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Computer Code for Off-Design Performance Analysis of Radial-Inflow Turbines With Rotor Blade Sweep

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Summary

The analysis procedure of an existing computer program for predicting the off-design performance of radial-inflow turbines was extensively modified to improve its accuracy and to extend its applicability. The rotor inlet slip factor correlation was replaced by one that includes rotor blade sweep. At the rotor exit a sector analysis was used in place of the original mean-line analysis to model the large radial variations occurring there. Disk friction, clearance, and vaneless space losses were added to the loss model, and the method of computing trailing-edge loss was changed. Finally, the entire analysis procedure was recoded.

The resulting computer program, called RTOD (Radial Turbine Off-Design), predicts the performance of a radial turbine (with or without rotor blade sweep) as a function of pressure ratio, speed, and stator setting. The program models the flow with stator viscous and trailing-edge losses; a vaneless space loss between the stator and the rotor; and rotor incidence, viscous, trailing-edge, clearance, and disk friction losses. The stator and rotor viscous losses each represent the combined effects of profile, endwall, and secondary flow losses. The stator inlet and exit and rotor inlet flows are modeled by a mean-line analysis, but a sector analysis is used at the rotor exit. For a pivoting stator the leakage flow through the clearance gap is accounted for.

This report describes the program modeling changes and, for completeness, presents all analysis equations. The program input and output, including error messages, are also described and are illustrated with an example case.

Comparing calculated results with experimental data shows that the revised program predicts mass flow rate better than does the original program. Calculated results are also used to illustrate the potential improvement in off-design performance offered by rotor back-sweep for high-work-factor radial turbines.

Introduction

The radial turbine has been used extensively in automotive turbochargers and aircraft auxiliary power units and has recently been given serious consideration for primary powerplant application for both automobiles and rotorcraft. In automotive and rotorcraft applications the turbine operates over a wide range of power settings and requires accurate off-design performance prediction for a successful design.

A computer code for the off-design performance analysis of radial-inflow turbines is described in reference 1, and additions to this program for variable-area stators are reported in reference 2. Although this program has been used extensively, it still requires modifications to extend its accuracy and applicability. Current interest in highly loaded radial turbines has resulted in designs having rotor back-sweep, a feature that this program cannot accommodate. Also, the analysis model, as shown in reference 1, predicts too low a value for mass flow in the region of rotor choke. In addition, the loss model does not include all of the losses present in the design program of reference 3, and the trailing-edge loss does not properly account for the effect of variable stator angle.

To overcome these deficiencies, extensive modifications were made to the analysis model. The rotor slip factor equation was replaced by one that includes rotor blade sweep. A sector analysis, similar to that of reference 3, was used at the rotor exit in place of the mean-line calculation. Disk friction, clearance, and vaneless space losses were added to the loss model, and the stator and rotor trailing-edge loss equations were changed. Finally, the entire procedure was recoded.

The resulting computer program, called RTOD (Radial Turbine Off-Design), which is described in this report, computes off-design performance of a radial turbine by modeling the flow with stator viscous and trailing-edge losses; with a vaneless space loss between the stator and the rotor; and with rotor incidence, viscous, clearance, trailing-edge, and disk friction losses. RTOD can analyze a variable stator, including the leakage flow in the clearance gap. Furthermore, RTOD determines the rotor exit flow variation from hub to tip and can account for the effect of swept rotor blading. The user must specify the stator and rotor viscous losses (which represent the combined effects of profile, endwall, and secondary flow loss) to match a reference performance point. RTOD then calculates the turbine performance over a specified operating range.

For the sake of completeness this report describes all of the modeling assumptions, but with emphasis on recent modifications, comprised in the computer program RTOD. The program input and output are described and the use of the program is illustrated by example problems. Results obtained from this program and from the program of reference 1 are compared with experimental data. RTOD can be obtained from the Computer Software Management Information Center (COSMIC), University of Georgia, Athens, Georgia 30602.

Analysis Modification

This section describes the changes that were made to the previous analysis method and loss model of references 1 and 2. The described losses are applied at or between the flow stations shown in figure 1. Station 0 is the station upstream of the stator, station 1 is just inside the stator trailing edge, and station 2 is immediately downstream of the stator trailing edge. Station 3 is upstream of the rotor, station 4 is just inside the rotor trailing edge, and station 5 is immediately downstream of the rotor trailing edge.

Rotor Analysis

The rotor analysis was extended to include a slip factor correlation that can account for swept rotor blading. Also, at the rotor exit a sector analysis replaces the previous mean-line analysis. The details are as follows (all symbols defined in appendix A):

Slip factor. – In reference 1 the optimum rotor inlet tangential velocity is calculated from the Stanitz slip factor correlation

$$V_{u,3,\mathrm{opt}} = U_3\left(1 - \frac{1.98}{N_{\mathrm{bl}}}\right)$$

The Stanitz correlation is limited to radial blading only. RTOD uses the Wiesner slip factor correlation of reference 4, which is a function of blading (sweep) angle. The correlation, when adapted to a turbine geometry, takes the form

$$V_{u,3,\mathrm{opt}} = U_3\left(\frac{\left[1 - \sqrt{\cos\alpha_{\mathrm{bl}}}/(N_{\mathrm{bl}})^{0.7}\right]\left\{1 - \left[(r_{4,\mathrm{av}}/r_3 - \epsilon_{\mathrm{lim}})/(1 - \epsilon_{\mathrm{lim}})\right]^3\right\}}{1 - \tan\alpha_{\mathrm{bl}}/\tan\alpha_3}\right)$$

For radial-bladed rotors (no blade sweep) the difference between the two correlations is quite small. For the example 1 turbine the Wiesner correlation predicts an optimum inlet tangential velocity 2.7 percent less than that predicted by the Stanitz correlation. The full incidence loss calculation procedure is shown by equations (B71) to (B82).

Exit sectors. – The program of reference 1 analyses the rotor exit at a mean radius only. In RTOD the rotor exit is divided into a specified number of sectors that are coupled by simple radial equilibrium

$$\frac{dp}{dr} = \frac{\rho V_u^2}{r g_c}$$

Temperature, pressure, and velocity are calculated for each sector (eqs. (B94) to (B106)), and overall values of mass flow and work are obtained by summing the individual sector values. These calculations take into account the viscous loss, since a viscous rotor loss coefficient is specified for each sector. The disk friction and tip clearance losses, however, are of an overall nature and do not reflect hub-to-tip variations. The shaft output work is obtained from the velocity diagram work by subtracting the losses due to disk friction and tip clearance. This means that the sector velocity diagrams are not consistent with the shaft output work.

Loss Model

In reference 1 the losses through the stator and the rotor are each modeled by a single overall loss. In RTOD a vaneless space loss (between stations 2 and 3) was added, as well as a rotor disk friction and tip clearance loss. Furthermore, the method of calculating trailing-edge pressure drop was changed. These changes are discussed in this section.

Vaneless space loss. – The program of reference 1 assumes no pressure drop from the stator exit to the rotor inlet (between stations 2 and 3). In RTOD the total pressure drop is obtained from

$$\Delta p' = \frac{4fl\rho V^2}{2D_h g_c}$$

where f is the Fanning friction factor for parallel walls and l is a length based on an average flow angle between stator outlet and rotor inlet. The calculation procedure is given by equations (B45) to (B54).

Rotor disk friction loss. – The program of reference 1 does not calculate a separate disk friction loss but lumps all losses (viscous, disk friction, clearance, and trailing edge) into one overall loss coefficient. In RTOD the following expression from reference 3 is used for the disk friction loss (eq. (B129)):

$$L_{\mathrm{df}} = \frac{0.02125\,\rho_3 U_3^2 r_3^2}{g_c J w_{\mathrm{tot}}(\rho_3 U_3 r_3/\mu)^{0.2}}$$

Rotor tip clearance loss. – The following expression from reference 3 is used in RTOD to calculate the rotor tip clearance loss (eq. (B130)):

$$L_{\mathrm{cl}} = \frac{2\Delta\bar{h}'_{3-5}(h_{\mathrm{cl}}/2r_{4,\mathrm{tip}})}{(1 - r_{4,\mathrm{hub}}/r_{4,\mathrm{tip}})}$$

The formulation assumes that the fractional loss due to clearance is equal to the ratio of clearance to passage height at the rotor exit.

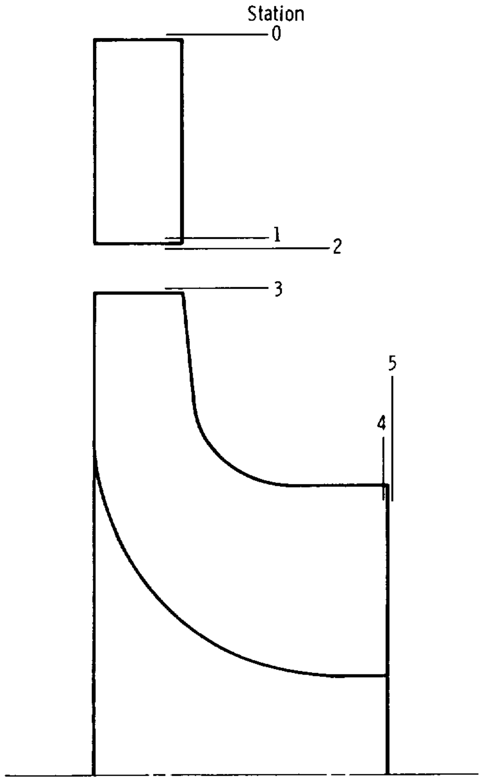

Figure 1. - Radial turbine schematic and station identification.

Trailing-edge pressure drop. – Reference 1 assumes no loss in total pressure across the stator or rotor trailing edges (stations 1 to 2 and 4 to 5, respectively). This is contrary to observed results, since it is known that a total pressure drop does occur, although it is usually small. Reference 2 assumes that static pressure remains constant across trailing edges. Although this assumption gives a drop in total pressure, the predicted total pressure drop decreases as the flow angle is increased. This is again contrary to observed results and will cause errors when analyzing a pivoting stator. The expressions for total pressure drop across the stator and rotor trailing edges used in RTOD were obtained from the expression for one-dimensional, compressible fluid flow with sudden area increase. The assumed expressions, although not based on rigorous analysis, predict the correct trend of total pressure loss with flow angle. Furthermore, the predicted levels of loss appear reasonable. For the stator the drop in total pressure is expressed by (eq. (B39))

$$\Delta p' = \left(1 - \frac{V_{r,2}}{V_{r,1}}\right)^2 \left(\frac{\rho_1 V_1^2}{2g_c}\right)$$

and for the rotor it is (eq. (B113))

$$\Delta p' = \left(1 - \frac{V_5 \cos \alpha_5}{V_{x,4}}\right)^2 \left(\frac{\rho_4 V_4^2}{2g_c}\right)$$

Program Input

The program input consists of a title card, the gas properties, the turbine geometry description, and the parameters that specify the desired calculation options. Except for the title card, all data input is in NAMELIST format. Data may be input in either SI or U.S. customary units.

Figure 2 identifies the angle and the angle sign convention used in RTOD. At the stator and the rotor inlet, all angles are defined from a radial line; at the rotor exit, angles are defined from an axial line. Figure 3 is a graphic representation of the geometry input variables, and figure 4 shows the example 1 data input.

All input variables are listed in table I, along with the required units, the default values, and the type of variables needed (real or integer).

The user must specify the viscous stator and viscous rotor losses (PR10 or EBR3D for the stator and XKR for

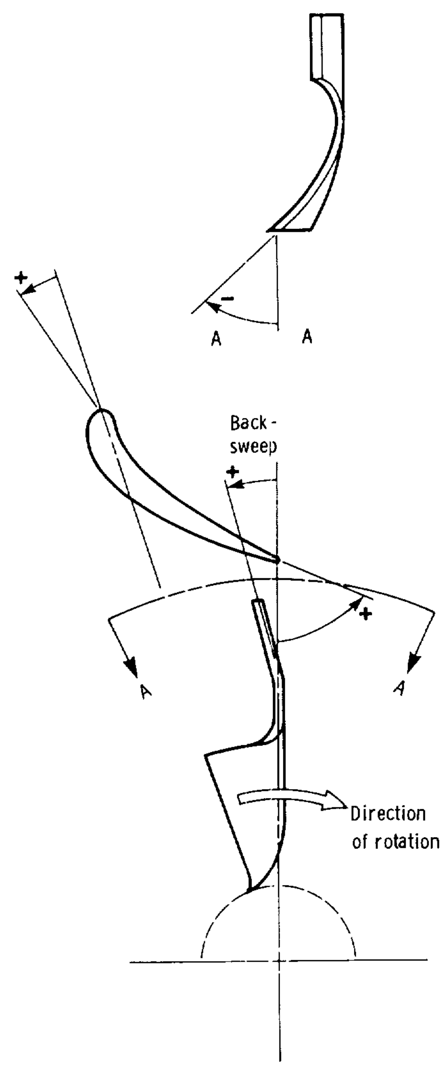

Figure 2. - RTOD angle sign convention.

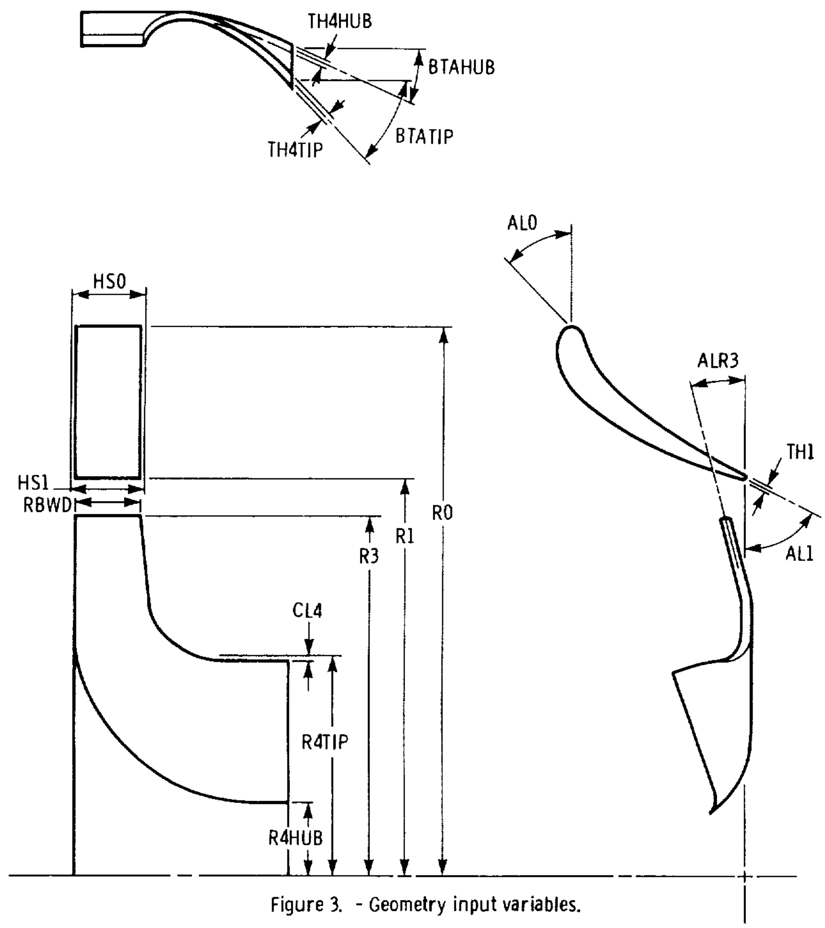

Figure 3. - Geometry input variables.

the rotor) such that the RTOD prediction matches two performance parameters (usually flow and work or flow and efficiency) at a particular pressure ratio. The parameters to be matched can either be design-point values obtained from a design-point program or test data.

Program Output

The program output is in either SI or U.S. customary units, corresponding to the chosen input units. The output consists of a printout of the title card, all input data, and one of three chosen levels of output as described in this section. The RTOD data printout corresponding to the input data of figure 4 is shown in figure 5.

Short Output

The short output consists of a tabulation of 12 overall parameters for each calculated operating point. Figure 6 shows the short output corresponding to the input data of figure 4. Note that the rotor chokes and that the value of stator exit critical velocity ratio $(V/V_{cr})_1$ at rotor choke is printed out. Also, the rotor limit load point is identified. The parameters in the short output are as follows (with the corresponding equations in parentheses):

(1) Equivalent flow (B148)
(2) Equivalent work (B150)
(3) Actual flow (B44)
(4) Actual work (B131)
(5) Equivalent power (B151)
(6) Equivalent torque (B152)
(7) Equivalent p_0'/p_5 (B153)
(8) Equivalent p_0'/p_5' (B154)
(9) Blade-jet speed ratio (B162)
(10) Work factor (B161)
(11) Static efficiency (B134)
(12) Total efficiency (B135)

Medium Output

The medium-output option gives the same tabulation of overall parameters as the short output and also gives tabulated velocity, temperature, pressure, and flow angle information at stations 0 to 5. At stations 3 to 5 the information is presented for both the absolute and relative frames of reference, and at stations 4 and 5 (rotor exit) the information is presented for the hub, mid, and tip sectors only. If an even number of sectors is specified, the midpoint sector is taken to be the sector above the mean radius. The medium-output option also presents additional information for stations 1, 3, and 5. For the most part this information is self-explanatory, except for these items:

Station 1

EBR3D — three-dimensional kinetic energy loss coefficient, e_{3D}, as specified; or, if total pressure ratio was specified, calculated from equation (B3)

P1T/P0T — total pressure ratio, p_1'/p_0' (input parameter PR10), as specified; or if e_{3D} was specified, calculated from equation (B4)

V/VCR AT MAX FLOW — critical velocity ratio $(V/V_{cr})_1$ that produces maximum value of mainstream flow $(\rho V)_1$ (equal to 1.0 if p_1'/p_0' was specified; less than 1.0 if e_{3D} was specified).

BL1 — blockage, defined by ratio of vane trailing-edge tangential thickness to vane trailing-edge spacing

Station 3

REL. T. PR. AFTER INCID. LOSS — relative total pressure after incidence loss, a fictitious pressure that shows the effect of incidence

U3 — rotor inlet tip speed

Figure 7 shows the medium output for a $(V/V_{cr})_1$ of 0.6 only (for the example input of fig. 4, except that OPT = 1). The output shown is produced for every value of $(V/V_{cr})_1$ in its specified range.

TABLE I. - INPUT VARIABLES

Input parameter	Description and units	Variable type[a]	Default value
	Overall input variables		
TITLE	Title card (up to 80 alphanumeric characters)	---	------
IUNTS	Type of units (0 for U.S. customary, 1 for SI)	I	0
T0T	Total temperature at station 0, K (°R)	R	------
P0T	Total pressure at station 0, N/cm^2 (psia)	↓	------
G	Specific heat ratio	↓	------
R	Gas constant, J/kg-K (ft-lbf/lbm-°R)	↓	------
XMU	Gas viscosity, kg/m-s (lbm/ft-s)	↓	------
OPT	Output option: 0 - short output 1 - medium output 2 - long output	I	0
	Stator input variables		
NSTV	Number of stator vanes	R	------
CLFR1	Stator total clearance (hub and tip) as fraction of passage height at trailing edge	↓	0.0
AL0	Flow inlet angle at station 0, deg	↓	------
AL1	Stator exit blade angle at station 1, deg	↓	------
R0	Radius at station 0, cm (in.)	↓	------
R1	Radius at station 1, cm (in.)	↓	------
HS0	Passage height (including clearance) at stator inlet, cm (in.)	↓	------
HS1	Passage height (including clearance) at stator exit, cm (in.)	↓	------
TH1	Vane trailing-edge normal thickness, cm (in.)	↓	------
IOPTS	Option for stator loss calculation: 1 - user specifies p_1'/p_0' 2 - user specifies $\bar{e}_{3D}$	I	1
PR10	Specified ratio of p_1'/p_0' (input only if IOPTS = 1)	R	0.98
EBR3D	Specified value of $\bar{e}_{3D}$ (input only if IOPTS = 2)	↓	0.055
VRSTRT	Initial value of $(V/V_{cr})_1$ at station 1 (must be subsonic)	↓	------
VRSTOP	Final value of $(V/V_{cr})_1$ (must be greater than VRSTRT)	↓	------
DELVR	Incremental change in $(V/V_{cr})_1$ between VRSTRT and VRSTOP	↓	------

TABLE I. - Concluded.

Input parameter	Description and units	Variable type[a]	Default value
	Rotor input variables		
NRBL	Number of full rotor blades extending from inlet to output	R	------
ISPL	Splitter blades (0 for no splitters, 1 for splitters)	I	0
RPM	Rotative speed, rpm	R	------
ALR3	Rotor inducer sweep angle (0.0 for radial blade), deg		0.0
R3	Radius at station 3, cm (in.)		------
RBWD	Rotor "b" width at station 3, cm (in.)		------
R4HUB	Rotor exit hub radius, cm (in.)		------
R4TIP	Rotor exit tip radius, cm (in.)		------
CL4	Clearance height at rotor exit, cm (in.)		------
NSCT4	Specified number of equal height sectors at rotor exit (25 max)	I	------
OPBTA4	Rotor exit blade angle distribution option 0 - user specifies exit blade angle for each sector 1 - program linearly interpolates for exit blade angle at each sector from input BTAHUB and BTATIP	I	1
BETA4	Rotor exit blade angle for each sector (hub to tip; input only if OPBTA4 = 0), deg	R	
BTAHUB	Rotor exit blade angle at hub (input only if OPBTA4 = 1), deg	R	
BTATIP	Rotor exit blade angle at tip (input only if OPBTA4 = 1), deg	R	
OPTH4	Rotor exit normal thickness distribution option: 0 - user specifies normal thickness for each sector 1 - program linearly interpolates for normal thickness at each sector from input TH4HUB and TH4TIP	I	1
TH4	Rotor exit normal thickness for each sector (hub to tip; input only if OPTH4 = 0), cm (in.)	R	
TH4HUB	Rotor exit normal thickness at hub (input only if OPTH4 = 1), cm (in.)		
TH4TIP	Rotor exit normal thickness at tip (input only if OPTH4 = 1), cm (in.)		
XKR	Rotor viscous loss coefficient for each sector (hub to tip)		
PNEGI	Power in incidence loss expression (for negative incidence)		2.50
PPOSI	Power in incidence loss expression (for positive incidence)		1.75
P4RDPC	Incremental percentage reduction in pressure at rotor exit after rotor choke		10.0

[a] I denotes integer; R denotes real.

```
 4.59 INCH RADIAL TURBINE IN AIR
&DATT TOT=518.7, POT=14.696, G=1.4, R=53.35, XMU=1.216E-5,
NSTV=14., AL0=45., AL1=72.0, R0=2.888, R1=2.345, HS0=.5771, HS1=.5771,
IOPTS=1, PR10=.987, VRSTRT=.30, VRSTOP=1.20, DELVR=.05, TH1=.02293,
NRBL=11., ISPL=1, RPM=29550., R3=2.295, RBWD=.5771, R4HUB=.572, R4TIP=1.646,
CL4=.0145, NSCT4=11, OPBTA4=1, BTAHUB=-32.50, BTATIP=-56.70, OPTH4=1,
TH4HUB=.077, TH4TIP=.038, XKR=11*.240, OPT=0 &END
```

Figure 4. - Example 1 data input.

Long Output

The long output is similar to the medium output, except that for stations 4 and 5 the tabulated inputs are presented for each sector. Also, additional rotor loss information is presented. For the most part this information is again self-explanatory, except for the following items:

EQUIVALENT SPEED	defined by equation (B149)
SPECIFIC DIAM.	specific diameter; defined by equation (B160)
SPECIFIC SPEED	defined by equation (B159)

The long output also presents a further tabulation of the following items for each sector at stations 4 and 5:

Station 4

R4	radius associated with midpoint of sector
BL4	blockage for each sector; defined by ratio of free flow area at station 4 to free flow area at station 5
W/WCR AT	value of $(W/W_{cr})_4$ that produces

```
4.59 INCH RADIAL TURBINE IN AIR

  IUNTS=  0            ,        TOT=0.5187E 03          R,      POT=0.1470E 02            PSIA,
     G=0.1400E 01      ,        XMU=0.1216E-04 LBM/FT*S,          R=0.5335E 02 FT*LBF/(LBM*R),
   OPT=  0             ,

  NSTV=0.1400E 02      ,      CLFR1=     0.0000        ,
   AL0=0.4500E 02   DEG,         R0=0.2888E 01       IN,       HS0=0.5771E 00      IN,
   AL1=0.7200E 02   DEG,         R1=0.2345E 01       IN,       HS1=0.5771E 00      IN,
   TH1=0.2293E-01    IN,
 IOPTS=  1             ,       PR10=0.9870E 00         ,     EBR3D=     0.0000        ,
VRSTRT=0.3000E 00      ,     VRSTOP=0.1200E 01         ,     DELVR=0.5000E-01        ,

  NRBL=0.1100E 02      ,       ISPL=  1                ,       RPM=0.2955E 05     RPM,
  ALR3=     0.0000  DEG,         R3=0.2295E 01       IN,      RBWD=0.5771E 00      IN,
R4HUB=0.5720E 00     IN,     R4TIP=0.1646E 01       IN,       CL4=0.1450E-01      IN,
NSCT4= 11              ,
OPTH4=  1              ,
TH4HUB=0.7700E-01    IN,     TH4TIP=0.3800E-01       IN,
OPBTA4=  1             ,
BTAHUB=-.3250E 02   DEG,     BTATIP=-.5670E 02      DEG,
 PNEGI=0.2500E 01      ,
 PPOSI=0.1750E 01      ,
P4RDPC=0.1000E 02

BETA4( 1)=-.3360E 02 DEG,     TH4( 1)=0.7523E-01    IN,     XKR( 1)=0.2400E 00
     ( 2)=-.3580E 02             ( 2)=0.7168E-01               ( 2)=0.2400E 00
     ( 3)=-.3800E 02             ( 3)=0.6814E-01               ( 3)=0.2400E 00
     ( 4)=-.4020E 02             ( 4)=0.6459E-01               ( 4)=0.2400E 00
     ( 5)=-.4240E 02             ( 5)=0.6105E-01               ( 5)=0.2400E 00
     ( 6)=-.4460E 02             ( 6)=0.5750E-01               ( 6)=0.2400E 00
     ( 7)=-.4680E 02             ( 7)=0.5395E-01               ( 7)=0.2400E 00
     ( 8)=-.4900E 02             ( 8)=0.5041E-01               ( 8)=0.2400E 00
     ( 9)=-.5120E 02             ( 9)=0.4686E-01               ( 9)=0.2400E 00
     (10)=-.5340E 02             (10)=0.4332E-01               (10)=0.2400E 00
     (11)=-.5560E 02             (11)=0.3977E-01               (11)=0.2400E 00
```

Figure 5. - RTOD data printout (corresponding to fig. 4 data input).

(V/VCR)1	EQUIV. FLOW (LBM/ SEC)	EQUIV. WORK (BTU/ LBM)	ACTUAL FLOW (LBM/ SEC)	ACTUAL WORK (BTU/ LBM)	EQUIV. POWER (HP)	EQUIV. TORQUE (FT*LBF)	EQUIV. POT/P5	EQUIV. POT/P5T	BLADE-JET SPEED RATIO	WORK FACTOR	STATIC EFFIC.	TOTAL EFFIC.
0.3000	0.3769	4.6418	0.3769	4.6421	2.4756	0.4400	1.2412	1.2099	0.9689	0.3425	0.6230	0.7041
0.3500	0.4337	6.1643	0.4337	6.1646	3.7830	0.6724	1.2874	1.2571	0.8983	0.4520	0.7112	0.7827
0.4000	0.4878	7.6727	0.4878	7.6732	5.2962	0.9414	1.3395	1.3085	0.8375	0.5606	0.7695	0.8338
0.4500	0.5389	9.1776	0.5388	9.1782	6.9979	1.2438	1.3998	1.3660	0.7832	0.6692	0.8049	0.8649
0.5000	0.5865	10.6883	0.5865	10.6890	8.8703	1.5766	1.4712	1.4320	0.7336	0.7783	0.8223	0.8808
0.5500	0.6306	12.2155	0.6305	12.2162	10.8994	1.9373	1.5575	1.5090	0.6875	0.8886	0.8255	0.8850
0.6000	0.6707	13.7671	0.6706	13.7679	13.0643	2.3221	1.6623	1.5989	0.6448	1.0008	0.8185	0.8815
0.6500	0.7066	15.3593	0.7066	15.3602	15.3556	2.7293	1.7930	1.7058	0.6048	1.1160	0.8032	0.8721
0.7000	0.7381	17.0213	0.7381	17.0223	17.7751	3.1594	1.9637	1.8373	0.5661	1.2362	0.7799	0.8573
0.7500	0.7651	18.8252	0.7651	18.8263	20.3776	3.6219	2.2084	2.0099	0.5267	1.3668	0.7467	0.8365
0.8000	0.7873	21.1710	0.7873	21.1723	23.5866	4.1923	2.7154	2.3076	0.4757	1.5366	0.6851	0.8004
0.8062	0.7898	21.9757	0.7897	21.9769	24.5496	4.3635	3.0194	2.4488	0.4556	1.5949	0.6522	0.7821

ROTOR HAS CHOKED AT V1VCR1=0.8062E 00 (4 ITERATIONS)
MAX P4(1) FOR CHOKED ROTOR=0.4182E 01 PSIA (14 ITERATIONS)

0.8062	0.7898	22.3802	0.7897	22.3814	25.0014	4.4438	3.2415	2.5399	0.4438	1.6242	0.6301	0.7691
0.8062	0.7898	22.7041	0.7897	22.7055	25.3634	4.5081	3.4813	2.6337	0.4330	1.6477	0.6085	0.7548
0.8062	0.7898	22.9340	0.7897	22.9353	25.6201	4.5538	3.7395	2.7322	0.4231	1.6644	0.5869	0.7382
0.8062	0.7898	22.9930	0.7897	22.9943	25.6860	4.5655	3.8505	2.7747	0.4193	1.6686	0.5779	0.7305

ROTOR LIMIT LOADING ACHIEVED AT P4(1)=0.2919E 01 PSIA (13 ITERATIONS)

Figure 6. - RTOD short output for figure 4 data input.

(V/VCR)1	EQUIV. FLOW (LBM/SEC)	EQUIV. WORK (BTU/LBM)	ACTUAL FLOW (LBM/SEC)	ACTUAL WORK (BTU/LBM)	EQUIV. POWER (HP)	EQUIV. TORQUE (FT*LBF)	EQUIV. POT/P5	EQUIV. POT/P5T	BLADE-JET SPEED RATIO	WORK FACTOR	STATIC EFFIC.	TOTAL EFFIC.
0.6000	0.6707	13.7671	0.6706	13.7679	13.0643	2.3221	1.6623	1.5989	0.6448	1.0008	0.8185	0.8815

STATION (AND SECTOR)	VCR OR WCR (FT/S)	V/VCR OR W/WCR	VU/VCR OR WU/WCR	VR(VX)/VCR OR WR(WX)/WCR	TT (R)	T (R)	PT (PSIA)	P (PSIA)	FLOW ANGLE (DEG)
0	1019.1802	0.1693	0.1197	0.1197	518.7000	516.2205	14.6960	14.4516	45.0000
1	1019.1802	0.6000	0.5706	0.1854	518.7000	487.5779	14.5050	11.6806	71.9999
2	1019.1802	0.5960	0.5706	0.1721	518.7000	487.9883	14.4916	11.7042	73.2137
3	1019.1802	0.6095	0.5831	0.1775	518.7000	486.5876	14.4669	11.5673	73.0723
3(R)	989.8862	0.1827	0.0025	0.1827	489.3105	486.5876	11.7954	11.5673	0.7704
4(1)	959.0784	0.3021	-0.0326	0.3003	459.3274	452.3423	9.2163	8.7350	-6.1953
4(6)	960.0049	0.3009	0.0012	0.3009	460.2151	453.2712	9.2145	8.7370	0.2269
4(11)	960.7744	0.2855	0.0120	0.2853	460.9534	454.6897	9.1659	8.7373	2.4163
4(1)(R)	962.1702	0.3594	-0.1989	0.2993	462.2937	452.3423	9.4262	8.7350	-33.6000
4(6)(R)	967.0215	0.4195	-0.2945	0.2987	466.9673	453.2712	9.6964	8.7370	-44.5999
4(11)(R)	974.5627	0.4978	-0.4108	0.2812	474.2788	454.6897	10.1273	8.7373	-55.5999
5(1)	959.0784	0.2231	-0.0326	0.3003	459.3274	455.5159	9.1831	8.9192	-8.4009
5(6)	960.0049	0.2603	0.0012	0.3009	460.2151	455.0198	9.2060	8.8474	0.2624
5(11)	960.7744	0.2622	0.0120	0.2853	460.9534	455.6709	9.1631	8.8008	2.6314

```
STATION 1
    PRIMARY WEIGHT FLOW = 0.6706E 00  (LBM/S)          SPECIFIED PRIMARY FLOW ANGLE = 0.7200E 02  (DEG)
  CLEARANCE WEIGHT FLOW =     0.0000  (LBM/S)             ACTUAL PRIMARY FLOW ANGLE = 0.7200E 02  (DEG)
      TOTAL WEIGHT FLOW = 0.6706E 00  (LBM/S)                   CLEARANCE FLOW ANGLE =     0.0000  (DEG)
                  EBR3D = 0.5523E-01                            V/VCR AT MAX FLOW = 0.1000E 01
                P1T/POT = 0.9370E 00                                           BL1 = 0.7051E-01

STATION 3
           OPTIMUM BETA = -.2060E 02  (DEG)        REL. T. PR. AFTER INCID. LOSS = 0.1177E 02  (PSIA)
        INCIDENCE ANGLE = 0.2137E 02  (DEG)                                   U3 = 0.5918E 03  (FT/S)

STATION 5
     MASS AVERAGED  T5 = 0.4553E 03  (R)       MASS AVG. DELH  (STA. 0 TO 5) = 0.1682E 02  (BTU/LBM)
     MASS AVERAGED T5T = 0.4604E 03  (R)       MASS AVG. DELHT (STA. 0 TO 5) = 0.1562E 02  (BTU/LBM)
        P5 AT THE HUB = 0.8918E 01  (PSIA)     MASS AVG. POWER (STA. 0 TO 5) = 0.1482E 02  (HP)
     P5 AT THE SHROUD = 0.8801E 01  (PSIA)     MASS AVG. TORQUE(STA. 0 TO 5) = 0.2634E 01  (FT*LBF)
           AVERAGE PS = 0.8860E 01  (PSIA)
          AVERAGE PST = 0.9211E 01  (PSIA)
```

Figure 7. - RTOD medium output at $(V/V_{cr})_1$ = 0.6 for figure 4 data input.

MAX FLOW — maximum (choking) value of $(\rho W)_4$ (always less than 1.0)

U4 — rotor sector speed

Station 5

AERØD. ETA — sector aerodynamic static efficiency; defined by equation (B127)

AEROD. ETAT — sector aerodynamic total efficiency; defined by equation (B128)

WM(K)/WMTOT — fraction of total weight flow passed by each sector

Figure 8 shows the long output for a $(V/V_{cr})_1$ of 0.6 only (for the example input of fig. 4, except that OPT = 2). The long output shown is produced for every value of $(V/V_{cr})_1$ in its specified range.

Error Messages

All DO-loops in RTOD have appropriate error messages that are printed out if convergence is not achieved. The specified convergence criteria are defined by the ratio of the absolute value of the new value minus the old value to the old value and are usually required to be less than 1×10^{-4}. These are tight tolerances, but no DO-loop has ever failed to converge for physically possible flow.

(V/VCR)1	EQUIV. FLOW (LBM/ SEC)	EQUIV. WORK (BTU/ LBM)	ACTUAL FLOW (LBM/ SEC)	ACTUAL WORK (BTU/ LBM)	EQUIV. POWER (HP)	EQUIV. TORQUE (FT*LBF)	EQUIV. POT/P5	EQUIV. POT/P5T	BLADE-JET SPEED RATIO	WORK FACTOR	STATIC EFFIC.	TOTAL EFFIC.
0.6000	0.6707	13.7671	0.6706	13.7679	13.0643	2.3221	1.6623	1.5989	0.6448	1.0008	0.8185	0.8815

STATION (AND SECTOR)	VCR OR WCR (FT/S)	V/VCR OR W/WCR	VU/VCR OR WU/WCR	VR(VX)/VCR OR WR(WX)/WCR	TT (R)	T (R)	PT (PSIA)	P (PSIA)	FLOW ANGLE (DEG)
0	1019.1802	0.1693	0.1197	0.1197	518.7000	516.2205	14.6960	14.4516	45.0000
1	1019.1802	0.6000	0.5706	0.1854	518.7000	487.5779	14.5050	11.6806	71.9998
2	1019.1802	0.5960	0.5706	0.1721	518.7000	487.9883	14.4916	11.7042	73.2136
3	1019.1802	0.6095	0.5831	0.1775	518.7000	486.5876	14.4669	11.5673	73.0722
3(R)	989.8862	0.1827	0.0025	0.1827	489.3105	486.5876	11.7955	11.5673	0.7704
4(1)	959.0784	0.3021	-0.0326	0.3003	459.3274	452.3423	9.2163	8.7350	-6.1954
4(2)	959.2175	0.3015	-0.0237	0.3006	459.4604	452.4976	9.2159	8.7362	-4.5008
4(3)	959.3916	0.3015	-0.0159	0.3011	459.6274	452.6646	9.2163	8.7368	-3.0167
4(4)	959.5884	0.3015	-0.0092	0.3014	459.8159	452.8477	9.2166	8.7370	-1.7400
4(5)	959.7964	0.3014	-0.0035	0.3014	460.0151	453.0496	9.2163	8.7370	-0.6631
4(6)	960.0049	0.3009	0.0012	0.3009	460.2151	453.2712	9.2145	8.7370	0.2268
4(7)	960.2041	0.2997	0.0049	0.2997	460.4063	453.5137	9.2107	8.7370	0.9447
4(8)	960.3867	0.2977	0.0078	0.2976	460.5813	453.7766	9.2043	8.7371	1.5058
4(9)	960.5454	0.2948	0.0099	0.2946	460.7336	454.0603	9.1949	8.7371	1.9278
4(10)	960.6760	0.2908	0.0113	0.2906	460.8589	454.3647	9.1822	8.7372	2.2257
4(11)	960.7744	0.2855	0.0120	0.2853	460.9534	454.6897	9.1659	8.7373	2.4162
4(1)(R)	962.1702	0.3594	-0.1989	0.2993	462.2937	452.3423	9.4262	8.7350	-33.6000
4(2)(R)	962.9231	0.3692	-0.2160	0.2995	463.0173	452.4976	9.4680	8.7362	-35.8000
4(3)(R)	963.7849	0.3803	-0.2341	0.2997	463.8467	452.6646	9.5158	8.7368	-38.0000
4(4)(R)	964.7554	0.3925	-0.2533	0.2998	464.7813	452.8477	9.5697	8.7370	-40.2000
4(5)(R)	965.8345	0.4056	-0.2735	0.2995	465.8215	453.0496	9.6299	8.7370	-42.4000
4(6)(R)	967.0215	0.4195	-0.2945	0.2987	466.9673	453.2712	9.6964	8.7370	-44.5999
4(7)(R)	968.3162	0.4341	-0.3164	0.2972	468.2185	453.5137	9.7694	8.7370	-46.7999
4(8)(R)	969.7185	0.4493	-0.3391	0.2948	469.5754	453.7766	9.8489	8.7371	-48.9999
4(9)(R)	971.2268	0.4650	-0.3624	0.2914	471.0376	454.0603	9.9350	8.7371	-51.1999
4(10)(R)	972.8420	0.4812	-0.3863	0.2869	472.6055	454.3647	10.0277	8.7372	-53.3999
4(11)(R)	974.5627	0.4978	-0.4108	0.2812	474.2788	454.6897	10.1273	8.7373	-55.5998
5(1)	959.0784	0.2231	-0.0326	0.3003	459.3274	455.5159	9.1831	8.9192	-8.4009
5(2)	959.2175	0.2341	-0.0237	0.3006	459.4604	455.2632	9.1920	8.9015	-5.8013
5(3)	959.3916	0.2429	-0.0159	0.3011	459.6274	455.1062	9.1984	8.8856	-3.7447
5(4)	959.5884	0.2501	-0.0092	0.3014	459.8159	455.0222	9.2029	8.8715	-2.0980
5(5)	959.7964	0.2558	-0.0035	0.3014	460.0151	454.9966	9.2055	8.8588	-0.7812
5(6)	960.0049	0.2603	0.0012	0.3009	460.2151	455.0198	9.2060	8.8474	0.2623
5(7)	960.2041	0.2634	0.0049	0.2997	460.4063	455.0840	9.2039	8.8368	1.0751
5(8)	960.3867	0.2650	0.0078	0.2976	460.5813	455.1843	9.1988	8.8270	1.6909
5(9)	960.5454	0.2656	0.0099	0.2946	460.7336	455.3169	9.1905	8.8178	2.1399
5(10)	960.6760	0.2646	0.0113	0.2906	460.8589	455.4800	9.1786	8.8091	2.4458
5(11)	960.7744	0.2622	0.0120	0.2853	460.9534	455.6709	9.1631	8.8008	2.6313

Figure 8. - RTOD long output at $(V/V_{cr})_1$ = 0.6 for figure 4 data input.

Main Program

The user can specify geometries and operating conditions that have no flow solutions. For low flow rates (small specified values of V/V_{cr} at station 1) it may not be possible to pass the flow at station 4 while maintaining radial equilibrium. In a real turbine the flow would be passed but with separated or reversed flow in portions of the rotor exit. However, RTOD cannot account for that situation, and if it occurs, the following error message is printed out:

STATION 4 HAS NOT CONVERGED IN
25 ITERATIONS FOR (V/VCR)1 = x.xxxxx ± xx

ROTOR WEIGHT FLOW = xxxxxx ± xx,
STATOR WEIGHT FLOW = x.xxxxx ± xx

After this message, program execution starts over with the next specified value of stator exit critical velocity ratio.

Subroutine MAXVAL

Subroutine MAXVAL calculates the maximum value of density times velocity ρV and the associated critical velocity ratio at the stator and rotor exits (stations 1 and 4). The input points must define a curve that shows a maximum somewhere in the specified interval (other than at the end points). If unrealistically large values of stator or rotor loss are specified, the shape of the generated curve may not exhibit the required characteristic. If this happens, the following error message is printed out:

WARNING--THE CURVE TO BE MAXIMIZED
IN SUBROUTINE MAXVAL IS NOT OF THE
PROPER FORM

```
STATION 1
    PRIMARY WEIGHT FLOW = 0.6706E 00  (LBM/S)         SPECIFIED PRIMARY FLOW ANGLE = 0.7200E 02  (DEG)
   CLEARANCE WEIGHT FLOW =     0.0000  (LBM/S)            ACTUAL PRIMARY FLOW ANGLE = 0.7200E 02  (DEG)
      TOTAL WEIGHT FLOW = 0.6706E 00  (LBM/S)                 CLEARANCE FLOW ANGLE =     0.0000  (DEG)
                  EBR3D = 0.5523E-01                            V/VCR AT MAX FLOW = 0.1000E 01
                P1T/POT = 0.9870E 00                                          BL1 = 0.7051E-01

STATION 3
         OPTIMUM BETA = -.2060E 02  (DEG)          REL. T. PR. AFTER INCID. LOSS = 0.1177E 02  (PSIA)
      INCIDENCE ANGLE = 0.2137E 02  (DEG)                                     U3 = 0.5918E 03  (FT/S)

STATION 5
      MASS AVERAGED  T5 = 0.4553E 03  (R)          MASS AVG. DELH  (STA. 0 TO 5) = 0.1682E 02  (BTU/LBM)
      MASS AVERAGED T5T = 0.4604E 03  (R)          MASS AVG. DELHT (STA. 0 TO 5) = 0.1562E 02  (BTU/LBM)
          P5 AT THE HUB = 0.8918E 01  (PSIA)       MASS AVG. POWER (STA. 0 TO 5) = 0.1482E 02  (HP)
       P5 AT THE SHROUD = 0.8801E 01  (PSIA)       MASS AVG. TORQUE(STA. 0 TO 5) = 0.2634E 01  (FT*LBF)
             AVERAGE P5 = 0.8860E 01  (PSIA)
            AVERAGE P5T = 0.9211E 01  (PSIA)

     DELHT ACROSS ROTOR = 0.1400E 02  (BTU/LBM)                 ACTUAL  DELHT = 0.1377E 02  (BTU/LBM)
     DISK FRICTION LOSS = 0.4306E-01  (BTU/LBM)                 ACTUAL  POWER = 0.1306E 02  (HP)
     TIP CLEARANCE LOSS = 0.1890E 00  (BTU/LBM)                 ACTUAL TORQUE = 0.2322E 01  (FT*LBF)

      EQUIVALENT SPEED = 0.2955E 05  (RPM)                    SPECIFIC DIAM. = 0.2677E 01
        SPECIFIC SPEED = 0.7071E 00
---------------------------------------------------------------------------------------------------------
```

	STATION 4				STATION 5		
SECTOR	R4 (IN)	BL4	W/WCR AT MAX FLOW	U4 (FT/S)	AEROD. ETA	AEROD. ETAT	WM(K)/ WMTOT
1	0.6208E 00	0.2547E 00	0.8653E 00	0.1601E 03	0.8608E 00	0.9105E 00	0.4412E-01
2	0.7185E 00	0.2154E 00	0.8653E 00	0.1853E 03	0.8557E 00	0.9102E 00	0.5381E-01
3	0.8161E 00	0.1855E 00	0.8653E 00	0.2104E 03	0.8505E 00	0.9089E 00	0.6354E-01
4	0.9137E 00	0.1620E 00	0.8653E 00	0.2356E 03	0.8453E 00	0.9069E 00	0.7325E-01
5	0.1011E 01	0.1431E 00	0.8653E 00	0.2608E 03	0.8403E 00	0.9044E 00	0.8289E-01
6	0.1109E 01	0.1275E 00	0.8653E 00	0.2860E 03	0.8354E 00	0.9014E 00	0.9237E-01
7	0.1207E 01	0.1144E 00	0.8653E 00	0.3112E 03	0.8309E 00	0.8980E 00	0.1016E 00
8	0.1304E 01	0.1031E 00	0.8653E 00	0.3363E 03	0.8267E 00	0.8943E 00	0.1104E 00
9	0.1402E 01	0.9340E-01	0.8653E 00	0.3615E 03	0.8230E 00	0.8904E 00	0.1187E 00
10	0.1500E 01	0.8482E-01	0.8653E 00	0.3867E 03	0.8197E 00	0.8862E 00	0.1263E 00
11	0.1597E 01	0.7717E-01	0.8653E 00	0.4119E 03	0.8170E 00	0.8818E 00	0.1331E 00

Figure 8. - Concluded.

Program execution continues after this message. The value of ρV used is the greater of the two at the interval end points. The program user should reexamine and change the specified stator or rotor losses if this error message occurs.

Example Problems

A radial turbine that was tested in air at pressure ratios higher than choking is analyzed in this section, and the results are compared with experimental data, together with the results from the program of reference 1. Also, the same turbine is analyzed for various values of rotor back-sweep to illustrate the potential off-design performance improvements in high-work-factor turbines.

Example 1 – Comparison with Test Data and Reference 1 Code Prediction

Figure 9 shows a comparison of calculated versus experimental mass flow at 100- and 60-percent design speeds for the 11.66-cm (4.59-in.) diameter turbine of reference 5. The test data are previously unpublished air data that were used because they covered a wider range of pressure ratio (beyond rotor choke) than the published argon data. Both the RTOD and reference 1 code predictions were matched with data at the design equivalent total-to-static pressure ratio of 1.54. The RTOD results more closely predict the experimental mass flow rates at high pressure ratios. At both 100- and 60-percent design speeds, RTOD predicts an approximately 1 percent greater mass flow ratio than the reference 1 code. The improved mass flow prediction is attributed to the more

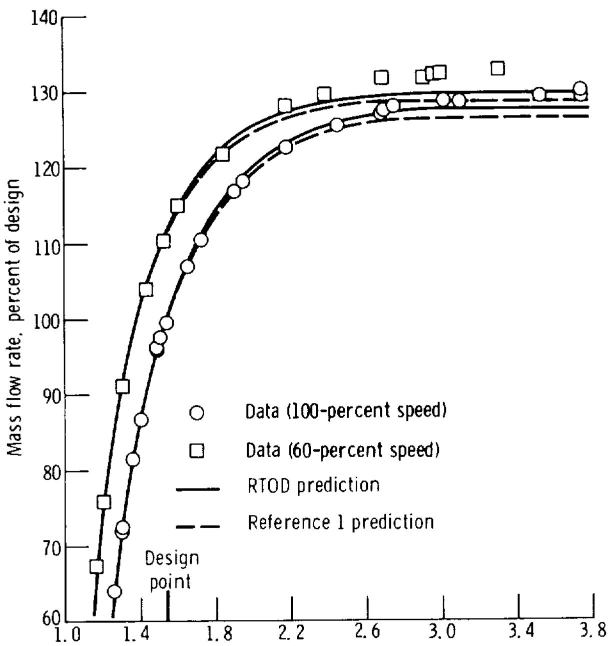

Figure 9. - Mass flow rate comparison.

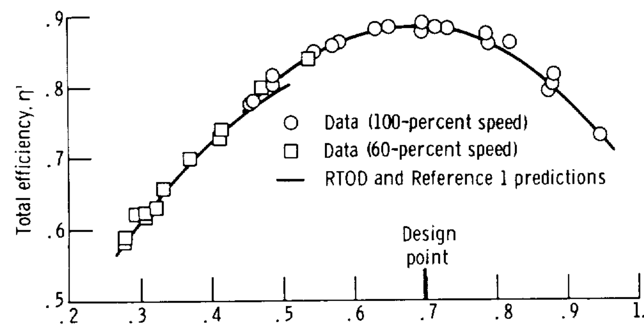

Figure 10. - Total efficiency comparison.

accurate geometry and flow modeling at the rotor trailing edge. Both the RTOD and reference 1 code total efficiency predictions agree closely with test data as shown in figure 10.

Although the RTOD mass flow predictions are an improvement over the reference 1 code predictions, they still underpredict choking weight flow. At 100-percent design speed the RTOD prediction is 1.63 percent below test results; while at 60-percent speed it is 2.28 percent low. This discrepancy could be caused by the basic rotor loss modeling scheme. Rotor viscous loss is taken to be a function of rotor relative inlet and exit velocities squared (eq. (B87)), multiplied by a loss coefficient (input parameter XKR) that is assumed to remain constant. This assumption forces the calculated maximum mass flow rate per unit area to occur at low values of rotor exit relative critical velocity ratio ($(W/W_{cr})_4 = 0.87$ in example 1). Proper variation of the rotor loss coefficient with relative flow velocity might improve the choking mass flow prediction.

Example 2 – Effects of Rotor Back-Sweep

To illustrate the effects of rotor back-sweep, the example 1 turbine was analyzed with assumed back-sweep angles of 15° and 30°. The predicted total efficiencies are shown in figure 11. For high values of work factor (the region where modern radial turbines are likely to operate) the falloff in efficiency is less with back-sweep than for a radial-bladed rotor (because the incidence is lower).

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio, March 25, 1983.

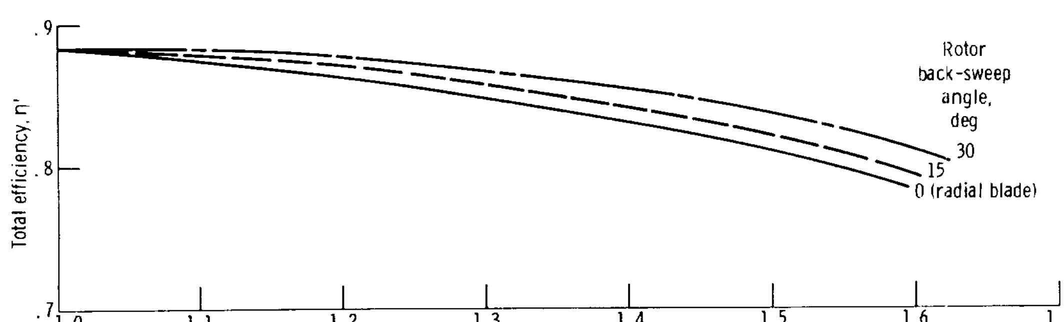

Figure 11. - Effect of rotor back-sweep on calculated total efficiency for example 1 turbine.

Appendix A
Symbols

A	area, m^2 (ft^2)
BL4	ratio of blocked area at station 4 to free flow area at station 5
CLFR1	ratio of total clearance area to total passage area at station 1
D_h	hydraulic diameter, m (ft); defined by eq. (B46)
D_s	specific diameter; defined by eq. (B160)
e_{3D}	three-dimensional kinetic energy loss coefficient; defined by eq. (B3)
f	friction factor
g_c	force-mass conversion constant, 1 (32.174 (lbm-ft)/(lbf-s^2))
h	height, m (ft); or enthalpy, J/kg (Btu/lbm)
J	dimensional constant, 1 (778 ft-lbf/Btu)
K	rotor viscous loss coefficient
k	sector number
L	kinetic energy loss, J/kg (Btu/lbm)
l	length, m (ft)
M	Mach number
N	number of sectors at station 4
N_{bl}	number of rotor blades (including splitters)
N_s	specific speed; defined by eq. (B159)
n	incidence exponent
P	power, W (hp)
p	pressure, N/m^2 (psia)
Q	parameter defined by eq. (B166)
R	gas constant, J/kg-K (ft-lbf/lbm-°R)
Re	Reynolds number; defined by eq. (B47)
r	radius, m (ft)
T	temperature, K (°R)
TRQ	torque, N-m (ft-lbf)
U	tangential velocity, m/s (ft/s)
V	velocity in absolute reference frame, m/s (ft/s)
W	velocity in relative reference frame, m/s (ft/s)
WRK	work, J/kg (Btu/lbm)
w	flow rate, kg/s (lbm/s)
α	fluid flow angle in absolute frame of reference; or blading angle, deg
β	fluid flow angle in relative frame of reference; or blading angle, deg
γ	ratio of specific heats
δ	parameter defined by eq. (B156)
ϵ	parameter defined by eq. (B157)
ϵ_{lim}	parameter defined by eq. (B73)
η	efficiency
θ_{cr}	parameter defined by eq. (B155)
μ	viscosity, kg/m-s (lbm/ft-s)
ν	loading coefficient (blade-jet speed ratio); defined by eq. (B162)
ρ	density, kg/m^3 (lbm/ft^3)
ψ	work factor; defined by eq. (B161)
Ω	rotational speed, rpm

Subscripts:

act	actual
aed	aerodynamic
av	average
bl	blading
ck	choked
cl	clearance
cr	critical
df	disk fraction
equ	equivalent
f	friction
hub	hub
i	incidence
id	ideal
mx	maximum
opt	optimum
p	primary
r	radial component
ref	reference
shr	shroud
std	standard
tip	tip
tot	total
u	tangential component
x	axial component
0,1,2, 3,4,5	station numbers defined by fig. 1

Superscripts:

$(\ )'$	total conditions in absolute reference frame
$(\ )''$	total conditions in relative reference frame
$\overline{(\ )}$	mass averaged

Appendix B
Equations

Main Program (MAINP)

The analysis of the flow through the turbine starts at the stator exit (station 1) for each specified value of $(V/V_{cr})_1$. Flow conditions are then calculated at station 0 and at stations 2 to 5. The procedure is described in this appendix.

Station 1 – primary flow. – Total temperature at station 1 is assumed to be equal to that at station 0. Thus

$$T'_{1,p}=T'_{0,p} \tag{B1}$$

The maximum value of primary flow per unit area $[(\rho V)_{1,p,\mathrm{mx}}]$ and the value of $(V/V_{cr})_1$ where that maximum occurs $[(V/V_{cr})_{1,p,\mathrm{mx}}]$ are calculated by subroutine MAXVAL. If the total pressure drop through the stator (station 0 to station 1) is modeled by specifying the total pressure ratio p'_1/p'_0, $(\rho V)_{1,p,\mathrm{mx}}$ occurs at $(V/V_{cr})_{1,p,\mathrm{mx}}=1.0$. If the total pressure drop is modeled by specifying $\bar{e}_{3D}$, it occurs at $(V/V_{cr})_{1,p,\mathrm{mx}} < 1.0$. If p'_1/p'_0 is specified, total pressure at station 1 is calculated from

$$p'_{1,p}=\left(\frac{p'_1}{p'_0}\right)p'_{0,p} \tag{B2}$$

and the three-dimensional kinetic energy loss coefficient $\bar{e}_{3D}$ is obtained from

$$\bar{e}_{3D}=\frac{\left(\dfrac{p'_1}{p'_0}\right)^{\frac{(\gamma-1)}{\gamma}}\left[1-\dfrac{\gamma-1}{\gamma+1}\left(\dfrac{V}{V_{cr}}\right)_1^2\right]+\dfrac{\gamma-1}{\gamma+1}\left(\dfrac{V}{V_{cr}}\right)_1^2-1}{\left(\dfrac{p'_1}{p'_0}\right)^{\frac{(\gamma-1)}{\gamma}}\left[1-\dfrac{\gamma-1}{\gamma+1}\left(\dfrac{V}{V_{cr}}\right)_1^2\right]-1} \tag{B3}$$

If $\bar{e}_{3D}$ is specified,

$$p'_{1,p}=p'_{0,p}\left\{\frac{1-\bar{e}_{3D}-\dfrac{\gamma-1}{\gamma+1}\left(\dfrac{V}{V_{cr}}\right)_{1,p}^2}{(1-\bar{e}_{3D})\left[1-\dfrac{\gamma-1}{\gamma+1}\left(\dfrac{V}{V_{cr}}\right)_{1,p}^2\right]}\right\}^{\gamma/(\gamma-1)} \tag{B4}$$

Both methods of calculating the total pressure drop are equivalent to specifying an overall viscous loss that takes into account the vane profile, endwall, and secondary flow losses.

From the isentropic relationships

$$p_{1,p}=p'_{1,p}\left\{1-\frac{\gamma-1}{\gamma+1}\left(\frac{V}{V_{cr}}\right)_{1,p}^2\right\}^{\gamma/(\gamma-1)} \tag{B5}$$

$$T_{1,p}=T'_{1,p}\left\{1-\frac{\gamma-1}{\gamma+1}\left(\frac{V}{V_{cr}}\right)_{1,p}^2\right\} \tag{B6}$$

and

$$\rho_{1,p}=\frac{p_{1,p}}{RT_{1,p}} \tag{B7}$$

$$V_{1,p}=\left(\frac{V}{V_{cr}}\right)_{1,p}\left(\frac{2\gamma g_c RT'_{1,p}}{\gamma+1}\right)^{1/2} \tag{B8}$$

For values of $(V/V_{cr})_{1,p}$ less than $(V/V_{cr})_{1,p,\mathrm{mx}}$ the primary flow exits at the specified stator exit blade angle α_1. Thus

$$w_{1,p}=\rho_{1,p}V_{1,p}A_1(1-\mathrm{CLFR1})\cos\alpha_{1,p} \tag{B9}$$

For values of $(V/V_{cr})_{1,p}$ greater than $(V/V_{cr})_{1,p,\mathrm{mx}}$ (choked flow) the exit angle is decreased to maintain maximum primary flow rate. The choked-flow exit angle is

$$\alpha_{1,p,\mathrm{ck}}=\cos^{-1}\left\{\frac{w_{p,\mathrm{mx}}}{\rho_{1,p}V_{1,p}A_1(1-\mathrm{CLFR1})}\right\} \tag{B10}$$

The radial and axial components of the primary flow are then defined by

$$V_{u,1,p}=V_{1,p}\sin\alpha_{1,p} \tag{B11}$$

$$V_{r,1,p}=V_{1,p}\cos\alpha_{1,p} \tag{B12}$$

Mach number is calculated from

$$M_{1,p}=\left[\frac{\frac{2}{\gamma+1}\left(\frac{V}{V_{cr}}\right)^2_{1,p}}{1-\frac{\gamma-1}{\gamma+1}\left(\frac{V}{V_{cr}}\right)^2_{1,p}}\right]^{1/2} \tag{B13}$$

If the radial component of Mach number ($M_1 \cos \alpha_{1,p}$) is greater than 1.0, the specified $(V/V_{cr})_{1,p}$ is reduced until $M_{1,p} \cos \alpha_{1,p} = 1.0$ (defined as limit loading).

Station 1 – clearance flow. – The clearance flow is assumed to have the same total and static conditions of temperature and pressure as the primary flow:

$$T'_{1,cl} = T'_{1,p} \tag{B14}$$

$$T_{1,cl} = T_{1,p} \tag{B15}$$

$$p'_{1,cl} = p'_{1,p} \tag{B16}$$

$$p_{1,cl} = p_{1,p} \tag{B17}$$

This implies that

$$V_{1,cl,cr} = V_{1,p,cr} \tag{B18}$$

$$V_{1,cl} = V_{1,p} \tag{B19}$$

$$\left(\frac{V}{V_{cr}}\right)_{1,cl} = \left(\frac{V}{V_{cr}}\right)_{1,p} \tag{B20}$$

$$\left(\frac{V}{V_{cr}}\right)_{1,cl,mx} = \left(\frac{V}{V_{cr}}\right)_{1,p,mx} \tag{B21}$$

$$\rho_{1,cl} = \rho_{1,p} \tag{B22}$$

The flow angle $\alpha_{1,cl}$ and the flow rate $w_{1,cl}$ are determined iteratively from conservation of moment of tangential momentum between stations 0 and 1. The tangential component of velocity is obtained from

$$V_{u,1,cl} = \frac{r_0 V_{u,0}}{r_1} \tag{B23}$$

The clearance flow angle and flow rate thus are

$$\alpha_{1,cl} = \sin^{-1}\left(\frac{V_{u,1,cl}}{V_{1,cl}}\right) \tag{B24}$$

$$w_{1,cl} = \rho_{1,cl} V_{1,cl} A_1 \text{ CLFR1} \cos \alpha_{1,cl} \tag{B25}$$

Since $\alpha_{1,cl}$ is uniquely defined by equation (B24), it cannot be changed to maintain the maximum clearance weight flow at values of $(V/V_{cr})_{1,cl}$ greater than $(V/V_{cr})_{1,cl,mx}$. Clearance weight flow therefore decreases above $(V/V_{cr})_{1,cl,mx}$.

From here on, all conditions at station 1 that are the same for both the primary and clearance flows (T, T', p, p', V, V_{cr}, and ρ) will have the subscripts p or cl deleted and will be identified only by the subscript 1.

Station 0. – Conditions at station 0 are evaluated from the specified geometry, inlet angle, and gas total conditions and by using mass continuity between stations 0 and 1. The ratio $(V/V_{cr})_0$ is determined iteratively from

$$\left(\frac{V}{V_{cr}}\right)_0 = \frac{(w_{1,p}+w_{1,cl})RT'_0}{A_0 \cos \alpha_0\, p'_0 V_{0,cr}\left[1-\frac{\gamma-1}{\gamma+1}\left(\frac{V}{V_{cr}}\right)^2_0\right]^{1/(\gamma-1)}} \tag{B26}$$

Also

$$p_0 = p'_0\left[1-\frac{\gamma-1}{\gamma+1}\left(\frac{V}{V_{cr}}\right)^2_0\right]^{\gamma/(\gamma-1)} \tag{B27}$$

$$T_0 = T'_0\left\{1-\frac{\gamma-1}{\gamma+1}\left(\frac{V}{V_{cr}}\right)^2_0\right\} \tag{B28}$$

$$\rho_0 = \frac{p_0}{RT_0} \tag{B29}$$

$$V_0 = \left(\frac{V}{V_{cr}}\right)_0\left(\frac{2\gamma R g_c T'_0}{\gamma+1}\right)^{1/2} \tag{B30}$$

$$V_{u,0} = V_0 \sin \alpha_0 \tag{B31}$$

$$V_{r,0} = V_0 \cos \alpha_0 \tag{B32}$$

Station 2. – The total temperature at station 2 is assumed to equal that at station 1

$$T'_2 = T'_1 \tag{B33}$$

and tangential momentum is conserved between stations 1 and 2. The tangential velocity is a mass-averaged value of the primary and clearance flows at station 1:

$$V_{u,2} = \frac{V_1}{w_{1,p}+w_{1,cl}}(w_{1,p}\sin\alpha_{1,p}+w_{1,cl}\sin\alpha_{1,cl}) \tag{B34}$$

For no clearance flow this reduces to

$$V_{u,2} = V_1 \sin \alpha_{1,p} = V_{u,1} \tag{B35}$$

By using an initial guess for $V_{r,2}$, the conditions at station 2 are evaluated iteratively as follows:

$$V_2 = \left(V_{r,2}^2 + V_{u,2}^2\right)^{1/2} \tag{B36}$$

$$\left(\frac{V}{V_{cr}}\right)_2 = \frac{V_2}{\left(\frac{2\gamma R g_c T_2'}{\gamma + 1}\right)^{1/2}} \tag{B37}$$

$$T_2 = T_2'\left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_2^2\right] \tag{B38}$$

$$p_2' = p_1' - \left(1 - \frac{V_{r,2}}{V_{r,1}}\right)^2\left(\frac{0.5\rho_1 V_1^2}{g_c}\right) \tag{B39}$$

$$p_2 = p_2'\left[1 - \frac{\gamma - 1}{\gamma + 1}\left(\frac{V}{V_{cr}}\right)_2^2\right]^{\gamma/(\gamma - 1)} \tag{B40}$$

$$\rho_2 = \frac{p_2}{RT_2} \tag{B41}$$

$$\alpha_2 = \cos^{-1}\left(\frac{V_{r,2}}{V_2}\right) \tag{B42}$$

$$w_2 = \rho_2 V_2 A_2 \cos \alpha_2 \tag{B43}$$

The assumed $V_{r,2}$ is updated until continuity is satisfied; that is,

$$w_2 = w_{1,p} + w_{1,cl} \tag{B44}$$

Station 3. – The total pressure drop between stations 2 and 3 is obtained from the laminar and turbulent equation for flow between parallel plates by using an average flow angle between stations 2 and 3. Furthermore, moment of tangential momentum is conserved ($r_2 V_{u,2} = r_3 V_{u,3}$). From an initial guess of α_3 an average turning angle, a hydraulic diameter, a Reynolds number, and a flow path length are calculated:

$$\alpha_{2\text{-}3} = \frac{\alpha_2 + \alpha_3}{2} \tag{B45}$$

$$D_h = 2h_1 \cos \alpha_{2\text{-}3} \tag{B46}$$

$$\mathrm{Re}_2 = \frac{\rho_2 V_2 D_h}{\mu} \tag{B47}$$

$$l_{2\text{-}3} = \frac{r_1 - r_3}{\cos \alpha_{2\text{-}3}} \tag{B48}$$

Friction factor f is defined as follows:

$$f = 0.24 \qquad \text{for } \mathrm{Re}_2 < 100 \tag{B49}$$

$$f = \frac{24}{\mathrm{Re}_2} \qquad \text{for } 100 \le \mathrm{Re}_2 < 3000 \tag{B50}$$

$$f = 3.3368 \times 10^{-7}\,\mathrm{Re}_2^{1.2596} \qquad \text{for } 3000 \le \mathrm{Re}_2 < 3700 \tag{B51}$$

$$f = 0.0014 + \frac{0.125}{\mathrm{Re}_2^{0.32}} \qquad \text{for } 3700 \le \mathrm{Re}_2 \tag{B52}$$

The change in total pressure and the total pressure at station 3 are obtained from

$$\Delta p' = \frac{4 f l_{2\text{-}3} \rho_2 V_2^2}{2 D_h g_c} \tag{B53}$$

$$p_3' = p_2' - \Delta p' \tag{B54}$$

Assuming that

$$T_3' = T_2' \tag{B55}$$

$$r_2 = r_1 \tag{B56}$$

and

$$V_{u,3} = \frac{r_2/r_3}{V_{u,2}} \tag{B57}$$

all other conditions at station 3 are evaluated from

$$V_3 = \frac{V_{u,3}}{\sin \alpha_3} \tag{B58}$$

$$T_3 = T_3' - V_3^2\left(\frac{\gamma - 1}{2\gamma R g_c}\right) \tag{B59}$$

$$p_3 = p_3'\left(\frac{T_3}{T_3'}\right)^{\gamma/(\gamma - 1)} \tag{B60}$$

$$\rho_3 = \frac{p_3}{RT_3} \tag{B61}$$

$$V_{r,3} = \frac{V_{r,2}\rho_2 A_2}{\rho_3 A_3} \tag{B62}$$

Finally, a new exit angle

$$\alpha_3 = \tan^{-1}\left(\frac{V_{u,3}}{V_{r,3}}\right) \tag{B63}$$

is calculated and compared with the previously used value. This procedure is repeated until successive values of α_3 are within a specified tolerance.

Relative conditions at station 3 are calculated from the following equations:

$$W_{u,3} = V_{u,3} - U_3 \tag{B64}$$

$$W_{r,3} = V_{r,3} \tag{B65}$$

$$W_3 = \left(W_{u,3}^2 + W_{r,3}^2\right)^{1/2} \tag{B66}$$

$$T_3'' = T_3' - \frac{\gamma - 1}{2\gamma R g_c}\left(V_3^2 - W_3^2\right) \tag{B67}$$

$$p_3'' = p_3'\left(\frac{T_3''}{T_3'}\right)^{\gamma/(\gamma - 1)} \tag{B68}$$

$$\beta_3 = \sin^{-1}\left(\frac{W_{u,3}}{W_3}\right) \tag{B69}$$

$$W_{cr,3} = \left(\frac{2\gamma R g_c T_3''}{\gamma + 1}\right)^{1/2} \tag{B70}$$

The rotor incidence angle (from optimum) is calculated as follows: the optimum $V_{u,3}$ is obtained from the Wiesner slip factor correlation (ref. 4), which, when adapted to a turbine geometry, takes the form

$$V_{u,3,opt} = U_3\left(\frac{\left[1 - \sqrt{\cos\alpha_{bl}}/(N_{bl})^{0.7}\right]\left\{1 - \left[(r_{4,av}/r_3 - \epsilon_{lim})/(1 - \epsilon_{lim})\right]^3\right\}}{1 - \tan\alpha_{bl}/\tan\alpha_3}\right) \tag{B71}$$

where

$$r_{4,av} = \frac{r_{4,hub} + r_{4,tip}}{2} \tag{B72}$$

$$\epsilon_{lim} = \frac{1}{e^{(8.16\cos\alpha_{bl}/N_{bl})}} \tag{B73}$$

The last term in the numerator of equation (B71) is a correction factor that is applied only if the ratio $r_{4,av}/r_3$ is greater than ϵ_{lim}. Then

$$W_{u,3,opt} = V_{u,3,opt} - U_3 \tag{B74}$$

$$\beta_{3,opt} = \tan^{-1}\left(\frac{W_{u,3,opt}}{V_{r,3}}\right) \tag{B75}$$

$$\alpha_{3,i} = \beta_3 - \beta_{3,opt} \tag{B76}$$

The loss in total pressure due to incidence is calculated as follows: the incidence loss is modeled as in reference 2 by

$$L_i = \frac{W_3^2\left(1 - \cos^n\alpha_{3,i}\right)}{2g_c J} \tag{B77}$$

where the exponent n is given by the input parameters PNEGI and PPOSI for negative and positive incidence, respectively. The loss due to incidence can also be expressed as a difference between ideal and actual velocity:

$$W_3^2 - W_{3,i}^2 = W_3^2\left(1 - \cos^n\alpha_{3,i}\right) \tag{B78}$$

Solving for $W_{3,i}^2$ gives

$$W_{3,i}^2 = W_3^2\cos^n\alpha_{3,i} \tag{B79}$$

The relationship

$$W_{3,i}^2 = \frac{2\gamma g_c R T_3''}{\gamma - 1}\left[1 - \left(\frac{p_3}{p_{3,i}''}\right)^{(\gamma-1)/\gamma}\right] \tag{B80}$$

implies that

$$p_{3,i}'' = \frac{p_3}{\left[1 - \dfrac{(\gamma-1)W_{3,i}^2}{2\gamma g_c R T_3''}\right]^{\gamma/(\gamma-1)}} \tag{B81}$$

Substituting equation (B79) for $W_{3,i}^2$ then gives

$$p_{3,i}'' = \frac{p_3}{\left[1 - \dfrac{(\gamma-1)W_3^2 \cos^n\alpha_{3,i}}{2\gamma g_c R T_3''}\right]^{\gamma/(\gamma-1)}} \tag{B82}$$

Station 4. – Flow conditions at station 4 are calculated by considering the effects of radius change, incidence, and viscous loss. By assuming an initial value of static pressure at the hub sector the pressure at each successive sector is obtained from simple radial equilibrium. The total mass flow of all sectors is then compared with the mass flow at station 1, and the next guess of hub-sector static pressure is changed accordingly. This is repeated until mass flow has converged within a given tolerance. The procedure is given here (with calculations performed for each sector at the midpoint radius).

The changes in total temperature, critical velocity, and ideal relative total pressure due to radius change are calculated from

$$T_4'' = T_3'' + \frac{\gamma - 1}{2\gamma R g_c}\left(U_4^2 - U_3^2\right) \tag{B83}$$

$$W_{cr,4} = W_{cr,3}\left(\frac{T_4''}{T_3''}\right)^{1/2} \tag{B84}$$

$$p_{4,id}'' = p_3''\left(\frac{T_4''}{T_3''}\right)^{\gamma/(\gamma-1)} \tag{B85}$$

Note that the ideal relative total pressure at station 4 is based on the relative total pressure at station 3 and not on the relative total pressure due to incidence loss $p_{3,i}''$. At station 3, $p_{3,i}''$ was calculated for information purposes only. As will be shown, the relative total pressure at station 4 p_4'' is obtained from $p_{4,id}''$ by considering the combined effects of incidence and rotor viscous loss.

The value of maximum flow rate per unit area $(\rho W)_{4,mx}$, as well as the value of $(W/W_{cr})_4$ where it occurs $\left[(W/W_{cr})_{4,mx}\right]$, is calculated by subroutine MAXVAL. The total relative pressure at station 4 is formulated as follows: the incidence loss is modeled by (see previous section)

$$L_i = \frac{W_3^2\left(1 - \cos^n\alpha_{3,i}\right)}{2g_c J} \tag{B86}$$

and the loss due to viscous friction in the rotor (rotor blade profile, endwall, and secondary flow losses) is modeled as in reference 1 by

$$L_f = \frac{K\left(W_3^2 \cos^2\alpha_{3,i} + W_4^2\right)}{2g_c J} \tag{B87}$$

where K is the specified rotor viscous loss coefficient (input parameter XKR). The sum of these two losses is expressed as the difference between the squares of an ideal and an actual velocity to give

$$W_{4,id}^2 - W_4^2 = W_3^2\left(1 - \cos^n\alpha_{3,i}\right) + K\left(W_3^2\cos^2\alpha_{3,i} + W_4^2\right) \tag{B88}$$

This equation can be solved for W_4

$$W_4 = \left[\frac{W_{4,id}^2 - W_3\left(1 - \cos^n\alpha_{3,i}\right) - K W_3^2 \cos^2\alpha_{3,i}}{1 + K}\right]^{1/2} \tag{B89}$$

with $W_{4,id}^2$ obtained from the relationship

$$W_{4,id}^2 = \frac{W_{cr,4}^2(\gamma+1)}{\gamma - 1}\left[1 - \left(\frac{p_4}{p_{4,id}''}\right)^{(\gamma-1)/\gamma}\right] \tag{B90}$$

For a given value of static pressure the relative velocity is thus calculated by equation (B89). Also,

$$T_4 = T_4'' - \frac{(\gamma-1)W_4^2}{2\gamma R g_c} \tag{B91}$$

$$\rho_4 = \frac{p_4}{RT_4} \tag{B92}$$

$$p_4'' = p_4 \left(\frac{T_4''}{T_4}\right)^{\gamma/(\gamma-1)} \tag{B93}$$

The static pressure at each radial sector above the hub sector is obtained from

$$\frac{dp}{dr} = \frac{\rho_4(k-1)\{[V(k-1)_{u,4} + V(k)_{u,4}]/2\}^2}{g_c r(k-1)_4} \tag{B94}$$

$$p_4(k) = p_4(k-1) + \frac{dp}{dr}\Delta r \tag{B95}$$

where k designates the given sector and $(k-1)$ designates the sector immediately below it.

For $(W/W_{cr})_4$ less than $(W/W_{cr})_{4,mx}$ the flow follows the specified exit angle β_4; for $(W/W_{cr})_4$ greater than $(W/W_{cr})_{4,mx}$ the exit angle is decreased to maintain the maximum flow rate. The reduced exit angle is calculated from

$$\beta_4 = \cos^{-1}\left[\frac{(\rho W)_{4,mx}}{\rho_4 W_4 A_5(1-\mathrm{BL4})}\right] \tag{B96}$$

where BL4 is the ratio of the blocked area at station 4 to the free flow area at station 5. Weight flow and velocity components are calculated from

$$w_4 = \rho_4 W_4 A_5(1-\mathrm{BL4}) \cos\beta_4 \tag{B97}$$

$$W_{u,4} = W_4 \sin\beta_4 \tag{B98}$$

$$W_{x,4} = W_4 \cos\beta_4 \tag{B99}$$

If the rotor is fully choked (all sectors), the program solves for that value of stator $(V/V_{cr})_1$ where rotor choke occurred. Also, the maximum value of station 4 hub-sector static pressure for rotor choke is calculated and printed out. Once the rotor has fully choked, the hub-sector static pressure is incrementally decreased by a specified amount (input parameter P4RDPC) until limit loading is achieved (axial Mach number equal to 1.0). The axial component of Mach number is calculated from

$$M_{x,4} = \left[\frac{\left(\frac{2}{\gamma+1}\right)\left(\frac{W}{W_{cr}}\right)_4^2}{1-\left(\frac{\gamma-1}{\gamma+1}\right)\left(\frac{W}{W_{cr}}\right)_4^2}\right]^{1/2} \cos\beta_4 \tag{B100}$$

It is not allowed to exceed 1.0. If it does, the value of hub-sector static pressure is adjusted until $M_{x,4}$ equals 1.0. That value of hub-sector static pressure is also printed out.

The absolute conditions are calculated from the relative conditions by

$$V_{u,4} = W_{u,4} + U_4 \tag{B101}$$

$$V_{x,4} = W_{x,4} \tag{B102}$$

$$V_4 = \left(V_{u,4}^2 + V_{x,4}^2\right)^{1/2} \tag{B103}$$

$$\alpha_4 = \tan^{-1}\left(\frac{V_{u,4}}{V_{x,4}}\right) \tag{B104}$$

$$T_4' = T_4'' + \frac{(\gamma-1)\left(V_4^2 - W_4^2\right)}{2\gamma R g_c} \tag{B105}$$

$$p_4' = \frac{p_4''}{\left(T_4''/T_4'\right)^{\gamma/(\gamma-1)}} \tag{B106}$$

Station 5. – At station 5 the flow area is divided into the same sectors as at station 4. All calculations shown below are performed for each sector at the midpoint radius. The total temperature and tangential momentum at station 5 are assumed to be equal to those at station 4:

$$T_5' = T_4' \tag{B107}$$

$$V_{u,5} = V_{u,4} \tag{B108}$$

Equation (B107) implies that

$$V_{cr,5} = V_{cr,4} \tag{B109}$$

From an initial guess of α_5 the flow conditions are established iteratively as follows:

$$V_5 = \frac{V_{u,5}}{\sin\alpha_5} \tag{B110}$$

$$\left(\frac{V}{V_{cr}}\right)_5 = \frac{V_5}{V_{cr,5}} \tag{B111}$$

$$T_5 = T_5' - (\gamma-1)\frac{V_5^2}{2\gamma R g_c} \tag{B112}$$

$$p_5' = p_4' - \left(1 - \frac{V_5 \cos\alpha_5}{V_{x,4}}\right)^2\left(\frac{0.5\rho_4 V_4^2}{g_c}\right) \tag{B113}$$

$$p_5 = p_5' \left[1 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\left(\frac{V}{V_{cr}}\right)_5^2\right]^{\gamma/(\gamma - 1)} \tag{B114}$$

$$\rho_5 = \frac{p_5}{RT_5} \tag{B115}$$

$$V_{x,5} = \frac{V_{x,4}\rho_4 A_5(1 - \mathrm{BL4})}{\rho_5 A_5} \tag{B116}$$

A new exit angle is then calculated and compared with the previous value.

$$\alpha_5 = \tan^{-1}\left(\frac{V_{u,5}}{V_{x,5}}\right) \tag{B117}$$

The procedure is repeated until successive values of α_5 are within a specified tolerance.

Performance parameters. – The performance parameters are calculated as follows: for each sector the static and total ideal enthalpy change from station 0 to station 5 is calculated from

$$\Delta h_{0\text{-}5} = \frac{\gamma R T_0'}{(\gamma - 1)J}\left[1 - \left(\frac{p_5}{p_0'}\right)^{(\gamma - 1)/\gamma}\right] \tag{B118}$$

$$\Delta h_{0\text{-}5}' = \frac{\gamma R T_0'}{(\gamma - 1)J}\left[1 - \left(\frac{p_5'}{p_0'}\right)^{(\gamma - 1)/\gamma}\right] \tag{B119}$$

Mass-averaged values of ideal enthalpy change, power, and torque are established from

$$w_{tot} = \sum_{k=1}^{N} w(k) \tag{B120}$$

$$\Delta \bar{h}_{0\text{-}5} = \frac{\sum_{k=1}^{N} \Delta h(k)_{0\text{-}5} w(k)}{w_{tot}} \tag{B121}$$

$$\bar{h}_{0\text{-}5}' = \frac{\sum_{k=1}^{N} \Delta h'(k)_{0\text{-}5} w(k)}{w_{tot}} \tag{B122}$$

$$P = \frac{w_{tot} J \, \Delta \bar{h}_{0\text{-}5}'}{550} \tag{B123}$$

$$\mathrm{TRQ} = \frac{w_{tot} J \, \Delta \bar{h}_{0\text{-}5}'(60)}{2\pi\Omega} \tag{B124}$$

The change in total enthalpy across the rotor (station 3 to station 5) for each sector is calculated from

$$\Delta h_{3\text{-}5}' = \frac{U_3 V_{u,3} - U_4 V_{u,4}}{J g_c} \tag{B125}$$

and its mass-averaged value, from

$$\Delta \bar{h}_{3\text{-}5}' = \frac{\sum_{k=1}^{N} \Delta h'(k)_{3\text{-}5} w(k)}{w_{tot}} \tag{B126}$$

The aerodynamic efficiencies for each sector are calculated from

$$\eta_{aed} = \frac{\Delta h_{3\text{-}5}'}{\Delta h_{0\text{-}5}} \tag{B127}$$

$$\eta_{aed}' = \frac{\Delta h_{3\text{-}5}'}{\Delta h_{0\text{-}5}'} \tag{B128}$$

Disk friction and exit tip clearance loss per unit mass flow are obtained from (ref. 3)

$$L_{df} = \frac{0.02125 \rho_3 U_3^3 r_3^2}{g_c J w_{tot}\left(\frac{\rho_3 U_3 r_3}{\mu}\right)^{0.20}} \tag{B129}$$

$$L_{cl} = \frac{2\Delta h_{3\text{-}5}'\left(\frac{h_{cl}}{2r_{4,tip}}\right)}{1 - \frac{r_{4,hub}}{r_{4,tip}}} \tag{B130}$$

Actual total enthalpy, actual power, and actual torque across the rotor are

$$\Delta h_{act}' = h_{3\text{-}5}' - L_{df} - L_{cl} \tag{B131}$$

$$P_{act} = \frac{w_{tot} J \, \Delta h_{act}'}{550} \tag{B132}$$

$$\mathrm{TRQ}_{act} = \frac{60 w_{tot} J \, \Delta h_{act}'}{2\pi\Omega} \tag{B133}$$

and actual overall efficiencies are

$$\eta = \frac{\Delta h'_{act}}{\Delta h_{0-5}} \quad \text{(B134)}$$

$$\eta' = \frac{\Delta h'_{act}}{\Delta h'_{0-5}} \quad \text{(B135)}$$

Mass-averaged static and total temperatures are calculated by

$$T_5 = \frac{\sum_{k=1}^{N} T(k)_5 w(k)}{w_{tot}} \quad \text{(B136)}$$

$$T'_5 = \frac{\sum_{k=1}^{N} T'(k)_5 w(k)}{w_{tot}} \quad \text{(B137)}$$

Static pressures at the hub and shroud and average static and total pressures are obtained from

$$\left(\frac{dp}{dr}\right)_{hub} = \frac{\rho(1)_5 V^2(1)_{u,5}}{g_c r(1)_4} \quad \text{(B138)}$$

$$\left(\frac{dp}{dr}\right)_{shr} = \frac{\rho(N)_5 V^2(N)_{u,5}}{g_c r(N)_4} \quad \text{(B139)}$$

$$p_{5,hub} = p(1)_5 - \left(\frac{dp}{dr}\right)_{hub} \frac{\Delta r}{2} \quad \text{(B140)}$$

$$p_{5,shr} = p(N)_5 + \left(\frac{dp}{dr}\right)_{shr} \frac{\Delta r}{2} \quad \text{(B141)}$$

$$p_{5,av} = \frac{p_{5,hub} + p_{5,shr}}{2} \quad \text{(B142)}$$

$$p'_{5,av} = p_{5,av} \left(\frac{\bar{T}'_5}{\bar{T}_5}\right)^{\gamma/(\gamma-1)} \quad \text{(B143)}$$

Equivalent parameters are calculated on the basis of the following standard conditions:

$$T'_{std} = 288.2 \text{ K } (518.7\ ^\circ\text{R}) \quad \text{(B144)}$$

$$p'_{std} = 101\ 325 \text{ N/m}^2 \ (2116.22 \text{ lbf/ft}^2) \quad \text{(B145)}$$

$$R_{std} = 287.05 \text{ J/(kg-K)} \ (53.35 \text{ ft-lbf/lbm-}^\circ\text{R}) \quad \text{(B146)}$$

$$\gamma_{std} = 1.40 \quad \text{(B147)}$$

The equivalent parameters (flow, speed, work, power, torque, total-to-static pressure ratio, and total-to-total pressure ratio) are

$$w_{equ} = \frac{w_{tot}\sqrt{\theta_{cr}}\,\epsilon}{\delta} \quad \text{(B148)}$$

$$N_{equ} = \frac{\Omega}{\sqrt{\theta_{cr}}} \quad \text{(B149)}$$

$$\text{WRK}_{equ} = \frac{\Delta h'_{act}}{\theta_{cr}} \quad \text{(B150)}$$

$$P_{equ} = \frac{P_{act}\,\epsilon}{\delta\sqrt{\theta_{cr}}} \quad \text{(B151)}$$

$$\text{TRQ}_{equ} = \frac{\text{TRQ}_{act}\,\epsilon}{\delta} \quad \text{(B152)}$$

$$\left(\frac{p'_0}{p_5}\right)_{equ} = \left[1 - \frac{(\gamma_{std}-1)J\,\Delta\bar{h}_{0-5}}{\theta_{cr}\gamma_{std}R_{std}T_{std}}\right]^{-\gamma_{std}/(\gamma_{std}-1)} \quad \text{(B153)}$$

$$\left(\frac{p'_0}{p'_5}\right)_{equ} = \left[1 - \frac{(\gamma_{std}-1)J\,\Delta\bar{h}'_{0-5}}{\theta_{cr}\gamma_{std}R_{std}T_{std}}\right]^{-\gamma_{std}/(\gamma_{std}-1)} \quad \text{(B154)}$$

where

$$\theta_{cr} = \frac{\left(\frac{\gamma}{\gamma+1}\right) R T'_0}{\left(\frac{\gamma_{std}}{\gamma_{std}+1}\right) R_{std} T'_{std}} \quad \text{(B155)}$$

$$\delta = \frac{p'_0}{p'_{std}} \quad \text{(B156)}$$

$$\epsilon = \frac{\gamma_{std}\left(\frac{2}{\gamma_{std}+1}\right)^{\gamma_{std}/(\gamma_{std}-1)}}{\gamma\left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}} \quad \text{(B157)}$$

The overall dimensionless parameters of specific speed, specific diameter, work factor, and loading coefficient (blade-jet speed ratio) are defined as follows:

$$\rho_{5,av} = \frac{p_{5,av}}{RT_{5,av}} \tag{B158}$$

$$N_s = \frac{\left(\frac{2\pi\Omega}{60}\right)\left(\frac{w_{tot}}{\rho_{5,av}}\right)^{1/2}}{(g_c J\ \Delta \bar{h}'_{0\text{-}5})^{0.75}} \tag{B159}$$

$$D_s = \frac{2r_3(g_c J\ \Delta \bar{h}'_{0\text{-}5})^{0.25}}{(w_{tot}/\rho_{5,av})^{1/2}} \tag{B160}$$

$$\psi = \frac{g_c J\ \Delta h'_{3\text{-}5}}{U_3^2} \tag{B161}$$

$$\nu = \frac{U_3}{(2g_c J\ \Delta h_{0\text{-}5})^{1/2}} \tag{B162}$$

Subroutine MAXVAL

Subroutine MAXVAL finds the maximum value of a function over an interval. The chosen function must be of a form such that a maximum exists within the interval (other than at the end points). MAXVAL is used to calculate the maximum values of flow rate per unit area at the stator exit $(\rho V)_1$ and rotor exit $(\rho W)_4$ sectors. The actual mass flow calculations are performed by the called subroutines ROVST and ROVROT for the stator and rotor, respectively. Given the interval, the function to be maximized, and an initial number of divisions of the interval, MAXVAL determines the function's maximum value and its location in the interval.

Subroutine ROVST

Subroutine ROVST calculates the value of density times velocity at the stator exit (station 1) for the assumed stator loss model and a specified $(V/V_{cr})_1$. The formulation is as follows:

$$V_{cr,1} = \left(\frac{2\gamma R g_c T'_1}{\gamma + 1}\right)^{1/2} \tag{B163}$$

$$V_1 = \left(\frac{V}{V_{cr}}\right)_1 V_{cr,1} \tag{B164}$$

$$T_1 = T'_1\left[1 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\left(\frac{V}{V_{cr}}\right)_1^2\right] \tag{B165}$$

$$Q = \left(\frac{\gamma - 1}{\gamma + 1}\right)\left(\frac{V}{V_{cr}}\right)_1^2 \tag{B166}$$

If the total pressure at station 1 is calculated by specifying IOPTS = 1 (specifying the pressure ratio p'_1/p'_0),

$$p'_1 = \left(\frac{p'_1}{p'_0}\right)p'_0 \tag{B167}$$

If the total pressure at station 1 is calculated by specifying IOPTS = 2 (specifying $\bar{e}_{3D}$),

$$p'_1 = p'_0\left[\frac{1 - \bar{e}_{3D} - Q}{(1 - \bar{e}_{3D})(1 - Q)}\right]^{\gamma/(\gamma - 1)} \tag{B168}$$

Finally,

$$p_1 = p'_1\left[1 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\left(\frac{V}{V_{cr}}\right)_1^2\right]^{\gamma/(\gamma - 1)} \tag{B169}$$

$$\rho_1 = \frac{p_1}{RT_1} \tag{B170}$$

$$\rho V = \rho_1 V_1 \tag{B171}$$

Subroutine ROVROT

Subroutine ROVROT calculates the value of density times velocity at the rotor exit sectors (station 4) for the assumed rotor loss model and a specified $(W/W_{cr})_4$. The equations used are as follows:

$$W_4 = \left(\frac{W}{W_{cr}}\right)_4 W_{cr,4} \tag{B172}$$

$$T_4 = T''_4 - \frac{(\gamma - 1)W_4^2}{2\gamma g_c R} \tag{B173}$$

The static pressure is calculated from equation (B88), which expresses the sum of the two rotor losses (incidence and viscous friction) as the difference between the squares of an ideal and an actual velocity

$$W_{4,\mathrm{id}}^2 - W_4^2 = W_3^2\left(1 - \cos^n\alpha_{3,\mathrm{i}}\right) + K\left(W_3^2 \cos^2\alpha_{3,\mathrm{i}} + W_4^2\right) \qquad \text{(B174)}$$

By using

$$W_{4,\mathrm{id}}^2 = \left(\frac{\gamma+1}{\gamma-1}\right) W_{\mathrm{cr},4}^2 \left[1 - \left(\frac{p_4}{p''_{4,\mathrm{id}}}\right)^{(\gamma-1)/\gamma}\right] \qquad \text{(B175)}$$

$$W_4^2 = \left(\frac{\gamma+1}{\gamma-1}\right) W_{\mathrm{cr},4}^2 \left[1 - \left(\frac{p_4}{p''_4}\right)^{(\gamma-1)/\gamma}\right] \qquad \text{(B176)}$$

equation (B174) can be rewritten as

Then

$$p_4 = p''_4 \left[1 - \left(\frac{\gamma-1}{\gamma+1}\right)\left(\frac{W}{W_{\mathrm{cr}}}\right)_4^2\right]^{\gamma/(\gamma-1)} \qquad \text{(B178)}$$

$$\rho_4 = \frac{p_4}{RT_4} \qquad \text{(B179)}$$

$$\rho V = \rho_4 W_4 \qquad \text{(B180)}$$

$$p''_4 = p''_{4,\mathrm{id}} \left[\frac{\left(\frac{\gamma+1}{\gamma-1}\right) - \left(\frac{W_4}{W_{4,\mathrm{cr}}}\right)^2 (1+K) - K\left(\frac{W_3}{W_{4,\mathrm{cr}}}\right)^2 \cos^2\alpha_{3,i} - \left(\frac{W_3}{W_{4,\mathrm{cr}}}\right)^2 \left(1 - \cos^n\alpha_{3,i}\right)}{\left(\frac{\gamma+1}{\gamma-1}\right) - \left(\frac{W_4}{W_{\mathrm{cr},4}}\right)^2}\right]^{\gamma/(\gamma-1)} \qquad \text{(B177)}$$

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16. Abstract

The analysis procedure of an existing computer program was extended to include rotor blade sweep, to model the flow more accurately at the rotor exit, and to provide more detail to the loss model. This report describes the modeling changes and presents all analysis equations and procedures. Program input and output are described and are illustrated by an example problem. Results obtained from this program and from a previous program are compared with experimental data.

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