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KINEMATIC EQUATIONS FOR RESOLVED-RATE CONTROL OF AN INDUSTRIAL ROBOT ARM

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SUMMARY

An operator can use kinematic, resolved-rate equations to dynamically control a robot arm by watching its response to commanded inputs. In a tutorial fashion, this paper derives known resolved-rate equations for the control of a particular six-degree-of-freedom industrial robot arm and proceeds to simplify the equations for faster computations. Methods for controlling the robot arm in regions which normally cause mathematical singularities in the resolved-rate equations are discussed.

INTRODUCTION

In the Intelligent Systems Research Laboratory at the Langley Research Center, an operator sits at a remote console with a three-axis controller in each hand and commands the motions of an industrial robot arm. The operator has optional control modes. In particular, resolved-rate control (ref. 1) enables the operator to directly control the robot hand. The operator views the robot hand, decides that he wants it to move in a certain direction, and deflects a controller. The robot hand then moves accordingly with a velocity proportional to the amount of deflection of the controller. Commanded hand velocities are transformed (resolved) into requisite movements (velocities) of the individual joints in the robot arm to effect the commanded hand motion.

The intent of this paper is to: (1) derive the resolved-rate equations in reference 2 from a point of view of an operator remotely controlling a robot arm; (2) simplify these equations for real-time application; (3) leave additional parameters unspecified in the final equations for more flexibility in modeling the robot arm; and (4) further document a set of resolved-rate equations.

SYMBOLS

A^i_{i-1}	homogeneous transformation matrix from coordinate system i to coordinate system i-1
Ai, a_i	common normal between Z_{i-1} and Z_i
CAL3	Cos α_3
Ci	Cos θ_i
C23	Cos $(\theta_2 + \theta_3)$
D1, D2, D3, D4, D5	functions defined by equations (52) to (56), respectively

$\underline{d}_{k6}$, $\underline{d}_{k,6}$	vector from coordinate system k to hand coordinate system
$\hat{\underline{d}}_{k6}$, $\hat{\underline{d}}_{k,6}$	$\underline{d}_{k6}$ in base coordinates
ES	elbow-to-shoulder length
F1, F3, F4, F5, F6, F7	functions defined by equations (27), (29), (30), (31), (41) and (37), respectively
F2	constant defined by equation (28)
$\underline{g}^1$, $\underline{g}^2$	vectors defined by equation (42) and (59), respectively
HW	hand-to-wrist length
i	integer which indicates different axis systems and associated parameters
K2, K3	constant gains
k	integer
M, M1, M2	matrices defined by equations (26), (43), and (58), respectively
M^*, $M1^*$, $M2^*$	generalized matrix inverse of M, M1, and M2, respectively
NO	neck-to-base length
$\underline{P}_{0k}$, $\underline{P}_{0,k}$	position vector in base coordinates from base coordinate system to coordinate system k
Q	point in cartesian coordinates
$\underline{q}$	vector to point Q
$\underline{RVEL}$	rotational velocity of robot hand, expressed in hand axis system
$\hat{\underline{RVEL}}$	$\underline{RVEL}$ in base coordinates
$\tilde{\underline{RVEL}}$	resultant rotational velocity of that commanded and that induced by rotations of joints 1, 2, and 3 (eq. 48)
RVEL(1), RVEL(2), RVEL(3)	components of $\underline{RVEL}$
$\hat{RVEL}(1)$, $\hat{RVEL}(2)$, $\hat{RVEL}(3)$	components of $\hat{\underline{RVEL}}$
$\tilde{RVEL}(1)$, $\tilde{RVEL}(2)$, $\tilde{RVEL}(3)$	components of $\tilde{\underline{RVEL}}$
R^i_{i-1}	rotational transformation matrix from coordinate system i to coordinate system i - 1

Ri, r_i	relative distance between coordinate system i-1 and coordinate system i, measured along Z_{i-1}
SAL3	Sin α_3
Si	Sin θ_i
S23	Sin $(\theta_2 + \theta_3)$
SN	shoulder-to-neck length
$\widetilde{SN}$	SN + R3
$\underline{TVEL}$	translational velocity of robot hand in hand axis system
$\hat{\underline{TVEL}}$	$\underline{TVEL}$ in base coordinates
TVEL(1), TVEL(2), TVEL(3)	components of $\underline{TVEL}$
$\hat{TVEL}(1)$, $\hat{TVEL}(2)$, $\hat{TVEL}(3)$	components of $\hat{\underline{TVEL}}$
Δt	time increment
$\hat{\underline{V}}_i$	translational velocity of hand axis system caused by rotation of joint i, expressed in base coordinates
WE	wrist-to-elbow length
X_i	axis directed along common normal between Z_{i-1} and Z_i
Y_i	axis directed to complete right-hand axis system with X_i and Z_i
Z_i	axis of rotation of joint i-1
X_0, Y_0, Z_0	base coordinate system
X_6, Y_6, Z_6	hand coordinate system
$\underline{x}_i$, $\underline{y}_i$, z_i	unit vectors along X_i, Y_i, Z_i
$\hat{\underline{x}}_i$, $\hat{\underline{y}}_i$, $\hat{\underline{z}}_i$	unit vectors $\underline{x}_i$, $\underline{y}_i$, $\underline{z}_i$, expressed in base coordinates
α_i	angle between Z_{i-1} and Z_i, measured positive about X_i
θ_i	joint angle with initial value corresponding to initial position of robot arm in figure 3

θ'_i	joint angle between X_{i-1} and X_i, measured positive about Z_{i-1}
$\underline{\theta}$	vector of joint angles (eq. 12)
$\underline{\theta}_{New}$, $\underline{\theta}_{Old}$	next and previous vector of joint angles, respectively
$\underline{\omega}_i$	rotational velocity of joint i about Z_{i-1}
$\hat{\underline{\omega}}_i$	$\underline{\omega}_i$ expressed in base coordinates

Use of a dot over a symbol indicates first derivative with respect to time, a vector is underlined, and a caret (^) above a vector means that the vector is expressed with respect to the base coordinate system.

ANALYSIS

Figure 1 represents a six-degree-of-freedom industrial manipulator. In reference 2, three of these manipulators served as legs in simulating the locomotion of a three-legged robot over structural beams. In the current paper, based on reference 2, an operator controls the motions of a robot arm.

Controlling individual joints in a robot arm to accomplish a complex task is difficult, especially if time to complete the task is critical or if part of the operator's attention is needed elsewhere. A more natural approach is for an operator to command the motion of the robot hand and then automate the requisite coordination of the individual joints in the arm (ref. 1). The relative joint geometry dictates the basic transformation equations.

Joint Axis Systems and Transformation Matrices

Consecutive joint axis systems in robotic manipulators can be related by the Denavit-Hartenberg parameters (ref. 3). For rotational joints, (figure 2) these parameters consist in three constant parameters a_i, r_i, α_i and a variable joint angle θ'_i. By definition, joints always rotate about their Z-axis. The Y_{i-1}- and Y_i- axis (not shown) complete right-handed coordinate systems. (Although not considered here, r_i is the variable for prismatic joints.)

The homogeneous transformation matrix (based on figure 2) from coordinate system i to coordinate system i-1 is (refs. 2 or 4, for example)

$$A^i_{i-1} = \left[\begin{array}{ccc:c} \cos\theta'_i & -\cos\alpha_i \sin\theta'_i & \sin\alpha_i \sin\theta'_i & a_i \cos\theta'_i \\ \sin\theta'_i & \cos\alpha_i \cos\theta'_i & -\sin\alpha_i \cos\theta'_i & a_i \sin\theta'_i \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \qquad (1)$$

The parameter r_i plays the role of s_i in reference 2 and d_i in reference 3.

Let the location of a point Q with respect to the coordinate system (X_i, Y_i, Z_i) be described by the vector $\underline{q}_i$. Then, the location of Q from coordinate system (X_{i-1}, Y_{i-1}, Z_{i-1}) is the vector $\underline{q}_{i-1}$, where

$$\begin{bmatrix} \underline{q}_{i-1} \\ \hline 1 \end{bmatrix} = A^i_{i-1} \begin{bmatrix} \underline{q}_i \\ \hline 1 \end{bmatrix} \tag{2}$$

in which A^i_{i-1} accounts for both rotation and displacement of (X_i, Y_i, Z_i) with respect to (X_{i-1}, Y_{i-1}, Z_{i-1}). However, if one is only interested in the components of $\underline{q}_i$ in directions parallel to (X_{i-1}, Y_{i-1}, Z_{i-1}), such as velocity, then it is sufficient to compute:

$$\underline{q}_{i-1} = R^i_{i-1} \underline{q}_i \tag{3}$$

where

$$R^i_{i-1} = \begin{bmatrix} \cos\theta'_i & -\cos\alpha_i \sin\theta'_i & \sin\alpha_i \sin\theta'_i \\ \sin\theta'_i & \cos\alpha_i \cos\theta'_i & -\sin\alpha_i \cos\theta'_i \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix} \tag{4}$$

is the submatrix of A^i_{i-1} which accounts for the rotation of (X_i, Y_i, Z_i) with respect to (X_{i-1}, Y_{i-1}, Z_{i-1}).

The Denavit-Hartenberg parameters which have been specifically assigned numerical values and those parameters which are carried symbolically in subsequent equations for assignment by researchers are shown in the table. In reference 2, ES = WE, $\alpha_3 = 90°$, and $a_3 = r_3 = 0$. However, in the present paper, ES and WE may be different; and different constant values (refs. 2 and 5) for α_3 and r_3 can be used. Also, since preliminary measurements indicate that α_3 may not be exactly 90°, α_3 is left unspecified in the equations. A method to calculate the Denavit-Hartenberg parameters for an assembled robot arm is developed in reference 6. (The parameter r_2 would be chosen as zero in reference 6 because joints (2) and (3) produce parallel rotations; however, the nonzero value of r_2 used in reference 2 (and here) can also be obtained by the same basic method in reference 6.)

Notice in the table that θ'_i is related to another joint angle θ_i (unprimed). The joint angles θ_i ($i = 1,2,\ldots,6$) are referenced to the initial position of the robot arm in figure 3.

The transformation matrices A^i_{i-1} (in terms of θ_i, $i = 1,2,\ldots,6$) are given in reference 2, except for A^3_2, which is different because of the three unspecified parameters α_3, r_3, and a_3. However, for convenience, all these transformation matrices are contained in appendix A. The rotational matrices R^i_{i-1} are simply the 3x3 submatrices in the upper left-hand corner of A^i_{i-1}.

Resolved-Rate Control Equations

Figure 4 shows the axis system (X_6, Y_6, Z_6) of the robot hand. (The hand itself is not shown.) With one three-axis controller, an operator commands translational speeds TVEL(1), TVEL(2), and TVEL(3) along X_6, Y_6, and Z_6, respectively; and, with the other three-axis controller, he commands rotational speeds RVEL(1), RVEL(2), and RVEL(3) about X_6, Y_6, and Z_6, respectively. That is, the operator commands the translational velocity

$$\underline{\text{TVEL}} = \begin{bmatrix} \text{TVEL}(1) \\ \text{TVEL}(2) \\ \text{TVEL}(3) \end{bmatrix} \tag{5}$$

and the rotational velocity

$$\underline{\text{RVEL}} = \begin{bmatrix} \text{RVEL}(1) \\ \text{RVEL}(2) \\ \text{RVEL}(3) \end{bmatrix} \tag{6}$$

$\underline{x}_6$, $\underline{y}_6$, and $\underline{z}_6$ are unit vectors along X_6, Y_6, and Z_6, respectively. These commands are then resolved by the computer into individual joint rotations in the robot arm to produce the commanded hand motion.

The axis system for the robot hand may be located wherever desired for convenience; for example, near the tip of the robot hand (fig. 5) or, as in this paper, at the robot hand mounting (fig. 1). In the sequel, commands of rotational and translational velocity to the robot hand are expressed in terms of joint velocities. First, a word about notation: a vector is underlined and an overhead caret (^) indicates a vector expressed with respect to the base coordinate system (X_0, Y_0, Z_0). For example, $\hat{\underline{x}}_i$ is a unit vector along X_i but expressed in base coordinates.

Rotational velocity of robot hand in base coordinates.- By convention, joint i in the robot arm rotates with angular speed $\dot{\theta}'_i$ about Z_{i-1} (fig. 2). However, since θ_i only differs from θ'_i by a constant offset, $\dot{\theta}_i = \dot{\theta}'_i$. Thus, the rotational velocity of joint i is (fig. 6)

$$\underline{\omega}_i = \dot{\theta}_i \underline{z}_{i-1} \tag{7}$$

or, with respect to the base-coordinate system,

$$\hat{\underline{\omega}}_i = R_0^{i-1} \dot{\theta}_i \underline{z}_{i-1} \tag{8}$$

The vector sum of these individual joint rotational velocities is the resultant rotational velocity of the robot hand:

$$\hat{\underline{\text{RVEL}}} = \sum_{i=1}^{6} R_0^{i-1} \dot{\theta}_i \underline{z}_{i-1} \tag{9}$$

In vector-matrix form, equation (9) is

$$\underline{\hat{RVEL}} = \left[\hat{\underline{z}}_0 \vdots \hat{\underline{z}}_1 \vdots \cdots \vdots \hat{\underline{z}}_5 \right] \dot{\underline{\theta}} \tag{10}$$

where

$$\hat{\underline{z}}_{i-1} = R_0^{i-1} \underline{z}_{i-1} = R_0^{i-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{11}$$

is the unit vector $\underline{z}_{i-1}$ expressed in base coordinates and is simply the third column of R_0^{i-1}, and where

$$\dot{\underline{\theta}} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_6 \end{bmatrix} \tag{12}$$

is a vector of joint velocities. (A programming notation in reference 2 is $JVEL(i) = \dot{\theta}_i$)

Translational velocity of robot hand in base coordinates.- The resultant translational velocity of the robot hand caused by individual joint relations in the robot arm is

$$\underline{\hat{TVEL}} = \sum_{i=1}^{6} \hat{\underline{V}}_i \tag{13}$$

where joint i induces the translational velocity

$$\hat{\underline{V}}_i = \dot{\theta}_i \hat{\underline{z}}_{i-1} \times \hat{\underline{d}}_{i-1,6} \tag{14}$$

which is the cross product of the joint rotational velocity (with respect to the base coordinate system) and the vector moment arm

$$\hat{\underline{d}}_{i-1,6} = \underline{p}_{06} - \underline{p}_{0,i-1} \tag{15}$$

from the origin of coordinate system i-1 to the origin of the hand axis system. For example, the vector moment radii $\hat{\underline{d}}_{06}$ and $\hat{\underline{d}}_{16}$ are shown in figure 7. $\underline{p}_{0k}$ is the 3x1 vector in the upper-right corner of $A^k{}_0 = A^1{}_0 A^2{}_1 \ldots A^k{}_{k-1}$; that is, the first three entries in the fourth column of $A^k{}_0$. In vector-matrix form, equation (13) becomes

$$\underline{\hat{TVEL}} = \left[\hat{\underline{z}}_0 \times \hat{\underline{d}}_{06} \vdots \hat{\underline{z}}_1 \times \hat{\underline{d}}_{16} \vdots \cdots \vdots \hat{\underline{z}}_5 \times \hat{\underline{d}}_{56} \right] \dot{\underline{\theta}} \tag{16}$$

Jacobian matrix.- Equations (10) and (16) are combined as

$$\begin{bmatrix} \hat{\underline{TVEL}} \\ \hline \hat{\underline{RVEL}} \end{bmatrix} = J\,\dot{\underline{\theta}} \tag{17}$$

using the jacobian matrix

$$J = \left[\begin{array}{c|c|c|c} \hat{\underline{z}}_0 \times \hat{\underline{d}}_{06} & \hat{\underline{z}}_1 \times \hat{\underline{d}}_{16} & \cdots & \hat{\underline{z}}_5 \times \hat{\underline{d}}_{56} \\ \hline \hat{\underline{z}}_0 & \hat{\underline{z}}_1 & \cdots & \hat{\underline{z}}_5 \end{array}\right] \tag{18}$$

Symbolically, for nonsingular J,

$$\dot{\underline{\theta}} = J^{-1} \begin{bmatrix} \hat{\underline{TVEL}} \\ \hline \hat{\underline{RVEL}} \end{bmatrix} \tag{19}$$

constitutes the set of joint angles which will produce commanded values of translational velocity $\hat{\underline{TVEL}}$ and rotational velocity $\hat{\underline{RVEL}}$. For the robot arm in figure 1, there are six joint angles so that J is a 6x6 matrix.

Control inputs in hand axis system.- In equations (17) and (19) the control inputs are with respect to the base coordinate system (as indicated by the overhead caret). However, in application, an operator watches the robot hand move and issues commands to the hand itself. Therefore, the auxiliary equation

$$\begin{bmatrix} \hat{\underline{TVEL}} \\ \hline \hat{\underline{RVEL}} \end{bmatrix} = \begin{bmatrix} \underline{R}\,{}^{6}_{0} & \underline{TVEL} \\ \hline R\,{}^{6}_{0} & \underline{RVEL} \end{bmatrix} \tag{20}$$

is needed to transform the operator's inputs from the hand axis system to the base coordinate system. For specific elements in R^6_0 see appendix B. Now, as indicated in figure 8, an operator inputs translational and rotational velocities in the hand axis system to make the robot arm move. These inputs are then transformed (eq. 20) to the base coordinate system for use in the resolved-rate equations (eqs. 17 or 19) to compute the joint velocity ($\dot{\underline{\theta}}$) to drive the robot arm. These joint velocities must be integrated to obtain joint angles; for example, the arm moves to a new position θ_{New}, which is related by Euler integration to its old position θ_{Old} by the equation

$$\underline{\theta}_{New} = \underline{\theta}_{Old} + \dot{\underline{\theta}}\,\Delta t \tag{21}$$

where Δt is the time increment for computationally updating the joint angles in the robot arm. The operator varies his inputs to dynamically drive the robot arm by using feedback, such as visual, graphical, or force.

Location of Hand Axis System

Simplification of J matrix.- In general, one is faced with solving equation (17) for $\dot{\theta}$, given $\hat{TVEL}$ and $\hat{RVEL}$. Reduction in the computational complexity is beneficial for real-time operation. Toward this end, the origin of the hand coordinate system (X_6, Y_6, Z_6) is chosen to coincide with the origins of (X_4, Y_4, Z_4) and (X_5, Y_5, Z_5) in figure 3 (ref. 2). Consequently, $\hat{d}_{45} = \hat{d}_{56} = 0$; and $\hat{z}_3$ and $\hat{d}_{36}$ are parallel. Therefore, the three cross-product terms in the upper-right corner of J are zero, that is

$$J = \left[\begin{array}{c|c|c|c|c|c} \hat{z}_0 \times \hat{d}_{06} & \hat{z}_1 \times \hat{d}_{16} & \hat{z}_2 \times \hat{d}_{26} & 0 & 0 & 0 \\ \hline \hat{z}_0 & \hat{z}_1 & \hat{z}_2 & \hat{z}_3 & \hat{z}_4 & \hat{z}_5 \end{array}\right] \tag{22}$$

Equations for translational and rotational velocities of robot hand resulting from simplified J matrix.- From equations (17) and (22),

$$\hat{TVEL} = (\hat{z}_0 \times \hat{d}_{06})\dot{\theta}_1 + (\hat{z}_1 \times \hat{d}_{16})\dot{\theta}_2 + (\hat{z}_2 \times \hat{d}_{26})\dot{\theta}_3 \tag{23}$$

$$\hat{RVEL} - \hat{z}_0\dot{\theta}_1 - \hat{z}_1\dot{\theta}_2 - \hat{z}_2\dot{\theta}_3 = \hat{z}_3\dot{\theta}_4 + \hat{z}_4\dot{\theta}_5 + \hat{z}_5\dot{\theta}_6 \tag{24}$$

Hence, equation (23) is solved for $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$; and, with these solutions, equation (24) is solved for $\dot{\theta}_4$, $\dot{\theta}_5$, and $\dot{\theta}_6$.

Solving for Joint Rates $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$

Equation (23) can be expressed as (see appendix C)

$$\begin{bmatrix} \hat{TVEL}\ (1) \\ \hat{TVEL}\ (2) \\ \hat{TVEL}\ (3) \end{bmatrix} = M \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{25}$$

where

$$M = \begin{bmatrix} -S1\ F1 - C1\ F2 & C1\ F3 & C1\ F4 \\ C1\ F1 - S1\ F2 & S1\ F3 & S1\ F4 \\ 0 & -F1 & -F5 \end{bmatrix} \tag{26}$$

$$F1 = S2\ ES + F5 \tag{27}$$
$$F2 = \widetilde{SN} + CAL3\ WE \tag{28}$$
$$F3 = C2\ ES + F4 \tag{29}$$
$$F4 = C23\ SAL3\ WE - S23\ A3 \tag{30}$$
$$F5 = S23\ SAL3\ WE + C23\ A3 \tag{31}$$

Singularities associated with M.- The determinant of M (appendix D), equated to zero, supplies the following singularity conditions:

$$S2\ ES + S23\ SAL3\ WE + C23\ A3 = 0 \tag{32}$$

$$WE\ SAL3\ S3 + A3\ C3 = 0 \tag{33}$$

With variations in θ_1 (fig. 3), the origin of the hand axis system generates a circle about Z_0. Equation (32) implies that the minimum radius for this circular motion has been reached; that is, no further motion normal to Z_0 is possible. For nominal values $a_3 = 0$, ES = WE, and $\alpha_3 = 90°$, equation (32) reduces to the singular condition S2 + S23 = 0 in reference 2.

Equation (33) implies that the robot arm is at its maximum (or minimum) extension with respect to the joint angle θ_3. For nominal values $a_3 = 0$ and $\alpha_3 = 90°$, equation (33) corresponds to the singular condition $\theta_3 = 0$ in reference 2. ($\theta_3 = 180°$ is not achievable with the robot arm depicted in figure 3.) With $\theta_3 = 0$ in figure 3, the robot hand can be extended no further along Z_6. In figure 3, both singularity conditions are satisifed with all the translational velocity along X_1 and none along Y_1 and Z_1.

Solution for nonsingular conditions (det (M) ≠ 0).- When not in a singular condition, equation (25) is solved directly as

$$\dot{\theta}_1 = \{[\hat{TVEL}(1)\ S1 - \hat{TVEL}(2)\ C1]\ [F1\ F4 - F3\ F5]\}/\det(M) \tag{34}$$

$$\dot{\theta}_2 = -\{F5\,[\hat{TVEL}(1)\ F7 + \hat{TVEL}(2)\ F6] + \hat{TVEL}(3)\ F1\ F4\}/\det(M) \tag{35}$$

$$\dot{\theta}_3 = -F1\,[\hat{TVEL}(1)\ F7 - \hat{TVEL}(2)\ F6 + \hat{TVEL}(3)\ F3]/\det(M) \tag{36}$$

where

$$F7 = C1\ F1 - S1\ F2 \tag{37}$$

Generalized matrix inverse solution.- Near a singular condition (det (M) = 0), the generalized inverse matrix solution to equation (25) can be used rather than equations (34) to (36). From equation (25),

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = M^* \begin{bmatrix} \hat{TVEL}\ (1) \\ \hat{TVEL}\ (2) \\ \hat{TVEL}\ (3) \end{bmatrix} \tag{38}$$

where M* denotes the generalized inverse of M. For nonsingular M, $M^* = M^{-1}$. (In reference 2, expressions are generated for generalized matrix inverses corresponding to the singularity conditions S3 = 0 and S2 + S23 = 0; however, the double singularity which happens at S2 = S3 = 0 is not accounted for).

To reduce the computational burden and benefit real-time operation, a further reduction in the matrix to be inverted is suggested. Multiply the first row of equation (25) by C1, the second row by S1, and add the results to get

$$\dot{\theta}_1 = [F3\ \dot{\theta}_2 + F4\ \dot{\theta}_3 - C1\ \hat{TVEL}(1) - S1\ \hat{TVEL}(2)]/F2 \tag{39}$$

For the present robot arm in mind, CAL3≈0 and $\tilde{SN}\neq 0$ so that, from equation (28), F2≠0. Therefore, given $\dot{\theta}_2$ and $\dot{\theta}_3$, one computes $\dot{\theta}_1$ without difficulty in equation (39). With equation (39), the first and third rows of equation (25) can be written as

$$\underline{g1} = M1 \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{40}$$

where

$$F6 = S1\ F1 + C1\ F2 \tag{41}$$

$$\underline{g1} = \begin{bmatrix} F2\ \hat{TVEL}(1) - F6(C1\ \hat{TVEL}(1) + S1\ \hat{TVEL}(2)) \\ \hat{TVEL}(3) \end{bmatrix} \tag{42}$$

$$M1 = \begin{bmatrix} (C1\ F2 - F6)F3 & (C1\ F2 - F6)F4 \\ -F1 & -F5 \end{bmatrix} \tag{43}$$

Equation (40) can be solved as

$$\begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = M1^*\ \underline{g1} \tag{44}$$

where M1* is the generalized inverse of the 2x2 matrix M1. Then, with solutions for $\dot{\theta}_2$ and $\dot{\theta}_3$, one computes $\dot{\theta}_1$ with equation (39).

Variables held constant near singular points.- A method for avoiding singular points is to "freeze" variables when they come within a specified region of a singular point. A variable is held constant until a new value for the variable is computed outside the specified region. For example, if θ_2 and θ_3 approach the singular condition in equations (32) and (33) then the angles maintain their current values until new values are computed outside the singular region. This type of control excludes certain positions of the robot arm which may or may not be satisfactory, depending upon the task, and may also cause the arm to jerk.

Auxiliary hand control.- People are limited in the speed with which they can move their arms and hands, and there are geometric constraints which disallow certain motions and positions. Yet, people have excellent control of their arms and hands without being consciously aware of these limitations, which can appear as singularities in mathematical equations. Therefore, it appears that some type of auxiliary hand control is warranted near singularities to help an operator gain additional control of the robot arm. Given a singular arm position, the auxiliary control scheme should take into consideration what the operator would most likely want to do in the given situation. Consider the following rudimentary scheme which may have some favorable characteristic motions for an operator.

In the vicinity of $\theta_3 = 0$, let θ_3 be proportional to the component of $\underline{\hat{TVEL}}$ parallel to X_2 in figure 3. Thus,

$$\dot{\theta}_3 = \begin{cases} 0 & \text{if } \theta_3 = 0 \text{ and } (\underline{\hat{TVEL}})_{X_2} > 0 \\ K3\ (\underline{\hat{TVEL}})_{X_2} & \text{otherwise} \end{cases} \tag{45}$$

where K3 is a constant to be specified. This means that θ_3 will not vary in response to an impossible outward (radial) motion command for the robot arm; whereas, if there is a component of velocity toward the shoulder of the arm, θ_3 will cause the arm to retract. Now, make

$$\dot{\theta}_2 = -[WE/(WE + ES)]\dot{\theta}_3 + K_2[(\underline{\hat{TVEL}})_{Y_2}/(WE + ES)] \tag{46}$$

where K_2 is a constant to be specified. The first term in equation (46) will cause θ_2 to vary so that the arm moves back in nearly a straight line from the hand towards the shoulder. The second term will allow the arm to pitch in the extended position in proportion to the component of $\underline{\hat{TVEL}}$ in the pitching direction for the arm. The components of $\underline{\hat{TVEL}}$ needed in equations (45) and (46) can be extracted in the process of computing the transformation from hand to base (appendix B). Equation (39) specifies azimuth movement.

Another feature which should be incorporated is the ability to bend the robot arm at the elbow in either the up or down direction. To do this, change the sign on K3 in equation (45) each time the arm enters the singularity mode. Consequently, an operator simply extends the arm and backs up again to reverse the directions of the elbow bend. This scheme has not yet been evaluated. There is the prospect that this "in-and-out" motion may be a desirable feature as the robot arm nears its maximum extension. Perhaps, this feature may be desirable in a larger region than just in a very small neighborhood of the singularity at the full extension of the robot arm. Whether or not the control superimposed by this scheme will be consistent with an operator's desired control in singular situations remains to be seen from experiment.

Solving for Joint Rates $\dot{\theta}_4$, $\dot{\theta}_5$, $\dot{\theta}_6$

Equation (24) can be written as

$$\underline{\tilde{RVEL}} = \underline{\hat{z}}_3\dot{\theta}_4 + \underline{\hat{z}}_4\dot{\theta}_5 + \underline{\hat{z}}_5\dot{\theta}_6 \tag{47}$$

where

$$\underline{\tilde{RVEL}} = \underline{\hat{RVEL}} - \underline{\hat{z}}_0\dot{\theta}_1 - \underline{\hat{z}}_1\dot{\theta}_2 - \underline{\hat{z}}_2\dot{\theta}_3 \tag{48}$$

is the resultant rotational velocity of that commanded and that induced by the rotation of the first three joints. Equation (47) simplifies to the following three simultaneous equations in $\dot{\theta}_4$, $\dot{\theta}_5$, and $\dot{\theta}_6$: (see appendix C)

$$\dot{\theta}_5 = -C4\ D5 - S4\ D4 \tag{49}$$

$$S5\ \dot{\theta}_6 = -S4\ D5 + C4\ D4 \tag{50}$$

$$\dot{\theta}_4 + C5\ \dot{\theta}_6 = SAL3\ D3 - CAL3\ D2 \tag{51}$$

where

$$D1 = C1\ \widetilde{RVEL}(1) + S1\ \widetilde{RVEL}(2) \tag{52}$$

$$D2 = S1\ \widetilde{RVEL}(1) - C1\ \widetilde{RVEL}(2) \tag{53}$$

$$D3 = S23\ D1 + C23\ \widetilde{RVEL}(3) \tag{54}$$

$$D4 = C23\ D1 - S23\ \widetilde{RVEL}(3) \tag{55}$$

$$D5 = CAL3\ D3 + SAL3\ D2 \tag{56}$$

Equation (49) clearly shows that there is no difficulty in computing $\dot{\theta}_5$; and equations (50) and (51) show that the only difficulty in computing $\dot{\theta}_6$ and then $\dot{\theta}_4$ is when $\theta_5 = 0$. In this case the mathematics cannot decide which angle to vary to produce a rotation about Z_6 (fig. 3). Some type of maximum angular rate penality may be needed to avoid excessive rates near singular points; that is, when $S5 \approx 0$ in equation (50).

Generalized matrix inverse.- Write equations (50) and (51) as

$$\underline{g}2 = M2 \begin{bmatrix} \dot{\theta}_4 \\ \dot{\theta}_6 \end{bmatrix} \tag{57}$$

where

$$M2 = \begin{bmatrix} 0 & S5 \\ 1 & C5 \end{bmatrix} \tag{58}$$

$$\underline{g}2 = \begin{bmatrix} -S4\ D6 + C4\ D4 \\ SAL3\ D3 - CAL3\ D2 \end{bmatrix} \tag{59}$$

Then,

$$\begin{bmatrix} \dot{\theta}_4 \\ \dot{\theta}_6 \end{bmatrix} = M2^*\ \underline{g}2 \tag{60}$$

where M2* is the generalized inverse of the 2x2 matrix M2. Again, $\dot{\theta}_5$ is computed with equation (49).

A total solution for $\dot{\theta}_i$ (i = 1, 2, ..., 6) is provided by equations (39), (44), (49), and (60). Hence, only the generalized inverses of two 2x2 matrices are required. For some kinematically redundant manipulators, control based on generalized matrix inverses can lead to undesirable arm configurations (ref. 7).

Choose which angle to rotate near singularity.- When $\theta_5 = 0$, equation (51) becomes

$$\dot{\theta}_4 + \dot{\theta}_6 = \text{SAL3 D3} - \text{CAL3 D2} \tag{61}$$

and equation (50) is useless. The generalized matrix inverse solution actually splits the rotational task equally between θ_4 and θ_6 when $\theta_5 = 0$. Another approach is to make $\dot{\theta}_4 = 0$ when θ_5 is less than a prescribed amount and let

$$\dot{\theta}_6 = \text{SAL3 D3} - \text{CAL3 D2} \tag{62}$$

Then, for additional rotational capability when θ_6 reaches a limit, continue rotation with

$$\dot{\theta}_4 = \text{SAL3 D3} - \text{CAL3 D2} \tag{63}$$

Uncoupled arm and wrist motions.- An approach to avoid the wrist singularity that occurs when $\theta_5 = 0$ is to: (1) position the wrist with $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$ (eq. 23); and (2) individually command the joint velocities $\dot{\theta}_4$, $\dot{\theta}_5$, and $\dot{\theta}_6$ at the wrist of the robot arm.

DISCUSSION

Visual observation of robot hand orientation.- In implementing resolved-rate control on an industrial robot manipulator with a symmetrical hand, the following annoyances were observed:

1. Keeping track of the positive hand axes: (Painting the hand might help alleviate this problem.)

2. Keeping track of wrist orientation angles to avoid hitting limits. (Displaying the angles to an operator will help, but this only increases his workload. Perhaps, a better solution is to add appropriate hierarchical control to handle this "out-of-range" problem for the operator.)

Large angular rates.- Another nuisance experienced in the teleoperator control of a robot arm is the occurrence of large angular rates near mathematical singularities. Hopefully, a hierarchical control structure on top of the operator's control will eliminate this problem.

Different designs of robot arm.- Other robot arm designs may offer certain control advantages.

Ability to record trajectory segment.- Current robot manipulators can be "taught" a trajectory to be repeated later upon command. Hence, a recent trajectory segment can be reversed to provide an operator with the option of a speedy and effortless evacuation of the robot hand from a congested workspace.

It appears that some type of auxiliary hand control structure would be useful in aiding a human operator (or computer) in the control of a robot arm. For instance, such control should be formulated to produce the motions that are probably

what an operator would like to do in a current predicament and to allow alternatives based on his responding inputs. Of course, in future path planning with constraints, hierarchical control would be of additional assistance to the operator.

CONCLUDING REMARKS

Kinematic equations for resolved-rate control of a robot arm are simplified to allow faster computations, and control in singular regions is discussed.

APPENDIX A

HOMOGENEOUS TRANSFORMATION MATRICES

When the parameters in the table are introduced into the general transformation matrix (eq. 1), the following six transformation matrices result. These matrices are the same as those used in reference 2, except for notation and A^3_2, which has three additional unspecified parameters a_3, r_3, and α_3. In the following matrices, A3 = a_3, R3 = r_3, SAL3 = $\sin\alpha_3$, CAL3 = $\cos\alpha_3$, C1 = $\cos\theta_1$, S1 = $\sin\theta_1$, etc.

$$A^1_0 = \begin{bmatrix} -C1 & 0 & -S1 & 0 \\ -S1 & 0 & C1 & 0 \\ 0 & 1 & 0 & NO \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A1)$$

$$A^2_1 = \begin{bmatrix} -S2 & -C2 & 0 & -ES\ S2 \\ C2 & -S2 & 0 & ES\ C2 \\ 0 & 0 & 1 & SN \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A2)$$

$$A^3_2 = \begin{bmatrix} -S3 & -C3\ CAL3 & C3\ SAL3 & -A3\ S3 \\ C3 & -S3\ CAL3 & S3\ SAL3 & A3\ C3 \\ 0 & SAL3 & CAL3 & R3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A3)$$

$$A^4_3 = \begin{bmatrix} -C4 & 0 & -S4 & 0 \\ -S4 & 0 & C4 & 0 \\ 0 & 1 & 0 & WE \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A4)$$

$$A^5_4 = \begin{bmatrix} -C5 & 0 & -S5 & 0 \\ -S5 & 0 & C5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A5)$$

$$A^6_5 = \begin{bmatrix} C6 & -S6 & 0 & 0 \\ S6 & C6 & 0 & 0 \\ 0 & 0 & 1 & HW \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A6)$$

APPENDIX B

TRANSFORMATION FROM HAND TO BASE (THTOB)

Operator inputs for controlling the robot arm are resolved in directions parallel to the base-coordinate axes by

$$\begin{bmatrix} \hat{\underline{TVEL}} \\ \hat{\underline{RVEL}} \end{bmatrix} = \begin{bmatrix} R_0^6 & \underline{TVEL} \\ R_0^6 & \underline{RVEL} \end{bmatrix} \tag{B1}$$

where the rotational matrix can be associatively grouped into products of matrices as

$$R_0^6 = [R_0^1(R_1^2R_2^3)][R_3^4(R_4^5R_5^6)] \tag{B2}$$

where

$$R_3^4(R_4^5R_5^6) = \begin{bmatrix} Q1 & Q3 & C4\ S5 \\ Q2 & Q4 & S4\ S5 \\ -S5\ C6 & S5\ S6 & C5 \end{bmatrix} \tag{B3}$$

$$R_1^2R_2^3 = \begin{bmatrix} P1 & -P2\ CAL3 & P2\ SAL3 \\ P2 & P1\ CAL3 & -P1\ SAL3 \\ 0 & SAL3 & CAL3 \end{bmatrix} \tag{B4}$$

$$R_0^1(R_1^2R_2^3) = \begin{bmatrix} -C1\ P1 & C1\ P2\ CA3 - S1\ SAL3 & -C1\ P2\ SAL3 - S1\ CAL3 \\ -S1\ P1 & S1\ P2\ CA3 + C1\ SAL3 & -S1\ P2\ SAL3 + C1\ CAL3 \\ P2 & P1\ CAL3 & -P1\ SAL3 \end{bmatrix} \tag{B5}$$

where

$$Q1 = C4\ C5\ C6 - S4\ S6 \tag{B6}$$
$$Q2 = S4\ C5\ C6 + C4\ S6 \tag{B7}$$
$$Q3 = -C4\ C5\ S6 - S4\ C6 \tag{B8}$$
$$Q4 = -S4\ C5\ S6 + C4\ C6 \tag{B9}$$
$$P1 = -C23 \tag{B10}$$
$$P2 = -S23\ CAL3 \tag{B11}$$

Multiplying equation (B3) on the left by equation (B5) produces a matrix L (identical to R^6_0). Let

$$T1 = C1\ CAL3\ P2 - S1\ SAL3 \tag{B12}$$
$$T2 = -C1\ P2\ SAL3 - S1\ CAL3 \tag{B13}$$
$$T3 = S1\ CAL3\ P2 + C1\ SAL3 \tag{B14}$$
$$T4 = -S1\ P2\ SAL3 + C1\ CAL3 \tag{B15}$$

Then the elements of L are:

$$L(1,1) = -C1\ P1\ Q1 + T1\ Q2 - T2\ S5\ C6 \quad \text{(B16)}$$
$$L(2,1) = -S1\ P1\ Q1 + T3\ Q2 - T4\ S5\ C6 \quad \text{(B17)}$$
$$L(3,1) = P2\ Q1 + P1\ CAL3\ Q2 + P1\ SAL3\ S5\ C6 \quad \text{(B18)}$$

$$L(1,2) = -C1\ P1\ Q3 + T1\ Q4 + T2\ S5\ S6 \quad \text{(B19)}$$
$$L(2,2) = -S1\ P1\ Q3 + T3\ Q4 + T4\ S5\ S6 \quad \text{(B20)}$$
$$L(3,2) = P2\ Q3 + P1\ CAL3\ Q4 - P1\ SAL3\ S5\ S6 \quad \text{(B21)}$$

$$L(1,3) = -C1\ P1\ C4\ S5 + T1\ S4\ S5 + T2\ C5 \quad \text{(B22)}$$
$$L(2,3) = -S1\ P1\ C4\ S5 + T3\ S4\ S5 + T4\ C5 \quad \text{(B23)}$$
$$L(3,3) = P2\ C4\ S5 + P1\ CAL3\ S4\ S5 - P1\ SAL3\ C5 \quad \text{(B24)}$$

The translational and rotational velocities of the robot hand are then resolved in directions parallel to the base coordinate system by equation (B1), with ${}^{6}R_{0} = L$.

The rotational transformation matrices (${}^{i}R_{i-1}$) are the 3x3 submatrices in the upper left-hand corner of the homogeneous transformation matrices (${}^{i}A_{i-1}$) in appendix A. The inverses of these rotational matrices are also their transposes. Thus,

$$R_1^0 = \begin{bmatrix} -C1 & -S1 & 0 \\ 0 & 0 & 1 \\ -S1 & C1 & 0 \end{bmatrix} \quad \text{(B25)}$$

$$R_2^1 = \begin{bmatrix} -S2 & C2 & 0 \\ -C2 & -S2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(B26)}$$

$\hat{TVEL}(1)$, $\hat{TVEL}(2)$, and $\hat{TVEL}(3)$ are the components of $\underline{\hat{TVEL}}$ with respect to the base coordinate system (X_0, Y_0, Z_0). Let $(\underline{\hat{TVEL}})_{X_2}$, $(\underline{\hat{TVEL}})_{Y_2}$, and $(\underline{\hat{TVEL}})_{Z_2}$ be the components of $\underline{\hat{TVEL}}$ relative to the coordinate system (X_2, Y_2, Z_2). Then, these components are related by the transformation equation.

$$\begin{bmatrix} (\underline{\hat{TVEL}})_{X_2} \\ (\underline{\hat{TVEL}})_{Y_2} \\ (\underline{\hat{TVEL}})_{Z_2} \end{bmatrix} = R_2^1\ R_1^0 \begin{bmatrix} \hat{TVEL}(1) \\ \hat{TVEL}(2) \\ \hat{TVEL}(3) \end{bmatrix} \quad \text{(B27)}$$

Therefore, in the text, the components of $\underline{\hat{TVEL}}$ in equation (45) and (46) are as follows:

$$(\underline{\hat{TVEL}})_{X_2} = S2\left[C1\ \hat{TVEL}(1) - S1\ \hat{TVEL}(2) + C2\ \hat{TVEL}(3)\right] \quad \text{(B28)}$$

$$(\underline{\hat{TVEL}})_{Y_2} = C2\left[C1\ \hat{TVEL}(1) - S1\ \hat{TVEL}(2) - S2\ \hat{TVEL}(3)\right] \quad \text{(B29)}$$

APPENDIX C

RESOLVED-RATE EQUATIONS

Joint rates are computed by solving equations (23) and (47) for commanded velocities of the robot hand. These equations, repeated here for convenience, are

$$\underline{\hat{TVEL}} = (\underline{\hat{z}}_0 \times \underline{\hat{d}}_{06})\dot{\theta}_1 + (\underline{\hat{z}}_1 \times \underline{\hat{d}}_{16})\dot{\theta}_2 + (\underline{\hat{z}}_2 \times \underline{\hat{d}}_{26})\dot{\theta}_3 \tag{C1}$$

and

$$\underline{\tilde{RVEL}} = \underline{\hat{z}}_3\dot{\theta}_4 + \underline{\hat{z}}_4\dot{\theta}_5 + \underline{\hat{z}}_5\dot{\theta}_6 \tag{C2}$$

where

$$\underline{\tilde{RVEL}} = \underline{\hat{RVEL}} - \underline{\hat{z}}_0\dot{\theta}_1 - \underline{\hat{z}}_1\dot{\theta}_2 - \underline{\hat{z}}_2\dot{\theta}_3 \tag{C3}$$

is the resultant rotational velocity of that commanded and that induced by rotations of joints 1, 2, and 3.

The third column of $R^k{}_0$ is $\underline{\hat{z}}_k$, which is easily seen from

$$\underline{\hat{z}}_k = R^k_0\,\underline{z}_k = R^k_0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{C4}$$

Equivalently, $\underline{\hat{z}}_k$ is the third column of

$$A^k_0 = A^1_0 A^2_1 \cdots A^k_{k-1} \tag{C5}$$

if the fourth element in the third column is disregarded.

The identity homogeneous transformation matrix is

$$A^0_0 = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \tag{C6}$$

From equation (C6),

$$\underline{\hat{z}}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{C7}$$

From equation (A1) in appendix A,

$$\underline{\hat{z}}_1 = \begin{bmatrix} -S1 \\ C1 \\ 0 \end{bmatrix} \tag{C8}$$

The third column (ignoring the fourth element) of the product

$$A_0^2 = A_0^1 A_1^2 = \left[\begin{array}{ccc|c} C1\ S2 & C1\ C2 & -S1 & ES\ C1\ S2 - SN\ S1 \\ S1\ S2 & S1\ C2 & C1 & ES\ S1\ S2 + SN\ C1 \\ C_2 & -S2 & 0 & ES\ C2 + NO \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{C9}$$

yields

$$\hat{\underline{z}}_2 = \begin{bmatrix} -S1 \\ C1 \\ 0 \end{bmatrix} \tag{C10}$$

In a similar manner,

$$\hat{\underline{z}}_3 = \begin{bmatrix} C1\ S23\ SAL3 - S1\ CAL3 \\ S1\ S23\ SAL3 + C1\ CAL3 \\ C23\ SAL3 \end{bmatrix} \tag{C11}$$

$$\hat{\underline{z}}_4 = \begin{bmatrix} -C1\ C23\ S4 - C1\ S23\ CAL3\ C4 - S1\ SAL3\ C4 \\ -S1\ C23\ S4 - S1\ S23\ CAL3\ C4 + C1\ SAL3\ C4 \\ S23\ S4 - C23\ CAL3\ C4 \end{bmatrix} \tag{C12}$$

$$\hat{\underline{z}}_5 = \begin{bmatrix} C1\ C23\ C4\ S5 - C1\ S23\ CAL3\ S4\ S5 - S1\ SAL3\ S4\ S5 \\ \quad + C1\ S23\ SAL3\ C5 - S1\ CAL3\ C5 \\ S1\ C23\ C4\ S5 - S1\ S23\ CAL3\ S4\ S5 + C1\ SAL3\ S4\ S5 \\ \quad + S1\ S23\ SAL3\ C5 + C1\ CAL3\ C5 \\ -S23\ C4\ S5 - C23\ CAL3\ S4\ S5 + C23\ SAL3\ C5 \end{bmatrix} \tag{C13}$$

The fourth column $A^k{}_0$ (again, exclude the fourth element) provides $\underline{p}_{0k}$, a vector in base coordinates from the base-coordinate system to coordinate system k. From equation (A1) in appendix A,

$$\underline{p}_{01} = \begin{bmatrix} 0 \\ 0 \\ NO \end{bmatrix} \tag{C14}$$

From equation (C9),

$$\underline{p}_{02} = \begin{bmatrix} ES\ C1\ S2 - SN\ S1 \\ ES\ S1\ S2 + SN\ C1 \\ ES\ C2 + NO \end{bmatrix} \tag{C15}$$

In a similar manner,

$$\underline{p}_{03} = \begin{bmatrix} C1\ C23\ A3 + C1\ ES\ S2 - S1\ \tilde{SN} \\ S1\ C23\ A3 + S1\ ES\ S2 + C1\ \tilde{SN} \\ -S23\ A3 + ES\ C2 + NO \end{bmatrix} \tag{C16}$$

$$\underline{p}_{04} = \begin{bmatrix} C1\ S23\ SAL3\ WE - S1\ CAL3\ WE + C1\ C23\ A3 + C1\ ES\ S2 - S1\ \tilde{SN} \\ S1\ S23\ SAL3\ WE + C1\ CAL3\ WE + S1\ C23\ A3 + S1\ ES\ S2 + C1\ \tilde{SN} \\ C23\ SAL3\ WE - S23\ A3 + ES\ C2 + NO \end{bmatrix} \tag{C17}$$

$$\underline{p}_{05} = \begin{bmatrix} C1\ S23\ SAL3\ WE - S1\ CAL3\ WE + C1\ C23\ A3 + C1\ ES\ S2 - S1\ \tilde{SN} \\ S1\ S23\ SAL3\ WE + C1\ CAL3\ WE + S1\ C23\ A3 + S1\ ES\ S2 + C1\ \tilde{SN} \\ C23\ SAL3\ WE - S23\ A3 + ES\ C2 + NO \end{bmatrix} \tag{C18}$$

where

$$\tilde{SN} = SN + R3 \tag{C19}$$

Since coordinate systems 5 and 6 coincide,

$$\underline{p}_{06} = \underline{p}_{05} \tag{C20}$$

A vector (in base coordinates) from coordinate system k to the robot hand-axis system is

$$\hat{\underline{d}}_{k6} = \underline{p}_{06} - \underline{p}_{0k} \tag{C21}$$

Hence,

$$\hat{\underline{d}}_{06} = \underline{p}_{06} = \begin{bmatrix} C1\ S23\ SAL3\ WE - S1\ CAL3\ WE + C1\ C23\ A3 \\ \quad + C1\ ES\ S2 - S1\ \tilde{SN} \\ S1\ S23\ SAL3\ WE + C1\ CAL3\ WE + S1\ C23\ A3 \\ \quad + S1\ ES\ S2 + C1\ \tilde{SN} \\ C23\ SAL3\ WE - S23\ A3 + ES\ C2 + NO \end{bmatrix} \tag{C22}$$

$(\underline{p}_{00} = 0)$

$$\underline{d}_{16} = \underline{p}_{06} - \underline{p}_{01} = \begin{bmatrix} C1\ S23\ SAL3\ WE - S1\ CAL3\ WE + C1\ C23\ A3 \\ \quad + C1\ ES\ S2 - S1\ \tilde{SN} \\ S1\ S23\ SAL3\ WE + C1\ CAL3\ WE + S1\ C23\ A3 \\ \quad + S1\ ES\ S2 + C1\ \tilde{SN} \\ C23\ SAL3\ WE - S23\ A3 + ES\ C2 \end{bmatrix} \qquad (C23)$$

$$\underline{d}_{26} = \underline{p}_{06} - \underline{p}_{02} = \begin{bmatrix} C1\ S23\ SAL3\ WE - S1\ CAL3\ WE + C1\ C23\ A3 \\ \quad - S1\ R3 \\ S1\ S23\ SAL3\ WE + C1\ CAL3\ WE + S1\ C23\ A3 \\ \quad + C1\ R3 \\ C23\ SAL3\ WE - S23\ A3 \end{bmatrix} \qquad (C24)$$

$$\underline{d}_{36} = \underline{p}_{06} - \underline{p}_{03} = \begin{bmatrix} C1\ S23\ SAL3\ WE - S1\ CAL3\ WE \\ S1\ S23\ SAL3\ WE + C1\ CAL3\ WE \\ C23\ SAL3\ WE \end{bmatrix} \qquad (C25)$$

By design, coordinate systems 4, 5, and 6 coincide so that $\hat{\underline{d}}_{45} = \hat{\underline{d}}_{56} = \hat{\underline{d}}_{66} = 0$.

Translational velocity equations.- The required cross products in equation (C1) are:

$$\hat{\underline{z}}_0 \times \hat{\underline{d}}_{06} = \begin{bmatrix} -S1\ S23\ SAL3\ WE - C1\ CAL3\ WE - S1\ C23\ A3 \\ \quad -S1\ ES\ S2 - C1\ \tilde{SN} \\ C1\ S23\ SAL3\ WE - S1\ CAL3\ WE + C1\ C23\ A3 \\ \quad + C1\ ES\ S2 - S1\ \tilde{SN} \\ 0 \end{bmatrix} \qquad (C26)$$

$$\hat{\underline{z}}_1 \times \hat{\underline{d}}_{16} = \begin{bmatrix} C1(ES\ C2 + SAL3\ C23\ WE) - C1\ S23\ A3 \\ S1(ES\ C2 + SAL3\ C23\ WE) - S1\ S23\ A3 \\ -SAL3\ WE\ S23 - ES\ S2 - C23\ A3 \end{bmatrix} \qquad (C27)$$

$$\hat{\underline{z}}_2 \times \hat{\underline{d}}_{26} = \begin{bmatrix} C1\ WE\ C23\ SAL3 - C1\ S23\ A3 \\ S1\ WE\ C23\ SAL3 - S1\ S23\ A3 \\ -S23\ SAL3\ WE - C23\ A3 \end{bmatrix} \qquad (C28)$$

With equations (C26), (C27), and (C28), equation (C1) can be expressed as

$$\begin{bmatrix} \hat{TVEL}\ (1) \\ \hat{TVEL}\ (2) \\ \hat{TVEL}\ (3) \end{bmatrix} = \begin{bmatrix} -S1\ F1 - C1\ F2 & C1\ F3 & C1\ F4 \\ C1\ F1 - S1\ F2 & S1\ F3 & S1\ F4 \\ 0 & -F1 & -F5 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{C29}$$

where

$$F1 = S2\ ES + F5 \tag{C30}$$
$$F2 = \tilde{SN} + CAL3\ WE \tag{C31}$$
$$F3 = C2\ ES + F4 \tag{C32}$$
$$F4 = C23\ SAL3\ WE - S23\ A3 \tag{C33}$$
$$F5 = S23\ SAL3\ WE + C23\ A3 \tag{C34}$$

Rotational velocity equation.- With equations (C11), (C12), and (C13), equation (C2) becomes

$$\begin{bmatrix} \tilde{RVEL}\ (1) \\ \tilde{RVEL}\ (2) \\ \tilde{RVEL}\ (3) \end{bmatrix} = \begin{bmatrix} C1\ S23\ SAL3 - S1\ CAL3 & -C1(C23\ S4 + C4\ S23\ CAL3) - S1\ C4\ SAL3 \\ S1\ S23\ SAL3 + C1\ CAL3 & -S1\ (C23\ S4 + C4\ S23\ CAL3) + C1\ C4\ SAL3 \\ C23\ SAL3 & S23\ S4 - C23\ C4\ CAL3 \end{bmatrix}$$

$$\begin{bmatrix} (C1\ C23\ C4 - S1\ S4\ SAL3 - C1\ S23\ S4\ CAL3)S5 + (C1\ S23\ SAL3 - S1\ CAL3)C5 \\ (S1\ C23\ C4 + C1\ S4\ SAL3 - S1\ S23\ S4\ CAL3)S5 + (S1\ S23\ SAL3 + C1\ CAL3)C5 \\ -(S23\ C4 + C23\ S4\ CAL3)S5 + C23\ SAL3\ C5 \end{bmatrix} \begin{bmatrix} \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} \tag{C35}$$

Equation (C35) is now simplified considerably as follows. Multiply the first row of equation (C35) by C1, the second row by S1, and add the results to get

$$D1 = S23\ SAL3\ \dot{\theta}_4 - (C23\ S4 + C4\ S23\ CAL3)\dot{\theta}_5 + [(C23\ C4 - S23\ S4\ CAL3)S5 + S23\ SAL3\ C5]\dot{\theta}_6 \tag{C36}$$

where

$$D1 = C1\ \tilde{RVEL}\ (1) + S1\ \tilde{RVEL}\ (2) \tag{C37}$$

Now, multiply equation (C36) by S23, row three of equation (C35) by C23, and add to get

$$D3 = SAL3(\dot{\theta}_4 + C5\ \dot{\theta}_6) - C_4 CAL3\ \dot{\theta}_5 - S4\ CAL3\ S5\ \dot{\theta}_6 \tag{C38}$$

where

$$D3 = S23\ D1 + C23\ \widetilde{RVEL}(3) \tag{C39}$$

Multiply row 1 in equation (C35) by S1 and subtract C1 times row 2 to obtain

$$D2 = -CAL3(\dot{\theta}_4 + C5\ \dot{\theta}_6) - C4\ SAL3\ \dot{\theta}_5 - S4\ SAL3\ S5\ \dot{\theta}_6 \tag{C40}$$

where

$$D2 = S1\ \widetilde{RVEL}(1) - C1\ \widetilde{RVEL}(2) \tag{C41}$$

Multiply equation (C36) by C23 and subtract S23 times row 3 of equation (C35) to produce

$$D4 = -S4\ \dot{\theta}_5 + C4\ S5\ \dot{\theta}_6 \tag{C42}$$

where

$$D4 = C23\ D1 - S23\ \widetilde{RVEL}(3) \tag{C43}$$

Multiply equation (C38) by CAL3 and add to SAL3 times equation (C40) to get

$$D5 = -C4\ \dot{\theta}_5 - S4\ S5\ \dot{\theta}_6 \tag{C44}$$

where

$$D5 = CAL3\ D3 + SAL3\ D2 \tag{C45}$$

Finally, multiply equation (C44) by -C4 and add to -S4 times equation (C42) to obtain

$$\dot{\theta}_5 = -C4\ D5 - S4\ D4 \tag{C46}$$

Then, multiply equation (C44) by -S4 and add C4 times equation (C42) to obtain

$$S5\ \dot{\theta}_6 = -S4\ D5 + C4\ D4 \tag{C47}$$

The end result is obtained by multiplying equation (C38) by SAL3 and subtracting CAL3 times equation (C40). Thus,

$$\dot{\theta}_4 + C5\ \dot{\theta}_6 = SAL3\ D3 - CAL3\ D2 \tag{C48}$$

Therefore, equation (C35) has been simplified to the three scalar equations (C46), (C47), and (C48).

APPENDIX D

DETERMINANT OF M

In this appendix, the determinant of M (eq. (26)) is expanded explicitly in terms of the joint angles and robot arm parameters. The determinant

$$\det(M) = \det \begin{bmatrix} -S1\ F1 - C1\ F2 & C1\ F3 & C1\ F4 \\ C1\ F1 - S1\ F2 & S1\ F3 & S1\ F4 \\ 0 & -F1 & -F5 \end{bmatrix} \tag{D1}$$

expands to

$$\det\ (M) = F1[F5(F3 - F4) - F4\ S2\ ES] \tag{D2}$$

But, from equations (28) and (29),

$$F3 - F4 = C2\ ES \tag{D3}$$

Therefore,

$$\det(M) = F1\ ES(F5\ C2 - F4\ S2) \tag{D4}$$

From equations (30) and (31),

$$F5\ C2 - F4\ S2 = SAL3\ WE(S23\ C2 - C23\ S2) + A3(C23\ C2 + S23\ S2) \tag{D5}$$

which, with the trigonometric identities

$$S23\ C2 - C23\ S2 = S3 \tag{D6}$$

$$C23\ C2 + S23\ S2 = C3 \tag{D7}$$

becomes

$$F5\ C2 - F4\ S2 = SAL3\ WE\ S3 + A3\ C3 \tag{D8}$$

With equations (D8) and equation (27) for F1 in the text, the determinant of M in equation (D4) is expressed simply as

$$\det(M) = (S2\ ES + F5)(SAL3\ WE\ S3 + A3\ C3)ES \tag{D9}$$

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TABLE - ASSUMED RELATIVE JOINT PARAMETERS

Joint, i	α_i, deg	a_i, in.	r_i, in.	θ_i', deg	θ_i limits, deg
1	90	0	NO	$\theta_1 + 180$	± 160
2	0	ES	SN	$\theta_2 + 90$	± 165
3	α_3	a_3	r_3	$\theta_3 + 90$	± 135
4	90	0	WE	$\theta_4 + 180$	± 135
5	90	0	0	$\theta_5 + 180$	± 105
6	0	0	HW	θ_6	± 270

Neck-to-base length (NO).

Elbow-to-shoulder length (ES).

Shoulder-to-neck length (SN).

Wrist-to-elbow length (WE).

Hand-to-wrist length (HW).

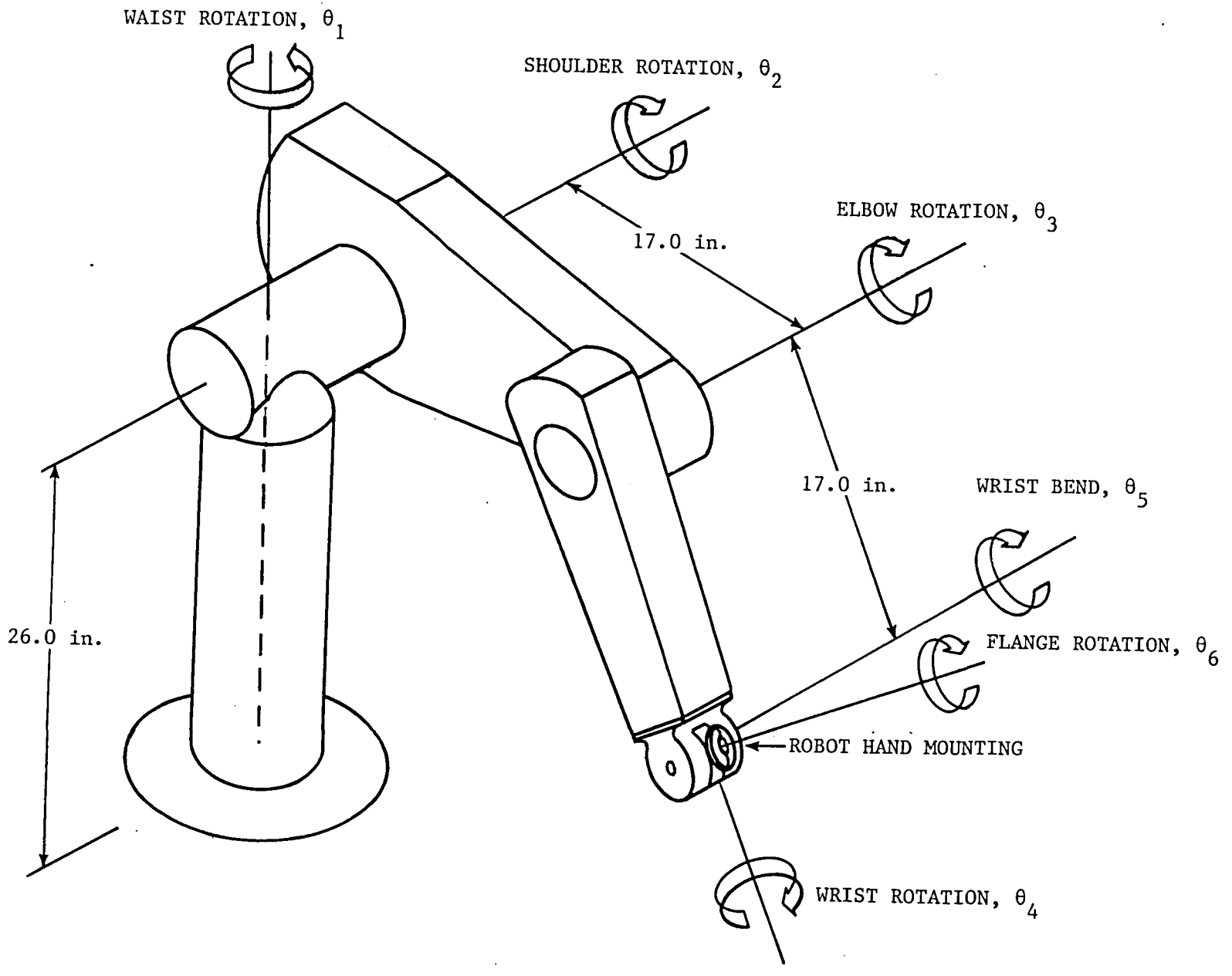

Figure 1.- Robot arm with rotational joints.

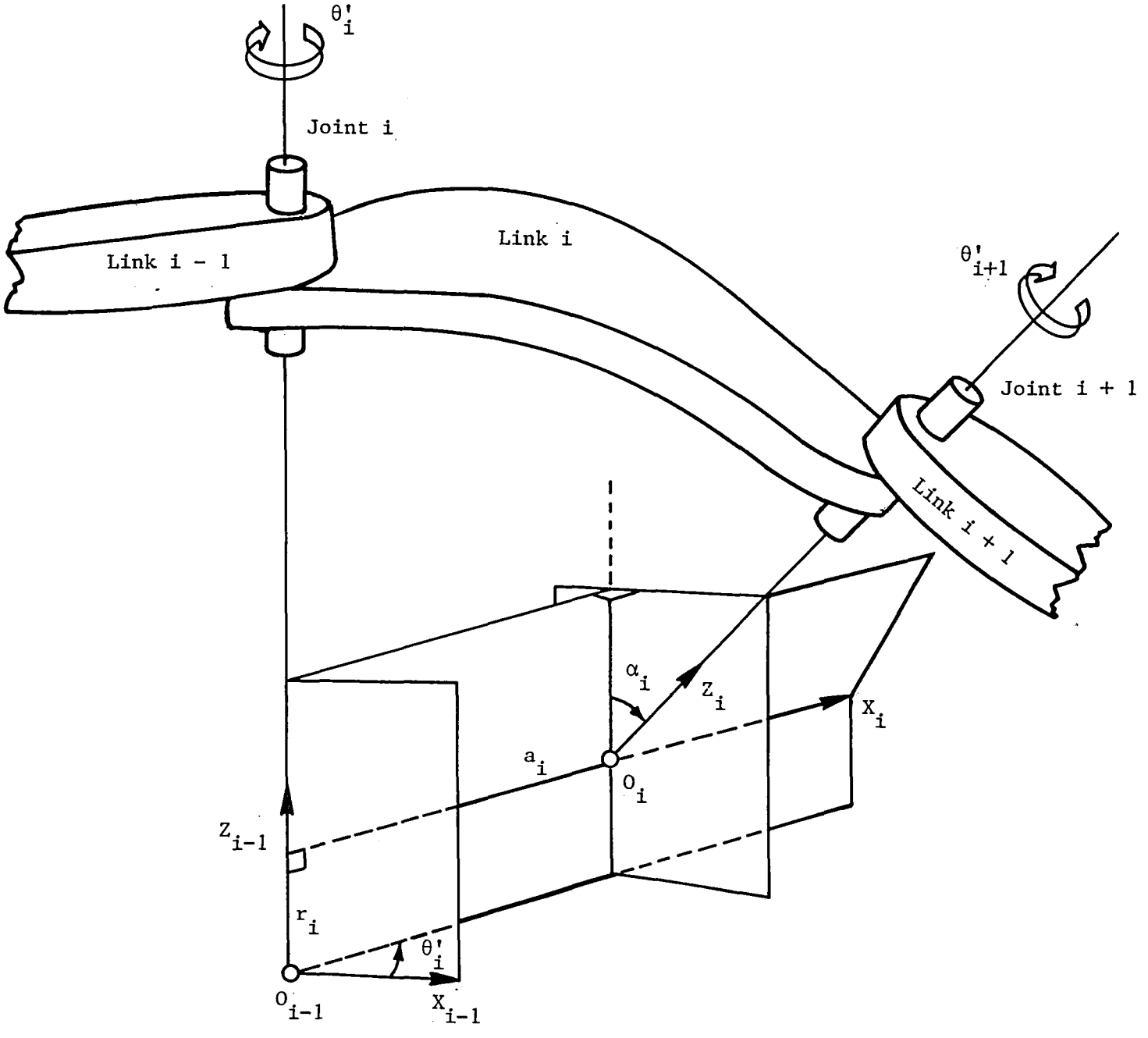

Figure 2.- Joint axis systems defined by Denavit-Hartenberg parameters a_i, α_i, r_i, and θ'_i.

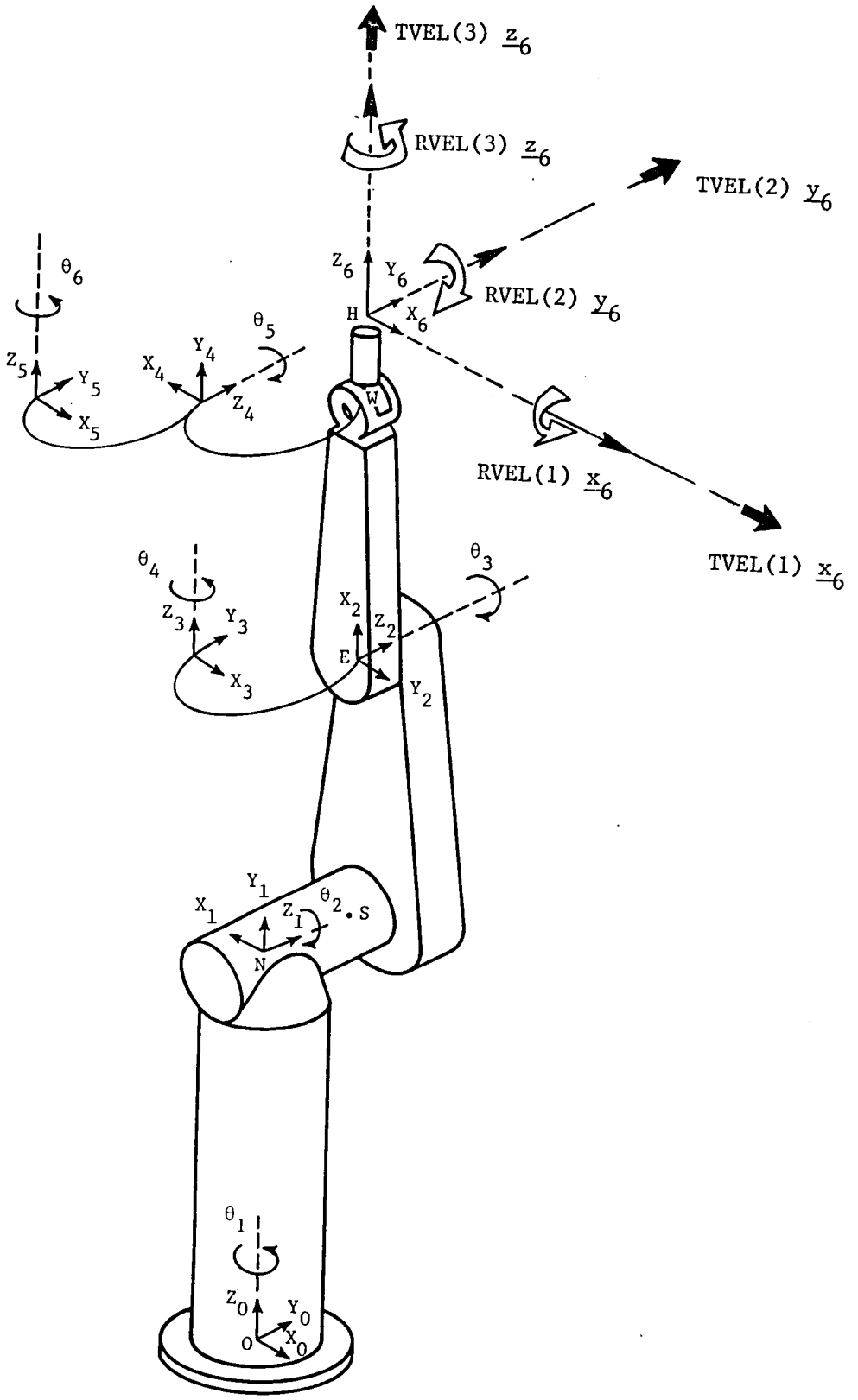

Figure 3.- Initial position of robot arm, joint axis systems, and commanded robot hand velocities.

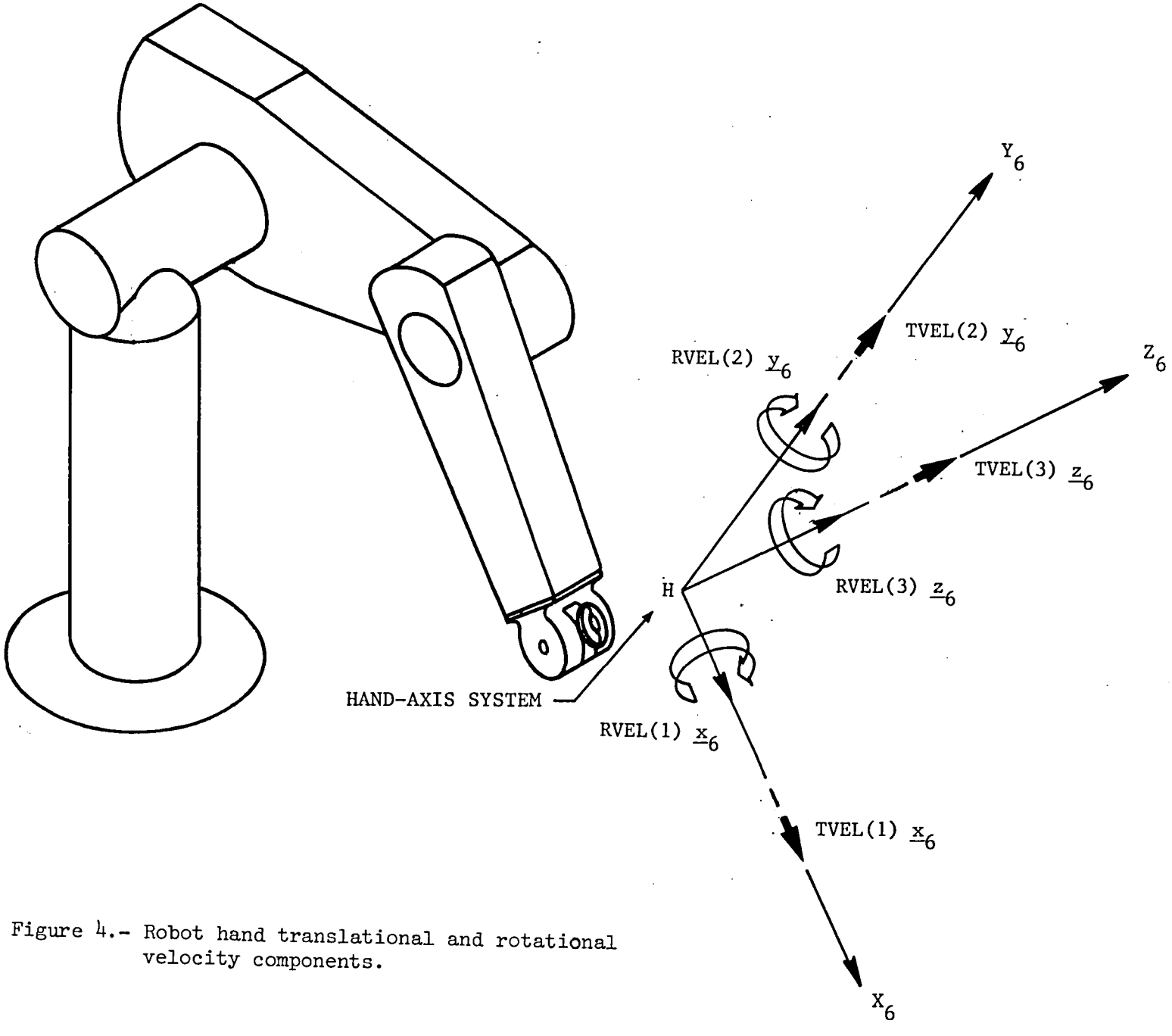

Figure 4.- Robot hand translational and rotational velocity components.

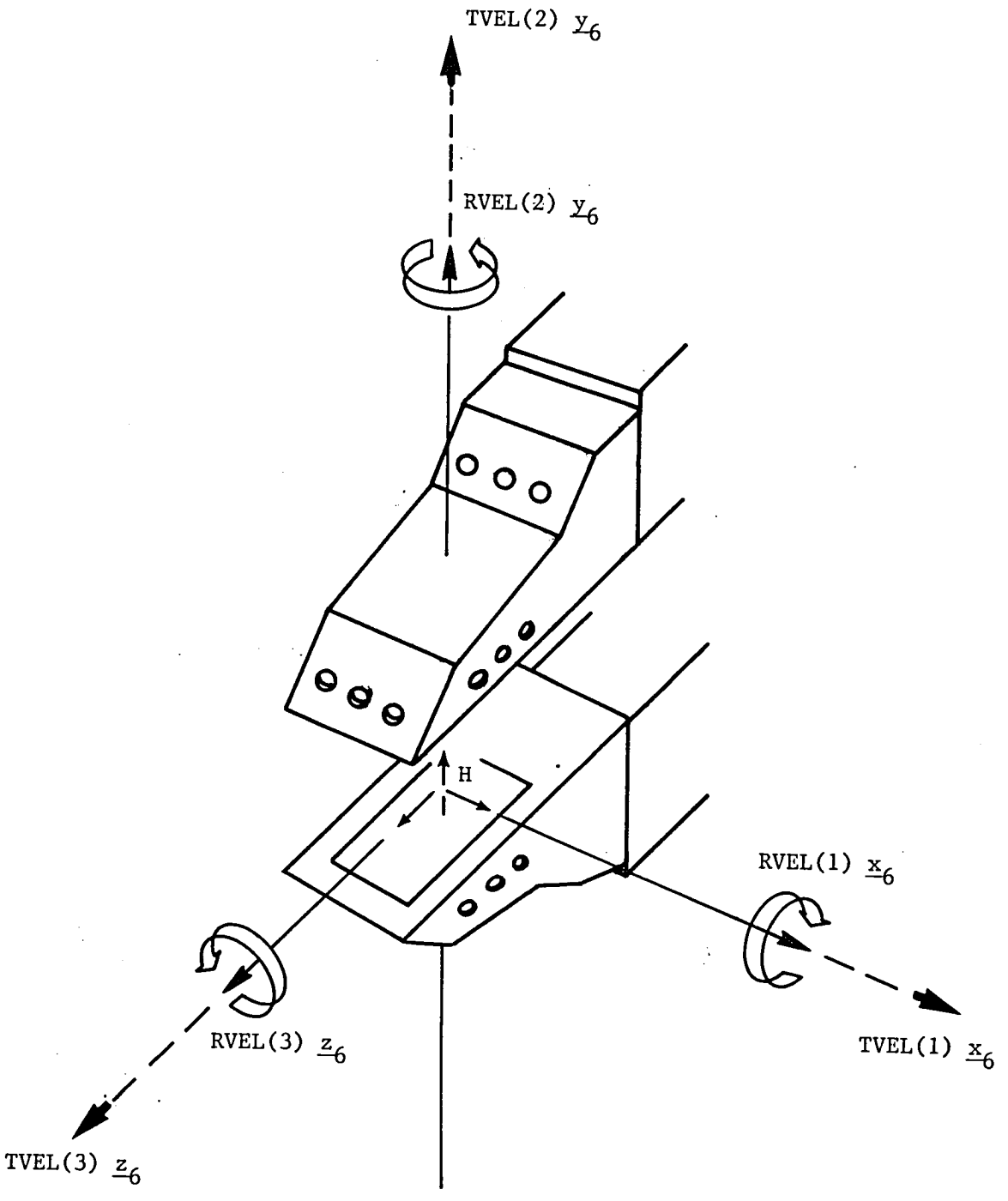

Figure 5.- Robot hand axis system.

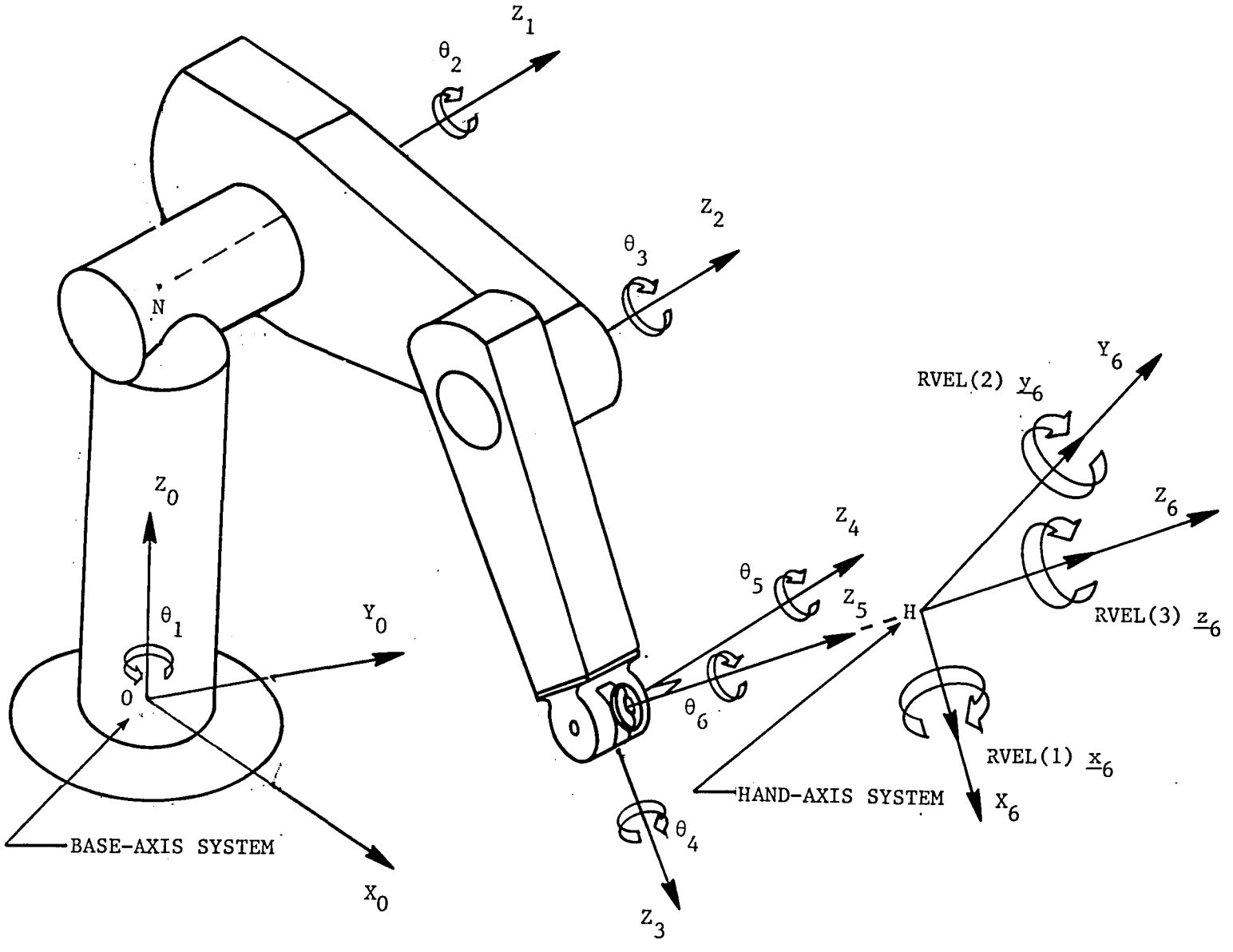

Figure 6.- Rotational velocity components of robot hand induced by individual joint rotations.

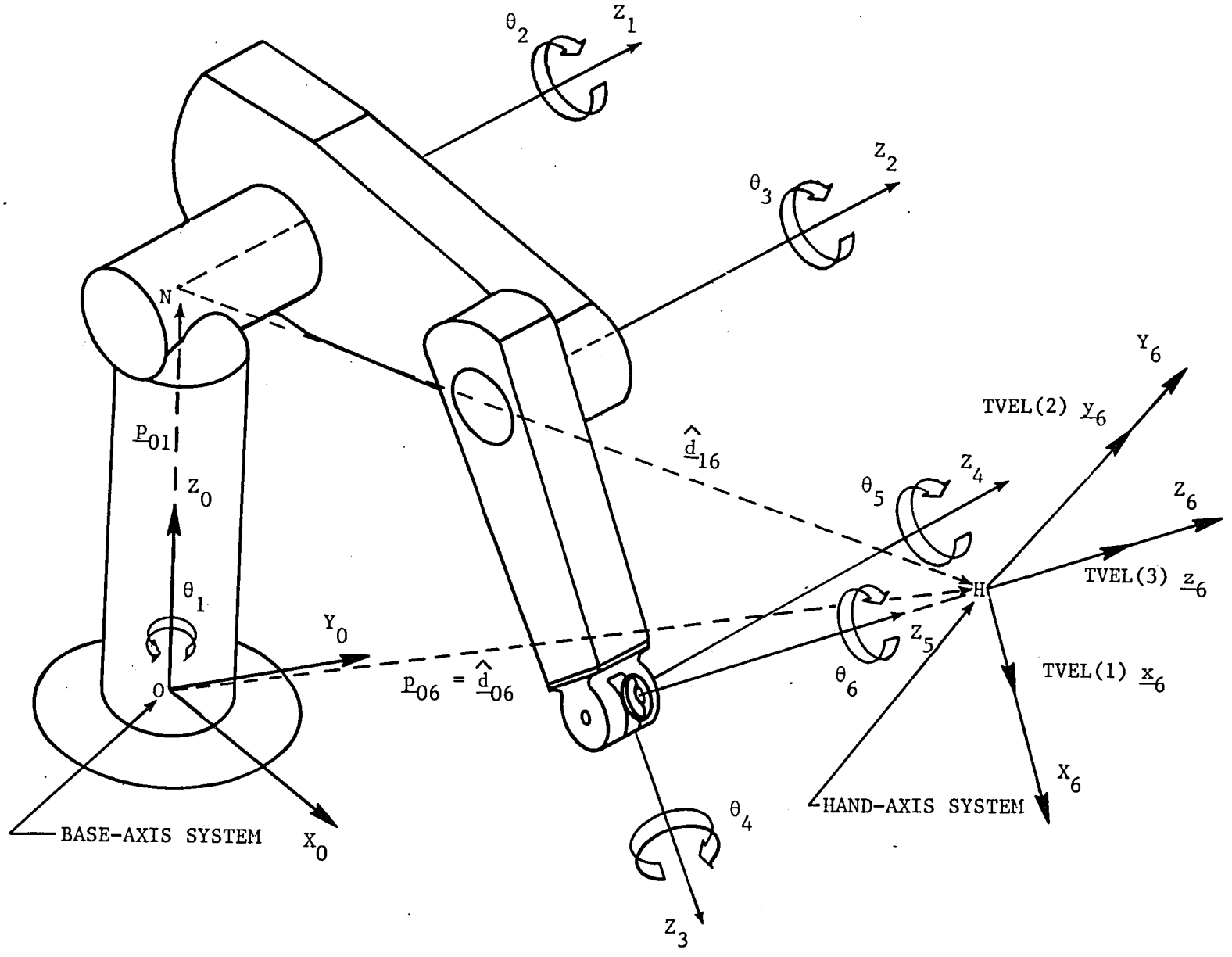

Figure 7.- Illustration of moment radius vectors $\hat{d}_{06}$ and $\hat{d}_{16}$ needed in calculating contributions to translational velocity of robot hand caused by rotations of joints 1 and 2.

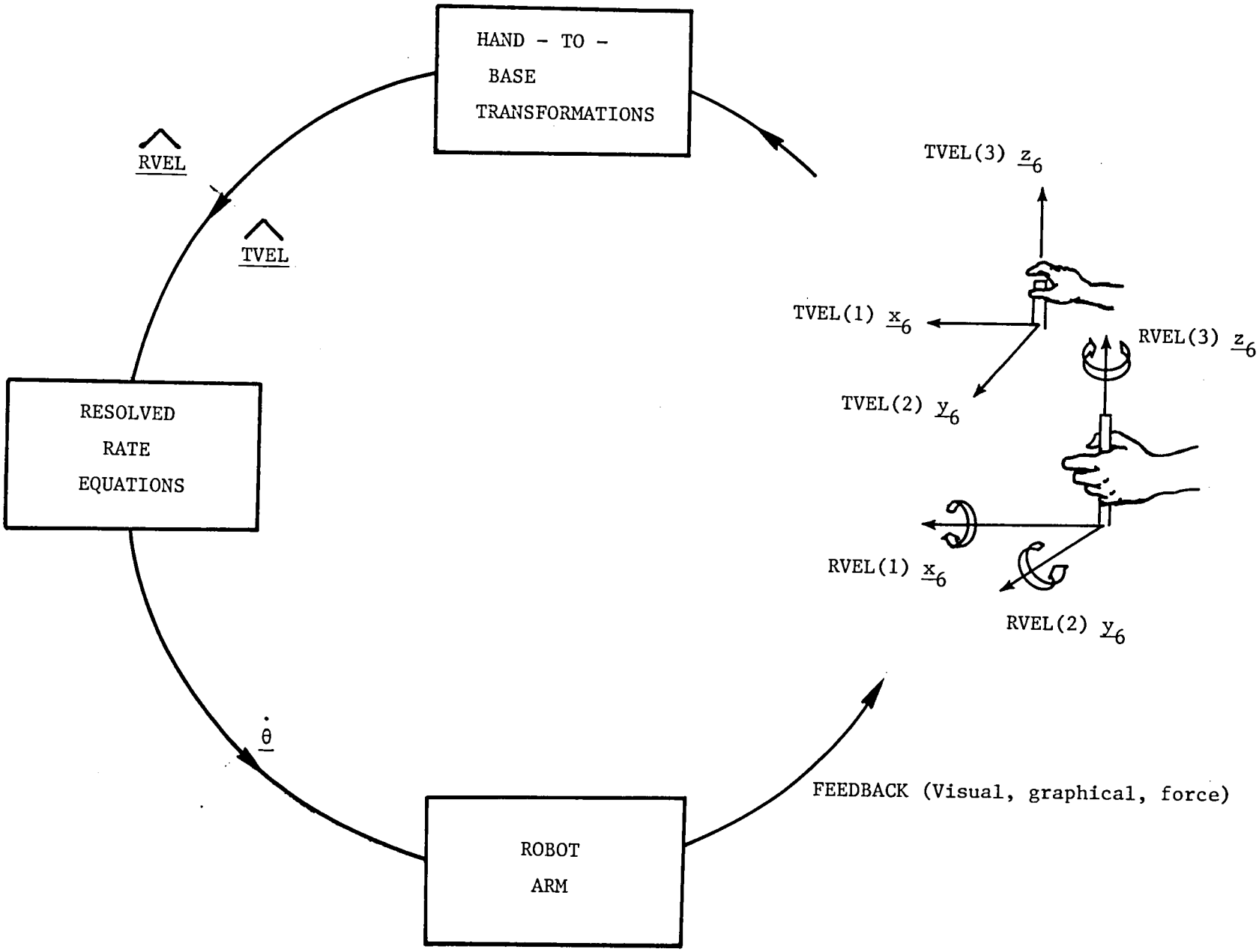

Figure 8.- Illustration of information flow in robot arm control.

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15. Supplementary Notes

16. Abstract

An operator can use kinematic, resolved-rate equations to dynamically control a robot arm by watching its response to commanded inputs. In a tutorial fashion, this paper derives known resolved-rate equations for the control of a particular six-degree-of-freedom industrial robot arm and proceeds to simplify the equations for faster computations. Methods for controlling the robot arm in regions which normally cause mathematical singularities in the resolved-rate equations are discussed.

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