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Modal Analysis of a Nonuniform String With End Mass and Variable Tension

Mario H. Rheinfurth
and Zachary J. Galaboff

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Modal Analysis of a Nonuniform String With End Mass and Variable Tension

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National Aeronautics
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MODAL ANALYSIS OF A NONUNIFORM STRING WITH END MASS AND VARIABLE TENSION

I. INTRODUCTION

Dynamic systems occasionally contain slender flexible elements which are commonly categorized as strings, ropes, cables, tethers, etc. These elements share the common feature that their bending stiffness can be neglected. The structural usefulness of these elements derives strictly from their being in axial tension during operation. This tension provides the necessary restoring force against lateral loads. The dynamic analysis of these systems usually incorporates partial differential equations for describing the lateral motion of these elements. A drawback of such a dynamic analysis is that the numerical solution can be quite complicated. Considerable simplification can often be achieved by using a modal synthesis technique, which describes the lateral motion of these flexible elements by a superposition of appropriately chosen shape functions. This approach yields a set of ordinary differential equations which are more amenable to numerical solution. The success of such a modal synthesis depends largely on the proper choice of the shape functions. They are usually selected from the natural modes (eigenfunctions) of the isolated structural elements using boundary conditions which are geometrically and dynamically resembling the actual ones.

The following analysis describes an iterative algorithm of the Stodola type which furnishes the natural modes and frequencies of a nonuniform string with one fixed end and the other having an attached end mass. The tension is allowed to vary along the string. If desirable the algorithm can be easily modified to accommodate other boundary conditions.

II. DIFFERENTIAL EQUATION OF THE LATERAL STRING VIBRATIONS

An element of a flexible string, as shown in Figure 1, with T(x) being the tension, p(x,t) an external transverse load per unit length, and $\rho(x)$ the mass per unit length at station x, is considered. If the lateral displacement y(x,t) is assumed to be small, the equation of motion in the transverse direction is obtained by applying Newton's law:

$$\left[T(x) + \frac{\partial T(x)}{\partial x}\,dx\right]\left[\frac{\partial y(x,t)}{\partial x} + \frac{\partial^2 y(x,t)}{\partial x^2}\,dx\right] - T(x)\,\frac{\partial y(x,t)}{\partial x} + p(x,t) = \rho(x)\,dx\,\frac{\partial^2 y(x,t)}{\partial t^2} \quad . \qquad (1)$$

This expression can be reduced to yield the differential equation:

$$\frac{\partial}{\partial x}\left[T(x)\,\frac{\partial y(x,t)}{\partial x}\right] + p(x,t) = \rho(x)\,\frac{\partial^2 y(x,t)}{\partial t^2} \quad . \qquad (2)$$

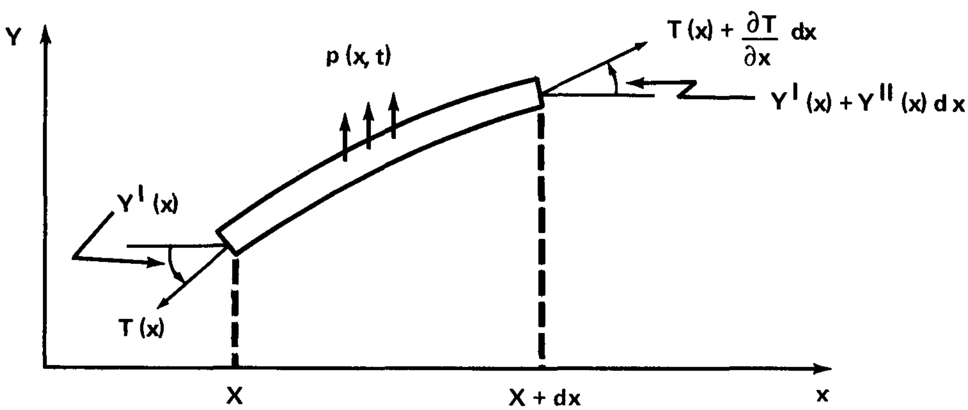

Figure 1. Differential element of a string.

Setting the external loading p(x,t) to zero, the differential equation for the free lateral vibrations is obtained:

$$\frac{\partial}{\partial x}\left[T(x)\,\frac{\partial y(x,t)}{\partial x}\right] = \rho(x)\,\frac{\partial^2 y(x,t)}{\partial t^2} \quad . \tag{3}$$

This differential equation must be satisfied over the entire length of the string.

The boundary conditions associated with the differential equation depend on the particular application. A very common situation is one in which one end is fixed and the other has an attached mass M. This will be used in the following analysis.

If the string is fixed at x = 0,

$$y(0,t) = 0 \tag{4}$$

must be satisfied. The boundary condition for the end $x = \ell$, where the mass is attached, is obtained by Newton's law:

$$M\,\frac{\partial^2 y(\ell,t)}{\partial t^2} = -\,T(\ell)\,\frac{\partial y(\ell,t)}{\partial x} \quad . \tag{5}$$

The minus sign on the right-hand side can be explained by observing that the acceleration of the end mass is caused by the string tension at $x = \ell$. Both boundary condition (4) and (5) are homogeneous.

III. NATURAL MODES AND FREQUENCIES

To determine the natural modes, apply the customary separation of variables method according to which the displacement is written in product form as:

$$y(x,t) = Y(x)\, F(t) \quad , \tag{6}$$

where Y(x) is a function of the station x and F(t) a function of time t only.

Assuming the time function to be a harmonic oscillation with frequency ω, the partial differential equation (3) reduces to the eigenvalue problem

$$\frac{d}{dx}\left[T(x)\, Y'(x)\right] = -\,\omega^2 \rho(x)\, Y(x) \quad , \tag{7}$$

with the boundary conditions

$$Y(0) = 0 \tag{8a}$$

$$\omega^2 M\, Y(\ell) = T(\ell)\, Y'(\ell) \quad . \tag{8b}$$

The following numerical algorithm employs a "sweeping" technique which uses the orthogonality property of the natural modes (eigenfunctions). To obtain this orthogonality property notice that the n natural frequencies and the corresponding n eigenfunctions satisfy the differential equation (7) such that

$$\frac{d}{dx}\left[T(x)\, Y_n{}'(x)\right] = -\,\omega_n{}^2 \rho(x)\, Y_n(x) \quad . \tag{9}$$

If the differential equation is now multiplied by the m-th eigenfunction $Y_m(x)$ and integrated by parts, then

$$\int_0^\ell Y_m(x) \frac{d}{dx}\left[T(x)\, Y_n{}'(x)\right] dx = Y_m(x)\left[T(x)\, Y_n{}'(x)\right]\Big|_0^\ell - \int_0^\ell T(x)\, Y_m{}'(x)\, Y_n{}'(x)\, dx$$

$$= -\,\omega_n{}^2 \int_0^\ell \rho(x)\, Y_m(x)\, Y_n(x)\, dx \quad . \tag{10}$$

Using the boundary conditions (8a) and (8b), the following relationship can be established:

$$\int_0^\ell T(x)\, Y_m'(x)\, Y_n'(x)\, dx = \omega_n^2 \left[\int_0^\ell \rho(x)\, Y_m(x)\, Y_n(x)\, dx + M\, Y_m'(\ell)\, Y_n'(\ell) \right] . \tag{11}$$

Reversing n and m in the above steps which lead to equation (11), equation (12) is obtained:

$$\int_0^\ell T(x)\, Y_m'(x)\, Y_n'(x)\, dx = \omega_m^2 \left[\int_0^\ell \rho(x)\, Y_m(x)\, Y_n(x)\, dx + M\, Y_m'(\ell)\, Y_n'(\ell) \right] . \tag{12}$$

The desired orthogonality property is now obtained by subtracting equation (12) from equation (11) and letting $\omega_m^2 \neq \omega_n^2$ (distinct eigenvalues)

$$\int_0^\ell T(x)\, Y_m'(x)\, Y_n'(x)\, dx = 0 \quad , \tag{13}$$

$$m \neq n$$

$$\int_0^\ell \rho(x)\, Y_m(x)\, Y_n(x)\, dx + M\, Y_m(\ell)\, Y_n(\ell) = 0 \quad . \tag{14}$$

If n = m is set in equation (11), an expression for the natural frequency known as Rayleigh's quotient is obtained:

$$\omega_n^2 = \frac{\displaystyle\int_0^\ell T(x)\, Y_n'^2(x)\, dx}{\displaystyle\int_0^\ell \rho(x)\, Y_n^2(x)\, dx + M\, Y_n^2(\ell)} \tag{15}$$

This expression could also have been obtained by equating the maximum potential energy to the maximum kinetic energy of the system in harmonic motion.

IV. ITERATIVE ALGORITHM

The eigenvalue problem given by equations (7), (8a), and (8b) can be numerically solved in various ways. In the following an iterative algorithm is used whose basic mechanism was first introduced by Stodola [1].

The algorithm proceeds along the following steps:

Step I:

Selection of trial functions which resemble the mode functions to be expected. The exact shape of these trial functions is, however, not very important, because the number of iteration steps necessary for convergence is only slightly affected by the choice of these functions.

For the system being analyzed a suitable set of trial functions is

$$Y_m^{(0)}(x) = (-1)^{m+1} \sin \frac{(2m-1)}{2\ell} \pi x \qquad m = 1, 2, \ldots \quad . \tag{16}$$

Step II:

Starting with m = 1 insert a trial function on the right-hand side of equation (7) and integrate to obtain the transverse force due to the inertial loading on the string. This yields

$$Q_m^{*}(x) = \int_x^{\ell} \rho(x)\, Y_m(x)\, dx + M\, Y_m(\ell) \quad . \tag{17}$$

The factor ω^2 has been omitted because the mode functions have an arbitrary amplitude. The last term of this equation represents the inertial load caused by the end mass M.

Step III:

Calculate the slope of the mode function by dividing the transverse force by the tension in the string according to equation (7):

$$Y_m'^{*}(x) = \frac{Q_m^{*}(x)}{T(x)} \quad . \tag{18}$$

Step IV:

Obtain a new improved mode function by integrating the corresponding slope:

$$Y_m^*(x) = \int_o^x Y_m'(x)\, dx \quad . \tag{19}$$

Step V:

To avoid convergence to the first mode function when higher modes are calculated, it is necessary to introduce a "sweeping" procedure by which lower mode components are eliminated. Mathematically this is achieved by making the mode function under consideration orthogonal to all lower mode functions using the Gram-Schmidt orthogonalization process [2] with the orthogonality relation of equation (14).

Omitting the details the end result is

$$\tilde{Y}_m(x) = Y_m^*(x) - \sum_{i=1}^{m-1} a_{m,i}\, Y_i(x) \quad , \tag{20}$$

where $Y_i(x)$ are all the final lower mode functions obtained earlier.

The coefficients $a_{m,i}$ are

$$a_{m,i} = \frac{\int_o^\ell \rho(x)\, Y_m^*(x)\, Y_i(x)\, dx + M\, Y_m^*(\ell)\, Y_i(\ell)}{\int_o^\ell \rho(x)\, Y_i^2(x)\, dx + M\, Y_i^2(\ell)} \quad . \tag{21}$$

Step VI:

It remains to normalize the new improved mode function, which has an arbitrary scale factor, by a convenient normalization factor. Sometimes the generalized mass

$$M_i = \int_o^\ell \rho(x)\, Y_i^2(x)\, dx + M\, Y_i^2(\ell) \quad , \tag{22}$$

is used as a normalization factor, i.e., the arbitrary scale factor is chosen to obtain a generalized mass of unity. Another normalization procedure sets either the maximum deflection or the deflection at the free end to unity. If the first procedure is used the present normalization step is

$$Y_i(x) = \frac{\widetilde{Y}_i(x)}{M_i} \quad . \tag{23}$$

Step VII:

At this point one iteration cycle is complete and a decision has to be made whether to repeat another cycle or to terminate the process. This decision is usually based upon the accuracy requirements demanded for the natural frequency as given by equation (15). That is, if two successive values of this frequency are within the desired accuracy limits the iteration is stopped.

V. APPLICATIONS

A. Uniform String in a Constant Gravity Field, No End Mass (M = 0)

A uniform string of length ℓ and mass per unit length ρ is suspended by one end in a constant gravitational force field. The free end has no end mass. To simplify the ensuing differential equation, the fixed end is at $x = \ell$ and the free end at $x = 0$ (Fig. 2). In this case the tension is not constant, but equal to the weight of the string below a general point x. Therefore the tension is

$$T(x) = g \int_0^x \rho \, dx = \rho \, gx \quad . \tag{24}$$

Substituting this tension equation in equation (7) leads to

$$x \, Y''(x) + Y'(x) + \lambda^2 \, Y(x) = 0 \quad , \tag{25}$$

where $\lambda^2 = \omega^2/g$.

This is a Bessel-type differential equation whose solution is [3]

$$Y(x) = C_1 \, J_o \, (2 \, \lambda \sqrt{x}) + C_2 \, Y_o \, (2 \, \lambda \sqrt{x}) \quad . \tag{26}$$

The corresponding boundary conditions are taken from equation (8a) and (8b). For the fixed end obtain (remember: 0 and ℓ are reversed):

$$Y(\ell) = 0 \quad . \tag{27}$$

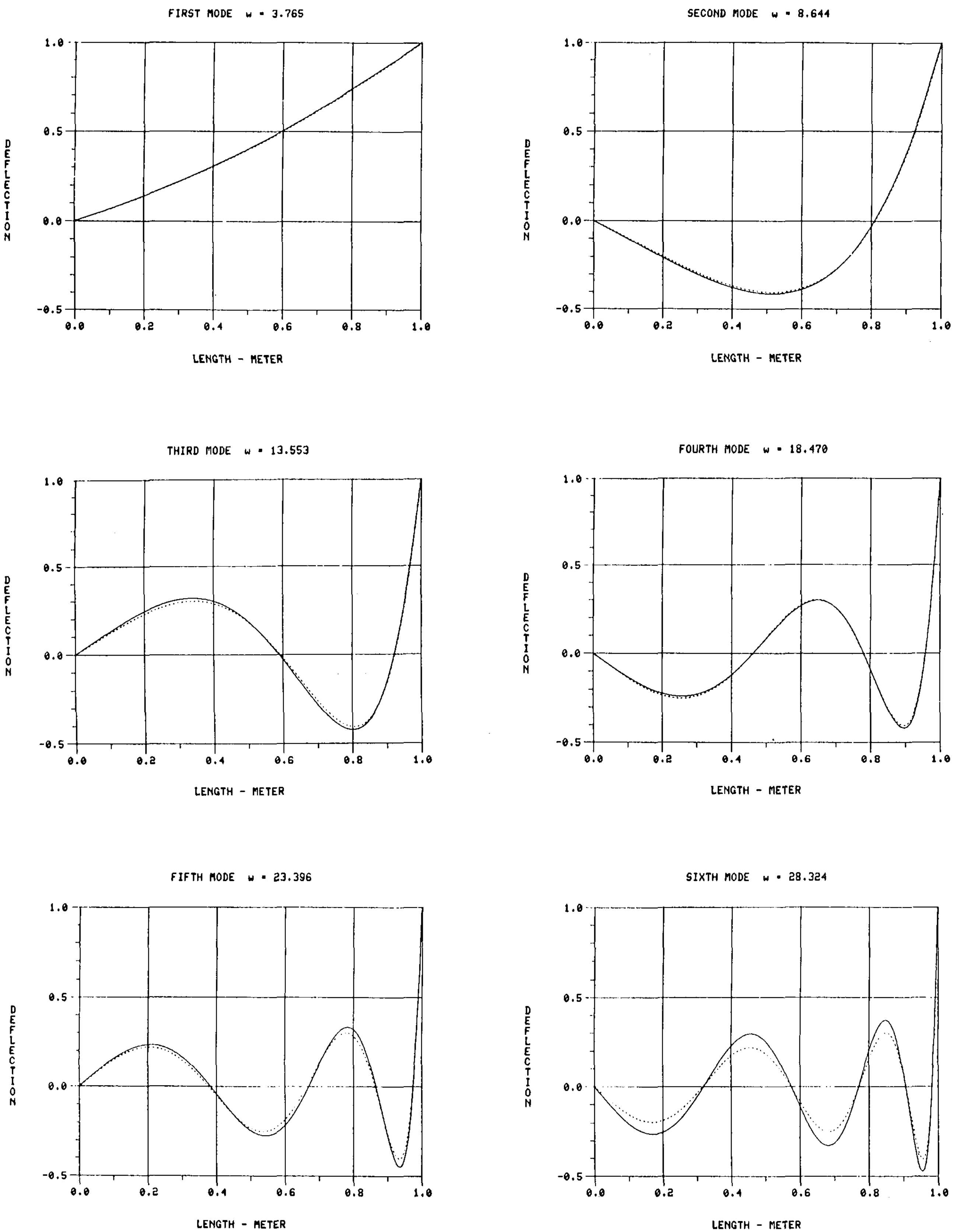

Figure 2. Mode functions for constant gravity field (M = 0), length = 1 m.

For the free end the mass M and the tension in the string are both zero. Consequently there are no restrictions on the deflection and the slope at the free end; only that the deflection must be finite. Because the function $Y_0(2 \lambda \sqrt{x})$ becomes infinite for $x = 0$, it is obvious that the coefficient C_2 must be zero. For the fixed end the deflection is zero. That means:

$$J_0 (2 \lambda \sqrt{\ell}) = 0 \quad . \tag{28}$$

This equation determines the natural frequencies of the string in the uniform gravity field. There are many roots to equation (28) because they are the zeroes of the Bessel function of the first kind of order zero. Using the appropriate mathematical tables of Reference 3 the first six natural frequencies are obtained as:

$$\begin{aligned} \omega_1 &= 1.2024 \sqrt{g/\ell} & \omega_2 &= 2.7600 \sqrt{g/\ell} \\ \omega_3 &= 4.3268 \sqrt{g/\ell} & \omega_4 &= 5.8958 \sqrt{g/\ell} \\ \omega_5 &= 7.4655 \sqrt{g/\ell} & \omega_6 &= 9.0355 \sqrt{g/\ell} \quad . \end{aligned} \tag{29}$$

The associated mode functions are shown in Figure 2 for a length $\ell = 1$ m.

It is interesting to note that the string behaves in its first mode dynamically almost like a rigid slender rod, whose frequency is

$$\omega = 1.2247 \sqrt{g/\ell} \quad , \tag{30}$$

and whose mode function is a straight line. Because the string is less stiff than the rod it is not surprising that its first mode frequency is smaller than the frequency of the rigid rod.

To validate the iterative algorithm outlined in the preceding paragraph a computer program was written and a numerical solution obtained for the above example. Using a simple trapezoidal integration procedure with 500 integration points resulted in an accuracy of 0.4 percent for the 6th natural frequency. An increase to 1000 integration points reduced the error of the 6th natural frequency to 0.1 percent. Since the error increases with the mode number the above given error is smaller for the lower modes. Table 1 shows a comparison between the exact natural frequencies of equation (29) and the frequencies from the Stodola method using 1000 integration points for the first six modes. (The units are in radians per second).

Figure 2 depicts the first six mode functions obtained by the iterative algorithm (solid curves) and for the purpose of comparison the corresponding exact mode functions, which are given by the Bessel functions of equation (26) (dashed curves). It is seen that with increasing mode number the accuracy erodes more rapidly for the mode functions than for the natural frequencies. It must be

TABLE 1. COMPARISON OF NATURAL FREQUENCIES
(CONSTANT GRAVITY FIELD)

	Stodola	Exact
ω_1	3.765	3.766
ω_2	8.644	8.645
ω_3	13.553	13.552
ω_4	18.470	18.466
ω_5	23.396	23.383
ω_6	28.324	28.300

remembered, however, that the present algorithm uses the very simple trapezoidal rule for integration and that substantial improvement in accuracy would result by using a more precise quadrature rule.

B. Uniform String in a Constant Gravity Field With End Mass

This case cannot be solved in closed form. Numerical solutions are presented for two end masses, one in which the end mass is equal to the string mass (M = m), the other in which the end mass is twice the string mass (M = 2m). The first six mode functions and associated natural frequencies are shown in Figure 3 and Figure 4. It is interesting to note that the mode functions do not change much with increasing end mass and quickly approach those of a fixed string under constant tension. With increasing mass the free end becomes almost a nodal point with the exception of the first mode. The first mode becomes more and more a straight line, that is, the dynamic behavior is very close to that of a rigid rod with an attached end mass. For end mass M = m the rigid rod has a natural frequency of $\omega = 3.322$ s^{-1} which is slightly higher than the first mode frequency of $\omega_1 = 3.308\ s^{-1}$ in Figure 3. This is consistent with the observation made above for the case of no end mass. The rigid rod being stiffer than the flexible string has to exhibit a higher frequency. For end mass M = 2 m the rigid rod has a natural frequency of $\omega = 3.242\ s^{-1}$. For comparison with the first natural frequency $\omega_1 = 3.236\ s^{-1}$ of Figure 4, it is again seen that the rigid body frequency is higher, but that the difference has become much smaller because of the increased end mass.

C. Uniform String in a Gravity Gradient Force Field, No End Mass (M = 0)

In recent years a number of space missions have been proposed which incorporate tether-mediated orbital maneuvers. The dynamic analysis of such systems via a modal synthesis technique requires a set of suitable shape functions for describing the lateral tether deflections. The next two paragraphs are, therefore, dedicated to calculating the lateral mode functions and natural frequencies of a string in orbit. It is assumed that the string is in a circular orbit and aligned with the local vertical in its undeformed state. The tension in the string is determined by the combined effect of the centrifugal force and the first-order gravity gradient force term of a spherically symmetric gravity field. For a uniform string without an end mass the tension in the string is then for a fixed end at x = 0:

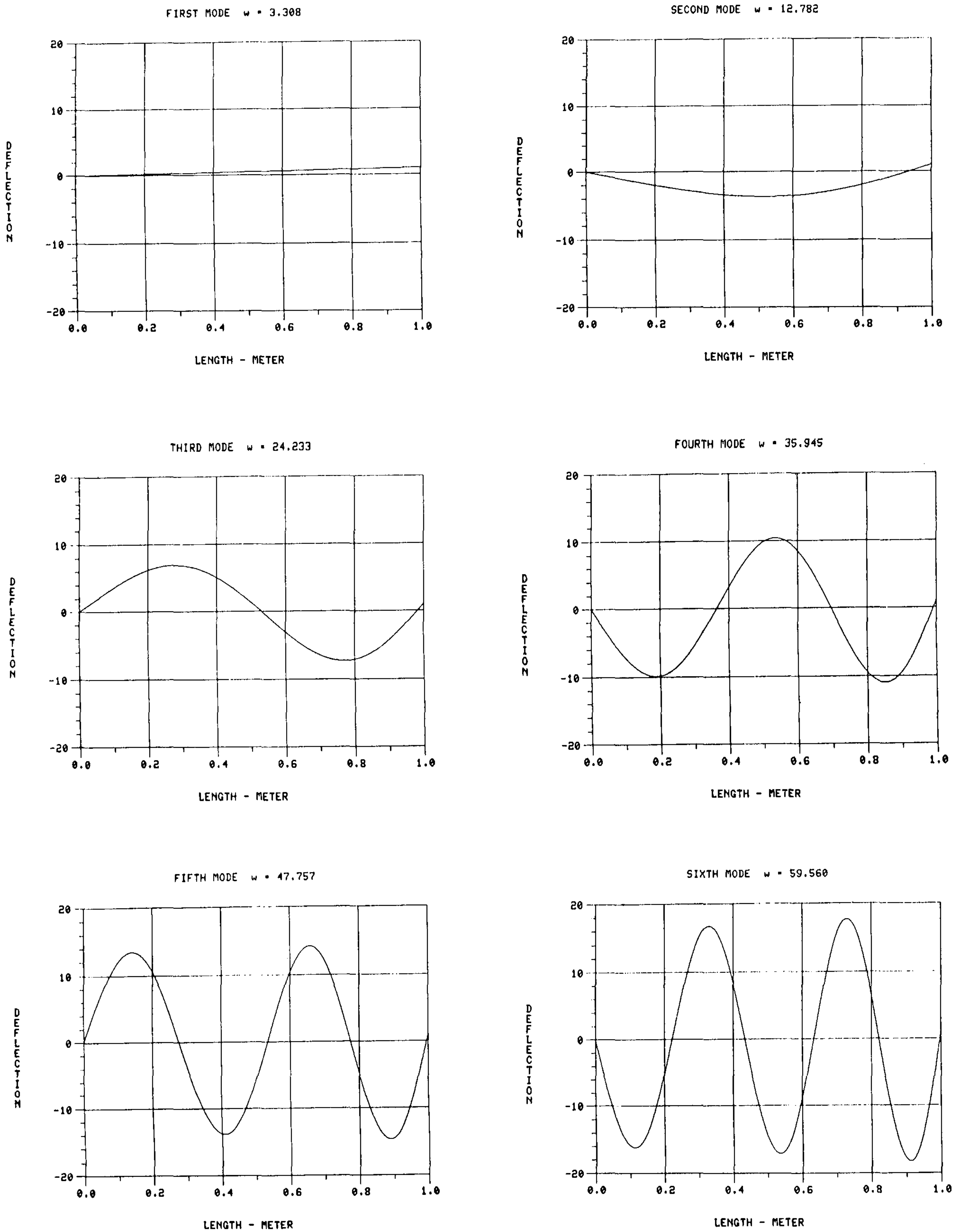

Figure 3. Mode functions for constant gravity field (M = m), length = 1 m.

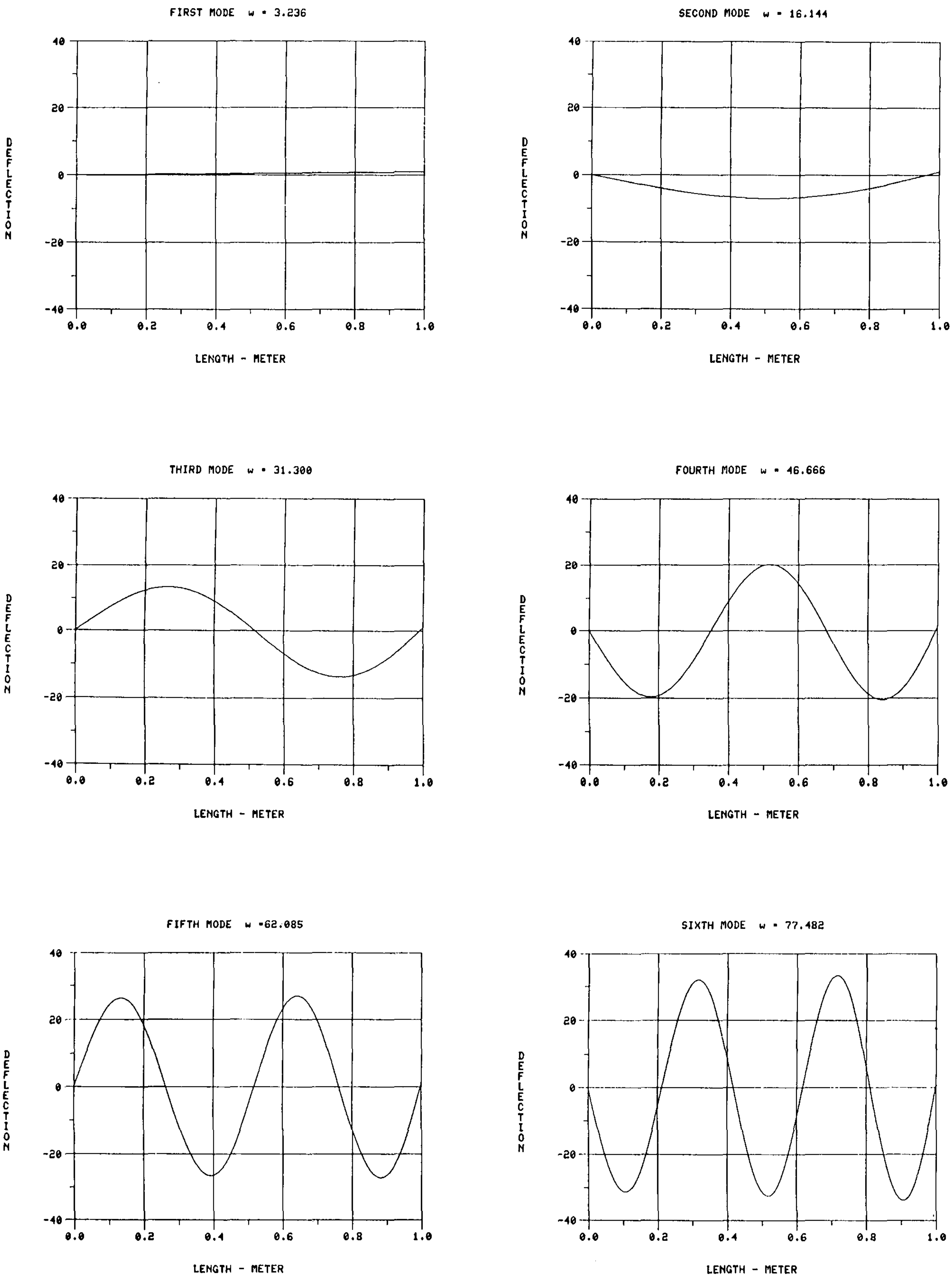

Figure 4. Mode functions for constant gravity field (M = 2m), length = 1 m.

$$T(x) = \frac{3}{2} \omega_c^2 \rho (\ell^2 - x^2) \tag{31}$$

where

ω_c = orbital angular velocity

$\rho = m/\ell$ = mass per unit length .

If only first-order gravity gradient terms are used, the tension is the same for a string hanging "down" or hanging "up."

Substituting equation (31) in equation (7) yields the eigenvalue problem:

$$\frac{d}{dx} [(\ell^2 - x^2) Y'(x)] + \frac{2}{3} \frac{\omega^2}{\omega_c^2} Y(x) = 0 \tag{32}$$

or with $z = x/\ell$

$$(1 - z^2) \frac{d^2 Y(z)}{dz^2} - 2z \frac{d Y(z)}{dz} + n(n+1) Y(z) = 0 \quad , \tag{33}$$

where

$$n(n+1) = \frac{2}{3} \frac{\omega^2}{\omega_c^2} \quad , \qquad n = \text{integer} \quad .$$

This equation is known as Legendre's differential equation and its solution as Legendre polynomials. The general solution is given by

$$Y_n(z) = \sum_{k=0}^{N} \frac{(-1)^k (2n-k)!}{2^n k! (n-k)! (n-2k)!} z^{n-2k} \tag{34}$$

with

$N = n/2 \quad$ if n is even $\qquad 0 < z < 1$

$N = n-1/2 \quad$ if n is odd .

It is readily seen that only the odd polynomials satisfy the boundary condition at x = 0. Their associated natural frequencies are obtained from equation (33) as:

$$\omega = \sqrt{\frac{3}{2} n(n+1)}\ \omega_c \qquad n = 1, 3, 5, \ldots \tag{35}$$

In contrast to the string in a constant gravity field, the natural frequencies are independent of the length of the string. They are directly proportional to the orbital angular velocity. The first six natural frequencies are then:

$$\begin{aligned}
\omega_1 &= \sqrt{3}\,\omega_c = 1.7321\ \omega_c \\
\omega_2 &= \sqrt{18}\ \omega_c = 4.2426\ \omega_c \\
\omega_3 &= \sqrt{45}\ \omega_c = 6.7082\ \omega_c \\
\omega_4 &= \sqrt{84}\ \omega_c = 9.1652\ \omega_c \\
\omega_5 &= \sqrt{135}\ \omega_c = 11.6190\ \omega_c \\
\omega_6 &= \sqrt{198}\ \omega_c = 14.0712\ \omega_c
\end{aligned} \tag{36}$$

The first six mode functions of equation (34) are (normalized to unity at z = 1):

$$Y_1(z) = z$$

$$Y_2(z) = \frac{1}{2}(5\ z^3 - 3\ z)$$

$$Y_3(z) = \frac{1}{8}(63\ z^5 - 70\ z^3 + 15\ z) \tag{37}$$

$$Y_4(z) = \frac{1}{16} (429\ z^7 - 693\ z^5 + 315\ z^3 - 35\ z)$$

$$Y_5(z) = \frac{1}{128} (12155\ z^9 - 25740\ z^7 + 18018\ z^5 - 4620\ z^3 + 315\ z)$$

$$Y_6(z) = \frac{1}{256} (88179\ z^{11} - 230945\ z^9 + 218790\ z^7 - 90090\ z^5 + 15015\ z^3 - 693\ z) \quad .$$

(37)
(concluded)

Plots of these mode functions are shown in Figure 5 for a length ℓ = 100 km.

It is interesting to note that the gravity gradient string behaves in its first mode dynamically exactly like a rigid slender rod. This is even true for a string with arbitrary mass density as can be verified by setting Y(x) = x in equation (7) and the tension T(x)

$$T(X) = 3\ \omega_c{}^2 \int_x^{\ell} \rho(x)\ x\ dx \quad . \tag{38}$$

The natural frequency is, of course, $\omega_1 = \sqrt{3}\ \omega_c$. Table 2 shows a comparison between the exact natural frequencies as calculated from equation (36) and the ones obtained from the Stodola method. The altitude of the circular orbit above the Earth surface was set at h = 200 km which yields an orbital rate $\omega_c = 1.1847 \times 10^{-3}$ rad/sec. The Stodola method for this case was set up to sweep out the straight line first mode before starting the iteration cycle. The frequency units are again in radians per second.

TABLE 2. COMPARISON OF NATURAL FREQUENCIES
(GRAVITY-GRADIENT FIELD)

	Stodola	Exact
ω_1	2.0520×10^{-3}	2.0520×10^{-3}
ω_2	5.0264×10^{-3}	5.0263×10^{-3}
ω_3	7.9503×10^{-3}	7.9472×10^{-3}
ω_4	1.0858×10^{-3}	1.0858×10^{-3}
ω_5	1.3775×10^{-2}	1.3765×10^{-3}
ω_6	1.6680×10^{-2}	1.6670×10^{-3}

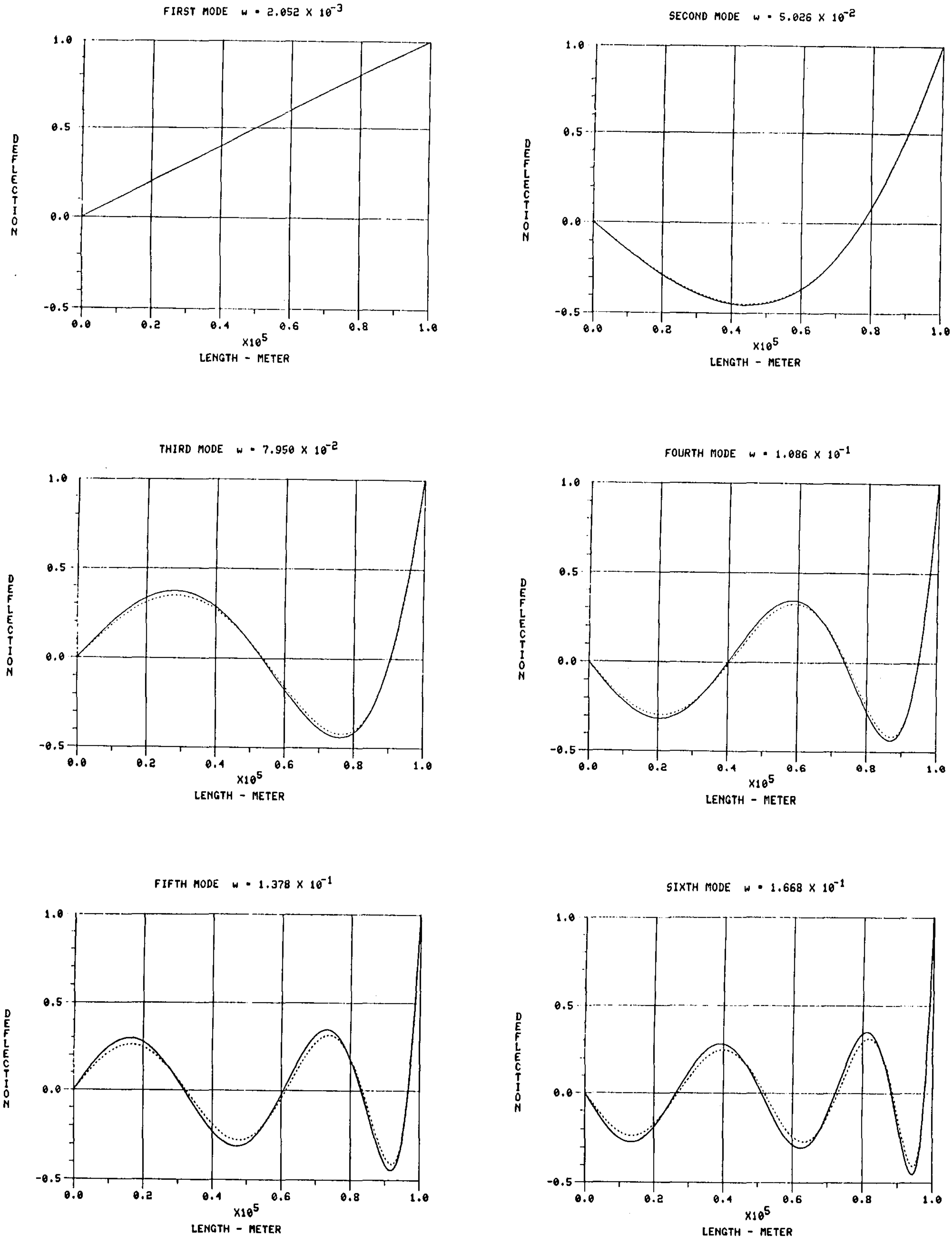

Figure 5. Mode functions for gravity gradient field (M = 0), length = 100 km.

The error is smaller than 0.1 percent for the 6th natural frequency. Comparison of the exact and the numerical mode functions in Figure 5 again reveals the increasing degradation of mode function accuracy with increasing mode number.

D. Uniform String in a Gravity-Gradient Force Field With End Mass

This case cannot be solved in closed form. Again, numerical solutions are presented for two end masses. In one case the end mass is equal to the string mass (M = m), in the other the end mass is twice the string mass (M = 2m). The tension in the string is:

$$T(x) = 3\,\omega_c^2 \left[\frac{\rho}{2} (\ell^2 - x^2) + M\ell \right] \quad . \tag{39}$$

The first six mode functions and the associated natural frequencies are shown in Figure 6 and Figure 7. Comparing these mode functions with the ones of the uniform gravity field shown in Figures 4 and 5, it is seen that they are very similar.

It can be concluded from this that differences in the tension profile along the string have a small effect on the basic character of the mode functions. The dynamic response of the string/end mass system is essentially determined by the mean tension level and its inertial properties. It is again noticed that with increasing end mass the mode functions rapidly approach those of a fixed string under constant tension. Another point of interest is the fact that the first mode frequency remains the same regardless of the end mass value. This is in concord with the statement made in the preceding paragraph that the first mode frequency is independent of the mass distribution of the string. The argument given there is still valid for a nonzero end mass if the integral of equation (38) is looked upon as a Stieltjes integral.

VI. CONCLUSION

The Stodola method provides an effective means for calculating mode functions and frequencies of a string with an attached end mass and nonuniform tension. The "sweeping" technique allows the computation of higher modes but with progressing numerical effort and decreasing accuracy. However, it is quite feasible to calculate at least 10 modes, especially if a more accurate integration procedure is chosen rather than the simple trapezoidal rule used in the present algorithm.

Differences in the tension profile along the string have only a small effect on the mode functions, whose basic character is essentially determined by the mean tension level and the inertial property of the string/end mass system. For increasing end mass the mode functions rapidly approach those of a fixed string under constant tension.

End conditions different from the ones investigated can be easily accommodated. The FORTRAN computer code used for the above examples is available upon request.

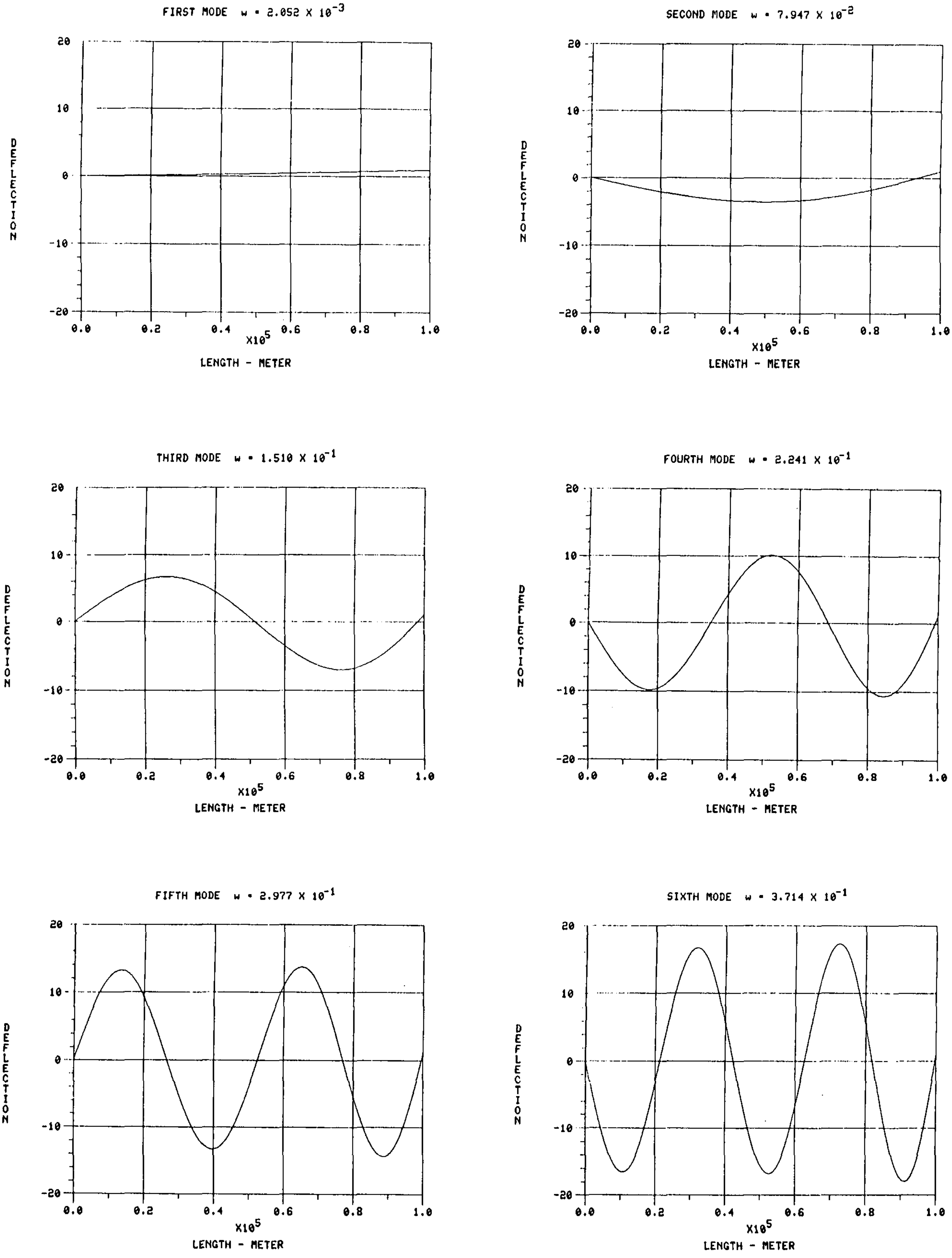

Figure 6. Mode functions for gravity gradient field (M = m), length = 100 km.

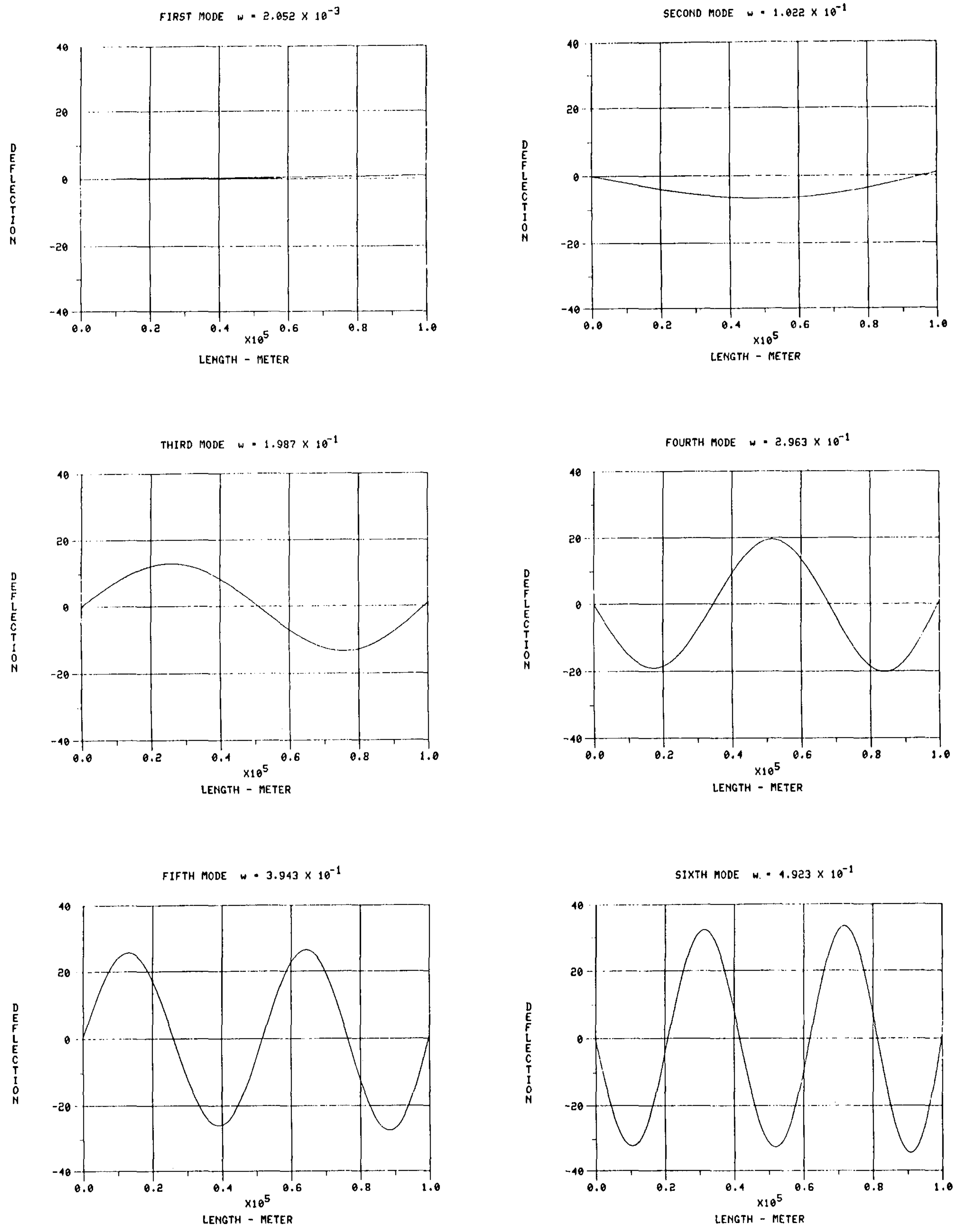

Figure 7. Mode functions for gravity gradient field (M = 2m), length = 100 km.

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16. ABSTRACT

Modal synthesis techniques for dynamic systems containing strings describe the lateral displacements of these strings by properly chosen shape functions. This report provides an iterative algorithm to calculate the natural modes of a nonuniform string and variable tension for some typical boundary conditions including one end mass. Numerical examples are given for a uniform string in a constant and a gravity gradient force field.

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