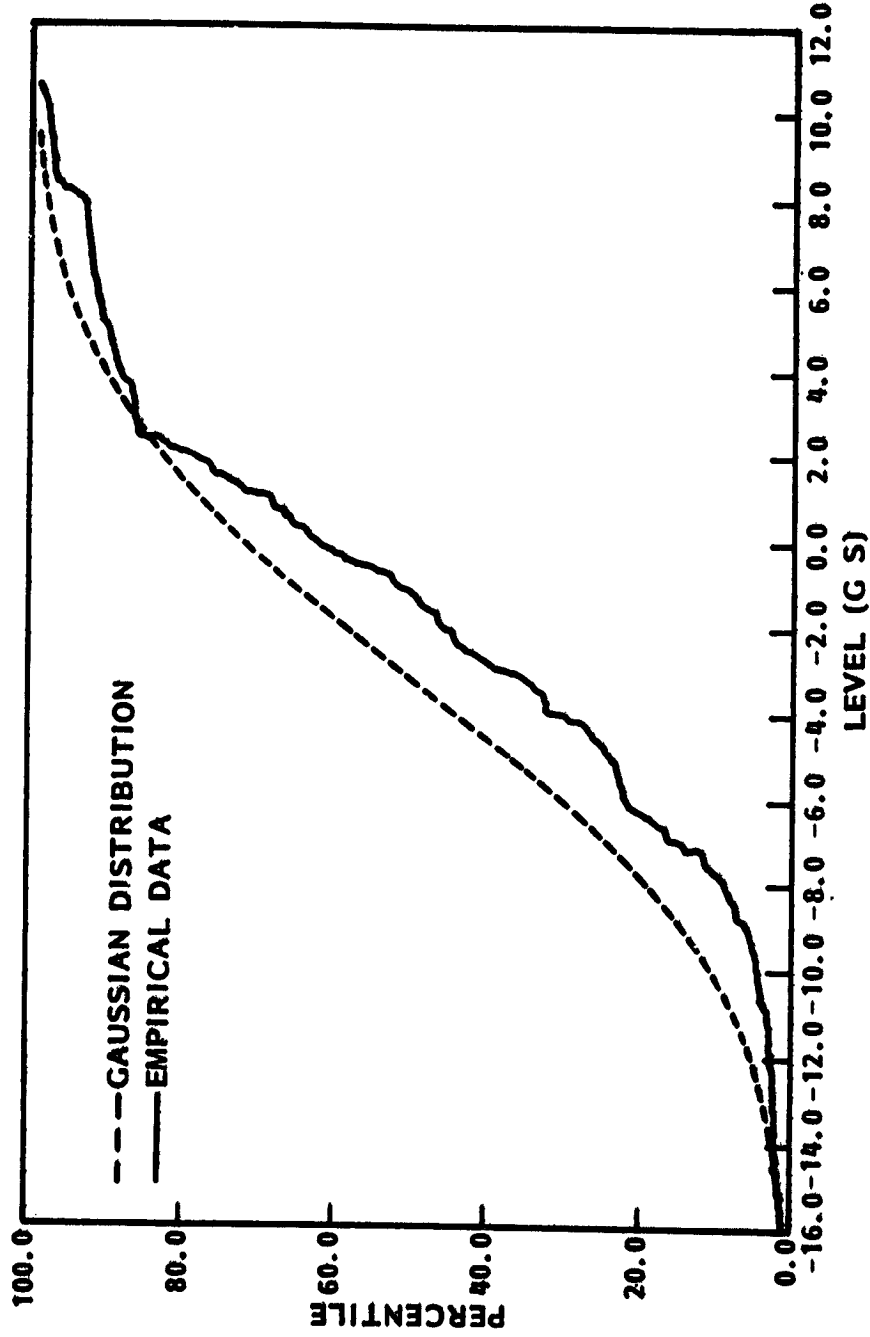
Fig. B-7   Gaussian Distribution Comparison
Accelerometer 6/Transient 2 + Spectrum
B/T-3 Second

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Fig. B-8   Gaussian Distribution Comparison
Accelerometer 6/Transient 2 + Spectrum
B/Time Averaged

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GAUSSIAN DISTRIBUTION COMPARISON

ACCELEROMETER 6 / TRANSIENT 3 + SPECTRUM B / T-.25 SECOND

- - - GAUSSIAN DISTRIBUTION
——— EMPIRICAL DATA

LEVEL (G S)

PERCENTILE

Fig. B-9   Gaussian Distribution Comparison
Accelerometer 6/Transient 3 + Spectrum
B/T- .25 Second

70
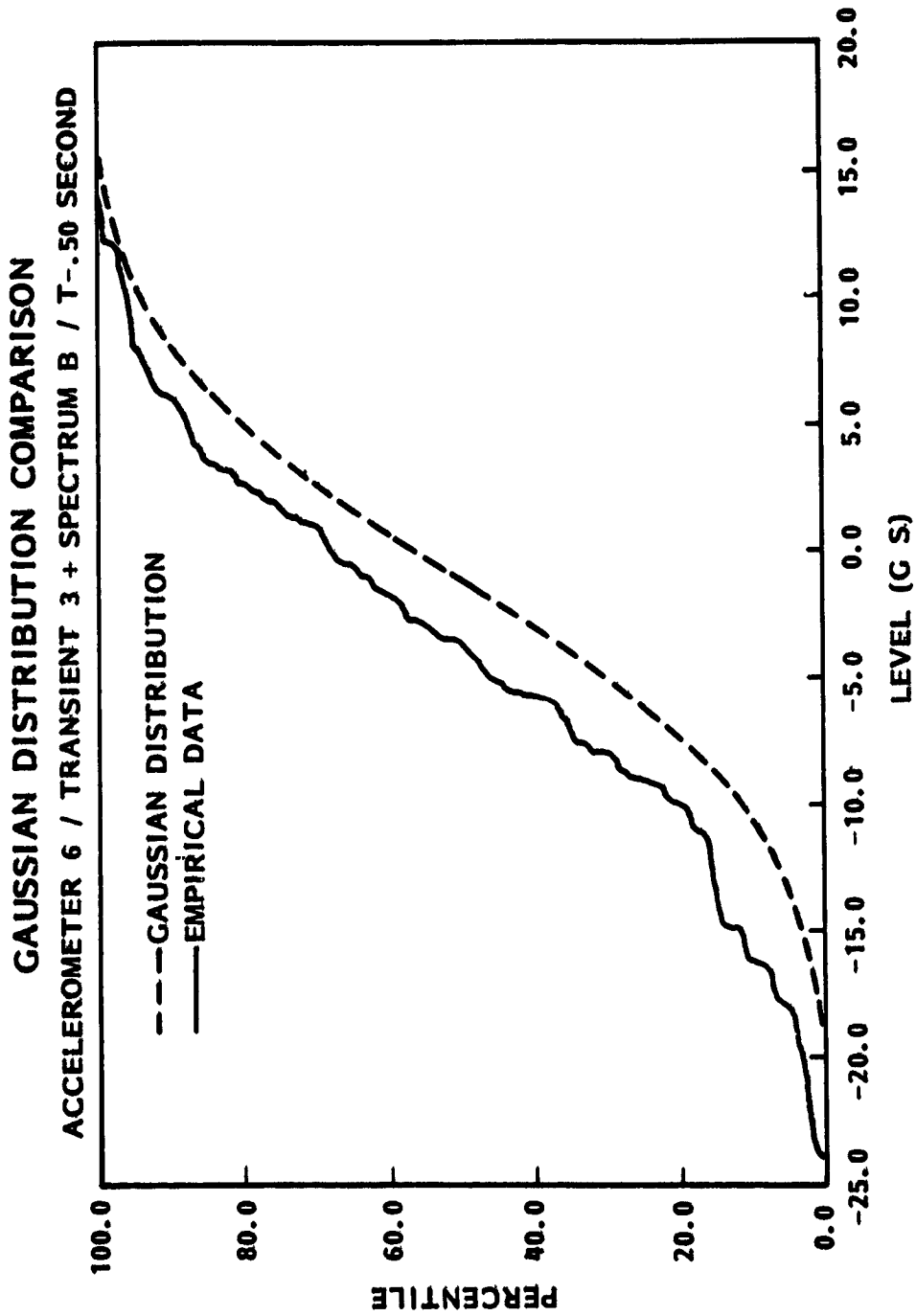
Fig. B-10   Gaussian Distribution Comparison
Accelerometer 6/Transient 3 + Spectrum
B/T- .50 Second

Fig. B-11 Gaussian Distribution Comparison
Accelerometer 6/Transient 3 + Spectrum
B/T- .75 Second

72
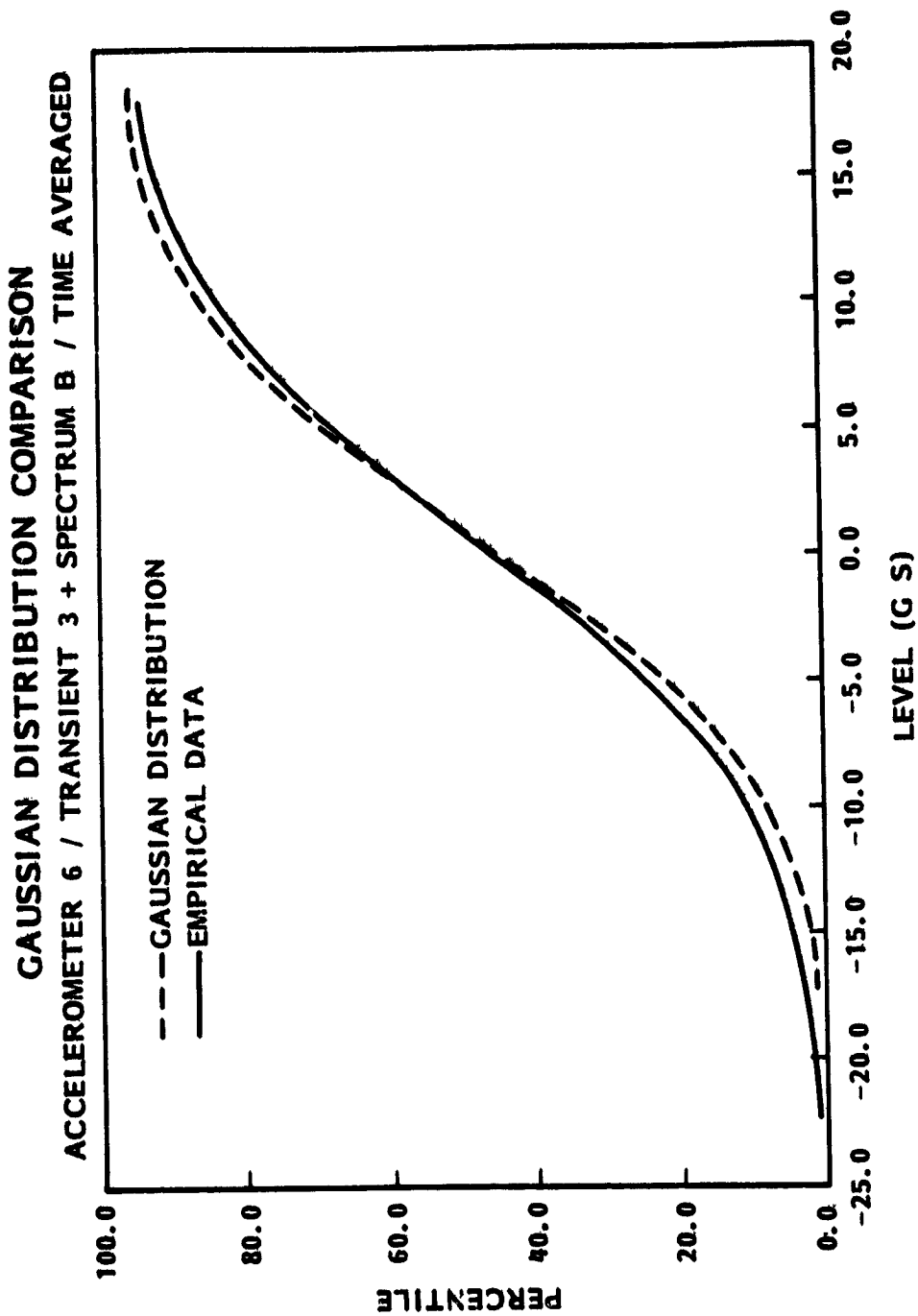
Fig. B-12   Gaussian Distribution Comparison
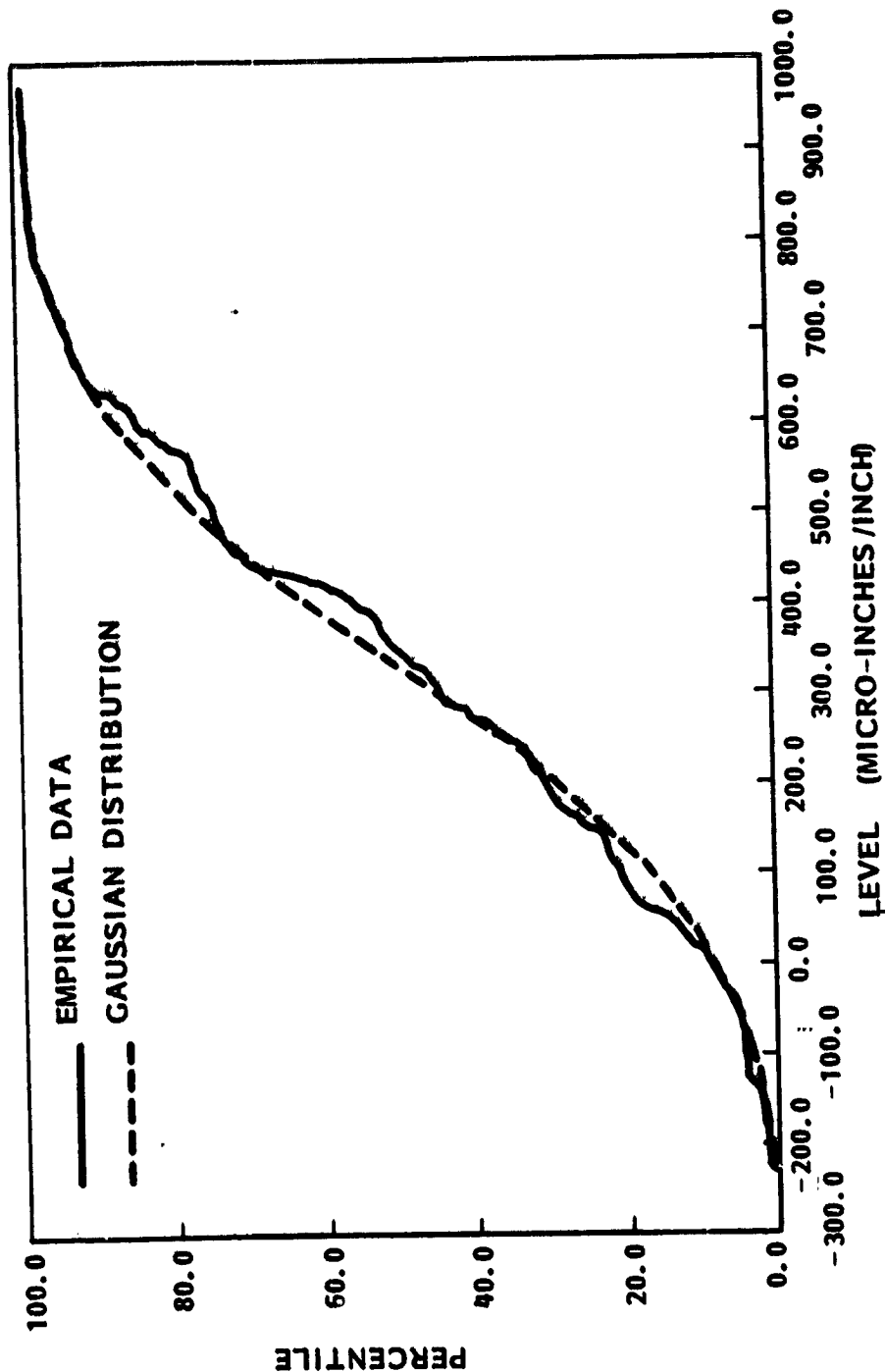Accelerometer 6/Transient 3 + Spectrum
B/Time Averaged

73

Fig. B-13   Gaussian Distribution Comparison
Strain Gage 2/Transient 1 + Spectrum
A/T-1 Second

GAUSSIAN DISTRIBUTION COMPARISON
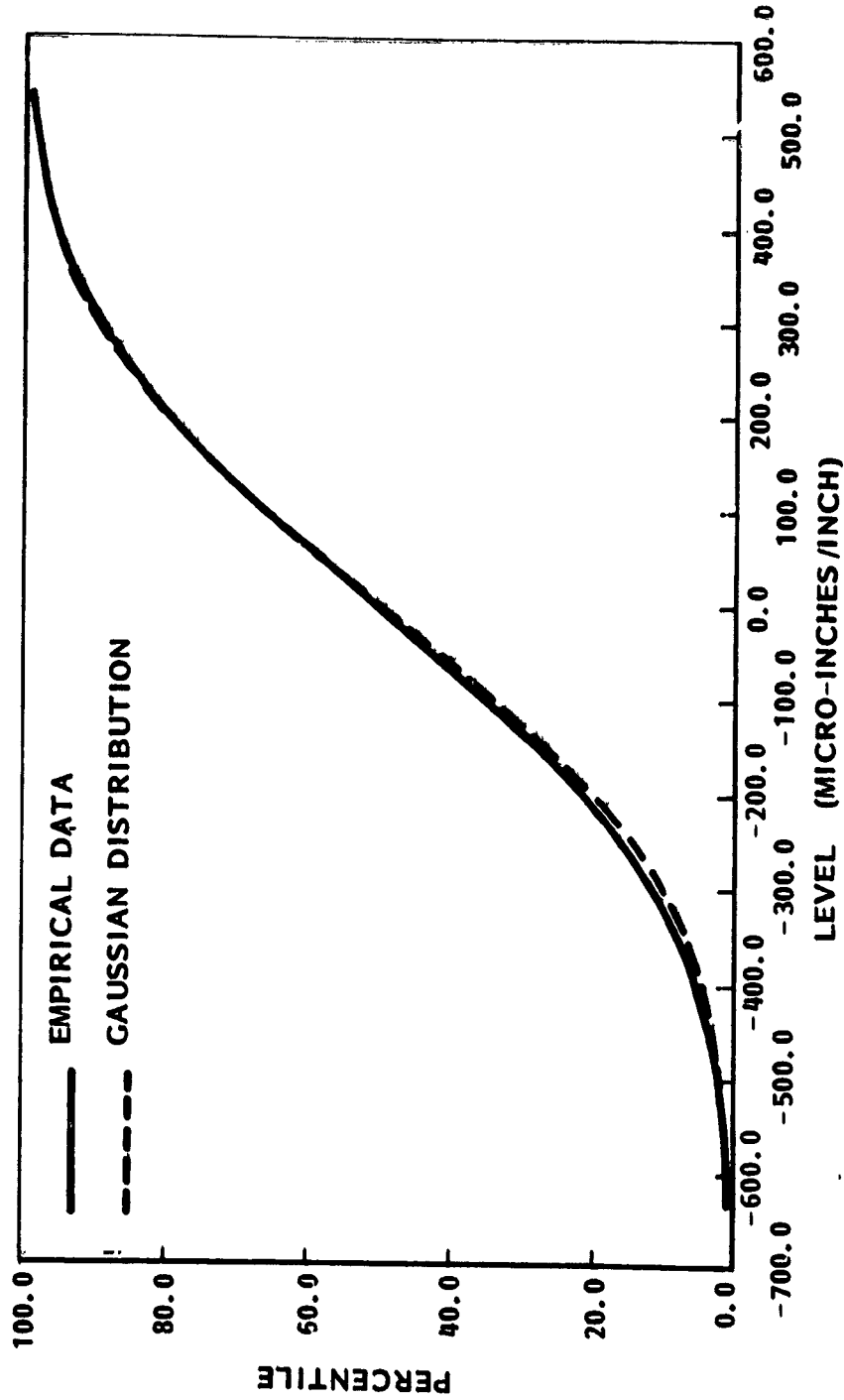
STRAIN GAGE 2 / TRANSIENT 1 + SPECTRUM A / T-2 SECOND

- - - - GAUSSIAN DISTRIBUTION

——— EMPIRICAL DATA

LEVEL (MICRO-INCHES/INCH)

PERCENTILE
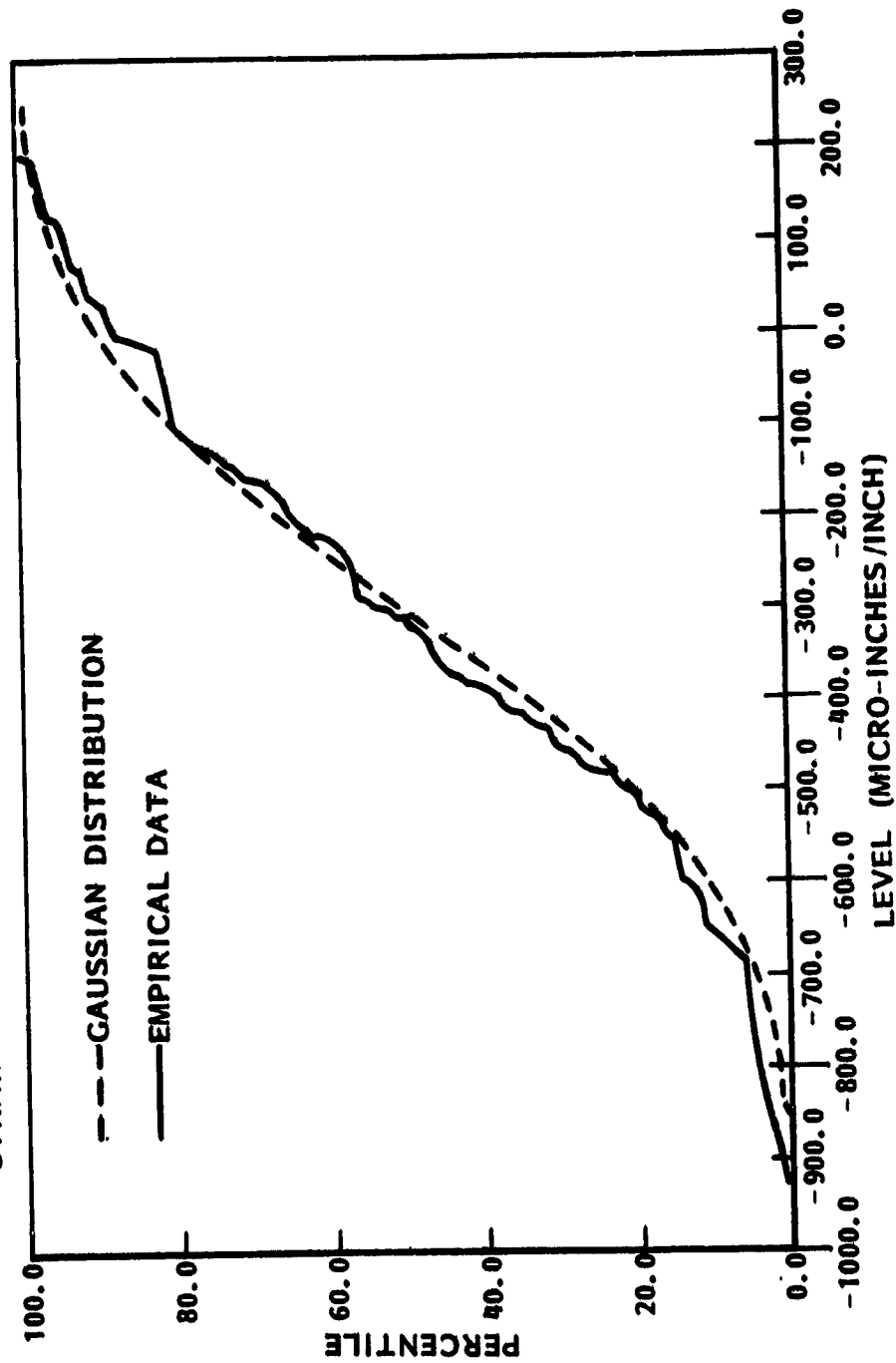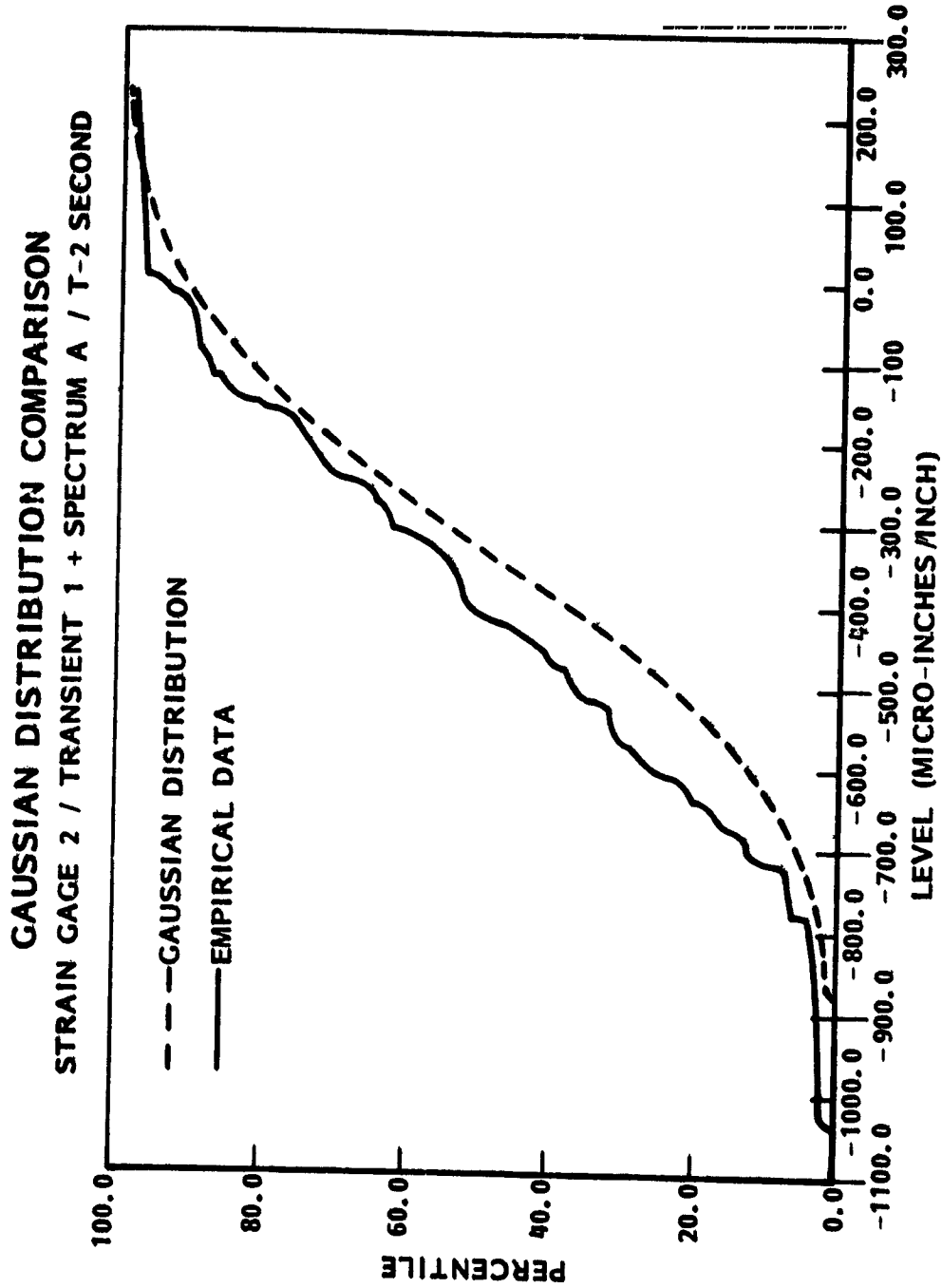
Fig. B-14   Gaussian Distribution Comparison
Strain Gage 2/Transient 1 + Spectrum
A/T-2 Second

75

GAUSSIAN DISTRIBUTION COMPARISON
STRAIN GAGE 2 / TRANSIENT 1 + SPECTRUM A / T-3 SECOND

Fig. B-15   Gaussian Distribution Comparison
Strain Gage 2/Transient 1 + Spectrum
A/T-3 Second

76

GAUSSIAN DISTRIBUTION COMPARISON

STRAIN GAGE 2 / TRANSIENT 1 + SPECTRUM A / TIME AVERAGED

Fig. B-16

Fig. B-16   Gaussian Distribution Comparison
Strain Gage 2/Transient 1 + Spectrum
A/Time Averaged

Fig. B-17   Gaussian Distribution Comparison
            Strain Gage 2/Transient 2 + Spectrum
            B/T-1 Second

GAUSSIAN DISTRIBUTION COMPARISON
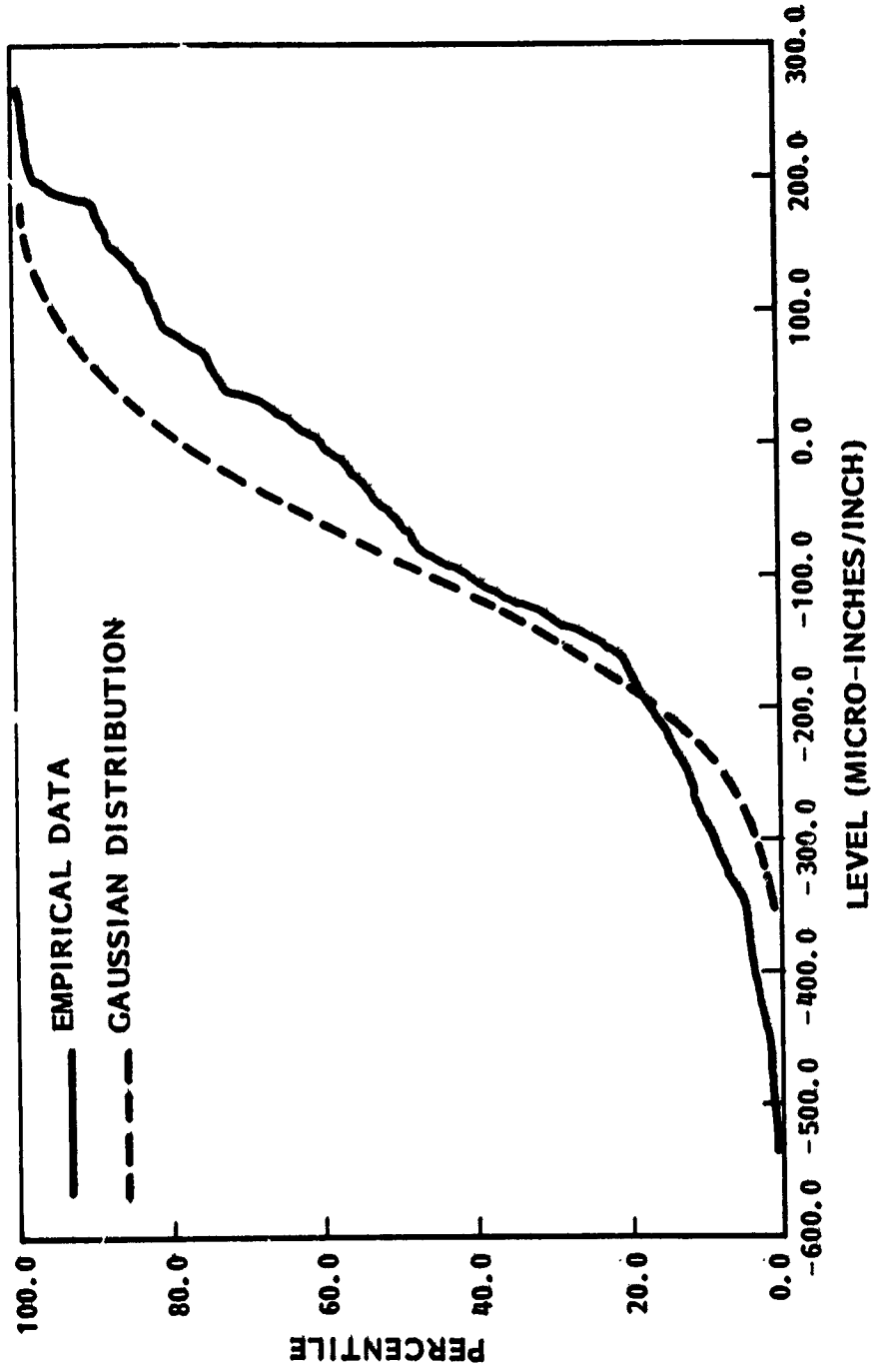STRAIN GAGE 2 / TRANSIENT 2 + SPECTRUM B / T-2 SECOND

EMPIRICAL DATA
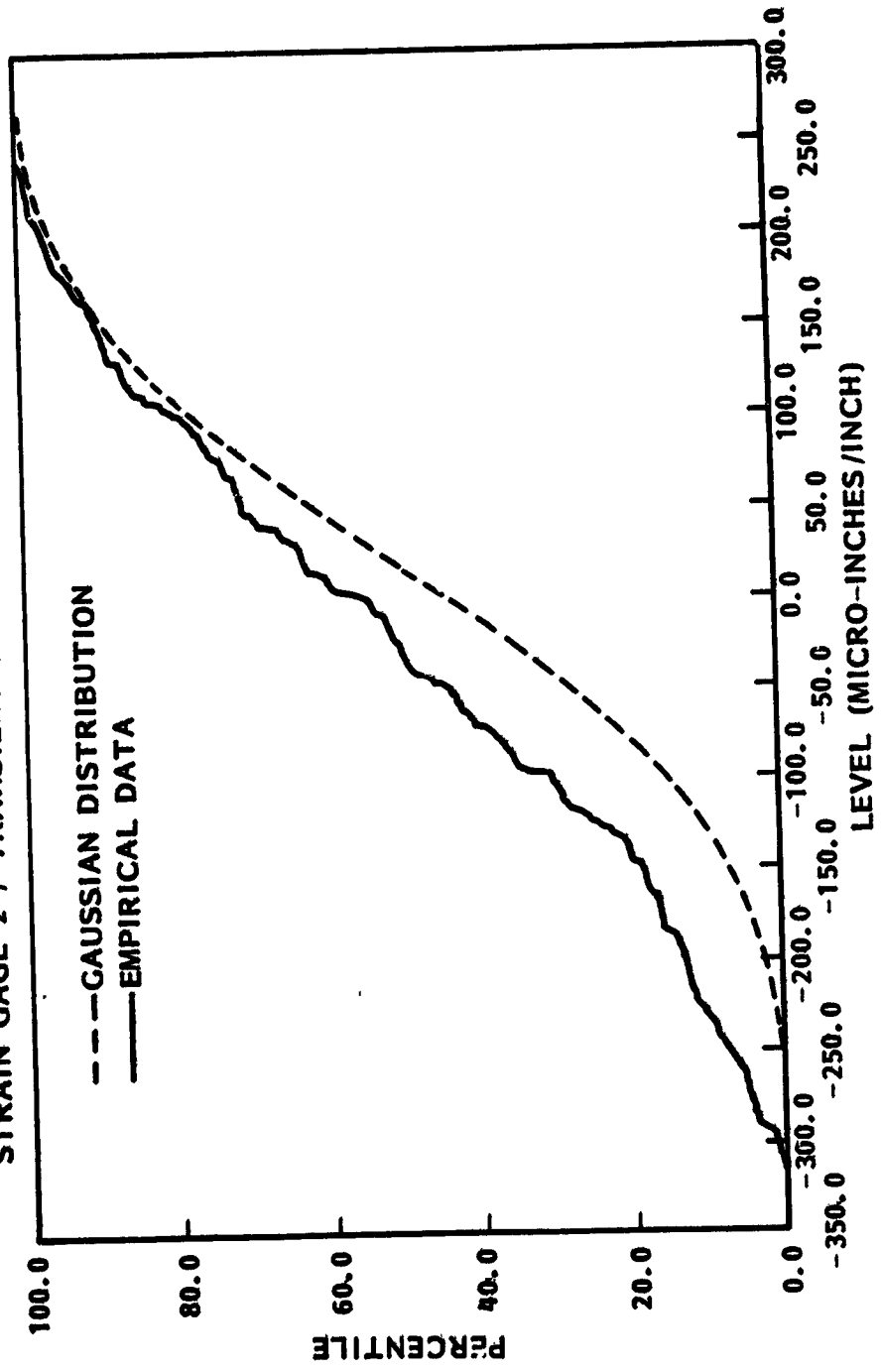GAUSSIAN DISTRIBUTION

LEVEL (MICRO-INCHES/INCH)

PERCENTILE

Fig. B-18   Gaussian Distribution Comparison
Strain Gage 2/Transient 2 + Spectrum
B/T-2 Second

GAUSSIAN DISTRIBUTION COMPARISON

STRAIN GAGE 2 / TRANSIENT 2 + SPECTRUM B / T-3 SECOND

- - - GAUSSIAN DISTRIBUTION
—— EMPIRICAL DATA

LEVEL (MICRO-INCHES/INCH)

PERCENTILE

Fig. B-19 Gaussian Distribution Comparison
Strain Gage 2/Transient 2 + Spectrum
B/T-3 Second

80

GAUSSIAN DISTRIBUTION COMPARISON
STRAIN GAGE 2 / TRANSIENT 2 + SPECTRUM B /. TIME AVERAGED

EMPIRICAL DATA
GAUSSIAN DISTRIBUTION

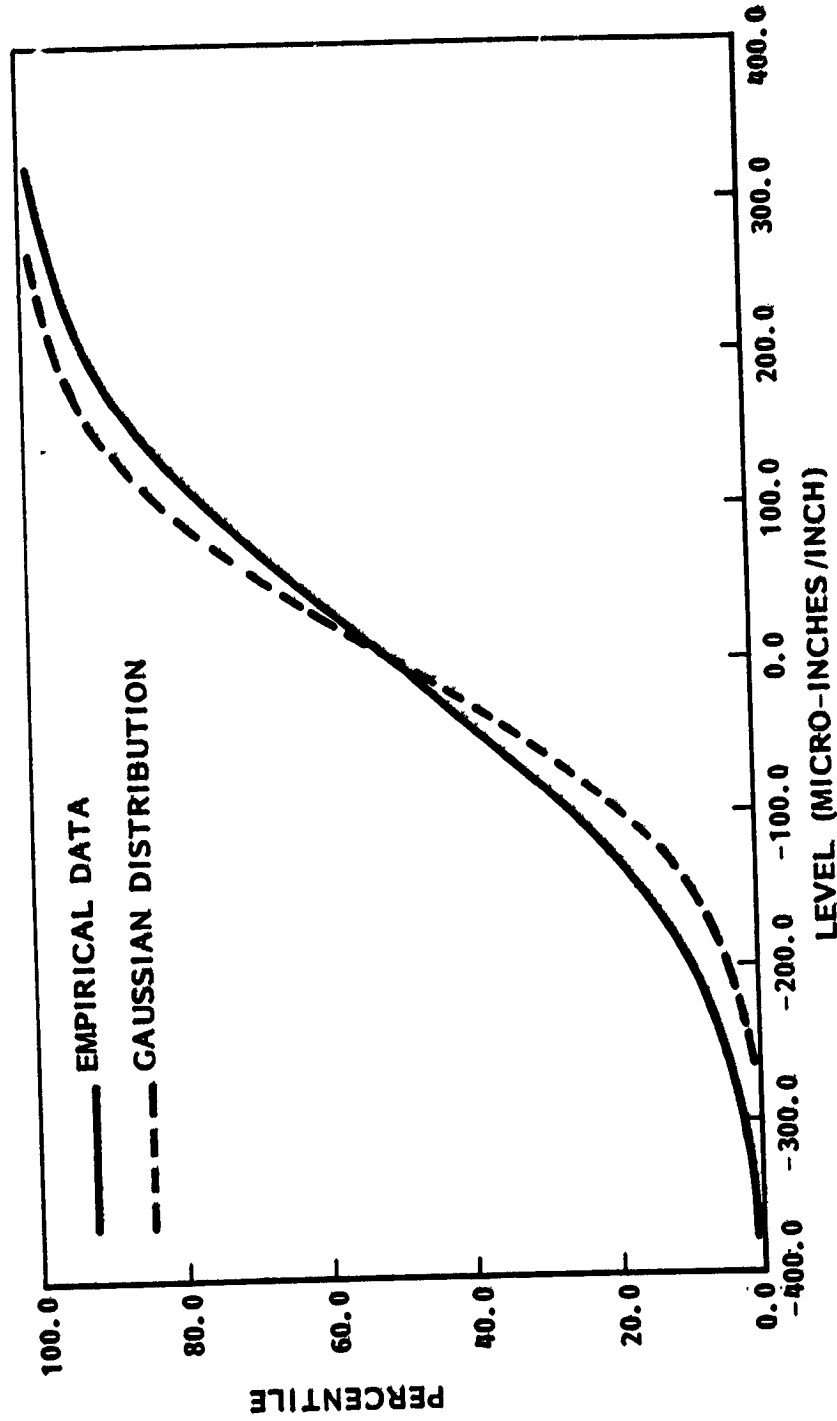LEVEL (MICRO-INCHES/INCH)

PERCENTILE

Fig. B-20   Gaussian Distribution Comparison
Strain Gage 2/Transient 2 + Spectrum
B/Time Averaged

81