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EXPERTMENTAL DETERMINATION OF THE PARTICLE MOTIONS ASSOCIATED WITH THE LOW ORDER ACOUSTIC MODES IN ENCLOSURES

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# EXPERIMENTAL DETERMINATION OF THE PARTICLE MOTIONS ASSOCIATED WITH THE LOW ORDER ACOUSTIC MODES IN ENCCOSURES 

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EXPERIMENTAL DETERMTNATION OF THE PARTICLE MOTIONS associated with the low order acoustic modes in enclosures

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## ABSIRACT

This paper describes a procedure for experimentally determining, in terms of the particle motions, the shapes of the low order acoustic modes in enclosures. The procedure is based on finding differentiable functions which approximate the shape functions of the low order acoustia medes when these modes are defined in terms of the acoustic pressure. The differentiable zpproximating functions are formed from polynomials which are fitted by a least squares procedure to experimentally determined values which define the shapes of the low order acoustic modes in terms of the acoustic pressure. These experimentally determined values are found by a conventional technique in which the transfer functions, which relate the acoustic pressures at an array of points in the enclosure to the volume velocity of a fixed point source, are measured. The gradient of the function which approximates the shape of a particular mode in terms of the acoustic pressure is evaluated to give the mode shape in terms of the particle motion. The procedure was tested by using it to experimentally determine, in texms of the particle motions, the shapes of the low order acoustic modes in a small rectangular enclosure. Details of the experimental technique used and some of the results obtained are given.

## INTRODUCTION

The excitation of one more of the low order acoustic modes in an enclosure can generate in the enclosure sound pressures with distinct tonal qualities which can annoy the occupants of the enclosure. A commonly experienced example of this situation is the "cavity boom" which sometimes occurs in the cabins of passenger cars. Cavity boom arises when a low order acoustic mode, which may be coupled to a complaint portion of the enclosing structure, is excited. Usually the cavity boom is excited by the engine and it often occurs when the engine firing frequency coincides with the frequency of a low order mode. Many studies, both theoretical and experimental, have been made to investigate the low order acoustic modes in passenger cars ${ }^{1 / 2 / 3}$. Annoying sound pressures associated with the presence of low order acoustic modes arise in the cabins of agricultural and industrial machines.'. A similar effect san occur in the cabins of ships and aircraft. It is usually a simple task to experimentally detemine the frequencies: of the. low order acoustic modes of an enclosure. However, it is not a simple. task to experimentally determine the mode shapes, particularly if they vary with more than one spatial co-ordinate and if the modeshapes are required in terms of the particle motions. A knowledge of the modeshapes, and in particular the modeshapes in terms of the particle motions, is useful in devising appropriate noise control measures.

Over the last decade there have been considerable developments in experimental methods for determining the modeshapes of structures ${ }^{5 / 6}$. The ability to describe the motion of a structure when it is vibrating in a particular mode is one of the most useful developments which has occurred. Whe ability to describe the motion has been facilitated by the fact that a motion transducer, usually an accelerometer, is one of the two transducers
which a;ce used to generate the electrical signals from which the transfer functions, used in the experimental modal analysis of the structure, are obtained. Although considerable developments have been made in experimental methods for determining the modeshapes of structures, it is only in recent years that attempts have been made to experimentally determine the shapes of acoustic modes ${ }^{1 / 7 / 0 \text {. One possible reason for the relative lack of progress }}$ in the acoustic case is that the most conveniently neasured acoustic quantity, the acoustic pressure, cannot be used directly to determine the particle motion associated with a particular mode.

This paper describes a procedure whereby the particle motions associated with low ordar acoustic modes in enclosures can be derived from transfer function measurements which relate thi acoustic pressures at an array of points in an enclosure to the volume velucity of a fixed point source in the enclosure. The theory of the procedure is described and then the results of a numerical evaluation of the procedure are given. Finally, a description of how the procedure was used to experimentally determine the low order acoustic modes in a small rectangular enclosure is given.

## I. DESCRIPTION OF THE METHOD

Let an xyz axes system be located in the enclosure as shown in Fig. 1. The axes are not necessarily orthogonal. Suppose that $I L \times M M \times N N$ transfer functions which relate the acoustic pressures at the $工 \times \mathrm{MM} \times \mathrm{NN}$ points shown in Fig. 1 to the volume velocity of a fixad point source are measured. The points shown in Fig. I are on lines parallel to the $x, y$ and $z$ axes. The distances between the points on a line parallel to a particular axis need not be equal but the distances between corresponding points on different lines are equal. Suppose, that ly a convertional modal analysis method, such as the method described by Nieter and Singh ${ }^{8}$, the acoustic pressure at the point $x_{I I}, Y_{J J}, z_{K K}$ per unit volume velocity of the source is measured as $P_{r}\left(X_{I X}, Y_{J J}, z_{K K}\right)$ in the $r^{\text {th }}$ mode whose angular frequency is $\omega_{r}$. This measured quantity is composed of the true value, denoted $P_{r} *\left(X_{I I}, Y_{J J}, z_{K K}\right)$ and an error term, denoted $F_{\text {. }}$. Thus:

$$
\begin{equation*}
P_{x}\left(x_{I I}, Y_{J J}, z_{K K}\right)=P_{x} *\left(x_{I I}, Y_{J J}, z_{K K}\right)+E \tag{1}
\end{equation*}
$$

The $I L \times M M \times N N$ values of $P_{r}\left(X_{I I} / Y_{\Omega S}, Z_{K K}\right)$ derived from the $L L \times M M \times N N$ measured transfer functions can be used to determine a function which. approximates $P_{r}{ }^{*}(x, y, z)$. Let this approximating function, which is denoted $p_{r}(x, y, z)$, be given by:

$$
\begin{align*}
\operatorname{pr}_{r}(x, y, z)= & C\left(1+x_{2} x+\ldots+x_{I} x^{I-1}+\ldots+x_{L} x^{I-1}\right) \times \\
& \left(1+Y_{2} y+\ldots+y_{J} Y^{J-1}+\ldots+y_{M Y} Y^{M-1}\right) \times \\
& \left(I+z_{2} z+\ldots+z_{K} z^{X-1}+\ldots+z_{N} z^{N-1}\right) \tag{2}
\end{align*}
$$

The constants $C, \ldots, X_{I}, \ldots, X_{J} \ldots, Z_{K}, \ldots$ which define the approximating function can be derived from the measured values, $P_{r}\left(X_{I I}, Y_{J J}, z_{K K}\right.$, by the following procedure. A total of $M M \times N N$ polynomials of the form:

$$
x_{1, J J, K K}^{\prime}+x_{2, J J, K K}^{\prime} x+\ldots \ldots+x_{I, J J, K X^{\prime}}^{x^{J-1}}+\ldots .+X_{L, J J, K K}^{\prime} x^{L-1}
$$

can be fitted by a least squares procedure to tho wh values of $p_{I}\left(x_{I I}, X_{J J}, z_{K K}\right)$ along each of the $M M \times N N$ lines which are paralled to the $x$ axis. The coefficients $X_{I}^{\prime}, J J, K K$ derived from a least squares fit along a particular line can be nosnalized by dividing all of the coefficient:s by one of the coefficientis. If the first coefficient is used as the normalizing coefficient, a polywomial of the following form is obtained:

$$
I+X_{2, J J, K K} x+\ldots \ldots+X_{I, J J, K K X^{I-1}}+\ldots \ldots+X_{L}, J J, K K^{I-1}
$$

$X_{I, J J, K K}$ is given by $X_{I, J J, K K}^{\prime} / X_{i, J J, J J}$ - Ideally, corresronding normalized coefficients for each of the MMX NN polynomials will be equal. However, since, these coefficients are derived from experimental data they will differ and su $X_{I}$ can be estimated by averaging the $X_{I, J J_{1}} K X$ coefficients. Thus $X_{I}$ is given by:

$$
X_{I}=\sum_{J J=1}^{M M} \sum_{K K=1}^{N N} X_{I, J J, K K} / M M \times N N
$$

Along some lines parallel to the $x$ axis the experimentally derived values $P_{x}\left(K_{I I}, Y_{J J}, z_{K K}\right)$ may be small and so the coefficients and hence the normalized coefficienes may not be representative and it is prudent to discard such coeffioients in the averaging. This can be done by evaluating the "size" of each of the fitted polynonials in terms of a measure such as the root meari square value of the fitted polynomial and then discarding coefficients associated with polynomials whose "size" is, for example, less than the average size.

A similar procedure involving the fitting of polynomials in the $y$ and $z$ directions can be used to determine the normalized coefficients ... $\mathrm{Y}_{\mathcal{J}} \ldots \mathrm{Z}_{\mathrm{K}} \ldots$... Once the normalized coefficients $\ldots X_{I} \ldots X_{J} \ldots Z_{K} \ldots$ have been found, the coefficient $C$ which appears in Eqn. (2) can b and to minimise the sum of the squares of the differences between $P_{Y}\left(X_{I I}, Y_{J J}, z_{K K}\right)$ and $P_{I}\left(X_{I I}, Y_{J J}, z_{K K}\right)$.

It is found, in the development of the acoustic wave equation, that the acceleration of a gas particle in a particular diraction is given by the negative of the gradient of the pressure in that direction divided by the density of the gas. Thus, for the $\mathrm{r}^{\text {th }}$ mode, the amplitude of the particle acceleration in the $x$ direction'per unit volume velocitiy of the scurce $a_{r x}(x, y, z)$, can be approximated by:

$$
\begin{equation*}
a_{r x}(x, y, z) \approx \frac{1}{P} \frac{\partial p_{r}(x, y, z)}{\partial x} \tag{3}
\end{equation*}
$$

It can be seen frow this equation that the first partial derivative of the approximating function with respect to $x$ must be found. This can be readily done when $p_{x}(x, y, z)$ is defined in the form of Eqn. (2). The amplitudes of the particleivelocity and the particle displacement can be readily found from the amplitude of the acceleration. Similarly, the amplitudes of the acceleration, velocity and displacement in the $y$ and $z$ directions can be estimated from the partial derivatives of the approximating function with yespect to $y$ and $z$.

## II. CALCULATIONS WITH THE FITTING PROCEDURE

It is evident that the approximating function $p_{x}(x, y, z)$ must approximate $P_{x} *(x, y, z)$ sufficiently well so that the fixst partial differentials of $P_{r}(x, y, z)$ and $P_{r} *(x, y, z)$ are in satisfactory agreement. An indication of how many terms should be incorporated in the approximating function can be obtained by using a suitable differentiable function to generate data at the pointes $X_{I I}, Y_{J J}, z_{K K} ; I I=I, L L ; J J=1, M M ; K K=I, N N$, fitting approximating functions with various numbers of terms and then comparing the first partial differentials of the original and the approwimating functions.

A suitable differentiable function is given by:

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$$
\begin{equation*}
P_{x^{*}}^{*}(x, y, z)=\cos (\ell \pi x) \cos (m \pi y) \cos (n \pi z) \tag{4}
\end{equation*}
$$

$$
\ell_{p} n, n=0,1,2, \ldots
$$

This function, in addition to being reudily differentiable, delines in terms of the acoustic pressure, the shapes of the modes in a rectangular enclosure whose sides have unit lengths in the $x, y$ and $z$ directions. There are $\ell, m$ and $n$ half waves in the $x, y$ and $z$ directions.

Fig. $2(a)$ shows the values of the oxiginal and the approximating functions aliong the $x$ axis after an approximating function with $y=M=N=3$ was fitted to $L L=M M=N N=4$ values derived from the original function, Eqn. (4), with $\ell=m=n=1$. The gradients along this line in the $x, y$ and $z$ directions are shown in Fig. $2(b)$ for the original and the approsimating functions. Figs. 3 (a) and 3 (b) show the corresponding results for the case of $L=M=N=4$, $L L=M M=N N=5$ and $\ell=m=n=1$. Comparison of Figs. $2(b)$ and $3(b)$ indicates. that if a mode in an enclosure involves a half wave in a given direction, the corresponding approximating polynomial should incorporate at least four : coefficients. It is of interest that, for a rectangular enclosure, seven modes can occur when there is no more than one half wave in each of the $x, y$ and $z$ directions.

Fig. $4(\mathrm{a})$ shows the values of the original and approximating functions along the $x$ axis after an approximating function with $L=M=N=6$ was fitted to $L I=M M=N N=7$ values derived from the original function, Eqn. (4), with $\ell=m=n=2$. The gradients along this line in the $x, y$ and $z$ directions are shown in Fig. $4(b)$, for the original and the approximating functions. Figs, 5 (a) and 5 (b) show the corresponding results for the case of $L=M=N=7$, LL $=M M=N N=8$ and $\ell=m=n=2$. Comparison of Figs. $4(b)$ and $5(b)$ indicates that if a mode in an enclosure involves two half waves in a given direction, the corresponding approximating polynomial should incorporate at least seven coefficients.

It is of interost, that, for a rectangular onclosuxs, fifteon modes can occur when there are no more than two half waves in oach ois tha $x, y$ and $z$ directions.

Although it is possible to fit polynomials to modes which involve more than two half waves in each direction, the number of coefficients required in the approximating polynomial becomes largo as doas the number of points at which measurements must be made. The number of data points at which measurements must be made depends upon the magnitude of the error term $E$ in Eqn. (1) relative to the magnitude of the true value of the function. If more than two half waves occur in a given direction, the approximating function can be fully or partially constructed from harmonic functions instead of the polynomials used in Eqn. (2).

## III. EXPERIMENTS

The previously described procedure was applied to experimentally determining the shapes of the particle velocity medes in a small rectangular enclosure. These modeshapes can be compared with the theoretically determined modeshapes.

The internal dimensions of the rectangular enclosure and the orientation of the xyz axes system are shown in Fig. 6. The acoustic field in the enclosure was excited by a horn driver which was located in the $x=0, y=0$, $z=L_{z}$ corner of the enclosure. The horn driver, which was driven by noise which was white over the 50 Hz to 500 Hz range contained a microphone in the magnet cavity so that the volume velocity of the source could be measured. Details of the source and how it was calibrated are given in Appendix A.

Transfer functions which related the acoustic pressuros at points along Lines parallel to the $x, y$ and $z$ axis to the volume velocity of the source were measured. The points, which were equally spaced along the lines, were
 ware datermined for the first ten resonance frequencies by the coincidentquadrature procedure ${ }^{\circ}$. An approximating function was fitted to each of the ten sets of data by the previously described procedure. First, all of the sets waxe fitted with polynomials defined by $L=M=N=6$ so that the general shape of the mode in the direction of each axis could be estimated. The appropriate values of the polynomials in each direction for each set of data ", could then be fixed. If there were two half waves in a given djrection the polymomial index $L, M$ or $N$ for that direction was set to 7 on the basis of the results given in the previous section. If there was one half wave in a given direction, the polynomial index $L, M$ or $\cdot N$ for that direction was set to 4. If the acoustic pressure amplitude was nominally constant: in a given direction, the polynomial index $L, M$ or $N$ for that direction was set: to 2.

A sample of the results derived is presented in Figs. 7 and 8. The results given in Fig. 7 relate to the first mode, whose frequency was 123.8 Hz . It was found, with reference to Fig. 6, that there was one half wave in the $x$ direction and no variaition in the $y$ and $z$ directions. The amplitude of the acoustic pressure per unit volume velocity of the source varied along the $x$ axis as shown in Fig. $7(a)$. Values derived from the approximating function and from the measurements along a line close to the axis are shown. The particle velocities derived from the approximating function varied along the $x$ axis as shown in Fig. $7(b)$. Whe results given in Fig. 8 relate to the tenth mode whose frequency was 302.8 Hz . It was found, with reference to Fig, 6, that for this mode, there were two half waves in the $x$ direction; one in the $y$ direction and there was no variation

In the $z$ direction, The ninth mola, which involved two half waves in tho $z$ direction and no variation in the $x$ and $y$ directions occurred at a Erequency of 296.2 dz , only 2 lower than the frequency of the tenth mode. The anplitudes of the acoustic pressure per unit volume velocity of the source varied along the $x$ and $y$ axes as shown in Figs. $8(a)$ and $g(b)$. Values derived from the approximating function and from tho measurementa along lines close to these axes are shown, The poxticle velocities derived from the approximating function are shown in Figs. $8(c)$ and $8(d)$.

Although the results given in Figs. $7(b), 8(c)$ and $\theta(d)$ contain the expected features of themodeshapes in terms of the particle motions, they do not oxhibit the expected results at the boundaries. Thus, for example, in Fig. 7 (b), it can be seen that although the particle velocity in the $x$ direction is zero at $x=0$, it is not zero at $x=1.482 \mathrm{~m}$. This feature was found to present to a lesser extent in the corresponding results for ot war met. This situation is probably due to a number of factors.such as the non-rigid boundary of the enclosure used in the experiments and the irregular data which defined the shapes of the modes in terms of the acoustic pressure. A comparison of Figs. $7(a)$ and $8(a)$ shows that the experimental data for the first mode was in gentral more irregular than that for the tenth mode. However, if it is certain that a particular boundary is rigid, the least squares fitting procedure can be readily modified to make the gradient of the approximating function at and normal to that boundary zexo. The particle velocity at and normal to that boundary will then be zero.

## IV. CONCLUDING REMARKS

The experimental results derived from the procedure described in this paper suggest that the procedure can be developed to allow the particle motions associated with low order acoustic modes in enclosures to be found. The possibility of aisplaying these modes in animated form as is presently dons in displaying the vibrational modes of a structure then exists. Undoubtedly, superior approximating functions than the one defined by Eqn. (2) exist and several types of approximating functions are currently being investigated.

Particular attention is being paid to the ability of these approximating furctions to extract useful information fromirregular data. However, the approximating function defined by Eqn. (2) can be rapidly determined with basic computationa\% facilities; a fenture not possessed by other approximating functions which were considered.

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## APPENDIX A

## THE CALIBRATED VOLUME VELOCITY SOURCE

The calibrated volume velocity source was constructed from the driver of a horn type loudspeaker. The horn driver was modified. by grinding slits into the magnet so that the volume between the diaphragm and the magnet was freely connected to the volume axound the magnet. Pressure changes in the volume around the magnet due to the motion of the diaphragm of the driver could be measured by a microphone which was insexted into the volume around the magnet. A cross-section of the arrangement is shown in Fig. A.I. Thus by measuring the pressure with the microphone, the volume velocity produced by the source could be determined.

It can be easily shown that, if the total voluma onclosed by the diaphragm, $V_{0}$, changes by a small amount, $\delta V$, the absolute pressure, $P_{0}$, in this volume changes by a small amount, $\delta \mathrm{P}=-\delta \mathrm{V} \times\left(\gamma \mathrm{P}_{\mathrm{O}} / \mathrm{V}_{\mathrm{O}}\right)$, if the gas in the volume behaves in an adiabatic manner. The specific heats ratio is given by $\gamma$. Thus the volume velocity $(\sim)$ of the source is related to the acoustic pressure $p(t)$ in the volume behind the diaphragm by Eqn. (A.I):

$$
\begin{equation*}
U(t)=-\left(V_{0} / \gamma P_{0}\right) \times d p(t) / d t \tag{A.1}
\end{equation*}
$$

The volume enclosed by the diaphragm, $V_{o}$, being of very irregular shape, could not be accurately determined and so the calibration was made by measuring the acoustic pressure in a small accurately known volume which was attached to the throat of the driver along with the voltage generated by the microphone which was used to measure the acoustic pressure in the volume enclosed by the diaphragm. The volume velocity was derived from the acoustic pressure measured in the small accurately known volume by use of an equation of the form of Eqn. (A.1). The amplitude of the frequency response function relating the volume velocity produced by the source and the voltage generated
by the microphone is shown in Fig. A.2. This frequency response function was derived from a transfer function measurement with a white noise electrical signal being applied to the coil of the driver.

## CAPTIONS

Fig. 1 Nomenclature for measurement points.

Fig. $2(a)$ Magnitudes of the original $(-\longrightarrow)$ and the approximating ( - - $-\infty$ ) functions along the $x$ axis.
(b) Gradients in the $x, y$ and $z$ directions of the original ( $\longrightarrow$ ) and the approxtmating ( $-\infty-\infty$ ) furctions along the $x$ axis.
$L=M=N=3 ; L L \square M M=N N=4 ; \ell=m=n=1$.

Fig. 3 (a) Magnitudes of the original ( - ) and the approximating ( $-\infty-\infty$ ), functions along the x axis.
(b) Gradients in the $x, y$ and $z$ directions of the original
 and the approximating ( $-\ldots-\infty$ ) functions along the $x$ axis: $X=M=N=4 ; L L=M M=N N=5 ; \ell=m=n=1$.

Fig. $4(a)$ Magnitudes of the original $(\longrightarrow)$ and the approximating ( $-\infty-)^{-}$) functions along the x axis.
(b) Gradients in the $x, y$ and $z$ direcitions of the original $(\longrightarrow)$ and the approximatin ( $-\ldots-$ ) functions along the x axis. $L=M=N=6 ; L L=M M=N N=7 ; \ell=m \Rightarrow n=2$.

Fig. 5(a) Magnitudes of the original $(\longrightarrow$ and the approximating ( - ------) functions along the x axis.
(b) Gradients in the $x, y$ and $z$ directions of the original $(\longrightarrow$ and the approximating (-----) functions along the $x$ axis. $\mathrm{L}=\mathrm{M}=\mathrm{N}=7 ; \mathrm{LL}=\mathrm{M}=\mathrm{NN}=8 ; \ell=\mathrm{m}=\mathrm{n}=2$.

Fig. 6 , Enclosure used in experiments showing intexnal dimensions and axes oriantation.

Fig. 7 (a) Pressure amplitude per unit volume velocity amplitude along the $x$ axis from the measured values (•) and from the approximating function (-).
(b) Particle velocity amplitudes par unit volume veluaity amplitude along the x axis.
$L=4, M=N=2 ; L L=11, M M=7, N N=9$.

Fig. $8(a)$ Pressure amplitude per unit volume velocity amplitude along the $x$ axis from the measured values (•) and from the approximating functiun $\xrightarrow{?}$.
(b) Pressure amplitude per unit volume velocity amplitude along the $y$ axis from the measured values (*) and from the approximating function $\qquad$
(c) Particle velocity amplitudes per unit volume velocity amplitude along the $\mathbf{x}$ axis.
(d) Particle velocity amplitudes per unit volume velocity amplitude along the $y$ axis.
$L=7, M=4, N=2 ; L L=11, M M=7, N N=9$.

Fig. A. Cross-section of calibrated volume velocity source.

Fig. A. 2 Amplitude of frequency response curve for source.

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Fig. 1' Nomenclature for measurement points.


Fig. $2(a)$ Nagnitudes of the original $(-$ ) and the approximating ( ---- ) runctions ajong the $x$ axis.
(b) Gradients in the $x, y$ and $z$ directions of the original (-m) and the approximating (---m--) functions along the $x$ axis. $L \approx M=N=3 ; L L=M M=N N \approx 4 ; \ell=m=n=1$.

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Fig. $2(a)$ Magnitudes of the original $(\longrightarrow-$ and tho approximating ( $-\infty-\infty$ ) functions along the $x$ axis.
(b) Gradients in the $x, y$ and $z$ directions of the original $\longrightarrow \longrightarrow$ and the approximating (-m-m- functions along the $i$ axis.
$I=M=N=3 ; L L=M M=N N=4 ; 2 m m=n=1$.


Fig. $3(a)$ Magnitudes of the original $(-$ ) and the approaimating ( $-\ldots-\ldots$ )
functions along the $x$ axis.
$(b)$ Gradients in the $x, y$ and $z$ directions of the original $\longrightarrow$ and the approximating (------) functions along the $x$ axis.



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Fig. $3(a)$ Magnitudes of the original ( - ) and the approximating ( $-m-m$ )
functions 'along the $\times$ axis.
(b) Gradients in the $x, y$ and $z$ directions of the original (-) and the approximating (--m--) functions along the $\%$ axis.

$$
\mathrm{I}_{\mathrm{A}}=\mathrm{M}=\mathrm{N}=4: \mathrm{LL}=\mathrm{MM}=\mathrm{NN}=5 ; \ell=\mathrm{m}=\mathrm{n}=1 .
$$



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Fig. $4(a)$ Nagnitudes of tho oxiginal ( $-\infty$ ) and the approxinating (-mm-m) functions along the $x$ axis.
(b) Gradiants in the $x, y$ and $z$ directions of the original (-) and the apprnximating.(-mm-m) Eunctions along tho 25 axis.



$$
\begin{aligned}
& \text { Fig. } 4(a) \text { Magnitudes of tho original ( }-\infty \text { ) and the approximating ( }-m-m m \text { ) } \\
& \text { functions along the } x \text { axis. } \\
& \text { (b) Gradients in tho } x, y \text { and } z \text { dinccitions of the original } \\
& \text { and the approximating (m-m-m) functions along the } x \text { axis. } \\
& \text { 工 } m=M=N=6 ; 工 L=M A=N N=7 ; 2=m=n=2:
\end{aligned}
$$



Fig. $5(a)$ Magnitudos of tho oxiginal ( - - and the approximating ( $----\infty$ )
functions along the $x$ axis.
(b) Gradients in the $x, y$ and $z$ directions of the original $\longrightarrow$ and the approwimating (---m-m) functions along the $x$ axis.
. $L=M=N=7 ; L L m M M=N N=8 ; \ell=m=n=2$.


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Fig. 5(a) Magnitudes of tho original (-m) and the approximating functions along the $x$ axis.
(b) Gradienes in the $x, y$ and $z$ directions of the original and the approximating (-m----) functions along the $x$ axis.:
$L=M=N=7 ; L L=M M=N N=8 ; \ell=m=n=2$.


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Fig. 6
Enclosuro used in experiments showing internal dimensions and . axes orientation.


Fig. 7 (a) pressurc amplituda per unit volume velocity amplitudo along the $x$ nxis from the measured values ( $\cdot$ ) and from the approximating function $\qquad$
(b) particle velocity amplitudes per init volume velocity amplitude along the $x$ axis.
$L=4, M=N=2 ; H=11, M M=7, N N=9$.


- Fig. 7 (a) pressure amplitude per unit volume velocity amplitude along the $x$ axis from the measured values ( $($ ) and from the approximating function $(\xrightarrow{-}$.
(b) Particle velocity amplitudes per unit'volume velocity amplitude along the $x$ axis.
$L=4, M=N=2 ; C H=11, M M=7, N N=9$.


Fig. $8(a) \quad$ Pressure amplitude per unit volume velocity amplitude along the $x$ axis from the measured values (.) and from the approximating function ( ).*



## ORIGINAL. PAGE IE

 OF POOR QUALITYFig. 8(b) Pressure amplitude per unit volume velocity amplitude along the $y$ axis from the measured values (.) and from the approximating function ( ).



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Fig. $8(c)$ Particle velocity' amplitudes per unit volume velocity amplitude along the $x$ 'axis.

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## Fig. 8(d) Particle velocity amplitudes per unit volume velocity amplitude along the $y$ axis.



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Fig. A.l Cross-section of calibrated volume velocity source.
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Fig. A. $\dot{2}$ Amplitude of frequency response curve for source. ORIGINAL PAGE IS OF POOR QUALITY


