

# A NEW CALIBRATION METHOD FOR TIME DELAY STANDARD AND ITS APPLICATION

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## ABSTRACT

This paper presents a new method which is used to measure time delay accurately by using a Type 900-LB Slotted Line. The accuracy for calibrating time delay of a precision coaxial air line Type 900-L is about  $\pm(0.4-0.6)$ ps, and for coaxial cables with VSWR less than 1.5 and time delay  $t$  less than 50ns is about  $\pm(3-5)$ ps.

Theoretical analysis and mathematical derivation of microwave networks in cascade are given in this paper. Methods to eliminate the errors which are caused by the discontinuities and the error analysis of the measuring system are presented.

In the last part of this paper, skin effect analysis of the transient characteristic of coaxial transmission line are discussed in detail. Methods to eliminate the errors which are resulted from using the calibrated time delay standard to calibrate time interval measurement instruments are presented. The estimation of errors and formulae for correction of those errors are described.

## 1. INTRODUCTION

Owing to the rapid growth of the requirement of measuring time delay accurately nowadays, a lot of commercial precision electronic instruments such as time delay generators with an accuracy  $\pm 1$ ns, electronic counters for time interval measurement with an accuracy of  $\pm 100$ ps, and coaxial time delay standard with

an accuracy of  $\pm 50$ ps have been developed and manufactured. Since frequency  $f$  is the reciprocal of period  $T$  and there is no difficulty to set a frequency within the accuracy of  $\pm 1 \times 10^{-9}$ , it is possible to measure the time delay of a coaxial transmission line accurately by means of frequency domain measurement method. By using the method described in this paper a coaxial cable with VSWR less than 1.5 and time delay  $t$  less than 50ns has been measured. The accuracy of measuring the time delay of that cable is about  $\pm(3-5)$ ps. Skin effect analysis of the transient characteristic of coaxial transmission line have been discussed in detail. The effect of the inductive component and resistive component of the coaxial line on pulse transient characteristic have been analyzed and estimated also. This paper indicates how to correct the error which is caused by skin effect, in this consequence measuring and transfer accuracy has been increased. When the calibrated time delay standard is used to calibrate a precision time measurement counter, the total calibration uncertainty is about  $\pm 10$ ps.

## II Principle and Method of Measurement

2.1 The principle of measurement is based on the method of measuring the displacement of the minimum of the standing wave pattern as shown in Fig.1.

When the device under test (DUT) is disconnected and connected between the Slotted Line and Short, the displacement of the minimum of the standing wave pattern is  $\Delta d$ . The time delay  $t$  of the device under test at measuring frequency is known as

$$t = \frac{n}{2} \frac{1}{f} \pm \frac{\Delta d}{c} \quad (1)$$

Where  $c$  is the velocity of the electromagnetic wave and  $n$  is any positive integer.

### 2.2 Method of Measurement

There are two methods to measure the time delay of coaxial transmission line accurately by using a Type 900-LB Slotted Line.

They are the periodic and the nonperiodic method.

### Periodic Method

When the time delay of the DUT is exactly  $n/2$  multiples of the period of the measuring frequency, the  $\Delta d$  in eq.(1) equals zero. Through properly adjusting the measuring frequency the displacement of the minimum of the standing wave pattern  $\Delta d$  should approach zero, when the DUT is disconnected and connected between the Slotted Line and Short. Then the time delay of the DUT at measuring frequency is

$$t = \frac{n}{2} \frac{1}{f}$$

This method is called "Periodic Method". It is suitable to measure the time delay of two port devices with small VSWR.

If  $S_{11}$ ,  $S_{22}$  and  $S_{21}$  are the scattering parameters of the reversible two ports DUT then the maximum measured phase and time delay errors can be represented as

$$\begin{aligned} \Delta \phi &\approx \pm \sin^{-1} \left( \frac{|S_{11}| + |S_{21}|^2 |S_{22}|}{|S_{21}|^2} \right) \quad (\text{degree}) \\ \Delta t &= \frac{\Delta \phi}{720} \cdot \frac{1}{f_{GHz}} \quad (ns) \end{aligned} \quad (2)$$

### Nonperiodic Method

If the VSWR of the DUT is rather large, the measured phase error  $\Delta \phi$  in eq.(2) will be quite large and it is difficult to adjust the measuring frequency properly to make the displacement of minimum of the standing wave pattern  $\Delta d$  approach zero, so that the measured time delay error  $\Delta t$  would be rather large. In such condition it is impossible to acquire an accurate time delay measurement by making measurement at one frequency only; the nonperiodic method which makes time delay measurement with a series of measuring frequencies is required.

### 2.3 Example of measurement

Use the nonperiodic method to measure the time delay of a piece of coaxial cable with  $N$  connectors.

In order that the coaxial cable can be inserted into the Type

900-LB Slotted Line system, the coaxial cable with Type N connectors has to be connected with the Type 900-QNJ adaptors as shown in Fig. 2.

The system of connection is as in Fig. 1. When the DUT is inserted in system at terminal 1-1, the minimum of the standing wave pattern would shift. The method, which evaluate the time delay of the DUT by means of measuring the displacements of minimum and the measuring frequencies, is called nonperiodic method, and eq. (1) is suitable for this method to calculate the time delay.

The measurement procedure is as follows.

1. According to a certain test frequency  $f_1$ , there is a corresponding displacement of minimum  $\Delta d_1$ , and the time delay (including the two Type 900-QNJ adaptors in this example) can be evaluated.
2. Repeat a series of time delay measurement by changing the test frequency in fixed frequency intervals, and obtain a set of time delay data which would be changing periodically with frequency.
3. As shown in Fig.2. the measuring terminal is changed owing to the adding of Type 900-QNJ adaptors. They are shifted from terminals  $T_1$  and  $T_2$  to terminals 1-1 and 2-2 respectively. There would be discontinuities at terminal  $T_1$ , and  $T_2$  owing to the mismatch between the coaxial cable and Type N connectors. In general the VSWR of conventional coaxial cables are about 1.3 to 1.5 and the VSWR of Type 900-QNJ adaptors are 1.1016 at 3 GHz so that it can be neglected as compared with the coaxial cable.

There are rapid periodic variations in the time delay data, owing to the discontinuity at  $T_1$ , and the error of measurement can be minimized by taking the arithmetical mean value of the maximum and minimum time delay values in the frequency range of interest. But the discontinuity at  $T_2$  would introduce unnoticed systematic error if the terminal  $T_2$  is connected directly to the Short Type 900-WNC. In our experimental system terminal  $T_2$  is connected through a Type 900-QNJ adaptor to the Short Type 900-WNC. The electric length of the Type 900-QNJ adaptor is 5cm, so that there are slow periodic variations (about 3000MHz) superimposed on the

rapid periodic variations in the time delay data. In order to increase the electric length a piece of Type 900-L30 Air Line is added between terminal 2-2 and the Type 900-WNC Short, then the frequency of slow periodic variation becomes 428.6 MHz and it is acceptable in this experiment. By using the method described above, a set of time delay data with two kinds of periodic variations would be present; take the arithmetical mean value of the maximum and minimum time delay values in the frequency range of interest as the required time delay datum. Then the systematic error owing to the discontinuities at Terminal  $T_1$  and  $T_2$  can be eliminated.

4. Why are there two kinds of periodic variation in the time delay data? There is a  $e^{-j2\beta(l+l_{20})}$  term in eq.(4) (section III), which is introduced by the reflection at the M network in Fig 3. Let  $l_1 = l + l_{20}$ , then  $e^{-j2\beta(l+l_{20})}$  changes into  $e^{-j2\beta l_1}$ . The condition for producing a periodic variation is  $2\beta l = 2n\pi$ , therefore the frequency interval for a rapid periodic variation is

$$\Delta f = \frac{c}{2l_1} \quad (3)$$

For example  $l_1 = l + (l_{20} + l_{30}) = l + l'_{20} = 267.5 \text{ cm}$

$l$  --electric length of the coaxial cable with Type N connectors

$l_{20}$  --electric length of Type 900-QNJ Adaptor

$l_{30}$  --electric length of Type 900-L30 Air Line

$l'_{20}$  --electric length of Type 900-QNJ Adaptor and 900-L30 Air Line.

put the value of  $l_1$  into eq. (3) and obtain  $\Delta f_1 = 56 \text{ MHz}$

In the same way the slow periodic variation which is caused by the reflections at N network in Fig 3. can be analyzed.

If the Type 900-L30 Air Line is connected in the measuring system the frequency interval for it would become

$$\Delta f_2 = 428.6 \text{ MHz}$$

The calculated values of  $\Delta f_1$  and  $\Delta f_2$  are well matched with the experimental curve shown in Fig. 6.

5. Evaluate the time delay of the coaxial cable under test.

In this paper a Type N connector coaxial cable with a Type 900-QNJ

adaptor at each end is used as a device under test (DUT). Take the arithmetical mean value of the maximum and minimum time delay value to obtain the accurate time delay value of the DUT.

$$t = \frac{1}{2} (t_{\max} + t_{\min}) = 8209.6 \text{ ps.}$$

The time delay of the air line has been discounted from this value.

### III Theoretical Analysis of Microwave Networks in Cascade

Let  $l$  represent the electric length of the coaxial cable,  $l_{10}$  and  $l_{20}$  the electric length of Type 900-QNJ adaptors,  $l'_{20}$  is the electric length of Type 900-QNJ adaptor and Type 900-L30 Air Line as shown in Fig.3. Terminal 1-1 is connected to the output terminal of the 900-LB Slotted Line and Terminal 2-2 (or 2'-2') is connected to the Type 900-WNC Short.

Use  $m_{11}$ ,  $m_{21}$ ,  $m_{12}$  and  $m_{22}$  to represent the scattering matrix elements of the connector pair M and  $n_{11}$ ,  $n_{21}$ ,  $n_{12}$  and  $n_{22}$  to represent the scattering matrix elements of the connector pair N. Use  $S_{11}$ ,  $S_{21}$ ,  $S_{12}$  and  $S_{22}$  to represent the scattering matrix elements of the network between the terminals  $T_1$  and  $T_2$  and  $S'_{11}$ ,  $S'_{21}$ ,  $S'_{12}$  and  $S'_{22}$  to represent the scattering matrix elements of the network between the terminals 1-1 and 2-2 (or 2'-2').

When terminal 2-2 (or 2'-2'), is connected to the Type 900-WNC Short, the reflection coefficient  $\Gamma'_1$  at terminal 1-1 can be represented as (see appendix I)

$$\Gamma'_1 \simeq -m_{12}n_{12}m_{21}n_{21}e^{-j2\beta(l+l_{10}+l_{20})}(1+2\delta)\left[1 - \frac{m_{11}e^{j2\beta(l+l_{20})}}{m_{21}m_{21}n_{12}n_{21}}(1-2\delta) - \frac{n_{11}e^{j2\beta l_{20}}}{n_{12}n_{21}}(1-\delta) - n_{22}e^{-j2\beta l_{20}} - n_{12}n_{21}m_{22}e^{-j2\beta(l+l_{20})}(1+\delta)\right] \quad (4)$$

The  $m_{12}n_{12}m_{21}n_{21}e^{-j2\beta(l+l_{10}+l_{20})}$  term in eq. (4) represents the required quantity for the time delay measurement of the DUT, and the other four terms in the bracket represent the rapid and slow periodic variations of the time delay measurement data. If there is no  $l_{20}$ ,  $n_{11}$  and  $n_{22}$  would introduce an unnoticed systematic error. If  $l_{20}$  is very short, the frequency of the slow periodic variation would be very high (as described in Section II it is 3000MHZ). After adding

a 30 Cm precision air line the frequency decreases to 428.6MHZ. In the point of view of our experiment it is acceptable and easy to be realized.

The phase angle  $\beta(l+l_{10}+l'_{20})$  in eq.(4) corresponds to the time delay between terminals 1-1 and 2'-2', from which the time delay of the DUT can be evaluated by subtracting 1.0012 ns, the time delay of the precision air line Type 900-L30.. If the time delay of the two 900-QNJ adaptors are subtracted, the time delay of the coaxial cable with Type N connector can be obtained.

It doesn't introduce systematic error in time delay measurement when the DUT has loss, but it increases the random error as the sharpness of the minimum of the standing wave pattern decreases.

Proceed the error analysis according to eq.(4) as follows

1. Let the VSWR of the DUT be 1.5, then  $|S_{11}| = |S_{22}| = 0.2$ ,  $|m_{22}| = |n_{11}| = 0.1$  so that  $|\delta| = 0.01$ . Under the worst phase combination it corresponds to a phase angle of  $1.15^\circ$ . When the measuring frequency is 4 GHz, it just introduces an error of 0.4 ps.

2. The effect of  $m_{12}$   $n_{12}$   $m_{21}$   $n_{21}$ .

Evaluate  $m_{12}$   $m_{21}$  as shown in Fig. 4.

Evaluate  $n_{12}$   $n_{21}$  by the same way.

If the transmission line has an impedance discontinuity at a certain interface, we use network M to represent this discontinuity and let  $m_{11}$ ,  $m_{21}$ ,  $m_{12}$  and  $m_{22}$  represent the elements of the M matrix. For convenience use normalized impedance in  $Z_1$  and  $Z_2$ . Let  $Z_1 = 1$ ,  $Z_2 = 1+x+jy$  There is the following relation

$$m_{12}m_{21} = \frac{1}{1 + \frac{(x+jy)^2}{4(1+x+jy)}} = 1 + \delta' \quad \text{where } \delta' = \frac{-(x+jy)^2}{4(1+x+jy)}$$

since  $|S_{11}| = |S_{22}| = 0.2$   $x = 0.1$ ,  $y = 0.1$  so that  $\delta' \leq 0.01$  In the same way  $n_{12}$   $n_{21} = 1 + \delta'$  then  $m_{12}$   $m_{21}$   $n_{12}$   $n_{21} \approx 1 + 2\delta'$  (5)

From eq.(5) it introduces a 0.4 ps time delay uncertainty, but in precision time delay measurement, this uncertainty can be eliminated by interchanging the input and output of the DUT in two measurements and taking the mean value.

3. Analysis about the effects of the other four terms in the bracket

Consider the  $\frac{m_{11} e^{j2\beta(l+l_{20})}}{m_{12} m_{21} n_{12} n_{21}} (1-2\delta)$  term and  $n_{12} n_{21} m_{22} e^{-j2\beta(l+l_{20})} (1+\delta)$

term first. Neglect  $\delta$  as a small quantity, and assume the M matrix is lossless, then the extrema of the expression

$$1 - \frac{m_{11} e^{j2\beta(l+l_{20})}}{m_{12} m_{21} n_{12} n_{21}} - n_{12} n_{21} m_{22} e^{-j2\beta(l+l_{20})} \quad (6)$$

are  $1+g+jh$  and  $1-g-jh$ , they correspond to the phase angles

$$\theta_1 = \tan^{-1} \frac{h}{1+g}, \quad \theta_2 = \tan^{-1} \frac{-h}{1-g} \quad (7)$$

Then the expression for the phase error  $\Delta\theta$ , while taking the mean value of the extrema as the time delay, is

$$\Delta\theta \approx \frac{1}{2} \tan^{-1} (-2gh) \quad (8)$$

In the same way, the effect of the two terms  $\frac{n_{11} e^{j2\beta l_{20}}}{n_{12} n_{21}} (1-\delta)$

and  $n_{22} e^{-j2\beta l_{20}}$  can be considered.

Since  $|S_{11}| = |S_{22}| = 0.2$ ,  $g \leq 0.14$   $h \leq 0.14$

then  $\Delta\theta/2 = 35'$  it corresponds to a 0.4ps time delay uncertainty.

When taking the mean value of the extrema as the time delay of the DUT, the four terms in the bracket would introduce an uncertainty of  $2 \times 0.4ps$ .

#### IV Measurement Errors

There are eight kinds of errors source for the time delay measurements of two ports DUT in the Type 900 system.

1. The frequency instability of the signal generator introduce the error  $\Delta t_1$

$$\Delta t_1 = \frac{\Delta f}{f} \cdot t$$

in example above  $\Delta t_1 = \pm 0.4ps$ .

2. The inaccuracy of the slotted line scale introduces the error  $\Delta t_2$ . The inaccuracy of the slotted line scale is 0.1mm, (with the micrometer it can be decreased to 0.01 mm).

so that  $\Delta t_2 = \pm 0.33 ps$ .



3. The Type 900-L30 precision Air Line introduces time delay error  $\Delta t_3$ . In the measuring system, a Type 900-L30 Precision Air Line is added to increase the electric length of  $\lambda_{20}$ , and its time delay is  $1001.2 \pm 0.4$  ps, so that it introduces the error  $\Delta t_3$ .

$$\Delta t_3 = \pm 0.4 \text{ ps.}$$

4. The M and N networks of the connector pairs would introduce the error  $\Delta t_4$ .

This error comes from the  $\delta$  term in eq. (4) and it has been described in part III

$$\Delta t_4 = \pm 0.4 \text{ ps.}$$

5. The test frequency interval can't be infinitesimal, it would introduce the error  $\Delta t_5$ .

Since a series of measured time delay data of the required measuring frequencies are required to obtain the extrema, the smaller the measuring frequency interval, the more accurate extrema can be obtained. In the example described in Part II, the measuring frequency interval is 10MHZ, and a frequency interval of two to three MHZ can be used in the neighborhood of the extrema. According to the data analysis, the error  $\Delta t_5$  is less than 1ps.

6. The reflection of the Type 900-BT connector would introduce the error  $\Delta t_6$ .

Type 900-BT connectors are used in both the terminal 1-1 and 2-2 of the measuring system, and the time delay error introduced by them is less than 0.8 ps.

7. While taking the mean value of the extrema to evaluate the time delay it would introduce the error  $\Delta t_7$  and  $\Delta t_8$ .

As described in part III,  $\Delta t_7 = \pm 0.4$  ps.  $\Delta t_8 = \pm 0.4$ ps.

From  $\Delta t_1$  to  $\Delta t_8$  the total r.m.s. error is about 1.6 ps.

#### V Skin Effect Analysis of the Transient Characteristic of Coaxial Transmission Line.

The time delay of a coaxial transmission line changes with frequency. Since the series inductance changes with frequency owing to skin effect. For example if the time delay of a piece of coaxial cable

is 4996.64 ps at 1000MHZ it would change to 5000 ps at 3000MHZ. So that, when it is operated in a pulse condition, the risetime of the output pulse would be degenerated owing to the frequency response of time delay. If an input pulse with 1ns risetime is passing through a coaxial cable with 50ns time delay the rise time of the output pulse would change to 1.14ns. On the other hand the changing of series resistance owing to skin effect degenerates the risetime of the output pulse too. The magnitude of degeneration is a function of the attenuation of the coaxial cable and the risetime of the input pulse. If an input pulse with 1ns risetime is passing through a coaxial cable with 0.4 dB attenuation, the risetime of the output pulse would change to 1.03ns.

5.1 The relation between the frequency response of the series inductance and the pulse time delay and transient characteristic. As described above, the series inductive component increases as the frequency decreases so that the wave propagation speed through the cable decreases and the time delay increases. For this reason the calibrated time delay standard should be used at the calibrated frequency. If it is used on the other frequencies, correction should be taken. According to the relation between the electromagnetic wave propagation velocity and the inductive component, the formula of time delay as a function of frequency can be derived. (see appendix II)

$$t = t_0 \left( 1 + \frac{b}{\sqrt{f_{GHz}}} \% \right) \text{ ns.} \quad (9)$$

As the existence of skin effect, the phase frequency characteristic is no longer linear. When working in a pulse condition, since the time delays are different for each of the harmonic components of the pulse, the resultant output waveform would change and the risetime degenerates as shown on Fig. 5.

This means, the frequency response of the inductive component would also degenerate the pulse output waveform besides the frequency response of the resistive component but, when using the calibrated time delay standard to calibrate time measurement in-

struments the effect of the inductive component is very small and can be neglected, provided the calibrating frequency of the time delay standard and the bias voltage of the calibrated equipment are properly selected.

$$\text{Let } U_1(t) = \frac{E}{2} + \frac{2E}{\pi} \sum_{\nu=0}^{\nu=\infty} \frac{\sin(2\nu+1)\Omega t}{2\nu+1} \quad (10)$$

where  $\Omega = \frac{2\pi}{T}$  — fundamental angular frequency

E -- pulse (or square wave) amplitude

If the coaxial transmission line has ideal linear phase frequency characteristic and the module of the transmission coefficient is constant throughout the passband and equal to zero outside the passband, then the output can be represented as

$$U_2(t) = \frac{KE}{2} + \frac{2KE}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t-t_c)}{2\nu+1} \quad (11)$$

where K -- the module of transmission coefficient

$t_c$  -- delay time

Since the phase frequency characteristic is nonlinear owing to skin effect, the delay time is no longer constant and should be expressed as

$$t_x = t_0 \left( 1 + \sqrt{\frac{b}{f_x \epsilon_h}} \right) \quad (12)$$

where  $t_0$  -- delay time of zero depth of penetration

$t_x$  -- delay time at frequency  $f_x = x \cdot f$ ,

f -- the highest harmonic frequency of the input pulse

$f \approx 0.35/d_0$

b -- correction coefficient

under this condition eq. (11) should change to

$$U_2(t) = \frac{KE}{2} + \frac{2KE}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t-t_x)}{2\nu+1} \quad (13)$$

When the bias voltage equals to  $KE/2$ , there is a delay time,  $t=t_A$  which satisfies the following formula (14)

$$\sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t_A-t_x)}{2\nu+1} = 0 \quad (14)$$

$$t_A - t_x = t_0 b \left( \frac{1}{\sqrt{A}} - \frac{1}{\sqrt{X}} \right) \frac{1}{\sqrt{f_0 \text{ GHz}}} \% \quad (15)$$

$$f_A = A \cdot f_0 \quad (16)$$

Substituting formulae (15) and (16) into formula (14) and taking integrating we get

$$A \approx 0.3 \quad (17)$$

so that 
$$t_A = t_0 \left( 1 + \frac{b}{\sqrt{0.3 f_0 \text{ GHz}}} \% \right) \quad (18)$$

According to the preceding analysis, when  $KE/2$  is used as the bias voltage,  $0.3 f_0$  should be taken as the calibrating frequency for the time delay standard. For example, if the input pulse  $E=0.5V$  and the pulse risetime  $\sigma_0=0.1ns$ , then  $f_0=3.5GHZ$ . When use  $0.25V$  as the bias voltage, the time delay data should be taken at the measuring frequency  $f=1.05GHZ$  as the standard time delay value for this specified condition.

Besides, from eq. (12) and Fig. 5. it is evident that the time of the lower part of the leading edge of the delayed pulse corresponds to delay time of its higher harmonics. So that, when the risetime and the amplitude of the input pulse are rather large  $KE/10$  (or smaller) may be used as the bias voltage, and  $f_0$  is taken as the calibrating frequency for the time delay standard. When using the calibrated time delay standard to calibrate time measurement instruments the error which is caused by the frequency response of the inductive component, can be eliminated by using the method described above.

## 5.2 The distortion of pulse transient characteristic owing to the skin effect of resistive component.

When a pulse passes through a coaxial cable, the output waveform would be degenerated owing to the changing of the high frequency loss of the coaxial cable. But in general the dielectric loss is much smaller than the resistive loss.

1. The expression of resistive component in coaxial cable may be expressed as

$$R = \frac{\rho_b}{2\pi\delta_b b_o} + \frac{\rho_a}{2\pi\delta_a a_o} \quad (19)$$

where  $\delta_b$ ,  $\delta_a$  are the skin depths of the inner and outer conductors  $\rho_b$ ,  $\rho_a$  are the resistivities of the materials of the inner and outer conductors.

$b_o$ ,  $a_o$  are the radius of the inner and outer conductors

It shows that the changing of the resistive component owing to skin effect of the coaxial cable increases with frequency. It doesn't introduce time delay measurement error when the standard is calibrated at a single frequency but it would degenerate the output pulse. Since the higher harmonics have larger attenuation so the risetime of the output pulse would increase. If the bias voltage used is not zero, time delay measurement error is introduced. The larger the bias voltage used, the larger the time delay measurement error that would be introduced, but by using the correction formula (22) given below, this error term can be reduced by a factor of 10.

2. Correction formula for reducing the time delay measurement error owing to the resistive component

Considering the resistive loss of the coaxial cable, the increment of the risetime can be evaluated as (see appendix IV)

$$\Delta\sigma' = \delta_o a_s$$

where 
$$a_s = \frac{2}{3}\alpha_{f_o} - \frac{1}{4}(\alpha_{f_o})^2 + \frac{1}{15}(\alpha_{f_o})^3 - \dots \quad (20)$$

$\alpha_{f_o}$  -- the attenuation nB at the highest frequency  $f_o = 0.35/\delta_o$

$\delta_o$  -- the risetime of the input pulse

As described above if 0.1KE or 0.5KE is used as bias voltage the correction value should be

$$\begin{aligned} \Delta\sigma_1 &= 0.1\delta_o a_s && \text{for 0.1KE bias voltage} \\ \Delta\sigma_2 &= 0.5\delta_o a_s && \text{for 0.5KE bias voltage} \end{aligned} \quad (21)$$

So that in a pulse condition the effective standard delay time of the calibrated time delay standard should be

$$t_s = t_h + 0.1 \delta_0 a_s \approx t_h + \frac{\alpha_{fs}}{15} \delta_0$$

$$t'_s = t'_h + 0.5 \delta_0 a_s \approx t'_h + \frac{\alpha_{fs}}{3} \delta_0 \quad (22)$$

where  $t_s$ --effective standard delay time of the calibrated time delay standard, when the input pulse risetime is  $\delta_0$  and the bias voltage is 0.1KE.

$t'_s$ -- the same as  $t_s$ , except 0.5KE is used as bias voltage

$t_h$ --delay time of the calibrated time delay standard at frequency  $f_0$  .

$t'_h$ -- the same as  $t_h$ , except it is at frequency  $0.3 f_0$  .

$\delta_0$ -- the risetime of the input pulse

$\alpha_{fs}$ -- the attenuation in neper of the time delay standard at frequency  $f_0$  .

## VI Conclusion

1. This paper presents a new method to calibrate the time delay standard (coaxial cable). The accuracy of calibration is  $\pm(3-5)$ ps.
2. When we use the calibrated time delay standard to calibrate precision time measurement instruments, we must use formula (22) to evaluate effective standard delay time for correcting the errors owing to skin effect.
3. When we use the calibrated time delay standard to calibrate precision time measurement instruments, two additional errors should be considered as follows.
  - a. The evaluation of the highest limiting frequency may have a tolerance of 10%, and it introduces an error  $\Delta t'_1$  .

$$\Delta t'_1 = 0.054 \frac{t_0 b}{\sqrt{f_0 \epsilon_{th}}}$$

when  $f_0 = 0.35$ GHZ,  $b = 0.0816$ ,  $t_0 = 8200$ ps,  $\Delta t'_1 = 0.61$ ps

when  $f_0 = 0.35$ GHZ  $b = 0.0816$ ,  $t_0 = 50.000$ ps  $\Delta t'_1 = 3.72$ ps

It is evident the higher the  $f_0$ , the smaller the  $\Delta t'_1$  .

b. 10% tolerance of the bias voltage used may introduce an error  $\Delta t_2'$  .

$$\Delta t_2' = 0.0067 \alpha_f \delta_0 \quad \text{for } 0.1\text{KE bias voltage}$$

$$\Delta t_2' = 0.033 \alpha_f \delta_0 \quad \text{for } 0.5\text{KE bias voltage}$$

Since  $\alpha_f$  is proportion to  $\sqrt{f_0}$  , and  $\delta_0 \approx 0.35/f_0$  , then  $\Delta t_2'$  decreases as  $\delta_0$  decreases in the rate of  $\sqrt{f_0}$  .

In general, for an input pulse with  $E \geq 2\text{V}$ ,  $\delta_0 \leq 1\text{ns}$  or  $E \geq 0.5\text{V}$ ,  $\delta_0 \leq 0.1\text{ns}$ , using the calibrated time delay standard to calibrate time measurement instruments, the additional error owing to skin effect is less than  $\pm 5\text{ps}$ .

So that, when the calibrated time delay standard is used to calibrate a precision time measurement instrument (counter), the total calibration uncertainty is less than  $\pm 10\text{ps}$ .

## Appendix I

Formula derivation for the reflection coefficient at the input terminal of the microwave networks in cascade.

As shown in Fig. 3. the microwave networks has the following relation

$$\begin{aligned} b_1 &= m_{11} a_1 + m_{12} A_2 \\ B_2 &= m_{21} a_1 + m_{22} A_2 \end{aligned} \quad (1)$$

and

$$\begin{aligned} A_1 &= B_2 e^{-j\beta l} \\ A_2 &= B_1 e^{-j\beta l} \end{aligned} \quad (2)$$

$$\begin{aligned} B_1 &= n_{11} A_1 + n_{12} a_2 \\ b_2 &= n_{21} A_1 + n_{22} a_2 \end{aligned} \quad (3)$$

where  $\beta$  --phase constant, thus

$$\begin{aligned} S_{11} &= m_{11} + \frac{m_{12} m_{21} n_{11} e^{-j2\beta l}}{1 - m_{22} n_{11} e^{-j2\beta l}} \\ S_{22} &= n_{22} + \frac{n_{12} n_{21} m_{22} e^{j2\beta l}}{1 - m_{22} n_{11} e^{-j2\beta l}} \\ S_{21} &= \frac{m_{21} n_{21} e^{-j\beta l}}{1 - m_{22} n_{11} e^{-j2\beta l}} \\ S_{12} &= \frac{n_{12} m_{12} e^{-j\beta l}}{1 - m_{22} n_{11} e^{-j2\beta l}} \end{aligned} \quad (4)$$

Since the Type 900-QNJ Adaptors have electric length the S' matrix of the network between terminal 1-1 and 2-2

becomes

$$\begin{aligned} S'_{11} &= S_{11} e^{-j2\beta l_{10}} \\ S'_{22} &= S_{22} e^{-j2\beta l_{20}} \\ S'_{12} &= S_{12} e^{-j\beta(l_{10} + l_{20})} \\ S'_{21} &= S_{21} e^{-j\beta(l_{10} + l_{20})} \end{aligned} \quad (5)$$

Let  $\delta = m_{22} n_{11} e^{-j2\beta l}$ , eq. (5) can be rewritten as

$$\begin{aligned} S'_{11} &\approx m_{11} e^{-j2\beta l_{10}} + m_{12} m_{21} n_{11} e^{-j2\beta(l + l_{10})} (1 + \delta) \\ S'_{22} &\approx n_{22} e^{-j2\beta l_{20}} - n_{12} n_{21} m_{22} e^{-j2\beta(l + l_{20})} (1 + \delta) \\ S'_{12} &\approx n_{12} m_{12} e^{-j\beta(l + l_{10} + l_{20})} (1 + \delta) \\ S'_{21} &\approx m_{21} n_{21} e^{-j\beta(l + l_{10} + l_{20})} (1 + \delta) \end{aligned} \quad (6)$$



According to theory of the network analysis, the expression for the reflection coefficient at the input terminal of the S' network is

$$\Gamma' = S'_{11} + \frac{S'_{12} S'_{21} \Gamma_L}{1 - S'_{22} \Gamma_L}$$

where  $\Gamma_L$  is the reflection coefficient of the termination of the network. When this S' network is terminated a Short,  $\Gamma_L = -1$

$$\Gamma' = S'_{11} - \frac{S'_{12} S'_{21}}{1 + S'_{22}} \quad (7)$$

Substituting eq. (6) into eq. (7) and neglecting  $\delta^2$  and higher order terms of the small quantities the expression for the reflection coefficient at terminal 1-1 becomes

$$\begin{aligned} \Gamma'_1 = & -m_{12} n_{12} m_{21} n_{21} e^{-j2\beta(l+l_{20})} (1+2\delta) \left[ 1 - \frac{m_{11} e^{j2\beta(l+l_{20})}}{m_{12} m_{21} n_{12} n_{21}} (1-2\delta) - \frac{n_{11} e^{j2\beta l_{20}}}{n_{12} n_{21}} (1-\delta) - \right. \\ & \left. - n_{12} n_{21} m_{22} e^{-j2\beta(l+l_{20})} (1+\delta) - n_{22} e^{-j2\beta l_{20}} \right] \quad (8) \end{aligned}$$

This is the eq. (4) in this paper.

## Appendix II

Formula derivation for the relation between delay time and measuring frequency.

In lossless coaxial transmission line the propagation velocity of the electromagnetic wave is

$$V = \frac{1}{\sqrt{LC}} \quad (9)$$

where L --inductance per unit length

C --capacitance per unit length

The delay time for a piece of transmission line is inversely proportional to the propagation velocity

$$t \propto \frac{1}{V} \quad (10)$$

$$\text{Thus } t \propto \sqrt{L} \quad (11)$$

the expression of inductance for zero skin depth is

$$L_0 = 2 \ln \frac{D_0}{d_0} \times 10^{-9} \text{ H/cm} \quad (12)$$

where  $D_0, d_0$  --the inner diameter of the outer conductor and the outer diameter of the inner conductor respectively.

In those frequencies, where skin effect can not be neglected, the effective diameters of the inner and outer conductors of the coaxial transmission line are no longer their mechanical dimensions  $d_0$  and  $D_0$ . In this case, the effective diameter of the outer conductor increases with the amount of skin depth, on the contrary, the effective diameter of the inner conductor decreases with the amount of the skin depth. So that, the value of inductance is no longer expressed as eq. (12) it should change to

$$L = 2 \ln \frac{D}{d} \times 10^{-9} \text{ H/cm} \quad (13)$$

where  $D, d$  --the effective diameters of the outer and inner conductors considering the skin effect.

From eq. (11), (12) and (13) we get

$$\frac{\Delta t}{t_0} = \frac{\sqrt{L} - \sqrt{L_0}}{\sqrt{L_0}} \% \quad (14)$$

From eq. (14) we can express the time delay as the function of measuring frequency

$$t = t_0 \left( 1 + \frac{b}{\sqrt{f_{GHz}}} \% \right) \quad (15)$$

where b -- correction coefficient for the inductive component variation owing to skin effect.

Coefficient b is the function of the diameter of the coaxial transmission line, and the resistivity of the conductors. Here are some examples for the Type 900-LZ Reference Air Line b is 0.0284, and for the Type SYV-50-5 and SYV-50-2-2 coaxial cables made in China b is 0.0816 and 0.1711 respectively.

### Appendix III

The relation between the risetime of the output pulse and the bandwidth.

Let the expression for input pulse (square wave)

$$\text{as } U_1(t) = \frac{E}{2} + \frac{2E}{\pi} \sum_{\nu=0}^{\nu=\infty} \frac{\sin(2\nu+1)\Omega t}{2\nu+1} \quad (16)$$

where  $\Omega = \frac{2\pi}{T}$  — fundamental angular frequency

E -- amplitude of the input pulse (square wave)

When the input pulse has passed through an ideal network which has linear phase frequency characteristic and constant transmission coefficient inside the pass band and zero transmission coefficient outside the pass band, the output pulse becomes

$$U_2(t) = \frac{KE}{2} + \frac{2KE}{\pi} \sum_{\nu=0}^{\nu=\mu_m} \frac{\sin(2\nu+1)\Omega(t-t_c)}{2\nu+1} \quad (17)$$

where K -- transmission coefficient

$t_c$  -- delay time

$\mu_m$  -- the maximum value which depends on the highest order  $(2\mu_m+1)$  of harmonics which can pass through the network.

$$\text{It is evident that } (2\mu_m+1)\Omega \leq \omega_0 \quad (18)$$

where  $\omega_0$  -- the highest angular frequency of the passband.

Differentiate with respect to eq. (17)

$$\frac{dU_2(t)}{dt} = \frac{2KE\Omega}{\pi} \sum_{\nu=0}^{\nu=\mu_m} \cos((2\nu+1)\Omega(t-t_c)) \quad (19)$$

$$\text{when } t = t_c, \quad \left[ \frac{dU_2(t)}{dt} \right]_{\max} = 2KEf_0 \quad (20)$$

where  $f_0$  -- the highest frequency of the passband.

So that the risetime of the output pulse can be expressed

$$\text{as } \delta_0 = \frac{KE}{\left[ \frac{dU_2(t)}{dt} \right]_{\max}} = \frac{0.5}{f_0} \quad (21)$$

In practice, when  $\omega > \omega_0$ , the transmission coefficient is not zero, but it decreases rapidly as the frequency increases. So that in general the following formula is used instead of (21)

$$\delta_0 \approx \frac{0.35}{f_0} \quad (22)$$

## Appendix IV

Formula derivation for the output pulse risetime increment owing to the variation of coaxial transmission line loss.

Assume that the coaxial transmission line has linear phase frequency characteristic, When an input pulse expressed as eq. (16) passes through a piece of coaxial transmission line with loss, the amplitude frequency characteristic of the output pulse can be expressed as

$$U_2(t) = \frac{E}{2} + \frac{2E}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \frac{\sin(2\nu+1)\Omega(t-t_c)}{2\nu+1} e^{-\alpha} \quad (23)$$

where  $\alpha$  --attenuation of coaxial transmission line in neper

$$\alpha = \frac{\alpha'}{8.68} l \quad (\text{Neper}) \quad (24)$$

For conventional coaxial cable

$$\alpha' = 2.98 \times 10^{-9} \sqrt{f} \frac{1}{a_o} \left( \frac{a_o}{b_o} + 1 \right) \frac{\sqrt{\epsilon_r}}{\ln \frac{a_o}{b_o}} \quad (\text{dB/cm}) \quad (25)$$

Where  $b_o$ ,  $a_o$  --the radius of the inner and outer conductors respectively

$\epsilon_r$  --relative dielectric constant of the dielectric medium

$f$  -- frequency

According to eq. (25), can rewrite eq. (24) to

$$\alpha = K \sqrt{f} \quad (26)$$

Differentiate with respect to eq. (23)

$$\frac{dU_2(t)}{dt} = \frac{2E\Omega}{\pi} \sum_{\nu=0}^{\nu=\nu_m} \cos(2\nu+1)\Omega(t-t_c) e^{-\alpha(f)} \quad (27)$$

since  $\alpha_{f_x} = \sqrt{x} \alpha_{f_o}$ , make transformation  $x = \nu/\nu_m$  and integrate when  $t=t_c$  (assumed  $t_c$  is constant), there is

$$\left[ \frac{dU_2(t)}{dt} \right]_{\max} = \frac{E\omega_o}{\pi} \left( 1 - \frac{2}{3} \alpha_{f_o} + \frac{1}{4} (\alpha_{f_o})^2 - \frac{1}{15} (\alpha_{f_o})^3 + \dots \right) = 2E f_o a \quad (28)$$

$$\begin{aligned}
 a &= 1 - \frac{2}{3}\alpha_f + \frac{1}{4}(\alpha_f)^2 - \frac{1}{15}(\alpha_f)^3 + \dots \\
 a_s &= 1 - a = \frac{2}{3}\alpha_f - \frac{1}{4}(\alpha_f)^2 + \frac{1}{15}(\alpha_f)^3 - \dots
 \end{aligned}
 \tag{28}$$

where  $\alpha_f$  --attenuation in neper at the highest frequency  $f$ ,  
 $f$ , -- the highest harmonic frequency of the input pulse

take  $f_s = 0.35/\delta_s$

$\delta_s$  --risetime of the input pulse

thus, the increment of the output pulse risetime can be obtained

$$\Delta\delta' = \delta_s a_s \tag{29}$$

#### Reference

1. "Type 900 Reference Coaxial Air Line", General Radio Company, U.S.A.
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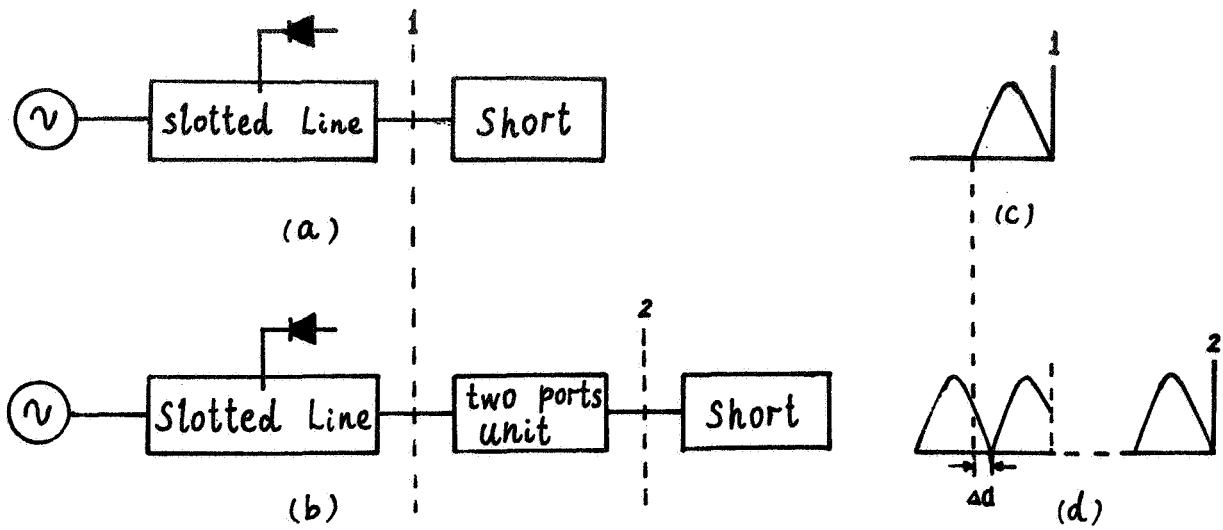


Fig.1 principle diagram for time delay measurement with microwave system

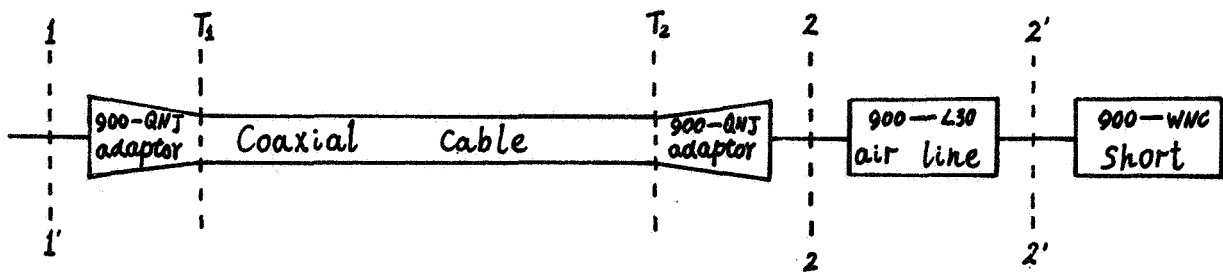


Fig.2 Using the nonperiodic method to measure coaxial cables with Type N connectors

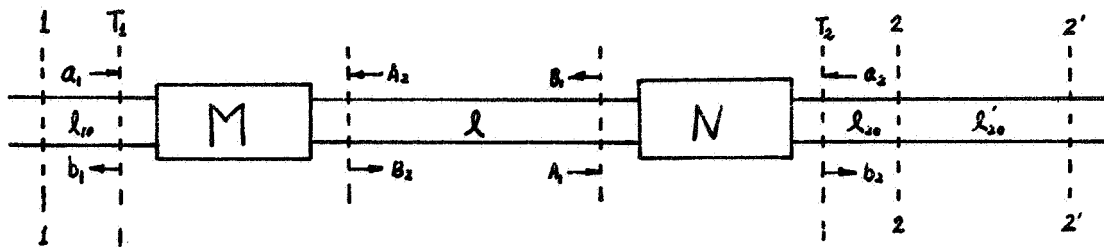


Fig.3 Theoretical analysis of microwave networks  
in cascade



Fig. 4

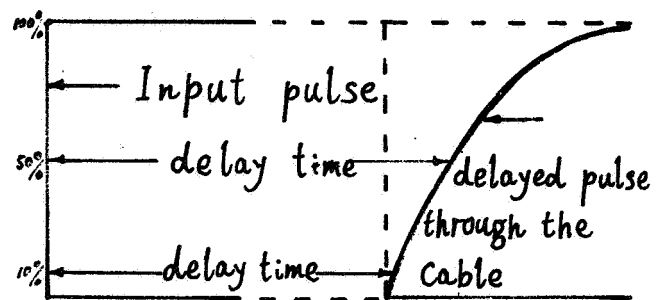


Fig. 5

Fig.4 Analysis at the discontinuity of network M

Fig.5 Step response distortion owing to non-linear phase  
frequency characteristic of skin effect



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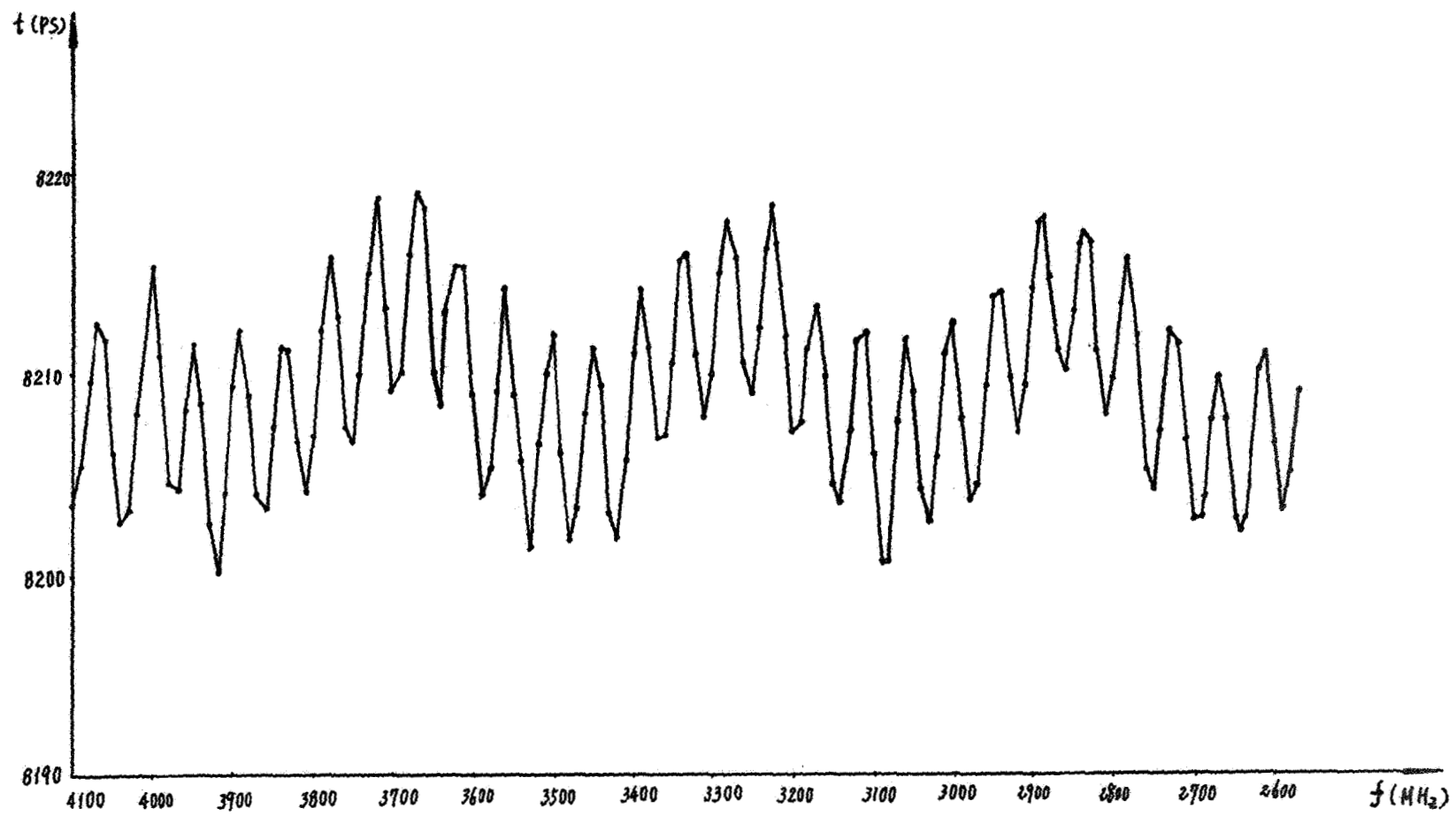


Fig.6 Delay Time Versus frequency

QUESTIONS AND ANSWERS

None for Paper #8