DRA

VIBRATION RESPONSE OF GEOMETRICALLY NONLINEAR ELASTIC BEAMS TO PULSE AND IMPACT LOADING

End of the Year Progress Report

NASA Grant NAG 1-158

Principal Investigator: H. Liebowitz

Senior Research Engineer: E. T. Moyer, Jr.

October 7, 1983

School of Engineering and Applied Science

The George Washington University

Washington, D.C. 20052

(NASA-CR-174495) VIBRATION RESPONSE OF GEOMETRICALLY NONLINEAR ELASTIC BEAMS TO PULSE AND IMPACT LOADING Progress Report (George Washington Univ.) 33 p HC A03/MF A01 CSCL 20K G3/39

N84-12527

Unclas 15237

VIBRATION RESPONSE OF GEOMETRICALLY NONLINEAR ELASTIC BEAMS TO PULSE AND IMPACT LOADING

By

E. Thomas Moyer, Jr.
Senior Research Engineer

October 7, 1983

School of Engineering and Applied Science
The George Washington University
Washington, D.C. 20052

ABSTRACT

The governing equations for the geometrically non-linear deformation of elastic beams subjected to dynamic bending loads are developed and solved for various initial conditions. Of primary interest is the response to pulse loading and simulated impact. Both transient and several cycle solutions are generated for the free vibration response to pulse loading. The results obtained are compared to a first mode analysis approximation.

A new model is developed to simulate impact loading by the distribution of additional mass to the elastic system and subjecting it to a velocity pulse. The governing equations are solved using second order finite differences in space and time. The solutions obtained are in reasonable agreement with experimental results previously obtained [1].

INTRODUCTION

Of fundamental importance to the design and accurate assessment of aircraft and aerospace structures is the understanding of the response of composite laminates to foreign object impact. Accurate and consistent modeling of the damage caused by these conditions can only be achieved after the basic deformation response of the system has been characterized. Of primary interest is the response to low velocity impact of foreign objects (less than 10 meters/second at 10 Joules of impact energy or less) similar to tool drop problems.

The simple square beam was chosen for study since it possesses the fourth order bending effects of flat plate systems yet is a simpler mathematical system to solve. The response of the beam system was investigated both under sharp initial velocity pulse conditions and under simulated impact conditions to study both the basic free vibration response and the impact response.

Nonlinear deformation theory was employed due to the importance of membrane effects. The buoyancy terms were discovered to contribute significantly to the total energy response of the system even in a moderately small deflection regime. The analysis also demonstrates that though the basic responses are characterized by single mode envelopes, the higher order modes cause important fluctuations which cannot be ignored.

The material properties and dimensions chosen for this study simulate the graphite-epoxy plates being tested under impact loading at the NASA Langley Research Facility. While the results of this study are not expected to match those of the tests exactly (see, for example [1]), the basic characteristics of the system (contact time, maximum displacement, percentage of membrane energy) predicted by the analysis are consistent with the experimental observations. Further refinements of the modeling and introduction of damage assessment techniques should lead to predictive impact analysis.

The basic system is solved employing second order finite difference operators in space and an explicit time integrator. The initial transient response is predicted using a graded time step technique discussed in [2]. Convergence and accuracy was investigated both by checking the convergence of the displacements and independently monitoring the total energy of the system (which should remain constant during the motion). Other methods of solution involving different time integrators and finite element approximations for the spatial derivatives will be discussed in a subsequent publication.

NONLINEAR BEAM THEORY

Consider a straight beam of square cross section (cross-sectional dimension h) having a length L. Assume that the beam is inextensible and that shear effects can be neglected. Let the origin of the coordinate system and the coordinate measure, x, be defined by Figure 1. The equilibrium conditions can be written in differential form as

$$\frac{\partial N}{\partial x} = \rho \frac{\partial^2 U}{\partial t^2}$$

$$\frac{1}{h^2} \frac{\partial^2 M}{\partial x^2} + \frac{\partial}{\partial x}\left(N \frac{\partial W}{\partial x}\right) = \rho \frac{\partial^2 W}{\partial t^2}$$

where N is the average normal stress across the cross-sectional face (i.e., the membrane stress) and M is the cross-sectional moment. It is explicitly assumed from the outset that shear terms can be neglected. U and W represent the longitudinal and transverse displacements, respectively. For an elastic, isotropic beam, assuming inextensibility, the stress-displacement relations can be written as [3]

$$M = - E I \frac{\partial^2 W}{\partial x^2}$$

$$N = E \left[\frac{\partial U}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2\right]$$

where E is Young's modulus for the material and I is the cross-sectional moment of inertia. Utilizing the stress-displacement relations in the equilibrium conditions, the governing equations can be written as

$$\frac{\partial^2 U}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = -\frac{\partial W}{\partial x}\frac{\partial^2 W}{\partial x^2}$$

$$\frac{\partial^4 W}{\partial x^4} - \frac{1}{R^2}\left[\frac{3}{2}\left(\frac{\partial W}{\partial x}\right)^2\frac{\partial^2 W}{\partial x^2} + \frac{\partial W}{\partial x}\frac{\partial^2 U}{\partial x^2}\right.$$

$$\left. + \frac{\partial^2 W}{\partial x^2}\frac{\partial U}{\partial x}\right] = -\frac{1}{a^2}\frac{\partial^2 W}{\partial t^2}$$

where the following definitions have been adopted

$$c^2 = E/\rho$$

$$R^2 = I/h^2$$

$$a^2 = c^2 R^2$$

The quantity ρ is the mass density of the material and A is the cross-sectional area of the beam.

The total energy of the beam can be decomposed into bending energy, membrane energy and kinetic energy. By definition, these quantities can be written as

$$E_K = \begin{Bmatrix} \text{Kinetic} \\ \text{Energy} \end{Bmatrix} = \frac{1}{2} \int_0^L \rho \left[\left(\frac{\partial U}{\partial t}\right)^2 + \left(\frac{\partial W}{\partial t}\right)^2 \right] A \, dx$$

$$E_B = \begin{Bmatrix} \text{Bending} \\ \text{Energy} \end{Bmatrix} = \frac{1}{2} \int_0^L \frac{M^2}{E\ I} \, dx$$

$$E_M = \begin{Bmatrix} \text{Membrane} \\ \text{Energy} \end{Bmatrix} = \frac{1}{2} \int_0^L \frac{N^2}{E} \ A \, dx$$

The total energy is given by the sum of these components as

$$E_T = \begin{Bmatrix} \text{Total} \\ \text{Energy} \end{Bmatrix} = E_K + E_B + E_M$$

Utilizing the stress-displacement relations, the bending and membrane components can be written as

$$E_B = \frac{E\ I}{2} \int_0^L \left(\frac{\partial^2 W}{\partial x^2}\right)^2 dx$$

$$E_M = \frac{E\ A}{2} \int_0^L \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x}\right)^2 \right]^2 dx$$

The displacement formulation will be utilized for computational convenience.

BOUNDARY CONDITIONS AND MATERIAL MODELING

The boundary conditions of interest for impact problems are mainly those which simulate the transverse impact response of a beam or plate with fixed (built in) edges or pinned edges. The stress state and subsequent damage will be more severe in the fixed end problem, therefore, the analysis is restricted to these conditions. For fixed or built in edges, the boundary conditions for a geometrically nonlinear beam can be written as

$$W(x = 0) = 0. \qquad W(x = L) = 0.$$
$$\partial W(x = 0)/\partial x = 0. \qquad \partial W(x = L)/\partial x = 0.$$
$$U(x = 0) = 0. \qquad U(x = L) = 0.$$

The material properties used in this study are chosen to simulate the response of typical graphite-epoxy composite plates. Typical average quasi-isotropic properties are

$$E = 7.2135 \text{ E} + 10 \text{ Pascals}$$

$$\rho = 1.6000 \text{ E} + 03 \text{ Kg/m}^3$$

$$\nu = 0.33$$

For a circular plate with clamped edges the bending stiffness, K and natural frequency, ω_0 are given by [4]

$$K = \frac{4\pi\ E\ h^3}{3(1 - \nu^2)\ r^2}$$

$$\omega_0 = 3.125\ \frac{h}{r^2}\sqrt{\frac{E}{\rho}}$$

where r is the plate radius and h is the plate thickness. Assuming a plate with radius 2.54 cm and thickness 1.03 mm, the bending stiffness and natural frequencies are given by

$$K = 6.0843\ E + 05\ J/m^3$$

$$\omega_0 = 3.4150\ E + 04\ /sec$$

For a linear, double cantilevered beam of square cross section, the bending stiffness and natural frequency are given by [5]

$$K = 192\ \frac{E\ I}{L^3}$$

$$\omega_0 = \frac{(4.730)^2}{L^2}\sqrt{\frac{E\ I}{\rho\ A}}$$

Equating the beam and plate parameters, the equivalent beam length and thickness are

h = 3.5565E-03 meters

L = 6.7203E-02 meters

which are the dimensions employed in the present solution.

INITIAL CONDITIONS AND IMPACT MODELING

To investigate the stability and accuracy of the solution method, a beam subjected to pulse initial velocity conditions over a short portion of the midspan of the beam was studied. The initial conditions for this problem are

$$W(x,\ t = 0.) = 0.$$
$$\partial W(x,\ t = 0.)/\partial t = F(x)$$
$$U(x,\ t = 0.) = 0.$$
$$\partial U(x,\ t = 0.)/\partial t = 0.$$

where

$$F(x) = V_0 \sin\left(\frac{\pi x'}{L'}\right) \qquad x_1 \le x \le x_2$$
$$= 0 \qquad \text{otherwise}$$
$$x' = x - \left(\frac{L - L'}{2}\right) \ ; \qquad L' = \frac{L}{10}$$
$$x_1 = \frac{L - L'}{2} \qquad x_2 = \frac{L + L'}{2}$$

An initial velocity (V_0 = 243. m/s) was chosen corresponding to an initial energy of 4.06 Joules.

To simulate the response of a beam to foreign body impact, mass in excess of the beam's mass was uniformly distributed over a central sector of the beam of length $L' = L/10$. The mass added was 113 grams which is typical of impactor objects of interest in tool drop problems. The choice of L' was to simulate typical contact areas for foreign body impact problems.

An initial velocity profile using a sine squared distribution over the midsection of the beam was chosen to transfer the energy of impact. The initial conditions are given as

$$W(x,\ t = 0.) = 0.$$

$$\frac{\partial W}{\partial t}(x,\ t = 0.) = G(x)$$

$$U(x,\ t = 0.) = 0.$$

$$\frac{\partial U}{\partial t}(x,\ t = 0.) = 0$$

where

$$G(x) = V_0 \sin^2 \left(\frac{\pi x'}{L'}\right) \qquad x_1 \leq x \leq x_2$$

$$= 0 \qquad \text{otherwise}$$

$$x' = x - \left(\frac{L - L'}{2}\right) \ ; \qquad L' = \frac{L}{10}$$

$$x_1 = \frac{L - L'}{2} \ ; \qquad x_2 = \frac{L + L'}{2}$$

$V_0 = 7.0$ m/s was chosen corresponding to an impact energy of 2.01 Joules which is typical of low velocity impact energies of interest.

FINITE DIFFERENCE APPROACH

The governing differential equations for beam problems form a system of partial differential equations in one space variable (x) and time. To solve these equations numerically, the spatial derivatives are approximated using finite difference operators. The domain is divided into N discrete points. The derivative at any point, i, can be approximated by

$$\left.\frac{\partial Y}{\partial x}\right|_{x = x_i} = \frac{Y_{i+1} - Y_{i-1}}{2\,\Delta x} + 0(\Delta x^2) \quad 2 \leq i \leq (N - 1)$$

where

$$Y(x = x_i) = Y_i$$

$$\Delta x = x_{i+1} - x_i$$

to second order in Δx. Using the same order of approximation, the higher derivatives can be written as

$$\left.\frac{\partial^2 Y}{\partial x^2}\right|_{x = x_i} = \frac{Y_{i+1} - 2Y_i + Y_{i-1}}{(\Delta x)^2} \quad 2 \leq i \leq (N - 1)$$

$$\left.\frac{\partial^4 Y}{\partial x^4}\right|_{x = x_i} = \frac{Y_{i+2} - 4Y_{i+1} + 6Y_i - 4Y_{i-1} + Y_{i-2}}{(\Delta x)^4}$$

$$3 \leq i \leq (N - 2)$$

for an arbitrary function $Y(x,\ldots)$.

Using these central difference operators, the governing equations for the nonlinear beam vibrations can be written in the form

$$\frac{d^2 W_j}{d t^2} = F(W_k, U_\ell) \qquad\qquad j, k, \ell \leq N$$
$$\frac{d^2 U_j}{d t^2} = G(W_k, U_\ell)$$

Thus the problem is reduced to solving this set of coupled ordinary differential equations.

Second order central differences have been chosen for this work due to experience with the governing system obtained in the preliminary studies carried out to date. Employing higher order operators would produce a poorly conditioned system due to the high number of modes present in the problems of interest. Employing first order operators requires too many spatial points to be employed to achieve accuracy in a finite amount of computing time. Generally, both parabolic and hyperbolic systems are discretized most efficiently using second order difference operators in space [6].

The time integration was performed using an explicit second order central difference approach. The difference operator for a single variable, Y, can be written as

$$\frac{d^2\, Y(t)}{dt^2} = \frac{1}{(\Delta t)^2}\,[Y(t + \Delta t) - 2\,Y(t) + Y(t - \Delta t)]$$

Using this time integrator, the solution of the beam equations can be written in the form

$$\begin{aligned} W_j\,(t + \Delta t) &= 2\,W_j\,(t) - W_j\,(t - \Delta t) \\ &\quad + (\Delta t)^2\,F(W_k,\,U_\ell) \\ U_j\,(t + \Delta t) &= 2\,U_j\,(t) - U_j\,(t - \Delta t) \\ &\quad + (\Delta t)^2\,G(W_k,\,U_\ell) \end{aligned} \qquad j,\,k,\,\ell \leq N$$

The time step, Δt, is chosen to insure stability and accuracy in the solution. Choice of proper time steps is linked to the mesh density as discussed in [2].

The domain was divided into 1000 spatial increments for the initial transient response as is discussed in [2]. This density was reduced to 500 spatial increments after 1 milli-second of the response (less than .001 of the period). The error in the energy was calculated to establish consistency. Discretizations of 1000 increments were also carried for several time steps for comparison. The relative difference between the 2 discretizations was less than .01% after the first millisecond under both pulse and impact initial conditions. Further reductions in mesh incrementation led to errors unacceptable (i.e., over 1% relative difference). Five hundred (500) increments was established as the minimum density to achieve an accurate solution.

The initial time step chosen in both solutions was 1.E-14 seconds. After 100 time steps, this was increased to 1.E-12 seconds. After an additional 100 time steps the step size was increased to 1.E-10 seconds. This time step was continued until the mesh density was changed after 1.E-06 seconds. The final time step used for the remainder of the calculation was 1.E-09 seconds. The rationale for this approach and a full discussion of accuracy and stability are given in [2].

RESULTS AND DISCUSSION

The center point displacement for the first problem studied with a pulse initial velocity in the form of a sine wave (described previously) is shown in Figure 2. The calculation was carried out for more than 2 full periods of oscillation. Convergence was checked by running several portions of the calculation with both coarser and finer meshes. After the initial startup (i.e., the first microsecond), a mesh of 500 points was sufficient for convergence.

While the basic response is similar to a first mode analysis, many higher order modes are obviously involved. The period of the oscillation is about 1.73E-04 seconds. This is 6.2% smaller than would be predicted by a linear single-mode analysis. This is consistent with the known influence of buoyancy terms on vibration analysis [7]. The maximum center point displacement was 1.68 millimeters or 2.5% of the length of the beam. It is also 48% of the beam's thickness. Even in this relatively small deflection range, the nonlinearity is important.

As a measure of the accuracy and consistency of the solution, the total energy was calculated. It remained constant to within 0.5% during the entire solution. It is necessary to resolve the energy accurately for two reasons. First, the energy is a good measure of the performance of a finite difference solution [2]. Second, the energy and its components are important parameters when damage prediction is considered. It is interesting, therefore to look at the energy response of the beam system.

Figure 3 is a plot of the Kinetic Energy as a function of time. While the response is contained within a first mode envelope, the high frequency effects are extremely evident. The instantaneous fluctuations are as high as 30% of the envelope response. Figure 4 is a plot of the Bending Energy as a function of time. It also is contained within a single mode envelope but with significant fluctuations due to high mode effects. The basic modal response is approximately 180 degrees out of phase with the kinetic energy as would be expected. Close examination reveals that the fluctuations on the bending energy curve are also 180 degrees out of phase (approximately) with the kinetic energy fluctuations. This is a strong indication that the present solution technique is a good approach for resolving vibration problems with multiple mode responses.

Figure 5 is a plot of the Membrane Energy as a function of time. This energy component is assumed zero in a linear analysis. The maximum membrane energy is 17.9% of the total energy indicating the necessity of a nonlinear analysis. The curve indicates that the membrane response is not primarily a modal envelope response. No definitive periodicity is evident during the time period studied. The high order modes are responsible for a significant portion of the membrane energy as is evident by the highly oscillatory response.

The second problem involved studying the response of a nonlinear beam to impact loading. Impact was simulated by adding additional concentrated mass to a central portion of the beam and subjecting the same section to an initial velocity and added mass were chosen to supply the initial energy consistent with impact energy transfer occurring in tool drop problems [1]. The response of the beam was calculated until the center point displacement passed the initial zero point which would correspond to the point at which contract in an impact problem was lost. This corresponds is the first half cycle of the beams response.

Convergence was checked by varying the density of the grid and the time step size at various points in the calculation. Less than 0.5% local fluctuation in the displacements was chosen as the criteria for convergence. As an independent check on the calculation, the total energy of the system was monitored continuously. Using a density of 1000 mesh points for the first microsecond and subsequently a density of 500 mesh points, the above convergence criteria was satisfied. The initial time stepping technique (described in [2]) employed continually varying time steps from t = 1.E-14 seconds to t = 1.E-10 seconds during the first microsecond. With a mesh density of 500 points (after the initial startup) time steps of t = 1.E-09 seconds were sufficient for convergence. During the entire calculation, the total energy remained constant to within 3% of the energy transferred by the initial conditions.

Figure 6 is a plot of the center point displacement as a function of time during the first half cycle. The contact time (or half period) predicted was 1.858 milliseconds which is consistent with experimental observations for this type of problem [1]. The maximum displacement was 2.01 millimeters (3% of the length of the beam and 56.5% of the thickness). The response is obviously modal with no discernible fluctuations.

Figure 7 is a plot of the Kinetic Energy as a function of time. The response is contained within a first mode envelope but with significant fluctuations (on the order of 15-20%). While not evident in the center point response, the higher order effects have significant influence on the energetics of the vibrations. Figure 8 is a plot of the Bending Energy as a function of time. Again the response is contained within a first mode envelope with nontrivial fluctuations. A careful examination shows that both the modal response and the fluctuations on the bending energy curve are 180 degrees out of phase with the kinetic energy.

Figure 9 is a plot of the Membrane Energy as a function of time. For this problem, the membrane energy also follows a modal response. The fluctuations due to the higher modes area as much as 40% of the total membrane energy (at certain instances). The maximum membrane energy is 12% of the total energy response. The effects of the nonlinearity cannot be ignored even in this moderately small deflection regime.

CONCLUDING REMARKS

The results generated in this study demonstrate a simple approach to the solution of impact loading problems for low velocity and moderate deflection applications. A model is proposed which accounts for the energy and stiffness properties transferred by an impacting object on the system. While the method is computationally cumbersome and computer runtimes are significant, the approach is shown to be both consistent and accurate when compared with known experimental results.

This study also reinforced the necessity for an independent numerical check for accuracy and stability for nonlinear problems. It was necessary to check both the displacement convergence and the energy conservation to avoid erroneous results. The energy check is a strong approach for the finite difference method as it is a second order quantity in the displacement derivatives (the primary variables and, thus, the most accurate results are the displacement components).

Investigations are currently being carried out to determine optimal numerical approaches to the solution of the governing system. Refinements to the impact model and the direct comparison with experimental results are also being studied.

ACKNOWLEDGEMENTS

The authors would like to thank The NASA Langley Research facility and Dr. Wolf Elber for their encouragement and assistance in this work. This study was sponsored by NASA Grant #NAG 1-158.

REFERENCES

[1] W. Elber, "Deformation and Failure Mechanics in Low-Velocity Impacts on Thin Composite Plates." NASA Langley Research Center, May, 1983.

[2] E. T. Moyer, Jr., "Energy Conservation in the Transient Response of Nonlinear Beam Vibration Problems Subjected to Pulse Loading - A Numberical Approach." Accepted for publication in Computers and Structures, June, 1983.

[3] H. N. Chu and G. Herrmann, "Influence of Large Amplitudes on Free Flexural Vibrations of Elastic Plates." J. Appl. Mech., Vol. 23, pp. 532-540, 1956.

[4] S. Timoshenko and S. Woinowsky-Krieger. Theory of Plates and Shells, McGraw-Hill, New York, 1959.

[5] S. Timoshenko, D. H. Young and W. Weaver. Vibration Problems in Engineering, John Wiley and Sons, New York, 1974.

[6] B. Carnahan, H. A. Luther and J. O. Wilkes. Applied Numerical Methods, John Wiley and Sons, New York, 1969.

[7] C. Y. Chia. Nonlinear Analysis of Plates, McGraw-Hill, New York, 1980.

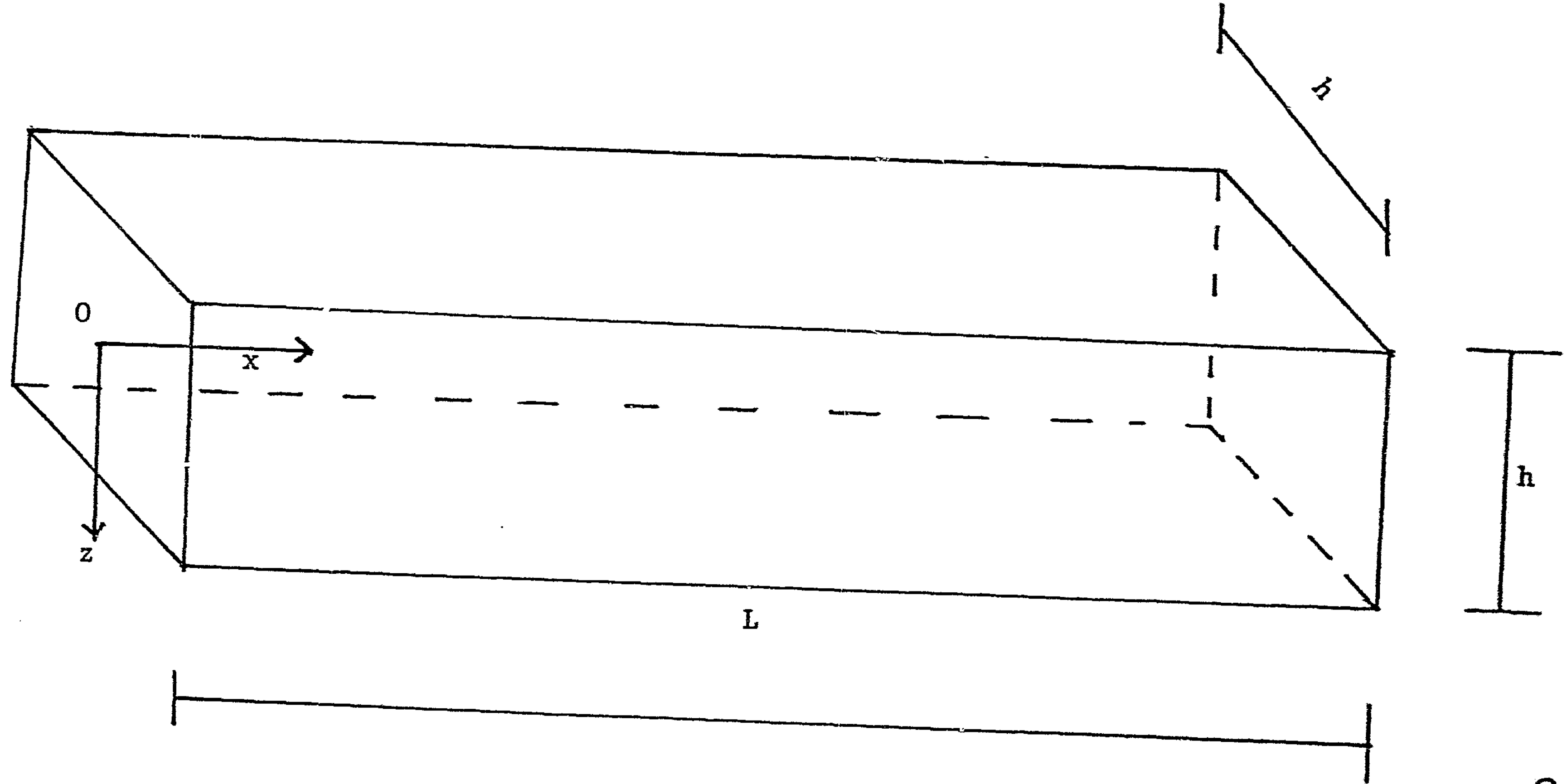

Figure 1: Geometry And Coordinates For Beam Vibration Studies.

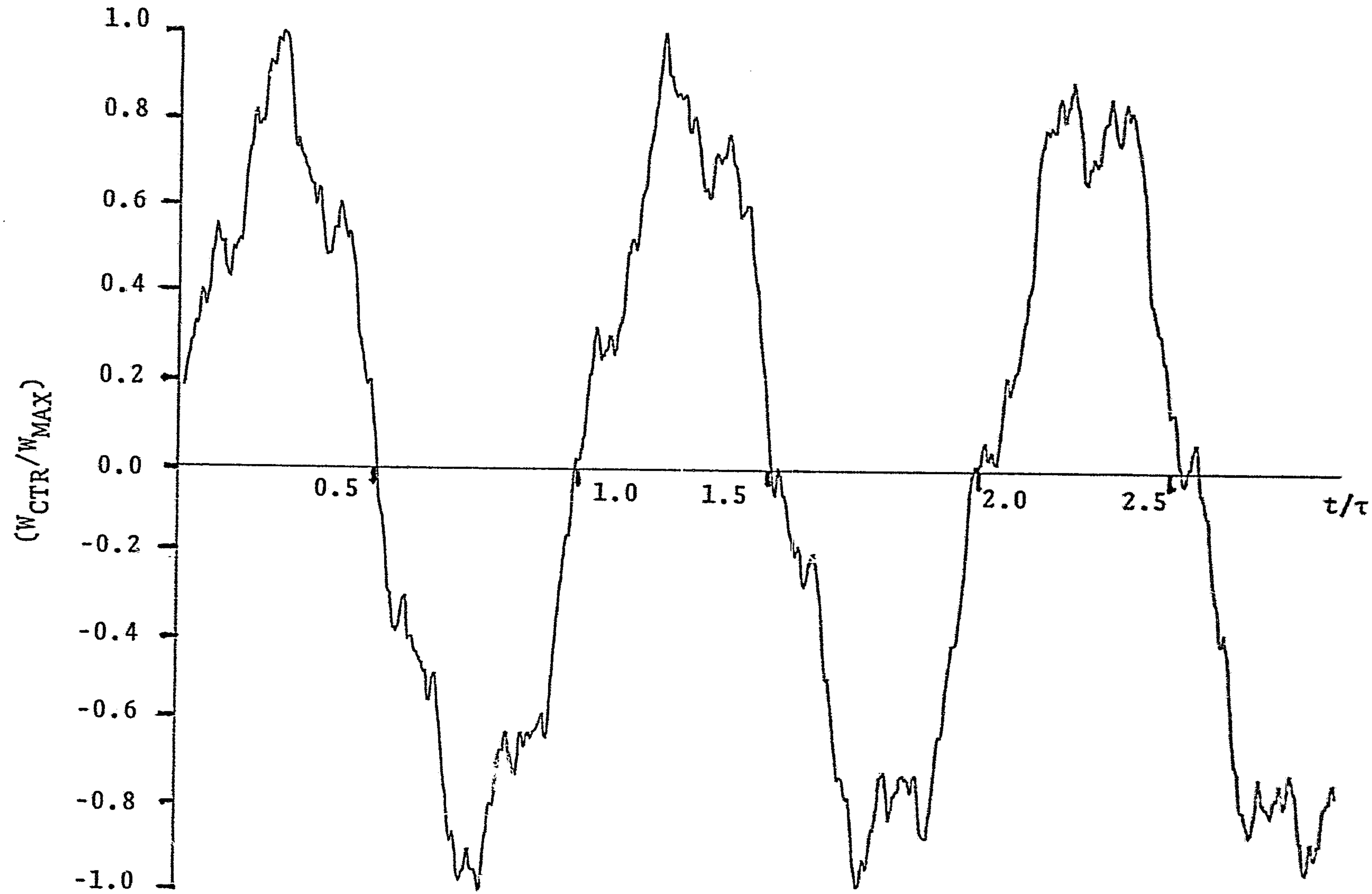

Figure 2: Center Point Displacement vs. Time.

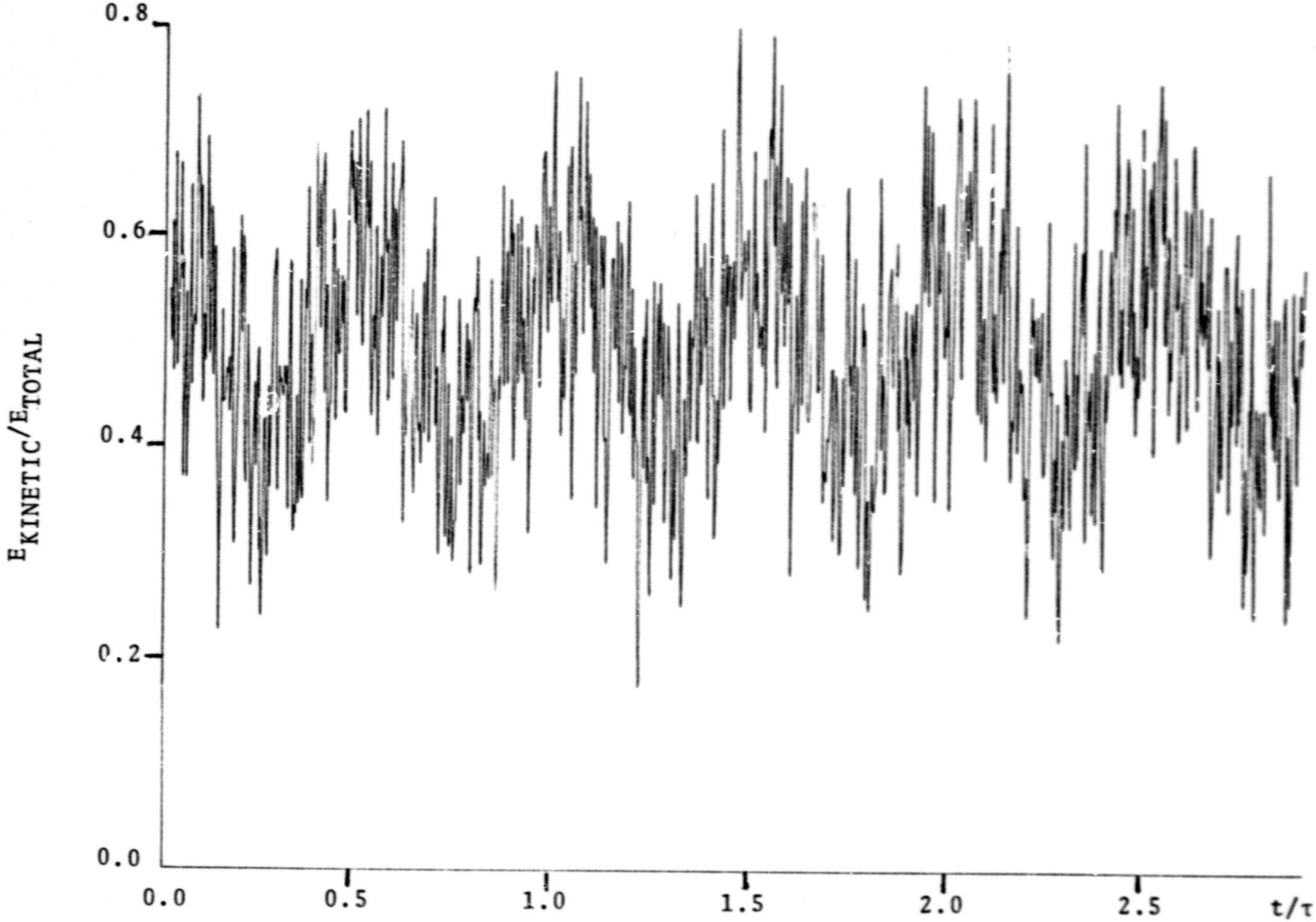

Figure 3: Kinetic Energy vs. Time.

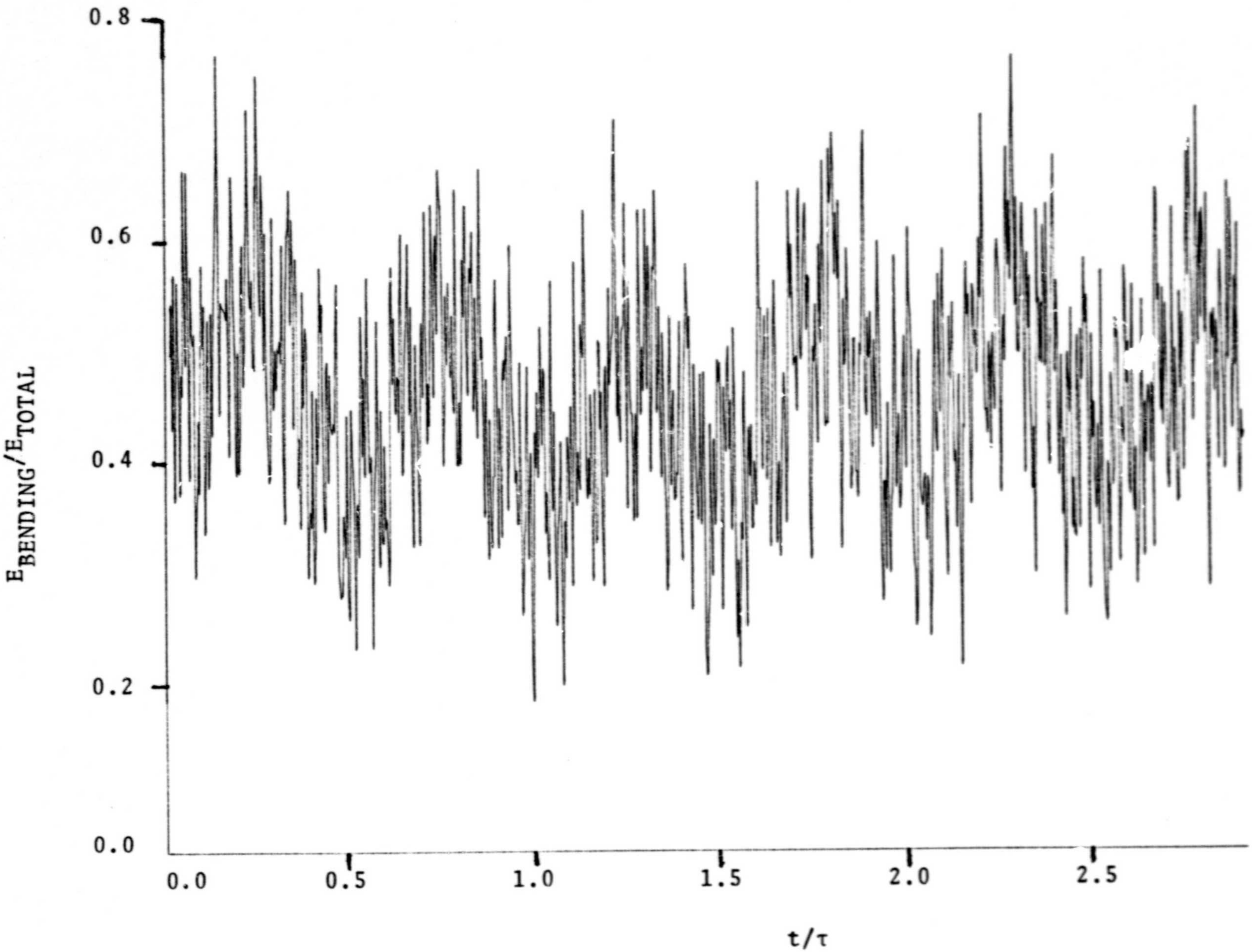

Figure 4: Bending Energy vs. Time.

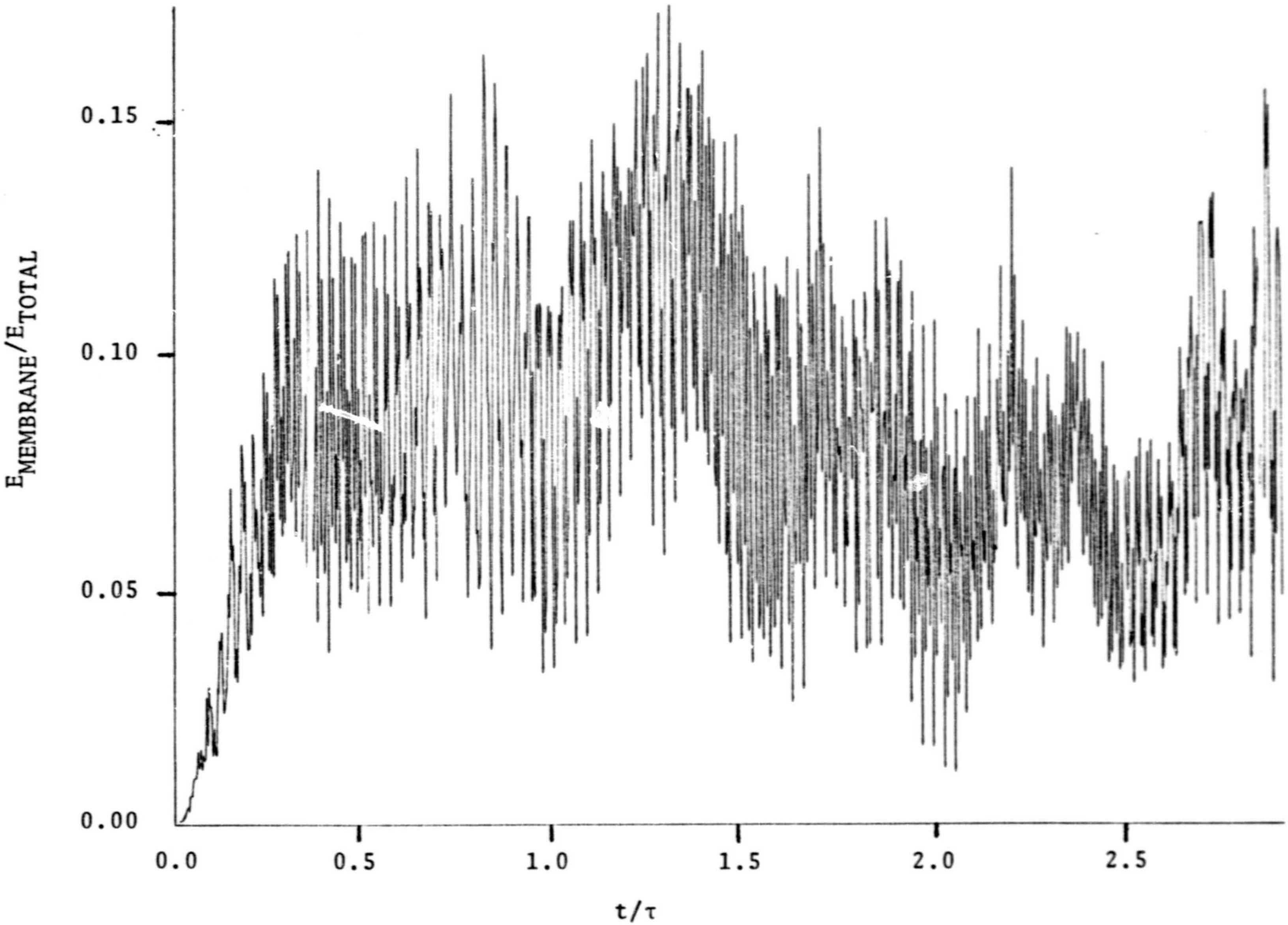

Figure 5: Membrane Energy vs. Time.

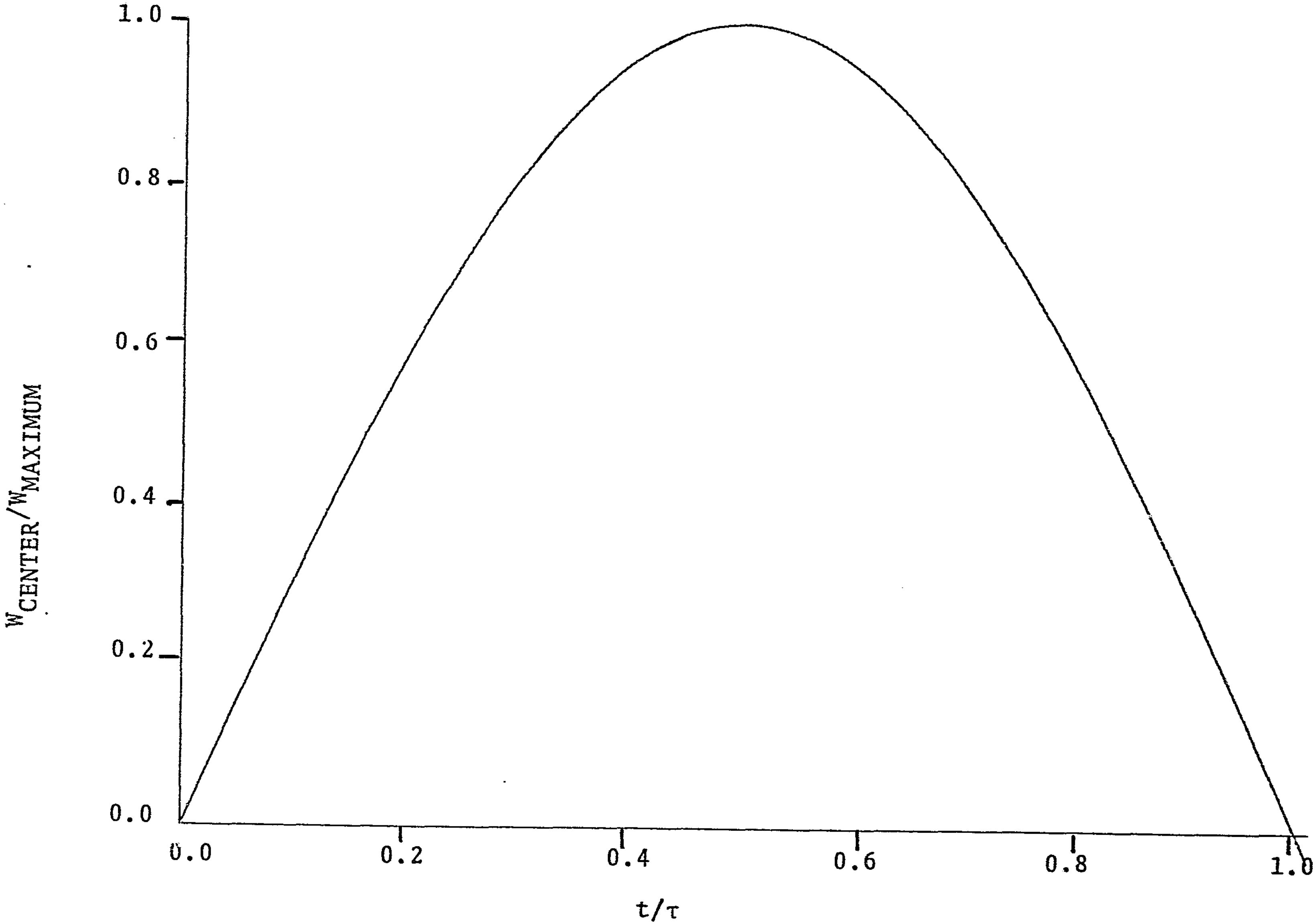

Figure 6: Center Point Displacement vs. Time Impact Loading.

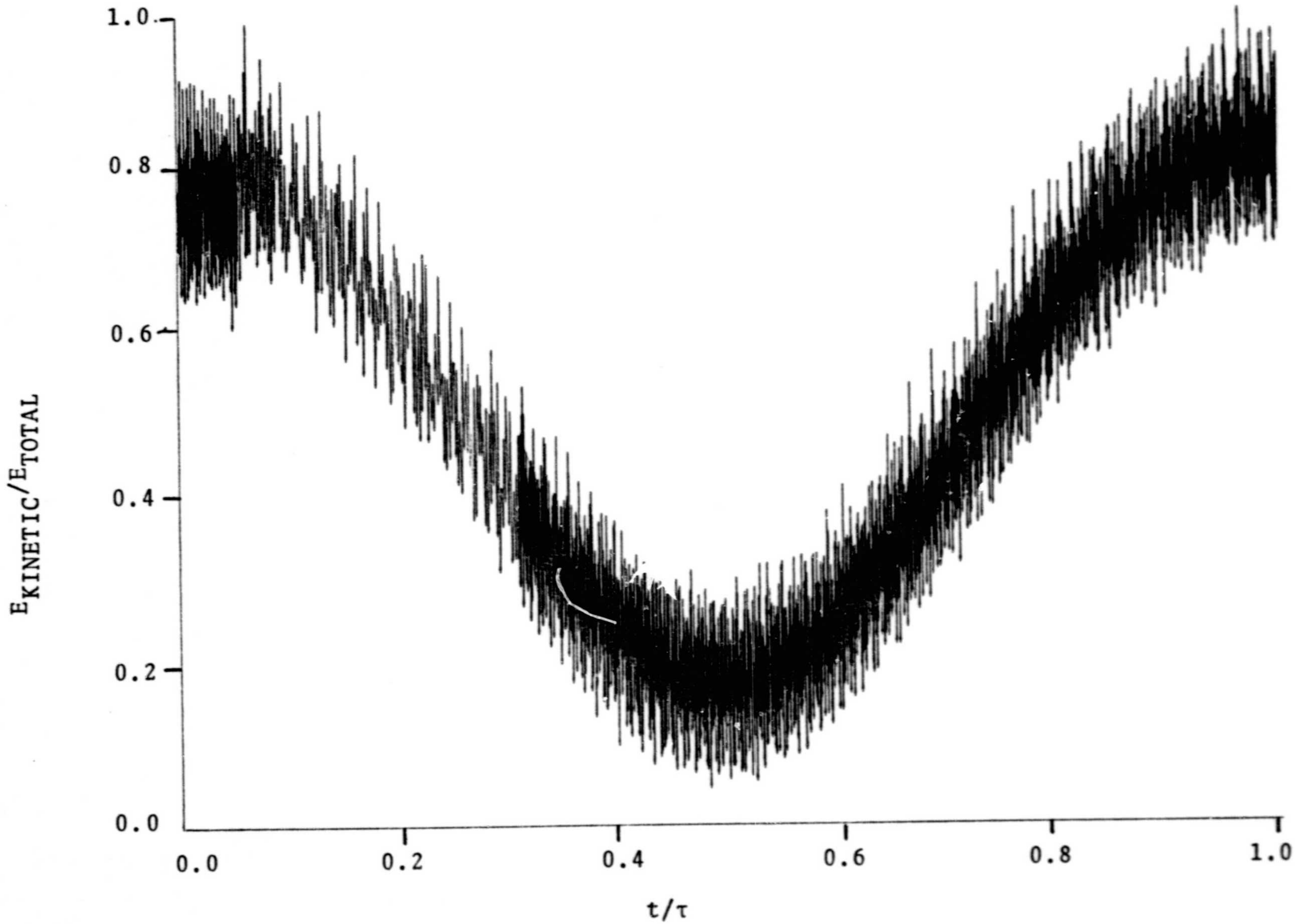

Figure 7: Kinetic Energy vs. Time Impact Loading.

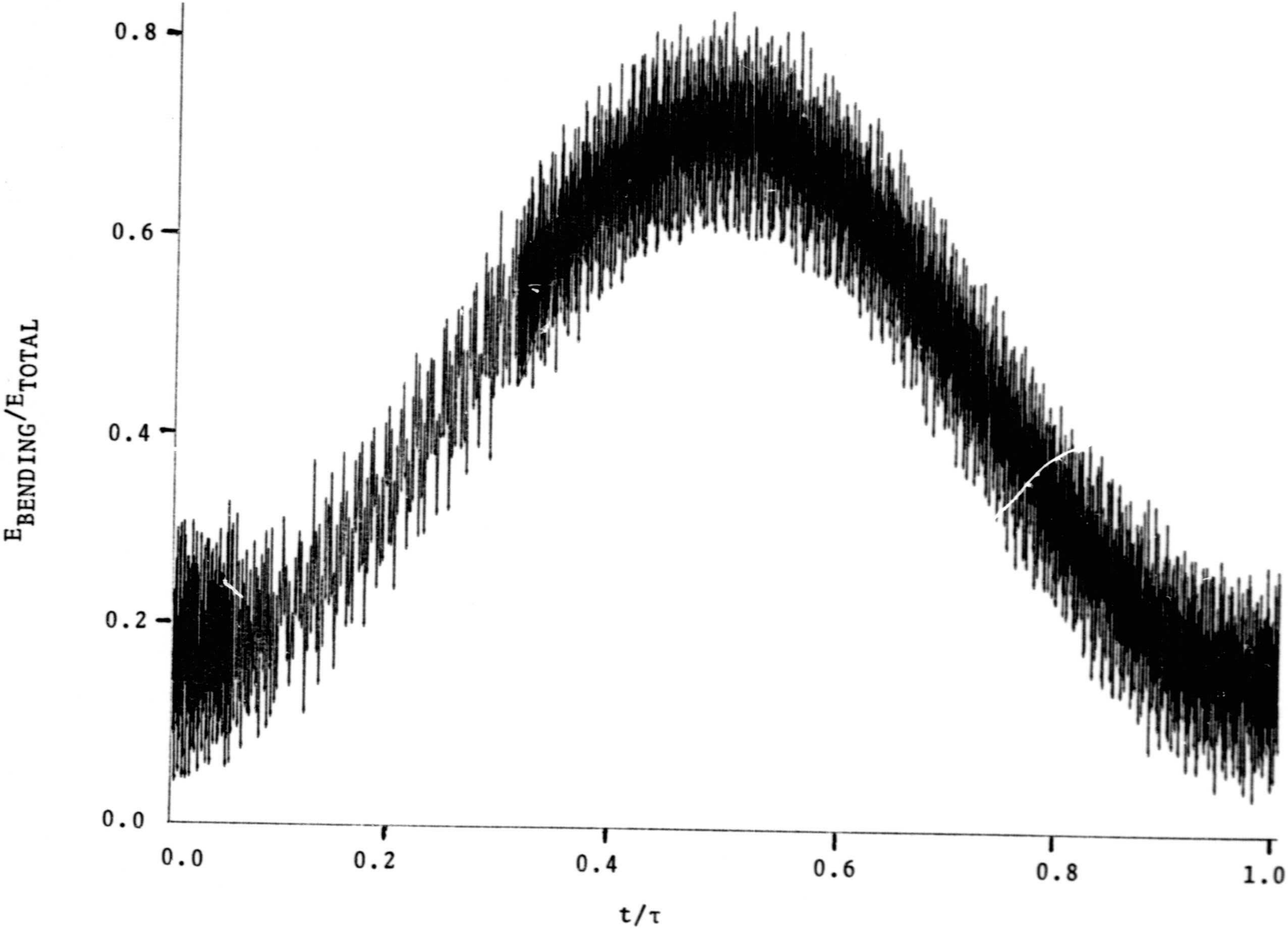

Figure 8: Bending Energy vs. Time Impact Loading.

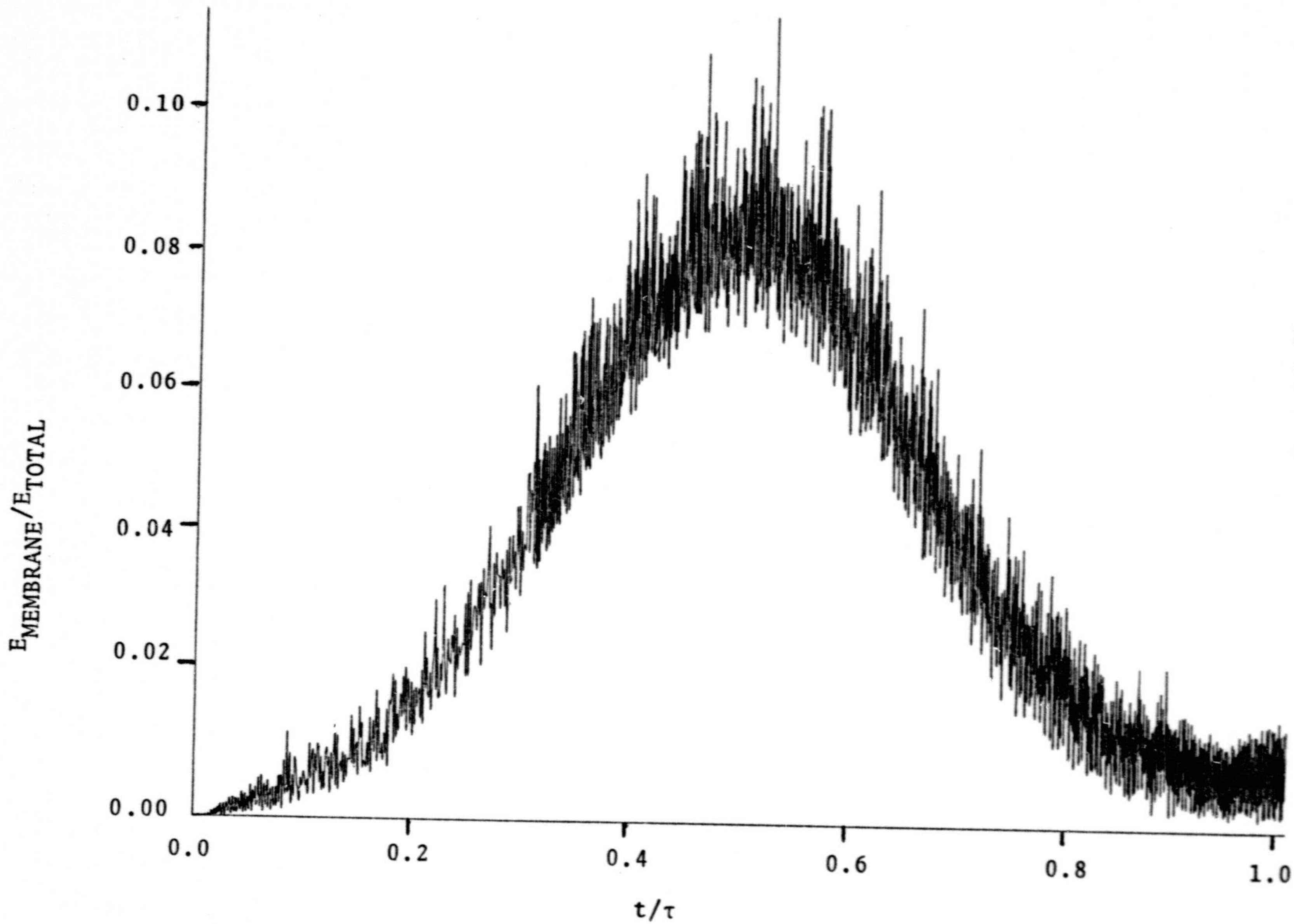

Figure 9: Membrane Energy vs. Time Impact Loading.