SDI
SYSTEM DYNAMICS INCORPORATED

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for

SPACE SHUTTLE PROPULSION PARAMETER ESTIMATION

USING

OPTIMAL ESTIMATION TECHNIQUES

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1219 N.W. 10 AVE. • POST OFFICE BOX 13687
GAINESVILLE, FLORIDA 32604 • (904)-376-3199

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1.0 INTRODUCTION

This third monthly progress report contains corrections and additions to previously submitted reports. The additions include the results of regression analyses on the tabular aerodynamic data provided. The objectives of the regression analysis were two-fold. The first was the establishment of a representative aerodynamic model for future coefficient estimation. The second was to reduce the storage requirements for the "nominal" model used to check out the estimation algorithms. The results of the regression analyses are presented in Appendix D.

The primary activities during this reporting period were the development of the computer routines for the filter portion of the estimation algorithm and the "bringing-up" of the NASA SRB predictive program on SDI's computer. For the filter program, approximately 54 routines have been developed. The routines have been highly subsegmented to facilitate overlaying program segments within the partitioned storage space on our computer. The attempt to bring up the NASA SRB predictive program on our computer has not been completely successful to date. With such a large program, overlaying techniques are required. Progress has been made and we are "hopefully" in the final stages of shifting subsegments around within the segments such that no segment is larger than the allocated partition.

2.0 FILTERING AND SMOOTHING ALGORITHM

The Space Shuttle Parameter Estimation Program utilizes optimal estimation techniques to provide estimates of the propulsion system parameters. The technique selected is the extended Kalman filter and the modified Bryson-Frazier smoother. By modeling the propulsion system parameters as time correlated random variables, improved estimates of these parameters are obtained and are properly time phased by removing the filter induced lag by using the combined filter/smoother. The smoother also provides improved estimates of the initial state estimates.

The system, in state-space notation, is modeled as the continuous dynamical system disturbed by additive Gaussian white noise

$$\dot{\underline{x}} = \underline{f}(\underline{x}(t), t) + G(t)\,\underline{w}(t) + \underline{u}(t), \quad \underline{x}(o) = \underline{x}_o \tag{1}$$

where

$\underline{x}$ = n-dimensional state vector

$\underline{x}_o$ = Gaussian initial condition vector with covariance P_o

$\underline{w}(t)$ = p-dimensional white, zero-mean white Gaussian noise with covariance

$$E[\underline{w}(t)\,\underline{w}^T(\tau)] = Q(t)\,\delta(t - \tau)$$

$\underline{u}(t)$ = n-dimensional control vector.

The elements of the vector $\underline{x}(t)$ represent vehicle position, velocities, attitudes, angular rates, aerodynamic and propulsion parameters, measurement biases, etc. Elements of $\underline{u}(t)$ include known control inputs such as SSME power level commands.

The system described by equation (1) is observed at discrete times, t_k, with not all states being directly measured. Some measurements are non-linear functions of the elements of the state vector $\underline{x}(t)$. In general the measurement process is described as

$$\underline{z}_k = \underline{h}_k(\underline{x}(t_k)) + \underline{v}_k \tag{2}$$

where

$\underline{z}_k$ = m-dimensional observation vector

$\underline{h}_k$ = functional representation of the measurements in terms of the states

$\underline{v}_k$ = m-dimensional, zero-mean, while Gaussian noise sequence with covariance

$$E[\underline{v}_i \; \underline{v}_j^T] = R_i \; \delta_{i,j}$$

Examples of the elements of the observation vector $\underline{z}_k$ include radar measurements of range, azimuth, and elevation from the radar site to the vehicle.

It is assumed that the system process noise vector $\underline{w}(t)$ and the measurement noise vector $\underline{v}_k$ are uncorrelated. Also, the system state initial condition vector $\underline{x}_o$ is not correlated with either of these two noise vectors. Therefore

$$E[\underline{w}(t) \; \underline{v}_k^T] = 0, \quad E[w(t) \; \underline{x}_o^T] = 0, \quad E[\underline{x}_o \; \underline{v}_k^T] = 0$$

where the superscript T denotes transpose. For later reference, the following matricies are defined

$$F(\underline{x}(t),\ t) = \frac{\partial \underline{f}(\underline{x}(t),\ t)}{\partial \underline{x}(t)} \tag{3}$$

and

$$H(\underline{x}_k(-)) = \frac{\partial \underline{h}(\underline{x}(t_k))}{\partial \underline{x}(t_k)} \ . \tag{4}$$

These matricies are linearizations of the dynamics and measurement models respectively, evaluated about either a nominal or reference value of the state, or about the state estimate.

2.1 Extended Kalman Filter Algorithm

The extended Kalman filter algorithm is in essence a conventional linear Kalman filter algorithm applied to a mathematical model resulting from the linearization of the system model equation (1), and measurement process, equation (2), about a current state estimate. The filter yields optimal estimates if the linearization is accurate, i.e., the state estimate closely approximates the true state. The derivation of the algorithm can be found in reference [1].

The algorithm proceeds as follows. After initialization of the state estimate and covariance, the state estimate and covariance are propagated forward in time until a measurement update is available, by

$$\dot{\hat{\underline{x}}} = \underline{f}(\hat{\underline{x}}(t),\ t) \qquad , \qquad t_{k-1} < t \leq t_k \tag{5}$$

and

$$\dot{P}(t) = F(\hat{\underline{x}}(t),\ t)\ P(t) + P(t)\ F(\hat{\underline{x}}(t),\ t)^T + G(t)\ Q(t)\ G(t)^T \tag{6}$$

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At the measurement time, the state estimate and covariance are updated by

$$\underline{\hat{x}}_k(+) = \underline{\hat{x}}_k(-) + K_k(\underline{z}_k - \underline{h}_k(\underline{\hat{x}}_k(-))) \tag{7}$$

and

$$P_k(+) = (I - K_k\, H_k(\underline{\hat{x}}_k(-)))\, P_k(-) \tag{8}$$

where the (-) and (+) represent the appropriate values just before and just after the update. The updated values are used to reinitialize the time propagation equations (3) and (4) for integrating up to the next measurement time. The Kalman gain matrix is computed as

$$K_k = P_k(-)\, H_k(\underline{\hat{x}}_j(-))^T\, (H_k(\underline{\hat{x}}_k(-))\, P_k(-)\, H_k(\underline{\hat{x}}_k(-))^T + R_k)^{-1} \tag{9}$$

This algorithm is repeated until the last time point, t_N, is processed. For later use in the smoother algorithm, various combinations of the state estimates ($\underline{\hat{x}}$), measurements ($\underline{z}$), linearized dynamics matrix (F) and measurement matrix (H), measurement noise covariance (R) and estimation error covariance matrix (P) must be stored for each time instant to be processed by the smoother algorithm.

2.2 Modified Bryson-Frazier Smoother Algorithm

The operation of the smoother algorithm is similar to the filter algorithm except in reverse time. The derivation of this smoother algorithm is found in reference [2]. This fixed interval smoothing algorithm provides optimal estimates given all the measurements in comparison to the filtering algorithm providing optimal estimates given the previous measurements processed. Therefore the smoother provides improved estimates in addition to removing the time lag induced by the filter algorithm.

The smoothing algorithm adjoint variables, $\underline{\lambda}$ and Λ are "initialized" at the final time processed by the filter, T,

$$\underline{\lambda}(T-) = -H_N^T(H_N P_N H_N^T + R_N)^{-1} (\underline{z}_N - \underline{H}_N(\hat{x}_N(-))) \delta_{t_{N,T}} \tag{10}$$

and

$$\Lambda(T-) = H_N(H_N P_N H_N^T + R_N)^{-1} H_N \delta_{t_{N,T}} \tag{11}$$

If T is not an observation time, $\underline{\lambda}$ and Λ are zero. The adjoint variables are propagated in reverse time to the next previous measurement time by

$$\dot{\underline{\lambda}} = - F(\underline{\hat{x}}(t), t) \underline{\lambda} \qquad , \qquad t_k \leq t < t_{k+1} \tag{12}$$

$$\dot{\Lambda} = - F(\underline{\hat{x}}(t), t)^T \Lambda - \Lambda F(\underline{\hat{x}}(t), t) \tag{13}$$

At the time of an available measurement, t_k, the adjoint variables are updated by

$$\underline{\lambda}(-) = \underline{\lambda}(+) - H_k^T(H_k P_k H_k^T + R_k)^{-1} ((\underline{z}_k - \underline{h}_k(\underline{x}_k(-)))$$

$$+ (H_k P_k H_k^T + R_k) K_k^T \underline{\lambda}(+)) \tag{14}$$

and

$$\Lambda(-) = (I - K_k H_k)^T \Lambda(+) (I - K_k H_k) + H_k^T(H_k P_k H_k^T + R_k)^{-1} H_k \tag{15}$$

The smoother state estimate and error covariance are obtained using the filter estimate and covariance and the adjoint variables by

$$\underline{x}^*(t) = \hat{\underline{x}}(t) - P(t)\,\underline{\lambda}(t) \tag{16}$$

and

$$P^*(t) = P(t) - P(t)\,\Lambda(t)\,P(t). \tag{17}$$

Due to the potential number of time points to be processed, smoother estimates may only be computed at the discrete measurement times. For this approach the propagation equations (10) and (11) are replaced by

$$\underline{\lambda}_k(+) = \Phi_k^T\,\underline{\lambda}(-)_{k+1} \tag{18}$$

and

$$\Lambda_k(+) = \Phi_k^T\,\Lambda_{k+1}(-)\,\Phi_k \tag{19}$$

where Φ_k^T is the state transition matrix formed with the linearized dynamics matrix F to propagate the adjoint variable from time t_{k+1} to time t_k. The algorithm continues in reverse time until the initial time is reached.

2.3 Iterations with the Filter/Smoother Algorithm

The performance of the filter/smoother algorithm is a direct result of the accuracy of the linearization. Repeated operations of the algorithms with adjustments in initial state estimates and covariance in each cycle can yield improved estimates. This technique is known as global iterated filtering as defined in reference [3]. Each cycle of operating the algorithms would yield increasing improvements in the state estimates.

This feature of the algorithm operation is of special interest to the propulsion parameter estimation problem using the NASA predictive models. Initial, or nominal, values of the parameters of interest can be used to obtain the necessary partial derivatives indicated earlier. From operating the algorithm improved estimates of those parameters are obtained. Using these improved estimates, more accurate partial derivatives are obtained for use in the algorithms. This process is continued until there is in essence no change in the partial derivatives or quality of the state estimates. If the linearization is accurate, the measurement residual should be a white noise process with known covariance.

3.0 FILTER/SMOOTHER ALGORITHM SYSTEM AND MEASUREMENT MODEL

The usefulness of the filter/smoother algorithm is to provide estimates of the system states from the observed motion and dynamics while the system is driven by known and unknown elements. These unknown elements are elements of the system state vector to be estimated. The evolution of motion resulting from these known and unknown elements is assumed to be suitably represented for this study by a six degree-of-freedom (6 DOF) rigid body equations of motion. These equations are presented and discussed in section 3.1.

To implement these equations into the filter/smoother algorithm presented in section 2.0, a linearization of the system state and measurement models is required. These linearized equations are presented in section 3.2.

3.1 Equations of Motion and Measurement Equations

3.1.1 Rigid Body Equations of Motion

The rate of change of vehicle velocity in body coordinates, $\underline{v}^{(B)}$, as a result of external forces acting on the vehicle is described by

$$\underline{\dot{v}}^{(B)} = \frac{\rho A v_m^2}{2m} \underline{c}_f + {}^{B}C^{I}\, \underline{g}^{(I)} \left(\underline{r}^{(I)}\right) - \underline{\omega} \times \underline{v}^{(B)} + \frac{\underline{f}_T^{(B)}}{m} + \frac{\underline{f}_P^{(B)}}{m} \tag{20}$$

where

ρ = atmospheric density

A = aerodynamic coefficient referenced area

v_m = magnitude of vehicle velocity relative to the surrounding air mass

m = vehicle mass

$\underline{c}_f$ = aerodynamic force coefficient vector

$\underline{g}^{(I)}\ (\underline{r}^{(I)})$ = gravity vector in inertial coordinates

$\underline{\omega}$ = angular rotation of the body relative to the inertial frame

$\underline{f}_T^{(B)}$ = resultant thrust force vector in body coordinates

$\underline{f}_P^{(B)}$ = resultant plume force vector in body coordinates

The rate of change of vehicle position in inertial coordinates, $\underline{r}^{(1)}$, is then obtained by

$$\dot{\underline{r}}^{(I)} = {}^{I}C^{B}\, v^{(B)} \tag{21}$$

where ${}^{I}C^{B}$ is the transformation matrix from body coordinates to inertial coordinates. The elements of the ${}^{I}C^{B}$ transformation matrix are obtained from the resulting Euler angles defined by

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

where φ, θ, and ψ are roll, pitch and yaw attitudes respectively. The roll, pitch and yaw rates of the body relative to inertial coordinates are p, q, and r respectively. Finally, the rate of change of the body rates relative to inertial is given by

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$$\underline{\dot{\omega}} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = [I]^{-1} \left[\frac{\rho A v_m^2}{2} \underline{\ell c}_m + \frac{\rho A v_m^2}{2} (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \underline{c}_f \right.$$

$$\left. - \underline{\omega} \times (I\underline{\omega}) + \underline{T}_T^{(B)} + T_P^{(B)} \right] \tag{23}$$

where

I = vehicle moments of inertia matrix

$\underline{\ell c}_m$ = aerodynamic coefficient referenced length and moment coefficient vector

$\underline{r}_{cg}^{(B)}$ = vehicle center-of-gravity vector in body coordinates

$\underline{r}_A^{(B)}$ = aerodynamic coefficient reference position in body coordinates

$\underline{T}_T^{(B)}$ = resultant thrust torque vector in body coordinates

$\underline{T}_P^{(B)}$ = resultant plane torque vector in body coordinates

The equations of motion represent the first twelve elements of the system state vector. These equations are summarized in Table 3.1.1-1.

The moment of inertia matrix I in general is given by

$$I = \begin{bmatrix} I_x & -I_{xy} & -I_{zx} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{zx} & -I_{yz} & I_z \end{bmatrix} \tag{24}$$

for the moment axis terms, i.e., I_y, and the product of inertia terms, i.e., I_{zx}.

TABLE 3.1.1-1

Equations of Motion

(first twelve system states)

$$\underline{\dot{r}}^{(I)} = {}^{I}C^{B}\,\underline{v}^{(B)}$$

inertial mean of 50

$$\underline{\dot{v}}^{(B)} = \frac{\rho v_m^2 A}{2m}\,\underline{c}_f + {}^{B}C^{I}\,\underline{g}^{(I)}(\underline{r}^{(I)}) - \underline{\omega} \times \underline{v}^{(B)} + \frac{\underline{f}_T^{(B)}}{m} + \frac{\underline{f}_P^{(B)}}{m}$$

$$\underline{\dot{\theta}} = \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\underline{\dot{\omega}}^{I\,B} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = [I]^{-1}\left[\frac{\rho v_m^2 A d}{2}\,\underline{c}_m + \frac{\rho v_m^2 A}{2}\,(\underline{r}_A - \underline{r}_{cg})^{(B)} \times \underline{c}_f - \underline{\omega} \times ([I]\,\underline{\omega}) + \underline{T}_T + \underline{T}_P\right]$$

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The aerodynamic force and moment coefficients and plume forces are defined as functions of angle-of-attack, α, and angle-of-sideslip, β, as shown in Figure 3.1.1-1. The body referenced relative velocity vector, removing the wind velocity, $\underline{v}_w$, from the vehicle velocity, is given by

$$\underline{v}_r = \underline{v}^{(B)} - {}^{B}C^{I}\ \underline{v}_w = \underline{v}^{(B)} - {}^{B}C^{LL}\ \underline{v}_w^{(LL)} \tag{25}$$

where $\underline{v}_w^{(LL)}$ is the local-level referenced wind velocity vector. The following equations define α and β in terms of the components of $\underline{v}_r$

$$\alpha = \tan^{-1}\left(\frac{v_{r_3}}{v_{r_1}}\right) \tag{26}$$

$$\beta = \sin^{-1}\left(\frac{v_{r_2}}{v_m}\right) \tag{27}$$

where

$$v_m = (v_{r_1}^{\,2} + v_{r_2}^{\,2} + v_{r_3}^{\,2})^{\frac{1}{2}} \tag{28}$$

The resultant thrust force $\underline{f}_T^{(B)}$ is expanded as

$$\underline{f}_T^{(B)} = \sum_{i=1}^{n} {}^{B}C_i^{\mathbb{C}} \begin{bmatrix} f_{T_i}^{\mathbb{C}} \\ 0 \\ 0 \end{bmatrix} \triangleq \sum_{i=1}^{n} {}^{B}C_i^{\mathbb{C}}\ \underline{f}_{T_i}^{(\mathbb{C})} \tag{29}$$

where the transformation matrix ${}^{B}C_i^{\mathbb{C}}$ transforms the magnitude of thrust for each thrusting device, $f_{T_i}^{\mathbb{C}}$, from its center-line to the body coordinates. The general eqaution for f_{T_i} is

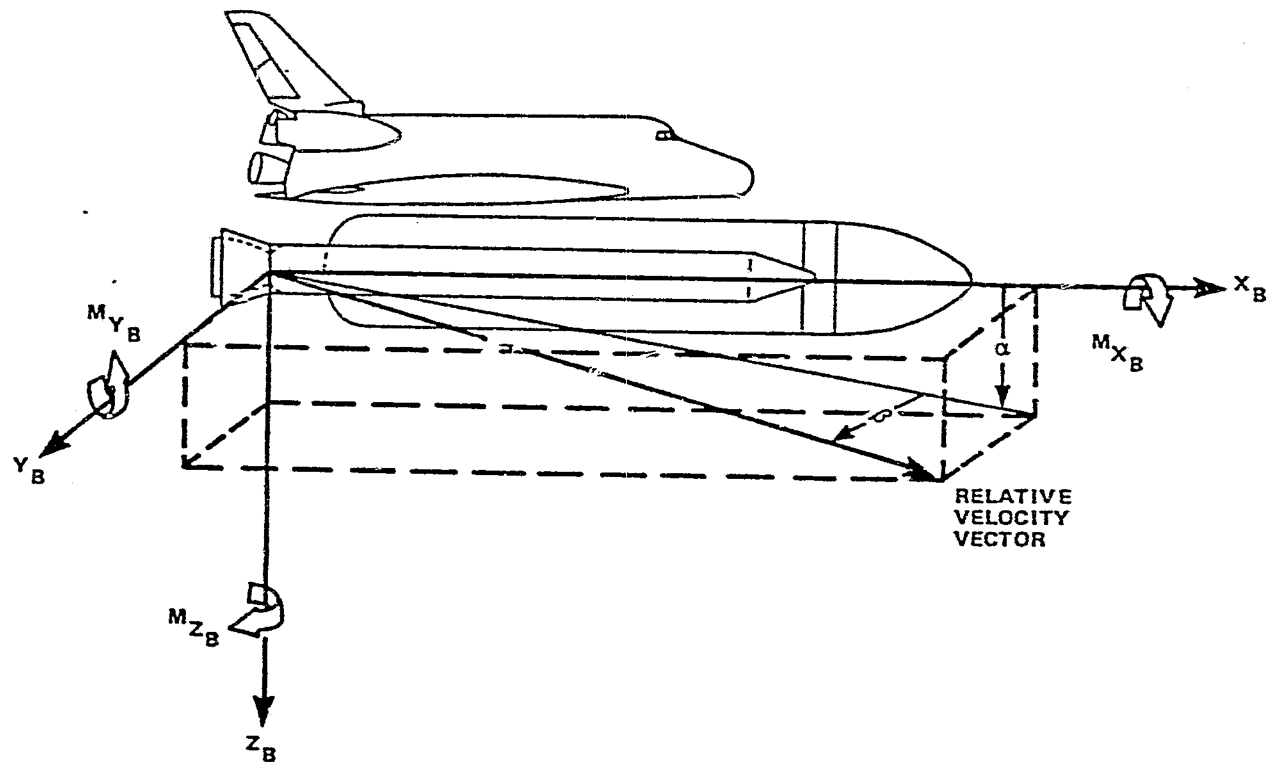

FIGURE 3.1.1-1. Body Referenced Wind Axis System

$$f_{T_i} = f_{T_{i\,vac}} - P_{s_i} A_e$$

where

$f_{T_{i\,vac}}$ = vacuum thrust

P_{s_i} = atmospheric pressure at motor exit

A_e = motor exit cone area

The matrix ${}^{B}C^{\text{℄}}_{i}$ is different for the SSME's and SRB's and is given by

$$ {}^{B}C^{\text{℄}_i} = \begin{cases} {}^{B}C^{MP}\ {}^{MP}C^{G}\ {}^{G}C^{\text{℄}} & \text{SSME} \\ \\ {}^{B}C^{\text{℄}} & \text{SRB} \end{cases} \qquad . \qquad (30)$$

where

${}^{B}C^{MP}$ = transformation from the engine mount plane to the body coordinates

${}^{MP}C^{G}$ = transformation from the gimbal reference plane to mount plane (structural deformation)

${}^{G}C^{\text{℄}}$ = transformation from enterline to the gimbal reference plane

${}^{B}C^{\text{℄}}$ = transformation from SRB nozzle centerline to the body coordinates (gimbal angles).

The resultant thrust torque is the summation of the torque contribution from each thrusting device and is given by

$$\underline{T}_T^{(B)} = \sum_{i=1}^{n} (\underline{r}_{T_i}^{(B)} - \underline{r}_{cg}^{(B)}) \times {}^{B}C_i^{\text{℄}} \begin{bmatrix} f_{T_i}^{\text{℄}} \\ 0 \\ 0 \end{bmatrix} \tag{31}$$

where

$\underline{r}_{T_i}^{(B)}$ = body coordinates of the thrust reference point for the i^{th} thrusting device.

3.1.2 Measurement Equations

The measurements assumed available for the filter/smoother algorithm include inertial platform acceleration and attitudes, ground based radar tracking, SRB's head pressure, SSME's chamber pressures, liquid H_2 flow rates, pressurant flow rates. The ET volumetric levels are available; however, due to their limited number (4), they may only be used for alternate checks of the filter/smoother algorithm performance.

The propulsion related measurements will be treated in a separate section. In the following, the inertial platform acceleration measurements, attitude measurements and ground based tracking measurements models will be described for later linearization.

3.1.2.1 Platform Acceleration Measurements

Accelerometers mounted orthogonally on an inertially stabilized platform, not located at the vehicle center of gravity, sense externally applied special forces and accelerations due to body rotation. The accelerometer measurement is modeled by

$$\underline{a}_m^{(S)} = {}^S C^P \; {}^P C^{P'} \; {}^{P'} C^B \left[\frac{\rho A v_m^2}{2m} \underline{c}_f + \frac{\underline{f}_T^{(B)}}{m} + \frac{\underline{f}_P^{(B)}}{m} + \underline{\omega} \times \underline{\omega} \times (\underline{r}_S^{(B)} - \underline{r}_{cg}^{(B)}) + \dot{\underline{\omega}} \times (\underline{r}_S^{(B)} - \underline{r}_{cg}^{(B)}) \right] + \underline{b}_a^{(S)} + \underline{v}_a^{(S)} \qquad (32)$$

where

${}^S C^P$ = transformation from platform coordinates to sensing coordinates

${}^P C^{P'}$ = transformation from misaligned platform coordinates to platform coordinates

${}^{P'} C^B$ = transformation from body to misaligned platform coordinates

$\underline{r}_S^{(B)}$ = body coordinates of the platform center

$\underline{b}_a^{(S)}$ = accelerometer bias vector

$\underline{v}_a^{(S)}$ = accelerometer measurement noise vector

3.1.2.2 Platform Attitude Measurements

The inertially stabilized platform for the STS is a four axis IMU with a redundant roll axis [4]. Vehicle body attitudes are generated via quaternions [5]. It is assumed that an equivalent representation can be made to obtain vehicle attitude by a three rotation sequence of roll, pitch, yaw to transform from inertial to body coordiantes. This approach has been used in reference [6].

The attitude angle measurement model is given by

$$\underline{\theta}_m^{(S)} = \underline{\theta} + \underline{b}_\theta^{(S)} + \underline{v}_\theta^{(S)} \tag{33}$$

where

$\underline{b}_\theta$ = platform misalignment bias vector (used to formulate ${}^{P}C^{P'}$)

$\underline{v}_\theta^{(S)}$ = attitude measurement noise vector.

The transformation matrix used to transform from body to inertial coordinates in terms of the elements of the $\underline{\theta}$ vector is given by

$$ {}^{I}C^{B} = \begin{bmatrix} \cos\theta\cos\psi & \sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi & \cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi \\ \cos\theta\sin\psi & \sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi & \cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi \\ -\sin\theta & \sin\varphi\cos\theta & \cos\varphi\cos\theta \end{bmatrix} \tag{34}$$

3.1.2.3 Ground Based Tracking Measurements

Ground based radar tracking devices can provide measurements of range, azimuth and elevation from the radar sight to the vehicle. Azimuth and elevation are established relative to the sight's local level. If the tracking device is a passive optical tracker (not laser) then only azimuth and elevation measurements are available requiring more than one to establish position information.

Defining x, y, and z as the local east, north and up position of the vehicle relative to the ground based tracking device, the radar measurement equations are given by

$$\rho = (x^2 + y^2 + z^2)^{\frac{1}{2}} + b_\rho + v_\rho \tag{35}$$

$$A = \tan^{-1}\left(\frac{x}{y}\right) + b_A + v_A \tag{36}$$

$$E = \tan^{-1}\left(\frac{z}{\sqrt{x^2 + y^2}}\right) + b_E + \Delta E + v_E \tag{37}$$

where

b_ρ, b_A, b_E = range, azimuth, elevation biases

ΔE = atmospheric refraction correction

v_ρ, v_A, v_E = range, azimuth, elevation measurement noise.

The position vector of the vehicle relative to the tracking device is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \triangleq \Delta \underline{r}_V^{(LL)} = {}^{LL}C^{ECF}\left[{}^{ECF}C^{ECI}\,\underline{r}^{(I)} - \underline{r}_{RDR}^{(ECF)}\right] \tag{38}$$

where

$^{LL}C^{ECF}$ = transformation from earth center fixed to local level

$^{ECF}C^{ECI}$ = earth centered inertial to earth centered fixed

$\underline{r}_{RDR}^{(ECF)}$ = position vector of tracking device in ECF coordinates.

The transformation matrix $^{LL}C^{ECF}$ is given by

$$^{LL}C^{ECF} = \begin{bmatrix} -\sin\lambda & -\sin L\cos\lambda & \cos L\cos\lambda \\ \cos\lambda & -\sin L\sin\lambda & \cos L\sin\lambda \\ 0 & \cos L & \sin L \end{bmatrix} \tag{39}$$

where L and λ are the geodetic latitude and east longitude of the device. The transformation matrix $^{ECF}C^{ECI}$ is given by

$$^{ECF}C^{ECI} = \begin{bmatrix} \cos[\omega_E(t - t_{RNP})] & \sin[\omega_E(t - t_{RNP})] & 0 \\ -\sin[\omega_E(t - t_{RNP})] & \cos[\omega_E(t - t_{RNP})] & 0 \\ 0 & 0 & 1 \end{bmatrix} [RNP] \tag{40}$$

where

ω_E = earth rotation rate

t_{RNP} = time tag for RNP matrix

The position vector, $\underline{r}_{RDR}^{(ECF)}$, of the tracking device is given by

$$\underline{r}_{RDR}^{(ECF)} = \begin{bmatrix} \left(\dfrac{R_E}{\sqrt{\cos^2 L + (1-e)^2 \sin^2 L}} + h\right) \cos L \cos\lambda \\ \left(\dfrac{R_E}{\sqrt{\cos^2 L + (1-e)^2 \sin^2 L}} + h\right) \cos L \sin\lambda \\ \left(\dfrac{R_E(1-e)^2}{\sqrt{\cos^2 L + (1-e)^2 \sin^2 L}} + h\right) \sin L \end{bmatrix} \tag{41}$$

where

R_E = equatorial radius of Fisher ellipsoid

e = flattening of Fisher ellipsoid

h = altitude of the device above Fisher ellipsoid

3.2 Linearized System State and Measurement Equations

The vehicle equations of motion are nonlinear functions of their motion variables and are implicit functions of other elements of the system states. The measurement equations involve similar function relationships. The linearizations for the filter/smoother algorithm require partial derivatives with respect to the motion variables, i.e., $\underline{v}^{(B)}$ and θ, and with respect to other elements of the state vector, yielding explicit functional relationships for the elements of interest.

For system state equations the partial derivatives will be presented in section 3.2.1 for the state elements in order of occurrence for the first twelve states. Other partial derivatives for candidate state elements will follow in section 3.3.1. The measurement equation partial derivatives for the first twelve states will be presented in section 3.2.2. Partial derivatives of the measurement equations for other candidate states will be presented in section 3.3.2.

The resulting partial derivatives are imbedded into the linearized system state matrix, $F(\underline{x}(t), t)$, as shown in Figure 3.2-1 . A corresponding linearized measurement matrix, $H(\underline{x}_K)$, is similarly formed with the measurement equations' partial derivatives.

3.2.1 System State Partial Derivatives

Partial derivatives of each of the equations listed in Figure 3.2-1 are developed in their order of occurrence with respect to the order of

FIGURE 3.2-1. Linearization for Filter/Smoother Model

(Dynamics)

	$\underline{r}^{(I)}$	$\underline{v}^{(B)}$	$\underline{\theta}$	$\underline{\omega}$
$\dot{\underline{r}}^{(I)}$	0	$\frac{\partial \dot{\underline{r}}^{(I)}}{\partial \underline{v}^{(B)}}$	$\frac{\partial \dot{\underline{r}}^{(I)}}{\partial \underline{\theta}}$	0
$\dot{\underline{v}}^{(B)}$	$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{r}^{(I)}}$	$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{v}^{(B)}}$	$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{\theta}}$	0
$\dot{\underline{\theta}}$	0	0	$\frac{\partial \dot{\underline{\theta}}}{\partial \underline{\theta}}$	$\frac{\partial \dot{\underline{\theta}}}{\partial \underline{\omega}}$
$\dot{\underline{\omega}}$	$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{r}^{(I)}}$	$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{v}^{(B)}}$	$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{\theta}}$	$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{\omega}}$

the corresponding states. Partial derivatives of thrust terms are presented as though for a single device.

Inertial Position Rate Equation

The first nonzero partial derivative of the $\dot{\underline{r}}^{(I)}$ equation is with respect to $\underline{v}^{(B)}$:

$$\frac{\partial}{\partial \underline{v}^{(B)}} (\dot{\underline{r}}^{(I)}) = {}^{I}C^{B}. \tag{42}$$

The second nonzero partial derivative is with respect to $\underline{\theta}$. This partial derivative results in a third order tensor and occurs frequently in later developments. The generalized form is presented in Appendix A.

Body Velocity Rate Equation

The partial derivative of $\dot{\underline{v}}^{(B)}$ with respect to $\underline{r}^{(I)}$ is given for altitude terms approximately as

$$\frac{\partial}{\partial \underline{r}^{(I)}} = \frac{\partial}{\partial h} \frac{\underline{r}^{(I)^T}}{|\underline{r}^{(I)}|} \tag{43}$$

where

$$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial h} = \frac{Av_m^2}{2m} \frac{\partial \rho}{\partial h} \underline{c}_f + \frac{\rho A v_m}{m} \underline{c}_f \frac{\partial v_m}{\partial h} + \frac{\rho A v_m^2}{2m} \frac{\partial \underline{c}_f}{\partial \alpha} \frac{\partial \alpha}{\partial h} + \frac{\rho A v_m^2}{2m} \frac{\partial \underline{c}_f}{\partial \beta} \frac{\partial \beta}{\partial h}$$

$$+ \frac{1}{m}\left({}^{B}C^{\mathfrak{C}} \frac{\partial \underline{f}_T^{(\mathfrak{C})}}{\partial p_s} \frac{\partial p_s}{\partial h} + \frac{\partial \underline{f}_p^{(B)}}{\partial h} + \frac{\partial \underline{f}_p^{(B)}}{\partial \alpha} \frac{\partial \alpha}{\partial h} + \frac{\partial \underline{f}_p^{(B)}}{\partial \beta} \frac{\partial \beta}{\partial h}\right) \tag{44}$$

The partial derivatives of $\frac{\partial v_m}{\partial \underline{v}^{(B)}}$, $\frac{\partial \alpha}{\partial \underline{v}^{(B)}}$, $\frac{\partial \beta}{\partial \underline{v}^{(B)}}$, $\frac{\partial \alpha}{\partial h}$, $\frac{\partial \beta}{\partial h}$ and $\frac{\partial v_m}{\partial h}$ occur frequently and are given in Appendix B.

The gravity vector ${}^{B}C^{I}\, \underline{g}^{(I)}(\underline{r}^{(I)})$ partial derivative with respect to $\underline{r}^{(I)}$ is

$$
{}^{B}C^{I}\,\frac{\partial \underline{g}^{(I)}(\underline{r}^{(I)})}{\partial \underline{r}^{(I)}} = {}^{B}C^{I}\,\frac{\mu}{|\underline{r}^{(I)}|^3}
\begin{bmatrix}
3\frac{r_1^2}{|\underline{r}|^2} - 1 & 3\frac{r_1 r_2}{|\underline{r}|^2} & 3\frac{r_1 r_2}{|\underline{r}|^2} \\
3\frac{r_1 r_2}{|\underline{r}|^2} & 3\frac{r_2^2}{|\underline{r}|^2} - 1 & 3\frac{r_2 r_3}{|\underline{r}|^2} \\
3\frac{r_1 r_3}{|\underline{r}|^2} & 3\frac{r_2 r_3}{|\underline{r}|^2} & 3\frac{r_3^2}{|\underline{r}|^2} - 1
\end{bmatrix}
\tag{45}
$$

where

μ = gravitational constant.

The partial derivative, $\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{r}^{(I)}}$, is the sum of the matricies in equations 43 and 45.

The partial derivative of $\dot{\underline{v}}^{(B)}$ with respect to $\underline{v}^{(B)}$ is given by

$$
\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{v}^{(B)}} = \frac{\rho A v_m}{m}\,\underline{c}_f\,\frac{\partial v_m}{\partial \underline{v}^{(B)}} + \frac{\rho A v_m^2}{2m}\,\frac{\partial \underline{c}_f}{\partial \alpha}\,\frac{\partial \alpha}{\partial \underline{v}^{(B)}} + \frac{\rho A v_m^2}{2m}\,\frac{\partial \underline{c}_f}{\partial \beta}\,\frac{\partial \beta}{\partial \underline{v}^{(B)}}
$$

$$
+ \frac{1}{m}\left[\frac{\partial \underline{f}_p}{\partial \alpha}\,\frac{\partial \alpha}{\partial \underline{v}^{(B)}} + \frac{\partial \underline{f}_p}{\partial \beta}\,\frac{\partial \beta}{\partial \underline{v}^{(B)}}\right] - \{\underline{\omega}\}
\tag{46}
$$

where

$\{\underline{\omega}\}$ = skew symmetric matrix made from the elements of the vector $\underline{\omega}$ and equivalent to the cross product operator $\underline{\omega} \times (\ )$.

The partial derivative of $\dot{\underline{v}}^{(B)}$ with respect to $\underline{\theta}$ is

$$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{\theta}} = \frac{\rho A v_m}{m} \underline{c}_f \frac{\partial v_m}{\partial \underline{v}_r} \frac{\partial \underline{v}_r}{\partial \underline{\theta}} + \frac{\rho A v_m^2}{2m} \frac{\partial \underline{c}_f}{\partial \alpha} \frac{\partial \alpha}{\partial \underline{v}_r} \frac{\partial \underline{v}_r}{\partial \underline{\theta}}$$

$$+ \frac{\rho A v_m^2}{2m} \frac{\partial \underline{c}_f}{\partial \beta} \frac{\partial \beta}{\partial \underline{v}_r} \frac{\partial \underline{v}_r}{\partial \underline{\theta}} + \frac{\partial}{\partial \underline{\theta}} [{}^{B}C^{I}\, \underline{g}^{(I)} (\underline{r}^{(I)})]$$

$$+ \frac{1}{m} [\frac{\partial \underline{f}_p^{(B)}}{\partial \alpha} \frac{\partial \alpha}{\partial \underline{v}_r} \frac{\partial \underline{v}_r}{\partial \underline{\theta}} + \frac{\partial \underline{f}_p^{(B)}}{\partial \beta} \frac{\partial \beta}{\partial \underline{v}_r} \frac{\partial \underline{v}_r}{\partial \underline{\theta}}]. \tag{47}$$

The partial derivatives of $\frac{\partial}{\partial \underline{v}_r}$ are given in Appendix B and the partial derivative $\frac{\partial \underline{v}_r}{\partial \underline{\theta}}$ is given in Appendix A. The last partial derivative is given in Appendix C.

The partial derivative of $\dot{\underline{v}}^{(B)}$ with respect to $\underline{\omega}$ is

$$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{\omega}} = \frac{\rho A v_m^2}{2m} \frac{\partial \underline{c}_f}{\partial \underline{\omega}} + \{\underline{v}^{(B)}\} \tag{48}$$

Euler Angle Rate Equation

The Euler angle rate equation is a function of both the Euler angles and the inertial rates. The linearization will yield the two associated matricies.

First with respect to the vector $\underline{\theta}$, the following matrix results

$$\frac{\partial \dot{\underline{\theta}}}{\partial \underline{\theta}} = \begin{bmatrix} q\cos\varphi\tan\theta - r\sin\varphi\tan\theta & q\sin\varphi\sec^2\theta + r\cos\varphi\sec^2\theta & 0 \\ -q\sin\varphi - r\cos\varphi & 0 & 0 \\ q\cos\varphi\sec\theta - r\sin\varphi\cos\theta & q\sin\varphi\sec\theta\tan\theta + r\cos\varphi\sec\theta\tan\theta & 0 \end{bmatrix} \tag{49}$$

The partial derivative of $\dot{\underline{\theta}}$ with respect to $\underline{\omega}$ is

$$\frac{\partial \dot{\underline{\theta}}}{\partial \underline{\omega}} = \begin{bmatrix} 1 & \sin\varphi\tan\theta & \cos\varphi\tan\theta \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi\sec\theta & \cos\varphi\sec\theta \end{bmatrix} \tag{50}$$

Inertial Angular Acceleration Equation

The first partial derivative of this equation is with respect to the vector $\underline{r}^{(I)}$. Using the approximation indicated in equation 43, this partial derivative is

$$\frac{\partial \underline{\dot{\omega}}}{\partial \underline{r}^{(I)}} = [I]^{-1} \Big\{\Big(\frac{Adv_m^{\,2}}{2}\frac{\partial \rho}{\partial h}\underline{c}_m + (\rho v_m Ad\underline{c}_m + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \rho v_m A\underline{c}_f)\frac{\partial v_m}{\partial h}$$

$$+ \Big(\frac{\rho v_m^{\,2} Ad}{2}\frac{\partial \underline{c}_m}{\partial \alpha} + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \frac{\rho v_m^{\,2} A}{2}\frac{\partial \underline{c}_f}{\partial \alpha}\Big)\frac{\partial \alpha}{\partial h} \tag{51}$$

$$+ \Big(\frac{\rho v_m^{\,2} Ad}{2}\frac{\partial \underline{c}_m}{\partial \beta} + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \frac{\rho v_m^{\,2} A}{2}\frac{\partial \underline{c}_f}{\partial \beta}\Big)\frac{\partial \beta}{\partial h}$$

$$+ \Big((\underline{r}_T^{(B)} - \underline{r}_{cg}^{(B)}) \times {}^{B}C^{\mathbb{C}}\frac{\partial \underline{f}_T^{(\mathbb{C})}}{\partial p_s}\frac{\partial p_s}{\partial h} + \frac{\partial \underline{T}_p}{\partial h} + \frac{\partial \underline{T}_p}{\partial \alpha}\frac{\partial \alpha}{\partial h} + \frac{\partial \underline{T}_p}{\partial \beta}\frac{\partial \beta}{\partial h}\Big)\Big\}\frac{\underline{r}^{(I)^T}}{|\underline{r}^{(I)}|}$$

Next, with respect to the vector $\underline{v}^{(B)}$, the partial derivative is

$$\frac{\partial \underline{\dot{\omega}}}{\partial \underline{v}^{(B)}} = [I]^{-1} \Big\{(\rho v_m Ad\,\underline{c}_m + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \rho v_m A\,\underline{c}_f)\frac{\partial v_m}{\partial \underline{v}^{(B)}}$$

$$+ \Big(\frac{\rho v_m^{\,2} Ad}{2}\frac{\partial \underline{c}_m}{\partial \alpha} + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \frac{\rho v_m^{\,2} A}{2}\frac{\partial \underline{c}_f}{\partial \alpha}\Big)\frac{\partial \alpha}{\partial \underline{v}^{(B)}} \tag{52}$$

$$+ \Big(\frac{\rho v_m^{\,2} Ad}{2}\frac{\partial \underline{c}_m}{\partial \beta} + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \frac{\rho v_m^{\,2} A}{2}\frac{\partial \underline{c}_f}{\partial \beta}\Big)\frac{\partial \beta}{\partial \underline{v}^{(B)}}$$

$$+ \frac{\partial \underline{T}_p}{\partial \alpha}\frac{\partial \alpha}{\partial \underline{v}^{(B)}} + \frac{\partial \underline{T}_p}{\partial \beta}\frac{\partial \beta}{\partial \underline{v}^{(B)}}\Big\} .$$

The partial derivative with respect to the vector $\underline{\theta}$ is

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{\theta}} = [I]^{-1} \{(\rho v_m A d\, \underline{c}_m + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \rho v_m A\, \underline{c}_f) \frac{\partial v_m}{\partial \underline{\theta}}$$

$$+ (\frac{\rho v_m^2 A d}{2} \frac{\partial \underline{c}_m}{\partial \alpha} + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \frac{\rho v_m^2 A}{2} \frac{\partial \underline{c}_f}{\partial \alpha}) \frac{\partial \alpha}{\partial \underline{\theta}} \tag{53}$$

$$+ (\frac{\rho v_m^2 A d}{2} \frac{\partial \underline{c}_m}{\partial \beta} + (\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}) \times \frac{\rho v_m^2 A}{2} \frac{\partial \underline{c}_f}{\partial \beta}) \frac{\partial \beta}{\partial \underline{\theta}}$$

$$+ \frac{\partial \underline{T}_p}{\partial \alpha} \frac{\partial \alpha}{\partial \underline{\theta}} + \frac{\partial \underline{T}_p}{\partial \beta} \frac{\partial \beta}{\partial \underline{\theta}} \}.$$

The final partial derivative for the first twelve states is with respect to the vector $\underline{\omega}$. This operation yields

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{\omega}} = [I]^{-1} \{\frac{\rho A d v_m^2}{2} \frac{\partial \underline{c}_m}{\partial \underline{\omega}} + \{\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}\} \frac{\rho A v_m^2}{2} \frac{\partial \underline{c}_f}{\partial \underline{\omega}}$$

$$+ \{I\underline{\omega}\} - \{\underline{\omega}\}\, I\} \tag{54}$$

3.2.2 Measurement Partial Derivatives

The measurements assumed to be available, as discussed earlier, include ground based radar tracking, inertially stabilized platform attitudes relative to the vehicle body, and stabilized platform mounted 3 axis orthogonal accelerations. As with the state dynamics matrix, the measurement equations are linearized about the best state estimates.

Radar Track Measurement Equation

Referring to the radar track measurement equations, the required partial derivatives are

$$\frac{\partial \rho}{\partial \underline{r}^{(I)}} = \frac{\partial \rho}{\partial \Delta \underline{r}_v^{(LL)}} \frac{\partial \Delta \underline{r}_v^{(LL)}}{\partial \underline{r}^{(I)}} \tag{55}$$

$$\frac{\partial A}{\partial \underline{r}^{(I)}} = \frac{\partial A}{\partial \Delta \underline{r}_v^{(LL)}} \frac{\partial \Delta \underline{r}_v^{(LL)}}{\partial \underline{r}^{(I)}} \tag{56}$$

$$\frac{\partial E}{\partial \underline{r}^{(I)}} = \frac{\partial E}{\partial \Delta \underline{r}_v^{(LL)}} \frac{\partial \Delta \underline{r}_v^{(LL)}}{\partial \underline{r}^{(I)}} . \tag{57}$$

The last partial derivative in each of these equations, $\frac{\partial \Delta \underline{r}_v^{(LL)}}{\partial \underline{r}^{(I)}}$, is

$$\frac{\partial \Delta \underline{r}_v^{(LL)}}{\partial \underline{r}^{(I)}} = LL_C{}^{ECF} \, ECF_C{}^{ECI} . \tag{58}$$

The rest of the required partial derivatives are

$$\frac{\partial \rho}{\partial \Delta \underline{r}_v^{(LL)}} = \Delta \underline{r}_v^{(LL)^T} / |\Delta \underline{r}_v| \tag{59}$$

$$\frac{\partial A}{\partial \Delta \underline{r}_v^{(LL)}} = [\frac{y}{x^2 + y^2}, \quad \frac{-x}{x^2 + y^2}, \quad 0] \tag{60}$$

$$\frac{\partial E}{\partial \Delta \underline{r}_v^{(LL)}} = [\frac{-xz}{\rho^2 \sqrt{x^2 + y^2}}, \quad \frac{-yz}{\rho^2 \sqrt{x^2 + y^2}}, \quad \frac{\sqrt{x^2 + y^2}}{\rho^2}] \tag{61}$$

Inertially Stabilized Platform Attitude Equation

The inertial platform is assumed to provide attitude angle measurements of the true attitude plus an attitude bias plus measurment noise. The partial derivative of the measured attitudes with respect to the vector $\underline{\theta}$ yields an identity matrix.

Accelerometer Measurement Equation

The accelerometer senses specific body forces excluding gravity along the sensing axes. With reference to the accelerometer equation, the partial derivative with respect to $\underline{r}^{(I)}$ is

$$\frac{\partial \underline{a}_m^{(S)}}{\partial \underline{r}^{(I)}} = {}^{S}C^{B}\Big[\frac{A v_m^{2}}{2m}\frac{\partial \rho}{\partial h}\underline{c}_f + \frac{\rho A v_m}{m}\underline{c}_f\frac{\partial v_m}{\partial h}$$

$$+ \frac{\rho A v_m^{2}}{2m}\frac{\partial \underline{c}_f}{\partial \alpha}\frac{\partial \alpha}{\partial h} + \frac{\rho A v_m^{2}}{2m}\frac{\partial \underline{c}_f}{\partial \beta}\frac{\partial \beta}{\partial h} \tag{62}$$

$$+ \frac{1}{m}\Big({}^{B}C^{\text{℄}}\frac{\partial \underline{f}_T^{(\text{℄})}}{\partial p_s}\frac{\partial p_s}{\partial h} + \frac{\partial \underline{f}_P^{(B)}}{\partial \alpha}\frac{\partial \alpha}{\partial h} + \frac{\partial \underline{f}_P^{(B)}}{\partial \beta}\frac{\partial \beta}{\partial h}\Big)\Big]\frac{\underline{r}^{(I)T}}{|\underline{r}^{(I)}|}$$

The partial derivative with respect to $\underline{v}^{(B)}$ yields

$$\frac{\partial \underline{a}_m^{(S)}}{\partial \underline{v}^{(B)}} = {}^{S}C^{B}\Big[\frac{\rho A v_m}{m}\underline{c}_f\frac{\partial v_m}{\partial \underline{v}^{(B)}} + \frac{\rho A v_m^{2}}{2m}\frac{\partial \underline{c}_f}{\partial \alpha}\frac{\partial \alpha}{\partial \underline{v}^{(B)}} + \frac{\rho A v_m^{2}}{2m}\frac{\partial \underline{c}_f}{\partial \beta}\frac{\partial \beta}{\partial \underline{v}^{(B)}}$$

$$+ \frac{1}{m}\Big[\frac{\partial \underline{f}_P}{\partial \alpha}\frac{\partial \alpha}{\partial \underline{v}^{(B)}} + \frac{\partial \underline{f}_P}{\partial \beta}\frac{\partial \beta}{\partial \underline{v}^{(B)}}\Big] \tag{63}$$

For the partial derivative of the accelerometer with respect to the vector $\underline{\theta}$, the measurement equation is temporarily rewritten as

$$\underline{a}_m^{(S)} = {}^{S}C^{P'}\,{}^{P'}C^{B}\,\underline{s}^{(B)} + \underline{b}_a^{(S)} + \underline{v}_a^{(S)} \tag{64}$$

where the vector $\underline{s}^{(B)}$ represents the sum of the aerodynamic, thrust, plume and rotational coupling terms. The matrix ${}^{P'}C^{B}$ is the same matrix as ${}^{I}C^{B}$. The required partial derivative results from

$$\frac{\partial \underline{a}_m^{(S)}}{\partial \underline{\theta}} = {}^S C^{P'} \frac{\partial}{\partial \underline{\theta}} {}^I C^B \underline{s}^{(B)}. \tag{65}$$

The partial derivative on the right hand side is developed in Appendix A with the vector $\underline{s}^{(B)}$ representing the sum of the terms indicated above.

The final partial derivative for the accelerometer measurement is with respect to the body rotation vector $\underline{\omega}$. Defining

$$\Delta \underline{r}_s \triangleq \begin{bmatrix} \Delta r_{1_s} \\ \Delta r_{2_s} \\ \Delta r_{3_s} \end{bmatrix} = (\underline{r}_s^{(B)} - \underline{r}_{cg}^{(B)}) \tag{66}$$

and denoting ω_i as the i^{th} element of the vector $\underline{\omega}$, the resulting matrix is

$$\frac{\partial \underline{a}_m}{\partial \underline{\omega}} = {}^S C^B \begin{bmatrix} \omega_2 \Delta r_2 + \omega_3 \Delta r_3 & \omega_1 \Delta r_2 - 2\omega_2 \Delta r_1 & \omega_1 \Delta r_3 - 2\omega_3 \Delta r_1 \\ \omega_2 \Delta r_1 - 2\omega_1 \Delta r_2 & \omega_1 \Delta r_1 + \omega_3 \Delta r_3 & \omega_2 \Delta r_3 - 2\omega_3 \Delta r_2 \\ \omega_3 \Delta r_1 - 2\omega_1 \Delta r_3 & \omega_3 \Delta r_2 - 2\omega_2 \Delta r_3 & \omega_1 \Delta r_1 + \omega_2 \Delta r_2 \end{bmatrix} \tag{67}$$

3.2.3 Additional Parameter Partial Derivatives

The mathematical developments are presented in this section for the partial derivatives of the system and measurement equations to allow for additional candidate parameters to be included in the estimation algorithms. These parameters include center-of-gravity, $\underline{r}_{cg}$, moments of inertia, I, wind velocity, $\underline{v}_w$, and inertial platform tilt errors. Aerodynamic and plume parameter partial derivatives are also presented.

The computer program is being structured to permit these parameters to be easily incorporated without significant impact on the program code.

3.2.3.1 Center-of-Gravity

From equation 23, the partial derivative of angular acceleration with respect to $\underline{r}_{cg}$ is

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{r}_{cg}} = [I]^{-1} \left[\frac{\rho A v_m^2}{2} \{\underline{c}_f\} + \sum_{i=1}^{n} \left\{ {}^{B}C_i^{\text{℄}} \begin{bmatrix} f_{T_i}^{\text{℄}} \\ 0 \\ 0 \end{bmatrix} \right\} \right] \tag{68}$$

From equation 32, the partial derivative of the measured acceleration with respect to $\underline{r}_{cg}$ is

$$\frac{\partial \underline{a}_m}{\partial \underline{r}_{cg}} = -{}^{S}C^{B} \{\underline{\omega} \times \underline{\omega}\} \tag{69}$$

3.2.3.2 Moments of Inertia

The moments-of-inertia are grouped into "principal" terms, $\underline{i}_p$, and cross product terms, $\underline{i}_{cp}$. From equation 24, these vectors are defined as

$$\underline{i}_p = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} \tag{70}$$

and

$$\underline{i}_{cp} = \begin{bmatrix} I_{xy} \\ I_{zx} \\ I_{yz} \end{bmatrix} \tag{71}$$

With these definitions, equation 23 is rewritten as

$$\dot{\underline{\omega}} = [I]^{-1} \; [\Sigma \underline{T} - \{\underline{\omega}\} \left\{ \begin{bmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{bmatrix} \underline{i}_p \right. \tag{72}$$

$$\left. + \begin{bmatrix} -\omega_2 & -\omega_3 & 0 \\ -\omega_1 & 0 & -\omega_3 \\ 0 & -\omega_1 & -\omega_2 \end{bmatrix} \underline{i}_{cp} \right\} \;]$$

where $\Sigma \underline{T}$ represents the sum of the nonrotational torques in equation 23. Defining an intermediate vector $\underline{a}$ as

$$\underline{a} = \Sigma \underline{T} - \underline{\omega} \times (I\underline{\omega}) \tag{73}$$

the partial derivatives of the angular acceleration with respect to $\underline{i}_p$ and $\underline{i}_{cp}$ are

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{i}_p} = \frac{\partial}{\partial \underline{i}_p} (I^{-1} \underline{a}) \bigg|_{\underline{a}\text{-fixed}} - (I)^{-1} \begin{bmatrix} 0 & -\omega_2\omega_3 & \omega_2\omega_3 \\ \omega_1\omega_3 & 0 & -\omega_1\omega_3 \\ -\omega_1\omega_2 & \omega_1\omega_2 & 0 \end{bmatrix} \tag{74}$$

and

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{i}_{cp}} = \frac{\partial}{\partial \underline{i}_{cp}} (I^{-1} \underline{a}) \bigg|_{\underline{a}\text{-fixed}} - [I]^{-1} \begin{bmatrix} \omega_1\omega_2 & -\omega_1\omega_2 & \omega_3^2 - \omega_2^2 \\ -\omega_2\omega_3 & \omega^2 - \omega_3^2 & \omega_1\omega_2 \\ \omega_2^2 - \omega_1^2 & \omega_2\omega_3 & -\omega_1\omega_3 \end{bmatrix} \tag{75}$$

where

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$$\frac{\partial}{\partial \underline{i}_p} (I^{-1}\ \underline{a}) = \frac{1}{\Delta} \begin{bmatrix} 0 & I_z a_1 + I_{zx} a_3 & I_y a_1 + I_{xy} a_2 \\ I_z a_2 + I_{yz} a_3 & 0 & I_x a_2 + I_{xy} a_1 \\ I_y a_3 + I_{yz} a_2 & I_x a_3 + I_{zx} a_1 & 0 \end{bmatrix}$$

$$- [I]^{-1} \frac{1}{\Delta} \begin{bmatrix} (I_y I_z - I_{yz}{}^2) a_1 & (I_x I_z - I_{zx}{}^2) a_1 & (I_x I_y - I_{xy}{}^2) a_1 \\ (\quad " \quad) a_2 & (\quad " \quad) a_2 & (\quad " \quad) a_2 \\ (\quad " \quad) a_3 & (\quad " \quad) a_3 & (\quad " \quad) a_3 \end{bmatrix} \tag{76}$$

and

$$\frac{\partial}{\partial \underline{i}_{cp}} (I^{-1}\ a) =$$

$$= \frac{1}{\Delta} \begin{bmatrix} I_z a_2 + I_{yz} a_3 & I_y a_3 + I_{yz} a_2 & -2 I_{yz} a_1 + I_{zx} a_2 + I_{xy} a_3 \\ I_z a_1 + I_{zx} a_3 & I_{yz} a_1 - 2 I_{zx} a_2 + I_{xy} a_3 & I_x a_3 + I_{zx} a_1 \\ I_{yz} a_1 + I_{zx} a_2 - 2 I_{xy} a_3 & I_y a_1 + I_{xy} a_2 & I_x a_2 + I_{xy} a_1 \end{bmatrix} \tag{77}$$

$$
- [I]^{-1}\,\frac{1}{\Delta}\begin{bmatrix} -2(I_z I_{xy} + I_{zx} I_{yz})a_1 & -2(I_y I_{zx} + I_{xy} I_{yz})a_1 & -2(I_x I_{yz} + I_{xy} I_{xz})a_1 \\ -2(\quad '' \quad)a_2 & -2(\quad '' \quad)a_2 & -2(\quad '' \quad)a_2 \\ -2(\quad '' \quad)a_3 & -2(\quad '' \quad)a_3 & -2(\quad)\quad)a_3 \end{bmatrix}
$$

and

$$
\Delta = I_x I_y I_z - I_{xy} I_{yz} I_{zx} - I_{zx} I_{xy} I_{yz} - I_y I_{zx}^2 - I_z I_{xy}^2 - I_x I_{yz}^2 \qquad (78)
$$

3.2.3.3 <u>Wind Velocity</u>

From equations 20 and 25, the partial derivative of the vehicle acceleration with respect to $\underline{v}_w$ is

$$
\frac{\partial \underline{v}^{(B)}}{\partial \underline{v}_w} = \frac{\rho A}{2m}\,\underline{c}_f\,\frac{\partial v_m^2}{\partial \underline{v}_w} + \frac{\rho A v_m^2}{2m}\,\frac{\partial \underline{c}_f}{\partial \alpha}\,\frac{\partial \alpha}{\partial \underline{v}_w} + \frac{\rho A v_m^2}{2m}\,\frac{\partial \underline{c}_f}{\partial \beta}\,\frac{\partial \beta}{\partial \underline{v}_w}\,. \qquad (79)
$$

The first of the partial derivatives in equation 79 can be obtained from the following equation

$$
v_m^2 = \underline{v}_r^T\,\underline{v}_r = \underline{v}^{(B)^T}\,\underline{v}^{(B)} - 2\underline{v}^{(B)^T}\,{}^B C\;\underline{v}_w + \underline{v}_w^T\;C^B\,{}^B C\;\underline{v}_w \qquad (80)
$$

From equation 80, the following is obtained

$$
\frac{\partial v_m^2}{\partial \underline{v}_w} = -\,2\,\underline{v}_r^T\,{}^B C. \qquad (81)
$$

Denoting the elements of the matrix ${}^{B}C$ as c_{11}, c_{12}, etc., the following equations are obtained for the partial derivatives of α and β with respect to $\underline{v}_w$;

$$\frac{\partial\alpha}{\partial\underline{v}_w} = \frac{-1}{v_{r_1}^{\,2} + v_{r_3}^{\,2}} \begin{bmatrix} (v_{r_1}c_{31} - v_{r_3}c_{11}) \\ (v_{r_1}c_{32} - v_{r_3}c_{12}) \\ (v_{r_1}c_{33} - v_{r_3}c_{13}) \end{bmatrix}^T \tag{82}$$

and

$$\frac{\partial\beta}{\partial\underline{v}_w} = \frac{-1}{v_m\sqrt{v_{r_2}^{\,2} + v_{r_3}^{\,2}}} \begin{bmatrix} v_m c_{21} - \dfrac{v_{r_2}}{v_m}(v_{r_1}c_{11} + v_{r_2}c_{21} + v_{r_3}c_{31}) \\ v_m c_{22} - \dfrac{v_{r_2}}{v_m}(v_{r_1}c_{12} + v_{r_2}c_{22} + v_{r_3}c_{32}) \\ v_m c_{23} - \dfrac{v_{r_2}}{v_m}(v_{r_1}c_{13} + v_{r_2}c_{23} + v_{r_3}c_{33}) \end{bmatrix}^T \tag{83}$$

3.2.3.4 Inertial Platform Tilt

Temporarily rewriting equation (32) as

$$\underline{a}_m^{(S)} = {}^{S}C^{P}(I + \underline{\delta\theta}\ \times)\ \underline{s} \tag{84}$$

where

$\underline{\delta\theta}$ = vector whose elements are the axes misalignments

$\underline{s}$ = sum of the bracketed terms in equation 32 multiplied by ${}^{P'}C^{B}$.

The following partial derivative of the measured acceleration with respect to $\underline{\delta\theta}$ is obtained

$$\frac{\partial \underline{a}_m^{(S)}}{\partial \underline{\delta\theta}} = {}^{S}C^{P} \{\underline{s}\}, \tag{85}$$

3.2.3.5 Aerodynamic and Plume Parameters

A linear model for the aerodynamic and plume characteristics is used. This model is expanded as

$$\underline{c}_f = \underline{c}_{fo} + \underline{c}_{f\alpha}\,\alpha + \underline{c}_{f\beta}\,\beta + \ldots \tag{86}$$

and $$\underline{c}_m = \underline{c}_{mo} + \underline{c}_{m\alpha}\,\alpha + \underline{c}_{m\beta}\,\beta + \ldots \tag{87}$$

$$\underline{f}_p = \underline{f}_p + \underline{f}_{p\alpha}\,\alpha + \underline{f}_{p\beta}\,\beta + \ldots \tag{88}$$

where additional terms to represent rates, cross couplings, and controls can be included.

The basic approach of establishing the partial derivatives will be illustrated for a couple of terms, $\underline{c}_{f\alpha}$ and $\underline{c}_{m\alpha}$. Using these example illustrations, the rest of the candidate parameters can be similarly obtained. From equation 20, the following partial derivative is obtained

$$\frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{c}_{f_\alpha}} = \frac{\partial \dot{\underline{v}}^{(B)}}{\partial \underline{c}_f} \frac{\partial \underline{c}_f}{\partial \underline{c}_{f_\alpha}} = \frac{\rho A v_m^{\ 2}}{2m} \alpha[U] \tag{89}$$

where

[U] = unit 3 x 3 matrix with one's (1) on the diagonal and zeros off the diagonal

From equation 23, the partial derivative of angular acceleration with respect to $\underline{c}_{m_\alpha}$ is

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{c}_{m_\alpha}} = \frac{\partial \dot{\underline{\omega}}}{\partial \underline{c}_m} \frac{\partial \underline{c}_m}{\partial \underline{c}_{m_\alpha}} = [I]^{-1} \frac{\rho A v_m^{\ 2}}{2} \alpha[U]. \tag{90}$$

The corresponding partial derivative with respect to $\underline{c}_{f_\alpha}$ is

$$\frac{\partial \dot{\underline{\omega}}}{\partial \underline{c}_{f_\alpha}} = \frac{\partial \dot{\underline{\omega}}}{\partial \underline{c}_f} \frac{\partial \underline{c}_f}{\partial \underline{c}_{f_\alpha}} = [I]^{-1} \frac{\rho A v_m^{\ 2}}{2} \{\underline{r}_A^{(B)} - \underline{r}_{cg}^{(B)}\} \alpha[U]. \tag{91}$$

The static aerodynamic coefficient model has been obtained by a multiple regression analysis of the current aerodynamic tubular data. This model is presented in Appendix D with the associated regression coefficients.

3.3 Propulsion Parameter States and Measurements

A candidate approach for incorporating the NASA propulsion model's capabilities has been identified. This approach utilizes nominal predicted values of thrust, pressure, propellant and pressurant mass flow rates, and utilizes sensitivities or partial derivatives of these variables with respect to the independent parameters selected for estimation by the algorithm.

The approach is to include deviations from nominal values of measured chamber pressure, power level, propellant and pressurant mass flow rates as states. The models assumed for these deviations are time correlated random processes. Then as states, partial derivatives of the first twelve states with respect to these variables will be required.

For the SSME and SRB, this modeling approach is discussed in the following. Additionally, the necessary partial derivatives of the first twelve state variables with resepct to the additional states are presented.

3.3.1 SSME Propulsion Parameter Model

For the SSME, the total actual values of vacuum thrust and oxidizer mass flow rates are modeled by

$$f_T = f_{T_{nom}} + \Delta f_T \quad . \tag{92}$$

and

$$\dot{m}_{O_2} = \dot{m}_{O_{2_{nom}}} + \Delta\dot{m}_{O_2} . \tag{93}$$

The measurements of fuel mass flow rate, pressurant mass flow rates and power level are modeled as

$$\dot{m}_{H_2} = \dot{m}_{H_{2_{nom}}} + \Delta\dot{m}_{H_2} + b_{\dot{m}_{H_2}} + s_{\dot{m}_{H_2}} \tag{94}$$

$$\dot{m}_{H_{2_p}} = \dot{m}_{H_{2_{p_{nom}}}} + \Delta\dot{m}_{H_{2_p}} + b_{\dot{m}_{H_{2_p}}} + s_{\dot{m}_{H_{2_p}}} \tag{95}$$

$$\dot{m}_{O_{2_p}} = \dot{m}_{O_{2_{p_{nom}}}} + \Delta\dot{m}_{O_{2_p}} + b_{\dot{m}_{O_{2_p}}} + s_{\dot{m}_{O_{2_p}}} \tag{96}$$

and

$$PL = PL_{nom} + \Delta PL + b_{PL} + s_{PL}. \tag{97}$$

These measured quantities include measurement noise $s_{(\ )}$ and potential bias states $b_{(\ )}$ modeled as random constants. In these measurements, the Δ'd variables are to be included as states in the estimation algorithm. If the nominal values are zero or unknown, then the Δ'd variables absorb the entire estimate. Where required, the estimate for the variables used in the estimation algorithm is formed using the nominal and the estimate of the deviation, etc. In example, thrust and fuel mass flow rate estimates are formed as

$$\hat{f}_T = f_{T_{nom}} + \Delta\hat{f}_T \tag{98}$$

and

$$\hat{\dot{m}}_{H_2} = \dot{m}_{H_{2_{nom}}} + \Delta\hat{\dot{m}}_{H_2} + \hat{b}_{\dot{m}_{H_2}} . \tag{99}$$

The deviation or Δ'd measurement variables are modeled as time correlated random variables. This permits these variables to vary within a band of frequencies. The typical model is then given as

$$\frac{d}{dt} \Delta(\ ) = -\frac{1}{\tau_{(\ )}} \Delta(\ ) + \frac{1}{\tau_{(\ )}} s_{(\ )} \tag{100}$$

where the parenthesis ( ) would be replaced by the variables, i.e., $\dot{m}_{H_2}$. For the SSME, an additional variable Δc^*_{mult} is modeled as in equation 100 and included as a state variable with the Δ'd measurement variables.

The thrust deviation is expanded as in the following truncated Taylor series as a function of the independent parameters.

$$\Delta f_T = \frac{\partial f_T}{\partial \dot{m}_{H_{2_p}}} \Delta\dot{m}_{H_{2_p}} + \frac{\partial f_T}{\partial \dot{m}_{O_{2_p}}} \Delta\dot{m}_{O_{2_p}} + \frac{\partial f_T}{\partial \Delta c^*} \Delta c^*$$

$$+ \frac{\partial f_T}{\partial PL} \Delta PL + \frac{\partial f_T}{\partial MR} \Delta MR. \tag{101}$$

In the $\dot{\underline{v}}^{(B)}$ and $\dot{\underline{\omega}}$ equations, with equation 101 replacing f_{T_i}, the partial derivatives of f_T with respect to the Δ'd variables are obtained directly from equation 101.

It is desirable to include vehicle mass bias as a state. The SSME's system contribution to the mass deviation is given by

$$\Delta\dot{m}_{SSME's} = \sum_i (\Delta\dot{m}_{H_{2_i}} + \Delta\dot{m}_{O_{2_i}} - \Delta\dot{m}_{H_{2_{p_i}}} - \Delta\dot{m}_{O_{2_{p_i}}}). \tag{102}$$

In equation 101, the Δm_{O_2} contribution to the mass deviation is not available from measurements. As with the thrust deviation, this quantity is formed as

$$\Delta\dot{m}_{O_2} = \frac{\partial\dot{m}_{O_2}}{\partial\dot{m}_{H_{2_p}}} \Delta\dot{m}_{H_{2_p}} + \frac{\partial\dot{m}_{O_2}}{\partial\dot{m}_{O_{2_p}}} \Delta\dot{m}_{O_{2_p}} + \frac{\partial\dot{m}_{O_2}}{\partial\Delta c^*} \Delta c^*$$
$$+ \frac{\partial\dot{m}_{O_2}}{\partial PL} \Delta PL + \frac{\partial\dot{m}_{O_2}}{\partial MR} \Delta MR. \tag{103}$$

which is in terms of other estimated state variables. In equations 101 and 103 the deviation in mixture ratio, ΔMR, is obtained algebraically from

$$\Delta MR = \frac{\dot{m}_{H_{2_{nom}}} - \hat{\dot{m}}_{H_2}}{\frac{\partial\dot{m}_{H_2}}{\partial MR}} \tag{104}$$

3.3.2 SRB Propulsion Parameter Model

The approach for the SRB modeling follows closely that used for the SSME. Candidate independent parameters include propellant burn rate exponent, a, and motor efficiency coefficient, c_m. Others can be added using this technique.

The actual value of vacuum thrust is given by equation 92. The only measurement available for the SRB is the total pressure at the forward head end of the motor case and is modeled as

$$P_{O_H} = P_{O_{H_{nom}}} + \Delta P_{O_H} + b_{P_{O_H}} + s_{P_{O_H}} \tag{105}$$

where $b_{(\ )}$ and $s_{(\ )}$ represents a bias and measurement noise respectively.

The independent parameters, Δa and Δc_m, are included in the model as states. The model assumed can be as given by equation 100 or another suitable dynamical process, i.e., random constant.

The thrust deviation is given by the following truncated Taylor series as a function of the candidate independent parameters.

$$\Delta f_T = \frac{\partial f_T}{\partial a}\Delta a + \frac{\partial f_T}{\partial c_m} \Delta c_m + \ldots \tag{106}$$

The partial derivatives for the $\dot{\underline{v}}^{(B)}$ and $\dot{\underline{\omega}}$ equations with respect to the independent parameters are obtained directly from equation 106. The mass deviation equation for the SRB is given as

$$\Delta\dot{m}_{SRB} = \sum_i (\Delta\dot{m})_i \tag{107}$$

where

$$\Delta \dot{m}_i = \frac{\partial \dot{m}}{\partial a} \Delta a + . . . \tag{108}$$

The head pressure deviation, ΔP_{O_H}, is expanded similarly

$$\Delta P_{O_H} = \frac{\partial P_{O_H}}{\partial a} \Delta a + . . . \tag{109}$$

3.3.3 Vehicle Mass

The total rate of change of vehicle mass is given by

$$\frac{d}{dt}(m) = \dot{m}_{SSME_{nom}} + \dot{m}_{SRB_{nom}} + \Delta\dot{m}_{SSME} + \Delta\dot{m}_{SRB} + \dot{m}_{NON-CONSUME} \tag{110}$$

The first two terms in this equation are the apriori assumed nominal values. The third and fourth terms were discussed earlier. The last term should be zero; however, it can include a mass bias uncertainty Δm_b

The equations, state and measurement, in which mass occurs are the $\underline{\dot{v}}^{(B)}$ and $\underline{a}_m$ equations. Assuming equation 110 can be summarized as $\dot{m} + \Delta\dot{m}_b$ then the mass can be written as $m + \Delta m_b$. Replacing this expression for the mass in the two indicated equations yields the following partial derivatives with respect to the Δm_b.

$$\frac{\partial \underline{\dot{v}}^{(B)}}{\partial \Delta m_b} = - \frac{1}{(m + \Delta m_b)^2} \left(\frac{\rho v_m^2 A}{2} \underline{c}_f + \underline{f}_p^{(B)} + {}^B C^{\mathcal{L}} \underline{f}_{T_i}^{(\mathcal{L})}\right) \tag{111}$$

and

$$\frac{\partial \underset{\sim}{a}_m^{(S)}}{\partial \Delta m_b} = - \frac{1}{(m + \Delta m_b)^2} \; {}^S C^B \left(\frac{\rho v_m^{\,2} A}{2} \underset{\sim}{c}_f + \underset{\sim}{f}_p^{(B)} + {}^B C^{\mathbb{L}} \underset{\sim}{f}_{T_i}^{(\mathbb{L})} \right) \tag{112}$$

4.0 PROJECTED ACTIVITIES DURING UPCOMING MONTH

During the next period, the filter portion of the estimation algorithm should be checked out. Additionally, the smoother portion routines should have been coded. The smoother routines are more straightforward and represent less developments than the filter routines.

The ability to bring up the NASA SRB predictive model on our computer will be determiend. If not successful, two other possibilities exist. The first, and least desirable, is to utilize the computer facility at the University of Florida under NASA sponsorship. The second is to have provided the necessary partial derivatives or allow SDI personnel access to these programs resident on the NASA computers for the purposes of generating the partial derivatives. There may be other possibilities than those mentioned here.

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APPENDIX A

PARTIAL DERIVATIVE OF THE VECTOR ${}^{I}C^{B}\,\underline{v}$ wrt $\underline{\theta}$

This partial derivative is one of several that occurs frequently in the formulation of the linearized system state and measuremetn eqautions. The desired partial derivative is

$$\frac{\partial}{\partial \underline{\theta}} \begin{bmatrix} (\cos\theta\cos\psi)v_1 + \begin{pmatrix}\sin\varphi\sin\theta\cos\psi \\ -\cos\varphi\sin\psi\end{pmatrix} v_2 + \begin{pmatrix}\cos\varphi\sin\theta\cos\psi \\ +\sin\varphi\sin\psi\end{pmatrix} v_3 \\ (\cos\theta\sin\psi)v_1 + \begin{pmatrix}\sin\varphi\sin\theta\sin\psi \\ +\cos\varphi\cos\psi\end{pmatrix} v_2 + \begin{pmatrix}\cos\varphi\sin\theta\sin\psi \\ -\sin\varphi\cos\psi\end{pmatrix} v_3 \\ (-\sin\theta)v_1 + (\sin\varphi\cos\theta)v_2 + (\cos\varphi\cos\theta)\, v_3 \end{bmatrix} . \qquad \text{A-1}$$

The resulting matrix is given in Table A-1.

TABLE A-1. Partial Derivative of ${}^{I}C^{B}\,\underline{v}$ wrt θ

$(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)v_2$ $-(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi)v_3$	$-\sin\theta\cos\psi\ v_1$ $+\sin\varphi\cos\theta\cos\psi\ v_2$ $+\cos\varphi\cos\theta\cos\psi\ v_3$	$-\cos\theta\sin\psi\ v_1$ $-(\sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi)v_2$ $-(\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi)v_3$
$(\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi)v_2$ $-(\sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi)v_3$	$-\sin\theta\sin\psi\ v_1$ $+\sin\varphi\cos\theta\sin\psi\ v_2$ $+\cos\varphi\cos\theta\sin\psi\ v_3$	$\cos\theta\cos\psi\ v_1$ $+(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi)v_2$ $+(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)v_3$
$\cos\varphi\cos\theta\ v_2$ $-\sin\varphi\cos\theta\ v_3$	$-\cos\theta\ v_1$ $-\sin\varphi\sin\theta\ v_2$ $-\cos\varphi\sin\theta\ v_3$	

APPENDIX B

$\frac{\partial v_m}{\partial \underline{v}^{(B)}}$, $\frac{\partial \alpha}{\partial \underline{v}^{(B)}}$, $\frac{\partial \beta}{\partial v^{(B)}}$, $\frac{\partial \alpha}{\partial h}$, $\frac{\partial \beta}{\partial h}$ and $\frac{\partial v_m}{\partial h}$ Expressions

These partial derivatives occur frequently and will be developed in this appendix. The equation for $\underset{\sim}{v}_r$ is

$$\underset{\sim}{v}_r = \underline{v}^{(B)} - {}^{B}C^{LL}\, \underset{\sim}{v}_w^{(LL)} \qquad \text{B-1}$$

Since the wind velocity, $\underset{\sim}{v}_w^{(LL)}$, is only a function of altitude then

$$\frac{\partial}{\partial \underset{\sim}{v}_r} = \frac{\partial}{\partial \underline{v}^{(B)}} \,. \qquad \text{B-2}$$

The first partial derivative, $\frac{\partial v_m}{\partial \underline{v}^{(B)}}$, is

$$\frac{\partial v_m}{\partial \underline{v}^{(B)}} = \frac{1}{v_m} \begin{bmatrix} v_{r_1} \\ v_{r_2} \\ v_{r_3} \end{bmatrix}^T . \qquad \text{B-3}$$

The second, $\frac{\partial \alpha}{\partial \underline{v}^{(B)}}$, is given by

$$\frac{\partial \alpha}{\partial \underline{v}^{(B)}} = \begin{bmatrix} \dfrac{-v_{r_3}}{v_{r_1}^{2} + v_{r_3}^{2}} \\ 0 \\ \dfrac{v_{r_1}}{v_{r_1}^{2} + v_{r_3}^{2}} \end{bmatrix}^T \qquad \text{B-4}$$

The equation for $\frac{\partial \beta}{\partial \underline{v}^{(B)}}$ is

$$\frac{\partial \beta}{\partial \underline{v}^{(B)}} = \begin{bmatrix} \dfrac{-v_{r_1} v_{r_2}}{v_m^{3}\sqrt{v_{r_1}^{2} + v_{r_3}^{2}}} \\ \dfrac{\sqrt{v_{r_1}^{2} + v_{r_3}^{2}}}{v_m^{3}} \\ \dfrac{-v_{r_2} v_{r_3}}{v_m^{3}\sqrt{v_{r_1}^{2} + v_{r_3}^{2}}} \end{bmatrix}^T \qquad \text{B-5}$$

The following equations define the last three required partial derivatives

$$\frac{\partial\alpha}{\partial h} = - \frac{\partial\alpha}{\partial \underline{v}^{(B)}} \; {}^{B}C^{LL} \; \frac{\partial \underline{v}_w^{(LL)}}{\partial h} \qquad \text{B-6}$$

$$\frac{\partial\beta}{\partial h} = - \frac{\partial\beta}{\partial \underline{v}^{(B)}} \; {}^{B}C^{LL} \; \frac{\partial \underline{v}_w^{(LL)}}{\partial h} \qquad \text{B-7}$$

$$\frac{\partial v_m}{\partial h} = - \frac{\partial\beta}{\partial \underline{v}^{(B)}} \; {}^{B}C^{LL} \; \frac{\partial \underline{v}_w^{(LL)}}{\partial h} \qquad \text{B-8}$$

APPENDIX C

PARTIAL DERIVATIVE OF THE VECTOR ${}^{B}C^{I}\,\underline{v}$ wrt $\underline{\theta}$

The third of the frequently occurring required partial derivatives is

$$\frac{\partial}{\partial\underline{\theta}}\begin{bmatrix} \cos\theta\cos\psi\ v_1 & + & \cos\theta\sin\psi\ v_2 & - & \sin\theta\ v_3 \\ \begin{pmatrix}\sin\varphi\sin\theta\cos\psi \\ -\cos\varphi\sin\psi\end{pmatrix} v_1 & + & \begin{pmatrix}\sin\varphi\sin\theta\sin\psi \\ +\cos\varphi\cos\psi\end{pmatrix} v_2 & + & \sin\varphi\cos\theta\ v_3 \\ \begin{pmatrix}\cos\varphi\sin\theta\cos\psi \\ +\sin\varphi\sin\psi\end{pmatrix} v_1 & + & \begin{pmatrix}\cos\varphi\sin\theta\sin\psi \\ -\sin\varphi\cos\psi\end{pmatrix} v_2 & + & \cos\varphi\cos\theta\ v_3 \end{bmatrix} \qquad \text{C-1}$$

The resulting matrix is given in Table C-1.

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TABLE C-1. Partial Derivative of ${}^{B}C^{I}\,\underline{v}$ wrt $\underline{\theta}$

0	$-\sin\theta\cos\psi\ v_1$ $-\sin\theta\sin\psi\ v_2$ $+\cos\theta\ v_3$	$-\cos\theta\sin\psi\ v_1$ $+\cos\theta\cos\psi\ v_2$
$(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)v_1$ $-(\cos\varphi\sin\theta\sin\psi - \sin\varphi\cos\psi)v_2$ $+ \cos\varphi\cos\theta\ v_3$	$\sin\varphi\cos\theta\cos\psi\ v_1$ $+\sin\varphi\cos\theta\sin\psi\ v_2$ $-\sin\varphi\sin\theta\ v_3$	$-(\sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi)v_1$ $+(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi)v_2$
$-(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi)v_1$ $-(\sin\varphi\sin\theta\sin\psi + \cos\varphi\cos\psi)v_2$ $- \sin\varphi\cos\theta\ v_3$	$\cos\varphi\cos\theta\cos\psi\ v_1$ $+\cos\varphi\cos\theta\sin\psi\ v_2$ $-\cos\varphi\sin\theta\ v_3$	$-(\sin\varphi\sin\theta\cos\psi - \cos\varphi\sin\psi)v_1$ $+(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)v_2$

APPENDIX D

AERODYNAMIC MODELING REGRESSION ANALYSIS AND RESULTS

The aerodynamic data tables provided as IVBC3 data has been incorporated into an aerodynamic coefficient polynomial model. This modeling effort reduces the dimensionality of the numerical tables to one and reduces the storage requirements for the aerodynamic model.

The coefficient model used for the two stages differ slightly as a result of the available data. The regression analysis led in the selection of the form of the aerodynamic model. Terms with insignificant correlation were eliminated from the model.

In equation form, the first stage static coefficients of axial force, C_A; normal force, C_N; pitching moment, C_m; rolling moment, C_ℓ; side force, C_Y; and yawing moment, C_n; are given below

$$C_A = C_{A_o} + C_{A_\alpha}\,\alpha + C_{A_{\alpha^2}}\,\alpha^2 + C_{A_{\alpha\beta^2}}\,\alpha\beta^2 + C_{A_{\beta^2}}\,\beta^2 \tag{D-1}$$

$$C_N = C_{N_o} + C_{N_\alpha}\,\alpha + C_{N_{\alpha\beta^2}}\,\alpha\beta^2 \tag{D-2}$$

$$C_m = C_{m_o} + C_{m_\alpha}\,\alpha + C_{m_{\alpha\beta^2}}\,\alpha\beta^2 \tag{D-3}$$

$$C_\ell = C_{\ell_o} + C_{\ell_\beta}\,\beta + C_{\ell_{\alpha\beta}}\,\alpha\beta + C_{\ell_{\alpha^2\beta}}\,\alpha^2\beta \tag{D-4}$$

$$C_Y = C_{Y_o} + C_{Y_\beta}\beta + C_{Y_{\alpha\beta}}\alpha\beta + C_{Y_{\alpha^2\beta}}\alpha^2\beta \qquad \text{D-5}$$

$$C_n = C_{n_o} + C_{n_\beta}\beta + C_{n_{\alpha\beta}}\alpha\beta + C_{n_{\alpha^2\beta}}\alpha^2\beta \qquad \text{D-6}$$

The corresponding second stage model is given by the following equations

$$C_A = C_{A_o} + C_{A_\alpha}\alpha + C_{A_{\alpha^2}}\alpha^2 \qquad \text{D-7}$$

$$C_N = C_{N_o} + C_{N_\alpha}\alpha + C_{N_{\alpha^2}}\alpha^2 \qquad \text{D-8}$$

$$C_m = C_{m_o} + C_{m_\alpha}\alpha + C_{m_{\alpha^2}}\alpha^2 \qquad \text{D-9}$$

$$C_\ell = C_{\ell_o} + C_{\ell_\beta}\beta + C_{\ell_{\alpha\beta}}\alpha\beta + C_{\ell_{\alpha^2\beta}}\alpha^2\beta \qquad \text{D-10}$$

$$C_Y = C_{Y_o} + C_{Y_\beta}\beta + C_{Y_{\alpha\beta}}\alpha\beta + C_{Y_{\alpha^2\beta}}\alpha^2\beta \qquad \text{D-11}$$

$$C_n = C_{n_o} + C_{n_\beta}\beta + C_{n_{\alpha\beta}}\alpha\beta + C_{n_{\alpha^2\beta}}\alpha^2\beta \qquad \text{D-12}$$

For the first stage, data from an angle-of-attack range of -6 to +6 degrees was used in the regression analysis. Data from a range of -8 to +4 degrees was used for the second stage. The results, TCXX..., from the regression analysis is presented below for each of the coefficients, $C_{XX...}$, above.

TABLE D-1. First Stage Axial Force Coefficient

PROBLEM SET: CAFA

MACH #	TCA0	TCAA	TCAAB2	TCAB2	T CAA2
.6	0.14888462	-0.14939267E-02	0.52950736E-05	-0.19513888E-03	-0.36778764E-03
.8	0.14096716	-0.16073198E-02	0.80431100E-05	-0.15666662E-03	-0.29035713E-03
.9	0.15895303	-0.87178504E-03	-0.79241381E-05	-0.10849207E-03	-0.17375973E-03
.95	0.18570921	-0.80946332E-03	-0.97817874E-05	-0.14827383E-03	-0.22280747E-03
1.0	0.24431951	-0.68517815E-03	-0.85392085E-05	-0.20781747E-03	-0.32113091E-03
1.1	0.25983587	-0.53749926E-03	0.31472900E-05	-0.17412701E-03	-0.28126000E-03
1.15	0.26992825	-0.20928589E-03	-0.46378850E-05	-0.13521819E-03	-0.23748010E-03
1.25	0.28716668	-0.47267828E-03	-0.17535077E-05	-0.12434516E-03	-0.26166640E-03
1.4	0.30411604	-0.53303648E-03	-0.35093940E-05	-0.19692448E-03	-0.16787715E-03
1.55	0.30857432	-0.51839335E-03	-0.30256535E-06	-0.21642850E-03	-0.12008918E-03
1.8	0.30735463	-0.11330346E-02	0.91515847E-06	-0.18192461E-03	-0.79305413E-04
2.2	0.28761023	-0.97124977E-03	-0.10002519E-04	-0.89146888E-04	-0.40277255E-05
2.5	0.28410554	-0.12258916E-02	-0.14255994E-04	-0.38432758E-04	0.16438302E-04
3.5	0.28069207	-0.27267837E-02	-0.18130060E-04	0.14365064E-03	0.11210326E-03
4.5	0.28001353	-0.40642847E-02	-0.13194465E-04	0.15932534E-03	0.14424580E-03

TABLE D-2. First Stage Normal Force Coefficient

PROBLEM SET: CNFA

MACH #	TCNO	TCNA	TCNAB2
.6	0.06175191	0.47317315E-01	-0.34953373E-06
.8	0.08705191	0.51266242E-01	-0.21971118E-04
.9	0.06880905	0.55013947E-01	0.37438092E-05
.95	0.06304239	0.55984642E-01	0.12846999E-04
1.0	0.05150905	0.57926252E-01	0.31073960E-04
1.1	0.04629619	0.58463752E-01	0.18769655E-04
1.15	0.05327953	0.56734987E-01	0.49979085E-05
1.25	0.06288191	0.56824278E-01	0.19573376E-04
1.4	0.05508667	0.56955535E-01	0.44349854E-04
1.55	0.04784143	0.56824278E-01	0.24865894E-04
1.8	0.02033000	0.57594139E-01	-0.13175871E-04
2.2	-0.00378286	0.52612297E-01	0.82223523E-05
2.5	-0.03606287	0.51134817E-01	0.49027673E-04
3.5	-0.04565714	0.39903577E-01	0.62995699E-04
4.5	-0.05337614	0.36437500E-01	0.43675380E-04

TABLE D-3. First Stage Pitching Moment Coefficient

PROBLEM SET: CMFA

MACH #	TCMO	TCMA	TCMAB2
.6	-0.08278762	-0.15543575E-01	0.16693601E-05
.8	-0.10786715	-0.18000530E-01	0.21009337E-04
.9	-0.09791238	-0.19910170E-01	0.46228233E-05
.95	-0.08543953	-0.20583043E-01	0.85657601E-04
1.0	-0.07620762	-0.21927318E-01	0.21230151E-05
1.1	-0.06862953	-0.20931607E-01	-0.26527516E-04
1.15	-0.07373904	-0.20659992E-01	0.65993113E-05
1.25	-0.07957237	-0.20311428E-01	-0.95234293E-06
1.4	-0.07712714	-0.20116061E-01	-0.17661379E-04
1.55	-0.06862857	-0.20584987E-01	0.12942927E-05
1.8	-0.05165952	-0.22142328E-01	0.42312208E-05
2.2	-0.02029048	-0.19702138E-01	-0.14325634E-04
2.5	0.00235619	-0.18991778E-01	-0.52894680E-04
3.5	0.01028095	-0.13785718E-01	-0.24528768E-04
4.5	0.01356667	-0.11521422E-01	-0.27753043E-04

TABLE D-4. First Stage Rolling Moment Coefficient

PROBLEM SET: CLLA

MACH #	TCLLO	TCLLB	TCLLAB	TCLLA2B
.6	0.00009800	-0.55153607E-02	-0.18314904E-03	0.25525271E-05
.8	-0.00100400	-0.51660468E-02	-0.24067984E-03	-0.47074923E-05
.9	0.00119100	-0.53059668E-02	-0.17415131E-03	0.40663767E-06
.95	0.00113500	-0.57055471E-02	-0.18708523E-03	0.26400435E-05
1.0	0.00102300	-0.65046088E-02	-0.21297160E-03	0.71316317E-05
1.1	0.00255850	-0.73706545E-02	0.23723571E-04	0.41424133E-04
1.15	0.00142150	-0.64209271E-02	-0.10909401E-03	0.19684549E-04
1.25	0.00088950	-0.61848979E-02	-0.15336619E-03	0.76067136E-05
1.4	0.00162650	-0.60260380E-02	-0.40835867E-03	-0.20967154E-04
1.55	0.00162500	-0.57253810E-02	-0.20703886E-03	0.41421335E-05
1.8	0.00160650	-0.55433516E-02	-0.17273379E-03	0.71404006E-05
2.2	0.00164350	-0.53707338E-02	-0.55630488E-04	0.12803971E-04
2.5	0.00175700	-0.46027126E-02	-0.66665525E-04	0.22202585E-05
3.5	0.00078500	-0.38413065E-02	-0.45428569E-04	-0.94020070E-05
4.5	-0.00035500	-0.32225023E-02	-0.65075328E-04	-0.85059000E-05

TABLE D-5. First Stage Side Force Coefficient

PROBLEM SET: CYA

MACH #	TCYO	TCYB	TCYAB	TCYA2B
.6	0.00040550	-0.34736611E-01	-0.24396958E-03	-0.34743120E-04
.8	0.00611750	-0.35624493E-01	-0.19144324E-03	-0.68390669E-04
.9	0.00208050	-0.37405483E-01	-0.96200623E-04	-0.65476503E-04
.95	0.00340600	-0.37516881E-01	-0.16656239E-03	-0.65729291E-04
1.0	0.00659450	-0.37924547E-01	-0.28471148E-03	-0.51625058E-04
1.1	0.00389250	-0.35857856E-01	-0.67672569E-04	-0.67499590E-04
1.15	0.00896900	-0.34745876E-01	-0.10685262E-03	-0.56302903E-04
1.25	0.01099600	-0.33643633E-01	-0.21200742E-03	-0.70274968E-04
1.4	0.00895500	-0.32880556E-01	-0.30565454E-03	-0.94529889E-04
1.55	0.00684850	-0.36689218E-01	-0.21161194E-03	-0.58715108E-04
1.8	0.00122250	-0.37921809E-01	0.23066868E-03	0.11196062E-04
2.2	-0.00312350	-0.39925124E-01	0.12333597E-03	-0.23788327E-04
2.5	0.00190050	-0.38449090E-01	0.28201332E-03	-0.50999923E-04
3.5	0.00643500	-0.34494311E-01	0.69754681E-03	-0.20773427E-04
4.5	0.00000500	-0.30585317E-01	0.73324196E-03	-0.23277323E-05

TABLE D-6. First Stage Yawing Moment Coefficient

PROBLEM SET: CLNA

MACH #	TCLNO	TCLNB	TCLNAB	TCLNA2B
.6	0.00101900	0.14767525E-01	0.56945679E-04	0.81995468E-05
.8	-0.00196700	0.15323062E-01	0.11934886E-03	0.30770094E-04
.9	0.00001700	0.15911508E-01	-0.12992905E-04	0.26990148E-04
.95	-0.00033150	0.16263550E-01	-0.21390285E-05	0.21246726E-04
1.0	-0.00102900	0.16967123E-01	0.19518508E-04	0.97513848E-05
1.1	-0.00056750	0.15625395E-01	0.11725974E-04	0.28817041E-04
1.15	-0.00357700	0.15465586E-01	0.40781717E-04	0.19518948E-04
1.25	-0.00415650	0.15171567E-01	0.17625467E-03	0.30797459E-04
1.4	-0.00269050	0.14852828E-01	0.32387782E-03	0.55297049E-04
1.55	-0.00395800	0.15761562E-01	0.23744810E-03	0.29008272E-04
1.8	-0.00297900	0.17182207E-01	0.21290279E-03	0.23987946E-05
2.2	-0.00073950	0.18076595E-01	0.11312054E-03	0.10975672E-04
2.5	-0.00348650	0.16406817E-01	-0.58733131E-04	0.34038654E-04
3.5	-0.00259800	0.13792288E-01	-0.43780621E-03	0.47138979E-05
4.5	0.00097000	0.11427909E-01	-0.46182974E-03	-0.30127571E-05

TABLE D-7. Second Stage Longitudinal Coefficients

PROBLEM SET: CAF2

MACH #	TCA20	TCA2A	TCA2A2
3.5	0.17786667	-0.38053570E-02	0.11815455E-03
4.5	0.16868572	-0.42267856E-02	0.16160712E-03
6.0	0.16880476	-0.43714279E-02	0.12023804E-03
8.0	0.17327619	-0.46357140E-02	0.10416679E-03
10.0	0.16560951	-0.47821430E-02	0.14226201E-03

PROBLEM SET: CNF2

MACH #	TCN20	TCN2A	TCN2A2
3.5	-0.08282144	0.27379462E-01	0.24330383E-03
4.5	-0.07680953	0.24214283E-01	0.18452371E-03
6.0	-0.06490477	0.21875000E-01	0.38690865E-04
8.0	-0.06376192	0.20946428E-01	-0.32738029E-04
10.0	-0.06909525	0.20232143E-01	-0.29761522E-05

PROBLEM SET: CMF2

MACH #	TCM20	TCM2A	TCM2A2
3.5	0.05514286	-0.10821427E-01	-0.17857179E-04
4.5	0.04723810	-0.86785713E-02	0.11904769E-03
6.0	0.04276191	-0.68392851E-02	0.86309483E-04
8.0	0.03900000	-0.56071421E-02	0.89285677E-04
10.0	0.03719047	-0.50357138E-02	0.11309532E-03

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TABLE D-8. Second Stage Lateral Coefficients

PROBLEM SET: CY2

MACH #	TCY20	TCY2B	TCY2AB	TCY2A2B
3.5	-0.00283400	-0.35310987E-01	0.67670009E-03	0.41799358E-04
4.5	-0.00476067	-0.32411996E-01	0.65709988E-03	0.32219676E-04
6.0	-0.00862133	-0.28534999E-01	0.60950016E-03	0.41402013E-05
8.0	-0.01420600	-0.25360515E-01	0.55275002E-03	-0.10169592E-04
10.0	-0.01559400	-0.23243498E-01	0.48189994E-03	0.23179970E-04

PROBLEM SET: CLN2

MACH #	TCLN20	TCLN2B	TCLN2AB	TCLN2A2B
3.5	0.00310133	0.14233000E-01	-0.41310006E-03	-0.87519926E-04
4.5	0.00433133	0.11893496E-01	-0.37830003E-03	-0.43759825E-04
6.0	0.00328200	0.90009952E-02	-0.31595002E-03	0.36501278E-05
8.0	0.00237733	0.77250013E-02	-0.28525002E-03	-0.22901133E-05
10.0	0.00358800	0.71255034E-02	-0.27645013E-03	-0.23530063E-04

PROBLEM SET: CLL2

MACH #	TCLL20	TCLL2B	TCLL2AB	TCLL2A2B
3.5	-0.00099533	-0.55765007E-02	0.17604997E-03	-0.12449930E-04
4.5	-0.00212667	-0.46779984E-02	0.11775004E-03	-0.12530111E-04
6.0	-0.00141267	-0.38770009E-02	-0.11815000E-03	0.31110012E-04
8.0	-0.00262400	-0.33380005E-02	0.46599998E-04	-0.91998515E-06
10.0	-0.00266133	-0.32379995E-02	0.14849989E-04	0.11949987E-04