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FINITE ELEMENT ANALYSIS
OF
WRINKLING MEMBRANES

by

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INTRODUCTION

The Department of Civil Engineering of the University of Southern California received a research grant from the Langley Research Center of the National Aeronautics and Space Administration to investigate the finite element analysis of wrinkling membranes. The determination of stresses and deformations within large partly-wrinkled membrane surfaces is a problem of significant technical interest in such areas as conceptual design and analysis of ultralightweight spacecraft structures.

When the research discussed herein was begun a three-phase project requiring continued funding for approximately three years was envisioned. However, shortly after funding for the first phase of the project was approved, plans for funding the remaining two phases of the project were cancelled by NASA due to a shortage of funds. Thus, what is reported herein is the result of one year of effort toward the development and implementation of an algorithm for the finite element analysis of quite general membrane surfaces. As such, the effort was divided into two major tasks, which are discussed below.

AXISYMMETRIC DEFORMATION OF A SHALLOW INFLATED MEMBRANE

A closed-form solution to an axisymmetric problem involving partial wrinkling of an inflated shallow membrane was obtained. In particular, a membrane in the shape of a sperical annulus was considered. The outer edge of the annulus was assumed to be fixed so that no displacements occur along the outer perimeter. The inner edge is assumed to be clamped to a rigid movable plug. Solutions for the complete stress, strain, and displacement fields under the assumption of inextensional material behavior are presented for the case of pure torsional loads applied to the plug, and for the case of pure axial loads applied to the plug.

The analytical solution to this three-dimensional problem was intended to serve as a "benchmark" against which comparisons could be made with future numerical solutions obtained from the finite element procedure.

IMPLEMENTATION OF A FINITE ELEMENT ALGORITHM FOR FLAT MEMBRANES

A recently-developed algorithm [1] for finite element constitutive modeling of wrinkling flat membranes based on the Stein-Hedgepeth [2] theory of wrinkling was implemented on the SAP 7 [3] general purpose finite element code at USC. The algorithm takes the form of a local strain-dependent relationship between stresses and strains within an element, and currently resides in the element library of nonlinear material behaviors accessible by all users. The algorithm has been made compatible with the full library of SAP 7 element types for two-dimensional stress analysis, including isoparametric quadrilateral and higher order elements. Furthermore, the algorithm has also been made compatible with the SAP 7 nonlinear analysis capabilities for problems involving finite deformations and large displacements.

NUMERICAL EXAMPLE OF PURE BENDING OF A RECTANGULAR SHEET

As a first numerical example for evaluation of the finite element code just described, the problem of pure bending of a stretched flat sheet was considered. A closed-form analytical solution for this problem is available [2] for comparison, and a finite element numerical model consisting of fifty isoparametric

8-node quadrilateral elements and 181 nodes was created. The edge conditions on the finite element model were chosen to approximate as closely as possible those of the stress-resultant boundary conditions in the analytical formulation, within the constraints that rigid body motion of the finite element model is unacceptable.

Comparisons between the nondimensional numerical and analytical results for the stress field within the membrane were made. A strip consisting of five elements near the center of the sheet and spanning the complete range of structural behavior from wrinkled to taut was chosen for comparison. It was found that the numerical solutions for stresses agree very closely with the analytical solutions with errors typically less than two or three percent. Comparisons were made for three different load cases, and the numerically obtained values for principal stresses in each case were interpolated to obtain an estimate of the location of the boundary of the wrinkled region within the sheet. These numerical estimates for boundary location were compared with analytical results and again found to be very accurate, with errors less than about one percent. Finally, numerical results for overall moment-curvature behavior were compared with analytical results, and were also found to be in very close agreement (less than about one percent error) over a very large range of load magnitudes.

NUMERICAL EXAMPLE OF PURE TORSION OF RIGID HUB IN AN INFINITE SHEET

A second numerical example consisted of the problem of pure torsion applied to a rigid circular hub bonded to a stretched infinite flat sheet. In order to facilitate the finite element model, an infinite sheet was considered, and the sheet was assumed to be pretensioned after the attachment of the hub. The problem was then considered as a composite of two component problems. The first component problem, which was modeled numerically with the finite element code, consisted of an annular region near the hub. The hub was considered to be fixed, and the outer edge of the annulus was subjected to prescribed uniform tensile and shear stresses. The radius of the outer edge was chosen to be large enough so that all wrinkling behavior occured within the annulus.

The second problem consisted of the linear elasticity problem for the exterior region. This problem was solved analytically. The behavior of the original composite problem was then derived by enforcing continuity of stresses and displacements at the interface between the annulus and exterior region.

A closed-form analytical solution for the composite problem was developed as a slight extension of the published solution in Ref. 4, Appendix B. Comparisons between nondimensional numerical and analytical results for the stress field within the membrane were made. A radial sector of the annulus, consisting of three elements which span the range of structural behavior from wrinkled to taut, was chosen for comparison. It was found that the numerical solutions for the maximum principal stress as a function of radius agreed very well with the analytical results, with errors typically less than about one percent. Results for the minimum principal stress, which vanishes at the wrinkle boundary, also compared favorably with the analytical results, but errors were typically a few percent larger than those for the maximum stress. Similar accuracy was obtained in comparisons of the location of the wrinkle boundary in both the numerical and analytical solutions.

CONCLUSIONS

The research proposal upon which the approval of the supporting NASA grant was based, states the primary objective of the research as "to develop a general finite element analysis capability for predicting the mechanical behavior of membrane structures, including nonlinear geometric effects and wrinkling in an averaged sense." In particular, for Phase I, the only phase which was funded, the stated objective is "Develop the stated primary objective for the special case of flat membranes, and evaluate the results for accuracy and efficiency in benchmark numerical examples."

It is felt that these stated objectives for the research have been met, and that significant progress has been made toward the objectives of Phase II, which include generalizing the analysis capability to include curved and three-dimensional membranes.

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