NASA
Technical
Paper
2245

December 1983

The Disturbance Flow Field Produced by an Evolving Vortex

T. B. Gatski

NASA

25th Anniversary
1958-1983

NASA
Technical
Paper
2245

1983

The Disturbance Flow Field Produced by an Evolving Vortex

T. B. Gatski

Langley Research Center
Hampton, Virginia

NASA
National Aeronautics
and Space Administration

Scientific and Technical
Information Branch

1983

SYMBOLS

a_0, a_1, a_2, a_3	numerical constants for parabolic velocity profile
b_0, b_1, b_2, b_3	numerical constants for parabolic velocity profile
c	phase velocity of vortex
c_o	amplitude of vortex
K	circulation constant
$O(\ )$	order of magnitude
Q	inner region independent variable
R	Reynolds number of undisturbed flow
t	time
u	velocity in streamwise direction
U_∞	free-stream velocity
v	velocity in transverse direction
x	streamwise coordinate direction
x'	streamwise coordinate in convective frame
$\tilde{x}$	streamwise coordinate in inner region
y	transverse coordinate
y'	transverse coordinate in convective frame
$\tilde{y}$	transverse coordinate in inner region
Γ	circulation of vortex
δ, δ_0	boundary layer thickness
ε	small matching parameter
μ	small parameter governing size of inner inviscid region
ν	kinematic viscosity
ϕ	arbitrary phase angle
ψ	stream function
Ω	vorticity

Subscripts:

c	center of vortex
i	inner region
np	nonparallel flow
o	outer region
p	parallel flow
s	+ when analysis is for $y > y_c$; - when analysis is for $y < y_c$
0,1,2	order of approximation

SUMMARY

The flow field of a vortex in a viscous shear flow is found by constructing a uniformly valid asymptotic expansion consisting of an inner solution field represented, to lowest order, by a two-dimensional, nonlinear, inviscid Stuart vortex and an outer solution field represented, to lowest order, by either a two-dimensional parallel or self-similar viscous flow. The technique involves scaling both the transverse and streamwise coordinates in the vicinity of the vortex as well as allowing for a "slow" variation of the outer viscous flow. Criteria are established for both the size of the vortical structure and proximity to the boundary surfaces. The composite solution is a consistent mathematical picture of the flow field at a fixed streamwise location as the vortical structure evolves past this point. Such a formulation is also useful in the specification of boundary or initial conditions in numerical fluid dynamic calculations, where an inconsistent setting of these conditions leads to spurious results for rather long computation times.

1. INTRODUCTION

Of general interest in a broad class of wall bounded flows is the dynamic evolution of vortical structures through the flow. In general, these structures are three-dimensional and their downstream evolution is exceedingly complex. In fact, as yet no overall mathematical description of such entities has been formulated. It is one of the intentions here, however, to establish a framework, based on first principles, which may form a basis for more detailed analytical studies. Another motivation concerns the establishment of boundary and initial conditions in numerical experiments. In such problems it is of great importance that any distribution of values, for the dependent variables, be consistent so that no extraneous or nonphysical results contaminate the solution field. This is especially true when such a variable as pressure is required from the numerical solution.

The mathematical framework that is used here is the method of matched asymptotic expansions. An inner solution field is constructed which consists of a two-dimensional vortical structure. The outer solution field is taken to be an otherwise undisturbed laminar two-dimensional parallel or self-similar viscous flow field. In order to allow for a finite-amplitude disturbance field, both the transverse and streamwise coordinate directions in the inner region are scaled by a suitably defined small parameter. Since such a short wavelength solution field must, in general, be coupled to the more slowly varying outer flow, a two-variable expansion procedure in the streamwise direction is necessitated. The coordinate basis allows for the complete description of the inner flow field. In the outer field, the scaled transverse coordinate is replaced by a transverse coordinate characterizing the overall size of the boundary layer flow, and the streamwise coordinates are left unaltered.

It has been argued that the dominant dynamics in the vicinity of the vortex are inviscid (e.g., ref. 1) and that viscous effects, which are omnipresent, play a secondary role in the overall features of the vortex. With this as a basis, an exact solution to the appropriately scaled nonlinear, inviscid vorticity transport equation, the Stuart vortex solution, is taken as the inner solution field to lowest order. It is found that, to lowest order, this solution is easily matched to a class of parallel and nonparallel viscous dominated flows. The composite solution field is constructed from the lowest order inner and outer solutions, and higher order equations are used to identify the inner variable scaling parameter. Finally, restrictions on the proximity of the vortex to boundary surfaces are derived from the requirement of zero component velocities on the boundaries.

2. INNER SOLUTION FIELD FOR EMBEDDED VORTEX

Consider a two-dimensional vortical structure evolving in an otherwise undisturbed two-dimensional viscous flow. The underlying dynamics of the vortex is inviscid and its evolution is nonlinear. With these basic requirements, one can deduce from first principles a vortical structure which is embedded in an otherwise viscous dominated flow. As indicated in section 1, the method of matched asymptotic expansions is used to construct a uniformly valid distribution of flow variables in a class of parallel and nonparallel wall bounded laminar flows.

An appropriate frame of reference must first be established in the flow. At some time t_o, the vortex with center at (x_c, y_c) is assumed to be moving uniformly with constant phase velocity c. The value c used is taken as the Eulerian velocity at the point (x_c, y_c) in the undisturbed flow field. The vortex flow is then stationary in the coordinate system

$$x' = x - ct \tag{1}$$

$$y' = y - y_c \tag{2}$$

where x' and y' are the streamwise and normal coordinate directions, respectively, and $x' = 0$ when $x = x_c$ and $t = t_o$. Such a description allows for the constructed vortex to be placed in a steady viscous outer flow field which is either independent of x itself or where the flow has reached some self-similar state. In addition, the coordinate lengths x' and y', appearing in equations (1) and (2), have been scaled by some characteristic length of the flow such as channel height or local boundary layer thickness. This, of course, is necessitated by the need for $O(1)$ scaling in both the inner and outer solution fields. The physical problem of interest requires vortical structures of small spatial extent relative to the characteristic scales of the overall flow. Such a requirement dictates a rescaling of the independent variables in the region dominated by the vortical structure. This yields a stretching of the y' coordinate and the decoupling of the streamwise dependence into a slow variable, representing the slow divergence of the overall flow, and a fast variable, representing the locally rapid streamwise variation of the disturbed vortical flow. The governing differential equations for the embedded vortex motion are then written in terms of the inner variables:

$$x', \qquad \tilde{x} = \frac{x'}{\mu}, \qquad \tilde{y} = \frac{y - y_c}{\mu} \tag{3}$$

where μ is a small scaling parameter which is a function of the inverse Reynolds number and is chosen such that in the vicinity of the vortex, $\tilde{x}$ and $\tilde{y}$ are $O(1)$. In general, $\tilde{x}$ can be expressed as a series expansion of x', which would account for any wave-number variations at higher order; however, to the order of the analysis, these higher order terms can be neglected. In order to apply equation (3) to slowly diverging flows and maintain a uniform phase velocity along the symmetry line $\tilde{y} = 0$, it is necessary to assume a dependence of y_c on the slow variable x',

$$\tilde{y} = \frac{1}{\mu}\left[y - y_c(x')\right] \tag{4}$$

This two-dimensional problem is formulated in terms of the stream function and vorticity, and in terms of the previously defined inner variables, the governing differential equations can be written as

$$u = \frac{1}{\mu}\frac{\partial \psi}{\partial \tilde{y}} \tag{5}$$

$$v = -\frac{\partial\psi}{\partial x'} - \frac{1}{\mu}\frac{\partial\psi}{\partial\tilde{x}} + \frac{1}{\mu}\frac{dy_c}{dx'}\frac{\partial\psi}{\partial\tilde{y}} \tag{6}$$

$$\Omega = -\frac{1}{\mu^2}\nabla^2\psi - \frac{\partial^2\psi}{\partial x'^2} - \frac{2}{\mu}\frac{\partial^2\psi}{\partial\tilde{x}\,\partial x'} + \frac{1}{\mu}\frac{d^2y_c}{dx'^2}\frac{\partial\psi}{\partial\tilde{y}} + \frac{2}{\mu}\frac{dy_c}{dx'}\frac{\partial^2\psi}{\partial\tilde{y}\,\partial x'}$$

$$+ \frac{2}{\mu^2}\frac{dy_c}{dx'}\frac{\partial^2\psi}{\partial\tilde{y}\,\partial\tilde{x}} - \frac{1}{\mu^2}\left(\frac{dy_c}{dx'}\right)^2\frac{\partial^2\psi}{\partial\tilde{y}^2} \tag{7}$$

$$\frac{\partial}{\partial\tilde{y}}(\psi - \mu c\tilde{y})\left[\frac{\partial\Omega}{\partial x'} + \frac{1}{\mu}\frac{\partial\Omega}{\partial\tilde{x}}\right] - \left[\frac{\partial\psi}{\partial x'} + \frac{1}{\mu}\frac{\partial\psi}{\partial\tilde{x}}\right]\frac{\partial\Omega}{\partial\tilde{y}} + c\,\frac{dy_c}{dx'}\frac{\partial\Omega}{\partial\tilde{y}}$$

$$= \frac{1}{\mu R}\left\{\nabla^2\Omega + \mu^2\frac{\partial^2\Omega}{\partial x'^2} + 2\mu\frac{\partial^2\Omega}{\partial\tilde{x}\,\partial x'} - \mu\frac{d^2y_c}{dx'^2}\frac{\partial\Omega}{\partial\tilde{y}} - 2\mu\frac{dy_c}{dx'}\frac{\partial^2\Omega}{\partial\tilde{y}\,\partial x'}\right.$$

$$\left. - 2\frac{dy_c}{dx'}\frac{\partial^2\Omega}{\partial\tilde{y}\,\partial\tilde{x}} + \left(\frac{dy_c}{dx'}\right)^2\frac{\partial^2\Omega}{\partial\tilde{y}^2}\right\} \tag{8}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial\tilde{x}^2} + \frac{\partial^2}{\partial\tilde{y}^2}$$

and R is a Reynolds number formed from a characteristic length and velocity scale of the flow, $\psi = \psi(x',\tilde{x},\tilde{y})$ is the stream function, $u = u(x',\tilde{x},\tilde{y})$ and $v = v(x',\tilde{x},\tilde{y})$ are the streamwise and normal velocity components, respectively, and $\Omega = \Omega(x',\tilde{x},\tilde{y})$ is the vorticity. Coupled with the governing differential equations, an appropriate asymptotic expansion of ψ in the inner vortex region is required. A suitably general expansion for ψ is given by

$$\psi = \psi_i(x',\tilde{x},\tilde{y}) = \mu R^{-1}\,\psi_{i0}(x',\tilde{x},\tilde{y}) + \mu^2 R^{-1}\,\psi_{i1}(x',\tilde{x},\tilde{y})$$

$$+ \mu^3 R^{-1}\,\psi_{i2}(x',\tilde{x},\tilde{y}) + O(\mu^4) \tag{9}$$

where the usual requirements for a valid asymptotic sequence are assumed to hold. Before obtaining the governing differential equations at the various orders of approximation, it is necessary to quantify the factor appearing in the equations.

In anticipation of the Blasius boundary layer example to be shown in section 5, one can estimate the size of dy_c/dx' for this boundary layer flow. If the transverse coordinate is scaled by the boundary layer thickness at some downstream location x_0, that is,

$$\delta_0 = 5\sqrt{\frac{\nu x_0}{U_\infty}} \tag{10}$$

where U_∞ is the free-stream velocity outside the boundary layer, then y_c is analogous to the usual Blasius similarity variable with $0 < y_c < 1$. Thus the factor dy_c/dx' can be written, to lowest order, as

$$\frac{dy_c}{dx'} \approx -\frac{5}{2}\, y_c R_x^{-1/2} = -\frac{25}{2}\, R^{-1} \tag{11}$$

where $R_x = \frac{U_\infty x_0}{\nu}$ in this boundary layer case. With this specification completed, the governing differential equations at the various orders can be derived.

The $O(\mu)$ equation is obtained by substituting the expansion equation (9) into equations (5) through (8):

$$\frac{\partial}{\partial \tilde{y}}(\psi_{i0} - c\tilde{y})\,\frac{\partial \Omega_{i0}}{\partial \tilde{x}} - \frac{\partial \psi_{i0}}{\partial \tilde{x}}\,\frac{\partial \Omega_{i0}}{\partial \tilde{y}} = -\frac{5}{2}\, c y_c R_x^{-1/2}\,\frac{\partial \Omega_{i0}}{\partial \tilde{y}} + \frac{1}{\mu R}\,\nabla^2 \Omega_{i0} \tag{12}$$

where

$$\Omega_{i0} = -\nabla^2 \psi_{i0} \tag{13}$$

Note that in the present formulation, the lowest order vorticity is not simply determined by gradients of the streamwise velocity but is determined by gradients of both the streamwise and transverse velocities. In order to be consistent with the initial physical assumptions concerning the dominant inviscid behavior of the vortex motion, it is appropriate that the limit $(\mu R)^{-1} \to 0$ be chosen in equation (12). The $O(\mu)$ equation reduces to

$$\frac{\partial \bar{\psi}_{i0}}{\partial \tilde{y}}\,\frac{\partial \Omega_{i0}}{\partial \tilde{x}} - \frac{\partial \bar{\psi}_{i0}}{\partial \tilde{x}}\,\frac{\partial \Omega_{i0}}{\partial \tilde{y}} = 0 \tag{14}$$

or

$$\frac{\partial(\bar{\psi}_{io},\Omega_{io})}{\partial(\tilde{y},\tilde{x})} = 0 \tag{15}$$

where

$$\bar{\psi}_{io} = \psi_{io} - c\tilde{y}, \qquad \Omega_{io} = -\nabla^2\bar{\psi}_{io} \tag{16}$$

The set of equations, equations (13), (15), and (16), were originally studied by Stuart (ref. 2) in the context of finite-amplitude oscillations in mixing layers. The solution to the set, appropriate to the physical problem at hand, is

$$\psi_{io}(\tilde{x},\tilde{y}) = c\tilde{y} + K\ \ln\left[\cosh\ \tilde{y} + c_o\ \cos(\tilde{x} + \phi)\right] \tag{17}$$

$$\Omega_{io}(\tilde{x},\tilde{y}) = \Omega_{io}(\bar{\psi}_{io}) = -K\left[1 - c_o^2\right]\ e^{-2\bar{\psi}_{io}/K} \tag{18}$$

where c_o is a constant constrained by (ref. 1)

$$o < c_o < 1 \tag{19}$$

and K is a constant associated with the circulation of the vortex having bounding contour, $-\phi < \tilde{x} < 2\pi - \phi$, $-\infty < \tilde{y} < \infty$, and ϕ is an arbitrary phase angle. The circulation of the vortex, as given by equation (18), is easily determined (ref. 2) as

$$\Gamma = -4\pi K \tag{20}$$

The distribution given in equations (17) and (18) is rather general, in the sense that the constant c_o governs the gross features of the vortex in a range varying between a point vortex distribution ($c_o \to 1$) and an unperturbed hyperbolic tangent velocity profile ($c_o = 0$). The stability characteristics of this vortex distribution have been studied in reference 3; however, the intent here is not to conduct such a stability study but to examine the composite flow structure at some point in space and time when such a vortex is evolving through the flow. This is also distinct from recent work by Pierrehumbert and Widnall (ref. 4) and the earlier work of Browand and Weidman (ref. 1), where such vortical distributions were studied in connection with free shear layers. There the objective was to examine the interaction dynamics and critical parameters of interacting vortices which spanned the width of the shear layer.

This lowest order solution leaves the two parameters μ and K unspecified. Higher order inner solutions are needed to fix μ, and the matching process will help structure the outer solution field. The $O(\mu^2)$ differential equation is

$$\frac{\partial(\psi_{i1},\Omega_{i0})}{\partial(\tilde{y},\tilde{x})} + \frac{\partial(\bar{\psi}_{i0},\Omega_{i1})}{\partial(\tilde{y},\tilde{x})} = \frac{1}{\mu R_x^{1/2}}\left[-\frac{5y_c}{2}\right]\left\{2\frac{\partial(\bar{\psi}_{i0},\bar{\psi}_{i0,\tilde{x}\tilde{y}})}{\partial(\tilde{y},\tilde{x})} + c\frac{\partial\Omega_{i0}}{\partial\tilde{y}}\right\} + \frac{1}{\mu^2 R}\nabla^2\Omega_{i0} \tag{21}$$

where

$$\Omega_{i1} = -\nabla^2\psi_{i1}, \qquad \bar{\psi}_{i0,\tilde{x}\tilde{y}} = \frac{\partial^2\bar{\psi}_{i0}}{\partial\tilde{x}\,\partial\tilde{y}} \tag{22}$$

The question that arises at this point is the choice of the appropriate limit for $(\mu R^{1/2})^{-1}$. Since the differential operator on the left-hand side of equation (21) is different than the lowest order differential operator, it is consistent to solve the homogeneous problem at this order; therefore,

$$(\mu R^{1/2})^{-1} \to 0, \qquad \mu \gg R^{-1/2} \tag{23}$$

is the chosen limit. The homogeneous form of equation (21) is readily solved by introducing the new independent variable

$$Q = e^{-\bar{\psi}_{i0}/K} \tag{24}$$

and transforming to the new coordinate system $(Q,\tilde{x})$. The resulting equation is

$$\frac{\partial}{\partial\tilde{x}}[F] = 0 \tag{25}$$

where

$$F(Q) = \Omega_{i1} - 2(1 - c_o^{\,2})Q^2\psi_{i1} \tag{26}$$

and, in general, Ω_{i1} and ψ_{i1} can both retain some periodic dependence on $\tilde{x}$. It is necessary to continue to the next order to further identify Ω_{i1} and ψ_{i1} and uniquely determine μ.

The $O(\mu^3)$ equation can be written as

$$\frac{\partial(\bar{\Psi}_{i0},\Omega_{i2})}{\partial(\tilde{y},\tilde{x})} + \frac{\partial(\psi_{i2},\Omega_{i0})}{\partial(\tilde{y},\tilde{x})} = -\frac{\partial(\psi_{i1},\Omega_{i1})}{\partial(\tilde{y},\tilde{x})} + \frac{1}{\mu^2 R^{1/2}}\left[-\frac{5}{2}\,y_c\right]\left\{2\,\frac{\partial(\bar{\Psi}_{i0},\bar{\Psi}_{i0,\tilde{x}\tilde{y}})}{\partial(\tilde{y},\tilde{x})} + c\,\frac{\partial\Omega_{i0}}{\partial\tilde{y}}\right\} + \frac{1}{\mu^3 R}\,\nabla^2\Omega_{i0} \tag{27}$$

where

$$\Omega_{i2} = -\nabla^2\psi_{i2} \tag{28}$$

Once again transforming to $(Q,\tilde{x})$ coordinates would place equation (27) in a more tractable form. In light of the $O(\mu^2)$ equation (eq. (25)), the right-hand side of equation (27) cannot retain any secular terms, that is, any periodic functions of $\tilde{x}$. In the absence of any decision on the limiting values of the terms continuing μ and the Reynolds number, it is necessary to examine the functional behavior of

$$-\frac{\partial(\psi_{i1},\Omega_{i1})}{\partial(\tilde{y},\tilde{x})}$$

or

$$-\frac{\partial(\psi_{i1},\Omega_{i1})}{\partial(Q,\tilde{x})} = G(Q,\tilde{x}) \tag{29}$$

in $(Q,\tilde{x})$ coordinates. If $\psi_{i1} = \psi_{i1}(Q,\tilde{x})$ and $\Omega_{i1} = \Omega_{i1}(Q,\tilde{x})$, then equation (29) becomes

$$\frac{\partial\psi_{i1}}{\partial\tilde{x}}\left[F' + 4Q(1 - c_o^2)\psi_{i1}\right] = G \tag{30}$$

where ' denotes differentiation with respect to Q. In order for this term to be independent of $\tilde{x}$ it is necessary for ψ_{i1}, and therefore Ω_{i1}, to be functions only of Q. The structure of these two functions can be further delineated by referring back to equation (26). Equation (26) can be expanded to give

$$F(Q) = Q(1 - c_o^{\,2})\left\{f(Q,\psi_{i1}) + Q\psi'_{i1} - 2\psi_{i1}\right\} - f(Q,\psi_{i1})$$

$$+ 2\,c_o \cos(\tilde{x} + \phi)\ f(Q,\psi_{i1}) \tag{31}$$

where

$$f(Q,\psi_{i1}) = Q^2\psi''_{i1} + Q\psi_{i1} \tag{32}$$

Clearly $f(Q,\psi_{i1})$ must equal zero and the resulting solution is ψ_{i1} = Constant. Without any loss of generality in the inner solution field, it is sufficient to take

$$\psi_{i1} = \Omega_{i1} = 0 \tag{33}$$

Now the limit processes in equation (27) can be performed. Since inhomogeneous terms should appear at this order, it is only necessary to determine whether nonparallel or purely viscous effects predominate. In the absence of any nonparallel effects the viscous limit dominates, that is,

$$\mu_p = R^{-1/3} \tag{34}$$

If the outer flow is slowly diverging, the appropriate limit is

$$\mu_{np} = \mu_p = R^{-1/3} \tag{35}$$

since

$$R_x^{1/2} > \mu R$$

Even though this analysis was not motivated by questions of stability, it is worth noting the viscous layer size obtained here (eq. (34)) and to compare these necessarily nonlinear results with previous nonlinear stability studies, such as those motivated by the ideas of Benney and Bergeron (ref. 5). In those studies nonlinear effects dominated in the critical layer and viscous effects were secondary. This, of course, is consistent with the structural ideas presented in this work. Finally, it should be pointed out that the nonparallel effects are sustained by terms originating in the nonlinear advection terms. In the absence of such stress gradient effects, the slow variation of the outer flow would not be felt by the vortex to this order of approximation.

3. INTERMEDIATE MATCHING REGION

In section 2, the inner solution field was established. This solution must be formally matched to an appropriate outer solution in order to form a uniformly valid composite expansion. The process involves the introduction of suitable intermediate variables (ref. 6) given by

$$y_\varepsilon = \frac{y - y_c(x')}{\varepsilon} \tag{36}$$

where

$$\frac{\varepsilon}{\mu} \to \infty \qquad \text{as} \qquad \varepsilon \to 0$$

and

$$\tilde{y} = \left|\frac{\varepsilon y_\varepsilon}{\mu}\right| \to \infty \qquad \text{as} \qquad |y - y_c| \to 0$$

Since the vortex is embedded in the flow there are two distinct matching regions: the region above the vortex $\tilde{y} \to \infty$ and the region below the vortex $\tilde{y} \to -\infty$. If the lowest order inner solution is written in terms of the intermediate variables, the inner solution (eq. (17)) becomes

$$\psi_i = c y_\varepsilon \varepsilon + \mu K \ln\left[\cosh \frac{\varepsilon y_\varepsilon}{\mu} + c_o \cos(\tilde{x} + \phi)\right] + O(\mu^3) \tag{37}$$

If $\left|\frac{\varepsilon y_\varepsilon}{\mu}\right| \to \infty$, the two matching limits of equation (37) can be written as

$$\psi_i \to (c + K)\varepsilon y_\varepsilon - \mu K \ln 2 + \mu K \ln\left[1 + c_o e^{-\varepsilon y_\varepsilon/\mu} \cos(\tilde{x} + \phi)\right]$$
$$+ O(\mu^3) \qquad \left(\frac{\varepsilon y_\varepsilon}{\mu} \to \infty\right) \tag{38}$$

$$\psi_i \to (c - K)\varepsilon y_\varepsilon - \mu K \ln 2 + \mu K \ln\left[1 + c_0 e^{-\varepsilon y_\varepsilon/\mu} \cos(\tilde{x} + \phi)\right]$$

$$+ O(\mu^3) \qquad \left(\frac{\varepsilon y_\varepsilon}{\mu} \to -\infty\right) \tag{39}$$

These stream function values will indeed match to the $O(1)$ outer stream function distribution, and the contributions to the respective outer flow regions are

$$\psi_i\Big|_{+\infty} \to (c + K)\varepsilon y_\varepsilon \tag{40}$$

$$\psi_i\Big|_{-\infty} \to (c - K)\varepsilon y_\varepsilon \tag{41}$$

It is important to note that in the present formulation, the periodic streamwise dependence is lost in the matching process because of the transcendentally small effect. This eases the restrictions on the interpretation of the present formulation, since the streamwise range of the vortex disturbance is limited here to a single wavelength. In addition, this allows the inner solution to be matched locally, point by point, to an outer flow which is either parallel or self-similar, but nevertheless slowly diverging. In the next two sections, an example of each type of flow will be given.

4. OUTER VISCOUS FLOW FOR PARALLEL FLOW CASE

The independent variables appropriate for the general outer problem are x', $\tilde{x}$, and y and are assumed to be scaled to $O(1)$. For the moment, the derivations will be rather general in order to lay a proper framework for both the parallel flow case considered in this section and the nonparallel case to be considered in the next section. The general form of the equations for the two-dimensional outer flow problem can be written as

$$u = \frac{\partial\psi}{\partial y} \tag{42}$$

$$v = -\frac{\partial\psi}{\partial x'} - \frac{1}{\mu}\frac{\partial\psi}{\partial\tilde{x}} \tag{43}$$

$$\Omega = -\frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial x'^2} - \frac{1}{\mu^2}\frac{\partial^2\psi}{\partial\tilde{x}^2} - \frac{2}{\mu}\frac{\partial^2\psi}{\partial\tilde{x}\,\partial x'} \tag{44}$$

$$\frac{\partial\psi}{\partial y}\left[\frac{\partial\Omega}{\partial x'} + \frac{1}{\mu}\frac{\partial\Omega}{\partial\tilde{x}}\right] - \left[\frac{\partial\psi}{\partial x'} + \frac{1}{\mu}\frac{\partial\psi}{\partial\tilde{x}}\right]\frac{\partial\Omega}{\partial y} = \frac{1}{R}\left[\frac{\partial^2\Omega}{\partial y^2} + \frac{\partial^2\Omega}{\partial x'^2} + \frac{1}{\mu^2}\frac{\partial^2\Omega}{\partial\tilde{x}^2} + \frac{2}{\mu}\frac{\partial^2\Omega}{\partial\tilde{x}\,\partial x'}\right] \tag{45}$$

Recalling the form of the outer limit of the inner solution, it is seen that the type of outer flow that is considered can be explicitly independent of both $\tilde{x}$ and x'. Then, if the outer flow is parallel or even dependent on some similarity variable, the lowest order outer solution and inner solution can be matched. The example problem to be considered in this section is plane Poiseuille flow. The outer dependent variables are independent of $\tilde{x}$ and x' to lowest order, that is

$$\psi_o = \psi_{os}(y) \tag{46}$$

where s is $+$ for $y > y_c$ and s is $-$ for $y < y_c$. Equation (45) reduces to the trivial form

$$\frac{\partial^2 \Omega_{os}}{\partial y^2} = 0 \tag{47}$$

with solutions

$$\psi_{o+} = \frac{a_0}{2}(y - 1)^2 + \frac{a_1}{6}(y - 1)^3 + a_2(y - 1) + a_3 \tag{48}$$

$$\psi_{o-} = \frac{b_0}{2} y^2 + \frac{b_1}{6} y^3 + b_2 y + b_3 \tag{49}$$

where the variables have been scaled by the channel height h and the maximum undisturbed inflow velocity U_∞. The boundary conditions imposed at lowest order are

$$\psi_{o+}(y = y_c) = \psi_{o-}(y = y_c) = 0 \tag{50}$$

$$\psi_{os} = \left\{ \begin{array}{ll} 2(1 - y_c^2) - \frac{4}{3}(1 - y_c^3) & (s + \text{ at } y = 1) \\ -2y_c^2 + \frac{4}{3} y_c^3 & (s - \text{ at } y = 0) \end{array} \right\} \text{(Undisturbed values)} \tag{51}$$

$$\left.\frac{\partial \psi_{o+}}{\partial y}\right|_{y=1} = \left.\frac{\partial \psi_{o-}}{\partial y}\right|_{y=0} = 0 \tag{52}$$

In anticipation of the matching, the value for the vortex phase velocity needs to be determined. This velocity is the velocity at $y = y_c$ if the flow did not contain the vortex. In the channel flow case

$$c = 4y_c(1 - y_c) \tag{53}$$

If the boundary conditions are applied to equations (48) and (49), the constant coefficients become

$$a_1 = \frac{6a_3}{(1 - y_c)^3} + \frac{3a_0}{(1 - y_c)}, \qquad b_1 = \frac{-6b_3}{y_c^{\ 3}} - \frac{3b_0}{y_c} \tag{54}$$

$$a_2 = 0.0, \qquad b_2 = 0.0 \tag{55}$$

$$a_3 = 2\left(1 - y_c^{\ 2}\right) - \frac{4}{3}\left(1 - y_c^{\ 3}\right), \qquad b_3 = -2y_c^{\ 2} + \frac{4}{3}\, y_c^{\ 3} \tag{56}$$

where the remaining coefficients, a_0 and b_0, are determined from the matching. The outer variables $(1 - y)$ and y can be written in terms of the matching variable y_ε and are given by

$$1 - y = (1 - y_c) - \varepsilon y_\varepsilon \tag{57}$$

$$y = y_c + \varepsilon y_\varepsilon \tag{58}$$

Substituting these forms into equations (48) and (49), one can obtain ψ_{o+} and ψ_{o-} in terms of powers of $\varepsilon y_\varepsilon$. From section 3, it is seen that the matching occurs with coefficients of $\varepsilon y_\varepsilon$. This process yields the coefficient values

$$a_0 = -\frac{6a_3}{(1 - y_c)^2} + \frac{2(c + K)}{(1 - y_c)} \tag{59}$$

$$b_0 = -\frac{6b_3}{y_c^{\ 2}} - \frac{2(c - K)}{y_c} \tag{60}$$

The remaining coefficients a_1 and b_1 are then determined from these values of a_0 and b_0. Note that the coefficients are all bounded in the present formulation. This results from the fact that the inner region is assumed to be of $O(\mu)$ in size; therefore, the vortex should be at least a distance of $O(\mu)$ away from the boundaries to be consistent with the basic assumption about the inner region. A uniformly valid composite solution, to lowest order in the dependent variables, can be formed by summing the lowest order inner and outer solutions and subtracting out the common part (ref. 6). The resulting composite solutions are

$$\psi \approx \begin{cases} \frac{a_0}{2}\left[(1-y)^2 - (1-y_c)^2\right] - \frac{a_1}{6}\left[(1-y_c)^3 - (1-y_c)^3\right] - K(y-y_c) + KR^{-1/3}\ln\left[\cosh\tilde{y} + c_o\cos(\tilde{x}+\phi)\right] & (y>y_c) \\ \frac{b_0}{2}\left[y^2-y_c^2\right] + \frac{b_1}{6}\left[y^3-y_c^3\right] + K(y-y_c) + KR^{-1/3}\ln\left[\cosh\tilde{y} + c_o\cos(\tilde{x}+\phi)\right] & (y<y_c) \end{cases} \quad (61)$$

$$u \approx \begin{cases} -a_0(1-y) + \frac{a_1}{2}(1-y)^2 - K + K\tanh\tilde{y}\left\{1 + \frac{c_o\cos(\tilde{x}+\phi)}{\cosh\tilde{y}}\right\}^{-1} & (y > y_c) \\ b_0 y + \frac{b_1}{2}y^2 + K + K\tanh\tilde{y}\left\{1 + \frac{c_o\cos(\tilde{x}+\phi)}{\cosh\tilde{y}}\right\}^{-1} & (y < y_c) \end{cases} \quad (62)$$

$$v \approx K\frac{c_o\sin(\tilde{x}+\phi)}{\cosh\tilde{y}}\left\{1 + \frac{c_o\cos(\tilde{x}+\phi)}{\cosh\tilde{y}}\right\}^{-1} \qquad (0 < y < 1) \quad (63)$$

$$\Omega \approx -KR^{1/3}\left[1 - c_o^2\right]\operatorname{sech}^2\tilde{y}\left\{1 + \frac{c_o\cos(\tilde{x}+\phi)}{\cosh\tilde{y}}\right\}^{-2} - \begin{cases} a_0 - a_1(1-y) & (y > y_c) \\ b_0 + b_1 y & (y < y_c) \end{cases} \quad (64)$$

where $\tilde{x}$, $\tilde{y}$, and ϕ were defined in section 2, $\mu = R^{-1/3}$ has been used, and the constant coefficients were defined earlier in this section. It should be pointed out that the vorticity is discontinuous at $y = y_c$ at first order, $O(1)$; however, to zeroth order, $O(R^{1/3})$, the vorticity is continuous across the channel. If higher order terms are included in the expansion procedure, this singularity would be removed. As can be seen from equations (62) and (63), the velocity boundary conditions at the wall will, in general, not be satisfied to the order of the approximation for all values of y_c and K. The task then is to delimit the range over which y_c and K may vary. At $y = 0$ and $y = 1$ the velocity fields reduce to

$$u(y=0) = K\left\{1 - \tanh R^{1/3}y_c\left[1 + \frac{c_o\cos(\tilde{x}+\phi)}{\cosh R^{1/3}y_c}\right]^{-1}\right\} + O(R^{-2/3}) \quad (65)$$

$$v(y=0) = K \frac{c_o \sin(\tilde{x} + \phi)}{\cosh R^{1/3} y_c} \left[1 + \frac{c_o \cos(\tilde{x} + \phi)}{\cosh R^{1/3} y_c} \right]^{-1} + O(R^{-2/3}) \tag{66}$$

$$u(y=1) = -K \left\{ 1 - \tanh R^{1/3}(1 - y_c) \left[1 + \frac{c_o \cos(\tilde{x} + \phi)}{\cosh R^{1/3}(1 - y_c)} \right]^{-1} \right\} + O(R^{-2/3}) \tag{67}$$

$$v(y=1) = K \frac{c_o \sin(\tilde{x} + \phi)}{\cosh R^{1/3}(1 - y_c)} \left[1 + \frac{c_o \cos(\tilde{x} + \phi)}{\cosh R^{1/3}(1 - y_c)} \right]^{-1} + O(R^{-2/3}) \tag{68}$$

where the accuracy of the approximation $O(R^{-2/3})$ in the velocity field is based on the order of approximation obtained in the inner solution field. Since the periodic functions are bounded, the restrictions on y_c must come from the hyperbolic functions. Note that the above does not put any restrictions on the value of K; however, restricting K to values less than 1 would improve the accuracy with which the boundary conditions are satisfied. A cursory examination of equations (65) through (68) reveals that the hyperbolic functions are bounded as

$$\tanh R^{1/3} y_c = 1.0 - O(R^{-2/3}) \tag{69}$$

$$\cosh R^{1/3} y_c = O(c_o R^{2/3}) \tag{70}$$

$$\tanh R^{1/3}(1 - y_c) = 1.0 - O(R^{-2/3}) \tag{71}$$

$$\cosh R^{1/3}(1 - y_c) = O(c_o R^{2/3}) \tag{72}$$

where the constant c_o is included in the restrictions on the hyperbolic cosine factor for generality. If an $O(1)$ constant of proportionality is assumed in these order estimates and finite-amplitude disturbances are considered, then the following limits on y_c can be extracted from equations (69) through (72)

$$R^{-1/3} \ln(2c_o R^{2/3}) \lessapprox y_c \lessapprox 1 - R^{-1/3} \ln(2c_o R^{2/3}) \tag{73}$$

where

$$2c_oR^{2/3} > 1 \tag{74}$$

With this determination of the limits on y_c as much information as possible has been extracted from the solution to this order. The only parameter in the problem that has not been delineated in any concise way is the circulation constant K. Constraints on K may arise at higher orders, either through the matching process or the application of the boundary conditions. Nevertheless, for the present study, K is a free parameter of O(1), whose value will be chosen to best suit the practical problem at hand.

As a test case, computations were carried out at a Reynolds number, based on channel height and maximum undisturbed inflow velocity, of 10^4. The amplitude of the Stuart vortex c_o was set at 0.25 (refs. 1, 3, and 4) and K was chosen to be 0.20. The value of μ corresponding to this Reynolds number is 0.046. The results of the streamwise velocity distribution at various values of $(\tilde{x} + \phi)$ are shown in figure 1. The results reflect, as $(\tilde{x} + \phi)$ varies from 0 to 2π, the effect of a passing vortex on an otherwise fully developed parallel flow at a fixed point in space. It is important to note that the vortex, which is of limited spatial extent in the present example, affects a rather large portion of the channel flow. This result indicates that quantities such as surface pressure and wall shear stress are modified by the passage of such a vortical distribution.

In the next section, the more relevant case of nonparallel flow is considered; however, the guidelines established in this section can be applied directly to this more general case.

5. OUTER VISCOUS FLOW FOR NONPARALLEL FLOW CASE

As indicated earlier, to the order of the approximation, nonparallel self-similar outer flows can be examined within the present context. Consider now, as the nonparallel flow example, a Blasius boundary layer flow. Since the inner Stuart vortex solution was found to be independent of x', the procedure used to obtain a composite solution for the channel flow can be applied to the boundary layer flow over a continuous range of x', with the only change at each x' location being the location of y_c. It should be recalled at this point that the nonparallel effects which appear in the inner region equations as dy_c/dx' were approximated to lowest order by the constant term $-5y_cR_x^{-1/2}/2$; thus, any x' variation of dy_c/dx' was removed. The lowest order equation in the outer region is obtained from equation (45) by using the following approximation to ψ:

$$\psi_o \approx \psi_{os}(x',y) \tag{75}$$

where, as before, no outer flow dependence on $\tilde{x}$ is assumed. The lowest order vorticity transport equation is

$$\frac{\partial(\psi_{os},\Omega_{os})}{\partial(y,x')} = \frac{1}{R}\left[\frac{\partial^2\Omega_{os}}{\partial x'^2} + \frac{\partial^2\Omega_{os}}{\partial y^2}\right] \tag{76}$$

Now, if the usual boundary layer approximations are made, equation (76) can be written in the familiar form

$$u\frac{\partial u}{\partial x'} + v\frac{\partial u}{\partial y} = \frac{1}{R}\frac{\partial^2 u}{\partial y^2} \tag{77}$$

where R is a Reynolds number formed from the free-stream velocity U_∞ and the boundary layer thickness at $x = x' + x_0$, that is

$$\delta(x) \approx 5\sqrt{\frac{\nu x_0}{U_\infty}} \tag{78}$$

In a completely analogous manner with the usual Blasius method, the resulting stream function equation is

$$2\psi'''_{os} + 25\psi''_{os}\psi_{os} = 0 \tag{79}$$

where the differentiation can be interpreted to be with respect to y in light of the nondimensionalization by $\delta(x)$. Equation (79) can be solved given the appropriate boundary conditions. As was the case in plane Poiseuille flow, the outer flow boundary layer is divided into two regions, $y < y_c$ and $y > y_c$. First the region $y < y_c$ is considered.

Since the solution to the ordinary differential equation (eq. (79)) will be determined by numerical integration, it is necessary, for matching purposes, to expand the outer stream function solution in a Taylor series expansion about y_c,

$$\psi_{o-}(y\to y_c) \approx \psi_{o-}(y=y_c) + (y - y_c)\left.\frac{\partial\psi_{o-}}{\partial y}\right|_{y=y_c} + \ldots \tag{80}$$

Note that the question of continuity of $\partial\psi_{o-}/\partial y$ across y_c does not enter into the present context, since the formulation of a composite solution valid over the whole domain is a superposition of two distinct outer fields. Each outer solution field is viewed as a distinct problem with appropriate boundary conditions applied at $y = 0$

and $y = y_c$ for $y < y_c$ and at $y = y_c$ and $y \to \infty$ for $y > y_c$. Thus, at a fixed x' the part of the outer solution which matches to the inner solution is

$$\psi'_{o-}(y=y_c) = c - K \tag{81}$$

Emboldened by the success in the channel flow, the same procedure used to determine the perturbed outer solution is followed in the present case. The boundary conditions associated with undisturbed stream function value and zero velocity at $y = 0$ are

$$\psi_{o-}(y=0) = 0 \qquad \text{(Undisturbed value)} \tag{82}$$

$$\psi'_{o-}(y=0) = 0 \tag{83}$$

Equations (81) through (83) are sufficient to determine the stream function, as well as its associated derivatives, from $y = 0$ to $y = y_c$. In this example, it should be noted that the stream function value at $y = y_c$ will not be 0, although it will be continuous since $\psi_{o-}(y = y_c)$ will be equated to $\psi_{o+}(y = y_c)$. Now the region $y > y_c$ will be considered.

From the outer limit of the inner solution (eq. (79)) and the series expansion (eq. (80)), the inner limit of the outer solution which matches with the inner solution is

$$\psi'_{o+}(y=y_c) = c + K \tag{84}$$

Since the stream function is assumed continuous across $y = y_c$,

$$\psi_{o+}(y=y_c) = \psi_{o-}(y=y_c) \tag{85}$$

the only remaining boundary condition to be applied is at the free stream. This condition is given by

$$\psi'_{o+} \to 1.0, \qquad y \to \infty \tag{86}$$

This, of course, is the usual free-stream condition, and the exact location where equation (86) holds is left intentionally arbitrary to account for any boundary layer thickening due to the presence of the vortex. This extra degree of freedom was not present in the channel flow problem due to the rigid bounding surfaces. Equation (79), coupled with the boundary conditions appropriate to the region of interest were solved with a fourth-order variable interval Runge-Kutta method. The composite solution for the dependent variables can be formally written as

$$\psi \approx \begin{cases} \psi_{o+}(y) - K(y - y_c) + KR^{-1/3} \ln[\cosh \tilde{y} + c_o \cos(\tilde{x} + \phi)] & (y > y_c) \\ \psi_{o-}(y) + K(y - y_c) + KR^{-1/3} \ln[\cosh \tilde{y} + c_o \cos(\tilde{x} + \phi)] & (y < y_c) \end{cases} \tag{87}$$

$$u \approx \begin{cases} \psi'_{o+}(y) - K + K \tanh \tilde{y}\left\{1 + \dfrac{c_o \cos(\tilde{x} + \phi)}{\cosh \tilde{y}}\right\}^{-1} & (y > y_c) \\ \psi'_{o-}(y) + K + K \tanh \tilde{y}\left\{1 + \dfrac{c_o \cos(\tilde{x} + \phi)}{\cosh \tilde{y}}\right\}^{-1} & (y < y_c) \end{cases} \tag{88}$$

$$v \approx \begin{cases} K \dfrac{c_o \sin(\tilde{x} + \phi)}{\cosh \tilde{y}}\left\{1 + \dfrac{c_o \cos(\tilde{x} + \phi)}{\cosh \tilde{y}}\right\}^{-1} & (y > y_c) \\ K \dfrac{c_o \sin(\tilde{x} + \phi)}{\cosh \tilde{y}}\left\{1 + \dfrac{c_o \cos(\tilde{x} + \phi)}{\cosh \tilde{y}}\right\}^{-1} & (y < y_c) \end{cases} \tag{89}$$

$$\Omega = -R^{1/3}K[1 - c_o^2] \operatorname{sech}^2\tilde{y}\left\{1 + \frac{c_o \cos(\tilde{x} + \phi)}{\cosh \tilde{y}}\right\}^{-2} - \begin{cases} \psi''_{o+}(y) & (y > y_c) \\ \psi''_{o-}(y) & (y < y_c) \end{cases} \tag{90}$$

where $\tilde{x}$, $\tilde{y}$, and ϕ were defined in section 2, $\mu = R^{-1/3}$ in this case, and terms of $O(R^{-2/3})$ were neglected. Note that once again the vorticity is discontinuous at $O(1)$. The restrictions on the location of the vortex can once again be derived from the hyperbolic functions. In light of the arbitrary location of the top free-stream boundary, y_c is only limited here to its proximity to the boundary at $y = 0$. The restrictions are

$$y_c > R^{-1/3} \ln(2\, c_o R^{2/3}) \qquad (2c_o R^{2/3} > 1) \tag{91}$$

Once again the parameter K is free to be chosen for the particular physical problem at hand.

As a test case, a computation was carried out at a Reynolds number, based on local boundary layer thickness and undisturbed free-stream velocity, of 10^3. The amplitude of the disturbance c_o was set at 0.25 and the constant K was set equal to 0.2. For the chosen Reynolds number, the small parameter μ was fixed at 0.1. Figure 2 shows the distortion of the streamwise velocity profile as the vortex passes by a fixed point streamwise location x_o. Once again the effect of the vortex is felt throughout most of the boundary layer, even though its spatial extent is bounded by μ. It is also worth noting that this type of disturbance can be present near the top of the boundary layer. The existence of such finite-amplitude disturbances near the top of the layer is a requisite for the receptivity of the boundary layer to any free-stream disturbances.

6. DISCUSSION

As was stated in section 1, the intent was to construct a uniformly valid flow field, to some order of approximation, that would mimic the structure of a real flow in the presence of a finite-amplitude vortical disturbance. The basic structure for the vortical field was taken to be a Stuart vortex. Such a field is derivable from the general vorticity transport equation when both the transverse and streamwise coordinates are scaled by the same small parameter. As was seen in the two test cases, however, the influence of the vortex spreads throughout most of the flow even though its dynamic features dominate a small spatial region of $O(\mu)$. In addition, the analysis that was presented necessarily restricts the results to large wave-number disturbances.

It is beneficial at this point to place the present work in context with previous work on vortices in the presence of solid boundaries. For example, Walker (ref. 7) studied the boundary layer produced by the presence of a single rectilinear vortex filament above an infinite plane wall. It was concluded that no steady flow field exists and that a recirculating eddy, which is formed in the boundary layer flow, causes rather explosive growth of the boundary layer itself. The problem presented in the present paper is fundamentally different. It is assumed here that the viscous flow already exists and that a vortex, which is necessarily of small spatial extent, is embedded within this flow. In the absence of the base undisturbed viscous flow, the present formulation is ill-posed since it would be impossible for the boundary condition at the boundaries to be satisfied, in general. For these reasons, it is felt that the present problem closely resembles a real flow situation of a vortical structure evolving through an otherwise undisturbed flow. It does remain an open question as to the behavior of the vortex as it evolves in time and space, but the present work shows that a stationary situation can exist and can be described in a consistent mathematical framework. This framework yielded some interesting features which are worth noting.

In the analysis, nonparallel effects were accounted for by requiring that the entire vortex move uniformly through the flow. Thus, for the laminar boundary layer flow, the vortex is inclined with respect to the wall, and as was mentioned in section 5, this region is sustained by nonlinear stress effects. The final comment concerns the effect of the vortex on the boundary surfaces. Recall that the wall shear stress, that is $\partial u/\partial y$, was allowed to vary in the calculation; only the stream function and velocity at the walls were taken as either undisturbed flow values or zero. This allowed for the effect of the passing vortex to be felt in both the wall shear stress and pressure signatures.

The work presented in this paper is intended as a first step in a more complete and accurate mathematical description of vortical structures, three-dimensional as well as two-dimensional, embedded in a broad class of viscous outer flows.

Langley Research Center
National Aeronautics and Space Administration
Hampton, VA 23665
October 31, 1983

REFERENCES

1. Browand, F. K.; and Weidman, P. D.: Large Scales in the Developing Mixing Layer. J. Fluid Mech., vol. 76, pt. 1, July 14, 1976, pp. 127-144.

2. Stuart, J. T.: On Finite Amplitude Oscillations in Laminar Mixing Layers. J. Fluid Mech., vol. 29, pt. 3, Sept. 6, 1967, pp. 417-440.

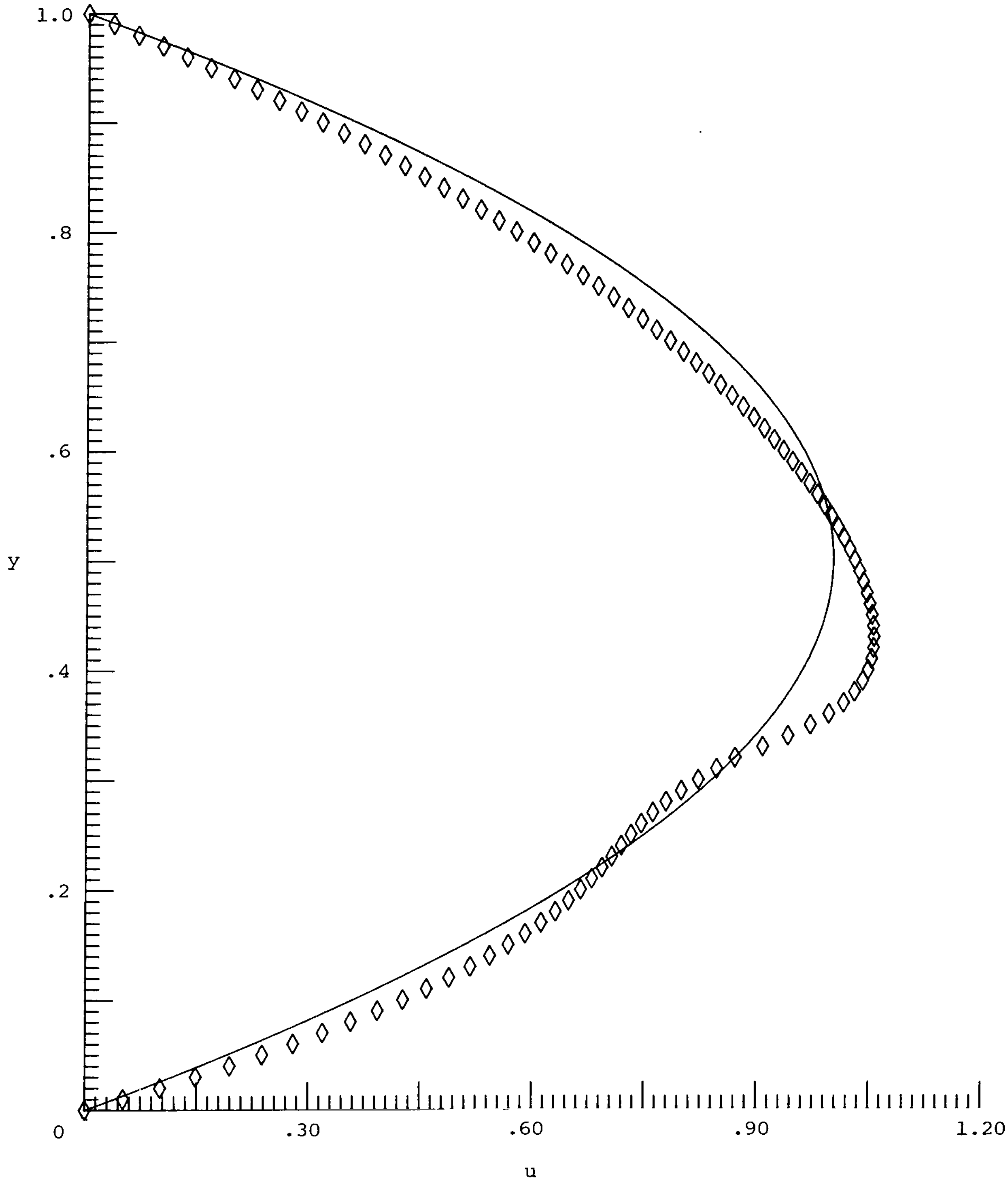
3. Pierrehumbert, Raymond Thomas: The Structure and Stability of Large Vortices in an Inviscid Flow. Ph. D. Thesis, Massachusetts Inst. Technol., 1980.

4. Pierrehumbert, R. T.; and Widnall, S. E.: The Structure of Organized Vortices in a Free Shear Layer. J. Fluid Mech., vol. 102, Jan. 1981, pp. 301-313.

5. Benney, D. J.; and Bergeron, R. F.: A New Class of Nonlinear Waves in Parallel Flows. Stud. Appl. Math., vol. 48, no. 3, Sept. 1969, pp. 181-204.

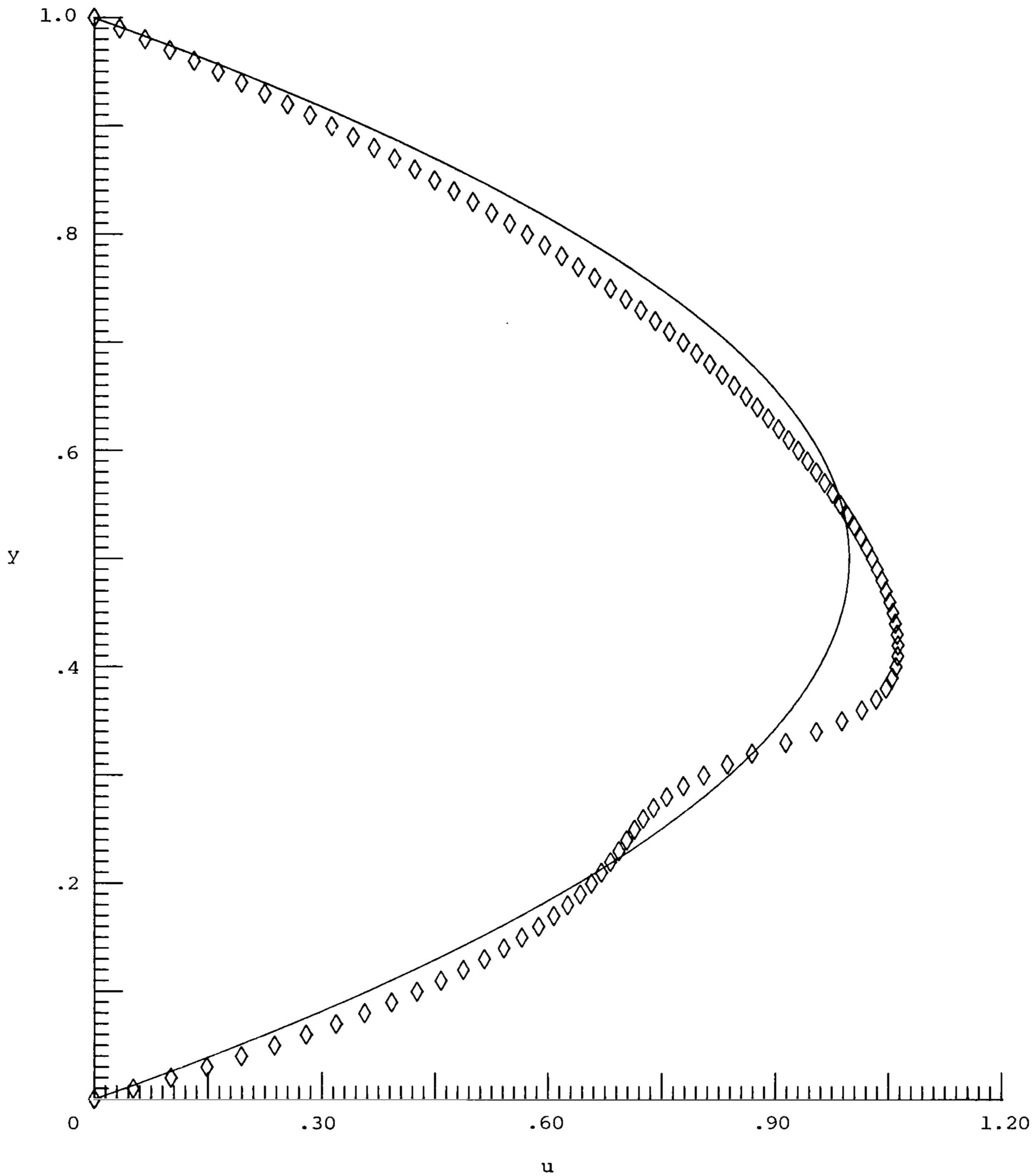
6. Cole, Julian D.: Perturbation Methods in Applied Mathematics. Blaisdell Pub. Co., 1968.

7. Walker, J. D. A.: The Boundary Layer Due to Rectilinear Vortex. Proc. R. Soc. London, ser. A, vol. 359, no. 1697, Feb. 15, 1978, pp. 167-188.



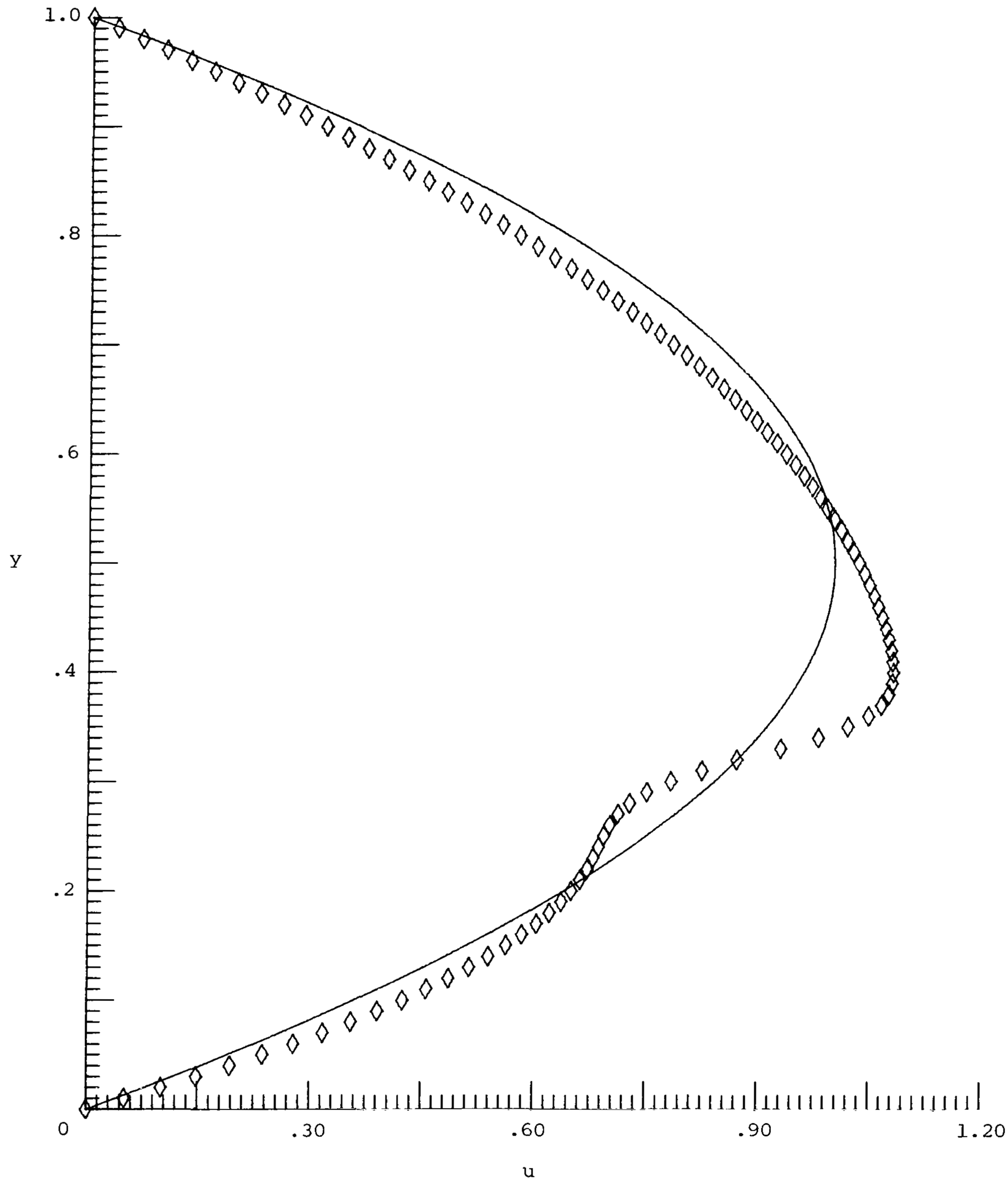
(a) $x + \phi = 0$.

Figure 1.- Effect of vortical structure on undisturbed velocity profiles $R = 10^4$; $c_o = 0.25$; $K = 0.2$.



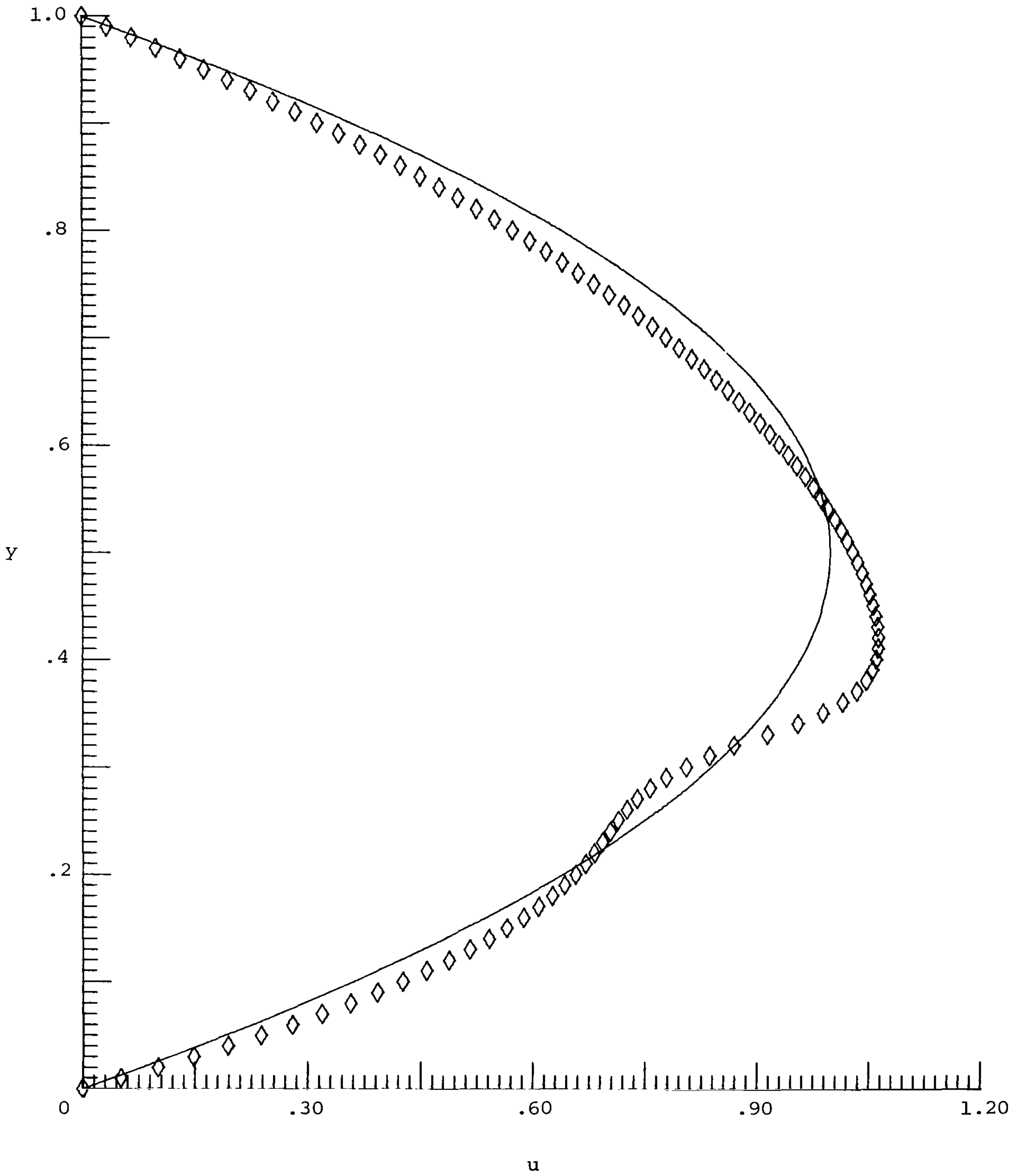
(b) $x + \phi = \frac{\pi}{2}$.

Figure 1.- Continued.



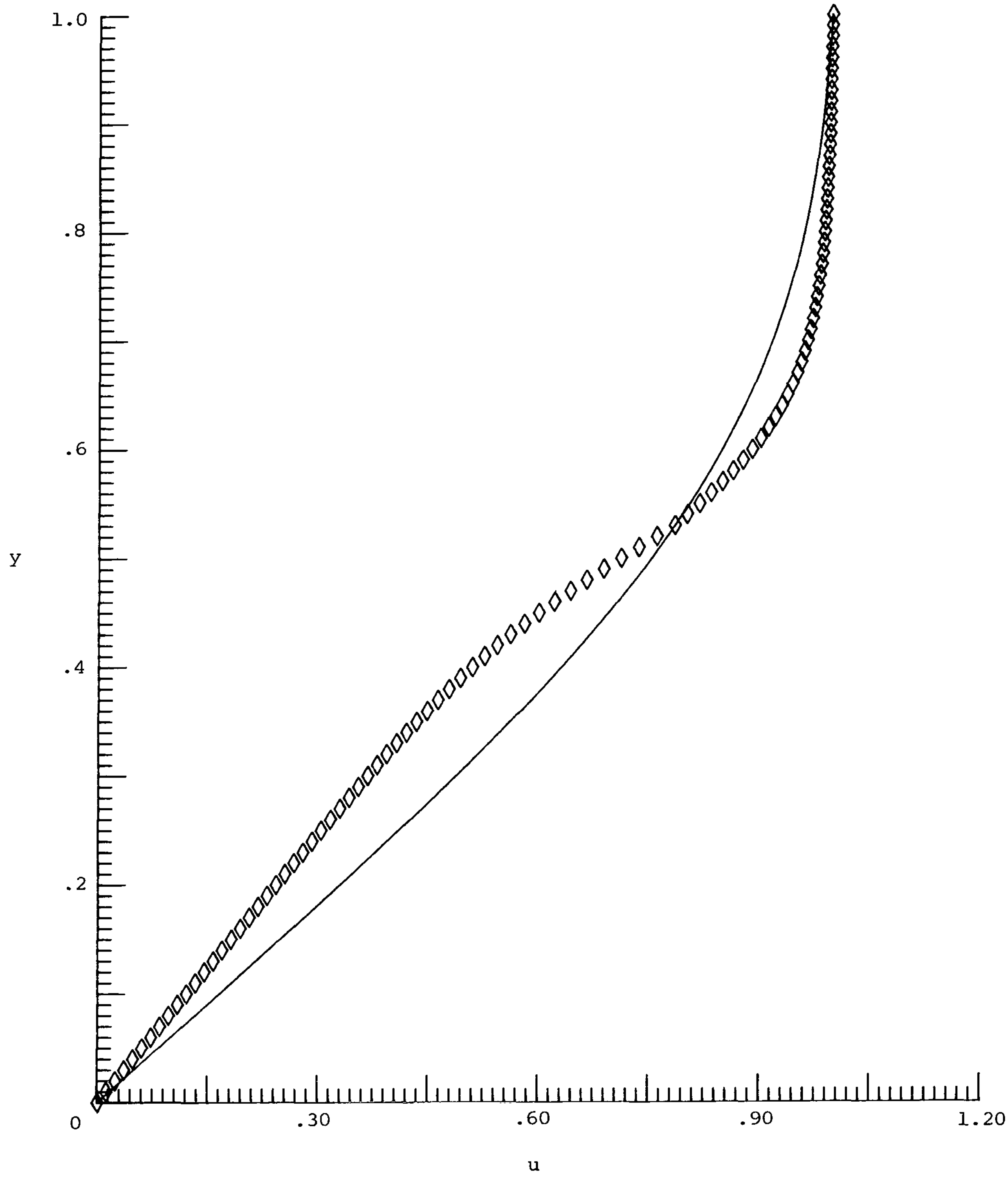
(c) $x + \phi = \pi$.

Figure 1.- Continued.



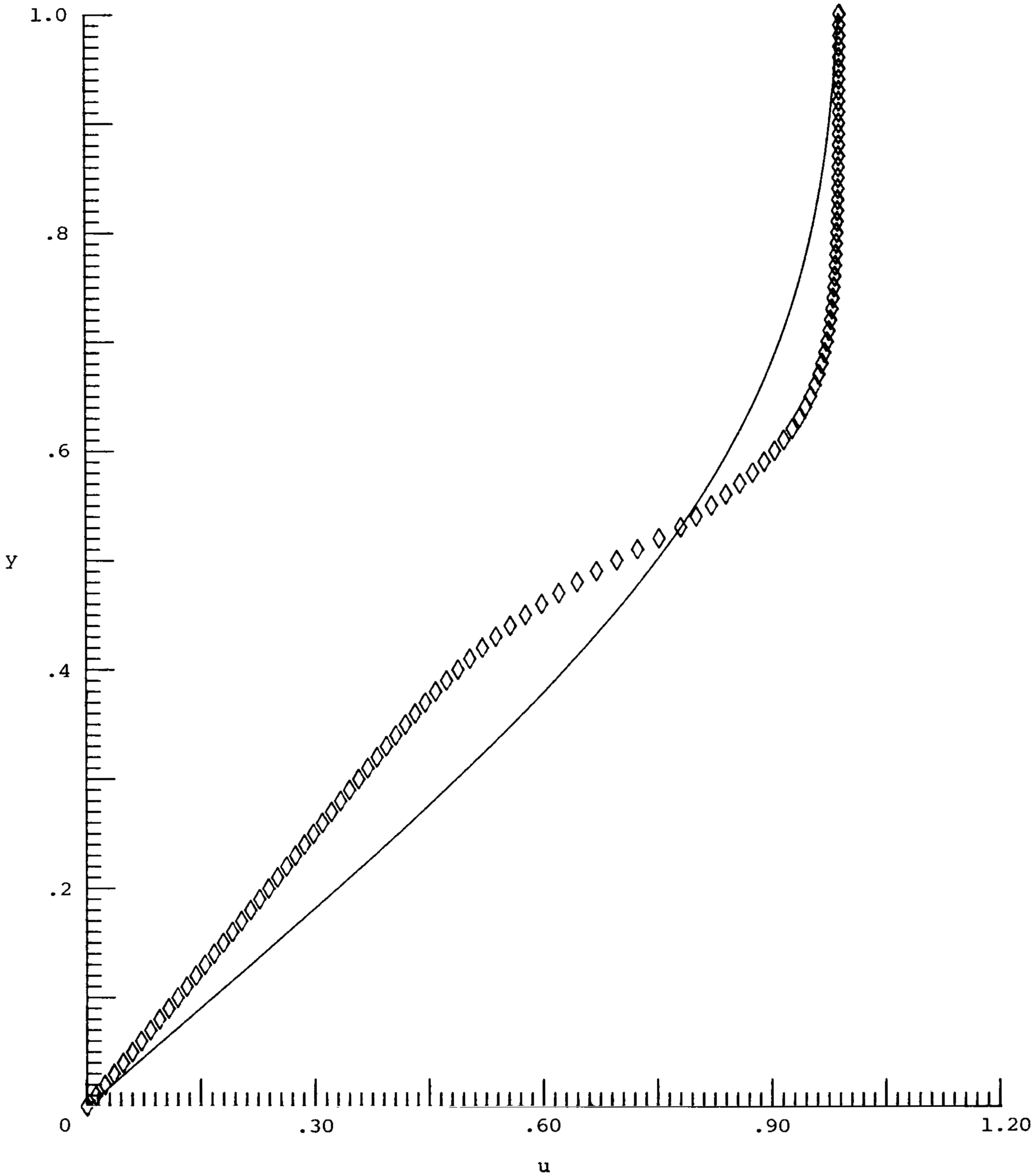
(d) $x + \phi = \frac{3\pi}{2}$.

Figure 1.- Concluded.



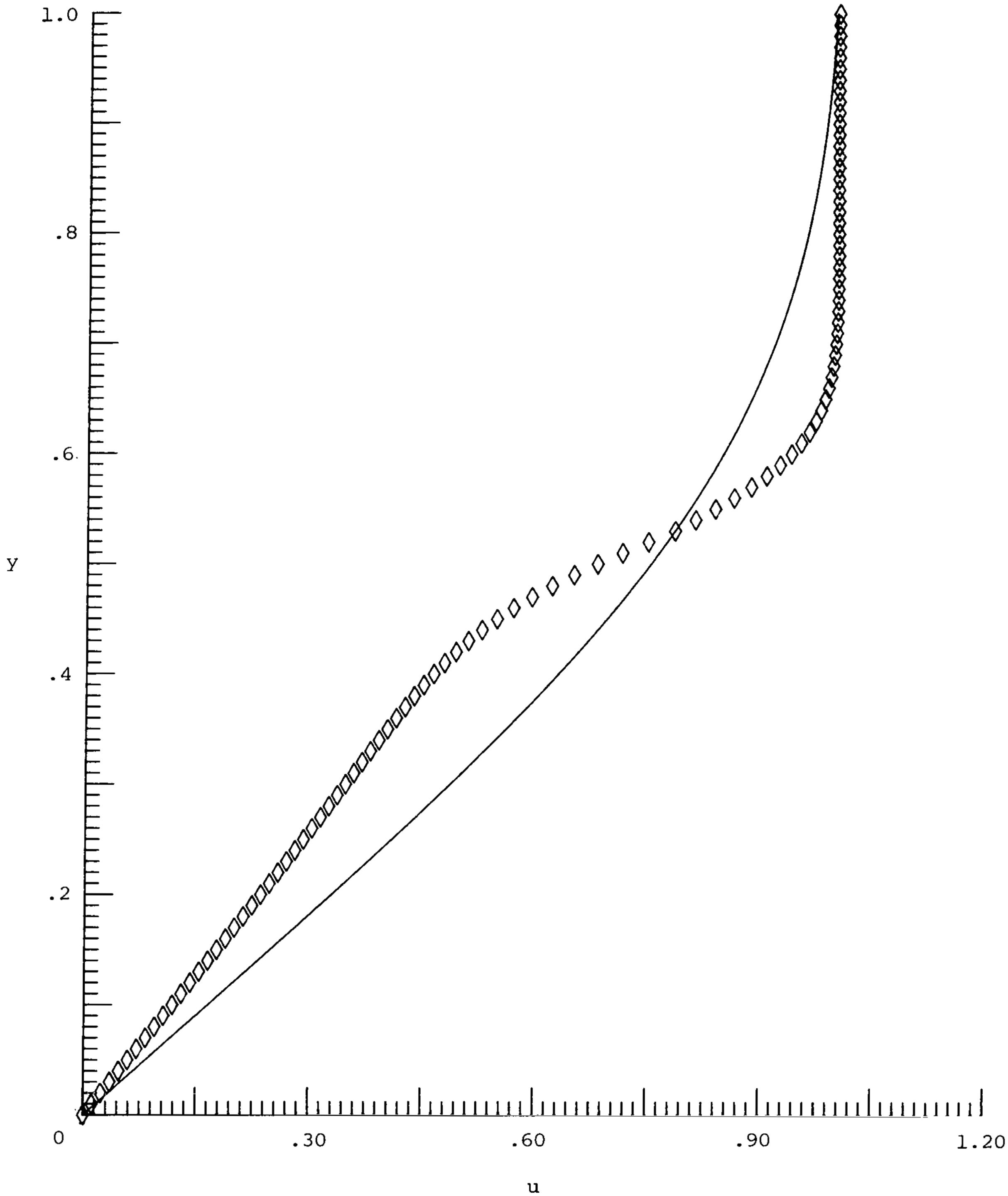
(a) $x + \phi = 0$.

Figure 2.- Disturbed boundary layer velocity profiles. $R = 10^3$; $c_o = 0.25$; $K = 0.2$.



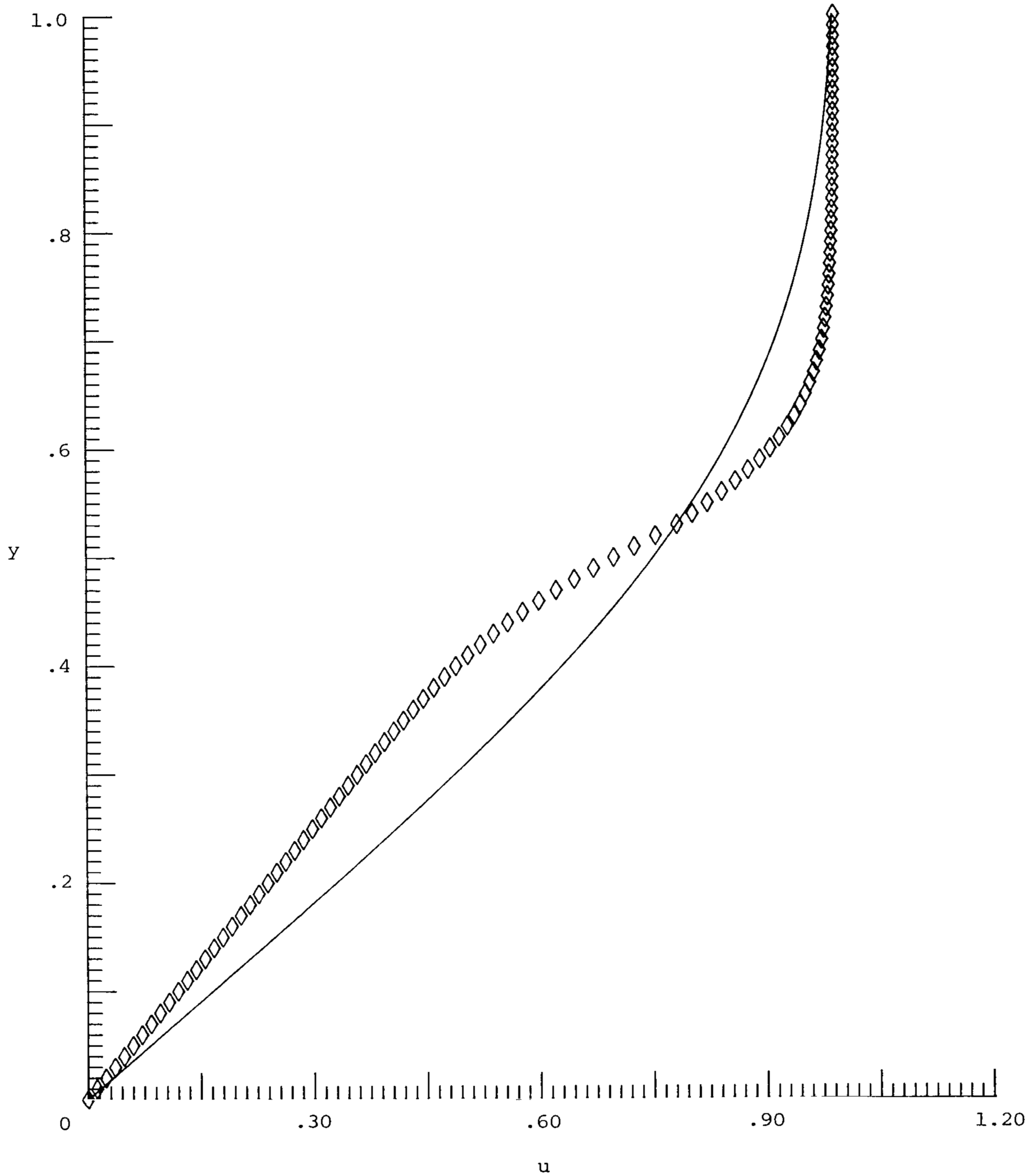
(b) $x + \phi = \frac{\pi}{2}$.

Figure 2.- Continued.



(c) $x + \phi = \pi$.

Figure 2.- Continued.



(d) $x + \phi = \frac{3\pi}{2}$.

Figure 2.- Concluded.

1. Report No. NASA TP-2245	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle THE DISTURBANCE FLOW FIELD PRODUCED BY AN EVOLVING VORTEX		5. Report Date December 1983
		6. Performing Organization Code 505-31-13-05
7. Author(s) T. B. Gatski		8. Performing Organization Report No. L-15670
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		10. Work Unit No.
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		13. Type of Report and Period Covered Technical Paper
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

The flow field of a vortex in a viscous shear flow is found by constructing a uniformly valid asymptotic expansion consisting of an inner solution field represented, to lowest order, by a two-dimensional, nonlinear, inviscid Stuart vortex and an outer solution field represented, to lowest order, by either a two-dimensional parallel or self-similar viscous flow. The technique involves scaling both the transverse and streamwise coordinates in the vicinity of the vortex as well as allowing for a "slow" variation of the outer viscous flow. Criteria are established for both the size of the vortical structure and proximity to the boundary surfaces. The composite solution is a consistent mathematical picture of the flow field at a fixed streamwise location as the vortical structure evolves past this point. Such a formulation is also useful in the specification of boundary or initial conditions in numerical fluid dynamic calculations, where an inconsistent setting of these conditions leads to spurious results for rather long computation times.

17. Key Words (Suggested by Author(s)) Vortex motion Coherent structure Asymptotic expansion		18. Distribution Statement Unclassified - Unlimited Subject Category 34	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 34	22. Price A03

For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1983

National Aeronautics and
Space Administration

Washington, D.C.
20546

Official Business
Penalty for Private Use, $300

THIRD-CLASS BULK RATE

Postage and Fees Paid
National Aeronautics and
Space Administration
NASA-451

U.S. MAIL

1 1U,D, 831215 S00903DS
DEPT OF THE AIR FORCE
AF WEAPONS LABORATORY
ATTN: TECHNICAL LIBRARY (SUL)
KIRTLAND AFB NM 87117

NASA

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return