NASA
Technical
Paper
2257

December 1983

Measurement of Rolling Friction by a Damped Oscillator

Moshe Dayan and
Donald H. Buckley

25

25th Anniversary
1958-1983

NASA

NASA
Technical
Paper
2257

1983

Measurement of Rolling Friction by a Damped Oscillator

Moshe Dayan and
Donald H. Buckley

Lewis Research Center
Cleveland, Ohio

Summary

An experimental method for measuring rolling friction is proposed. The method is mechanically simple. It is based on an oscillator in a uniform magnetic field and does not involve any mechanical forces except for the measured friction. The measured pickup voltage is Fourier analyzed and yields the friction spectral response. The proposed experiment is not tailored for a particular case. Instead, various modes of operation, suitable to different experimental conditions, are discussed.

Introduction

Rolling friction measurements usually involve elaborate mechanical arrangements and, in some cases, are not easy to make (refs. 1 to 4). Moreover, there is a suspicion that dissipative or nondissipative forces, apart from the friction to be measured, affect the results. In this report we are proposing a new method of rolling friction measurement that does not rely on mechanical devices. The only moving part is the rolling body whose friction (against a plane surface) is to be measured. It involves the use of an oscillator and the measurement of magnetic forces. Both the analysis of the concept and the experimental approach are given.

Analysis

Mechanically the apparatus is extremely simple, it includes a rolling oscillator over a horizontal plane. The restoring torque $\vec{\tau}$ is obtained by a magnetic field $\vec{B}$ interacting with a magnetic dipole $\vec{\mu}$ according to the following equation (ref. 5):

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{1}$$

The magnetic dipole results from dc current circulating in a coil placed inside the oscillating body. The directions of $\vec{B}$ and $\vec{\mu}$ are parallel at equilibrium. If we assume small-angle oscillations, the equation of motion for the angle $\theta(t)$ is

$$I\ddot{\theta} + B\mu\theta + f = 0 \tag{2}$$

where I is the moment of inertia and f is the rolling friction torque.

In cases of an electrically conducting oscillator, losses due to induced currents may be important. These losses, which are not included in equation (2), can be substantially reduced by reducing the conducting portions of the oscillating body. If the material whose friction is to be measured is metallic, it can be attached as a thin metallic foil on the surface of a bulk-insulating oscillator. Bringing the induced currents to a minimum is important for minimizing the experimental error. If that cannot be done directly, it can be done electrically by means of a feedback circuit, as discussed later under experimental considerations. Although it is possible to practically eliminate induced currents, we shall include their effect in our preliminary analysis. Their net effect is a magnetic moment normal to $\vec{B}$ and proportional to $B\dot{\theta}/\rho$, where ρ is the electrical resistivity. Thus the induced current term is a "Newtonian viscous damping" term. With it the equation of motion becomes

$$I\ddot{\theta} + \frac{\alpha}{\rho}B^2\dot{\theta} + \mu B\theta + f = 0 \tag{3}$$

where α is a constant dependent on the geometry. With $f = 0$, the "frictionless" or "pure viscous damping" case, the solution of equation (3) expresses exponentially decaying oscillations. When the friction is included, the equation is not soluble in the general case. Den Hartog (ref. 6) first solved accurately the externally driven (forced) oscillator with Coulomb (square wave) friction and combined Coulomb and viscous friction.

Equation (2) or (3) expresses f as a differential equation of θ. The angle $\theta(t)$ could be measured in different ways. We suggest the use of a magnetic pickup as the most suitable for our experimental approach. As is depicted in figure 1 a pickup coil L_p is placed in the oscillator so that its axis is normal to $\vec{B}$ at equilibrium. The coils L, L_p, and L_D are the restoring, pickup, and deriving coils, respectively.

The voltage $V(t)$ of the pickup coil is

$$V(t) = -\frac{1}{c}N_pA_pB\cos\theta\,\dot{\theta} \tag{4}$$

where N_p and A_p are, respectively, the number of turns and the cross-sectional area of L_p. For small-angle oscillations we can approximate

$$V(t) = -\frac{1}{c}N_pA_pB\dot{\theta} \tag{5}$$

and

$$a_0\int_0^t V(\tau)d\tau + a_1V(t) + a_2\dot{V}(t) + f = 0 \tag{6}$$

where $a_1 = 0$ with no induced currents.

We would like to stress that although the proposed experiment is introduced herein with specific features, the frictional oscillator is far more general. Thus, for example, the restoring force could be gravity or another force, and $\theta(t)$ could be measured by optical or
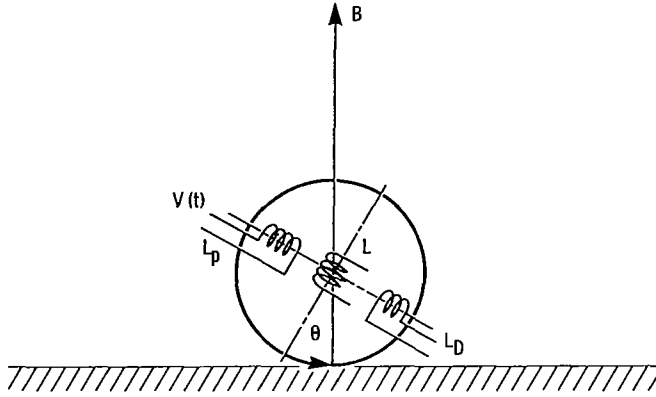
Figure 1. – Oscillator diagram.

mechanical means. Most of the discussion and analysis presented herein could be applied to any dissipative oscillator. Even the rolling friction is not essential and the method, in principle, is applicable to sliding friction as well. A specific apparatus is proposed because we believe it is the simplest way to apply the discussed method.

Even though equation (3) (or eq. (6)) is soluble in certain cases, it is not soluble in general. Rather than approximating it by using a model for f, we invert the equation of motion (i.e., use the measured velocity $\dot{\theta}(t)$ to numerically obtain the friction f). This function $f(t)$ can in principle be interpreted in terms of features inherent to the particular surfaces under study, ignoring features resulting from the main resonance $\sqrt{a_0/a_2}$. If the coefficients a_i are known together with $V(t)$, $f(t)$ can be found directly (by using numerical differentiation and integration). We believe that this straightforward approach, even though right in principle, does not yield the greatest accuracy. This is so because numerical differentiation usually yields rather "noisy" results unless incorporated with smoothening routines. The smoothening, however, is limited to the extent that it does not smear out real features. The alternative method of Fourier (or Laplace) transforms used herein is potentially better in terms of discrimination between signal and noise. Also, it provides a way of eliminating a_1, which is otherwise the main cause for systematic error. This is so because, as f, it represents a loss and affects the oscillatory decay.

The coefficient a_1 in equation (6) should be referred to as an experimentally adjustable parameter. The adjustment can be done by means of feedback electronic circuitry, which is described in the next section. If a_1 is adjustable, we would like to consider in detail two cases of special interest. One case is when $a_1 = 0$, which describes either the zero induced currents or the induced currents compensated by the feedback system. The other case of special interest is when the energy fed back compensates for both the induced current loss and the friction loss.

The Laplace transformation of equation (6) yields (ref. 7)

$$\alpha V(t) = \alpha V_0(t) \left[1 - \frac{1}{a_2 V_m} \alpha f \right] \tag{7}$$

where $\alpha y(t) = \int_0^\infty y(t)\, e^{-st}\, dt$ and $V_0(t)$ is the frictionless solution for $V(t)$ (what $V(t)$ would have been if $f \equiv 0$). In equation (7), $t = 0$ corresponds to the maximum pickup voltage V_m. The frictionless $V_0(t)$ for $a_1 = 0$ is an undamped harmonic oscillator. Its determination involves two parameters, the oscillation period and amplitude, which are well approximated by the measured period and first-cycle amplitude of $V(t)$. The constant a_2 should be determined independently. Thus for zero induced currents the experimental procedure involves exciting the oscillator by a pulse in L_D, measuring $V(t)$, extracting from it $\alpha V_0(t)$ and $\alpha V(t)$, and finally obtaining f by equation (7).

If viscous losses exist ($a_1 \neq 0$), determining $V_0(t)$ may be a problem, since the oscillation decay rate is affected by both f and a_1. It is always advantageous to minimize induced currents by reducing the metallic parts of the oscillator and enhancing the resistance of the current loops. In addition, we can compensate for these losses by feedback to L_D. We notice that the poles of $\alpha V_0(t)$ must be poles of $\alpha V(t)$. Thus the feedback should be adjusted to result in an imaginary pole of αV_0 (and αV). Practically this implies adjusting the feedback to get the best obtainable maximum of the half-line Fourier transform $\int_0^\infty e^{-i\omega\tau} V(\tau)\, d\tau$. To avoid being in the vicinity of the αV poles whose inherent source is αf (rather than αV_0), check for the maximum close to the expected V_0 resonance, namely $\omega_0 = \sqrt{a_0/a_2}$. We conclude that it is feasible to work in the zero a_1 mode in any practical case and to obtain the half-line Fourier transform of f.

It is quite interesting to deduce from equation (7) and a known theorem of Laplace transforms that (ref. 7)

$$V(t) = V_0(t) - \frac{1}{a_2 V_m} \int_0^t f(\tau)\, V_0(t-\tau)\, d\tau \tag{8}$$

Thus the deviation $[V(t) - V_0(t)]$ is proportional to the convolution of the friction f with the frictionless function V_0. This conclusion, even though not useful for the present analysis, improves our understanding. It can be generalized to nonoscillatory problems. It is valid for any frictional problem whose frictionless part can be written as a linear and homogeneous differential equation.

So far we have dealt with the decaying oscillation mode of operation. It is worthwhile to consider modes of

2

operation that maintain steady-state oscillations by providing the energy losses from an external source. At first we assume that the energy return is provided by a feedback force proportional to V, namely by adjusting a_1 in equation (6) to maintain steady-state oscillations. The periodicity suggests the use of the finite Fourier transform $E_n(V) = \int_{-\pi}^{\pi} e^{in\omega\tau} V(\tau) d(\omega\tau)$. Transforming equation (6) we get

$$a_0 E_n(V) + in\omega a_1 E_n(V)$$
$$- a_2 n^2 \omega^2 E_n(V) + in\omega E_n(f) = 0 \qquad (9)$$

By writing separate equations for the real and imaginary parts and denoting $A_n = \Re e E_n(V)$, $B_n = \Im m E_n(V)$, $C_n = \Re e E_n(f)$, $D_n = \Im m E_n(f)$, we have

$$C_n = -a_1 A_n - a_2 \frac{\omega_0^2 - n^2\omega^2}{n\omega} B_n \qquad (10a)$$

$$D_n = -a_1 B_n + a_2 \frac{\omega_0^2 - n^2\omega^2}{n\omega} A_n \qquad (10b)$$

where $n = 1, 3, 5, \ldots$ Even n's are excluded because of the problem symmetry, since $V(t)$ is antisymmetric with respect to a translation by a half-oscillation period. Equations (10) provide the spectral distribution of f versus the measured function $V(t)$ and the parameters ω_0, a_2, and a_1. For zero induced currents a_1 can be obtained by measuring the feedback current to L_D. For substantial induced currents, however, an accurate determination of a_1 is hard and this mode of operation is undesirable.

If induced currents do not exist, we can simplify the solution by filtering the feedback current and keeping only its first harmonic. Then $a_1 V$ in equation (6) is replaced by $b \sin \omega t$, and we get for odd n

$$C_n = -a_2 \frac{\omega_0^2 - n^2\omega^2}{n\omega} B_n \qquad (11a)$$

$$D_n = a_2 \frac{\omega_0^2 - n^2\omega^2}{n\omega} A_n + \pi b \delta_{n,1} \qquad (11b)$$

where $\delta_{n,1}$ is the Kronecker function. Similarly to the case of decaying oscillations, equation (11) suggests that $E_n(V)$ has a resonance at ω_0 and may have other resonances (or different features) inherent to the frictional response. Typical relaxation times associated with the response of the particular surfaces in study should be reflected in $E_n(f)$. Further insight can be achieved by varying ω_0 (by means of the restoring force dc current) and obtaining $E_n(f, \omega_0)$.

Finally, we would like to point out the possibility of external excitation with fixed amplitude and frequency (i.e., the "forced oscillator"). The equation of motion (without induced currents) is

$$a_0 \int_0^t V \, d\tau + a_2 \dot{V} + f = Q \cos \Omega t \qquad (12)$$

which transforms to

$$C_n = -a_2 \frac{\omega_0^2 - n^2\Omega^2}{n\Omega} B_n + \pi Q \delta_{n,\,\pm 1} \qquad (13a)$$

$$D_n = a_2 \frac{\omega_0^2 - n^2\Omega^2}{n\Omega} A_n \qquad (13b)$$

where we have used the same notation as in equations (10) and (11). Here A_n has an extra pole at Ω (and subharmonics). Despite this limitation, the forced oscillator is attractive because of its experimental simplicity. The choice of a particular mode of operation depends on the system under consideration, the requirements, and the instrumentation available.

Experimental

In keeping with former sections of this report, where we have discussed various possibilities for matching with specific situations, we shall outline here an experimental system flexible enough to be used in various modes of operation.

The main requirements from the magnetic field $\vec{B}$ are intensity and homogeneity in the region of oscillations. A strong magnetic field creates high fundamental frequency for a given magnetic moment and inertia and thus makes possible a wider range of velocities. Also, the stronger the field, the higher the pickup amplitude and consequently the signal-to-noise ratio. For a cylindrical oscillator of 1-g cm^2 moment of inertia, a magnetic field of 10^4 G, and a magnetic moment of 1 G (a coil of 1-cm^2 cross-sectional area and 10-A turns), the fundamental resonance frequency $\omega_0/2\pi$ is about 17 Hz. This frequency, which corresponds to a surface peak tangential velocity of the order of 50 cm sec^{-1}, is achievable with quite moderate magnetic fields and moments. Enhancing the frequency is

not technically difficult, but one should be careful to keep the inertia term $I\theta/R \propto I/R\,\omega^2$ smaller than the sliding friction force; otherwise, slip may occur.

Even though the sliding friction force puts an upper limit on the fundamental frequency of oscillation, the analysis of higher harmonics is limited only by the rate of data acquisition and the time constant of the electronics. With modern equipment a data acquisition rate of 10^{-3} relative resolution for both the oscillation period and voltage amplitude should not be difficult to achieve. Thus with careful shielding and grounding, very high order harmonics are resolvable. This feasible harmonics resolution provides valuable information on intrinsic frictional relaxation times much shorter than the oscillation period. Such may be, for example, the relaxation time needed for the friction to switch polarity around the zero velocity point. If this time is in the range 10^{-4} sec, it could be detected by relatively standard equipment. To go to the other extreme of long relaxation times, the fundamental frequency should be lowered (to the 0.1-Hz range).

With approximately the magnetic field and the frequency mentioned above, the pickup coil can be designed to yield a signal amplitude of a few millivolts without exceeding the time-constant limit. This would allow a signal resolution better then 10^{-3}. Altogether, instrumentation is not expected to be the cause of substantial experimental errors. All of the other errors foreseen can be classified into two groups: (1) errors stemming from gravitational-geometrical origins such as a possible lack of balance of either the oscillator or the planar surface and (2) errors resulting from uncertainty in determining the constants of the problem (the a_i's). The first group is not hard to reduce, by fine machining and mechanical balancing, to a minute value. Also, we can show that this type of error, when small, results in a small constant additive to a_0. Thus it translates to be an uncertainty of the second group but a very small one. The additive constant would not cause any additional error to that which exists anyway in measuring a_0 (or ω_0).

The other errors result from uncertainties in the values of the coefficients a_0, a_1, and a_2 (or ω_0, a_0, and a_1). For all nonconducting materials $a_1 = 0$ (or can be directly measured if caused by feedback). For conductors the best policy would be to start with minimal induced currents. This objective could be achieved if the bulk of the oscillator is made of an insulator, where the tested sample, as a thin foil of thickness d, is attached to the angular section that makes contact during oscillations. With this geometry the induced current is minimized to the order of $dB(R\theta_m)^2\sigma\dot\theta/c$ and the resulting force is $(dA\sigma)(B/c)^2\dot\theta\theta_mR$, where A is the foil's area, $2R\theta_m$ is the foil's width, and σ is the conductivity. The last force is somewhat different in character from the "Newtonian

viscous" term discussed so far, which approximates the homogeneous bulk conductor, or the full cylindrical foil. Inserting values as in the former example and $\sigma = 10^4$ $(\Omega\,\mathrm{cm})^{-1}$ and $d = 10^{-2}$ cm in the last expression, we get a force of 10^{-5} to 10^{-6} g. This value is negligible when compared with any typical rolling friction force, and it would not become significant even for the lowest resistivity metal. Only when the whole oscillator is made of a good conductor might the induced current produce a significant error, and then it should be treated along the lines described in the section Analysis.

Since in most practical cases we can reduce a_1 to a negligible value, there are two independent parameters to measure. One feasible possibility is to measure ω_0 and a_0. Note that $a_0 = -N_rA_rI_r/N_pA_p$, where N_r and A_r are the number of turns and cross-sectional area of the restoring coil and I_r is the current through the restoring coil. Thus a_0 is proportional to the ratio of the pickup voltages of these two coils when the cylinder is revolving at a constant angular velocity. This quantity can be measured to a very good accuracy. The parameter ω_0 can be approximated by the actual measured frequency. If this is not accurate enough, a correction could be made by taking into account the loss of the first interation and making a second iteration calculation. To conclude, we estimate that absolute accuracies of the order of ± 1 percent can be obtained. For a comparison between two samples, the relative accuracy could be higher by one order of magnitude.

A block diagram of the experimental system is shown in figure 2. The system makes possible the choice between feedback-maintained steady-state oscillations, pulse-excited decaying oscillations, or continuous-wave forced oscillations. The feedback control unit might include a filter if one chooses to feed back only the first harmonic.

The oscillating body cannot be a sphere because any rotational component around the axis of the pickup coil is forbidden. There should be a cylindrical symmetry around the axis mutually normal to the coil's axes. Apart from this, a great deal of freedom in the oscillator shape is allowed. It may be of any shape that has a sectional cylindrical symmetry. A measurement on a rolling ball could be done if only one degree of freedom is eliminated (e.g., by a rigid attachment of two identical balls). The finishing of the surfaces should be accurate enough to avoid microscopic slip. If the friction versus load has to be measured, we recommend varying the load by attaching weights on the oscillator. Loading by means of any mechanical system is undesirable since it may introduce tangential forces additional to those described in equation (3). Finally, since no assumption has been made on f, the experiment can be performed in any medium or in vacuum and on either dry or lubricated surfaces.
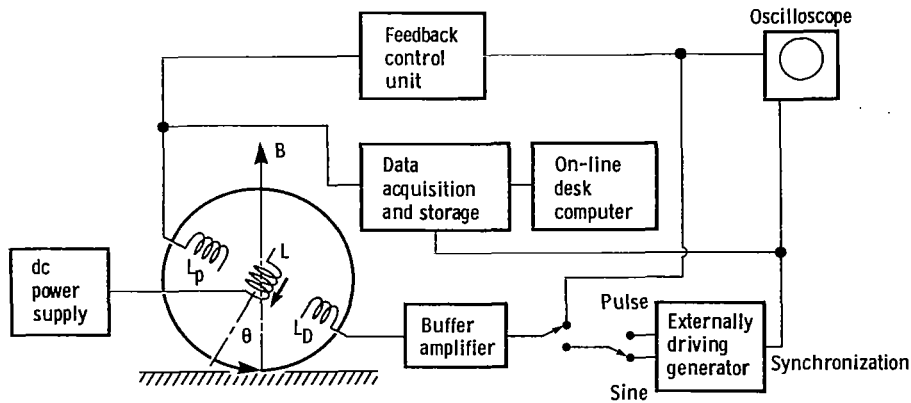
4

Figure 2. - Block diagram of experimental system.

Concluding Remarks

The method suggested herein for measuring rolling friction is characterized by mechanical simplicity and is compatible with friction measurements and detecting relaxation times in the range 10^1 to 10^{-4} sec. It is, however, a relatively low-velocity measurement because of possible slip at higher velocities. We believe it is capable of detecting friction polarity switching times and friction variations that may occur in the low speed range (0.1 cm/sec, ref. 2). Further analysis is needed for interpreting the output spectral response of rolling friction torque in terms of constant-velocity friction. At low vibrational frequencies, when the acceleration is small enough not to smear out friction velocity-dependence variations, translation to velocity dependency is easy. In this case we can write

$$f(\dot{\theta}) = \frac{-1}{\pi} \sum_{n=1,3,5...} D_n \sin n\omega t(\dot{\theta}) \qquad (14)$$

$$= \frac{1}{\pi} \sum_{n=1,3,5...} C_n \cos n\omega t(\dot{\theta})$$

where $t(\dot{\theta}) = t(V)$ is the inverse of $V(t)$ and t varies over one oscillation period. If high acceleration results in $f(\dot{\theta})$ different from the steady-state function, we can still learn a great deal about the relaxation time required to establish the steady-state function.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, August 15, 1983

References

1. Bowden, F. P.; and Tabor, D.: The Friction and Lubrication of Solids. Clarendon Press (Oxford), 1950.
2. Bowden, F. P.; and Tabor, D.: The Friction and Lubrication of Solids. Part II, Clarendon Press (Oxford), 1964.
3. Drutowski, R. C.: Energy Losses of Balls Rolling on Plates. J. Basic Eng., vol. 81, no. 2, June 1959, pp. 233-238.
4. Markho, P. H.: An Investigation of Free Rolling Resistance at Light Loads. J. Lubr. Technol., vol. 104, no. 3, July 1982, pp. 376-381.
5. Jackson, J. D.: Classical Electrodynamics. John Wiley Inc., 1962.
6. Den Hartog, J. P.: Forced Vibrations with Combined Coulomb and Viscous Friction. ASME Paper APM-53-9, 1931.
7. Churchill, R. V.: Modern Operational Mathematics in Engineering. 2nd. ed., McGraw-Hill, 1958.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TP-2257		

4. Title and Subtitle	5. Report Date
Measurement of Rolling Friction by a Damped Oscillator	December 1983
	6. Performing Organization Code
	506-53-1B

7. Author(s)	8. Performing Organization Report No.
Moshe Dayan and Donald H. Buckley	E-1583
	10. Work Unit No.

9. Performing Organization Name and Address	11. Contract or Grant No.
National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	
	13. Type of Report and Period Covered

12. Sponsoring Agency Name and Address	Technical Paper
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

An experimental method for measuring rolling friction is proposed. The method is mechanically simple. It is based on an oscillator in a uniform magnetic field and does not involve any mechanical forces except for the measured friction. The measured pickup voltage is Fourier analyzed and yields the friction spectral response. The proposed experiment is not tailored for a particular case. Instead, various modes of operation, suitable to different experimental conditions, are discussed.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Rolling friction Damped oscillations	Unclassified - unlimited STAR Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified	7	A02

THIRD-CLASS BULK RATE

U.S.MAIL

NASA