NASA Technical Memorandum 85725

Effect of Structural Flexibility on the Design of Vibration-Isolating Mounts for Aircraft Engines

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FEBRUARY 1984

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NASA
National Aeronautics
and Space Administration

Scientific and Technical
Information Office

1984

SUMMARY

In a pair of reports published in 1938 and 1939, E. S. Taylor and K. A. Browne point out the advantages of designing vibration-isolating engine mounts to decouple the rotational and linear oscillations of the power plant and present a technique for providing decoupling when the mounting points are located behind the center of gravity of the power plant. These authors assume that the mount structure is rigid. The present analysis extends this analysis to include the effects of structural flexibility. The results of this analysis show that the effects of structural flexibility increase as the distance between the center of gravity of the power plant and the plane of the mounting pads increases. Equations are presented to allow the design of mount systems, and data are presented to illustrate the results for a range of design conditions.

INTRODUCTION

In a pair of reports published in 1938 and 1939 (refs. 1 and 2), E. S. Taylor and K. A. Browne discuss the problem of vibration isolation of aircraft power plants and present a technique for decoupling linear and rotational oscillations of a power plant when it is attached at mounting points located behind the center of gravity of the engine-propeller combination. An example of decoupling is a condition such that a vertical force applied at the center of gravity of the power plant produces only vertical motion and no pitching, and a pitching moment applied to the power plant produces only pitching and no vertical movement of the center of gravity. The decoupled condition is very desirable because it allows separate consideration of the static deflections and the natural frequencies involving translation and rotation, and because it decreases the response of many resonant modes to linear or rotational excitations. This mounting technique has been widely used on aircraft radial engines and on horizontally opposed engines used in general aviation airplanes.

The mounting technique described in references 1 and 2 consists of using rubber mounts with different stiffnesses along three mutually perpendicular axes. An equation is derived in reference 1 for the angle at which these mounts should be placed to obtain the desired decoupling of linear and rotational motions. This technique of arranging the mounts so that their axes intersect at a point on a principal axis of the body to be isolated has been given the name "focal isolation system" and has been used in applications other than aircraft engines. A general discussion of this technique is given in reference 3.

In the derivation of reference 1, the airplane structure to which the mounts are attached is assumed to be infinitely stiff. In practice, the structure has some flexibility. In the present report, the analysis is extended to include the effects of structural flexibility. In addition, the analysis gives some insight into the sensitivity of the decoupling to the parameters of the mount design.

Thorough discussions of excitation forces, power-plant vibration modes, and practical engine-mount design considerations are given in references 1 and 2. The present report is concerned solely with the effect of structural flexibility on decoupling. The reader should refer to the reference reports for a more general discussion of the engine-mount design problem.

The overall problem of vibration isolation of an aircraft power plant is a problem in dynamics which can be solved exactly only by accounting for the distribution of mass and stiffness throughout the structure as well as such factors as engine and propeller gyroscopic effects. The present analysis, however, treats a problem in statics. The results of this simple approach are useful in giving equations for the design of the vibration-isolating mounts. These equations are believed to be applicable in the usual situation in which the engine-mount structure itself is relatively light and is placed between the large mass of the engine and other heavy components of the airplane.

ANALYSIS

The analysis given herein follows the notation of reference 1 (where possible) to facilitate a comparison of the results. For completeness, the derivation is given starting from basic principles. The analysis, therefore, repeats some of the steps given in reference 1.

Consider the engine-mount system shown in figure 1. The engine is supported by m equally spaced mounting pads, and each pad has its axis intersecting the centerline of the engine at a common point and making an angle α with the centerline. (A list of symbols used in this paper appears after the references.) Each pad has spring rates k_A in the tangential direction of the mounting circle, k_B in the axial direction of the pad, and k_C in the radial direction normal to the axis of the pad. The positive directions of the mounting pad axes are shown in the figure.

For analysis, the vertical and pitching motions of a power plant are considered. The same analysis is applicable to lateral decoupling of engine vibrations (lateral movement and yawing rotation) by redefining axes. A vertical force applied at the center of gravity of the power plant causes deflections of each of the mounting pads and a vertical deflection and pitching rotation of the airplane structure as shown in figure 2. If the structural member or ring to which the mounting pads are attached is assumed to be rigid, this ring will undergo a translation Z and a rotation θ. Positive directions of these quantities are shown in figure 2. In the subsequent derivation, however, the force F_Z is taken as the force applied by the mounting pads to the engine. This force has a direction opposite from the force applied to the mount. The rotation of the mount, therefore, is assumed to be given by the equation

$$\left.\begin{aligned} \theta &= -\frac{F_Z a}{C_\theta} \\ &\text{or} \\ C_\theta &= -\frac{F_Z a}{\theta} \end{aligned}\right\} \qquad (1)$$

where a is the horizontal distance from the engine center of gravity to the mounting pad plane.

This definition of C_θ is arbitrary. The actual relation between the force F_Z and the resulting deflection θ depends on how the structure of the aircraft is restrained. The deflection θ results from both the moment of the force F_Z about the mount ring location and the force F_Z itself. An experimental determination of C_θ would, therefore, give a value dependent on the distance a as well as on the method of restraint of the structure. In practice, engineering judgment would be required to select a reasonable value of C_θ. For this analysis, however, the preceding formula is adequate, provided the value of C_θ is applied to conditions similar to those under which it was measured or calculated.

The following procedure is used to derive a relation between the mounting pad angle α, the ratio a/r (called the overhang ratio), and the stiffness parameters of the system to produce the desired decoupling. The rear face of the power plant and the mounting ring itself are assumed rigid. A parallel vertical displacement of the engine is assumed to occur, as required for decoupling, which produces equal vertical displacements of the front faces of each mounting pad. The total vertical force corresponding to this displacement causes a rotation of the mounting ring which produces a horizontal displacement of the rear face of each mounting pad proportional to its distance above or below the centerline. The accompanying vertical displacement of the mounting ring has no effect on decoupling and can be neglected in the analysis. The lack of effect of the vertical displacement of the mounting ring results from the fact that it causes no additional relative displacement of the front and rear faces of the mounting pads because the power plant moves only vertically and is free to move the same amount as the mounting ring if the decoupling condition is satisfied.

From the relative displacements of the front and rear faces of each mounting pad, the axial, radial, and tangential forces produced on the engine by each pad are calculated. The forces are resolved along the power-plant axes, allowing the total vertical force and the moment about the center of gravity of the power plant to be calculated. The vertical force is set equal to the force used previously in calculating the rotation of the mounting ring, and the moment is set equal to zero as required by the assumption of a parallel vertical displacement of the engine. From the resulting equations, the stiffness components of the mount may be determined and a relation may be obtained between the mounting pad angle α, the overhang ratio a/r, and the stiffness parameters of the system.

The tangential, axial, and radial components of deflection of the front face of each pad caused by a displacement ΔZ of the engine are

$$\left.\begin{aligned} \Delta_{tan} &= -\Delta Z \sin \delta \\ \Delta_{ax} &= -\Delta Z \cos \delta \sin \alpha \\ \Delta_{rad} &= \Delta Z \cos \delta \cos \alpha \end{aligned}\right\} \qquad (2)$$

The tangential, axial, and radial components of deflection of the rear face of each mounting pad caused by a rotation θ of the mounting ring are

$$\left.\begin{aligned} \Delta_{tan} &= 0 \\ \Delta_{ax} &= -\theta r \cos\delta \cos\alpha \\ \Delta_{rad} &= -\theta r \cos\delta \sin\alpha \end{aligned}\right\} \quad (3)$$

The components of force produced by each mounting pad result from the components of relative displacement of the front and rear faces. These relative displacements multiplied by the corresponding spring rates give the components of force produced by the pad. These components of force are

$$\left.\begin{aligned} F_{tan} &= k_A \Delta Z \sin\delta \\ F_{ax} &= k_B \Delta Z \cos\delta \sin\alpha - k_B \theta r \cos\delta \cos\alpha \\ F_{rad} &= -k_C \Delta Z \cos\delta \cos\alpha - k_C \theta r \cos\delta \sin\alpha \end{aligned}\right\} \quad (4)$$

These force components may be resolved along the X-, Y-, and Z-axes as follows:

$$\left.\begin{aligned} F_X &= F_{ax} \cos\alpha + F_{rad} \sin\alpha \\ F_Y &= F_{tan} \cos\delta - F_{ax} \sin\alpha \sin\delta + F_{rad} \cos\alpha \sin\delta \\ F_Z &= -F_{tan} \sin\delta - F_{ax} \sin\alpha \cos\delta + F_{rad} \cos\alpha \cos\delta \end{aligned}\right\} \quad (5)$$

Substituting equations (4) into equations (5) gives the following equations for the X, Y, and Z components of force produced by each mounting pad:

$$\left.\begin{aligned}
F_X &= k_B\,\Delta Z \cos\delta \sin\alpha \cos\alpha - k_B\theta r \cos\delta \cos^2\alpha \\
&\quad - k_C\,\Delta Z \cos\delta \sin\alpha \cos\alpha - k_C\theta r \cos\delta \sin^2\alpha \\
F_Y &= k_A\,\Delta Z \sin\delta \cos\delta - k_B\,\Delta Z \sin\delta \cos\delta \sin^2\alpha \\
&\quad + k_B\theta r \sin\delta \cos\delta \sin\alpha \cos\alpha - k_C\,\Delta Z \sin\delta \cos\delta \cos^2\alpha \\
&\quad - k_C\theta r \sin\delta \cos\delta \sin\alpha \cos\alpha \\
F_Z &= -k_A\,\Delta Z \sin^2\delta - k_B\,\Delta Z \cos^2\delta \sin^2\alpha + k_B\theta r \cos^2\delta \sin\alpha \cos\alpha \\
&\quad - k_C\,\Delta Z \cos^2\delta \cos^2\alpha - k_C\theta r \cos^2\delta \sin\alpha \cos\alpha
\end{aligned}\right\} \qquad (6)$$

The values ΔZ and θ are the same for all mounting pads. The sum of the forces for m mounting pads equally spaced in the mounting ring ($m > 2$) may be shown to be

$$\begin{aligned}
\sum F_X &= 0 \\
\sum F_Y &= 0 \qquad\qquad (7) \\
\sum F_Z &= -\frac{m}{2}\,\Delta Z\,(k_A + k_B \sin^2\alpha + k_C \cos^2\alpha) - \frac{m}{2}\,\theta r (k_C - k_B) \sin\alpha \cos\alpha
\end{aligned}$$

These results depend on the relations

$$\sum_{i=0}^{m-1} \sin\delta_m = \sum_{i=0}^{m-1} \cos\delta_m = 0$$

and

$$\sum_{i=0}^{m-1} \cos^2\delta_m = \frac{m}{2}$$

The values δ_m, where $\delta_m = \delta_o + i\,\frac{2\pi}{m}$ for $i = 0, 1, \ldots, m - 1$ and $m > 2$, are the angles of m equally spaced mounting pads around the mounting ring. A proof of these relations is given in the appendix.

If the value of θ from equation (1) is substituted in the expression for $\sum F_Z$ in equations (7), an equation relating the vertical force to the vertical displacement of the engine is obtained. (The values of F_Z and F_X henceforth are the total values, and the summation signs are omitted.)

$$F_Z = -\frac{m}{2} \Delta Z \ (k_A + k_B \sin^2\alpha + k_C \cos^2\alpha) + \frac{m}{2} \frac{F_Z ar}{C_\theta}(k_C - k_B) \sin \alpha \cos \alpha \tag{8}$$

or

$$F_Z = -\frac{\frac{m}{2} \Delta Z \ (k_A + k_B \sin^2\alpha + k_C \cos^2\alpha)}{1 - \frac{m}{2} \frac{ar}{C_\theta}(k_C - k_B) \sin \alpha \cos \alpha} \tag{9}$$

The pitching moment exerted by a mounting pad about the center of gravity of the engine is

$$F_X r \cos \delta + F_Z a$$

For decoupling, this total pitching moment must be zero,

$$F_X r \cos \delta + F_Z a = 0$$

or

$$a/r = -\frac{F_X \cos \delta}{F_Z} \tag{10}$$

The total value of $F_X \cos \delta$ for m equally spaced mounting pads is

$$F_X \cos \delta = \frac{m}{2} \Delta Z \ (k_B - k_C) \sin \alpha \cos \alpha - \frac{m}{2} \theta r(k_C \sin^2\alpha + k_B \cos^2\alpha) \tag{11}$$

Substituting equation (11) into equation (10) and using the value of θ in terms of F_Z from equation (1) gives the following expression for a/r:

$$a/r = -\frac{\frac{m}{2}\,\Delta Z\,(k_B - k_C)\sin\alpha\cos\alpha - \frac{m}{2}\,\frac{F_Z a r}{C_\theta}(k_C\sin^2\alpha + k_B\cos^2\alpha)}{F_Z} \tag{12}$$

Substituting the value for F_Z from equation (9) into equation (12) gives

$$a/r = \frac{\frac{m}{2}\,\Delta Z\,(k_B - k_C)\sin\alpha\cos\alpha\left[1 - \frac{m}{2}\,\frac{ar}{C_\theta}(k_C - k_B)\sin\alpha\cos\alpha\right]}{\frac{m}{2}\,\Delta Z\left(k_A + k_B\sin^2\alpha + k_C\cos^2\alpha\right)} - \frac{m}{2}\,\frac{ar}{C_\theta}(k_C\sin^2\alpha + k_B\cos^2\alpha) \tag{13}$$

Simplifying gives

$$a/r = \frac{k_C\left(\frac{k_B}{k_C} - 1\right)\sin\alpha\cos\alpha\left[1 - \frac{m}{2}\,\frac{ar}{C_\theta}\,k_C\left(1 - \frac{k_B}{k_C}\right)\sin\alpha\cos\alpha\right]}{k_C\left(\frac{k_A}{k_C} + \frac{k_B}{k_C}\sin^2\alpha + \cos^2\alpha\right)} - \frac{m}{2}\,\frac{ar}{C_\theta}\,k_C\left(\sin^2\alpha + \frac{k_B}{k_C}\cos^2\alpha\right) \tag{14}$$

Equation (14) gives a relation between the overhang ratio a/r, the mounting pad angle α, the stiffness parameters k_A, k_B, and k_C, and the nondimensional ratio

$$f_s = \frac{m}{2}\,\frac{ar}{C_\theta}\,k_C \tag{15}$$

This quantity is the structural flexibility parameter f_s and is related to the relative stiffness of the mount system and the engine-mount structure.

As shown in reference 1, the values k_A, k_B, and k_C may be given arbitrary values by combining a system of linkages with rubber mounting units. In many practical cases, however, particularly for smaller engines, the mounting pads are used without the linkages. In these cases the tangential and radial stiffnesses k_A and k_C are equal, whereas the axial stiffness is larger by a factor f. In practice, f may be of the order of 10 to 50. Under these conditions, equations (12) and (14) may be placed in a simpler form. Let

$$\left.\begin{aligned} k_A &= k_C = k \\ k_B &= fk_C = fk \end{aligned}\right\} \tag{16}$$

Substituting equations (16) and f_s into equation (9) gives, for the vertical mount stiffness,

$$F_Z/\Delta Z = -\frac{\frac{m}{2}k\left[(f-1)\sin^2\alpha + 2\right]}{1 + f_s(f-1)\sin\alpha\cos\alpha} \tag{17}$$

Substituting equations (16) and f_s into equation (14) gives, for the value of a/r,

$$a/r = \frac{(f-1)\sin\alpha\cos\alpha\left[1 + f_s(f-1)\sin\alpha\cos\alpha\right]}{(f-1)\sin^2\alpha + 2} - f_s\left[(f-1)\cos^2\alpha + 1\right] \tag{18}$$

Another quantity of interest from the standpoint of vibration isolation is the torsional stiffness of the mounting system for rotation of the power plant about the longitudinal axis. This quantity is given by the equation

$$C_\phi = \frac{T_X}{\phi} = mr^2k_A \tag{19}$$

This stiffness is independent of the angle α of the mounting pads and may be used to establish the tangential stiffness k_A independently of decoupling considerations.

As pointed out previously, the preceding analysis may also be applied to decoupling lateral motion and yawing rotation by redefining the axis system.

RESULTS

The relation between the overhang ratio a/r, the mounting pad angle α, and the stiffness parameters k_A, k_B, and k_C (eq. (14)) may be compared with the following equation (eq. (11) of ref. 1) for the case of an infinitely stiff mounting structure:

$$a/r = \frac{1}{2} \frac{\sin 2\alpha}{\dfrac{k_A + k_C}{k_B - k_C} + \sin^2\alpha} \tag{20}$$

By algebraic rearrangement of terms, this equation may be put in the form

$$a/r = \frac{k_C\left(\dfrac{k_B}{k_C} - 1\right) \sin \alpha \cos \alpha}{k_C\left(\dfrac{k_A}{k_C} + \dfrac{k_B}{k_C} \sin^2\alpha + \cos^2\alpha\right)} \tag{21}$$

This equation is in agreement with equation (14) if the structural stiffness C_θ goes to infinity. The effect of the flexible structure is contained in two additional terms involving the flexibility parameter

$$f_s = \frac{m}{2} \frac{ar}{C_\theta} k_C \tag{22}$$

The relation between the mounting pad angle α and the stiffness ratio f for various values of the overhang ratio a/r and the structural flexibility parameter f_s, as obtained from equation (18), are presented in figure 3. This figure shows the relation between the parameters required to satisfy the decoupling condition. In addition, the variation of vertical mount stiffness with mounting pad angle when the decoupling condition is satisfied is shown in figure 4, which also includes replots of some of the curves of figure 3. The vertical mount stiffness is given as the quantity

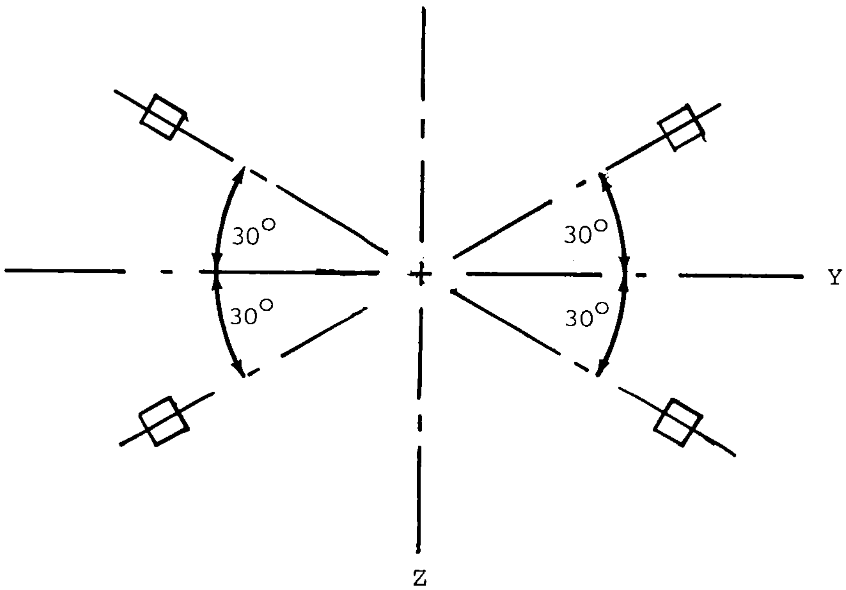
$$\frac{F_Z}{\frac{m}{2} k \, \Delta Z}$$

In order to determine the value of the stiffness ratio and the corresponding value of the vertical mount stiffness from figure 4, these values should be read on the abscissa for a given value of α on the ordinate. The stiffness presented is the ratio of force to the vertical deflection of the center of gravity of the power plant with respect to the center of the mount ring. Any additional compliance caused by vertical deflection of the airplane structure should be added to the deflection contributed by the mounting pads in order to calculate the overall stiffness of the mount. As noted previously, the vertical stiffness of the structure does not affect the decoupling calculations. Since no value was assigned to this stiffness, its effect cannot be included in the results plotted in figure 4.

Data are not presented for the case of unequal mounting pad stiffness parameters k_A and k_B, but similar results could be plotted for any desired values of these parameters by using equations (9) and (14).

The results presented in figures 3 and 4 are for the case of mounting pads equally spaced around the mount ring. Other arrangements of mounting pads may be studied by using equations (6) and (10) and adding the contribution of the individual mounting pads. The resulting equations for vertical mount stiffness and for the decoupling relation are found to have the same form as equations (17) and (18) but with different values for some of the constants.

For example, for the case of four mounting pads oriented ±30° from the horizontal (as shown in the accompanying sketch), the equations for $F_Z/\Delta Z$ and a/r corresponding to equations (17) and (18) are



$$F_Z/\Delta Z = \frac{-k[(f - 1)\sin^2\alpha + 4]}{1 + \frac{f_s}{2}(f - 1)\sin\alpha\cos\alpha} \qquad (23)$$

$$a/r = \frac{(f - 1)\sin\alpha\cos\alpha\left[1 + \frac{f_s}{2}(f - 1)\sin\alpha\cos\alpha\right]}{(f - 1)\sin^2\alpha + 4} - \frac{f_s}{2}[(f - 1)\cos^2\alpha + 1] \qquad (24)$$

where f_s is $\frac{2ark_C}{C_\theta}$ for the case of four mounting pads. These equations are for an arrangement of mounting pads typical of that used for the in-line, horizontally opposed engines often used in general aviation airplanes.

DISCUSSION

The results given in figure 3 show that for a given stiffness ratio f there are two values of the mounting pad angle α which will provide decoupling, provided the value of f is greater than some minimum value. For conditions of small overhang ratio, the value of f for decoupling is almost constant over a wide range of mounting pad angles. The decoupling property is, therefore, quite insensitive to mounting pad angle provided the stiffness ratio has the correct value. At larger values of overhang ratio, the range of suitable mounting pad angles is reduced.

The effects of structural flexibility, as shown by the curves for various values of the structural flexibility parameter f_s are small for small values of overhang ratio but increase rapidly with larger values. The required value of f increases rapidly with both overhang ratio and structural flexibility. Numerical values of the structural flexibility parameter may be interpreted by noting that with the pads oriented at $\alpha = 90°$ and an overhang ratio of 1.0, the angle through which the engine would pitch with respect to the mount ring as a result of a vertical force at its center of gravity would equal the angular deflection of the mount ring for $f_s = 1.00$. Normally, the stiffness of the mounting pads in their more flexible direction is expected to be much less than that of the mount structure. Therefore, values of f_s plotted are limited to 0.05. Furthermore, with large overhang ratios, mount flexibility prevents decoupling from being attained with practical values of f if the value of f_s is greater than about 0.05.

Though a wide range of mounting pad angles may be selected to provide decoupling, the data of figure 4 show that the vertical mount stiffness increases rapidly with increasing values of α. The designer, therefore, has the ability to select the vertical mount stiffness to provide the most effective vibration isolation and still meet the decoupling condition. As pointed out in reference 1, even greater flexibility in design is provided by mount systems in which the tangential and radial spring rates k_A and k_C are different.

A question may arise as to the method of calculating or measuring the mount stiffness parameter C_θ. If the stiffness is determined in a static test, this value varies depending on what point in the structure is considered fixed. A more rational way to determine C_θ would be to measure or calculate the response of the structure to a vibratory force applied at the location of the engine center of gravity in the frequency range of the engine vibrations. In this way the restraint on the structure, which arises mainly from the inertia of various structures and equipment aft of the mount structure, would be properly represented.

CONCLUDING REMARKS

In previous reports, a technique has been presented for the design of vibration-isolating mounts for a rear-mounted engine to decouple linear and rotational oscillations of the engine. This technique has been widely used. The present analysis extends this design procedure to account for the flexibility of the structure to which the mounts are attached.

Equations and curves are presented to allow the design of mount systems and to illustrate the results for a range of design conditions. The results of this analysis show that the structural flexibility has a relatively small effect when the center of gravity of the engine is close to the plane of the mounting points, but it becomes more important as the distance between the center of gravity and the plane of the mounting points increases.

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December 15, 1983

APPENDIX

PROOF OF RELATIONS INVOLVING ANGULAR POSITIONS OF MOUNTING PADS

This appendix contains proof of a theorem concerning the sum of sines and cosines of angles which divide a circle into m equal parts $(m > 2)$. The derivation of equations (7) of the main text depends on the relations

$$\sum_{i=0}^{m-1} \sin \delta_m = \sum_{i=0}^{m-1} \cos \delta_m = 0 \tag{A1}$$

$$\sum_{i=0}^{m-1} \sin^2\delta_m = \sum_{i=0}^{m-1} \cos^2\delta_m = \frac{m}{2} \tag{A2}$$

where

$$\delta_m = \delta_o + i\,\frac{2\pi}{m} \qquad (i = 0, 1, \ldots, m - 1 \quad \text{for} \quad m > 2)$$

Consider first

$$\sum_{i=0}^{m-1} \cos \delta_m = \sum_{i=0}^{m-1} \cos\left(\delta_o + i\,\frac{2\pi}{m}\right) = \sum_{i=0}^{m-1} \frac{\exp\left[j\left(\delta_o + i\,\frac{2\pi}{m}\right)\right] + \exp\left[-j\left(\delta_o + i\,\frac{2\pi}{m}\right)\right]}{2}$$

$$= \sum_{i=0}^{m-1} \frac{\exp(j\delta_o)}{2} \exp\left(ji\,\frac{2\pi}{m}\right) + \frac{\exp(-j\delta_o)}{2} \exp\left(-ji\,\frac{2\pi}{m}\right) \tag{A3}$$

where $j = \sqrt{-1}$. The terms $\exp\left(ji\,\frac{2\pi}{m}\right)$, when the values of i are substituted, constitute the terms of a geometric series. The sum of the terms of the series $1 + r + r^2 + \ldots + r^{m-1}$ is given by the formula $(1 - r^m)/(1 - r)$. The summation equation (A3) may, therefore, be written

$$\frac{\exp(j\delta_o)}{2}\left[\frac{1 - \exp(j2\pi)}{1 - \exp\left(\frac{j2\pi}{m}\right)}\right] + \frac{\exp(-j\delta_o)}{2}\left[\frac{1 - \exp(-j2\pi)}{1 - \exp\left(\frac{-j2\pi}{m}\right)}\right] \tag{A4}$$

But $\exp(j2\pi) = \cos 2\pi + j \sin 2\pi = 1$ and $\exp(-j2\pi) = \cos 2\pi - j \sin 2\pi = 1$. Hence, the numerator of each term goes to zero. The denominator remains finite for $m > 1$ in this case.

The same reasoning applies for

$$\sum_{i=0}^{m-1} \sin \delta_m$$

the only difference being in the coefficients of the terms which go to zero. Hence, equation (A1) applies for any $m > 1$.

Consider next

$$\sum_{i=0}^{m-1} \cos^2 \delta_m = \sum_{i=0}^{m-1} \cos^2\left(\delta_o + i\,\frac{2\pi}{m}\right) = \sum_{i=0}^{m-1} \left\{\frac{\exp\left[j\left(\delta_o + i\,\frac{2\pi}{m}\right)\right] + \exp\left[-j\left(\delta_o + i\,\frac{2\pi}{m}\right)\right]}{2}\right\}^2$$

$$= \sum_{i=0}^{m-1} \frac{\exp\left[2j\left(\delta_o + i\,\frac{2\pi}{m}\right)\right] + 2 + \exp\left[-2j\left(\delta_o + i\,\frac{2\pi}{m}\right)\right]}{4} \qquad \text{(A5)}$$

From the formula for the sum of a geometric series, the summation equation (A5) may be written

$$\frac{m}{2} + \frac{\exp(2j\delta_o)}{4}\left[\frac{1 - \exp(j4\pi)}{1 - \exp\left(\frac{j4\pi}{m}\right)}\right] + \frac{\exp(-2j\delta_o)}{4}\left[\frac{1 - \exp(-j4\pi)}{1 - \exp\left(\frac{-j4\pi}{m}\right)}\right] \qquad \text{(A6)}$$

But $\exp(j4\pi) = \cos 4\pi + j \sin 4\pi = 1$ and $\exp(-j4\pi) = \cos 4\pi - j \sin 4\pi = 1$. Hence, the numerators of the second and third terms go to zero. The denominators also go to zero for $m = 2$ but remain finite for $m > 2$. Similar reasoning applies for

$$\sum_{i=0}^{m-1} \sin^2 \delta_m$$

Hence, equation (A2) applies for any $m > 2$.

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SYMBOLS

a — horizontal distance from engine center of gravity to plane of mounting pads

C_θ — stiffness of engine mounting ring in pitch direction, $-aF_X/\theta$

C_ϕ — torsional stiffness of mounting system, T_X/ϕ

F_{tan}, F_{ax}, F_{rad} — components of force in tangential, axial, and radial directions applied to engine by a mounting pad

F_X, F_Y, F_Z — forces applied to engine by the mounting pads in the X, Y, and Z directions

f — ratio of axial stiffness of mounting pad to tangential or radial stiffness when $k_A = k_C = k$

f_s — structural flexibility parameter, $\frac{m}{2}\frac{ar}{C_\theta}k_C$

k — value of tangential or radial stiffness of mounting pad, $k = k_A = k_C$

k_A — tangential spring rate of mounting pad

k_B — axial spring rate of mounting pad

k_C — radial spring rate of mounting pad

m — number of mounting pads

r — radius to centerline of engine mounting pads

T_X — torque about longitudinal axis

X, Y, Z — translations along longitudinal, lateral, and vertical axes

α — angle between mounting pad axis and longitudinal axis

ΔZ — vertical displacement of engine relative to mount ring

$\Delta_{tan}, \Delta_{ax}, \Delta_{rad}$ — displacements of front or rear faces of mounting pad in tangential, axial, and radial directions

δ — angular position of mounting pad

θ — pitching rotation of engine-mount ring

ϕ — rotation of engine about longitudinal axis

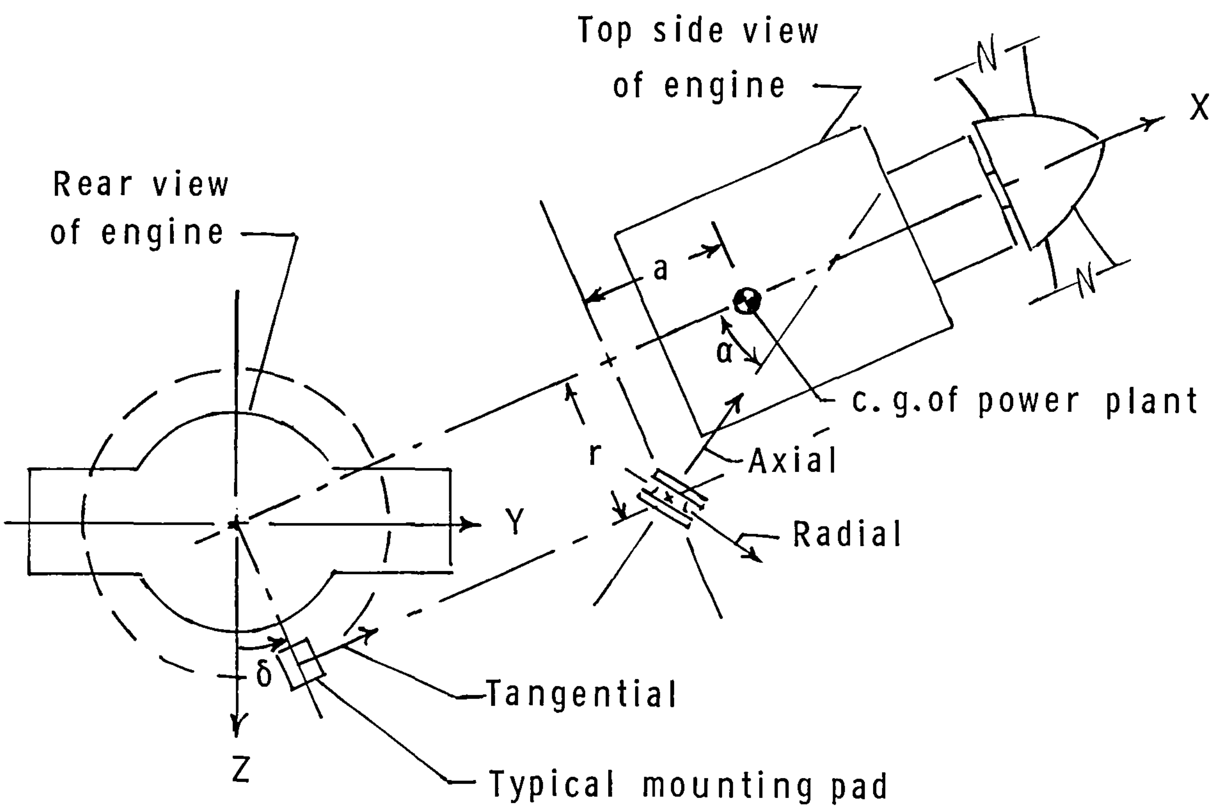


Figure 1.- Sketch of engine-mount system and definition of axes used in analysis.

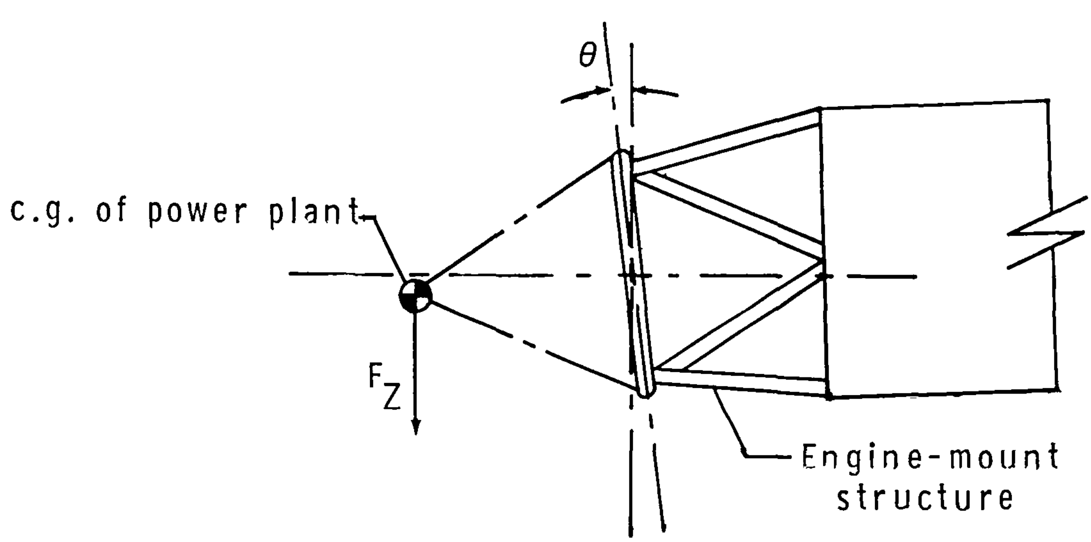
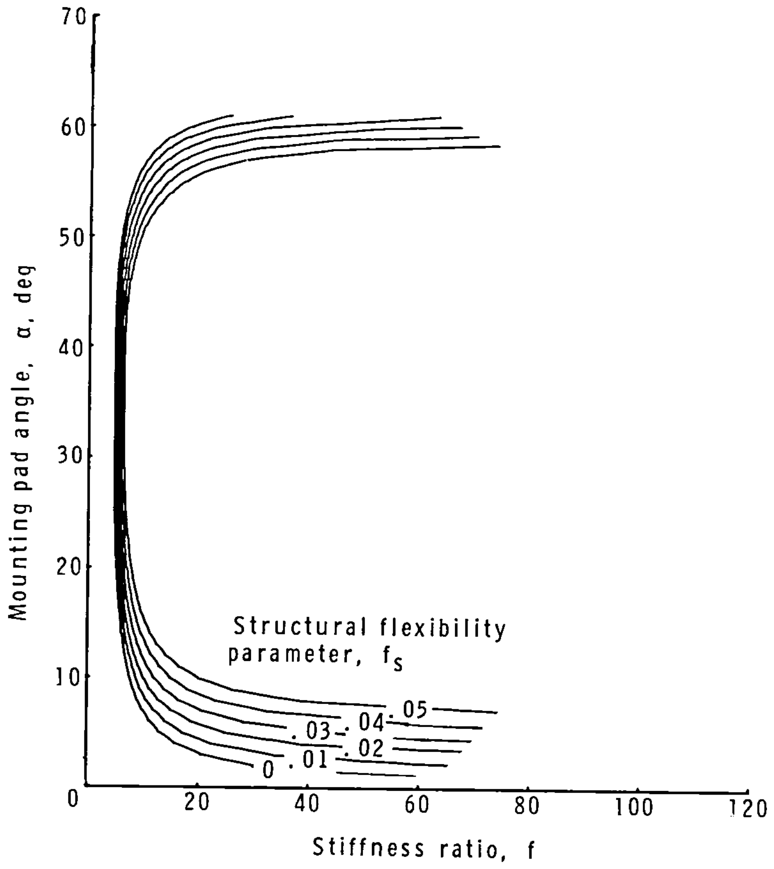
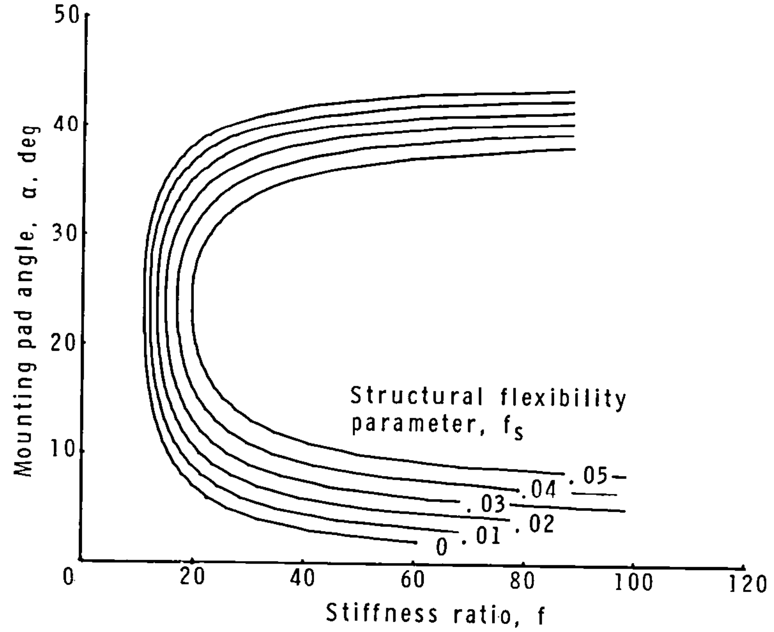


Figure 2.- Deflection of engine-mount structure caused by a force at the center of gravity of the power plant.

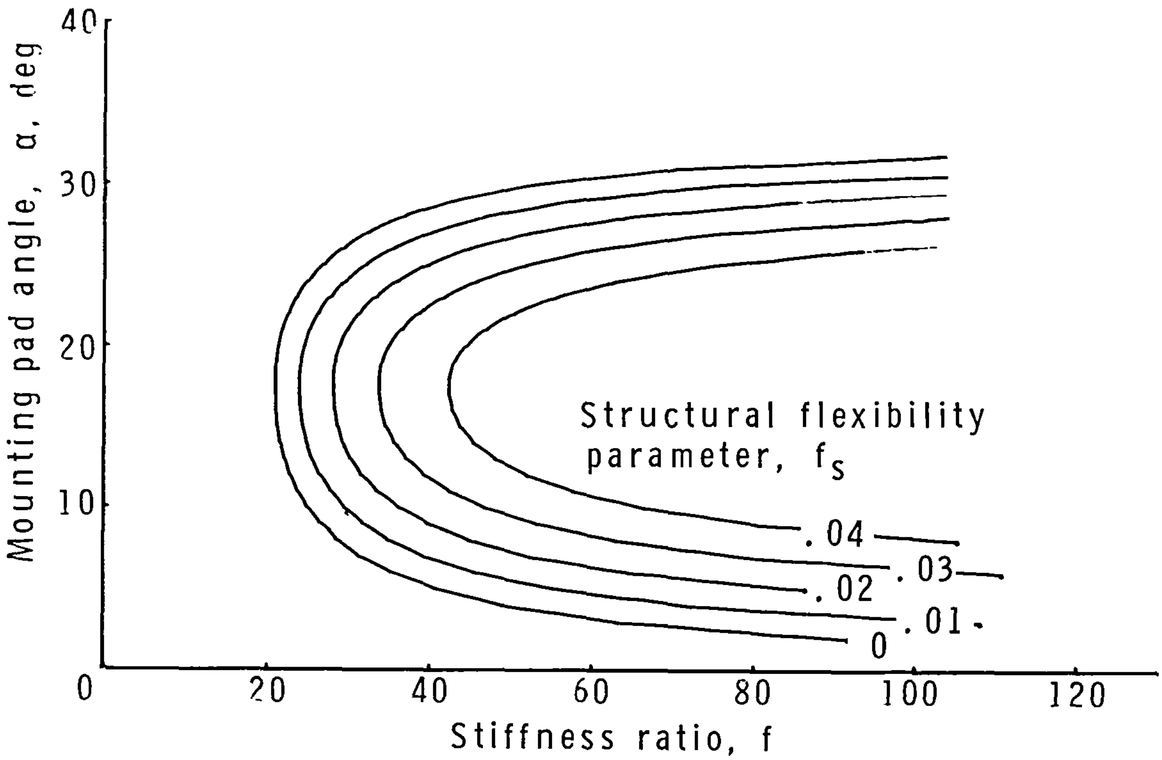


(a) a/r = 0.5.

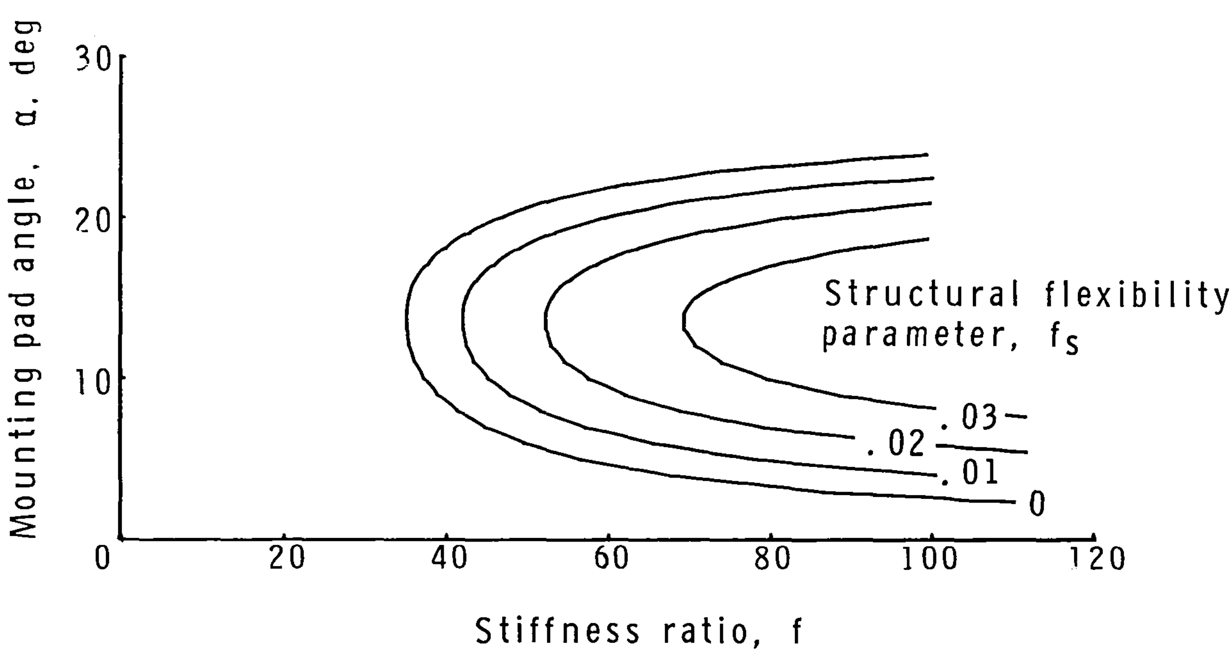


(b) a/r = 1.0.

Figure 3.- Variation of mounting pad angle α with stiffness ratio f for decoupling for various values of structural flexibility parameter f_s.

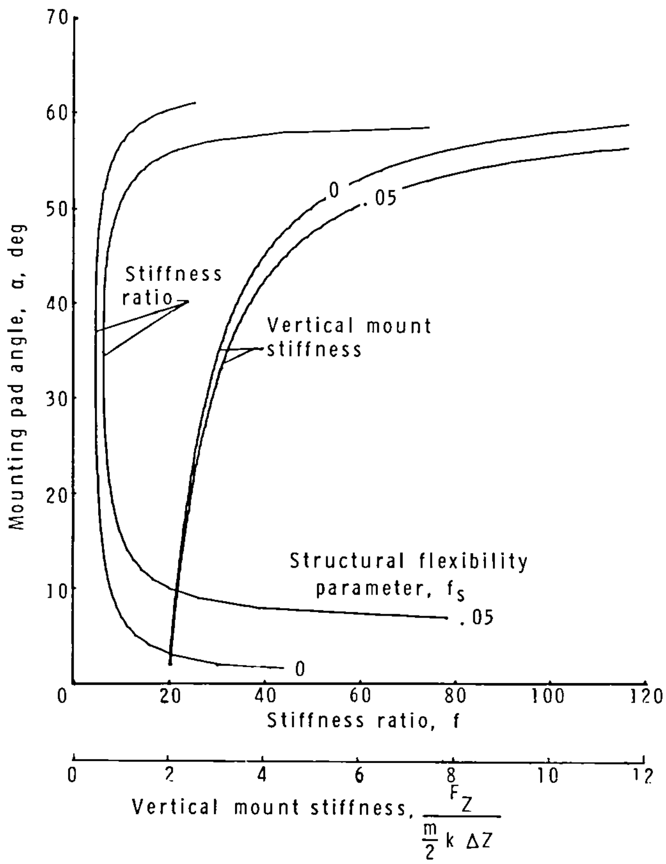


(c) a/r = 1.5.

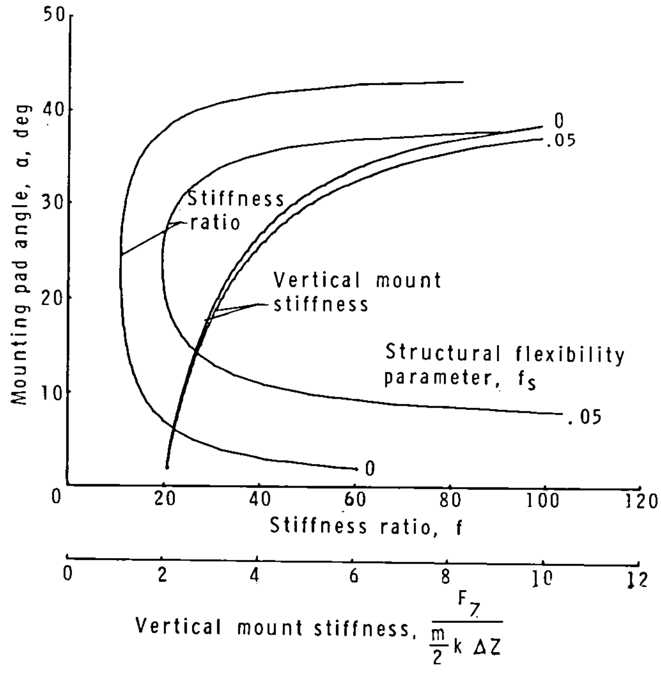


(d) a/r = 2.0.

Figure 3.- Concluded.

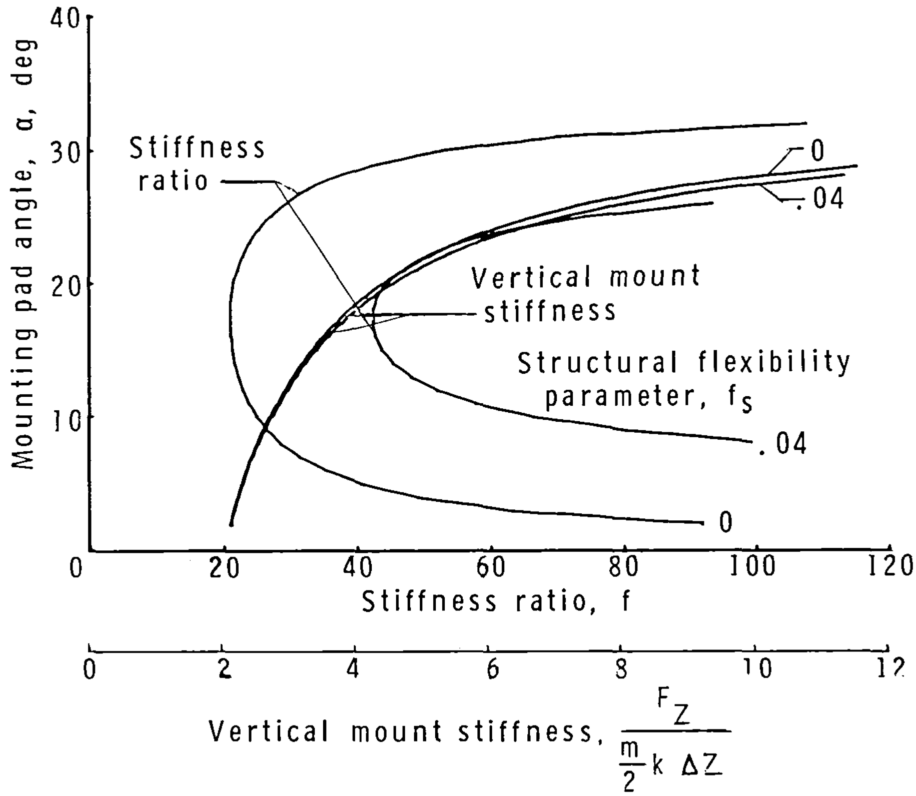


(a) a/r = 0.5.

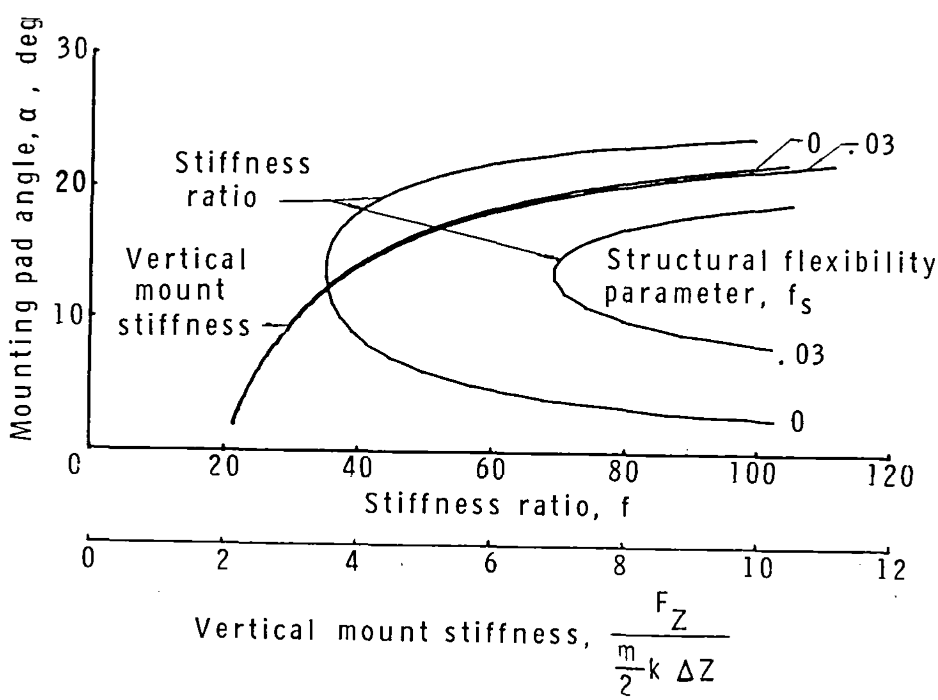


(b) a/r = 1.0.

Figure 4.- Variation of mounting pad angle α with stiffness ratio f for decoupling and corresponding values of vertical mount stiffness for two values of structural flexibility parameter f_s.



(c) a/r = 1.5.



(d) a/r = 2.0.

Figure 4.- Concluded.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-85725		

4. Title and Subtitle	5. Report Date
EFFECT OF STRUCTURAL FLEXIBILITY ON THE DESIGN OF VIBRATION-ISOLATING MOUNTS FOR AIRCRAFT ENGINES	February 1984
	6. Performing Organization Code 505-34-03-02
7. Author(s) William H. Phillips	8. Performing Organization Report No. L-15704
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665	10. Work Unit No.
	11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546	13. Type of Report and Period Covered Technical Memorandum
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

Previous analyses of the design of vibration-isolating mounts for a rear-mounted engine to decouple linear and rotational oscillations are extended to take into account flexibility of the engine-mount structure. Equations and curves are presented to allow the design of mount systems and to illustrate the results for a range of design conditions.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Aircraft design Aircraft propulsion Power plant vibration Vibration mode decoupling Focal isolation systems Vibration isolators	Unclassified - Unlimited Subject Category 39

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	22	A02

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