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THE EVOLUTION EQUATIONS FOR TAYLOR VORTICES  
IN THE SMALL GAP LIMIT

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# THE EVOLUTION EQUATIONS FOR TAYLOR VORTICES IN THE SMALL GAP LIMIT

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## Abstract

We consider the centrifugal instability of the viscous fluid flow between concentric circular cylinders in the small gap limit. The amplitude of the Taylor vortex is allowed to depend on a slow time variable, a slow axial variable, and the polar angle  $\theta$ . It is shown that the amplitude of the vortex cannot in general be described by a single amplitude equation. However, if the axial variations are periodic a single amplitude equation can be derived. In the absence of any slow axial variations it is shown that a Taylor vortex remains stable to wavy vortex perturbations. Furthermore, in this situation, stable non-axisymmetric modes can occur but do not bifurcate from the Taylor vortex state. The stability of these modes is shown to be governed by a modified form of the Eckhaus criterion.

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In two recent papers Tabeling [1] and Brand and Cross [2] have independently proposed an amplitude equation which governs the slow azimuthal and axial evolution of a Taylor vortex in the small gap limit. In this note we show that this amplitude equation corresponds to a velocity field which necessarily violates the no-slip condition at one of the cylinders. The remedy for this difficulty is well-known in hydrodynamic stability theory following the work of Davey, et al. [3] and requires the insertion of an eigenfunction in the expansion of the disturbance pressure field. The presence of this eigenfunction means that the evolution of a Taylor vortex cannot be described by a single amplitude equation.

We shall see that if axial variations are ignored then it is possible to describe the azimuthal evolution of a Taylor vortex by a single amplitude equation. However, even this reduced equation differs from the reduced form of the equation of Tabeling, Brandt and Cross. The appropriate amplitude equation is discussed in some detail and it is shown that in the small gap limit a Taylor vortex is stable to wavy vortex perturbations. Thus the evolution equation approach to describe the azimuthal evaluation of a Taylor vortex gives results which are not consistent with the classical results of Davey, et al. [5] and the available experimental results. The implications of this situation will be discussed later.

We consider then the stability of the flow of a viscous fluid of kinematic viscosity  $\nu$  between cylinders of radii  $R_1, R_1 + d$ . The outer cylinder is held fixed whilst the inner one rotates with angular velocity  $\Omega_1$ . We define the Reynolds number  $R$  and the parameter  $\delta$  by

$$R = \frac{U_0 d}{\nu}, \quad (1a)$$

$$\delta = \frac{d}{R_1} . \quad (1b)$$

In the limit  $\delta \rightarrow 0$  it is known that instability occurs when the Taylor number

$$T = R^2 \delta$$

is  $O(\delta^0)$ . Following Krueger, et al. [4] and Davey, et al. [5] it has been customary in the small gap limit to consider disturbances with azimuthal wavenumbers  $O(\delta^{-1/2})$  even though all the available experimental results suggest that only azimuthal wavenumbers of order  $\delta^0$  are important in the transition from Taylor vortex flow to wavy vortex flow. Hence we shall take  $\frac{\partial}{\partial \theta} \sim O(\delta^0)$  but the scalings of Davey, et al. [5] which were used by Tabeling, Brand and Cross can be recovered at a later stage by considering a further limiting process.

At this stage we restrict our attention to Taylor vortices of fixed axial wavelength with amplitude dependent on the polar angle  $\theta$  and time. We consider the limit  $\delta \rightarrow 0$  with

$$2 R^2 = \frac{1}{\delta} [T_0 + \delta T_1 + \delta^2 T_2 + \dots] = \frac{T}{\delta} ,$$

where  $T_0 = 3390$  is the critical Taylor number in the small gap limit. The velocity components in the radial, azimuthal and axial directions are scaled on  $v/d$ ,  $\Omega_1 R_1$  and  $v/d$ , respectively whilst the pressure is scaled on  $\rho v^2/d^2$ .

In the small gap limit the basic flow  $(0, \Omega_1 R_1 \bar{v}, 0)$  has the asymptotic form

$$\bar{v} = 1 - x + O(\delta)$$

where  $x$  is a radial variable scaled on  $d$ . The equations governing the stability of this basic flow can be written in the form

$$Lu = -\frac{\partial p}{\partial x} + Q_1 + T\bar{v} v + O(\delta) \quad (2a)$$

$$Lv = -\sqrt{\frac{2}{T}} \delta^{3/2} \frac{\partial p}{\partial \theta} + Q_2 + u \frac{d\bar{v}}{dx} + O(\delta) \quad (2b)$$

$$Lw = -\frac{\partial p}{\partial z} + Q_3 + O(\delta) \quad (2c)$$

$$\frac{\partial u}{\partial x} + \sqrt{\frac{T}{2}} \delta^{1/2} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = O(\delta), \quad (2d)$$

where  $z$  and  $t$  have been scaled on  $d$  and  $v/d^2$  respectively. The nonlinear functions  $Q_1$ ,  $Q_2$  and  $Q_3$  are  $O(\delta^0)$  whilst the operator  $L$  is defined by

$$L \equiv \frac{\partial}{\partial t} + \delta^{1/2} \sqrt{\frac{T}{2}} \bar{v} \frac{\partial}{\partial \theta} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2}. \quad (3)$$

Since the Taylor number differs from its critical value by  $O(\delta)$  we expect a finite amplitude motion of  $O(\delta^{1/2})$  and therefore expand  $\underline{u} = (U, V, W)$  in the form

$$\underline{u} = \delta^{1/2} \underline{u}_0 + \delta \underline{u}_1 + \delta^{3/2} \underline{u}_2 + \dots, \quad (4)$$

together with a similar expansion for the pressure. We then define the slow time scales  $\tau = \delta^{1/2} t$ ,  $\bar{t} = \delta t$ . The details of such an expansion procedure follow closely those of Tabeling, Brand and Cross and at order  $\delta^{1/2}$  it is found that

$$\underline{u}_0 = \frac{A(\theta, \tau, \bar{t}) e^{iaz}}{2} (U_0(x), V_0(x), W_0(x)) + \text{COMPLEX CONJUGATE}$$

where  $a$  is the critical axial wavenumber whilst  $(U_0, V_0, W_0)$  is the velocity eigenfunction corresponding to the critical point on the neutral curve. At order  $\delta$  it is found that  $A$  must satisfy the equation

$$\sqrt{\frac{T_0}{2}} s_0 \frac{\partial A}{\partial \theta} + \frac{\partial A}{\partial \tau} = 0$$

and Tabeling has calculated  $s_0$  numerically and found that  $s_0 = .5261$ . It follows from the above equation that  $A = A(\phi, \bar{t})$  where

$$\phi = \theta - \sqrt{\frac{T_0}{2}} s_0 \tau.$$

At order  $\delta$  the first harmonic and mean flow correction are determined and it is at this stage that the difficulty overlooked by Tabeling, Brand, and Cross arises. The mean flow correction at this order is in the  $\theta$ -direction and we denote it by  $v_M$ . However, we see from the equation of continuity that this mean flow drives a radial mean velocity field of  $O(\delta^{3/2})$  which we denote by  $u_M$ . The equation which determines  $u_M$  is

$$\frac{\partial u_M}{\partial x} = - \sqrt{\frac{T_0}{2}} \frac{\partial v_M}{\partial \theta},$$

and this equation must be integrated to satisfy  $u_M = 0$  at  $x = 0, 1$ . This cannot be achieved unless  $v_M$  contains some arbitrary function of  $\theta$  and  $\bar{t}$ . It is for this reason that the solution given by Tabeling, Brand and Cross does not satisfy the no-slip condition everywhere. The remedy is to allow for a pressure eigenfunction in the manner discussed by Davey, et al. [3] and DiPrima and Stuart [6]. Thus the perturbation pressure must be expanded in the form



$$p = \delta^{1/2} p_0 + \delta p_1 + \delta^{3/2} p_1 + \dots + \delta^{-1/2} \bar{p}_0(\theta, \bar{\tau}, \tau) + \dots,$$

where the relatively large size of the induced mean pressure field is, of course, a lubrication effect. The equation for  $v_M$  now becomes

$$\frac{\partial^2 v_M}{\partial x^2} = -\sqrt{\frac{2}{T_0}} \frac{\partial \bar{p}_0}{\partial \theta} - \frac{1}{2} |A|^2 \frac{d}{dx} (U_0 V_0), \quad (5a)$$

which can be integrated subject to  $v_M = 0$  at  $x = 0, 1$ . We can then substitute for  $v_M$  into the equation of continuity to find  $u_M$ . The condition that  $u_M$  should vanish at both  $x = 0$  and  $x = 1$  gives

$$\sqrt{\frac{2}{T_0}} \frac{\partial^2 \bar{p}_0}{\partial \theta^2} = 6 \frac{\partial}{\partial \theta} |A|^2 Q_0 \quad (5b)$$

where

$$Q_0 = \int_0^1 F_0(x) dx$$

with

$$F_0(x) = \int_0^x U_0 V_0 dx - x \int_0^1 U_0 V_0 dx.$$

The equation for  $\bar{p}_0$  is now integrated once and the arbitrary constant which appears in the resulting expression for  $\frac{\partial \bar{p}_0}{\partial \theta}$  is fixed by insisting that  $\bar{p}_0$  be periodic in  $\theta$ . The function  $v_M$  is then completely determined and we find

$$v_M = -\frac{1}{2} |A|^2 F_0(x) + \left\{ \frac{1}{2} \sigma_0 - 3Q_0 |A|^2 \right\} \{x^2 - x\} \quad (6)$$

where  $\sigma_0 = 3 \frac{Q_0}{\pi} \int_0^{2\pi} |A|^2 d\theta$ . The amplitude equation found by Tabeing, Brand and Cross corresponds to  $v_M = -\frac{1}{2} |A|^2 F_0(x)$  so that the radial mean

flow induced at higher order in their expansions cannot satisfy the no-slip condition at both cylinders. At order  $\delta^{3/2}$  we find that  $A$  satisfies the equation

$$\frac{\partial A}{\partial \tau} = T_1' A + c_3 \frac{\partial^2 A}{\partial \phi^2} - c_4 A |A|^2 + c_5 \left\{ \frac{1}{2} \sigma_0 - 3Q_0 |A|^2 \right\} A, \quad (7)$$

where  $T_1' = \frac{c_0}{2T_0} [T_1 - \bar{T}_1]$  with  $\bar{T}_1$  the order  $\delta$  correction to the axisymmetric critical Taylor number. The constants  $c_0, c_3, c_4$  are given by Tabeling as

$$c_0 = 26.16, \quad c_3 = 2.609, \quad c_4 = 40.2 \quad \text{with} \quad V_0(1/2) = 1$$

whilst  $c_5$  has been calculated by DiPrima and Stuart [5] who give

$$c_5 = -4.76.$$

The amplitude equation given by Tabeling, Brandt, and Cross corresponds to setting  $c_5 = 0$  in (7). The linearized form of (7) shows that the non-axisymmetric mode with wavenumber  $M$  is linearly unstable for

$$T_1' > T_{1c}' = c_3 M^2 \quad (8)$$

and the finite amplitude axisymmetric mode which bifurcates from  $T_{1c}'$  is

$$A = A_e = \sqrt{\frac{(T_1' - c_3 M^2)}{c_4}} e^{iM\phi} \quad (9)$$

and of course only integer values of  $M$  have any physical relevance. The

first mode to bifurcate is the Taylor vortex solution which has  $M = 0$ . In order to investigate the stability of (9) we write

$$A = A_e + b$$

and the linearized equation satisfied by  $b$  is

$$\begin{aligned} \frac{\partial b}{\partial \bar{t}} = & c_3 \frac{\partial^2 b}{\partial \phi^2} - (T_1' - c_3 M^2)(b + \bar{b}e^{2iM\phi})(1 + \epsilon) + c_3 M^2 b \\ & + \frac{e^{iM\phi}}{2\pi} \epsilon [T_1' - c_3 M^2] \int_0^{2\pi} (\bar{b}e^{iM\phi} + be^{-iM\phi}) d\phi \end{aligned} \quad (10)$$

where

$$\epsilon = \frac{3Q_0 c_5}{c_4}.$$

If we set  $M = 0$  in (10) we can study the stability of a Taylor vortex to  $\phi$  dependent perturbations. We can then see from (10) that the growth rate of a disturbance proportional to  $i\cos M\phi$  or  $i\sin M\phi$  is  $-c_3 M^2$  so that, in the small gap limit, there is no bifurcation from a Taylor vortex to a wavy vortex solution. The non-axisymmetric modes with  $M \neq 0$  are susceptible to the Eckhaus-Benjamin-Feir sideband instability mechanism. Following Stuart and DiPrima [7] it can be shown from (10) that the non-axisymmetric mode is unstable to sidebands with integer wavenumbers  $\gamma M, (2 - \gamma)M$  for  $-1 < \gamma < 3, \gamma \neq 1$ . The non-axisymmetric mode with wavenumber  $M$  which bifurcates from  $T_{1c}'$  is found to be unstable to such a sideband for

$$T_{1c}' < T_1' < \left[1 + \frac{(\gamma+1)(3-\gamma)}{2(1+\epsilon)}\right] T_{1c}'$$

which reduces to Eckhaus' result  $T_1' < 3T_{1c}'$  in the limit  $\gamma \rightarrow 1$  with  $\epsilon = 0$ . We see then that the Eckhaus criterion is altered if  $\epsilon \neq 0$  so that the pressure eigenfunction decreases the unstable regime. In the present problem only integer values of  $\gamma M$  are physically acceptable so that the non-axisymmetric mode is unstable for

$$T_{1c}' < T_1' < \left[1 + \frac{[4 - 1/M^2]}{2(1+\epsilon)}\right] T_{1c}'$$

It is of course possible that stable wavy vortex solutions to (7) exist but do not bifurcate from Taylor vortex flow. In order to investigate such a possibility (7) was integrated numerically using a fully implicit finite difference scheme. Several runs were made using difference initial distributions for  $A$  at different supercritical values of  $T_1'$ . At sufficiently large values of  $\bar{t}$  the numerical solutions approached one of the equilibrium solutions (9) for some value of  $M$ , we found no steady state solution other than those given by (9).

In previous descriptions of non-axisymmetric motion in the small gap limit it has become customary to take  $2R^2 \delta = T$  and  $\frac{\partial}{\partial \theta} \sim \delta^{-1/2}$  in the equations of motion. This procedure was followed by Tabeling, Brand, and Cross who then allowed  $T$  to differ from its critical axisymmetric value by  $O(\epsilon)$  and considered azimuthal variations with  $\frac{\partial}{\partial \theta} \sim O(\frac{\epsilon}{\delta})^{1/2} \gg 1$ . The procedure outlined in this paper has  $\frac{\partial}{\partial \theta} \sim O(1)$  but, by taking  $\epsilon = \delta$  in the work of Tabeling, Brand and Cross and inserting the required pressure eigenfunction, we can recover (7). Alternatively by taking the limit  $T_1' \rightarrow \infty$  with  $\frac{\partial}{\partial \theta} \sim T_1'^{1/2}$ ,  $A \sim T_1'^{1/2}$  in (7) we recover the corrected form of the equation of Tabeling, Brandt, and Cross. Thus the expansion procedures are equivalent and it is not necessary in the small gap limit to ignore the available experimental results and take  $\frac{\partial}{\partial \theta} \gg 1$ .

We shall now derive a generalized form of (7) which takes account of slow variations of the vortex amplitude in both the axial and azimuthal directions. Such an equation has been given by Tabeling, Brand, and Cross but the velocity field associated with that equation does not satisfy the no-slip condition at one of the cylinders. We again assume that  $\frac{\partial}{\partial \theta} \sim 0(1)$  and now choose to look for axial variations on the same length scale as those in the azimuthal direction. We therefore define

$$\zeta = \delta^{1/2} z$$

and retain the expansion (4). However we now write

$$\underline{u}_0 = \frac{A(\theta, \zeta, \tau, \bar{t})}{2} [U_0(x), V_0(x), W_0(x)] e^{iaz} + \text{COMPLEX CONJUGATE},$$

and note that in the absence of any pressure eigenfunction the radial mean flow at order  $\delta^{3/2}$  cannot satisfy the no-slip condition at both cylinders.

We therefore expand the pressure in the form

$$p = \delta^{1/2} p_0 + \delta p_1 + \dots + \delta^{-1/2} \bar{p}_0(\theta, \zeta, \tau, \bar{t}) + \bar{p}_1(\theta, \zeta, \tau, \bar{t}) \\ + \delta^{1/2} \bar{p}_2(\theta, \zeta, \tau, \bar{t}) + \dots, \quad (11)$$

where  $\bar{p}_0, \bar{p}_1, \bar{p}_2$ , etc. are pressure eigenfunctions introduced in order that the radial mean flow satisfies the required boundary conditions. However, it follows from (2c) that  $\bar{p}_0$  will drive an axial mean flow of order  $\delta^0$  if  $\frac{\partial \bar{p}_0}{\partial \zeta} \neq 0$ . Thus we set  $\frac{\partial \bar{p}_0}{\partial \zeta} = 0$  and  $\frac{\partial \bar{p}_1}{\partial \zeta} = 0$  in order that the azimuthal and

axial mean flows induced by the disturbance should be comparable. If we denote the mean part of  $w_1$  by  $w_M$  it follows from (2c) that

$$\frac{\partial^2 w_M}{\partial x^2} = -\frac{\partial \bar{p}_2}{\partial \zeta},$$

so that

$$w_M = -\frac{1}{2} \frac{\partial \bar{p}_2}{\partial \zeta} (x^2 - x), \quad (12)$$

whilst  $v_M$ , the order  $\delta$  azimuthal mean flow again satisfies (5a) so that

$$v_M = -\frac{1}{2} |A|^2 F_0(x) - \sqrt{\frac{2}{T_0}} \frac{\partial \bar{p}_0}{\partial \theta} \left( \frac{x^2 - x}{2} \right). \quad (13)$$

The radial mean flow at order  $\delta^{3/2}$  is now driven by both  $v_M$  and  $w_M$  and satisfies the no-slip condition at  $x = 0, 1$  if

$$\frac{\partial^2 \bar{p}_0}{\partial \theta^2} + \frac{\partial^2 \bar{p}_2}{\partial \zeta^2} = 6 \frac{\partial}{\partial \theta} |A|^2 Q_0 \sqrt{\frac{T_0}{2}}, \quad (14)$$

where  $Q_0$  is as defined earlier. We see that if  $\frac{\partial}{\partial \theta} \equiv 0$  then the pressure eigenfunctions  $\bar{p}_0$  and  $\bar{p}_2$  both vanish whilst if  $\frac{\partial}{\partial \zeta} \equiv 0$  we recover (5b). It is of interest to note that in the corresponding pressure equation in parallel flow stability theory  $\bar{p}_0 = \bar{p}_2$ .

The axial mean flow of order  $\delta$  interacts with the fundamental terms of order  $\delta^{1/2}$  to reproduce the fundamental so that the solvability condition at order  $\delta^{3/2}$  will depend on  $\frac{\partial \bar{p}_2}{\partial \zeta}$ . After some analysis we find that the appropriate solvability condition is

$$\begin{aligned} \frac{\partial A}{\partial \bar{t}} = & T_1^- A + c_3 \frac{\partial^2 A}{\partial \theta^2} + c_6 \frac{\partial^2 A}{\partial \zeta^2} + ic_7 \frac{\partial^2 A}{\partial \zeta \partial \theta} - c_4 A |A|^2 \\ & - c_5 \frac{\partial \bar{p}_0}{\partial \theta} A - ic_8 \frac{\partial \bar{p}_2}{\partial \zeta} A, \end{aligned} \quad (15)$$

where  $c_3$ ,  $c_4$ , and  $c_5$  are as defined previously whilst Tabeling has calculated  $c_6$ ,  $c_7$  and gives

$$c_6 = .9837, \quad c_7 = .3948.$$

The constant  $c_8$  satisfies

$$c_8 = \frac{a \int_0^1 \left[ \frac{1}{2} V_0^+(x^2 - x)V_0 - \frac{1}{2} U_0^+(x^2 - x)(U_0'' - a^2 U_0) + U_0 U_0^+ \right] dx}{\int_0^1 \left[ U_0^+(U_0'' - a^2 U_0) - V_0^+ V_0 \right] dx}, \quad (16)$$

where  $(U_0^+, V_0^+)$  is the function pair adjoint to  $(U_0, V_0)$ . We have not calculated  $c_8$  which is purely real but there is no reason to suppose that it is zero. The equations (14) and (15) are coupled so that in general it is not possible to describe the axial and azimuthal evolution of a Taylor vortex by a single amplitude equation.

In order to demonstrate how (14) and (15) can be solved let us suppose that all the disturbance quantities are periodic in  $\zeta$  with wavelength  $2L$ . We can integrate (14) once to give

$$\frac{\partial \bar{p}_2}{\partial \zeta} = 6Q_0 \sqrt{\frac{T_0}{2}} \frac{\partial}{\partial \theta} \int_{-L}^{\zeta} |A|^2 d\zeta' - (\zeta + L) \frac{\partial^2 \bar{p}_0}{\partial \theta^2} + F(\theta, \bar{t}), \quad (17)$$

where  $F$  is an unknown function of  $\theta$  and  $\bar{t}$ . However, if  $p_2$  is periodic

in  $\zeta$

$$\frac{\partial^2 \bar{p}_0}{\partial \theta^2} = \frac{3Q_0}{L} \sqrt{\frac{T_0}{2}} \frac{\partial}{\partial \theta} \int_{-L}^L |A|^2 d\zeta', \quad (18)$$

so that

$$\frac{\partial \bar{p}_2}{\partial \zeta} = 6Q_0 \sqrt{\frac{T_0}{2}} \frac{\partial}{\partial \theta} \int_{-L}^{\zeta} \left\{ |A|^2 - \frac{1}{2L} \int_{-L}^L |A|^2 d\zeta' \right\} d\zeta' + F(\theta, \bar{t}), \quad (19)$$

and if we integrate (19) once more with respect to  $\zeta$  and impose the condition that  $\bar{p}_2$  be periodic in  $\zeta$  we obtain

$$F(\theta, \bar{t}) = \frac{-3Q_0}{L} \sqrt{\frac{T_0}{2}} \frac{\partial}{\partial \theta} \int_{-L}^L \int_{-L}^{\zeta} \left\{ |A|^2 - \frac{1}{2L} \int_{-L}^L |A|^2 d\zeta' \right\} d\zeta' d\zeta. \quad (20)$$

The solution for  $\bar{p}_2$  obtained by this procedure is unique only up to an arbitrary function of  $\theta$  and  $\bar{t}$ . This nonuniqueness of  $\bar{p}_2$  can be seen from (14) and (15) directly, since only  $\frac{\partial \bar{p}_2}{\partial \zeta}$  and  $\frac{\partial^2 \bar{p}_2}{\partial \zeta^2}$  appear in these equations.

We now integrate (18) to give

$$\frac{\partial \bar{p}_0}{\partial \theta} = \frac{3Q_0}{L} \sqrt{\frac{T_0}{2}} \int_{-L}^L \left\{ |A|^2 - \frac{1}{2\pi} \int_0^{2\pi} |A|^2 d\theta \right\} d\zeta, \quad (21)$$

where the unknown functions of  $\bar{t}$  which results from integrating (18) has been fixed so that  $\bar{p}_0$  is periodic in  $\theta$ . The functions  $\frac{\partial \bar{p}_2}{\partial \zeta}$  and  $\frac{\partial \bar{p}_0}{\partial \theta}$  can then be substituted into (15) to give an equation depending only on  $A$  which must then be integrated numerically subject to the periodicity of  $A$  in  $\theta$  and  $\zeta$ . In the more general case where the perturbation is not periodic in  $\zeta$  such a simplification of (14) and (15) is not possible.

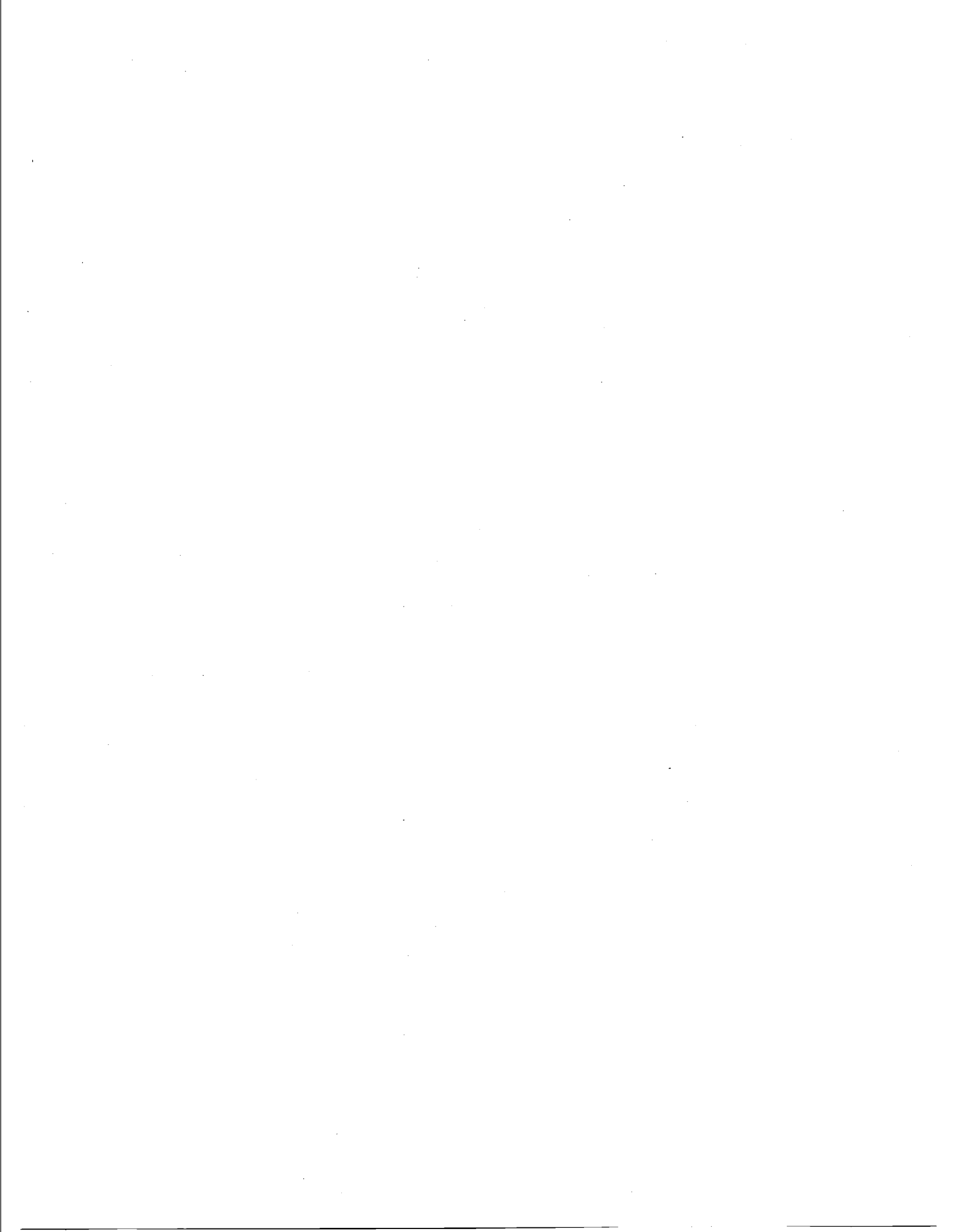


## CONCLUSION

We have given a self-consistent derivation of the equations governing the evolution of a Taylor vortex flow in the small gap limit. The equations which we have derived differ from those derived previously for the same problem. Our results for the situation where there is no slow axial variation suggest that, unlike the finite gap case, there is no bifurcation to a wavy vortex flow from a Taylor vortex flow. Furthermore we have found that non-axisymmetric modes are possible stable states in the small gap limit. This latter result is again different from the classical results of Davey et al. [5] for the finite (but small) gap problem. The expansion procedure used in the latter paper allowed the axisymmetric and non-axisymmetric eigenvalues to be split apart by taking  $\frac{\partial}{\partial \theta} \gg 1$  but results were obtained by subsequently taking  $\frac{\partial}{\partial \theta} \sim 0(1)$ . The expansion procedure used here has  $\frac{\partial}{\partial \theta} \sim 0(1)$  throughout and at first order in our expansions we have a multiple eigenvalue so that a self-consistent expansion can be developed. It is possible that the procedure we have used is valid only for very small values of  $\delta$  whilst that of Davey et al. [5], though not a formally self-consistent gives good agreement with experiment at moderately small values of  $\delta$ .

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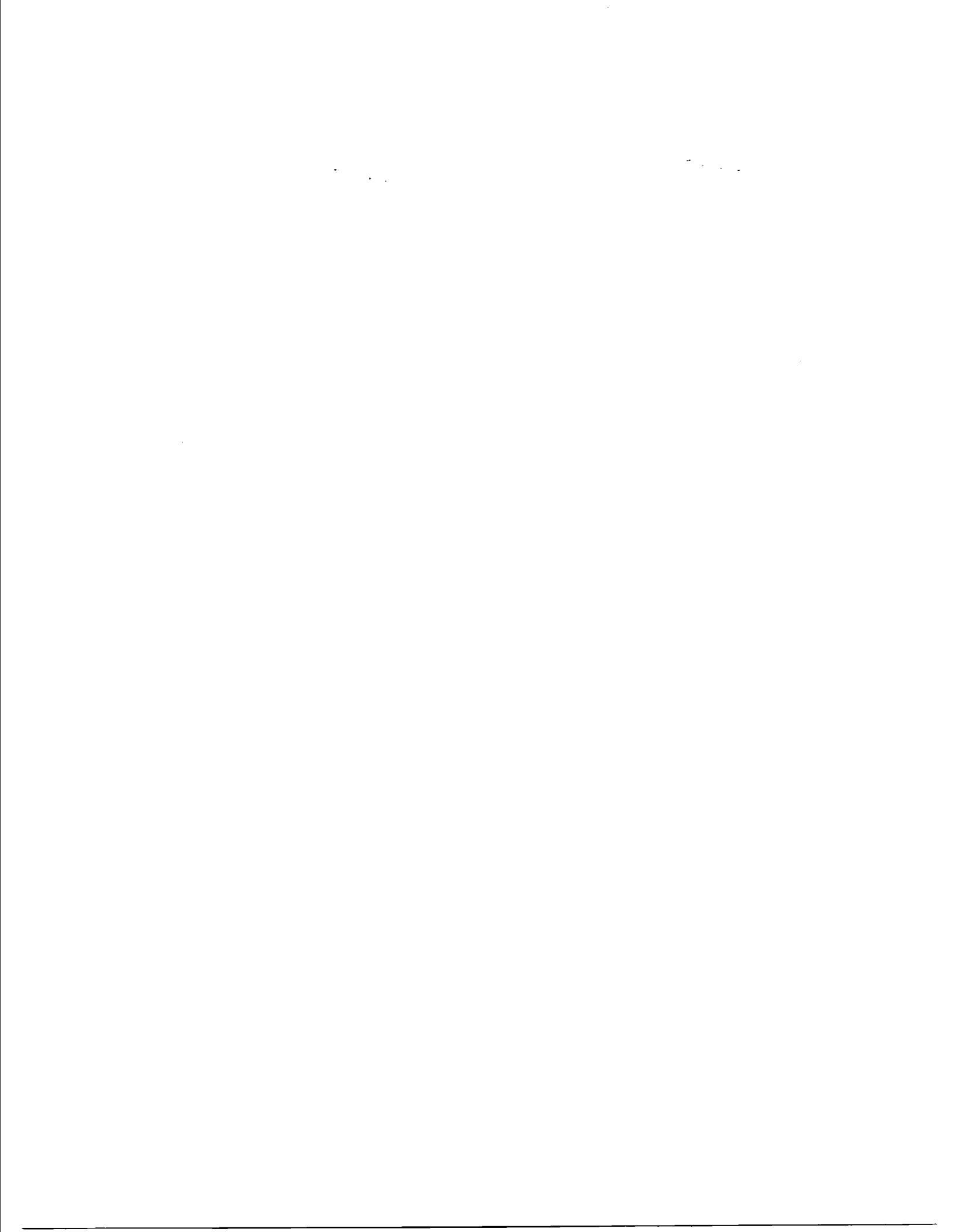
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16. Abstract  We consider the centrifugal instability of the viscous fluid flow between concentric circular cylinders in the small gap limit. The amplitude of the Taylor vortex is allowed to depend on a slow time variable, a slow axial variable, and the polar angle $\theta$ . It is shown that the amplitude of the vortex cannot in general be described by a single amplitude equation. However, if the axial variations are periodic a single amplitude equation can be derived. In the absence of any slow axial variations it is shown that a Taylor vortex remains stable to wavy vortex perturbations. Furthermore, in this situation, stable non-axisymmetric modes can occur but do not bifurcate from the Taylor vortex state. The stability of these modes is shown to be governed by a modified form of the Eckhaus criterion.					
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