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APPROACHES TO OPTIMIZATION OF SS/TDMA TIME SLOT ASSIGNMENT

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Abstract

Reduction techniques for traffic matrices are explored in some detail. These matrices arise in satellite switched time-division multiple access (SS/TDMA) techniques whereby switching of uplink and downlink beams is required to facilitate interconnectivity of beam zones. A traffic matrix is given to represent that traffic to be transmitted from n uplink beams to n downlink beams within a TDMA frame typically of 1 ms duration. The frame is divided into segments of time and during each segment a portion of the traffic is represented by a switching mode. This time slot assignment is characterized by a mode matrix in which there is not more than a single non-zero entry on each line (row or column) of the matrix. In this paper, investigation is confined to decomposition of an $n \times n$ traffic matrix by mode matrices with a requirement that the decomposition be 100 percent efficient or, equivalently, that the line(s) in the original traffic matrix whose sum is maximal (called critical line(s)) remain maximal as mode matrices are subtracted throughout the decomposition process. A method of decomposition of an $n \times n$ traffic matrix by mode matrices results in a number of steps that is bounded by $n^2 - 2n + 2$. It is shown that this upper bound exists for an $n \times n$ matrix wherein all the lines are maximal (called a quasi doubly stochastic (QDS) matrix) or for an $n \times n$ matrix that is completely arbitrary. That is, the fact that no method can exist with a lower upper bound is shown for both QDS and arbitrary matrices, in an elementary and straightforward manner. The upper bound of $n^2 - 2n + 2$ is itself of some interest. A least upper bound LUB(n) in line sum is established for this upper bound. Thus, an association of the maximum line sum of a matrix with the number of mode matrices required in its decomposition is made. A class ordering, LUB(n,k), is formed for QDS matrices which may be decomposed in k steps. These lower bounds are studied in relationship to methods of decomposition. A new method of decomposition is presented and simulation results will be given over some range of matrix size. It is anticipated that this range will be from n = 3 to n = 26. The simulation results will be compared to those which have been published as descriptive of expectations from earlier methods of decomposition. Random number generation is employed.

1. Introduction

"Satellite - switched, time division, multiple access (SS-TDMA) is a key technology needed for future communication satellites. SS-TDMA is an effective method of significantly increasing communication satellite channel capacity and improving satellite system flexibility." This statement begins a recent study done for NASA/Lewis Research Center on a spacecraft IF switch matrix by Ford Aerospace and Communications Corporation.[1]

As the demand for complex network equipment increases, both in satellite and large computer systems, the switching, multiplexing, and data concentrating design theory will depend more on matrix manipulation techniques than on pure circuit design. The 30/20 GHz ACTS (Advanced Communications Technology Satellite) program is among the first to have a high density TDMA switchboard in the sky capable of augmenting existing high density traffic trunking that links the major U.S. cities. The successful utilization of large segments of the network will depend on the efficient switching and reuse of communication channels as the traffic demand alters the reassignment of assets to higher density communication links.

It is the purpose of this paper to attack the problem of logically reducing the number of iterations that a matrix switch will make to most efficiently utilize the available assets by logically arriving at an optimal configuration in a minimum number of steps. We will thereby consider methodologies that are directed to the division of a matrix, representing, in each of its cells, a number of units of traffic to be switched, into a number of switching modes. Our concern will be confined to a square matrix and each cell will represent units of traffic to be transmitted from one zone (or beam) to another. A division of this nature is a requirement which arises in any system operating with a fixed TDMA frame structure with the frame representing an allocation of time to accommodate the traffic.

We shall minimize the duration within a frame that is required for passage of the specified traffic. This minimization corresponds identically to a maximization of transponder utilization. Upon satisfaction of this requirement, we shall seek to minimize the number of switchings required to divide the entirety of traffic into switching modes.

An optimal process for division of a square matrix, under the specified condition of efficient transmission, requires consideration of all possible sets of distinct combinations each of which may be made by selection of a single element of each line (row or column) of the matrix. The number of such combinations itself is a factorialization of the order of the matrix; thus, such a consideration rapidly ceases to be computationally feasible as the order of the matrix increases. However, under utilization of any particular method of division it is pertinent to discover the worst case that may occur in number of switching modes required. We shall begin an approach to this question by inquiring into the relation of the maximal line sum of a matrix to an upper bound on the number of switching modes entailed in its division.

2. Decomposition of a Matrix

As remarked, we will confine this consideration to decomposition of a nonnegative integer matrix and we will restrict the decomposition by requiring that any line of maximal sum remain maximal in the succession of residues following each step. This requirement corresponds to minimization of the time duration for transmission.

2.1 Definitions

Some terms need to be defined:

Line	a row or column of a matrix
Critical Line	a line of maximal sum
SDR	system of distinct representatives (For a square matrix of order n, this means a set of a non-zero elements such that one lies on each line of the matrix.)
QDS	quasi doubly stochastic (A doubly stochastic matrix has the sum of each row or column equal to 1. "Quasi" extends this equality to mutliplication by a scalar.)
Mode Matrix	a matrix having not more than a single non-zero element in each line (Some lines may thereby have all elements equal to zero.)

2.2 Reduction Bounds for Square Matrices

Two theorems are presented which will lead us to aspects of a generalized methodology of reduction.

Theorem

The upper bound (UB) in number of steps required to divide an arbitrary $n \times n$ matrix of nonnegative integers into mode matrices is $n^2 - 2n + 2$.

(a) The original matrix may be modified, by addition of dummy traffic, to form a QDS matrix with all critical lines unchanged.[2]

(b) The modified matrix is a sum of non-negative multiples of permutation matrices[3] (pg. 52).

(c) For the modified matrix (assumed in the following to contain no zero element), any schemata for reduction cannot exceed n^2 steps when a sequence of steps, each involving removal of an α multiple of a permutation having α as the smallest number in the elements it covers, is performed. (There are ONLY n^2 elements in the matrix.)

(d) The final step of (c) caused formation of a zero in n elements rather than 1 element; that is, $n - 1$ additional zeros were created.

(e) Each of those elements contained in the last reduction step of (c) lies within a permutation. All other elements of the matrix are contained in $n - 1$ row/column pairs crossing at distinct elements of that permutation. The removal of the last element contained in the row of each such crossing necessitated removal of the last element contained in the corresponding column of that crossing. Prior to the final step of (c), this removal in earlier steps caused removal of $n - 1$ column elements beyond those last row elements singly or multiply identifying each such step, since the residue matrix following each step is itself QDS. As a result, $n - 1$ additional zeros were created prior to the final step.

(f) From (d) and (e), $2(n - 1)$ additional zeros were created and the n^2 total of (c) must be modified by deleting them. We obtain $n^2 - 2(n - 1) = n^2 - 2n + 2$.

(g) Were the modified matrix assumed to contain any zero elements, the only instance in which an additional zero would not be created in (e) is that wherein there existed no "last element" in the row of a crossing. But in that case, the "n^2" of (f) is itself reduced by 1.

(h) Reduction of the original matrix parallels, step by step, reduction of the modified matrix. 1

Theorem

There exist matrices which are UB. This may be concluded through construction.

(a) Allow that construction will be of a QDS matrix. This does not subtract from the criterion of existence.

(b) For any $n \times n$ matrix form the following geometric sequence of $n^2 - 2n + 2$ elements: 1, 2, 4, 8, . . .

(c) Embed numbers equal to the members of this ascending sequence in the matrix, in a correspondent sequence in terms of number of elements as follows: 1, 1, 2, 2, 3, 3, . . ., n - 2, n - 2, n. Embedding from this sequence requires each embedded group to be contiguous on a line and such that a rotational whorl is formed as follows (for a 5 x 5 matrix):

.	2	2	.	5
3	.	.	2	5
3	1	.	2	5
3	.	1	.	5
.	3	3	3	5

We have shown the groups, by a notation indicating the number of members contained in each group, in the sequence above. We can now replace them by the elements in the geometric sequence as:

...	32	16	....	65536
64	...	....	8	32768
128	1	....	4	16384
256	...	2	....	8192
...	512	1024	2048	4096

(i) Each element in the sequence is greater than the sum of all prior to it.

(ii) Each element in the sequence except for the last group of n (in our case, 5) may be covered by a permutation containing no other member other than one of the last group.

(iii) Each element in the sequence of the last group may be covered by a permutation containing no other member of the sequence. A QDS matrix containing this sequence may be formed and, in fact, the sequence defines the QDS matrix containing it. It is clear that $n^2 - 2n + 2$ multiples of permutations are required to remove the sequence and that the construction performed is valid for any $n \times n$ matrix.

2.3 A Methodology for Reduction

A variety of methods for reduction of an arbitrary matrix have been proposed (2, 4). These methods do not follow a systematic point of view whereby successive steps address the characteristics of traffic still to be transmitted.

The method proposed encompasses a maximal reduction in each step performed. At the same time, it assures that the upper bound in steps is not exceeded. (The latter is assured by any method whose stepwise counterpart performed on any "dummied" matrix (cf. 2.2), which may be created from the matrix to be reduced, creates a new zero in the successive residues of that "dummied" matrix.)

A maximal reduction in a single step corresponds in intention to a maximal reduction in any sequence of 2 or more steps wherein an optimal solution is a maximal reduction in k steps such that following the kth step no residue exists or, equivalently, that after the $k - 1$ step the residue is a mode matrix. The recurrent problem of computational complexity is, however, incurred when a maximal reduction within a sequence in excess of that obtainable in a single step is attempted, for exhaustive, and self-defeating, consideration is required.

Algorithm: Matrix Decomposition Via Successive Maximal Single Step Reduction

For any square matrix, M, a super dummy matrix, M*, is formed as follows:

(a) Consider each element, $m_{ij} \epsilon M$, independently of all other cells of M and add sufficient traffic to it, forming m^*_{ij}, such that, were it to be so modified in M, it would exist on a critical line. When all cells of M have been addressed in this manner, M* has been formed.

(b) Choose an SDR in M* is accordance with the methodology of Appendix A. An SDR may always be chosen in M* since each element is greater than or equal to the corresponding cell in any QDS matrix that may be formed by the addition of dummy traffic to M.

(c) Form a mode matrix, D, as follows:

(i) Equate all elements to zero in D except for those corresponding to the chosen SDR in M*.

(ii) Determine λ where λ is equal to the smallest element of the chosen SDR in M*. (The maximal reduction in those elements of M, corresponding to the SDR taken on M*, is equal to λ.)

(iii) For each element of the SDR in M*, reduce the corresponding element in M having traffic m_{ij}, by the lesser of m_{ij} and λ and enter the amount of reduction obtained in the corresponding cell of the mode matrix, D.

(d) Upon removal of the traffic denoted in the mode matrix, D, formed in (c), a residue of M remains to which the procedure beginning at (a) is applied until that residue is itself a mode matrix. An example of this procedure is given in Appendix C.

Simulations of this algorithm will be presented covering square matrices from order 3 through order 26. Both QDS matrices over a range of line sums, and arbitrary matrices, over a range of maximal line sums, will be included in these simulations. A critical issue is the degree of computational complexity that implementation of this algorithm requires. The variation, such as occurs, in results obtained by utilizing the alternate methods of SDR choice described in Appendix A, will be presented as will the variation in degree of computational complexity.

3. Least Upper Bound of a Matrix

As we have earlier remarked in 2.2, an upper bound (UB) in the number of steps required for decomposition of an $n \times n$ matrix is $n^2 - 2n + 2$. We are interested here in discovering a least upper bound for n, LUB(n), which we define as follows:

LUB(n) (least upper bound for n): For a square matrix of any order there is a maximal line sum. When that line sum is of an amount such that there exists an $n \times n$ matrix, M, that will require $n^2 - 2n + 2$ steps for decomposition, then M is at an upper bound (UB). When that line sum is at a minimum over all $n \times n$ matrices for this to be true, the line sum is a least upper bound, LUB(n), for a square matrix of order n. That is, any $n \times n$ matrix which has a maximal line sum that is less than LUB(n) may be decomposed in fewer than $n^2 - 2n + 2$ steps.

Existence of an LUB(n) is clear by definition of decomposition. Since any $n \times n$ matrix may be embedded in a matrix of order larger than n and that larger matrix still require the same minimum in number of steps for decomposition as the embedded matrix, LUB(n) monotonically increases as n increases. There exists an $n \times n$ matrix each line of which is LUB(n). In the sequel, we will confine our discussion to QDS matrices since, by cause of the latter, any statement regarding LUB(n) for a QDS matrix will be true a fortiori for an $n \times n$ matrix that is not QDS.

We will call an $n \times n$ matrix which is QDS and whose lines are LUB(n) a "fat" matrix.

Fat Matrix Theorem (Wade)

A "fat" matrix for any $n > 5$ is unique within row or column rearrangement, a similarity transformation, and there is an expression in closed form determinable from n alone which specifies LUB(n) for all n.

Proof (1) Any QDS matrix may be expressed as a sum of nonnegative multiples of permutations. Thus, any QDS matrix may be decomposed by a sequence of steps each of which removes a multiple of a permutation and creates at least one new zero element.

(2) For a UB matrix the decomposition sequence of (1) is of length $n^2 - 2n + 2$.

(3) There exists a set of $n^2 - 2n + 2$ elements which may be employed in composition such that addition of a multiple of a permutation removes a single zero from the set. This is simply the compositional counterpart to (2). Further, the multiples of permutations employed are not in summation equal to the sum of a different set of multiples of permutations lesser in number by definition of UB invoked in (2).

(4) From the compositional set of (3), a sequence may be constructed such that for each succeeding element there exists a permutation containing it and containing no prior element of the sequence.

(5) A sequence $n^2 - 2n + 2$ in length, adhering to the criterion of (4), requires each row (column) but one

to contain $n - 2$ elements of the sequence, and a single row (column) to contain n elements of the sequence.

(6) The sequence of elements for $n > 5$ must be constructed so that the sum over all elements is maximal when, for each, the number of prior elements of the sequence which may reside in permutations containing it is taken. This maximization of the number of elements which may be included in permutations containing later elements of the sequence (each counted once for each later element which may be contained in the same permutation) maximizes the combinations possible over all elements of the sequence. Such maximization is equivalent to a construction which minimizes the sum over all elements, of the number of prior elements of the sequence on the column containing each element under consideration, when the construction is expressed in terms of a succession of rows. The latter is required for a minimal construct, described in (9) below, to be performed for n greater than 5 and duplicates any alternate construct to achieve mimimality for n less than or equal to 5.

(7) The maximization of combinations in (6) necessitates that any construction be within row rearrangement of the matrix illustrative of the spiral construct, described in 2.2.

(8) For a minimal construct yielding LUB(n), the construct of (6) which maximizes combinations must be arranged so that over all maximizations that choice is made which minimizes combinations as early as possible in the sequence. The latter is equivalent to a particular arrangement of the rows of a matrix illustrative of the spiral construct of 2.2. With n elements of the sequence on the last row (column), the sequence of rows (columns) preceding the last must be ordered so that the elements of each succeeding row (column) find a maximal number of prior elements in the columns (rows) containing them.

(9) For a minimal construct, each element of the sequence is one greater than the largest sum of all those prior elements each of which may reside in a permutation containing:

(a) the element under consideration,
(b) no smaller prior element, and
(c) no element already included in the sum.

(10) The LUB for n may now be determined by sequential computation of the elements in the $n^2 - 2n + 2$ sequence and this computation depends on n alone. The sum of the elements in that row (column) containing n members of the sequence is LUB(n).

(11) The elements of a "fat" matrix (a_{ij}) of order n are obtained by a recursive formulation.

$$i + j \geq n + 3: \; a_{ij}(i = n) = 1$$

$$a_{ij}(i \neq n) = \sum_{j'} a_{i+1,j'} \begin{Bmatrix} i + 1 + j' \neq \\ n + 1,\ n + 2 \end{Bmatrix}$$

$$i + j < n: \; a_{ij} = \sum_{j'} a_{i+1,j'} (j' = j \text{ } \underline{\text{or}} \text{ } j' + i \geq n + 2)$$

$$i + j = n: \; a_{ij} = \sum_{i'j'} a_{i'j'} + 1$$

$$\begin{Bmatrix} i' > i,\ j' > j,\ i' + j' \neq n + 2 \\ \underline{\text{or}}\ i' + j' \leq n,\ i' \geq i + 2 \end{Bmatrix}$$

$$a_{i,n} = \sum_{i'j'} a_{i'j'} + 1 (i' > 1,\ i' + j' \leq n)$$

$$\text{LUB}(n) = \sum_{j} a_{1,j}$$

All elements a_{ij} for $i + j = n + 1$ or $i + j = n + 2$, which have not been determined by this recursive formulation, complete the matrix through adherence to the QDS characteristic. 1

<u>Corollary</u> A fat matrix contains no zero.

<u>Proof:</u> Any element of the matrix that is not a member of the sequence from which it was constructed lies within at least one of those permutations forming any decomposition set. For any minimal construct, there are $n - 2$ elements of the sequence on each of $n - 1$ rows (columns). The remaining 2 on each such row (column) are necessarily so contained. 1

<u>Corollary</u> A fat matrix of order other than 5 is unique within rearrangement.

<u>Proof:</u> We need only show uniqueness for $n < 5$ and begin by noting that the first 5 steps of the fat matrix theorem are true for all n. For $n \leq 5$, the only alternate construct satisfying (4) and (5) is that in which every row (column) containing $n - 2$ elements of the sequence, finds, for each such element, a single element not of the sequence on the same column (row). For $n \leq 5$, a minimal construct yields LUB(n). For $n < 5$, a minimal construct yields a fat matrix identical within rearrangement to that obtained through minimization utilizing the spiral construct. 1

<u>Corollary</u> A fat matrix belongs to the class of primitive matrices.

<u>Proof:</u> A fat matrix is fully indecomposable, and there exists no collection of multiples of permutations of number less than $n^2 - 2n + 2$ which in summation are equal to it when it is of order n. The latter necessitates an index of imprimitivity of 1 (7 (Chap. II)). (The reference is recommended for further considerations.) 1

For $n = 6$, the fat matrix, unique within rearrangement (a similarity transformation) is:

119	152	710	287	123	119	(line sum = 1,510)
119	152	710	287	123	119	
51	371	68	914	55	51	
17	830	17	17	612	17	
897	4	4	4	596	5	{LUB = 1,510 for Order 6.}
307	1	1	1	1	1199	

4. Maximal Line Sum Relationship to Minimal Decomposition

We have shown in 3 that a fat matrix of order n may be decomposed in $n^2 - 2n + 2$ steps, and in no fewer for it is LUB(n). Any $n \times n$ matrix that has a maximal line sum of magnitude less than LUB(n) may be decomposed in fewer steps.

We do not know how to decompose an arbitrary $n \times n$ matrix so as to minimize the number of steps employed in that decomposition except by an exhaustive, and for larger n, computationally untenable consideration. However, for any $n \times n$ matrix, there exists a decomposition sequence of no more than k steps whenever the maximal line sum of that matrix is less than or equal to some constant. We define that maximal line sum as LUB(n,k) and state that any $n \times n$ matrix having a maximal line sum $M <$ LUB(n,k) may be decomposed in fewer than k steps. We further define an LUB(n,k) matrix as a QDS matrix of order n and line sum, LUB(n,k), for which no decomposition sequence of less than k steps exists. We claim existence of an LUB(n,k) matrix for every n,k pair and, finally, define a decomposition sequence of k steps as k - COMP.

We now suggest a trail of consideration which gives strong support to the proposed methodology of decomposition of Section 2. For, had we an optimal method for all LUB(n,k) matrices, we would have worst case bounds for any arbitrary matrix which we sought to decompose.

The consideration begins with the following conjecture, for which we know of no counter example.

Hypothesis Number 1

A k - COMP sequence exists which may be ordered such that it yields creation of at least one additional zero at each step of decomposition, if the matrix is of the LUB(n,k) class.

We will consider a matrix as, for example:

33	..	..	20	10
..	34	24	..	5
..	24	36	3	..
20	..	3	40	..
10	5	..	..	48

which has been constructed from multiples (m_1, m_2, m_3, m_4, m_5, m_6) of permutations with m_1, m_2, . . ., m_6 = 1, 2, 4, 8, 16, 32. This matrix, which may be decomposed in 6 steps only when no first step creates a new zero, is not of the LUB(5,6) class. That is, we have constructed a matrix for which no 6 - COMP zero creating sequence exists and which is clearly not of the LUB(5,6) class. We need to show that for all n, k, where k represents the minimum number of steps required for decomposition, the absence of any k - COMP sequence for which a zero creating order exists, implies that the matrix, decomposible by application of that sequence, is not of the LUB(n,k) class.

We now look at the number of zeros in an LUB(n,k) matrix and note that the following lemmas follow directly from the hypothesis just stated.

Lemma For $k \geq n$, an LUB(n,k) matrix contains at most $n^2 - 2n + 2 - k$ zeros.

Lemma For $k \geq n$, an LUB(n,k) matrix containing $n^2 - 2n + 2 - k$ zeros may be decomposed with any sequence of k steps whenever that sequence may be ordered such that each step creates at least one new zero.

To obtain a k - COMP sequence for an LUB(n,k) matrix with $k \geq n$, when that matrix contains less than $n^2 - 2n + 2 - k$ zeros, the consideration made must encompass more than a successive zero creating order. Again, we must resort to having found no counter example to the following conjecture.

Hypothesis Number 2

For $k \geq n$, for every LUB(n,k) matrix there exists a k - COMP sequence for which each step of decomposition not only creates at least one new zero but also achieves a maximal reduction to the original or residue matrix to which it is applied.

There exist several options in forming a maximal single step reduction in a decomposition process. Three of these have been described in Appendix A. We expect that the number of steps required for decomposition of an LUB(n,k) matrix will closely follow k. Thereby, the proposed method of decomposition will give an upper bound for number of steps required to decompose a QDS matrix of a specified line sum and, thus, necessarily for an arbitrary $n \times n$ matrix with critical line at any value.

Simulation studies are under development for decomposition of $n \times n$ matrices utilizing the methodology of 2.3. These studies will encompass both QDS and arbitrary square matrices of orders 3 through 26 over a substantial range of critical line sums (all line sums are critical in a QDS matrix).

5. Conclusions

We have reviewed reduction schemata for arbitrary square matrices of nonnegative integers and proposed a general model for decomposition that, while adhering to the criterion of efficiency, approaches optimality through encompassing those characteristics noted in Section 4. In addition, we have shown in Section 3 a method to obtain a least upper bound for a QDS matrix of any order.

Our direction now is to find:

(a) A closed form for LUB(n,k) for every n,k pair, and

(b) Proofs for Hypothesis Number 1 and Hypothesis Number 2 in Section 4 noting their intrinsic mathematical value.

Any consideration of efficient decomposition must attend to the issues of complexity and boundedness over a range (of order or line sum) of interest. These issues have been addressed though substantial work remains to be performed. Our proposed method in Section 2 is polynomial-bounded since each of those processes embedded in its various forms are themselves polynomial-bounded. When n is the order of the matrix to which they are applied, the computational complexity encountered is less than n^5.

Appendix A: Checks and Choices for Systems of Distinct Representatives (SDR's)

A system of distinct representatives (SDR) is a group of n non-zero elements of a square matrix of order n no two of which are on the same line. Our interest here is in selecting an SDR having the property that its smallest member is the largest element of a square matrix that can be so characterized. We are further interested in methods of selection of an SDR which not only has this property but also has secondary properties concerning the sum or sequence of values of the members which it comprises.

A methodology of SDR selection exhibiting the noted primary property is described under Part 1: Checks, and methodologies of SDR selection exhibiting the secondary properties are described in Part 2: Choices. Matrices considered are those for which an SDR is known to exist.

Part 1: Checks

For a square matrix of order n, we require an algorithm to determine the largest element which is simultaneously the smallest element in any SDR which contains this element as one of its representatives. The determination here is, coincidentally, a method for solving the "bottleneck assignment problem" that minimizes computational complexity. We note in passing that there may be more than one such element by cause of equality; our intent here is simply, in that circumstance, to find any one of them.

The following sequence of steps is required:

Step 1:

Count the number of zeros, j, in the matrix. Determine x from the following formulae:

$$\text{For } j \leq n - 1,\ x(x + 1)/2 \geq (n - 1)^2,$$
$$\text{For } j > n - 1,\ x(x + 1)/2 \geq n(n - 1) - j.$$

Choose the least x for which the applicable formula is true. The following table lists $x(x + 1)/2$. The number obtained from $(n - 1)^2$ or $n(n - 1) - j$ picks the smallest entry in this table that is larger than or equal to it. When the choice is made, record x.

TABLE FOR X

x	x(x + 1)/2	x	x(x + 1)/2
35	630	17	153
34	595	16	136
33	561	15	120
32	528	14	105
31	496	13	91
30	465	12	78
29	435	11	66
28	406	10	55
27	378	9	45
26	351	8	36
25	325	7	28
24	300	6	21
23	276	5	15
22	253	4	10
21	231	3	6
20	210	2	3
19	190	1	1
18	171	0	0

Step 2:

List all of the elements of the matrix in order of value excluding any zeros. (Elements of the same value may be in any order.)

Step 3:

Establish a sequence of "mark-out" groups. This sequence is as follows: $x + k$, $x - 1$, $x - 2$, $x - 3$, . . ., 1 wherein $k = 0$ for $j > n - 1$ and $k = (n - 1) - j$ for $j \leq n - 1$. Each "mark-out" group in the established sequence represents the number of elements to be marked out from the ordered list formed in Step 2 beginning with the smallest element.

Step 4:

Peform "mark-outs" by groups following the "mark-out" sequence of Step 3. After each "mark-out" group has been so marked, check for existence of an SDR[2,3] among all matrix elements which have not been marked out. (We note that since an SDR contains no zero, existence is determined among those elements neither marked out nor equal to zero.) As long as existence of an SDR is found following each group of "mark-outs," continue the "mark-out" sequence established in Step 3. If an SDR is not found following a "mark-out" group, restore the members of this group in descending order of element size. Upon restoration of each member of the group except the last, check for existence of an SDR among all matrix elements including any restored. Continue the process of restoration and SDR checks until an SDR is found or until the last member of the group has been restored.

The largest element that is simultaneously the smallest element in any SDR which contains this element as one of its representatives is either:

A. the next element in the ordered list when the "mark-out" sequence was completed successfully, or
B. the element restored when an SDR check yielded success following that restoration, or

C. the first element of the last "mark-out" group when total restoration of that group occurred.

Part 2: Choices

There exist 3 methods to choose an SDR which contains the element found in Part 1 or an element equal to it. In each method this element, or one equal to it, will be the smallest element, or one of the smallest elements, of the SDR chosen.

Method 1: Maximum SDR

By a "maximum SDR," we mean an SDR which is of maximal sum confined to those elements remaining in the matrix upon the completion of Step 1 below.

Step 1:

Restore all elements in the matrix that are equal to the element found in Part 1.

Step 2:

Form a matrix containing as additional zeros all those elements which have remained marked out after restoration and which is otherwise the same as the original matrix.

Step 3:

Determine a maximum SDR in this matrix utilizing the procedure detailed in Bourgeois[5] but restricted to non-zero elements. This procedure (for a maximum SDR) is given in Appendix B. (The elements forming the independent set obtained positionally identify the elements in the original matrix which constitute a "maximum SDR.")

Method 2: Downside SDR

By a "downside SDR," we mean an SDR formed through a sequence of elements, chosen as in Part 1, each of which subtends the matrix from which the succeeding element in the sequence is selected, and which, for "downside," is selected through utilizing the following procedure:

Step 1:

Form the matrix of order $n - 1$ from the elements of the current matrix excluding those elements on the same row and the same column as the element found in Part 1. If $n - 1 = 1$, the final member of the "downside SDR" is the single element left.

Step 2:

Equate to zero those elements which were marked out in the current matrix and mapped into the matrix formed in Step 1.

Step 3:

Find an element in the matrix of order $n - 1$ employing the procedure of Part 1, and return to Step 1 with this matrix and the element found as the next element in the sequence of elements which form the SDR.

The elements constituting the SDR chosen are identical to the elements in the original matrix prior to any "mark-out."

Method 3: Upside SDR

By an "upside SDR," we mean an SDR formed through a sequence of elements, each of which subtends the matrix from which the succeeding element in the sequence is selected, and which, for "upside," is selected through utilizing the following procedure:

Step 1:

Form the matrix of order $n - 1$ from the elements of the current matrix excluding those elements on the same row and the same column as the element found in Part 1. If $n - 1 = 1$, the final member of the "upside SDR" is the single element left.

Step 2:

Equate to zero those elements which were marked out in the current matrix and mapped into the matrix formed in Step 1.

Step 3:

List all of the elements of the matrix just formed in order of value excluding any zeros. (Elements of the same value may be in any order.)

Step 4:

Check for existence of an SDR containing the highest element on the list formed in Step 3 (or Step 6 when return to Step 4 is made from Step 6). If an SDR is not found, zero this element out in the matrix, delete it from the list, and repeat this step until a listed element is found which is a member of some SDR.

Step 5:

Denote the current matrix used in Step 4 as of order n. Form the matrix of order $n - 1$ from the elements of the current matrix excluding the elements on the same row and the same column as the element found in Step 4. If $n - 1 = 1$, the final member of the "upside SDR" is the single element left.

Step 6:

List all of the elements of the matrix just formed in order of value excluding any zeros; elements of the same value may be in any order. Note that this list is the list of Step 3 or the list formed in the preceding iteration of Step 6 after removal from that list of those elements that had been included but were contained in the row and column noted in the preceding iteration of Step 5. Return to Step 4.

The elements constituting the SDR chosen are identical to the elements in the original matrix prior to any "mark-out." These elements consist of that element found in Part 1, the sequence found in iterations of Step 4 (if any), and the culminating element of Step 1 or Step 5.

Appendix B: Maximum SDR Determination

The following procedure is employed to determine a maximum SDR in a matrix in which certain elements are restricted from containment.

Step 1:

Mark each original zero as restricted by removing it from the matrix. That is, make each such element a blank.

Step 2:

Transform each non-blank element of the matrix by substracting it from the largest element of a matrix.

Step 3:

For each row of the modified matrix (a_{ij}), subtract the value of the smallest element from each element in the row. For each column of the resulting matrix, subtract the value of the smallest element from each element in the column.

Step 4:

Find a zero, Z, of this matrix. If there is no starred zero in its row or its column, star Z. Repeat for each zero of the matrix. Go to Step 5.

Step 5:

Cover every column containing a 0*. If all columns are covered, the starred zeros form the desired independent set; Exit. Otherwise, go to Step 6.

Step 6:

Choose a noncovered zero and prime it; then consider the row containing it. If there is no starred zero Z in this row, go to Step 7. If there is a starred zero Z in this row, cover this row and uncover the column of Z. Repeat until all zeros are covered. Go to Step 8.

Step 7:

There is a sequence of alternating starred and primed zeros constructed as follows: let Z_0 denote the uncovered 0'. Let Z_1 denote the 0* in Z_0's column (if any). Let Z_2 denote the 0' in Z_1's row. Continue in a similar way until the sequence stops at a 0', Z_{2k}, which has no 0* in its column. Unstar each starred zero of the sequence, and star each primed zero of the sequence. Erase all primes and uncover every line. Return to Step 5.

Step 8:

Let λ denote the smallest noncovered element of the matrix; it will be positive. Add λ to each covered row; then subtract λ from each uncovered column. Return to Step 6 without altering any asterisks, primes, or covered lines.

This procedure is attributable to Munkres[6] and was employed by him in obtaining solutions to assignment problems.

Appendix C: Example of Matrix Decomposition

The sequence of steps performed in the complete decomposition of an n x n traffic matrix for n = 5 is shown. We begin with the original matrix, form a super dummy matrix, choose an SDR in the super dummy matrix, establish the reduction to be performed on the original matrix, and form the residue matrix and mode matrix which together represent this reduction. We then reinitiate the process on the residue matrix as long as it is not, itself, a mode matrix. In the example, the choice of "downside SDR" has been made. (Refer to Appendix A for methods of SDR selection.)

For the example shown, there are eight (8) steps taken to complete the decomposition and, as a consequence, there are nine (9) mode matrices generated. The sum of the latter, each of which represents traffic transmitted during the switching mode its nonzero elements imply, is the original matrix.

74	13	85	89	11
39	19	4	140	23
15	110	73	63	27
27	18	14	29	97
91	78	49	12	45
			C	

Matrix to Reduce (Original)

135	74	146	89	72
126	114	112	140	131
60	155	118	63	72
114	113	122	29	227
149	136	107	12	103

Super Dummy Matrix

0	0	85	0	0
0	0	0	140	0
0	110	0	0	0
0	0	0	0	97
91	0	0	0	0

Mode Matrix

Notes: (1) C refers to a critical line. A line which is critical remains so throughout the decomposition and a line which becomes critical remains so throughout succeeding steps in the decomposition continuing thereafter.

(2) The smallest element in the SDR shown on the super dummy matrix is the amount each element, in the matrix from which the super dummy was formed (corresponding to that element indentified by the SDR), is reduced to the limit possible.

74	13	0	89	11
39	19	4	0	23
15	0	73	63	27
27	18	14	29	0
0	78	49	12	45
			C	

Matrix to Reduce (1st Residue)

80	19	6	89	17
77	84	57	0	110
30	15	88	63	42
65	83	67	29	87
9	87	58	12	54

Super Dummy Matrix

0	0	0	77	0
39	0	0	0	0
0	0	73	0	0
0	0	0	0	0
0	77	0	0	0

Mode Matrix

74	13	0	12	11
0	19	4	0	23
15	0	0	63	27
27	18	14	29	0
0	1	49	12	45
C			C	

Matrix to Reduce (2nd Residue)

74	19	6	12	17
0	84	53	0	33
15	11	11	63	37
27	46	42	29	10
0	10	58	12	54

Super Dummy Matrix

46	0	0	0	0
0	0	4	0	0
0	0	0	46	0
0	18	0	0	0
0	0	0	0	45

Mode Matrix

28	13	0	12	11	
0	19	0	0	23	
15	0	0	17	27	
27	0	14	29	0	C
0	1	49	12	0	
C			C		

Matrix to Reduce (3rd Residue)

28	19	6	12	17
0	47	7	0	32
15	11	7	17	36
27	0	14	29	0
0	9	56	12	8

Super Dummy Matrix

28	0	0	0	0
0	19	0	0	0
0	0	0	0	27
0	0	0	28	0
0	0	28	0	0

Mode Matrix

0	13	0	12	11	
0	0	0	0	23	
15	0	0	17	0	
27	0	14	1	0	C
0	1	21	12	0	
C			C		

Matrix to Reduce (4th Residue)

0	19	6	12	17
0	19	7	0	31
15	10	7	17	8
27	0	14	1	0
0	9	28	12	8

Super Dummy Matrix

0	13	0	0	0
0	0	0	0	17
0	0	0	17	0
17	0	0	0	0
0	0	17	0	0

Mode Matrix

0	0	0	12	11	
0	0	0	0	6	
15	0	0	0	0	
10	0	14	1	0	C
0	1	4	12	0	
C			C		

Matrix to Reduce (5th Residue)

0	2	2	12	13
0	19	7	0	14
15	10	7	0	8
10	0	14	1	0
0	9	11	12	8

Super Dummy Matrix

0	0	0	0	11
0	0	0	0	0
12	0	0	0	0
0	0	12	0	0
0	0	0	12	0

Mode Matrix

0	0	0	12	0	
0	0	0	0	6	
3	0	0	0	0	
10	0	2	1	0	C
0	1	4	0	0	
C			C		

Matrix to Reduce (6th Residue)

0	1	1	12	1
0	7	7	0	13
3	10	7	0	7
10	0	2	1	0
0	8	11	0	7

Super Dummy Matrix

0	0	0	10	0
0	0	0	0	6
0	0	0	0	0
10	0	0	0	0
0	0	4	0	0

Mode Matrix

0	0	0	2	0	
0	0	0	0	0	
3	0	0	0	0	C
0	0	2	1	0	C
0	1	0	0	0	
C			C		

Matrix to Reduce (7th Residue)

0	1	1	2	1
0	2	1	0	3
3	0	0	0	0
0	0	2	1	0
0	3	1	0	2

Super Dummy Matrix

0	0	0	2	0
0	0	0	0	0
2	0	0	0	0
0	0	2	0	0
0	1	0	0	0

Mode Matrix

0	0	0	0	0
0	0	0	0	0
1	0	0	0	0
0	0	0	1	0
0	0	0	0	0

This matrix is itself a mode matrix. Therefore, decomposition is complete.

Matrix to Reduce (8th Residue)

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16. Abstract

Reduction techniques for traffic matrices are explored in some detail. These matrices arise in satellite switched time-division multiple access (SS/TDMA) techniques whereby switching of uplink and downlink beams is required to facilitate interconnectivity of beam zones. A traffic matrix is given to represent that traffic to be transmitted from n uplink beams to n downlink beams within a TDMA frame typically of 1 ms duration. The frame is divided into segments of time and during each segment a portion of the traffic is represented by a switching mode. This time slot assignment is characterized by a mode matrix in which there is not more than a single non-zero entry on each line (row or column) of the matrix. In this paper, investigation is confined to decomposition of an $n \times n$ traffic matrix by mode matrices with a requirement that the decomposition be 100 percent efficient or, equivalently, that the line(s) in the original traffic matrix whose sum is maximal (called critical line(s)) remain maximal as mode matrices are subtracted throughout the decomposition process. A method of decomposition of an $n \times n$ traffic matrix by mode matrices results in a number of steps that is bounded by $n^2 - 2n + 2$. It is shown that this upper bound exists for an $n \times n$ matrix wherein all the lines are maximal (called a quasi doubly stochastic (QDS) matrix) or for an $n \times n$ matrix that is completely arbitrary. That is, the fact that no method can exist with a lower upper bound is shown for both QDS and arbitrary matrices, in an elementary and straightforward manner. The upper bound of $n^2 - 2n + 2$ is itself of some interest. A least upper bound LUB(n) in line sum is established for this upper bound. Thus, an association of the maximum line sum of a matrix with the number of mode matrices required in its decomposition is made. A class ordering, LUB(n,k), is formed for QDS matrices which may be decomposed in k steps. These lower bounds are studied in relationship to methods of decomposition. A new method of decomposition is presented and simulation results will be given over some range of matrix size. It is anticipated that this range will be from $n = 3$ to $n = 26$. The simulation results will be compared to those which have been published as descriptive of expectations from earlier methods of decomposition. Random number generation is employed.

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