NASA Contractor Report 3693

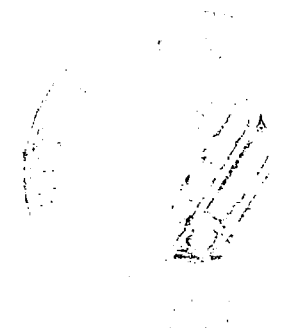# Ultrasonic Attenuation of a Void-Containing Medium for Very Long Wavelengths

James H. Williams, Jr., Samson S. Lee, and Hursit Yüce

25th Anniversary
1958-1983

NASA

NASA Contractor Report 3693

Ultrasonic Attenuation of a Void-Containing Medium for Very Long Wavelengths

James H. Williams, Jr., Samson S. Lee, and Hursit Yüce
Massachusetts Institute of Technology
Cambridge, Massachusetts

NASA

National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

1983

INTRODUCTION

In ultrasonic nondestructive evaluation (NDE), the nondestructive interrogation energy is transmitted by propagating stress waves. The parameters which govern the mechanical behavior of materials also affect their stress wave propagation characteristics.

A useful parameter in describing stress wave propagation in a medium is attenuation. Attenuation refers to the energy loss associated with the decrease in the stress wave amplitude due to both scattering and absorption. It has been observed that attenuation can be correlated with the tensile strength of flawed glass fiber composites [1] and the residual strength of impact-damaged graphite fiber composites [2]. It has also been observed that attenuation of as-fabricated graphite fiber composites can be correlated with the compression fatigue life [3] and flexural fatigue life [4]. In rock structures, attenuation of seismic waves is a major topic in exploration seismology [5].

The two major sources of ultrasonic attenuation are absorption due to viscoelastic effects of the medium and scattering from inhomogeneities in the ultrasonic beam path.

This study considers only attenuation due to scattering by voids in the medium. In the theoretical analysis, the geometries of the voids considered are spherical and ellipsoidal. The medium is assumed to be isotropic and the incident ultrasonic beam is assumed to be a plane longitudinal wave of wavelength much larger than the largest dimension of the void.

THEORETICAL ANALYSIS

An incident ultrasonic wave is scattered into various directions by voids present in the medium. In this analysis, the energy carried in the scattered wave is assumed to be entirely lost, thus contributing to ultrasonic attenuation.

It is also assumed that no interaction occurs between neighboring voids in the medium containing many randomly distributed voids. Large spacings between voids are not required in order to neglect void interactions [6,7]. It has been observed that longitudinal wave speed and attenuation developed using the void non-interaction assumption are valid up to 20% of void volume fraction [8]. Thus, scattering from an individual void can be treated independently for most void volume fractions of engineering interest. So, the combined effect of multiple voids is simply the arithmetic addition of their individual contributions.

An obstacle which scatters an incident ultrasonic wave can be described by its scattering cross section γ [9]. The scattering cross section of a scatterer is defined as the ratio of the total power scattered to the incident power flux. The scattering cross section has dimensions corresponding to area.

The scattering cross sections for a spherical void and an ellipsoidal void will be discussed below. Then an analysis is developed for evaluating the ultrasonic through-thickness attenuation based on scattering. The medium is assumed to be isotropic. The incident wave is assumed to be a plane longitudinal wave have a wavelength much larger than the largest dimension of the void.

Scattering from a Spherical Void

The exact solution for the scattering cross section γ_s having a spherical void of volume V for an incident longitudinal wave with wave number k_p is [9]

$$\gamma_s = V^2 k_p{}^4 g_c \qquad (1)$$

where

$$g_c = \frac{1}{4\pi}\left[\frac{4}{3} + 40\frac{2 + 3c^5}{(4 - 9c^2)^2} - \frac{3}{2}c^2 + \frac{2}{3}c^3 + \frac{9}{16}c^4\right] \qquad (2)$$

and where

$$c = \frac{c_p}{c_s} = \frac{k_s}{k_p} . \qquad (3)$$

The wave speed of the incident longitudinal wave is c_p, and c_s and k_s are the wave speed and wave number of the corresponding shear wave, respectively, having the same frequency as the incident longitudinal wave.

Eqn. (1) gives the exact scattering cross section of a spherical void for an incident longitudinal wave. From eqn. (1), it is observed that the scattering cross section is proportional to the square of the volume of the void and proportional to the fourth power of the incident wave number. Because the wave number k_p is related to the wavelength λ_p as

$$k_p = \frac{2\pi}{\lambda_p} , \qquad (4)$$

the scattering cross section is inversely proportional to the fourth power of the incident wavelength.

The scattering cross section γ_s in eqn. (1) also depends on the parameter c which is the ratio of the shear to longitudinal wave numbers through the term g_c defined in eqn. (2). Fig. 1 shows the scattering cross section of a spherical void for incident longitudinal waves versus c, based on eqns. (1), (2) and (3).

Scattering from an Ellipsoidal Void

No exact solution exists for the scattering cross section of an ellipsoidal void. An approximate solution will be derived below for the restrictions to be enumerated.

A schematic illustrating the scattering of ultrasonic wave by an ellipsoidal void is shown in Fig. 2. The center of the ellipsoidal void coincides with the origin of the x,y,z cartesian coordinate system. The ellipsoid can be described by

$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} + \frac{z^2}{a_3^2} = 1 \qquad (5)$$

where the principal dimensions of the ellipsoidal void are $2a_1$, $2a_2$ and $2a_3$ in the x, y and z directions, respectively. A plane ultrasonic longitudinal wave propagating in the z direction impinges upon the ellipsoidal void. The incident wave is scattered by the void into all directions described by the polar angles θ and ϕ as shown in Fig. 2. The medium is assumed to be isotropic.

The scattering of ultrasonic waves by an ellipsoidal flaw has been considered using the so-called Born approximation technique [10,11]. The Born approximation represents the first iteration on the integral equation formulation of the scattering problem [10]. The physical consequence of the use of the Born approximation in the

problem considered here is that the true displacement and strain fields inside the voids are replaced by the respective fields in the incident wave. When the Born approximation is applied to the scattering of an incident plane longitudinal wave having wave number k_p by a spherical void of radius a_o, good agreement is obtained with the exact solution given in eqn. (1), in backscattering for wavelengths such that

$$k_p a_o < 1 . \tag{6}$$

That is, for scattered waves that propagate back towards the incident wave and which satisfy eqn. (6) [11].

The differential scattering cross section $\frac{d\gamma_e}{d\Omega}$ of an ellipsoidal void for an incident longitudinal wave based on the Born approximation is* [11]

$$\frac{d\gamma_e}{d\Omega} = \frac{k_p^4}{(4\pi)^2} \left(-\cos\theta + \frac{\lambda + 2\mu \cos^2\theta}{\lambda + 2\mu} \right)^2 S_{pp}^2$$

$$+ \frac{k_p k_s^3}{(4\pi)^2} \left(-\frac{k_p \sin 2\theta}{k_s} + \sin\theta \right) S_{ps}^2 \tag{7}$$

where γ_e is the scattering cross section of an ellipsoidal void for an incident longitudinal wave; $d\Omega$ is the differential solid angle which is defined by $\sin\theta \, d\theta \, d\phi$ with reference to θ and ϕ as shown in Fig. 2 [12]; λ and μ are the Lamé constants of the parent material; and S_{pp} and S_{ps} are the shape factors of the differential scattering cross section for incident longitudinal wave scattering into longitudinal and shear waves, respectively. The shape factors are [11]

$$S_{pp} = \frac{4\pi \, a_1 a_2 a_3 \, (\sin \Delta K_{pp} - \Delta K_{pp} \cos \Delta K_{pp})}{(\Delta K_{pp})^3}$$

$$\tag{8}$$

$$S_{ps} = \frac{4\pi \, a_1 a_2 a_3 \, (\sin \Delta K_{ps} - \Delta K_{ps} \cos \Delta K_{ps})}{(\Delta K_{ps})^3}$$

---

* Note that the term $(4\pi)^2$ is missing from eqn. (2.12) in [11].

where

$$\Delta K_{pp} = k_p \left[a_1^2 \sin^2 \theta \cos^2 \phi + a_2^2 \sin^2 \theta \sin^2 \phi + a_3^2 (1 - \cos \theta)^2 \right]^{1/2}$$

$$\Delta K_{ps} = \left[k_s^2 a_1^2 \sin^2 \theta \cos^2 \phi + k_s^2 a_2^2 \sin^2 \theta \sin^2 \phi + a_3^2 (k_p - k_s \cos \theta)^2 \right]^{1/2}$$

(9)

The scattering cross section γ_e of the ellipsoidal void can be obtained by integrating eqn. (7) over a closed spherical surface centered at the flaw as

$$\gamma_e = \oiint_{\substack{\text{Surface} \\ \text{Enclosing} \\ \text{Ellipsoid}}} \frac{d\gamma_e}{d\Omega} d\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{d\gamma_e}{d\Omega} \sin \theta \, d\theta \, d\phi \ . \quad (10)$$

In order to examine the optimal utility of the Born approximation as used in [10,11], a further assumption is introduced by restricting this analysis to the very long wavelength regime such that

$$k_p a_i \ll 1 \qquad\qquad (11)$$

where a_i is the largest dimension of the ellipsoidal void. Eqn. (11) represents the regime in which the Born approximation is at its best. From eqn. (9), it can be observed that ΔK_{pp} and ΔK_{ps} are of the order of $k_p a_i$. Thus, the very long wavelength assumption in eqn. (11) requires ΔK_{pp} and ΔK_{ps} to be much less than unity. Then, the sine and cosine functions in eqn. (8) may be expanded into their respective Taylor series where only the first three terms will be retained.

After the Taylor series expansions are applied to eqns. (8), the result is substituted into eqn. (7), keeping only terms up to ΔK_{pp}^2 and ΔK_{ps}^2, which are evaluated using eqn. (9). Then, the resulting expression for $\frac{d\gamma_e}{d\Omega}$ is substituted into eqn. (10) and the indicated integrations are performed to give the scattering cross section of an ellipsoidal void γ_e as

$$\gamma_e = V^2 k_p^4 (g_{pp} \overline{S}_{pp} + g_{ps} \overline{S}_{ps}) \qquad\qquad (12)$$

where V is the volume of the ellipsoidal void given by

$$V = \frac{4}{3} \pi a_1 a_2 a_3 \tag{13}$$

and

$$g_{pp} = \frac{1}{15 \pi} \frac{(5c^4 - 10c^2 + 8)}{c^4}$$

and (14)

$$g_{ps} = \frac{1}{30 \pi} c(5 c^2 + 4) .$$

The corresponding shape factors for the scattering cross section are

$$\overline{S}_{pp} = 1 - \frac{k_p^2 a_3^2}{70} \left[\frac{(21c^4 - 56c^2 + 48) \eta + 8(21c^4 - 28c^2 + 16)}{(5c^4 - 10c^2 + 8)} \right]$$

(15)

$$\overline{S}_{ps} = 1 - \frac{k_p^2 a_3^2}{35} \left[\frac{2(7c^4 + 4c^2) \eta + (7c^4 - 9c^2 + 28)}{(5c^2 + 4)} \right]$$

where

$$\eta = \frac{a_1^2}{a_3^2} + \frac{a_2^2}{a_3^2} \tag{16}$$

Eqn. (12) gives the scattering cross section of an ellipsoidal void for an incident longitudinal wave with the very long wavelength assumption defined in eqn. (11), and based on the Born approximation.

The scattering cross section of an ellipsoidal void given in eqn. (12) is similar to that of a spherical void given in eqn. (1) because both scattering cross sections are proportional to the square of the volume of the void and proportional to the fourth power of the incident wave number. However, the shape of the void does not affect the scattering cross section of a spherical void beyond the contribution of its volume, whereas the void shape does affect the scattering cross section of an ellipsoidal void through the $\overline{S}_{pp}$ and $\overline{S}_{ps}$ shape factors.

It can be observed from eqns. (12) through (16) that the scattering cross section of an ellipsoidal void is the same if the dimensions a_1 and a_2 are interchanged. This is as expected because

interchanging a_1 and a_2 is equivalent to rotating the void about
the z-axis, which on physical grounds should not affect the scattered
power of a plane wave propagating in the z direction.

Additional observations on the effect of the shape of the void
can be made by considering voids of the same volume V and the same
dimension a_3 for an incident wave at a fixed wave number k_p. Then,
the only effect of the shape of the void on the scattering cross
section is through the term η defined in eqn. (16). For a void with a_1
equal to a_2; that is, a void that presents a circular cross-sectional
area orthogonal to the incident wave, a specific value of η is obtained
from eqn. (16) and the corresponding γ_e is obtained from eqn. (12) via
the $\bar{S}_{pp}$ and $\bar{S}_{ps}$ terms defined in eqn. (15). For a void such that a_1 is
much larger than a_2; that is, a void that presents an elongated
elliptical cross-sectional area orthogonal to the incident wave, a
larger value of η and thus smaller values of $\bar{S}_{pp}$ and $\bar{S}_{ps}$ and
correspondingly smaller γ_e are obtained. Although the two voids have
the same cross-sectional area ($\pi a_1 a_2$) orthogonal to the incident
wave, the elongated void has a smaller scattering cross section. This
is consistent with the expectation that an elongated void will provide
less attenuation than that of a less elongated void.

The very long wavelength assumption in eqn. (11) results in the
second terms in both $\bar{S}_{pp}$ and $\bar{S}_{ps}$ in eqn. (15) being much less than
the first terms, unity. In fact, when γ_e is evaluated from eqns. (12)
through (16) for a range of η from zero to 1,000,000, for $k_p a_3$ that
satisfies eqn. (11) and for c equal to 1.5, γ_e is found to vary only
by a maximum of 0.4%.

Thus, for an incident longitudinal wave having a wavelength that
is very long compared with the largest dimension of the void as stated
in eqn. (11), the exact shape of the void has little effect on the
scattered power or the scattering cross section, which depends
primarily on the volume of the void. In this case, the scattering
cross section of a spherical void as given in eqn. (1) having the same
volume as the arbitrarily shaped void can be used.

One further observation should be made regarding the accuracy
of the Born approximation. The scattering cross section of an
ellipsoidal void as given in eqn. (12) and obtained using the Born
approximation can be used to compute the scattering cross section of
a spherical void by letting a_1, a_2 and a_3 all be equal to a_0, the
radius of a spherical void. Fig. 3 shows the ratio of the scattering
cross section of a spherical void using the Born approximation in
eqn. (12) to that of a spherical void using the exact solution in
eqn. (1) versus c for two values of $k_p a_0$. The Born approximation
solution is observed to be insensitive to changes in $k_p a_0$ for very
long wavelengths satisfying eqn. (11). From Fig. 3, the approximate
solution obtained using the Born approximation is found to be at most
56% of that of the exact solution and this occurs at a value of $k_p a_0$
of zero and a value of c of approximately 1.7. Thus, the accuracy of

the Born approximation for evaluating scattering cross sections is
marginal, even for very long wavelengths. Nevertheless, the observation,
based on the Born approximation, regarding the negligible effect of the
exact shape of the void on the scattering cross section for very long
wavelengths is expected to remain valid.

Ultrasonic Through-Thickness Attenuation due to Scattering by Voids

Ultrasonic attenuation can be measured experimentally by the
through-thickness technique as illustrated schematically in Fig. 4.
The ultrasonic through-thickness attenuation will be derived in terms
of the scattering cross section of the voids and the void volume fraction
of the specimen.

As shown in Fig. 4, an input ultrasonic transducer is assumed to
apply a uniform pressure of amplitude p_0 that is sinusoidal in time
on the surface of a specimen. It is further assumed that the planar
area of the ultrasonic beam propagates through the specimen of thickness
ℓ without alternation. (Such an assumption is obviously an over-
simplification of the actual situation as a review of [13] will clearly
indicate. Nevertheless, this model is adequate for the input-output
relationship to be derived.) Portions of the input ultrasonic energy
are scattered by the randomly distributed voids, numbering n in the
ultrasonic beam path. The output transmitted pressure is measured by
a receiving ultrasonic transducer mounted on the opposite face of the specimen.
The output pressure is $p_0 e^{-\alpha\ell}$ where α is the attenuation which is a
function of frequency. It is assumed that energy carried in the
scattered waves is lost and that it represents the only energy dissipating
mechanism considered here.

The intensity I of a plane longitudinal travelling wave can be
related to the pressure p as [12]

$$I = \frac{p^2}{\rho c_p} \qquad (17)$$

where ρ is the mass density of the material and c_p is the longitudinal
wave speed. Thus, the incident ultrasonic intensity I_0 or power flux
in Fig. 4 is

$$I_0 = \frac{p_0^2}{\rho c_p} \qquad . \qquad (18)$$

In accordance with the definition of the scattering cross section
γ [9], the power lost due to scattering from a single void is γI_0.
Assuming no interaction between voids, the total power lost due to
scattering $P_{Scattered}$ by n voids is

$$P_{\text{Scattered}} = I_o \sum_{i=1}^{n} \gamma_i \qquad (19)$$

where n is the total number of voids in the ultrasonic beam and where the i-th void has scattering cross section γ_i.
Substituting eqn. (18) into eng. (19) gives

$$P_{\text{Scattered}} = \frac{p_o^2}{\rho c_p} \sum_{i=1}^{n} \gamma_i \; . \qquad (20)$$

Using the planar area A of the ultrasonic beam, the input ultrasonic power can be obtained from eqn. (18) as

$$P_{\text{In}} = \frac{p_o^2}{\rho c_p} \; A \; . \qquad (21)$$

Similarly, the output ultrasonic power can be obtained as

$$P_{\text{Out}} = \frac{(p_o e^{-\alpha \ell})^2}{\rho c_p} \; A \; . \qquad (22)$$

So, the equation for energy balance is

$$P_{\text{In}} - P_{\text{Scattered}} = P_{\text{Out}} \; . \qquad (23)$$

Substituting eqns. (20) to (22) into eqn. (23) gives

$$\frac{p_o^2}{\rho c_p} A - \frac{p_o^2}{\rho c_p} \sum_{i=1}^{n} \gamma_i = \frac{(p_o e^{-\alpha \ell})^2}{\rho c_p} A \qquad (24)$$

which simplifies to

$$A - \sum_{i=1}^{n} \gamma_i = A e^{-2\alpha\ell} . \qquad (25)$$

Eqn. (25) can be solved for the attenuation as

$$\alpha = -\frac{1}{2\ell} \ln\left(1 - \frac{1}{A} \sum_{i=1}^{n} \gamma_i\right) . \qquad (26)$$

Eqn. (26) gives the ultrasonic through-thickness attenuation of a specimen based on the scattering cross section γ_i of the i-th void. Fig. 5 shows $\alpha\ell$ versus the term $\left(1 - \frac{1}{A} \sum_{i=1}^{n} \gamma_i\right)$ based on eqn. (26). As indicated earlier, if the excitation frequency corresponds to a very long wavelength such that eqn. (11) is satisfied, the scattering cross section of each void can be evaluated from eqn. (1) for a spherical void having the same volume as the arbitrarily shaped void.

For the special case when the medium contains n identical voids, eqn. (26) can be simplified to

$$\alpha = -\frac{1}{2\ell} \ln\left(1 - \frac{n\gamma}{A}\right) \qquad (27)$$

where γ is the scattering cross section of a single void. For a specimen containing a void volume fraction V_v, the total volume of voids encompassed by the volume of the input-output ultrasonic beam is

$$nV = V_v A\ell \qquad (28)$$

where V is the volume of a single void. Eqn. (28) can be rearranged to give the number of voids in the ultrasonic input-output beam as

$$n = V_v \frac{A\ell}{V} . \qquad (29)$$

Substituting eqn. (29) into eqn. (27) gives

$$\alpha = -\frac{1}{2\ell} \ell n \left(1 - V_v \frac{\gamma \ell}{V}\right). \tag{30}$$

Eqn. (30) gives the ultrasonic through-thickness attenuation α due to scattering by voids in terms of the void volume fraction for the special case when the medium contains randomly distributed identical voids. Fig. 5 shows $\alpha\ell$ versus the term $\left(1 - V_v \frac{\gamma \ell}{V}\right)$ based on eqn. (30). It is reemphasized that the attenuation expressed in eqns. (26) and (30) is due only to scattering by voids. The total attenuation of the medium containing the voids is

$$\alpha_T = \alpha_b + \alpha \tag{31}$$

where α_T is the total attenuation and α_b is the "base" attenuation; that is, the attenuation of the material when no voids are present.

CONCLUSIONS

Ultrasonic longitudinal wave through-thickness attenuation of a void-containing medium due to scattering has been considered. The attenuation was evaluated assuming no interaction between voids. The scattered power was assumed to be entirely lost, thus contributing to the ultrasonic attenuation.

The scattered power due to the presence of a void was described by the scattering cross section of the void. An exact solution exists for the scattering cross section of a spherical void and was given in eqn. (1). The scattering cross section of an ellipsoidal void was derived in accordance with the Born approximation and was given in eqn. (12). A further assumption, restricting the approximate solution to very long wavelengths as given in eqn. (11), was made.

The most striking result, although not totally unexpected, was that the exact shape of a void has negligible effect on its scattering cross section in the very long wavelength regime as expressed in eqn. (11). The scattering cross sections of spherical and ellipsoidal voids are proportional to the square of the volume of the void and proportional to the fourth power of the incident wave number. Thus, it is suggested that when very long wavelengths are considered, the exact solution of the scattering cross section of a spherical void of the same volume should be used for arbitrarily shaped voids.

The ultrasonic through-thickness attenuation was derived in terms of the scattering cross section of the individual voids encountered by the ultrasonic beam path and was given in eqn. (26). For the special case when a medium contains identical voids, the attenuation was derived in terms of the void volume fraction and the scattering cross section as given in eqn. (30).

This study demonstrates the scattering effects of voids for waves of very long wavelengths satisfying eqn. (11). The results provide an analytical basis for the simple modelling of voids in attenuation measurements; that is, model arbitrarily shaped voids as spherical voids of the same volume.

Experimental verification of the results are encouraged. Also, it is observed that, as implemented here, the Born approximation is of somewhat marginal utility in evaluating scattering cross sections of voids. Thus, a different approximation procedure [14] may be employed to better estimate scattering cross sections, especially for shorter wavelengths as expressed by eqn. (6).

REFERENCES

[1] J.H. Williams, Jr., S.S. Lee and T.K. Wang, "Quantitative Non-destructive Evaluation of Automotive Glass Fiber Composites", Journal of Composite Materials, Vol. 16, January 1982, pp. 20-39.

[2] J.H. Williams, Jr., and N.R. Lampert, "Ultrasonic Evaluation of Impact-Damaged Graphite Fiber Epoxy Composite", Materials Evaluation, Vol 38, No. 12, December 1980, pp. 68-72.

[3] J.H. Williams, Jr., and B. Doll, "Ultrasonic Attenuation as an Indicator of Fatigue Life of Graphite Fiber Epoxy Composites", Materials Evaluation, Vol. 38, No. 5, May 1980, pp. 33-37.

[4] J.H. Williams, Jr., H. Yuce and S.S. Lee, "Ultrasonic and Mechanical Characterizations of Fatigue States of Graphite Epoxy Composite Laminates", Materials Evaluation, Vol. 40, No. 5, April 1982, pp. 560-565.

[5] M.N. Toksez and D.H. Jonston, ed. , Seismic Wave Attenuation, Geophysics Reprint Series, No. 2, Society of Exploration Geophysicists, Tulsa, Oklahoma, 1981.

[6] J.G. Berrymen, "Long-Wavelength Propagation in Composite Elastic Media, I. Spherical Inclusions", Journal of Acoustical Society of America, Vol. 68, No. 6, December 1980, pp. 1809-1819.

[7] J.G. Berrymen, "Long-Wavelength Propagation in Composite Elastic Media, II. Ellipsoidal Inclusions", Journal of Acoustical Society of America, Vol. 68, No. 6, December 1980, pp. 1820-1831.

[8] G.T. Kuster, "Seismic-Wave Propagation in Two-Phase Media and its Applications to the Earth's Interior", Ph.D. Thesis, Department of Earth and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, Mass., June 1972.

[9] C.F. Ying and R. Truell, "Scattering of a Plane Longitudinal Wave by a Spherical Obstacle in an Isotropically Elastic Solid", Journal of Applied Physics, Vol. 27, No. 9, September 1956, pp. 1086-1097.

[10] J.E. Gubernatis, E. Domany and J.A. Krumhansl, "Formal Aspects of the Theory of the Scattering of Ultrasound by Flaws in Elastic Materials", Journal of Applied Physics, Vol. 48, No. 7, July 1977, pp. 2804-2811.

[11] J.E. Gubernatis, E. Domany, J.A. Krumhansl and M. Huberman, "The Born Approximation in the Theory of the Scattering of Elastic Waves by Flaws", Journal of Applied Physics, Vol. 48, No. 7, July 1977, pp. 2812-2819.

[12] A.D. Pierce, Acoustic, McGraw-Hill Book Co., N.Y., 1981.

[13] J.H. Williams, Jr., H. Karagulle and S.S. Lee, "Ultrasonic Input-
      Output for Transmitting and Receiving Longitudinal Transducers
      Coupled to Same Face of Isotropic Elastic Plate", Materials
      Evaluation, Vol. 40, No. 6, May 1982, pp. 655-662.

[14] J.E. Gubernatis, "Long-Wave Approximations for the Scattering
      of Elastic Waves from Flaws with Applications to Ellipsoidal Voids
      and Inclusions", Journal of Applied Physics, Vol. 50, No. 6,
      June 1979, pp. 4046-4058.
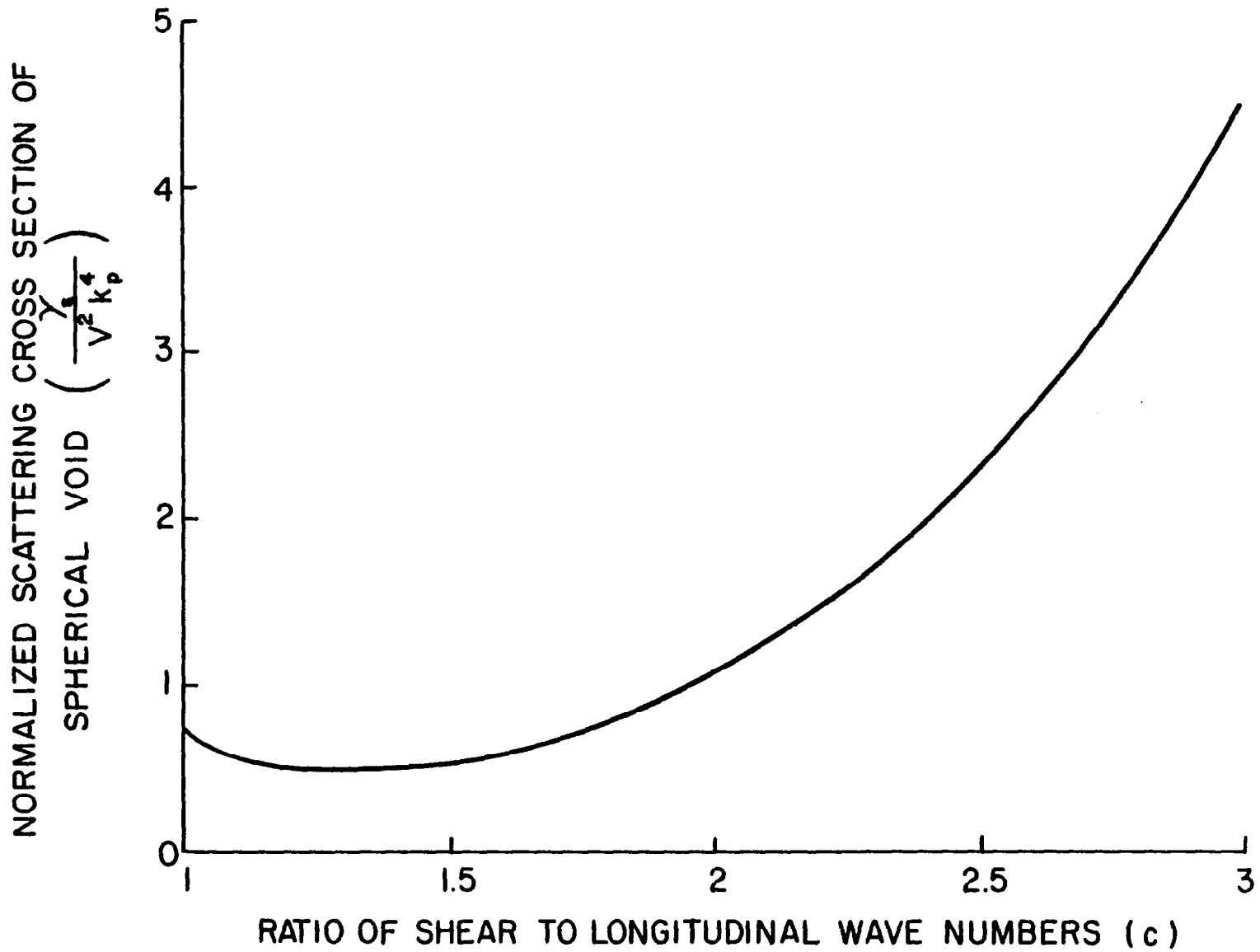
Fig. 1 Scattering cross section of a spherical void for incident longitudinal waves versus ratio of shear to longitudinal wave numbers.
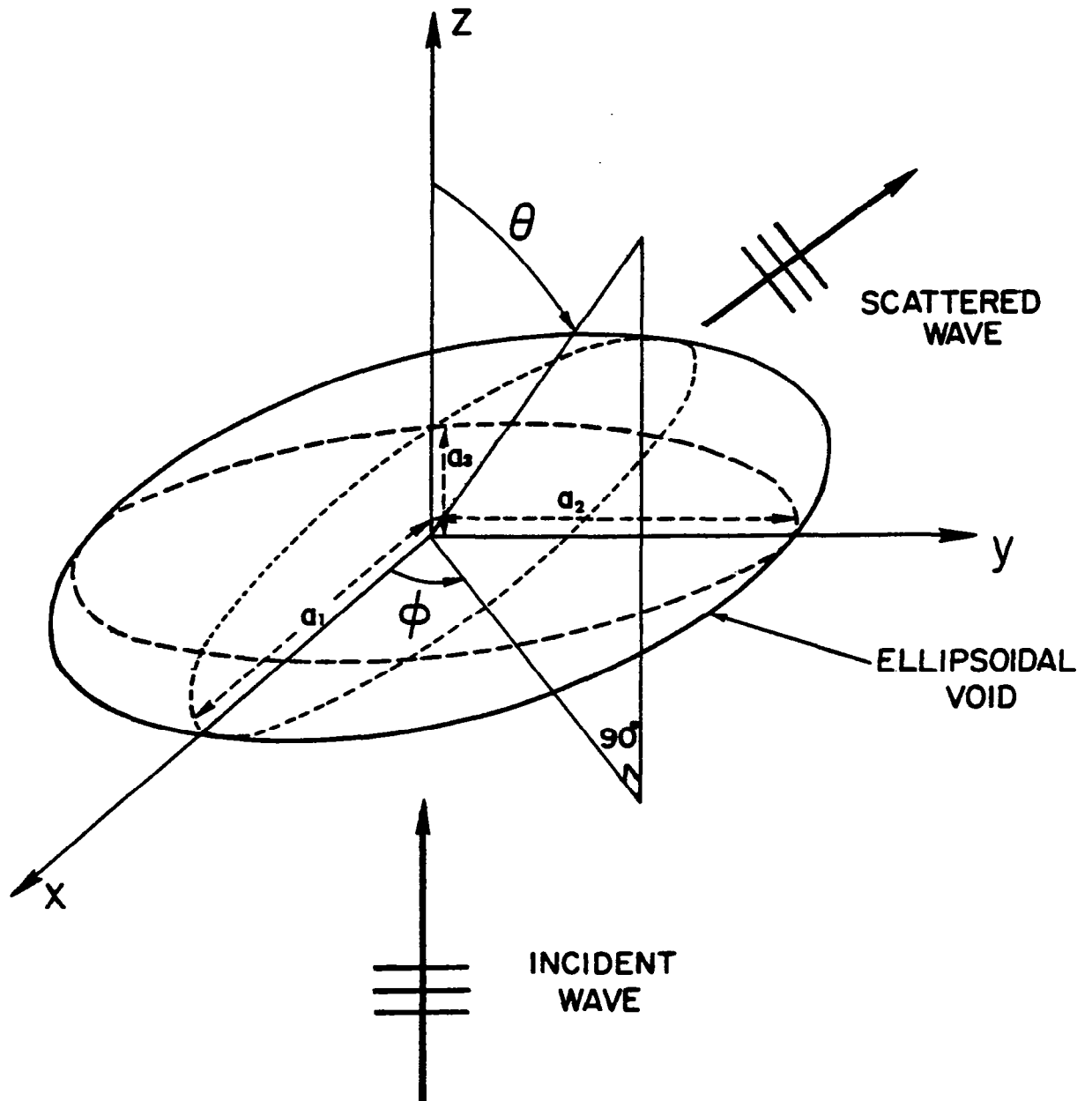
Fig. 2   Schematic of scattering of ultrasonic plane longitudinal wave
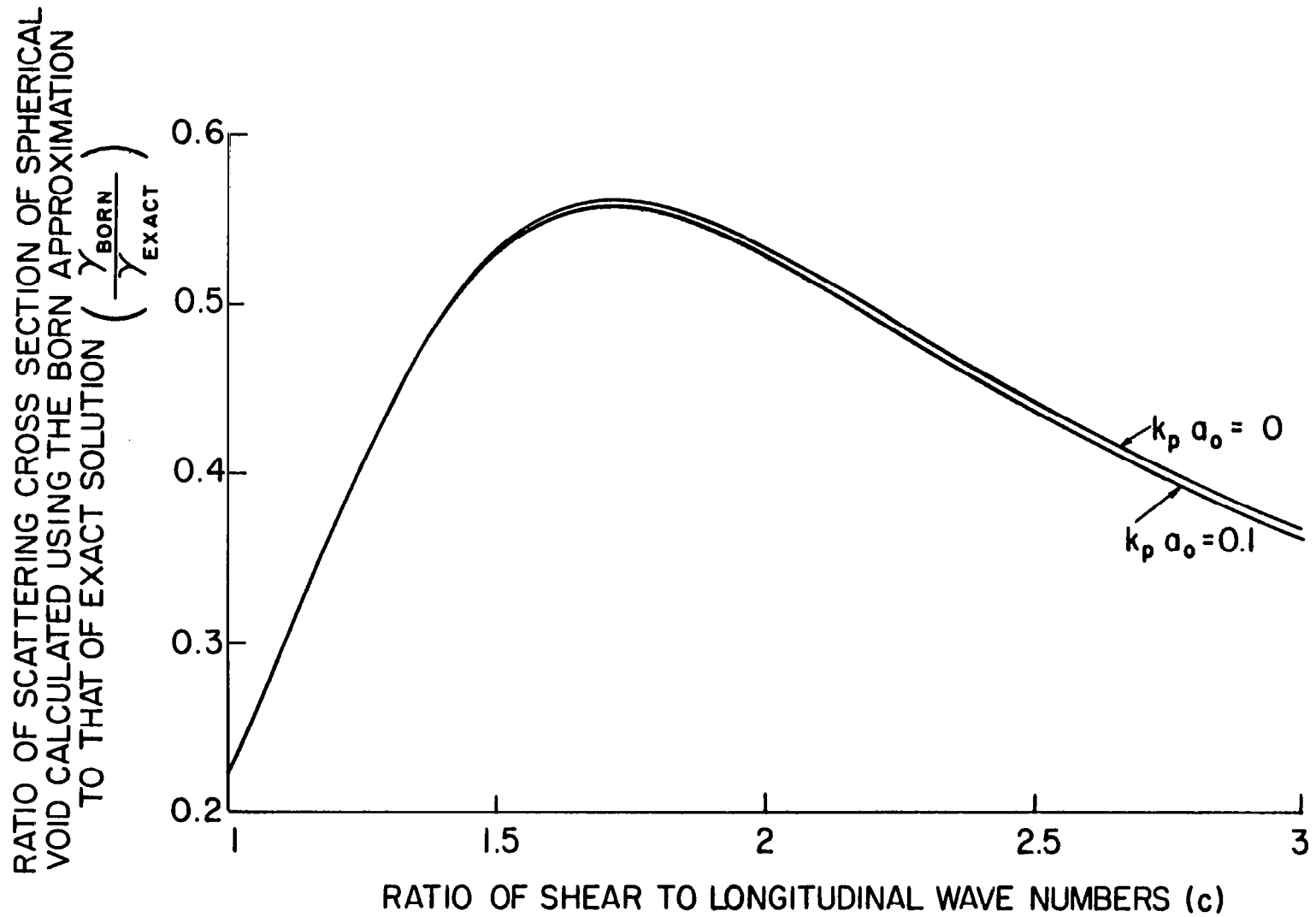by an ellipsoidal void.

Fig. 3 Ratio of scattering cross section of an ellipsoidal void based on the Born approximation reduced to a spherical void to that of the exact solution versus the ratio of shear to longitudinal wave numbers c for $k_p a_o \ll 1$.
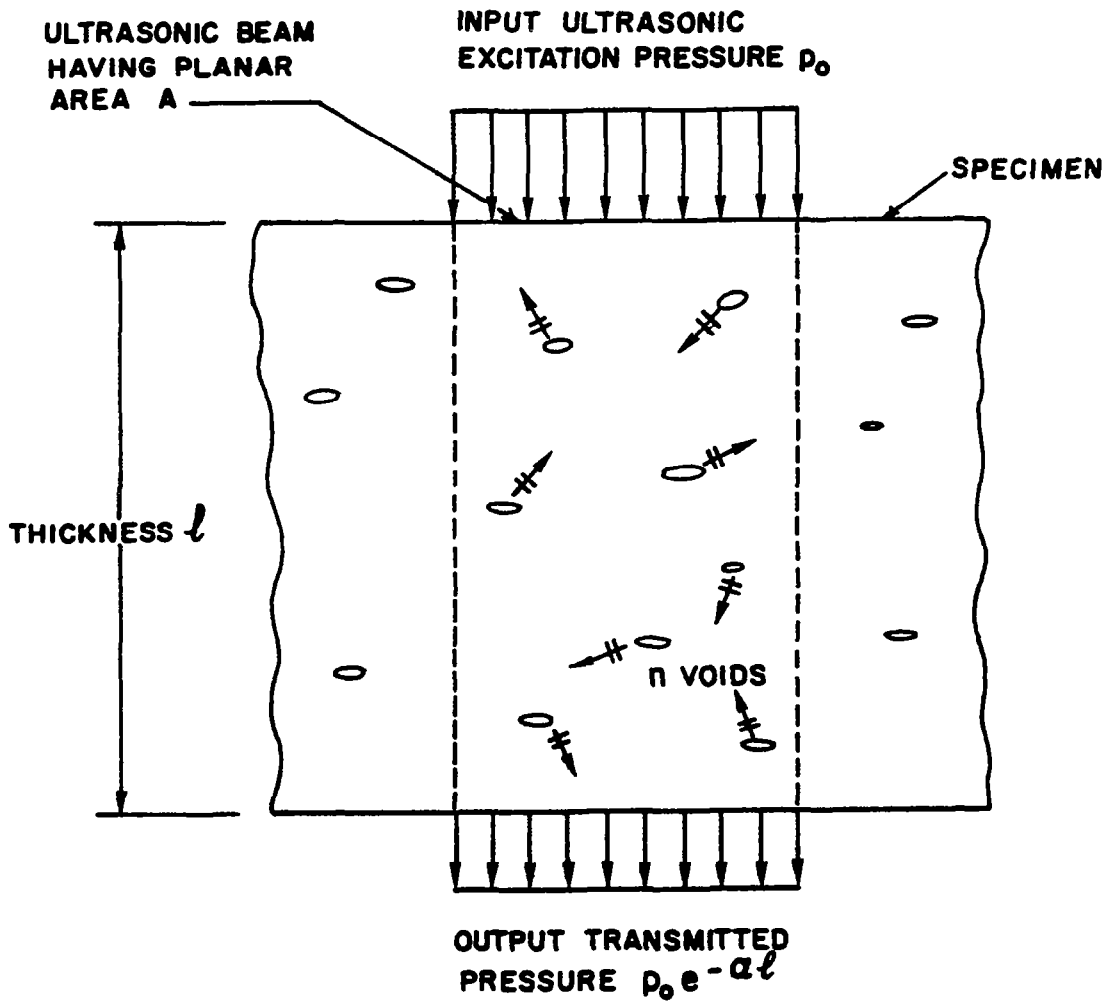
Fig. 4 Schematic illustrating ultrasonic through-thickness testing of a specimen containing voids, n of which are in the ultrasonic input-output beam path.
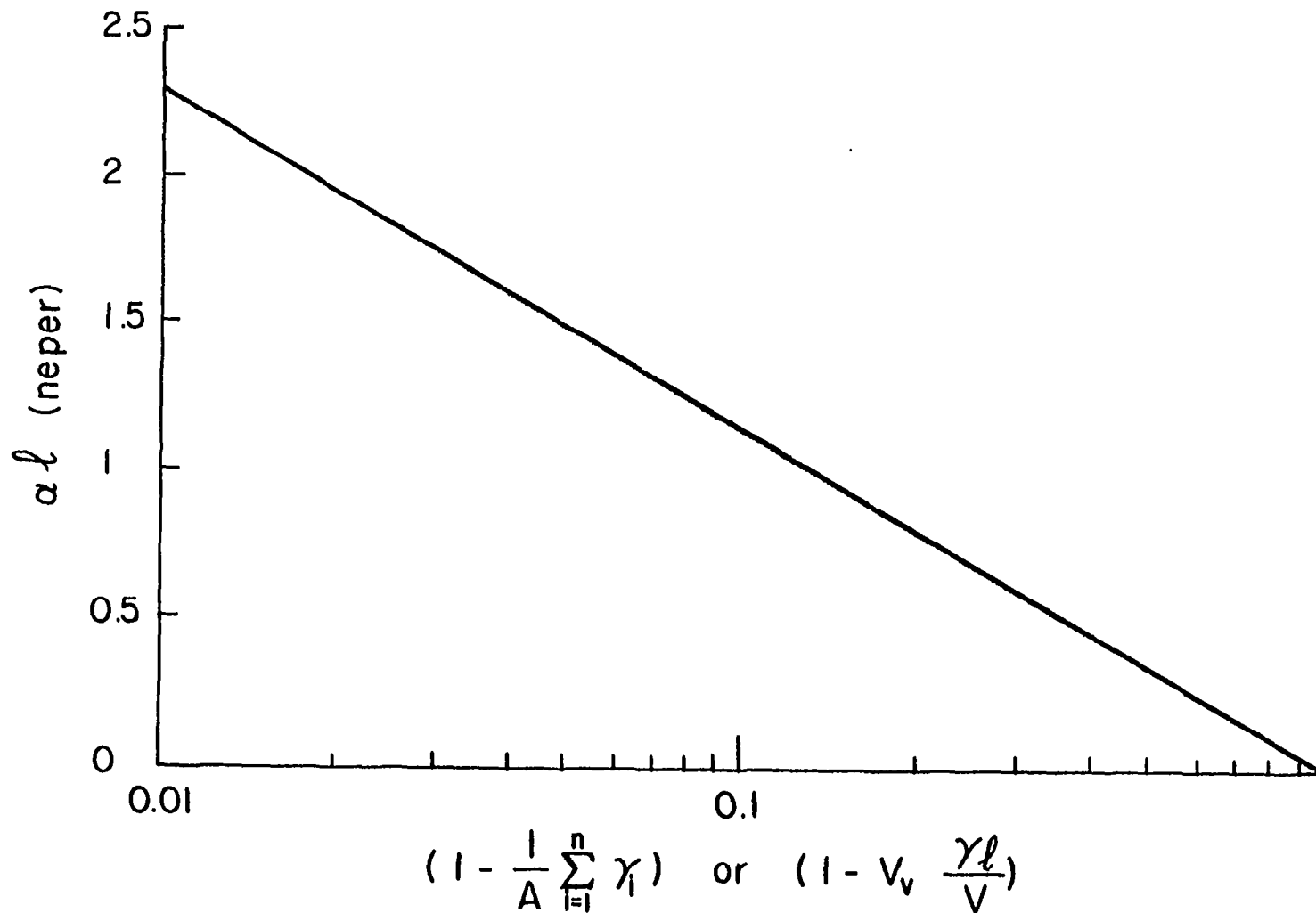
Fig. 5 Attenuation α of a specimen of thickness ℓ due to scattering by voids versus $(1 - \frac{1}{A} \sum_{i=1}^{n} \gamma_i)$ where A is the ultrasonic beam area and γ_i is the scattering cross section of the i-th void or $(1 - V_v \frac{\gamma \ell}{V})$ for the special case when the medium with void volume fraction V_v contains many identical voids in the beam path, each with scattering cross section γ and volume V.

1. Report No. NASA CR-3693	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle ULTRASONIC ATTENUATION OF A VOID-CONTAINING MEDIUM FOR VERY LONG WAVELENGTHS		5. Report Date June 1983
		6. Performing Organization Code
7. Author(s) James H. Williams, Jr., Samson S. Lee, and Hursit Yüce		8. Performing Organization Report No. None
		10. Work Unit No.
9. Performing Organization Name and Address Massachusetts Institute of Technology Department of Mechanical Engineering Cambridge, Massachusetts 02139		11. Contract or Grant No. NSG-3210
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code 505-33-22 (E-1639)

15. Supplementary Notes

Final report. Project Manager, Alex Vary, Materials Division, NASA Lewis Research Center, Cleveland, Ohio 44135.

16. Abstract

Ultrasonic longitudinal through-thickness attenuation in an isotropic medium due to scattering by randomly distributed voids is considered analytically. The attenuation is evaluated on the assumption of no interaction between voids. The scattered power is assumed to be entirely lost, thus accounting for the ultrasonic attenuation. The scattered power due to the presence of a void is described in terms of the scattering cross section of the void. An exact solution exists for the scattering cross section of a spherical void. An approximate solution for the scattering cross section of an ellipsoidal void is developed based on the so-called Born approximation commonly used in quantum mechanics. This approximate solution is valid for $k_p a_i \ll 1$, where k_p is the wave number of the incident longitudinal wave and a_i is the largest dimension of the void. It is found that the shape of the void has negligible effect on the scattering cross section and that only the volume of the void is important. Thus, it is noted that in cases where $k_p a_i \ll 1$, the exact scattering cross section of a spherical void having the same volume as an arbitrarily shaped void can be used for evaluating ultrasonic attenuation. This study provides an analytical basis for such a simple substitutional rule.

17. Key Words (Suggested by Author(s)) Nondestructive testing; Ultrasonics; Ultrasonic scattering; Ultrasonic attenuation; Scattering cross-section; Voids; Porosity; Void volume; Void content	18. Distribution Statement Unclassified - unlimited STAR Category 38

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages 21	22. Price* A02