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PARAMETER IDENTIFICATION

IN

CONTINUUM MODELS+

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ABSTRACT

We discuss approximation techniques for use in numerical schemes for estimating spatially varying coefficients in continuum models such as those for Euler-Bernoulli beams. The techniques are based on quintic spline state approximations and cubic spline parameter approximations. Both theoretical and numerical results are presented.

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PARAMETER IDENTIFICATION IN CONTINUUM MODELS

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ABSTRACT

We discuss approximation techniques for use in numerical schemes for estimating spatially varying coefficients in continuum models such as those for Euler-Bernoulli beams. The techniques are based on quintic spline state approximations and cubic spline parameter approximations. Both theoretical and numerical results are presented.

Introduction

We discuss here our efforts on the development of numerical algorithms for the estimation of parameters in variable structure elastic models. The class of problems we investigate is of fundamental importance in the use of continuum models (such as those for beams, plates, thin shells, etc.) to represent large flexible space structures [1-4]. Our own research has been motivated by the need to detect and/or estimate structural / material property changes in such structures while they are on orbit. In this regard the usefulness of parameter estimation techniques (i.e., inverse algorithms) for spatially varying parameters in models such as

$$\rho(x)\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 u}{\partial x^2}\right) = f(\beta,t,x) \qquad (1)$$

from the Euler-Bernoulli theory or in more sophisticated models arising in the Timoshenko theories should be rather obvious. In this presentation we describe spline-based methods that use and extend in a nontrivial way the ideas developed in [5-7] for constant parameter systems. These extensions allow one to treat variable parameters (such as mass density ρ, elastic modulus EI, load parameters β) in models such as (1) and were first developed in the context of variable coefficient first order parabolic systems in [8].

Although our ideas have much wider applicability in elastic structures, to illustrate the basic principles we, for simplicity, restrict our considerations here to a simply supported beam with constant mass density and variable stiffness, which is under a known load f. We assume that we have normalized our model and thus consider (with $D = \frac{\partial}{\partial x}$)

$$u_{tt} + D^2(\alpha(x)D^2u) = f(t,x) \quad 0 \le x \le 1, \quad t \geq 0, \qquad (2)$$

with boundary conditions

$$u(t,0) = u(t,1) = u_{xx}(t,0) = u_{xx}(t,1) = 0 \qquad (3)$$

and initial conditions

$$u(0,x) = \phi(x)$$
$$u_t(0,x) = \psi(x) \ . \qquad (4)$$

The methods presented here readily extend to more general cases (e.g., unknown parameters in more general boundary conditions, in initial conditions or load, variable mass and damping, etc.) as will be discussed in a forthcoming paper.

The problem we consider is that of estimating functions α in (2) from observations of the state u. To be more specific, we consider a least squares fit-to-data for (2). That is, given observations $\hat{y}_{ij}$ for for $u(t_i,x_j;\alpha)$, where u is the solution to (2)-(4) corresponding to the parameter α from the set Q of admissible parameters, we seek a parameter α^* that minimizes

$$J(\alpha) = \sum_{i,j} |u(t_i,x_j;\alpha) - \hat{y}_{ij}|^2 \qquad (5)$$

over Q.

We outline briefly the ideas of our approach to this problem. We first rewrite (2)-(4) as an abstract system in an appropriately chosen Hilbert space $Z = Z(\alpha)$, our state space. The resulting abstract identification problem is then approximated on finite dimensional state subspaces $Z^N(\alpha)$ which in our case here are generated by quintic spline elements. We then obtain estimates $\bar{\alpha}^N$ from minimizing a fit-to-data criterion using the approximate states.

The problem of obtaining $\bar{\alpha}^N$ is, however, not computationally feasible since it requires that a minimization procedure be carried out over an infinite dimensional function space or set Q. We therefore introduce a further approximation by employing sets Q^M (in the results reported below these involved cubic spline interpolations to the elements in Q) in the minimization problems. This results in a double approximation procedure: a state approximation (spaces Z^N to approximate Z) and a parameter approximation (sets Q^M to approximate Q); thus computationally we obtain least squares parameters $\bar{\alpha}^{N,M}$ and any convergence discussions must involve a double limit process for $N \to \infty$, $M \to \infty$. As we shall note below, the Trotter-Kato approximation theorem for linear semigroups, which we have successfully used in a number of other problems [5,6,7,9], can be used in the present case to establish a convergence theory. (For the convenience of the reader, we note that we are using the notation α for the unknown parameter function here whereas in the general formulations of [5-9] the symbol q is used to denote a vector of unknown parameters.) We proceed to outline the schemes we have used, discuss briefly convergence results, and present some of our numerical findings.

Quintic/Cubic Spline Approximation Schemes

Given (2)-(4), we rewrite this system in abstract form in the parameter dependent state space $Z(\alpha) \equiv (H^2_\alpha \cap H^1_0) \times H^0$. Here H^2 is the space of elements in $H^2(0,1)$ with inner product

$\langle z,w\rangle_\alpha = \langle \alpha D^2 z_1, D^2 w_1\rangle_0 + \langle z_2, w_2\rangle_0$ where $\langle\ ,\ \rangle_0$ is the usual inner product in $H^0(0,1) = L_2(0,1)$. Define the operator $A(\alpha)$ by

$$A(\alpha) = \begin{pmatrix} 0 & 1 \\ -D^2(\alpha D^2) & 0 \end{pmatrix}$$

on $\text{dom}(A) = \{z \in H^4 \times H^4 \mid z_1(n) = z_2(n) = D^2 z_1(n) = D^2 z_2(n) = 0,\ n = 0,1\}$. Then putting $z = (u,u_t)^T$, $z_0 = (\phi,\psi)^T$, and $F = (f,0)^T$, we can rewrite the system (2)-(4) as

$$\begin{aligned} \dot z(t) &= A(\alpha)z(t) + F(t) \qquad t > 0, \\ z(0) &= z_0. \end{aligned} \tag{6}$$

The estimation problem is then one of minimizing

$$J(\alpha) = \sum_{i,j} |z_1(t_i;\alpha)(x_j) - \hat{y}_{ij}|^2 \tag{7}$$

over Q, where we assume Q is a given subset of $H^2(0,1)$ such that $0 < a \le \alpha(x) \le b$ for each $\alpha \in Q$. (Here a and b are given bounds for elements in Q.)

Turning to approximations for (6), we let $S^5(\Delta^N)$ denote the set of quintic splines (C^4 functions that are piecewise polynomials of degree 5 -- see [6,10] for notation) corresponding to the partition $\Delta^N = \{x_j\}_{j=0}^N$, $x_j = j/N$, of $[0,1]$. We then define $Z^N(\alpha)$ as the subspace of $Z(\alpha)$ given by $Z^N = S_0^5(\Delta^N) \times S_0^5(\Delta^N)$ where $S_0^5(\Delta^N) \equiv \{s \in S^5(\Delta^N) \mid s(n) = D^2 s(n) = 0,\ n = 0,1\}$. Let $P^N(\alpha): Z \to Z^N$ be the orthogonal projection of $Z(\alpha)$ onto $Z^N(\alpha)$ and define approximations to A by $A^N(\alpha) \equiv P^N(\alpha)A(\alpha)P^N(\alpha)$. The approximations to (6) in $Z^N(\alpha)$ are then taken to be

$$\begin{aligned} \dot z^N(t) &= A^N(\alpha)z^N(t) + P^N F(t) \qquad t > 0, \\ z^N(0) &= P^N z_0 . \end{aligned} \tag{8}$$

The approximate estimation problem is that of minimizing

$$J^N(\alpha) = \sum_{i,j} |z_1^N(t_i;\alpha)(x_j) - \hat{y}_{ij}|^2 \tag{9}$$

over Q.

In order to establish a convergence theory for such Galerkin-type estimation schemes, it is not difficult (see [5,8,9]) to argue that it suffices to show that $z^N(t;\alpha^k) \longrightarrow z(t;\alpha^*)$, $N \to \infty$, $k \to \infty$, for <u>any</u> sequence $\{\alpha^k\}$ in Q converging to α^* in Q. The topology for this convergence depends on the problem at hand and in the present case, it is desirable to use the H^2 topology on Q. Using fundamental estimates from spline approximation theory (in particular, minor modifications of estimates given in Lemmas 1.18 and 1.19 of [11] suffice) and fundamental properties of dissipative operators, one can readily employ the Trotter-Kato theorem (see [5]) to establish the desired convergence $z^N(t;\alpha^k) \longrightarrow z(t;\alpha^*)$ whenever $\alpha^k \to \alpha^*$ in Q. Furthermore, letting $\bar\alpha^N$ be a solution of minimizing (9) over Q, if we have $\bar\alpha^N$ (or a subsequence $\bar\alpha^{N_j}$) converging to some $\bar\alpha$ in Q (this requires a compactness property for Q), it can be argued that $\bar\alpha$ is a solution to the original parameter estimation problem involving (6) and (7). Hence assuming that Q is compact in H^2, we have a sequence of problems which approximate in a desired sense our original problem.

As we have already observed, the problem of minimizing (9) over Q subject to (8) involves minimization over an infinite dimensional parameter space and a further approximation is necessary to obtain an implementable computational procedure. For this we use cubic interpolatory spline approximations to the elements of Q. For a given partition $\Delta^M = \{y_j\}_{j=0}^M$, $y_j = j/M$, of $[0,1]$, let I^M be the cubic spline interpolation map (see [12, p.48]) corresponding to Δ^M and define $Q^M \equiv I^M(Q)$. Then for a given function $\alpha \in Q$, $I^M(\alpha)$ involves the values $\alpha(y_j)$, $j = 0,1,\ldots,M$, and $D\alpha(0)$, $D\alpha(1)$, so that $I^M: Q \to Q^M$ is continuous in the C^1 topology on Q and the C^0 topology on Q^M. If Q is compact in the H^2 topology, then it is therefore easily seen that Q^M is compact in $C(0,1)$ and furthermore has a representation

$$Q^M = \left\{\alpha: [0,1] \to R^1 \;\middle|\; \alpha = \sum_{j=1}^{M+3} \gamma_j \beta_j^M,\ \gamma_j \in \Gamma_j^M\right\}$$

where $\{\beta_j^M\}$ are the cardinal cubic basis elements and the Γ_j^M are compact subsets of R^1. It follows that Q^M is compact in the H^2 topology and hence the problem of minimizing J^N over Q can be readily replaced by the finite dimensional state space - finite dimensional parameter space problem of minimizing J^N over Q^M. Under additional smoothness assumptions on the elements of Q, one can employ standard estimates for interpolatory splines to argue that Q^M approximates Q in an appropriate sense and that solutions of the problem for J^N over Q^M approximate those of the problem of minimizing J over Q. More precisely one can establish the following convergence results.

<u>Theorem</u>. Suppose $Q \subset H^4(0,1)$ with $\{|D^4\alpha|_0\}_{\alpha\in Q}$ bounded and suppose further that Q is compact in the $H^2(0,1)$ topology. Let $Q^M = I^M(Q)$ be the cubic spline interpolatory approximations to Q and let, for each N and M, $\bar\alpha^{N,M}$ denote a solution of the finite state-finite parameter space problem of minimizing J^N of (9) over Q^M. Let $\{\bar\alpha^{N_k,M_j}\}$ be any convergent (in the H^2 topology) subsequence with limit α^*; then α^* is a solution to the problem of minimizing J of (7) over Q.

<u>Numerical Results</u>

We have developed and tested software packages based on the quintic/cubic spline scheme discussed above. The packages were modifications of those (described in some detail in [6]) we have used for constant parameter estimation problems (e.g., the IMSL

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package DGEAR is used to solve the ordinary differential equations (8), while the IMSL version - ZXSSQ - of the Levenberg-Marquardt algorithm is used in minimizing J^N over Q^M). To test the method and software, "data" was generated in the following manner: A "true" parameter function α^* was chosen and an independent numerical algorithm (finite differences) was used to generate corresponding solution values $\hat{y}_{ij} = \hat{u}(t_i,x_j)$, a subset of which was then used as observations or data in our inverse algorithms. We present here the results from one of our test examples. Additional numerical results along with a complete theoretical treatment will be given in a paper currently in preparation.

Example. We take (2) with $\alpha^*(x) = .15 + .10 \tanh(5(x-.5))$, a constant load $f = 10$, and assume that the beam is initially at rest ($\phi=\psi=0$). Observations at points $x_j = .1,.2,\ldots,.9$ and times $t_i = .1,.2,\ldots,$ 1.0, were used in the least squares criterion (9). For a given value of M, Q^M involves $M+3$ cubic splines for the approximation of coefficients in (2) while the index N for the state involves $N+1$ quintic spline elements in the state approximation. We present in tabular form some of the results we obtained for several values of N and M. The values denoted by J in this table are $J^N(\bar{\alpha}^{N,M})$ while E denotes the L_2 norm of the error in the parameter estimates, i.e., $E = |\alpha^* - \bar{\alpha}^{N,M}|_0$. Several graphs of $\bar{\alpha}^{N,M}$ vs. α^* follow the references.

		N=2	N=4	N=6
M=1	J	$.21\times10^{-3}$	$.43\times10^{-4}$	$.20\times10^{-4}$
	E	.0207	.0106	.0055
M=2	J	$.18\times10^{-3}$	$.13\times10^{-4}$	$.16\times10^{-4}$
	E	.0213	.0115	.0060
M=3	J	$.18\times10^{-3}$	$.91\times10^{-5}$	$.34\times10^{-5}$
	E	.0048	.0146	.0010

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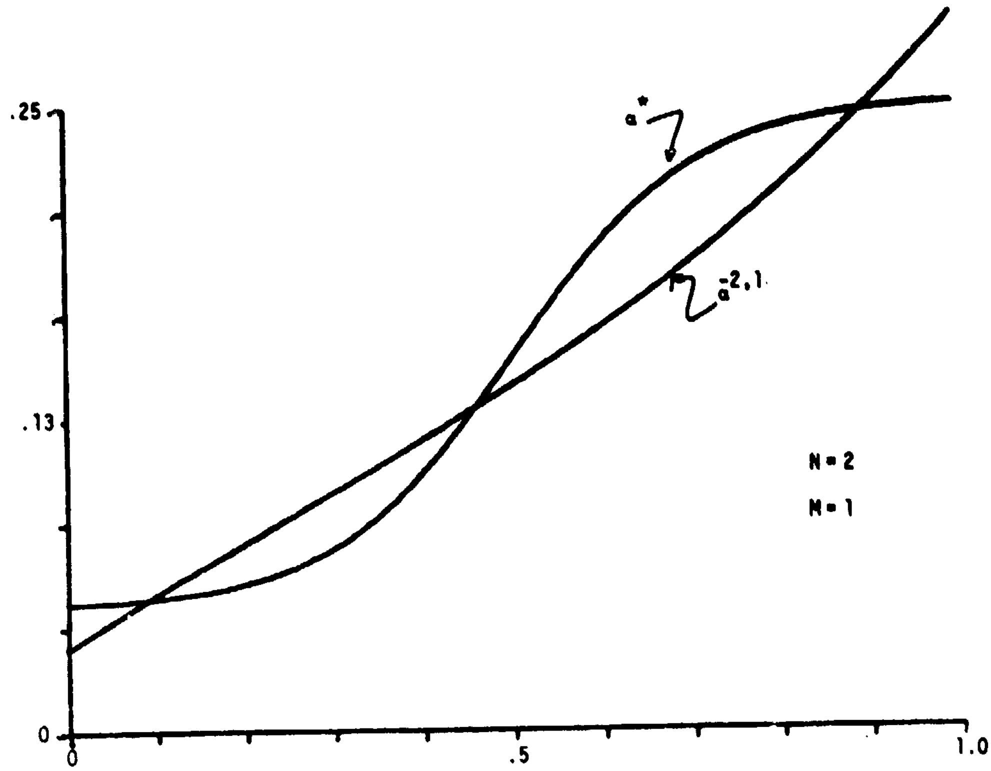

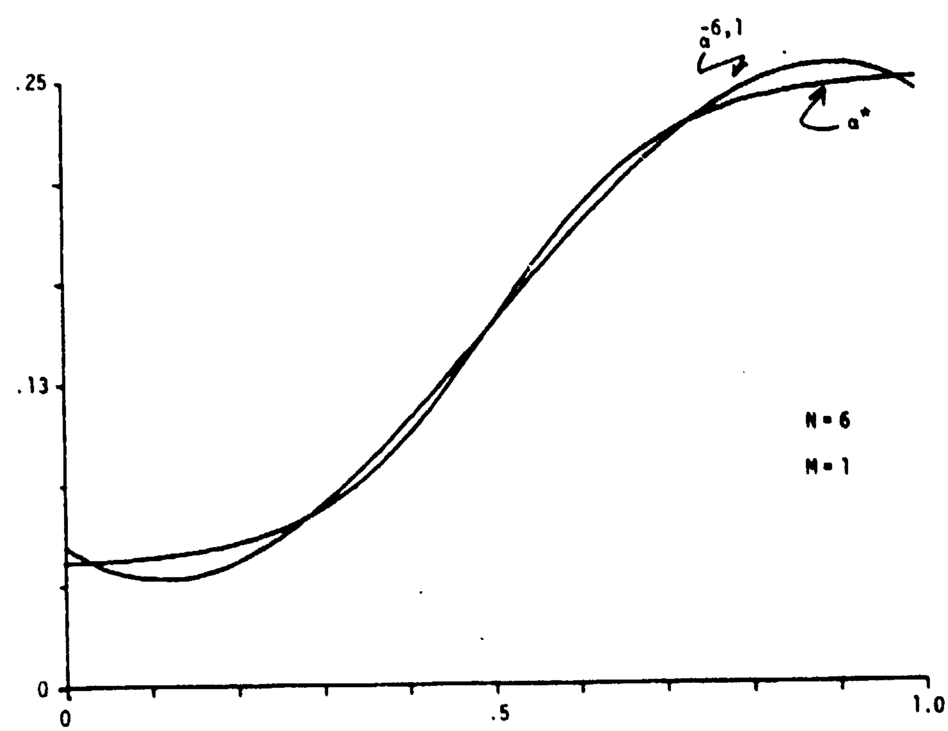

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