

NASA CR-172151

NASA Contractor Report 172151

NASA-CR-172151
19830021177

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Contract No. NAS1-17070
June 1983

INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING
NASA Langley Research Center, Hampton, Virginia 23665

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CAD OF CONTROL SYSTEMS: APPLICATION OF NONLINEAR PROGRAMMING TO A LINEAR-QUADRATIC FORMULATION

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ABSTRACT. The familiar suboptimal regulator design approach is recast as a constrained optimization problem and incorporated in a CAD package where both design objective and constraints are quadratic cost functions. This formulation permits the separate consideration of, for example, model-following errors, sensitivity measures and control energy as objectives to be minimized or limits to be observed. Efficient techniques for computing the interrelated cost functions and their gradients are utilized in conjunction with a nonlinear programming algorithm. The effectiveness of the approach and the degree of insight into the problem which it affords is illustrated in a helicopter regulation design example.

Keywords. Computer-aided design; control engineering computer applications; multivariable control systems; nonlinear programming; optimal control; optimization.

INTRODUCTION

Since the interest in control system optimization generated by the linear quadratic regulator (LQR) approach there have been a number of quite separate developments. Some workers followed the pattern set by Levine and Athans (1970) who imposed constraints on controller structure but retained the scalar quadratic cost function adopted in LQR design. Various features can be incorporated within this context such as sensitivity reduction and model-following and an efficient solution technique typically employs an unconstrained gradient minimization algorithm (see Fleming 1979). Computationally this is an attractive approach but from the design point of view it is dogged by the difficulty of using a scalar quadratic measure to describe all the desired facets of system performance.

Another line of attack was prompted by Zakian's Method of Inequalities (Zakian, 1973) in which system constraints and specifications are represented by a set of simultaneous algebraic inequalities. Control engineers found this approach attractive because the problem description could include such basic control design parameters as overshoot, damping, settling time, etc. Polak and Mayne advanced this idea by posing semi-infinite programming problems in which design objectives are realized by minimizing a function subject to a set of inequalities where these objectives can be expressed as infinite dimensional constraints (Mayne and co-workers, 1982).

Such problems require special mathematical programming algorithms to handle the infinite dimensional constraints and, sometimes, nondifferentiable functions.

The method described here strikes a compromise between these two approaches in that the quadratic measures of the suboptimal linear regulator approach are retained but the problem is recast as a constrained minimization problem in which the constraints may represent bounds on control energy, model-following errors, sensitivity, etc. The minimizing controller gain matrix is obtained efficiently using readily available software tools and a CAD package has been mounted on a 32-bit minicomputer. What appears at first sight to be a relatively straightforward extension of suboptimal regulator design yields a numerical solution technique which, from the designer's viewpoint, is much more efficient than the original design approach and provides results which reveal useful insights into the design problem. The method is exercised within the CAD context by means of a design example on helicopter regulation in which the identification of active constraints at the minimum plays a leading role in understanding the nature of the problem.

PROBLEM STATEMENT

For clarity a simplified statement of the problem follows:
Given the linear time-invariant plant description

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$$\dot{x}_p = A_p x_p + B_p u_p, \quad x_p(0) = x_{p0}, \quad (1)$$

$$y_p = C_p x_p, \quad (2)$$

and feedback control law

$$u_p = K y_p, \quad (3)$$

find the control, $u_p(t)$, which minimizes the quadratic cost function

$$J_0 = \int_0^{\infty} (x_p^T Q_0 x_p + u_p^T R_0 u_p) dt = \int_0^{\infty} x_p^T \bar{Q}_0 x_p dt, \quad (4)$$

$$\text{where } \bar{Q}_0 = Q_0 + C_p^T K^T R_0 K C_p,$$

subject to the inequality constraints

$$J_i < z_i, \quad i = 1, 2, \dots, q,$$

where J_i is a quadratic cost function of the form

$$J_i = \int_0^{\infty} x_p^T \bar{Q}_i x_p dt \quad (5)$$

$$\text{and } \bar{Q}_i = Q_i + C_p^T K^T R_i K C_p.$$

The inequality bounds, z_i , are designer-specified and the weighting matrices, Q_i, R_i , $i = 0, 1, \dots, q$, which are usually diagonal, are also selected by the designer.

For the full problem formulation, as incorporated in the CAD package, the plant state vector, x_p , may be augmented by a state compensator vector, x_c , a trajectory sensitivity vector, x_B , and a model state vector, x_m to accommodate a dynamic compensator option for the controller and the inclusion of sensitivity reduction and model-following terms in the cost functions (see Fleming, 1979).

One application, for example, might seek a controller gain matrix, K , to achieve a certain degree of model-following within some control energy constraints, i.e.,

$$\text{minimize } \int_0^{\infty} (y_p - y_m)^T Q_0 (y_p - y_m) dt,$$

w.r.t. K

subject to

$$\int_0^{\infty} u_p^2 dt < z_1, \quad i=1, 2, \dots, m,$$

where y_m represents the desired model output response. This example is typical of one which would be solved using the straightforward suboptimal linear regulator approach by successive minimization of

$$J = \int_0^{\infty} \{ (y_p - y_m)^T Q_p (y_p - y_m) + u_p^T R_p u_p \} dt,$$

where the designer strives to find the appropriate choice of Q_p and R_p to satisfy the control energy constraints. Thus what was previously solved in an approximate way in a number of unconstrained minimizations is now solved in a single constrained minimization.

FORMULATION OF THE NONLINEAR PROGRAMMING PROBLEM

If we let k represent the minimizing variables, i.e. the variable elements of gain matrix, K , then we have the nonlinear programming problem:

$$\text{minimize } J_0,$$

$$\text{w.r.t. } k,$$

subject to the inequality constraints

$$J_i - z_i < 0, \quad i=1, 2, \dots, q.$$

The cost functions, J_i , $i=0, 1, \dots, q$, Eqs. (4) and (5), can be computed from

$$J_i = \text{tr}(P_i X_0), \quad i=0, 1, \dots, q, \quad (6)$$

where $X_0 = x_{p0} x_{p0}^T$ and P_i satisfies the Liapunov matrix equation

$$P_i \bar{A} + \bar{A}^T P_i = -\bar{Q}_i, \quad i=0, 1, \dots, q, \quad (7)$$

where $\bar{A} = A_p + B_p K C_p$, (see Eqs. (1) - (3)).

Gradient information may be obtained from the gradient matrices

$$\partial J_i / \partial K = 2(B_p^T P_i + R_i K C_p) \Lambda C_p^T, \quad i=0, 1, \dots, q \quad (8)$$

where Λ satisfies the Liapunov matrix equation

$$\Lambda \bar{A}^T + \bar{A} \Lambda = -X_0. \quad (9)$$

For the case where initial conditions are not known matrix X_0 may be modified so that either the expected values, the average values or the worst-case values of x_{p0} in J_i , $i=0, 1, \dots, q$ are measured (see Fleming, 1979).

There are a variety of constrained optimization algorithms employing gradient information which will solve this problem. In the implementation (Vanderplaats, 1973) of Zoutendijk's Method of Feasible Directions (Zoutendijk, 1960) which was used, the algorithm initially finds a feasible point and then proceeds by iteratively searching along feasible directions, employing the conjugate direction method of Fletcher and Reeves (1964) if no constraints are violated. At each iteration gradient information is required only for the active or violated constraints.

IMPLEMENTATION OF THE METHOD

To compute the objective function, Eq. (4) and the constraint functions, Eq. (5), together with their associated gradients, Eq. (8), matrices P_i , $i=0, 1, \dots, q$ and Λ must be obtained from the set of Liapunov

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matrix equations (7) and (9) at each iteration of the minimization algorithm; also the set of equations (7) must be solved several times within each iteration to compute the objective and constraint functions at each step of the linear search. The method of Bartels and Stewart (1972), generally accepted as the most efficient Liapunov matrix equation solver, is a transformation method which reduces \bar{A} , Eq. (7), to its real Schur form and then obtains the solution by solving a set of low order ($q+2$) linear equations. It is well suited for this application where only one Schur reduction of \bar{A} per estimate of K is necessary for the solution of the ($q+2$) equations (7) and (9). The solution of these equations is further speeded up by exploiting certain structural and sparseness properties of Q_1 and X_0 .

The adoption of the Bartels-Stewart method has the added advantage that system eigenvalues may be determined for each new estimate of K from the diagonal and principal subdiagonal elements of the real Schur form of \bar{A} at little extra computational cost. Monitoring of eigenvalues is necessary since Eq. (7) is only valid for stable values of \bar{A} . Although the problem is such that, in general, the minimization routine will tend to generate stabilizing values of K , computational traps have been set to inhibit unstable excursions of K . Should such an excursion take place the line search step is repeatedly halved until a stable value of K is reached. Clearly the minimization routine also requires that the initial estimate of K stabilize \bar{A} , and, when the open-loop plant is unstable, should it be necessary, the program will search for a stabilizing value of K using a steepest descent technique similar to that of Koenigsberg and Frederick (1970).

The CAD program operates in conversational mode permitting easy modification of design parameters Q_1 , R_1 , and z_1 as well as specification of the gain parameters in matrix K to be optimized. Solution times are dependent on the number of constraints but typically the numerical minimization takes 2 - 4 times longer to compute than the corresponding suboptimal regulator approach. However this method is much more efficient in achieving the design goal thereby reducing design time as well as affording the designer increased understanding of the problem.

DESIGN EXAMPLE - HELICOPTER REGULATION

The plant describing helicopter longitudinal dynamics (Michael and Farrar, 1973) has four states and two controllers:

$$\dot{x}_p^T = [u_x, u_z, \theta, \delta]^T, \quad u_p = [u_1, u_2]^T.$$

Prior to this design a standard LQR design had been carried out and a satisfactory result obtained which gave good regulation for u_x and u_z (forward and vertical velocities) observing control magnitude constraints on u_1 and u_2 (longitudinal cyclic pitch and main rotor collective pitch). These controllers utilize feedback from all four plant states, i.e. $u = K_{opt} x$, and the resulting responses are characterized by

$$\dot{x}_p = (A_p + B_p K_{opt}) x_p - A_m x_m,$$

where A_m is defined to be the "model" matrix. Since feedback from u_x and u_z requires airspeed sensors which are undesirable, the objective here is to examine whether similar regulation responses can be achieved without sensing u_x and u_z and without violating control magnitude constraints.

We therefore wish to minimize an objective function which measures the error between the new plant response and the model response:

$$J_0 = \int_0^{\infty} (x_p - x_m)^T Q_0 (x_p - x_m) dt,$$

where $Q_0 = \text{diag}(1, 1, 0, 0)$,
subject to the constraints

$$J_1 < z_1, \quad i=1, 2,$$

where

$$J_1 = \int_0^{\infty} u_1^2 dt, \quad z_1 = 4.06,$$

$$J_2 = \int_0^{\infty} u_2^2 dt, \quad z_2 = 6.57$$

$$\text{and } x_{p0} = [1 \ 1 \ 1 \ 1]^T.$$

The values of z_1 and z_2 correspond to the values of J_1 and J_2 obtained for $u = K_{opt} x$ with the above initial conditions. The constraints therefore place limits on control energy and indirectly limit control magnitudes.

Various controller structures were considered:

$$\text{Controller 1: } \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \theta \\ \delta \end{bmatrix}$$

$$\text{Controller 2: } \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \theta$$

$$\text{Controller 3: } \begin{bmatrix} u_1 \\ u_2 \\ \dot{x}_{c1} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ 1 & k_{32} \end{bmatrix} \begin{bmatrix} \theta \\ x_{c1} \end{bmatrix}$$

TABLE 1 Design Results for Various Controller Configurations

Controller	Gain Matrix K	J_0	Active Constraints	$\max u_1(t) $	$\max u_2(t) $
LQR	$\begin{bmatrix} -2.56 & 0.29 & 0.50 & 0.24 \\ 0.38 & 1.98 & -0.26 & -0.06 \end{bmatrix}$	0	J_1, J_2	1.60	1.66 (2.04)
1	$\begin{bmatrix} 0.16 & 0.14 \\ -0.25 & 0.22 \end{bmatrix}$	0.072	J_2	1.07	1.60
2	$\begin{bmatrix} 0.18 \\ -0.25 \end{bmatrix}$	0.35	J_2	1.17	1.60
3	$\begin{bmatrix} 0.32 & -0.24 \\ -0.06 & -0.30 \\ 1 & -1.37 \end{bmatrix}$	0.10	---	1.13	1.47
4	$\begin{bmatrix} 0.25 & 0.36 \end{bmatrix}$	1.50	---	1.18	---
5	no feasible solution	60.2	$J_2 = 426.3$	---	8.43

$$\text{Controller 4: } u_1 = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$\text{Controller 5: } u_2 = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

Design results are tabulated above. Note that for the dynamic compensator, Controller 3, in order to obtain a minimal parameter form element k_{31} is nonvarying and set to unity.

Excellent model-following is achieved with Controller 1, pitch angle and pitch rate feedback, although there is some deterioration when Controller 2, pitch angle feedback alone, is used. This situation is remedied by using a first-order dynamic compensator together with pitch angle feedback, Controller 3. Graphical results, Figs. 1 and 2, illustrate the model-following capabilities of these three controllers. Controllers 4 and 5 explore the possibility of using u_1 or u_2 alone. Poor model-following is realized by Controller 4 and it was found to be impossible to stabilize the system using Controller 5 within the constraint bounds. The algorithm failed to find a feasible solution for this case in ten iterations; the results at this point are presented.

The last two columns of Table 1 and Fig. 3 indicate that the control energy constraints are effective in keeping the control magnitudes within bounds. The LQR controller has an initial value of 2.04 (bracketed in the table) which swiftly falls from this level. It is assumed, therefore, that the level $u_2(t) = -1.66$ is the more important magnitude to be observed since control effort is around this level for a significant period. Figure 3 illustrates

how well this bound is observed. If, however, u_2 magnitudes up to 2.04 can be accommodated then constraint 2 should be relaxed by increasing z_2 by an appropriate factor.

From the results we learn that the magnitude of u_2 is the limiting component which prevents further improvement in model-following capability with Controller Structures 1 and 2. With the dynamic compensator structure, Controller 3, its best performance is achieved within the constraints. Controller Structures 4 and 5 have proved themselves incapable of producing good results.

The implications, then, are clear:

- i) Both controls u_1 and u_2 are necessary,
- ii) It is desirable to feedback θ and $\dot{\theta}$ however, if $\dot{\theta}$ is unavailable, satisfactory results can be achieved with a first-order compensator,
- iii) A relaxation of the magnitude constraint on u_2 will lead to improved results.

It is in the nature of the problem formulation that designs for each controller structure were obtained in one optimization process enabling the designer to effectively experiment with different controllers structures. Further, using the gain-fixing facility of the program, system integrity can be taken into account in the manner of Fleming (1981).

CONCLUDING REMARKS

A new optimization approach based on the linear-quadratic formulation has been proposed for control system design.

Application to a Linear-Quadratic Formulation

Exploiting the mathematical tractability of the formulation an efficient CAD program has evolved for solving linear regulator problems. The use of constraint functions in the problem description frees the designer to a large extent from the chore of weighting matrix selection and presents additional information concerning the nature of the design problem.

Additional features may be incorporated within the program. For example, the inclusion of side constraints on gain values will be particularly useful in dynamic compensator design and the handling of large parameter variations may be implemented in a similar way to that of Vinkler and co-workers (1979). Also, since plant eigenvalues and eigenvectors are easily obtained from the Schur form of A arising from the approach these may also be incorporated in the design objective and constraints.

Although the package performs well current work is directed at further improving its effectiveness by identifying the best optimization algorithm in terms of speed of convergence and robustness for this particular nonlinear programming problem.

ACKNOWLEDGEMENT

The author is grateful to Nancy Shoemaker, ICASE, for her computational assistance and to A. A. Schy and D. P. Giesy, NASA Langley Research Center, for several valuable discussions relating to this work.

Research reported in this paper was supported by the National Aeronautics and Space Administration under Contract No. NAS1-17070 while the author was in residence at the Institute for Computer Applications in Science and Engineering, NASA Langley Research Center, Hampton, VA 23665.

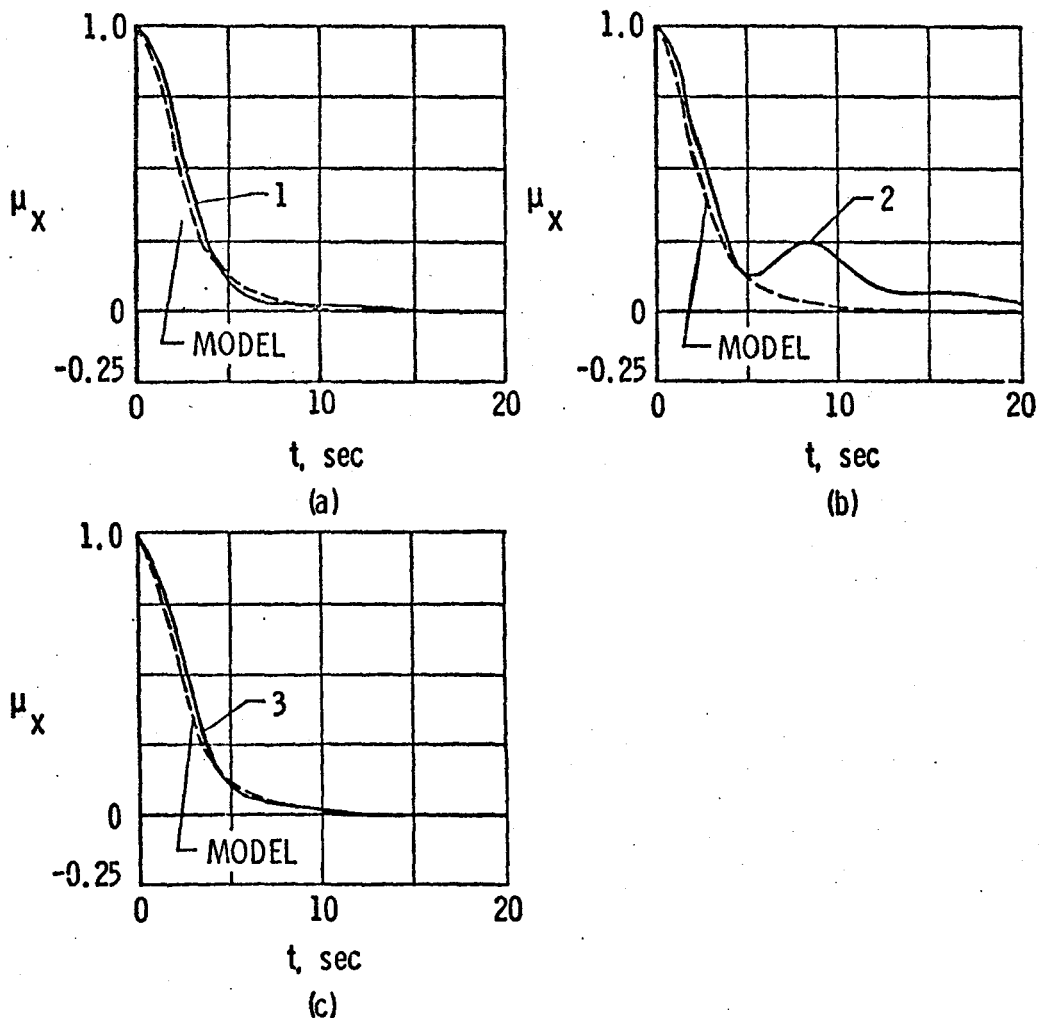


Fig. 1. Forward velocity responses, μ_x , illustrating model-following capabilities of a) Controller 1, b) Controller 2 and c) Controller 3.

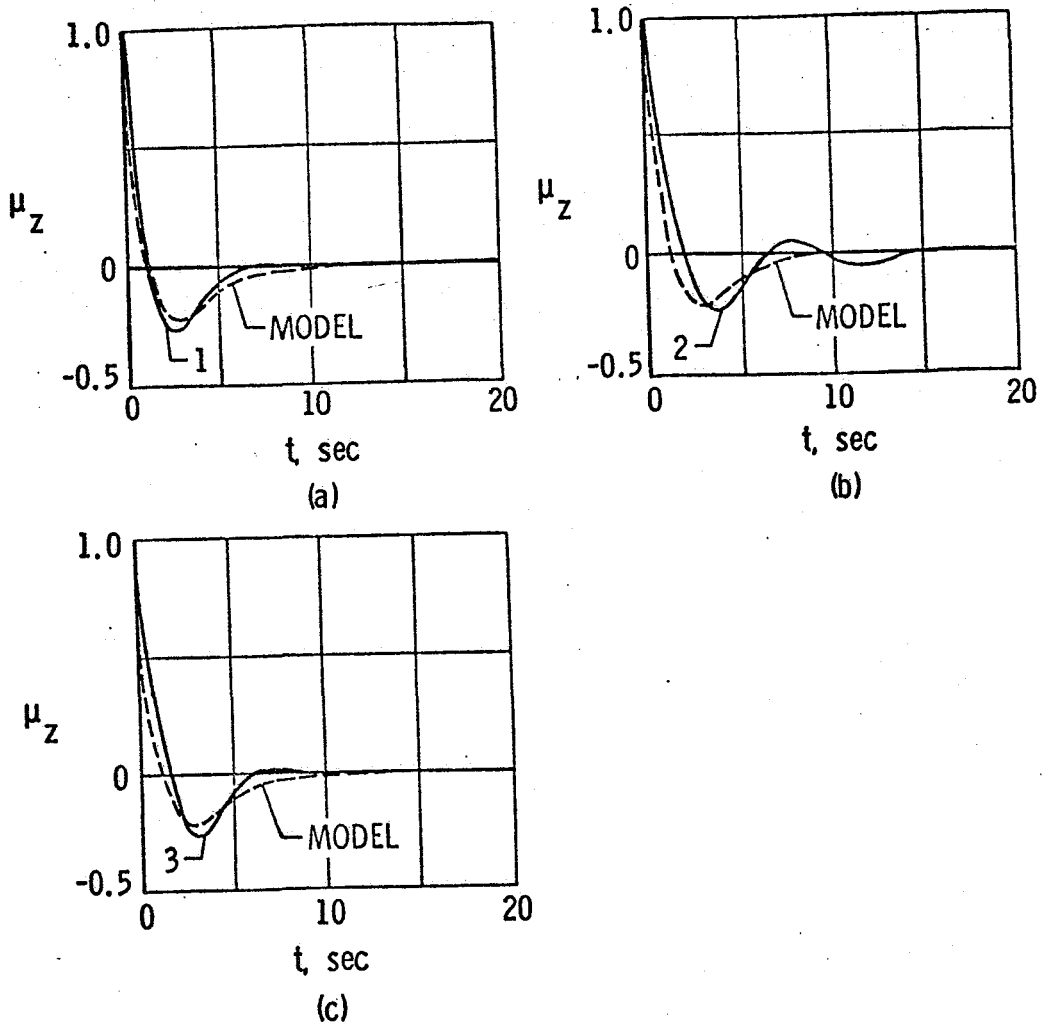


Fig. 2 Vertical velocity responses, μ_z , illustrating model-following capabilities of a) Controller 1, b) Controller 2 and c) Controller 3.

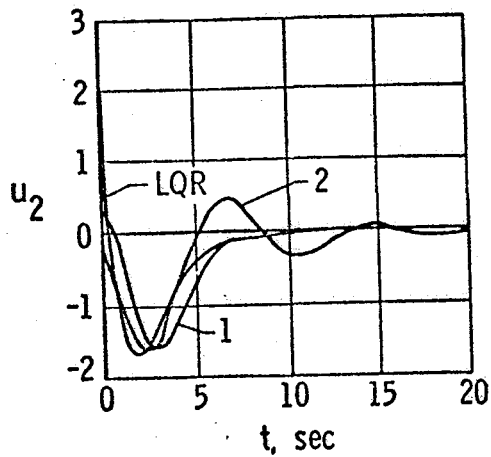


Fig. 3. Comparison of u_2 responses.

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1. Report No. NASA CR-172151		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle CAD of Control Systems: Application of Nonlinear Programming to a Linear Quadratic Formulation				5. Report Date June 1983	
				6. Performing Organization Code	
7. Author(s) Peter Fleming				8. Performing Organization Report No. 83-24	
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C/ NASA Langley Research Center Hampton, VA 23665				10. Work Unit No.	
				11. Contract or Grant No. NAS1-17070	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546				13. Type of Report and Period Covered Contractor report	
				14. Sponsoring Agency Code	
15. Supplementary Notes Langley Technical Monitor: Robert H. Tolson Final report					
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17. Key Words (Suggested by Author(s)) Computer-aided design Control engineering computer applications Nonlinear programming				18. Distribution Statement 31 Engineering (General) Unclassified-Unlimited	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 8	22. Price A02

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