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GRID SPACING CONTROL WITH VARIATION DIMINISHING SPLINES

By

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GRID SPACING CONTROL WITH VARIATION DIMINISHING SPLINES

By

Philip W. Smith*

The purpose of this research is to improve methods used to specify and control two and three dimensional grids on which numerical solutions of partial differential equations may be obtained. While initially focusing on grid generation, we anticipate that it will evolve into a consideration of the interaction of grid generation with the solution of a partial differential equation.

The research during this period concentrated on using the multi-surface method of grid generation, developed by Peter Eiseman, to continuously patch a grid onto an existing grid. In the resulting grid the elements of the Jacobian matrix must be continuous across the boundary between the original grid and the patched grid.

This research is motivated by the desire of engineers to account for the affects of wind tunnel walls in physical experiments by comparing numerical solutions using an original grid whose boundary represents the wind tunnel walls with numerical solutions using a much larger grid formed by patching a grid around the original with exterior free stream conditions.

Programs were written which accept as input the coordinates of the original grid and the desired new boundary, and then use the three-surface or four-surface version of the multi-surface method to extend the original grid out to a new boundary.

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The multi-surface transformation generates coordinates between an inner boundary surface $\vec{P}_1(\vec{s})$ and an outer boundary surface $\vec{P}_N(\vec{s})$. Intermediate surfaces $\vec{P}_2(\vec{s}),\ldots,\vec{P}_{N-1}(\vec{s})$ are introduced to control the coordinate representation between $\vec{P}_1(\vec{s})$ and $\vec{P}_N(\vec{s})$. Each surface representation is of the form

$$\vec{P}_K(\vec{s}) = \begin{bmatrix} x_k(\vec{s}) \\ y_k(\vec{s}) \\ z_k(\vec{s}) \end{bmatrix} , \quad k=1,2,\ldots,N \qquad \begin{matrix} 0 \le s_i \le 1 \\ i=1,2 \end{matrix}$$

The physical domain can be written

$$\vec{P}(\vec{s},\eta) = \begin{bmatrix} x(\vec{P}_1(\vec{s}),\ldots,\vec{P}_N(\vec{s}),\eta) \\ y(\vec{P}_1(\vec{s}),\ldots,\vec{P}_N(\vec{s}),\eta) \\ z(\vec{P}_1(\vec{s}),\ldots,\vec{P}_N(\vec{s}),\eta) \end{bmatrix}$$

where $0 \le \vec{s}_i \le 1$ for $i=1,2$ and $0 \le \eta \le 1$. The variable η is the independent variable which spans between surfaces.

The multi-surface method is explained in more detail in "Mesh Generation Using Algebraic Techniques" by Peter Eiseman and Robert Smith which can be found in <u>Numerical Grid Generation Techniques</u>, NASA Conference Publication 2166.

The intermediate surfaces are not coordinate surfaces, but are surfaces used to determine a field of tangent vectors to the coordinate curve spanning across the surface. The vector field of tangents is given by

$$V_k(s) = A_k(\vec{P}_{k+1}(\vec{s}) - \vec{P}_k(\vec{s})) \quad k=1,2,\ldots,N-1$$

where the A_k are scalars determining only the magnitude of the vectors

Using the independent variable η for the spanning direction we can choose $\eta_1 < \eta_2 < \ldots < \eta_{N-1}$ to correspond to the $\vec{V}_k$. A sufficiently smooth vector field $\vec{V}(\eta,\vec{s})$ is obtained by a smooth interpolation $\vec{V}(\eta_k,\vec{s}) = V_k(\vec{s})$. The first derivative of $\vec{P}(\vec{s},\eta)$ with respect to η is given by

$$1) \qquad \frac{\partial \vec{P}}{\partial \eta}(\vec{s},\eta) = \sum_{k=1}^{N-1} \psi_k(\eta) V_k(\vec{s}) \quad \text{where}$$

$$\psi_k(\eta_j) = \begin{cases} 1 \text{ if } j = k \\ 0 \text{ if } j \neq k \end{cases} .$$

Integrating 1) with an initial η_1 value of $\vec{P}_1(\vec{s})$ yielas

$$\vec{P}(\vec{s},\eta) = \vec{P}_1(\vec{s}) + \sum_{k=1}^{N-1} A_k G_k(\eta) (\vec{P}_{k+1}(\vec{s}) - \vec{P}_k(\vec{s}))$$

where

$$G_k(\eta) = \int_{\eta_1}^{\eta} \psi_k(x)\,dx.$$

If the interpolants ψ_k are polynomials in η then the coordinate curves which cannect the bounding surfaces are defined by polynomials η of one degree greater. So in particular, using the three-surface method with $\eta_1 = 0$, $\eta_2 = 1$, $\psi_1(\eta) = 1 - \eta$ and $\psi_2(\eta) = \eta$ we get

$$2) \qquad G_1(\eta) = \eta - \frac{\eta^2}{2}$$

$$G_2(\eta) = \frac{\eta^2}{2} \quad .$$

Also, note that if the A_k are chosen so that $A_k G_k(\eta_{N-1})$ is 1, then $\vec{P}(\vec{s},\eta_{N-1}) = \vec{P}_N(\vec{s})$.

We then have

$$\vec{P}(\vec{s},\eta) = \vec{P}_1(\vec{s}) + \sum_{k=1}^{2} \frac{G_k(\eta)}{G_k(\eta_2)}(\vec{P}_{k+1}(\vec{s}) - \vec{P}_k(\vec{s}))$$

which is Eiseman's three-surface transformation.

Substituting 2) for G_k, $k = 1,2$ and rewriting $\vec{P}(\vec{s},\eta)$ yields

$$\vec{P}(\vec{s},\eta) = \sum_{k=1}^{3} B_k(\eta)\vec{P}_k(\vec{s}) \qquad \text{where}$$

$$B_1(\eta) = 1 - 2\eta + \eta^2$$

$$B_2(\eta) = 2\eta - 2\eta^2$$

$$B_3(\eta) = \eta^2 .$$

Using the four-surface method with $\eta_1 = 0$, $\eta_2 = \frac{1}{2}$, $\eta_3 = 1$, $\psi_1(\eta) = (1-\eta)(\frac{1}{2}-\eta)$, $\psi_2(\eta) = \eta(1-\eta)$, and $\psi_3(\eta) = (\eta - \frac{1}{2})\eta$ yields

$$G_1(\eta) = \tfrac{1}{2}\eta - \tfrac{3}{4}\eta^2 + \tfrac{1}{3}\eta^3$$

$$G_2(\eta) = \tfrac{1}{2}\eta^2 - \tfrac{1}{3}\eta^3$$

$$G_3(\eta) = \tfrac{1}{3}\eta^3 - \tfrac{1}{4}\eta^2 .$$

So the four-surface transformation is given by

$$\vec{P}(\vec{s},\eta) = \vec{P}_1(\vec{s}) + \sum_{k=1}^{3} \frac{G_k(\eta)}{G_k(\eta_3)}(\vec{P}_{k+1}(\vec{s}) - \vec{P}_k(\vec{s})) .$$

After substitution and rewriting we have

$$\vec{P}(\vec{s},\eta) = \sum_{k=1}^{4} D_k(\eta)\vec{P}_k(\vec{s}) \quad \text{where}$$

$$D_1(\eta) = 1 - 6\eta + 9\eta^2 - 4\eta^3$$

$$D_2(\eta) = 6\eta - 12\eta^2 + 6\eta^3$$

$$D_3(\eta) = 6\eta^2 - 6\eta^3$$

$$D_4(\eta) = -3\eta^2 + 4\eta^3 \quad .$$

In order to use the three-surface method to patch a grid around an existing grid we choose $\vec{P}_1$ to be the next to the last row of the original grid and $\vec{P}_3$ to be the boundary of the new grid. The vector $\vec{s}$ becomes a scalar variable t, $0 \le t \le 1$, since we are dealing with curves. To obtain $\vec{P}_2$ we go through the following steps:

Compute $\eta_0(t) = \dfrac{d(\vec{P}_1(t)\ \vec{P}_2(t))}{d(\vec{P}_1(t),\vec{P}_3(t))}$ where

$d(\vec{A},\vec{B})$ = distance from $\vec{A}$ to $\vec{B}$.

Denote the last row of the original grid by $\vec{Q}$.

Pick $\vec{P}_2(t)$ to solve

$$\vec{Q}(t) = \sum_{k=1}^{3} B_k(\eta_0(t))\vec{P}_k(t).$$

We will then have $\vec{P}(t,0) = \vec{P}_1(t)$, $\vec{P}(t,\eta_0(t)) = \vec{Q}(t)$, and $\vec{P}(t,1) = \vec{P}_3(t)$.

Along each connecting curve C_t, we choose m equally-spaced values η_i, $i = 0,\ldots,m-1$ from $\eta_0(t)$ to 1 with $m = 1/\eta_0(0)$ and compute the corresponding physical points $\vec{P}(t,\eta_i) = \left\{ \begin{matrix} x(t,\eta_i) \\ y(t,\eta_i) \end{matrix} \right\}$. We then make a table consisting of accumulated arclengths $alpha_i$ $i = 0,\ldots,m-1$ along C_t from the point where $\eta = \eta_0(t)$ to where $\eta = 1$ versus the corresponding value of η.

Using the arclength, ARC, from $\vec{P}_1(t)$ to $\vec{Q}(t)$ we use the Newton-Raphson method to solve

$$ARC = (e^{k(t)\cdot\Delta\eta} - 1)/(e^{k(t)} - 1)$$

for $k(t)$ where $\Delta\eta = 1/(m-1)$.

Next we substitute m values of η from 0 to 1 in increments of $\Delta\eta$ into $(e^{k(t)\cdot\eta} - 1)/(e^{k(t)} - 1)$ to obtain an exponential distribution of arclength along C_t from $\vec{P}_1(t)$ to $\vec{P}_3(t)$. This is achieved by interpolating the values from the exponential expression into the table created above to find the appropriate $\eta^*(t,\eta)$ corresponding to a particular arclength. The value of $\eta^*(t,\eta)$ is then substituted into $\vec{P}(t,\eta)$ to obtain the physical coordinates for the patched grid. Our transformation can then be written

$$\vec{F}(t,\eta) = \vec{P}(t,\eta^*) = \sum_{k=1}^{3} B_k[\eta^*(t,\eta)]\vec{P}_k(t)$$

where $0 \le \eta \le 1$, $0 \le t \le 1$.

The function $\eta^*(t,\eta)$ is defined by $\eta^*(t,\eta) = u(\alpha)$ where

$$u(\alpha) = \begin{cases} \eta_i & \text{for } \alpha = \text{alpha}_i \\ [\eta_i(\alpha - \text{alpha}_{i-1}) + \\ \eta_{i-1}(\text{alpha}_i - \alpha)]/ \\ (\text{alpha}_i - \text{alpha}_{i-1}) & \text{for alpha}_{i-1} < \alpha < \text{alpha}_i. \end{cases}$$

This transformation guarantees continuity in the spacing between rows as one moves from the original grid to the patched grid, but continuity in the spacing between columns appears to be dependent on the distribution of points along the new boundary. Our current research centers on finding a reliable method for choosing the best boundary points. Hopefully, an interactive method for choosing the points can be inserted into the current programs.

All programs were done at NASA Langley Research Center on the Interactive Processing System using a Tektronix 4014 graphics terminal and a Prime 750 computer. This system allows one to interactively generate grids fairly easily.