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Thermal Electron Heating Rate, A Derivation

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ABSTRACT

The thermal electron heating rate, Q_e, is an important heat source term in the ionospheric electron energy balance equation, representing heating by photoelectrons or by precipitating higher energy electrons. A formula for the thermal electron heating rate is derived from the kinetic equation using the electron-electron collision operator as given by the unified theory of Kihara and Aono. This collision operator includes collective interactions to produce a finite collision operator with an exact Coulomb logarithm term. The derived heating rate Q_e is the sum of three terms, $Q_e = Q_p + S + Q_{int}$, which are respectively: 1) primary electron production term giving the heating from newly created electrons that have not yet suffered collisions with the ambient electrons, 2) a heating term evaluated on the energy surface $m_e v^2/2 = E_T$ at the transition between Maxwellian and tail electrons at E_T, and 3) the integral term representing heating of Maxwellian electrons by energetic tail electrons at all energies $> E_T$. Published ionospheric electron temperature studies have used only the integral term Q_{int} with differing lower integration limits. There can be a significant numerical difference between Q_e and Q_{int}. Use of the imcomplete heating rate could lead to erroneous conclusions regard electron heat balance, since Q_e is greater than Q_{int} by as much as a factor of two. The sensitivity of the heating rate to the method of calculating the energetic (tail) electron distribution function, using either a linear or a quadratic collision operator is demonstrated. Choice of the transition energy, E_T, between the thermal (Maxwellian) population and the energetic tail electrons significantly affects the magnitude of the individual heating rate terms. The net heating rate Q_e is less sensitive to the value of E_T then are the individual terms.

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1. INTRODUCTION

The Earth's ionospheric layer has been studied ever since its importance for long distance radio communication was discovered. The earliest ionospheric studies dealt mainly with its density variations with altitude, time and location. The thermal properties of the Earth's ionosphere first came under intensive study in the late 1950's and early 1960's when rockets became available for in situ measurements. The first in situ electron temperature measurements were made by Boggess et al. (1959), Spencer et al. (1962), and Brace et al. (1963); these measurements clearly demonstrated that the electron temperatures exceeded the neutral temperature in the dayside ionosphere. Predating these measurements by about 15 years, Drukarev (1946) predicted the existence of hot ambient electrons in the ionosphere due to heating from energetic photoelectrons. Early model calculations of ionospheric electron temperatures were made by Hanson and Johnson (1961), Dalgarno et al. (1963), and Hanson (1963). These calculations were made by balancing the ambient electron heating rate with the rate of electron cooling from collisions with ions and neutral particles. In their early work, Hanson and Johnson (1961) evaluated the ambient electron heating rate as the product of a constant heating efficiency multiplied by the total photoelectron production rate.

Electron heat balance studies after this pioneering work have calculated the heating rate as the product of the energetic electron (tail) distribution function and the energy loss rate between tail electrons and Maxwellian electrons, integrated over the energy range of the tail electrons. This formula for calculating the heating rate is an assumed formula. The derivation of the heating rate presented in this paper will show that it is one of the three terms contributing to the heating rate. The other two terms are significant and therefore can not be neglected.

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The reader is referred to the review paper by Schunk and Nagy (1978) for a general discussion of ionospheric electron temperature theory and observations, and for further references. The remainder of this paper is devoted to the thermal electron heating rate, how it is derived, a formula for it, and some numerical examples. In section 2 we give the basic equations (the electron kinetic equation and the electron fluid equation) from which the ambient heating rate is derived. An alternative electron heat balance equation is discussed and the need for partitioning the electrons into two populations is given. The formula for the ambient electron heating rate is derived in section 3 and some numerical examples and comparisons with earlier heating rates are given in section 4. Concluding remarks are given in section 5. Appendix A sketches a derivation of the electron-electron collision operator which is required in the heating rate derivation.

2. BASIC EQUATIONS

The starting point in the derivation of the thermal electron heating rate is the kinetic equation for the electron velocity distribution function, $f(v)$. We write the kinetic equation in the form (Burgers, 1969):

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\partial}{\partial \vec{v}} \cdot \vec{f}_m f = p + \frac{\delta f}{\delta t} \tag{1}$$

where $\vec{f}_m$ is the force per mass, p is the electron production rate from photo-ionization of neutrals, and $\delta f/\delta t$ is the collisional rate of change of f.

The electron heating rate is a term in the electron energy balance equation, which is obtained by taking the energy moment of the electron kinetic equation. If we follow the usual procedure of integrating over all velocities (including both populations of thermal and tail electrons) we obtain the energy equation:

$$N_e \frac{D}{Dt} 3/2 \, kT_e + N_e kT_e \, \mathrm{div} \, \vec{U}_e + \overset{\leftrightarrow}{P}_e : \frac{\partial \vec{U}_e}{\partial \vec{x}} +$$

$$+ \mathrm{div} \, \vec{q}_e = \langle (E - \frac{3}{2} kT_e) \, p \rangle + \langle (E - \frac{3}{2} kT_e) \frac{\delta f}{\delta t} \rangle , \qquad (2)$$

where $\langle \; \rangle$ denotes the integral over all velocities $\vec{v}$, $E = \frac{m_e u^2}{2}$ is the thermal energy, $\vec{u} = \vec{v} - \vec{U}_e$, N_e is the electron density, T_e is the electron temperature, $\vec{U}_e$ is the average electron velocity, $\overset{\leftrightarrow}{P}_e$ is the traceless momentum tensor, and $\vec{q}_e$ is the electron heat flux.

This equation is consistent with the energy equations of Shkarofsky et al. (1966) and Burgers (1969). The form appears different from the latter two references because the density, N_e, has been removed from the substantial derivative and we include the electron production term p. The density, temperature, average velocity, momentum tensor, and heat flux are defined (in Eq. 2) for the total electron population, since the moments of f were taken over the entire velocity space (see Chapman and Cowling, 1960 for the definitions).

The first term on the right side of the energy balance equation represents direct heating from electrons which are created by ionization of neutral particles. The second term is a net cooling rate from the totality of electron-ion and electron-neutral collisions. There is no thermal electron heating rate term in this equation because electron-electron collisions conserve energy and particles, and the integrals were carried out over all velocities. The heating of the thermal electron population (the Maxwellian electrons) balances the cooling of the energetic electron population (the enhanced tail electrons) so that the sum is zero. Thus the usual hydrodynamic energy equation, Eq. 2, does not contain the heating rate term that we wish to derive. Also, the transport terms in Eq. 2 such as $\mathrm{div} \, \vec{q}_e$ contain contributions from the tail electron population thay may be difficult to evaluate.

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For example, the tail population contribution to the heat flux may not have a simple representation as the gradient of a temperature, a local representation, but might be more accurately represented globally (nonlocally). The reader is referred to the papers of Scudder (1979) and Scudder and Olbert (1979) for a discussion of local versus global processes for the non-Maxwellian solar wind electrons.

In order to derive a thermal electron heating rate, it is necessary to split up the electron population into a thermal (Maxwellian) population with energies $E < E_T$ and an energetic (tail) population with energies $E > E_T$, where E_T is the transition energy between the two populations. One could derive spearate fluid equations for the thermal and energetic electron populations with integration limits 0 to E_T for the ambient electron quantities, and E_T to ∞ for the energetic electron quantities. We adopt this scheme only for the thermal electrons treating them as a fluid, and solve the kinetic equation numerically for the energetic (tail) electron distribution function which is to be added to the Maxwellian distribution function.

The thermal electron energy equation is given by Eq. 2 plus additional transport terms on the left side, and with < > modified to be the integral over all velocities such that $E < E_T$. The additional terms are approximately given by:

$$\frac{2E_T}{m_e} (E_T - 3/2 \ kT_e) \int d\Omega \ \hat{u} \ f(E_T, \ \hat{u}) \ [\frac{\partial \vec{U}_e}{\partial t} \cdot \hat{u}$$
$$+ \frac{\partial U_\beta}{\partial x_\alpha} \hat{u}_\beta \ (\sqrt{\frac{2E_T}{m_e}} \ \hat{u}_\alpha + \vec{U}_\alpha) - \vec{f}_m \cdot \hat{u} \ ], \qquad (2A)$$

where Ω is the solid angle of unit vector $\hat{u}$, $f(E_T, \ \hat{u})$ is the electron distribution function evaluated at $E = E_T$, and $\vec{f}_m$ is the force per unit mass. The approximation which gives the simple form for the additional terms (Eq. 2A) is to equate the density, temperature, and average velocity which

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characterize the Maxwell distribution function f^M, with the same parameters evaluated from the full distribution function $(f^M + f^T)$ for energy $E < E_T$. This is a good approximation for most of the ionosphere where $kT_e \ll E_T$, and $f^T \ll f^M$ for $E < E_T$.

We are interested in this paper primarily in the terms on the right side of Eq. 2, the collisional terms which lead to the thermal heating rate. We will not discuss further the time and space derivative (transport) terms which appear on the left side (Eqs. 2 and 2A).

With the partitioning of electrons into two populations, we rewrite the right side of the energy equation as the sum of two terms, Q_e the thermal heating rate, and L_e the thermal cooling rate. The thermal electron heating rate can be written as:

$$Q_e = \int d\vec{v} \; \Theta \; (E_T - E)(E - \tfrac{3}{2} kT_e)p + \int d\vec{v} \; \Theta \; (E_T - E)(E - \tfrac{3}{2} kT_e) \frac{\delta f}{\delta t}\|_{ee} \qquad (3)$$

where $\delta f/\delta t\|_{ee}$ is the electron-electron collision operator and Θ is the unit step function, $\Theta(x) = 1$ for $x \geq 0$, $\Theta(x) = 0$ for $x < 0$. The energy moments of the electron-ion and electron-neutral collision operators yield the thermal electron cooling rates which have been discussed in the literature (see Schunk and Nagy, 1978). The electron heating rate is derived by evaluating the integrals in Eq. 3. In order to proceed with this evaluation, we need the production rate of electrons, p, the transition energy E_T, an explicit expression for the electron-electron collision operator, and the total distribution function f. The production rate p is obtained during the numerical calculation of the energetic electron distribution function. The value chosen for E_T will affect the magnitude of the individual terms in the heat balance equation, Q_e, L_e, and the transport terms. The nominal value for E_T is the energy where f^T crosses f^M. This value usually has the advantage that: 1) it satisfies $E_T \gg kT_e$ so the moments of f^M over the limited energy range $E < E_T$

are nearly equal to the moments over the entire energy range; and 2) for $E <$ E_T, $f^T << f^M$ so that the cooling rates are nearly independent of the tail electrons.

An earlier derivation of the heating rate (Hoegy, 1977) used an electron-electron collision operator with simple Debye screening (Shkarofsky, 1961). The consequence of that derivation is that it led to an energetic-electron to ambient-electron loss rate, dE/dt, having the form of the Butler and Buckingham (1962) rate. The more exact energy loss rate of Itikawa and Aono (1966) which includes collective wave-particle and wave-wave effects, was substituted for the approximate loss rate without being included in an ab initio manner. The derivation presented here includes collective plasma interactions ab initio by employing the electron-electron collision operator of Kihara and Aono (1963), using their unified theory. Schunk and Hays (1971) demonstrated that use of the Butler and Buckingham (1962) energy loss rate would lead to as much as a 70% error in the loss rate calculation when compared with the exact loss rate formula containing collective interactions. Later, Swartz and Nisbet (1972) asserted that the heating rate is not very sensitive to the specific electron-electron loss rate used. However, the formula for the heating rate and the calculated distribution function both depend on the form chosen for the collision operator.

The electron-electron collision operator, $\frac{\delta f}{\delta t}$, is given in Fokker-Planck form (Kihara and Aono, 1963) as the first two terms in a velocity-change expansion of the Boltzmann collision operator

$$\frac{\delta f}{\delta t} = -\frac{\partial}{\partial \vec{v}} \cdot (<\frac{\Delta \vec{v}}{\Delta t}> f) + \frac{1}{2}\frac{\partial}{\partial \vec{v}}\frac{\partial}{\partial \vec{v}} : (<\frac{\Delta \vec{v} \Delta \vec{v}}{\Delta t}> f), \qquad (4)$$

where the explicit expressions for $<\frac{\Delta \vec{v}}{\Delta t}>$ and $<\frac{\Delta \vec{v} \Delta \vec{v}}{\Delta t}>$, are given in Kihara and Aono (1963). Their unified theory adds together the impact (binary) and collective interaction terms to obtain a non-divergent electron-electron

collision operator. As a result of adding together the binary and collective interactions, an exact, convergent term logarithmic in the density is obtained. The unified theory has been successfully applied in the calculation of transport coefficients (Kihara et al., 1963; Kihara, 1964; Itikawa and Aono, 1966; and Daybelge, 1969), however, an explicit formula for the electron-electron collision operator has not appeared in the literature. Appendix A sketches a derivation of the formula for the collision operator, using the unified theory. In the next section, this formula is used to derive the heating rate expression.

3. THERMAL ELECTRON HEATING RATE

The thermal electron heating rate was given in the previous section as the sum of two terms. The first term is a direct contribution from electrons produced by ionization of neutrals by solar EUV and particle precipitation,

$$Q_e^{(1)} = \int d\vec{v} \ \Theta \ (E_T - E) \ (E - \frac{3}{2} kT_e) \ p. \tag{5}$$

This term needs no further mathematical transformations, and is evaluated by numerical integration of the primary production rate, p.

The second term, an integral containing the electron-electron collision operator, represents heating of thermal electrons by energetic tail electrons,

$$Q_e^{(2)} = \int d\vec{v} \ \Theta \ (E_T - E) \ (E - \frac{3}{2} kT_e) \ \frac{\delta f}{\delta t} \|_{ee} \tag{6}$$

We simplify the form of the second term using the (unified theory) collision operator, as given by Eq. A26. Since electron-electron collisions conserve energy and mass (these moments of the collision operator over all velocity space are zero), we rewrite the collision integral, Eq. 6, as the negative of the integral over energies $E > E_T$ and use the collision operator evaluated for velocities large compared to the electron thermal velocity:

$$Q_e^{(2)} = - \ 4\pi \int_{v_T}^{\infty} dv \ (E - \frac{3}{2} kT_e) \ \frac{d}{dv} \{ \quad \}, \tag{7}$$

where the curly bracket represents the expression in curly brackets in Eq. A26. Further integration by parts yields the form:

$$Q_e^{(2)} = S + \int_{v_T}^{\infty} m_e \, v dv \, (-\frac{dE}{dt}) \frac{4\pi v f}{m_e}, \tag{8}$$

where dE/dt is the test electron energy loss rate given by Eq. A24 and where S summarizes all the terms evaluated on the energy surface $E = E_T$ and is given by,

$$S = (E_T - \frac{3}{2} kT_e) 4\pi Y \, [I_o^o \, f + \frac{v}{3} (I_2^o + J_{-1}^o) \frac{df}{dv}] - \frac{2}{3} E_T 4\pi Y (I_2^o + J_{-1}^o) f \tag{9A}$$

where Y and the integrals I_n^o and J_{-1}^o are defined by Eqs. A16-A19. Note that in deriving Eq. 8 we have retained only the dominant logarithm terms and have neglected derivatives of the velocity-dependent logarithm since they are an order of magnitude smaller. The surface term Eq. (9A) is valid for scattering particles having an arbitrary distribution function (the argument of the I and J integrals).

Next we evaluate the heating rate, $Q_e^{(2)}$, using the collision operator given by Eq. A29, valid for Maxwellian scatterers. The scattering particle distribution contributes to dE/dt, I, J, and F_2 in these formulas; the scattered particle distribution appears explicitly in the formulas. The result has the form of Eq. 8 with the surface term given by:

$$S^{MT} = 4\pi \, F_2(v) \, [m_e \, (\frac{E_T}{kT_e} - \frac{5}{2}) \, f + (E_T - \frac{3}{2} kT_e) \frac{1}{v} \frac{df}{dv}], \tag{9B}$$

where $F_2(v)$ is an integral of $\frac{dE}{dt}$ given by Eq. A28. The superscript MT signifies Maxwellian scatterers and tail population scattered particles. Use of the exact energy loss rate $\frac{dE}{dt}$ as given by Swartz et al. 1971 in Eqs. 8 and 9B provides an accurate evaluation of the heating rate for the MT contribution. For the TM and TT contributions (having tail electrons as the scatterers), the surface term of Eq. 9 and the $\frac{dE}{dt}$ expression of Eq. A24 must be used.

The thermal electron heating rate is the sum of the three terms given in Eqs. 5 and 8-9. The three terms were shown in the review paper by Schunk and Nagy (1978) and were first given by Hoegy (1977), however the early form of the surface term was an approximation of the complete form given by Eq. 9. Previous expressions for the heating rate have used only the integral term of Eq. 8, and have used a variety of lower bounds on the integration.

It was suggested by Hoegy (1977) that E_T should be the energy at which the calculated energetic (tail) distribution function, f^T, crosses the Maxwellian distribution function, f^M. It is not necessary that E_T have this crossing value, however it is necessary that all cooling terms in L_e be evaluated over the energies 0 to E_T, i.e. that the same energy range be used for both heating and cooling rates. As E_T is varied, both the heating rate Q_e and cooling rate L_e will change as will the transport terms. If the majority of the electron population is the Maxwellian population and if E_T is nearly equal to the crossing energy or greater, then the left hand side of the heat balance equation, Eq. 2, will be nearly constant and thus the sum $Q_e + L_e$ will be nearly independent of E_T, even though the individual terms are dependent on E_T. The dependence of Q_e on E_T will be demonstrated in the next section.

4. NUMERICAL EXAMPLE OF HEATING RATE

We give some numerical examples of the ambient electron heating rate, Q_e. It is first necessary to solve the kinetic equation for the energetic electron distribution function. Using the primary electron production rate, shown in Figure 1, calculated for the Venus ionosphere at 60° SZA and 180 km, we solve for the photoelectron distribution function f^{ph} such that the total distribution is the sum

$$f = f^M + f^{ph}, \tag{10}$$

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where f^M is the Maxwellian distribution function. Krinberg and Akatova (1978) showed that Eq. (10) is a good representation of the total distribution function in most of the ionosphere when f^{ph} is calculated according to their Eq. (3) and the Maxwell distribution is at temperature T_e determined by the heat balance equation. Their method of computing f^{ph} appears to be identical with the method used here when a linear collision operator is employed. A thorough discussion of the processes important in the calculation of the distribution function is given by Ashihara and Takayanagi (1974). In Figure 2 we plot the calculated photoelectron flux ϕ^{ph} which is related to f^{ph} by,

$$\phi^{ph}(E) = \frac{4\pi v^2}{m_e} f^{ph}(v), \tag{11}$$

and has the units, # electrons/(cm^2 sec eV). The solid line in Figure 2 is ϕ^{ph} calculated with the quadratic (in ϕ^{ph}) collision operator (Eq. A26); the plus signs represent ϕ^{ph} calculated with a linear (in ϕ^{ph}) collision operator given by the first term on the right hand side of equation A22, with dE/dt given by the Swartz et al. (1971) formula; and the M's represent ϕ^M for Maxwellian flux. The quadratic collision operator terms are non-negligible in the vicinity of the crossing of ϕ^M and ϕ^{ph} and have the effect of enhancing the calculated electron flux ϕ^{ph} as was first shown by Krinberg (1973). The total electron flux is the sum of the solid curve and the curve given by the M's, and is the flux to be compared with measured electron fluxes. The enhancement in ϕ^{ph} due to the quadratic collision operator may be important in such comparisons. Figure 3 is an expansion of the plot of Figure 2 for energies 0 to 10 eV, showing clearly the enhancement in the calculated flux at $E < 1.5$ eV. The higher solid line in Figure 3 is the sum of the Maxwellian and the calculated fluxes. In Figure 4 we show the fluxes at an altitude of 200 km.

In Figure 3, for 180 km altitude, the Venus ionosphere temperature is $1280°K$ and the crossing energy is $E_T = 1.45$ eV; while in Figure 4, for 200 km

altitude, the temperature is 2500°K and $E_T = 2.63$ eV. The Venus parameters are from the latest model ionosphere of Brace and Theis, to be published.

For contrast with the Venus results we show the primary electron production rate calculated for the Earth's ionosphere at an altitude of 200 km and solar zenith angle of 42° in Figure 5, and using the reference spectrum of Hinteregger (private communication). The calculated flux for this primary production rate is shown in Figures 6 and 7. The enhancement in Φ due to quadratic terms appears at energies of 2 eV and less. For $T_e = 2000$°K and $N_e = 1. \times 10^5$, the crossing energy is $E_T = 1.9$ eV. The steepness of the Maxwellian flux for the sample cases shown in the figures renders the enhancement in Φ due to the quadratic vs. the linear collision operator insignificant in these examples. However, in regions of the ionosphere having high temperature $T_e > 5000$° and a higher proportion of tail to Maxwellian electrons, the quadratic terms will be important.

The heating rate terms: Q_p, the primary production rate term; Q_{int}, the integral term; and S, the surface term are evaluated by substituting the calculated Φ^{ph} and the assumed Φ^M into Eqs. 5, 8, and 9. These terms are shown in Tables I through IV along with the total heating rate Q_e,

$$Q_e = Q_p + Q_{int} + S, \tag{12}$$

and the nominal heating rate Q_{nom} computed from Q_{int} using Φ^{ph} from the linear collision operator (the plus signs in Figure 2) and using the linear form of dE/dt. It is clear from the examples that the individual heating rate terms vary strongly with energy E_T, and that there is a significant difference between the nominal heating rate Q_{nom} computed only from the integral of dE/dx Φ^{ph} over energies greater than E_T, and the net heating rate Q_e, which includes the primary production term and the surface term. Also, the net heating rate is relatively insensitive to the value of E_T.

5. SUMMARY

A formula for the ambient electron heating rate, Q_e, has been derived from the electron kinetic equation. The heating rate depends on the transition energy, E_T, which defines the upper energy of the "ambient" Maxwellian electron population. The cooling rate, L_e, also depends on E_T; however, the sum $Q_e + L_e$ should be nearly independent of E_T. The heating rate is the sum of three terms,

$$Q_e = Q_p + S + Q_{int}, \qquad (13)$$

where Q_p is the heating from primary photoelectrons, S is a term evaluated on the energy boundary E_T, and Q_{int} is the familiar integral term

$$Q_{int} = \int_{E_T}^{\infty} dE \; \left(-\frac{dE}{dt}\right) F^{ph}(E),$$

$$= \int_{E_T}^{\infty} dE \; \left(-\frac{dE}{dx}\right) \Phi^{ph}(E), \qquad (14)$$

where $\Phi^{ph}(E) = vF^{ph}(E)$ is the photoelectron flux, # electrons/(cm^2 eVs), and F^{ph} is the photoelectron energy distribution function, # electrons/(cm^3 eV).

The value chosen for the transition energy, E_T, need not be the energy at which the Maxwellian and energetic distributions cross, however it must be chosen large enough so that the bulk of the electron population has energies $E < E_T$ so that the hydrodynamical parameters: temperature, density, bulk velocity, pressure, heat flux are well represented by integrals over the limits 0 to E_T.

The nominal heating rate calculated in many ionospheric temperature studies uses only Eq. 14 and does not generally include terms quadratic in Φ^{ph} either in the calculation of Φ or in the evaluation of the integral. In many instances the quadratic terms may not be important, and under some conditions, Q_p and S may not be signficiant compared to Q_{int}. It is important though to test the complete formula (Eq. 13) in case these terms are significiant. Some studies have used the heating rate of Eq. (14) with a

lower limit of $E_T \simeq kT_e$. Such a heating rate may better approximate Q_e than Q_{int} with E_T = transition energy, however there is no assurance the answer will be numerically accurate.

There is a way of avoiding the problem of the sensitivity of Q_e to the value chosen for E_T. This is to let $E_T \to \infty$ and treat the electrons as one population. In this case, $S \to 0$ and $Q_{int} \to 0$ so that $Q_e \to Q_p$, which becomes a large heating rate; then L_e also becomes large. One gains by not having to evaluate the surface term and not having to perform the integration in Q_{int}; however the cooling rate calculation requires the addition of integrals using the calculated ϕ^{ph} with the inelastic electron neutral excitation and ionization energy loss rates. Hence there is no way of avoiding, first the calculation of the energetic tail distribution, and second of performing integrals with the distribution, in a study of the electron heat balance equation, and in the determination of electron temperatures.

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Acknowledgements

Figure Captions

Figure 1. Primary photoelectron volume production rate p calculated for the Venus ionosphere at 180 km and 60° solar zenith angle. Production rate units are # electrons/(cm^3 eVs).

Figure 2. Photoelectron flux Φ calculated for the conditions of Figure 1. Plus signs represent the flux calculated with the linear collision operator; the M's represent the assumed Maxwell flux; the lower solid line is the flux calculated with the quadratic collision operator; and the upper solid line is the total electron flux. Flux units are # electrons/cm^2 eVs.

Figure 3. Expansion of the fluxes of Figure 2, for energies 0 to 10 eV.

Figure 4. Electron fluxes calculated for the Venus ionosphere at an altitude of 200 km and solar zenith angle of 60°. The symbols are the same as Figures 2 and 3.

Figure 5. Primary photoelectron volume production rate calculated for the Earth's ionosphere at 200 km and 42° solar zenith angle.

Figure 6. Photoelectron flux calculated with production rate of Fig. 5 and T_e = 2000^oK, N_e = 1. x 10^5 e/cm^3.

Figure 7. Photoelectron flux calculated with conditions of Fig. 6, but with N_e = 1. x 10^6.

TABLE I

E_T	Q_p	S	Q_{int}	Q_{nom}	Q_e
1.0	24	-454	3572	3549	3142
1.5	55	551	3264	3246	3853
2.0	79	744	2984	2966	3805
2.5	109	908	2725	2707	3741
3.0	139	1038	2485	2467	3662

Venus calculation at 180 km, 60° solar zenith angle. T_e = 1280°K, N_e = 1.3 x 10^5 e/cm^3. Photoelectron and Maxwellian distributions cross at E_T = 1.45 eV. Q_p = primary production rate term, S = surface term, Q_{int} = integral term using full quadratic collision operator, Q_{nom} = nominal integral term using linear collision operator, Q_e = net electron heating rate = Q_p + S + Q_{int}. The units of the heating rates are eV/cm^3/sec.

TABLE II

E_T	Q_p	S	Q_{int}	Q_{nom}	Q_e
2.0	11	0	702	691	714
2.5	17	150	646	637	813
3.0	23	197	595	587	815
3.5	29	230	548	540	806
4.0	34	253	504	496	791

Venus calculation at 200 km, 60° solar zenith angle. $T_e = 2500^\circ K$, $N_e = 1. \times 10^5$ e/cm^3. Photoelectron and Maxwellian distributions cross at $E_T = 2.63$ eV. Terms defined as in Table I.

TABLE III

E_T	Q_p	S	Q_{int}	Q_{nom}	Q_e
1.5	189	-297	1313	1235	1204
2.0	218	-134	907	878	991
2.5	289	436	676	664	1400
3.0	413	372	519	509	1304
3.5	603	290	393	386	1286

Earth calculation at 200 km, 42° solar zenith angle. $T_e = 2000^O K$, $N_e = 1. \times 10^5$ e/cm^3. Photoelectron and Maxwellian distributions cross at $E_T = 1.9$ eV. Terms defined as in Table I.

TABLE IV

E_T	Q_p	S	Q_{int}	Q_{nom}	Q_e
2.0	218	791	2765	2696	3774
2.5	289	1226	2336	2277	3852
3.0	413	1166	1958	1907	3537
3.5	603	1123	1629	1589	3354
4.0	777	1184	1358	1324	3318

Earth calculation at 200 km, $42°$ solar zenith angle. $T_e = 2000°K$, $N_e = 1. \times 10^6$ e/cm^3. Photoelectron and Maxwellian distributions cross at $E_T = 2.61$ eV. Terms defined as in Table I.

Appendix A

The explicit form of the electron collision operator is derived using the unified theory of Aono (1962), which unifies the impact and wave collision opertors to produce an exact Coulomb logarithm term. In Kihara and Aono (1963) Kihara et al. (1963), Itakawa (1963), and Kihara (1964) the exact logarithm term was derived for the energy loss rate of a test electron, the diffusion coefficient, and other transport coefficients using the unified theory, but the electron collision operator was not evaluated. Shkarofsky et al. (1966) developed a general form for the electron collision operator in a vector spherical harmonic expansion, but employed approximate logarithm terms. The derivation sketched here uses the unified theory and some of the methods of Shkarofsky et al. (1966), presents a straight forward method of evaluating the impact and wave collision integrals, and obtains formulas used in the derivation of the ambient electron heating rate.

The electron collision operator is taken in Fokker-Planck form as the first two terms in a Taylor series expansion of the Boltzmann collision operator. The expansion parameter, $\Delta \vec{v}$, is the velocity change due to collisions. Only terms in the first and second powers of $\Delta \vec{v}$ contribute to the Coulomb logarithm; terms with higher powers of $\Delta \vec{v}$ are nondivergent and also do not contribute to the transport coefficients. The collision operator for electron collisions with species s is:

$$\left(\frac{\delta f}{\delta t}\right)_{es} = -\frac{\partial}{\partial \vec{v}} \cdot \left(\frac{\langle \Delta \vec{v} \rangle}{\Delta t} s\, f\right)$$

$$+ \frac{1}{2}\frac{\partial^2}{\partial \vec{v}\partial \vec{v}} : \left(\frac{\langle \Delta \vec{v}\Delta \vec{v} \rangle}{\Delta t} s\, f\right) \tag{A1}$$

where the velocity change $\frac{\langle \Delta \vec{v} \rangle}{\Delta t} s$ is given in Kihara and Aono (1963) by their

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Eq. 2.3 for impact theory, and Eq. 3.25 for wave theory. The tensor term is given by their Eq. 2.2 for the impact theory and Eq. 3.24 for the wave theory. Evaluation of the vector and tensor terms is outlined below.

In the unified theory, the integral over impact parameter, b, has the convergence factor $\exp(-b^2/2b_0^2)$, where b_0 is a parameter obeying the inequality,

$$\text{close impact radius} \ll b_0 \ll \text{Debye length.} \tag{A2}$$

These inequalities of b_0 with the close impact radius and the Debye length allow us to make the mathematical simplifications that lead to the logarithm terms. This is illustrated by the impact parameter integral for the vector term:

$$2\pi \int_0^\infty \frac{bdb \; e^{-\frac{1}{2}b^2/b_0^2}}{b^2 + (\frac{ee_s}{\mu_s g^2})^2} = -\pi e^\xi E_1(-\xi), \tag{A3}$$

where ξ is proportional to the square of the ratio of the close impact radius to b_0:

$$\xi = (\frac{ee_s}{\mu_s g^2})^2/2b_0^2. \tag{A4}$$

Since $\xi \ll 1$, the exponential integral is well represented by the first term, $E_1(-\xi) = \ln \gamma\xi$, in an expansion in $1/\xi$. Thus the impact part of the vector collision term is:

$$\frac{\langle \Delta \vec{v} \rangle_s}{\Delta t}\Big|_{\text{impact}} = K \frac{m_e + m_s}{m_s} \int d\vec{v}_s f_s(\vec{v}_s) \frac{\vec{g}}{g^3} \frac{1}{2} \ln \frac{1}{\gamma\xi}, \tag{A5}$$

where $K = 4\pi e^2 e_s^2 / m_e^2$, $\vec{g} = \vec{v}_s - \vec{v}$ is the relative velocity, and γ is Euler's constant, $\gamma = 1.781072$.

Integration of the tensor term also involves the exponential integral and yields:

$$\frac{\langle \Delta\vec{v}\Delta\vec{v}\rangle_s}{\Delta t}\bigg|_{impact} = K \int d\vec{v}_s \frac{f_s(\vec{v}_s)}{g} \{(\hat{I}\hat{I} - \hat{g}\hat{g}) \frac{1}{2} \ln \frac{1}{\gamma\xi} + (3\,\hat{g}\hat{g} - \hat{I}\hat{I}) \frac{1}{2}\}, \tag{A6}$$

where $\hat{I}\hat{I}$ is the unit tensor, and $\hat{g} = \vec{g}/g$.

The wave (collective interaction) integrals are more complicated than the impact integrals. We consider first the tensor term (Eq. 3.24 of Kihara and Aono, 1963) and use the order of integration of Shkarofsky et al. (1966). The Dirac δ function in the ω integral sets $\omega = \hat{k} \cdot \vec{v}$ and simplifies the dependence of the integrand on the magnitude of $\vec{k}$; with the unified theory convergence factor, $\exp(-k^2 b_0^2/2)$, the k integral gives:

$$\frac{\langle \Delta\vec{v}\Delta\vec{v}\rangle_s}{\Delta t}\bigg|_{wave} = \frac{K}{2\pi} \iint \frac{d\vec{v}_s f_s(\vec{v}_s)}{g} d\hat{k}\, \delta\,(\hat{k} \cdot \hat{g})\, \hat{k}\hat{k}$$
$$[\,(\cos y - \frac{x}{y} \sin y\,) \ln (\frac{1}{\gamma\sqrt{x^2+y^2}})$$
$$+ (\sin y - \frac{x}{y} \cos y)\, \mathrm{arc\ tan}\,(\frac{y}{x})\,], \tag{A7}$$

where,

$$x = \sum_s \frac{b_0^2 k_s^2}{2}\, R\,(x_s)$$

$$y = \sum_s \frac{b_0^2 k_s^2}{2}\, I\,(x_s)$$

$$R(t) = 1 - 2t\, e^{-t^2} \int_0^t dz\, e^{z^2}$$

$$I(t) = \sqrt{\pi}t\, e^{-t^2}$$

$$x_s = \hat{k} \cdot \vec{v} \sqrt{\frac{m_s}{2kTs}}$$

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$$k_s^2 = \frac{4\pi n_s e_s^2}{kT_s},$$

and $b_0^2 k_s^2 \ll 1$ from the inequality, Eq. A2, where k_s is an inverse Debye length. From this inequality, $x \ll 1$ and $y \ll 1$, thus the square bracket expression in Eq. A7 reduces to

$$\ln \frac{1}{\gamma\sqrt{x^2 + y^2}} - \frac{x}{y} \arctan \left(\frac{y}{x}\right),$$

the same functional form obtained by Daybelge (1969). The remaining integral over all angles of $\hat{k}$ such that $\hat{k} \cdot \hat{g} = 0$ is similar to the integral over impact angles $\hat{b}$ such that $\hat{b} \cdot \hat{g} = 0$, however, the wave case is more complicated because of the dependence of x and y on $\hat{k}$. The $\hat{k}$ integral has the form:

$$\int d\hat{k}\, \delta\,(\hat{k} \cdot \hat{g})\, \hat{k}\, \hat{k}\, [\quad]$$
$$= \int_0^{2\pi} d\phi\, ((\vec{I}\vec{I} - \hat{g}\hat{g})\, \cos^2\phi + \hat{y}\hat{y}\,(1 - 2\cos^2\phi))\, [\cos^2\phi], \qquad (A8)$$

where $\hat{x}$, $\hat{y}$, and $\hat{g}$ form a right handed orthogonal coordinate system with $\hat{y}$ in the direction of $\vec{v} \times \vec{v}_s$, $\cos\phi = \hat{k} \cdot \hat{x}$, and $[\cos^2\phi]$ indicates that the square bracket expression is an even function of $\cos\phi$.

We are interested in evaluating the collision operator for energetic electrons, when $v\sqrt{\frac{m_s}{2kT_s}} \gg 1$; in this case $[\quad]$ has a logarithmic dependence on $\cos^2\phi$, and the k integral reduces to,

$$2\pi\, (\vec{I}\vec{I} - \hat{g}\hat{g})\, \frac{1}{2} \ln \frac{A}{4} - 2\, \pi\, \hat{y}\hat{y},$$

where A represents the argument of the logarithm multiplying $\cos^2\phi$. Therefore in the argument of the logarithm, $\cos\phi$ is replaced by $\frac{1}{2}$ using the integrals:

$$\int_0^{2\pi} d\phi \, \ln(\cos^2\phi) = 2\pi \ln\left(\frac{1}{4}\right)$$

$$\int_0^{2\pi} d\phi \, \ln(\cos^2\phi) \cos^2\phi = 2\pi \frac{1}{2}\left(\ln\left(\frac{1}{4}\right) + 1\right).$$

Eventually, in the integral over $\vec{v}_s$ with fixed $\vec{v}$, the $\hat{y}\hat{y}$ term becomes $\frac{1}{2}(\hat{I}\hat{I} - \hat{v}\hat{v})$. Therefore the tensor wave integral is:

$$\frac{\langle\Delta\vec{v}\Delta\vec{v}\rangle_s}{\Delta t}\bigg|_{wave} = K \int d\vec{v}_s \, \frac{f_s(\vec{v}_s)}{g} \left[\left(\frac{\hat{I}\hat{I}-\hat{g}\hat{g}}{2}\right) \ln\frac{1}{\gamma\eta} - \frac{\hat{I}\hat{I}-\hat{v}\hat{v}}{2}\right], \qquad (A9)$$

where,

$$\eta = \frac{b_0^2}{2} \frac{\omega_p^2}{v_\perp^2 \frac{1}{4}},$$

$$\omega_p^2 = \sum_s 4\pi \frac{n_s e_s^2}{m_s},$$

$$v_\perp = \frac{v v_s}{g} \sqrt{1-z^2},$$

$$z = \hat{v} \cdot \hat{v}_s$$

The evaluation of the wave vector term follows the above steps, except that the derivatives $\frac{\partial}{\partial\vec{v}}$ and $\frac{\partial}{\partial\vec{v}_s}$ (from Eq. 3.25 of Kihara and Aono (1963)) must be transformed using the relation

$$\vec{k}\,\vec{k} \cdot \frac{\partial}{\partial\vec{v}} \delta(\omega - \vec{k} \cdot \vec{v}) = k^2 \frac{\partial}{\partial\vec{v}}\delta(\vec{k} \cdot \vec{v} - \omega)$$

and integration by parts with $\vec{v}_s$. The result is:

$$\frac{\langle \Delta \vec{v} \rangle_s}{\Delta t} \Big|_{wave} = - K \frac{m_e + m_s}{m_s} \int d\vec{v}_s f_s(\vec{v}_s) \frac{\partial}{\partial \vec{g}} \frac{1}{g} [\frac{1}{2} \ln \frac{1}{\gamma \eta} - \frac{1}{2}] . \qquad (A10)$$

The derivative, $\frac{\partial}{\partial \vec{g}}$, acting on η produces nonlogarithm terms which we neglect, therefore the derivative acts only on $\frac{1}{g}$.

Summing the impact and wave contributions gives a net logarithm argument which is independent of b_o:

$$\frac{1}{2} \ln \frac{1}{\gamma \xi} + \frac{1}{2} \ln \frac{1}{\gamma \eta} = \ln \Lambda,$$

where

$$\Lambda = \frac{2 \mu_s g^2 V_\perp^{1/2}}{\gamma \, ee_s \omega_p} . \qquad (A11)$$

Thus the unified theory vector and tensor collisional velocity changes are:

$$\frac{\langle \Delta \vec{v} \rangle_s}{\Delta t} = K \frac{m_e + m_s}{m_s} \int d\vec{v}_s f_s(\vec{v}_s) \frac{\vec{g}}{g^3} [\ln \Lambda - \frac{1}{2}], \qquad (A12)$$

$$\frac{\langle \Delta \vec{v} \Delta \vec{v} \rangle_s}{\Delta t} = K \int d\vec{v}_s \frac{f_s(\vec{v}_s)}{g} [(\hat{I}\hat{I} - \hat{g}\hat{g}) \ln \Lambda - (\hat{I}\hat{I} - \hat{v}\hat{v})/2 + (3 \hat{g}\hat{g} - \hat{I}\hat{I})/2] . \qquad (A13)$$

This result, Eqs. A12 and A13, is valid when v is much greater than the thermal velocity. This restriction can be relaxed, however, the more general form is not needed here. In the following, we further develop the vector and tensor terms using the techniques of Shkarofsky et al. (1966).

The integration over $\vec{v}_s$ is carried out by expanding $f_s(\vec{v}_s)$ and the integrand in vector spherical harmonics. A convenient means of evaluating the

expansion is to employ the Rosenbluth potentials (Shkarofsky et al., 1966). To simplify the derivation we neglect the dependence of Λ on $\hat{v} \cdot \hat{v}_s$ and assume $f_s(\vec{v}_s)$, the distribution of scatterers is isotropic. Henceforth we retain only the dominant logarithm terms.

The results are:

$$\frac{1}{Y} \frac{\langle \Delta \vec{v} \rangle_s}{\Delta t} = - \frac{\vec{v}}{v^3} \frac{m_e + m_s}{m_s} I_0^o , \tag{A14}$$

$$\frac{1}{Y} \frac{\langle \Delta \vec{v} \Delta \vec{v} \rangle_s}{\Delta t} = \frac{\vec{I}\vec{I} - \hat{v}\hat{v}}{v} (I_0^o + J_{-1}^o)$$

$$+ (\frac{3 \hat{v}\hat{v} - \vec{I}\vec{I}}{3v}) (I_2^o + J_{-1}^o), \tag{A15}$$

$$Y = \frac{(4\pi)^2 e^2 e_s^2}{m_e^2} \ln \Lambda_o, \tag{A16}$$

$$\Lambda_o = \frac{2\mu_s v^3}{\gamma ee_s \omega_p}, \tag{A17}$$

$$I_n^o = \frac{1}{v^n} \int_0^v v_s^2 \, dv_s \, f_s^o(v_s) \, v_s^n \left(1 + \frac{\ln(\frac{1}{2} \frac{Vs}{v})}{\ln \Lambda_o}\right) \tag{A18}$$

$$J_n^o = \frac{1}{v^n} \int_v^\infty v_s^2 \, dv_s \, f_s^o(v_s) \, v_s^n \left(1 + \frac{\ln(\frac{1}{2} v_s^2/v^2)}{\ln \Lambda_o}\right). \tag{A19}$$

The collision operator, Eq. A1, can now be evaluated using Eqs. A14 and A15, and the following transformations:

$$\frac{\partial}{\partial \vec{v}} \cdot (\frac{\langle \Delta \vec{v} \rangle_s}{\Delta t} f_e) = \frac{1}{v^2} \frac{d}{dv} (v \vec{v} \cdot \frac{\langle \Delta \vec{v} \rangle_s}{\Delta t} f_e), \tag{A20}$$

$$\frac{\partial^2}{\partial \vec{v} \partial \vec{v}} : (\frac{\langle \Delta \vec{v} \Delta \vec{v} \rangle_s}{\Delta t} f_e) = - \frac{1}{v^2} \frac{d}{dv} (v \frac{\langle \Delta \vec{v} \cdot \Delta \vec{v} \rangle_s}{\Delta t} f_e)$$

$$+ \frac{1}{v^2} \frac{d}{dv} (\frac{1}{v} \frac{d}{dv} [v \vec{v}\vec{v} : \frac{\langle \Delta \vec{v} \Delta \vec{v} \rangle_s}{\Delta t} f_e]). \tag{A21}$$

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Combining the terms, we find:

$$\frac{\delta f}{\delta t}\Big)_{es} = -\frac{1}{v^2} \frac{d}{dv} \left(\frac{v}{m_e} \frac{dE}{dt} f_e\right)$$

$$+ \frac{1}{v^2} \frac{d}{dv} \left(\frac{1}{v} \frac{d}{dv} \left[v \, \vec{v}\vec{v} : \frac{1}{2} \frac{\langle \Delta\vec{v}\Delta\vec{v}\rangle_s}{\Delta t} f_e\right]\right),$$
(A22)

where $\frac{dE}{dt}$ is the energy loss rate of a test electron due to collisions with particles of specie s,

$$\frac{dE}{dt} = m_e \left[\vec{v} \cdot \frac{\langle \Delta\vec{v}\rangle_s}{\Delta t} + \frac{1}{2} \frac{\langle \Delta\vec{v}\cdot\Delta\vec{v}\rangle_s}{\Delta t}\right].$$
(A23)

Evaluation of the energy loss rate using Eqs. A14 and A15 gives:

$$\frac{v}{m_e} \frac{dE}{dt} = Y \left(-\frac{m_e}{m_s} I_0^0 + J_{-1}^0\right).$$
(A24)

This expression is a generalization of the fast test particle energy loss rate of Kihara and Aono (1963) since the full dependence of the integral on v, v_s, and $v \cdot v_s$ has been considered in the derivation. The cited work used the approximation of replacing g by v. The second term in the collision operator, Eq. A22, is:

$$\frac{1}{2} v \, \vec{v}\vec{v} : \frac{\langle \Delta\vec{v}\Delta\vec{v}\rangle_s}{\Delta t} = \frac{Y}{3} v^2 (I_2^0 + J_{-1}^0),$$
(A25)

thus the electron – s particle collision operator is given by:

$$\frac{\delta f}{\delta t}\Big)_{es} = \frac{1}{v^2} \frac{d}{dv} \left\{\frac{m_e}{m_s} I_0^0 Y f_e + \frac{v}{3} Y (I_2^0 + J_{-1}^0) \frac{df_e}{dv}\right\}.$$
(A26)

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This result is similar in form to Eq. 7-71b of Shkarofsky et al. (1966), differing only in that the present form contains the exact Coulomb logarithm.

In the course of the derivation we have made simplifications which amount to neglecting the non-logarithmic terms and we have assumed $v \gg$ the electron thermal velocity. These simplifications can be relaxed at any stage in the above derivation; also the case of anisotropic distribution functions can be included.

The argument of the logarithm in the final results Eqs. A16 and A17, is identical with the logarithm of Kihara and Aono (1963), but differs from the results of the later paper of Itikawa and Aono (1966) where arbitrary velocity v was allowed. In the latter, the logarithm is split into a term independent of v and a complicated function, G, containing the v dependence. At sufficiently high energies, a quantum mechanical evaluation of the collison operator is necesary. The quantum mechanical calculation of Kihara (1964) gives the logarithm argument:

$$mv^2/\hbar\omega_p .$$

The tedious evaluation required of the G function, prompted Swartz et al. (1971) to make an analytic fit to the energy loss rate $\frac{dE}{dt}$ for both the classical and quantum mechanical formulas, however, their analytic function best represents the classical formula with the logarithm argument of Eq. A17 at all energies and does not seem to have been fit to the quantum mechanical result.

It is useful to derive an alternate formula for the part of the electron-electron collision operator having Maxwellian scatterers. We begin with Eq. A22, with $s = e$, f_s = Maxwellian distribution, and dE/dt given by the general

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formula of Itikawa and Aono (1966) or the analytic formula of Swartz et al. (1971). Let

$$F_2(v) = v \; \vec{v}\vec{v} : \frac{1}{2} \frac{\langle \vec{\Delta v}\vec{\Delta v}\rangle}{\Delta t} \; s. \qquad (A27)$$

A simple equation is obtained for $F_2(v)$ from A22 when $f_e \rightarrow f_e{}^M$ (the Maxwell distribution function),

$$\frac{1}{v^2} \frac{d}{dv} \; [- \frac{v}{m_e} \frac{dE}{dt} f_e{}^M + \frac{1}{v} \frac{d}{dv} (F_2 \; f_e{}^M)] = 0,$$

since $F_2(v)$ is independent of the form of f_e.

The solution is given by:

$$F_2(v) = - \int_v^\infty v'^2 \; dv' \; \frac{1}{m_e} \frac{dE}{dt} \; e^{-\frac{m_e}{2kT_e} (v'^2 - v^2)}. \qquad (A28)$$

Thus the electron-electron collision operator for Maxwellian scatterers is given by:

$$\frac{\delta f}{\delta t}\Big)_{ee} = \frac{1}{v^2} \frac{d}{dv} \; \{F_2 \; (v) \; [\frac{m_e}{kT_e} f_e + \frac{1}{v} \frac{df_e}{dv}]\}, \qquad (A29)$$

and this formula is valid for arbitrary velocity v, since it contains the exact energy loss rate dE/dt. For tail distribution scatterers, Eq. A26 should be used.

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Figure 1

Figure2

Figure3

Figure4

Figure 5

Axis labels:
- X-axis: Energy (eV) — 0.00, 10.00, 20.00, 30.00, 40.00, 50.00
- Y-axis: Primary Volume Production Rate — 1.0E+00, 1.0E+01, 1.0E+02, 4.0E+02

Figure 6

Energy (eV)

Figure 7