

VHSH-111-85001

NASA Technical Memorandum 85881

NASA-TM-85881 19840009069

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

Bifurcation Analysis of Aircraft Pitching Motions Near the Stability Boundary

W.H. Hui and Murray Tobak

January 1984

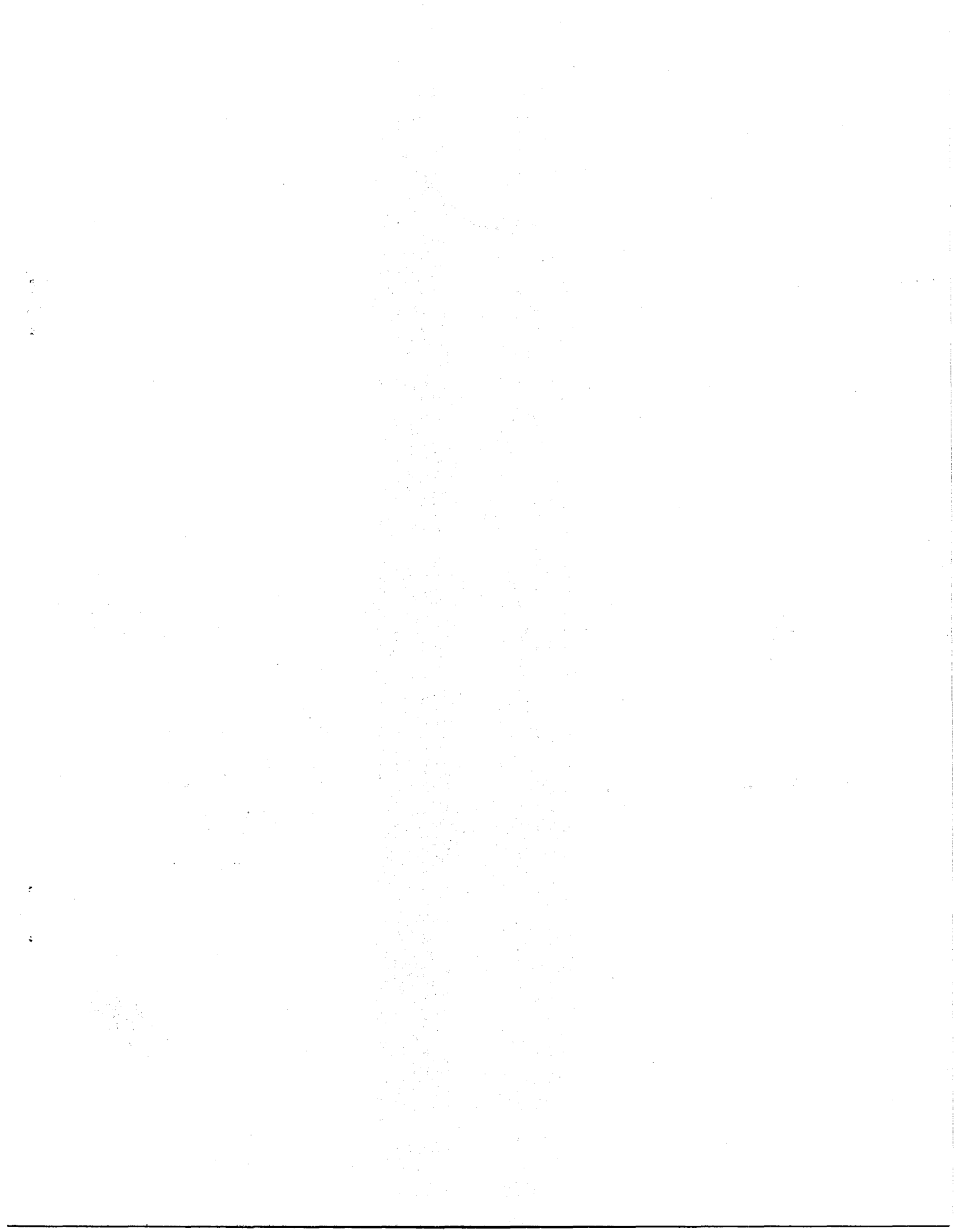
LIBRARY COPY

FEB 13 1984

LANGLEY RESEARCH CENTER
LIBRARY, NASA
HAMPTON, VIRGINIA

NASA

National Aeronautics and
Space Administration



ENTER:

6 1 1 RN/NASA-TM-85881
DISPLAY 06/6/1
84N17137**# ISSUE 8 PAGE 1097 CATEGORY 2 RPT#: NASA-TM-85881 A-9604
NAS 1.15:85881 CNT#: NAGW-130 84/01/00 33 PAGES UNCLASSIFIED
DOCUMENT

UTTL: Bifurcation analysis of aircraft pitching motions near the stability boundary

AUTH: A/HUI, W. H.; B/TOBAK, M. PAA: A/(Waterloo Univ.)

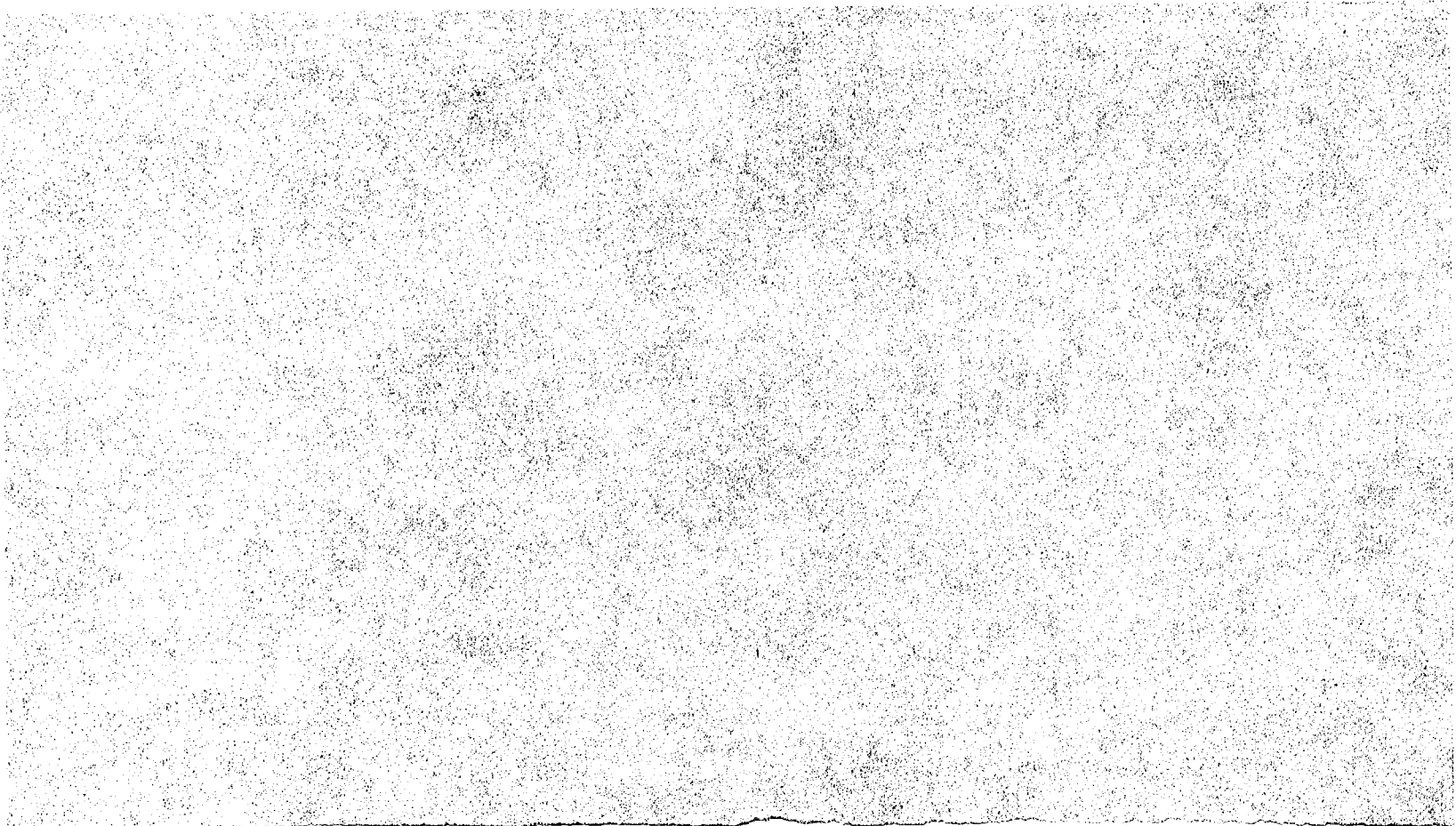
CORP: National Aeronautics and Space Administration. Ames Research Center, Moffett Field, Calif. AVAIL.NTIS SAP: HC A03/MF A01

Presented at the Conf. on Nonlinear Problems in Control and Fluid Dyn., Berkeley, Calif., 31 May - 10 Jun. 1983

MAJS: /*AIRCRAFT STABILITY/*ANGLE OF ATTACK/*BRANCHING (MATHEMATICS)/*EQUATIONS OF MOTION

MINS: / AIRFOILS/ DAMPING/ DEGREES OF FREEDOM/ DYNAMICS/ FLIGHT/ FLOW VELOCITY/ HYSTERESIS/ NONLINEARITY/ PERTURBATION THEORY/ PITCHING MOMENTS

ABA: Author



Bifurcation Analysis of Aircraft Pitching Motions Near the Stability Boundary

W. H. Hui, University of Waterloo, Waterloo, Ontario, Canada
Murray Tobak, Ames Research Center, Moffett Field, California

NASA

National Aeronautics and
Space Administration

Ames Research Center
Moffett Field, California 94035

N84-17137#

The following table shows the results of the survey conducted in the year 1998-1999. The data is presented in a tabular format, showing the number of respondents for each category. The categories are listed in the first column, and the number of respondents is listed in the second column.

Category	Number of Respondents
Male	120
Female	80
Age Group 18-25	150
Age Group 26-35	100
Age Group 36-45	90
Age Group 46-55	70
Age Group 56-65	60
Age Group 66-75	50
Age Group 76-85	40
Age Group 86-95	30
Age Group 96-105	20

The survey results indicate that the majority of respondents are male (120) and belong to the age group 18-25 (150). The number of respondents decreases as the age group increases, with the lowest number of respondents in the 96-105 age group (20).

BIFURCATION ANALYSIS OF AIRCRAFT PITCHING MOTIONS
NEAR THE STABILITY BOUNDARY

W. H. Hui

Professor of Applied Mathematics

University of Waterloo, Waterloo, Ontario, Canada

and

Murray Tobak

Research Scientist

NASA Ames Research Center, Moffett Field, CA 94035

ABSTRACT. Bifurcation theory is used to analyze the nonlinear dynamic stability characteristics of an aircraft subject to single-degree-of-freedom pitching-motion perturbations about a large mean angle of attack. The requisite aerodynamic information in the equations of motion can be represented in a form equivalent to the response to finite-amplitude pitching oscillations about the mean angle of attack. It is shown how this information can be deduced from the case of infinitesimal-amplitude oscillations. The bifurcation theory analysis reveals that when the mean angle of attack is increased beyond a critical value at which the aerodynamic damping vanishes, new solutions representing finite-amplitude periodic motions bifurcate from the previously stable steady motion. The sign of a simple criterion, cast in terms of aerodynamic properties, determines whether the bifurcating solutions are stable (supercritical) or unstable (subcritical). For flat-plate airfoils flying at supersonic/hypersonic speed, the bifurcation is subcritical,

implying either that exchanges of stability between steady and periodic motion are accompanied by hysteresis phenomena, or that potentially large aperiodic departures from steady motion may develop.

1. INTRODUCTION. Problems of aerodynamic stability of aircraft flying at small angles of attack have been studied extensively. With increasing angles of attack the problems become more complicated and typically involve nonlinear phenomena such as coupling between modes, amplitude and frequency effects, and hysteresis. The need for investigating stability characteristics at high angles of attack was clearly demonstrated by Orlik-Rückemann [1] in his survey paper which largely deals with experiments.

On the theoretical side, the greater part of an extensive body of work is based on the linearized theory, in which the unsteady flow is regarded as a small perturbation of some known steady flow (possibly nonlinear in, e.g., the angle of attack) that prevails under certain flight conditions. The question of the validity and limitations of such a linearized perturbation theory is of fundamental importance and yet has been investigated only rarely. One may argue that in principle, it should be possible to advance to higher and higher angles of attack σ by a series of linear perturbations, since the solution at each step should include a steady-state part which, when added to the previous steady-state solution, would provide the starting point for the next perturbation. This may well be true provided that at each step the steady motion is stable both statically and dynamically, and that the actual disturbances, e.g., the amplitude of oscillation, remain small. However, the linear perturbation procedure must eventually cease to be valid when the angle of attack exceeds a certain critical value σ_{cr} at which the steady motion is no longer stable.

In this paper we investigate the stability characteristics of an aircraft trimmed to a mean angle of attack σ_m near σ_{cr} at which the steady motion becomes unstable. Padfield [2] studied a similar problem, using the method of multiple scales, which is valid only for weakly nonlinear oscillations. We shall study the problem by means of bifurcation theory. This will allow us to draw on recent mathematical developments (e.g., [3]) that are particularly well suited to investigating fundamental questions in linear and nonlinear stability theory. A numerical scheme based on bifurcation theory was proposed earlier [4] for analyzing aircraft dynamic stability in a rather general framework. More recent work [5] demonstrates the considerable potential of bifurcation theory in flight dynamics studies, particularly toward establishing a method for the design of flight control systems to ensure protection against loss of control. On the other hand, while acknowledging the importance of the aerodynamic model in determining the aircraft stability characteristics, neither of these works contains an adequate assessment of the model requirements. The treatment of unsteady flow effects, in particular, receives no attention. In contrast, we shall focus on just this aspect of the problem at the expense of narrowing the scope of the motion analysis. Restricting the motion to a single-degree-of-freedom pitching oscillation will enable us to analyze a motion for which complete aerodynamic information is available, for certain aerodynamic shapes, in the form of exact solutions of the inviscid supersonic/hypersonic unsteady flow theory [6-10]. In this way it will be possible to establish a form revealing a precise analytical relationship between

the basic aerodynamic coefficients and the characteristics of the motion. For a more extensive account of the analysis presented here, see [11].

2. MATHEMATICAL FORMULATION.

A. The Coupled Dynamic/Aerodynamic System. We restrict attention to the single-degree-of-freedom pitching oscillations of an aircraft about a fixed trim angle. Before time zero, let the aircraft be in level, steady flight with its longitudinal axis inclined from the horizontal velocity vector by the fixed trim angle of attack σ_m . At time zero the aircraft is perturbed from its trim position, but during the subsequent motion the center of gravity continues to follow a rectilinear path at constant velocity V_∞ . The instantaneous angle of attack $\sigma(t)$ is again measured relative to the horizontal velocity vector, while the inclination of $\sigma(t)$ from the fixed trim angle σ_m , $\sigma(t) - \sigma_m$, will be designated $\xi(t)$. The equations of motion are

$$\frac{d}{dt} (\sigma - \sigma_m) = \dot{\xi} \quad (2.1a)$$

$$I \frac{d\dot{\xi}}{dt} = M(t) \quad (2.1b)$$

where I is the moment of inertia and $M(t)$ the instantaneous aerodynamic pitching moment, both referred to the center of gravity. We assume that the moment required to trim the aircraft at σ_m has been accounted for, so that $M(t)$ is a measure of the perturbation moment only.

Now we consider the equations governing the flow around the aircraft, assuming for simplicity that it is permissible to neglect viscous effects. The inviscid unsteady flow-field equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad (2.2a)$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} + \frac{\nabla p}{\rho} = 0 \quad (2.2b)$$

$$\frac{\partial}{\partial t} \frac{p}{\rho^\gamma} + \vec{v} \cdot \nabla \frac{p}{\rho^\gamma} = 0 \quad (2.2c)$$

where p , ρ , and \vec{v} are the pressure, the density, and the velocity vector, respectively, and γ is the ratio of specific heats of the medium. Denote the position vector originating from the center of gravity by \vec{r} and let the equation of the body surface be $B(\vec{r}, \xi(t)) = 0$, which, as noted, depends on the instantaneous displacement angle $\xi(t)$. The tangency condition at the body surface is then

$$\frac{\partial B}{\partial t} + \vec{v} \cdot \nabla B = 0 \quad \text{on } B = 0 \quad (2.3)$$

which also depends on $\xi(t)$. The far-field boundary conditions (relative to a coordinate system fixed in the aircraft) are

$$p = p_\infty, \quad \rho = \rho_\infty, \quad \vec{v} = \vec{V}_\infty \quad (2.4)$$

for subsonic flight, or the well-known shock jump conditions for supersonic flight.

It is clear from the above that the pitching motion of the aircraft $\xi(t)$ is coupled to the unsteady flow field (e.g., the pressure p) through (2.3), and, more directly, through $M(t)$ in (2.1a&b), which is determined from the instantaneous surface pressure through

$$\vec{M}(t) = \iint_{S:B=0} \vec{r} \times p \frac{\nabla B}{|\nabla B|} dS \quad (2.5)$$

The principal difficulty, however, is that the instantaneous surface pressure in (2.5) depends not only on the current state of the flow

field but also on its prior states. This means that past values of ξ figure in the determination of the current state of the flow field and $\vec{M}(t)$ is a functional, not a function, of $\xi(t)$. Thus, unless approximations are introduced, a determination of ξ starting from given initial conditions requires the simultaneous solution of the coupled equations (2.1-2.5).

Although the simultaneous solution of the coupled equations (2.1-2.5) in principle represents an exact approach to the problem of determining time-histories of maneuvers from given initial conditions, it is inevitably a difficult and costly approach (see the discussion of [12]). Approximate approaches leading to simpler, less costly computations are a practical necessity. To date, these computations have invoked the assumption of a slowly varying motion of known form (e.g., a harmonic motion) whose aerodynamic force and moment response at t could be calculated. This stratagem, in effect, uncouples the flow-field equations from the equations of motion. In a series of papers (see in particular [13]), Tobak and Schiff have shown how this stratagem can be rationalized to create mathematical models of the aerodynamic response at various levels of approximation of the dependence on the past motion. (See also the contribution of Tobak, Chapman, and Schiff in this collection.)

B. Uncoupling Near Neutral Dynamic Stability Boundary. Let us suppose that there is a mean angle of attack $\sigma_m = \sigma_{cr}$ at which the damping-in-pitch coefficient vanishes. Then, at and beyond this angle of attack, the steady motion of the aircraft will no longer be stable

so that, in response to a disturbance, the motion will seek a new stable state which usually will consist of a finite-amplitude periodic oscillation about the mean angle of attack. The changeover from a stable steady motion to a stable periodic motion is called a Hopf bifurcation (see [3, 14], also Sec. 4). Thus, in the vicinity of σ_{cr} , where the damping-in-pitch coefficient will be near zero, the consequent persistence of oscillatory motions will make it particularly appropriate to assume a periodic past motion about a mean position in calculating the aerodynamic pitching moment from the flow-field equations for use in (2.1). This assumption is consistent with the mathematical modeling approach of Tobak and Schiff [13] at the second level of approximation.

It will be convenient to write $M(t)$ in the form

$$M(t) = \frac{1}{2} \rho_{\infty} V_{\infty}^2 \bar{S} l [C_m(\xi, \dot{\xi}, \sigma_m) - C_m(0, 0, \sigma_m)] \quad (2.6)$$

The function $C_m(\xi, \dot{\xi}, \sigma_m)$ is the pitching-moment coefficient resulting from a finite-amplitude periodic pitching motion $\xi(t)$ about the center of gravity, and $C_m(0, 0, \sigma_m)$ is its steady-state value at the mean angle of attack σ_m . The function $C_m(\xi, \dot{\xi}, \sigma_m)$ depends on the instantaneous displacement angle $\xi(t)$, its rate of change $\dot{\xi}(t)$, and the mean angle of attack σ_m . It also depends, of course, on the location h of the center of gravity relative, say, to the wing leading edge, the flight Mach number M_{∞} , the ratio of the specific heats of the medium γ , and the aircraft shape. A great amount of work has been devoted to the theoretical determination of the function C_m for the case of pitching oscillations of infinitesimal amplitude. For example, for the cases of a wedge, a flat plate in supersonic/hypersonic flow [7, 8] and an airfoil

of arbitrary profile in the Newtonian limit [9], this function is available in exact analytical form for large, as well as small, mean angles of attack σ_m , and includes the critical angle σ_{cr} . On the other hand, little is known about the pitching-moment coefficient C_m when the amplitude of oscillation is finite, except for the special case of a slowly oscillating wedge [10]. In the next section we shall show how the function C_m for slow pitching oscillations of finite amplitude may be obtained from its behavior in the limit of oscillations of infinitesimal amplitude.

C. The Pitching-Moment Function for Slow Pitching Oscillations of Finite Amplitude. Consider a uniform flow \vec{V}_∞ past an aircraft that is undergoing a slow pitching oscillation of finite amplitude about a mean angle of attack σ_m . The instantaneous angular displacement from the mean position is measured by $\xi(t)$ so that $|\xi|_{\max}$ is the finite amplitude of the oscillation. It is required to calculate the unsteady pressure field, hence the form of the pitching moment $M(t)$ in (2.1). Let the cylindrical coordinates (r, ϕ, z) be such that the z -axis coincides with the lateral axis through the center of gravity. Let the equation of the body surface at the mean angle of attack σ_m be $\phi = \sigma_m + A(r, z)$. Then its equation at time t is

$$B(r, \phi, z, t) = A(r, z) + \sigma_m + \xi(t) - \phi = 0 \quad (2.7)$$

With velocity vector $\vec{v} = (v_r, v_\phi, v_z)$, the boundary condition (2.3) becomes

$$v_\phi = r \left(\dot{\xi} + v_r \frac{\partial A}{\partial r} + v_z \frac{\partial A}{\partial z} \right) \quad (2.8)$$

at $\phi = A(r, z) + \sigma_m + \xi(t)$. Equations (2.2), (2.4), and (2.8) complete the formulation for calculating the flow field in terms of $\xi(t)$. Now, for a long-established periodic motion of the aircraft, the aerodynamic response is also periodic. Furthermore, invoking the assumption of slow oscillations allows us to neglect terms of $O(\dot{\xi}^2, \ddot{\xi})$ and higher and to write

$$p = p_0 + \dot{\xi} p_1, \quad \rho = \rho_0 + \dot{\xi} \rho_1, \quad \vec{v} = \vec{v}_0 + \dot{\xi} \vec{v}_1 \quad (2.9)$$

where $()_0$ and $()_1$ are independent of $\dot{\xi}(t)$, but may depend on time through the function $\xi(t)$. Since the zeroth-order quantities $()_0$ represent the flow field when $\dot{\xi} = 0$, such a flow must be quasi-steady; i.e., the time variable t can only appear implicitly through the instantaneous displacement angle $\xi(t)$. Hence,

$$p_0 = p_0(\sigma_m + \xi(t), r, \phi, z), \quad \text{etc.} \quad (2.10)$$

To derive the mathematical problem for the first-order quantities $()_1$, we substitute (2.9) into (2.2), (2.4), and (2.8), use (2.10), and neglect terms of $O(\dot{\xi}^2, \ddot{\xi})$. Thus,

$$\nabla \cdot (\rho_0 \vec{v}_1 + \rho_1 \vec{v}_0) = - \frac{\partial \rho_0}{\partial \xi} \quad (2.11a)$$

$$\vec{v}_0 \cdot \nabla \vec{v}_1 + \vec{v}_1 \cdot \nabla \vec{v}_0 + \frac{1}{\rho_0} \left(\nabla p_1 - \frac{\rho_1}{\rho_0} \nabla p_0 \right) = - \frac{\partial \vec{v}_0}{\partial \xi} \quad (2.11b)$$

$$\vec{v}_0 \cdot \nabla \frac{p_1 - (\gamma p_0 / \rho_0) \rho_1}{\rho_0^\gamma} + \vec{v}_1 \cdot \nabla \frac{p_0}{\rho_0^\gamma} = - \frac{\partial}{\partial \xi} \frac{p_0}{\rho_0^\gamma} \quad (2.11c)$$

$$\vec{v}_1 = p_1 = \rho_1 = 0 \quad \text{as} \quad |\vec{r}| \rightarrow \infty \quad (2.11d)$$

and

$$v_{1\phi} = r \left(1 + v_{1r} \frac{\partial A}{\partial r} + v_{1z} \frac{\partial A}{\partial z} \right) \quad (2.11e)$$

at

$$\phi = A(r, z) + \sigma_m + \xi(t)$$

Since the time variable t appears in (2.11) only as a parameter through $\xi(t)$, the solution to (2.11) must be of the form

$$p_1 = p_1(\sigma_m + \xi(t), r, \phi, z), \quad \text{etc.} \quad (2.12)$$

The forms of p_0 and p_1 in (2.10) and (2.12) determine the form of the pressure p in (2.9) which, in turn, determines the form of the pitching-moment coefficient after the integrations indicated in (2.5) have been carried out. Thus

$$C_m(\xi, \dot{\xi}, \sigma_m) = f(\sigma_m + \xi) + \frac{\dot{\xi} \ell}{V_\infty} g(\sigma_m + \xi) \quad (2.13a)$$

In the case of oscillations of infinitesimal amplitude, (2.13a) reduces to

$$C_m = f(\sigma_m) + \xi f'(\sigma_m) + \frac{\dot{\xi} \ell}{V_\infty} g(\sigma_m) \quad (2.13b)$$

It is now clear that the functions $f'(\sigma_m)$ and $g(\sigma_m)$ are related to the stiffness derivative $S(\sigma_m)$ and the damping-in-pitch derivative $D(\sigma_m)$ at an angle of attack σ_m , as defined in classical aerodynamics, by

$$f'(\sigma_m) = -S(\sigma_m), \quad g(\sigma_m) = -D(\sigma_m) \quad (2.14)$$

Comparing (2.13a) and (2.13b), we conclude that knowing the stiffness and damping derivatives from the results of calculations for oscillations of

infinitesimal amplitude enables one to obtain immediately the pitching-moment coefficient C_m for the corresponding finite-amplitude case. This general conclusion is supported by the form of the exact solution for C_m for the wedge oscillating at large amplitude given in [10]. (To correct a misprint in [10], a term $hA_4[\cos(\theta - \theta_a) - 1]$ should be added to the right-hand side of Eq. (12b) in [10].)

Having determined an appropriate form of $M(t)$ for use in the inertial equations of motion (2.1), we now rewrite the equations introducing (2.13a) along with dimensionless time (i.e., characteristic lengths of travel) $\tau = V_\infty t/\ell$. (Note that we shall retain the symbol $(\dot{})$ to designate a time derivative. Henceforth, however, the derivative will be with respect to dimensionless time: $(\dot{}) \equiv d/d\tau$.) Let

$$\begin{aligned} F(\xi, \dot{\xi}, \sigma_m) &= \frac{M(\tau)}{I(V_\infty/\ell)^2} = \kappa[C_m(\xi, \dot{\xi}, \sigma_m) - C_m(0, 0, \sigma_m)] \\ &= \kappa[f(\sigma_m + \xi) - f(\sigma_m) + \dot{\xi}g(\sigma_m + \xi)] \end{aligned} \quad (2.15)$$

with

$$\kappa = \frac{\rho_\infty \bar{S} \ell^3}{2I}, \quad \dot{\xi} = \frac{d\xi}{d\tau}$$

The inertial equations of motion become

$$\frac{d\xi}{d\tau} = \dot{\xi} \quad (2.16a)$$

$$\frac{d\dot{\xi}}{d\tau} = F(\xi, \dot{\xi}, \sigma_m) \quad (2.16b)$$

An expansion of $F(\xi, \dot{\xi}, \sigma_m)$ in a Taylor series in ξ and $\dot{\xi}$ and a change of notation $u_1 = \xi$, $u_2 = \dot{\xi}$ yields for (2.16)

$$\begin{aligned} \dot{u}_i &= A_{ij}(\sigma_m)u_j + B_{ijk}(\sigma_m)u_j u_k + C_{ijkl}(\sigma_m)u_j u_k u_l \\ &+ O(|\vec{u}|^4), \quad (i = 1, 2) \end{aligned} \quad (2.17)$$

where

$$A = \begin{pmatrix} 0 & 1 \\ -\kappa S(\sigma_m) & -\kappa D(\sigma_m) \end{pmatrix} \quad (2.18a)$$

$$B_{1jk} = 0, \quad B_{2jk} = \frac{1}{2!} \left. \frac{\partial^2 F}{\partial u_j \partial u_k} \right|_{\vec{u}=0} \quad (2.18b)$$

$$C_{1jkl} = 0, \quad C_{2jkl} = \frac{1}{3!} \left. \frac{\partial^3 F}{\partial u_j \partial u_k \partial u_l} \right|_{\vec{u}=0} \quad (2.18c)$$

(Although (2.16a&b) have been derived on the assumption of slow oscillations (terms in C_m of $O(\dot{\xi}^2, \ddot{\xi})$ omitted), our subsequent bifurcation analysis of (2.16) will hold for general $F(\xi, \dot{\xi}, \sigma_m)$, i.e., as if no restriction had been placed on the magnitude of $\dot{\xi}$.)

In (2.18) the tensors B and C represent the effects of finite amplitude to the second and third order. We note that the following symmetry properties hold:

$$B_{2jk} = B_{2kj} \quad (2.19a)$$

$$C_{2jkl} = C_{2jlk} = C_{2kjl} = C_{2klj} = C_{2ljk} = C_{2llkj} \quad (2.19b)$$

On the basis of (2.17), we shall study the linear and nonlinear stability of the motion in subsequent sections.

3. LINEAR STABILITY THEORY. The stability of the steady motion at an angle of attack σ_m to infinitesimal disturbances is determined by the nature of the eigenvalues of the matrix A . They are

$$\lambda_{1,2}(\sigma_m) = \frac{1}{2} [-\kappa D(\sigma_m) \pm \sqrt{\kappa^2 D^2(\sigma_m) - 4\kappa S(\sigma_m)}] \quad (3.1)$$

Case I: $S(\sigma_m) < 0$. In this case $\lambda_1 > 0$, $\lambda_2 < 0$. The steady motion at this angle of attack σ_m is always unstable.

Case II: $S(\sigma_m) > 0$.

Case IIa: $D(\sigma_m) < 0$. In this case $\text{Re}(\lambda_1) > 0$ and the steady motion at σ_m is unstable.

Case IIb: $D(\sigma_m) > 0$. In this case $\text{Re}(\lambda_{1,2}) < 0$ and the steady motion at σ_m is stable.

Thus, only in Case IIb, when both stiffness and damping-in-pitch derivatives are positive, is the steady motion at angle of attack σ_m stable to infinitesimal disturbances. In fact, stability theory [3] can be used to show that stability of the steady motion in this case is assured only if the disturbance is sufficiently small.

In all cases in which the linearized theory predicts growth of the disturbance amplitude, the growth predicted is of exponential form and hence must cease to be valid after some finite time when the amplitude is no longer small. Thus, what eventually happens to a motion for which linearized stability theory predicts an initial growth of disturbances cannot be determined from the linearized theory itself. Instead, the full nonlinear inertial equations of motion, or a suitable approximation of them, such as (2.16), must be adopted to determine the ultimate state of the motion. Of particular interest is the dynamic stability boundary $\sigma_m = \sigma_{cr}$, where $S(\sigma_{cr}) > 0$, $D(\sigma_{cr}) = 0$. The stability characteristics near this boundary will be studied in the next section.

4. NONLINEAR STABILITY THEORY. At the dynamic stability boundary $\sigma = \sigma_{cr}$, we have $S(\sigma_{cr}) > 0$ and $D(\sigma_{cr}) = 0$, hence

$$\lambda_{1,2}(\sigma_{cr}) = \pm i\sqrt{\kappa S(\sigma_{cr})} \equiv \pm i\omega_0 \quad (4.1)$$

The existence of purely imaginary eigenvalues of the matrix A at $\sigma_m = \sigma_{cr}$ is the characteristic sign of a Hopf bifurcation [3, 14], signaling a changeover from stable steady motion to periodic motion. On crossing $\sigma_m = \sigma_{cr}$, the steady motion that had been stable for $\sigma_m < \sigma_{cr}$ will become unstable to disturbances, resulting (after a transient motion has died away) in the existence of a new motion, which (if it is stable) will be periodic. In the vicinity of $\sigma_m = \sigma_{cr}$, the circular frequency of the periodic motion will be nearly equal to ω_0 . We call the new solution of the equations of motion a bifurcation solution. In this section we shall determine its character and a criterion for its stability.

For σ_m slightly larger than σ_{cr} , the eigenvalues of the matrix A are

$$\lambda_{1,2} = -\frac{1}{2} \kappa D(\sigma_m) \pm i\Omega(\sigma_m) \quad (4.2)$$

where

$$\Omega(\sigma_m) = \sqrt{\kappa S(\sigma_m) - \kappa^2 D^2(\sigma_m)/4} \quad (4.3)$$

We shall assume that

$$D'(\sigma_{cr}) < 0 \quad (4.4)$$

which is the usual case in applications [6]. (The case $D'(\sigma_{cr}) > 0$ can be treated in exactly the same way.) The normalized eigenvector $\vec{\zeta}(\sigma_m)$ associated with the eigenvalue $\lambda(\sigma_m)$ is

$$\vec{\zeta}(\sigma_m) = \begin{pmatrix} \zeta_1(\sigma_m) \\ \zeta_2(\sigma_m) \end{pmatrix} = \frac{1-i}{2\sqrt{\Omega(\sigma_m)}} \begin{pmatrix} 1 \\ \lambda(\sigma_m) \end{pmatrix} \quad (4.5)$$

whereas the adjoint eigenvector $\vec{\zeta}^*(\sigma_m)$ with eigenvalue $\bar{\lambda}(\sigma_m)$, which is the complex conjugate of $\lambda(\sigma_m)$, is

$$\vec{\zeta}^*(\sigma_m) = \begin{pmatrix} \zeta_1^*(\sigma_m) \\ \zeta_2^*(\sigma_m) \end{pmatrix} = \frac{1+i}{2\sqrt{\Omega(\sigma_m)}} \begin{pmatrix} \bar{\lambda}(\sigma_m) + \kappa D(\sigma_m) \\ 1 \end{pmatrix} \quad (4.6)$$

A. Hopf Bifurcation. The bifurcation solution $\vec{u}(\tau, \sigma_m)$ may be written as

$$\vec{u} = a(\tau)\vec{\zeta} + \bar{a}(\tau)\bar{\vec{\zeta}} \quad (4.7)$$

Following Iooss and Joseph ([3], p. 125), we get

$$\left. \begin{aligned} a &= \varepsilon b_1(s) + \varepsilon^2 b_2(s) + \varepsilon^3 b_3(s) + O(\varepsilon^4) \\ s &= [\omega_0 + \varepsilon^2 \omega_2 + O(\varepsilon^4)]\tau \\ \sigma_m &= \sigma_{cr} + \varepsilon^2 \sigma_{m2} + O(\varepsilon^4) \end{aligned} \right\} \quad (4.8)$$

where, for brevity, we omit the lengthy solution forms for ε , b_n , ω_2 , and σ_{m2} (cf. [11]). The solution is periodic in τ with circular frequency equal to $\omega_0 + \varepsilon^2 \omega_2 + O(\varepsilon^4)$.

B. Stability of the Periodic Bifurcation Solution. According to Floquet theory [3], the stability of the periodic bifurcation solution (4.8) is determined by the sign of an index μ . To $O(\varepsilon^4)$, μ has the form

$$\mu = \kappa D'(\sigma_{cr}) \sigma_{m2} \varepsilon^2 + O(\varepsilon^4) \quad (4.9)$$

and the periodic bifurcation solution is stable if $\mu < 0$, unstable if $\mu > 0$. Since we have assumed $D'(\sigma_{cr}) < 0$, stability thus depends on the sign of σ_{m2} , with $\sigma_{m2} > 0$ denoting stability and $\sigma_{m2} < 0$ instability. It remains to cast σ_{m2} in more recognizable terms. After considerable manipulation, we get

$$\mu = \frac{\varepsilon^2}{4\omega_0^3} \left[\left(\frac{\partial^2 F}{\partial u_1^2} + \omega_0^2 \frac{\partial^2 F}{\partial u_2^2} \right) \frac{\partial^2 F}{\partial u_1 \partial u_2} + \omega_0^2 \left(\frac{\partial^3 F}{\partial u_1^2 \partial u_2} + \omega_0^2 \frac{\partial^3 F}{\partial u_2^3} \right) \right]_{\substack{\vec{u}=0 \\ \sigma_m = \sigma_{cr}}} \quad (4.10)$$

in terms of $F(u_1, u_2, \sigma_m)$. From (2.15) we see that the function F is directly related to the pitching-moment coefficient $C_m(\xi, \dot{\xi}, \sigma_m)$ acting on the aircraft which is performing a finite-amplitude pitching oscillation ξ around a mean angle of attack σ_m . Equation (4.10) demonstrates that the stability of the periodic motion near the dynamic stability boundary σ_{cr} is determined by the behavior of the aerodynamic response $C_m(\xi, \dot{\xi}, \sigma_m)$ in that vicinity.

With the assumption of slow oscillations under which the form of (2.15) was derived (terms of $O(\dot{\xi}^2, \ddot{\xi})$ neglected), we may substitute (2.15) into (4.10) to get

$$\mu = \frac{\varepsilon^2}{4\omega_0^3} \left(\kappa^2 \frac{\partial^2 f}{\partial \xi^2} \frac{\partial g}{\partial \xi} + \kappa \omega_0^2 \frac{\partial^2 g}{\partial \xi^2} \right)_{\substack{\xi=0 \\ \sigma_m = \sigma_{cr}}} \quad (4.11)$$

From (2.14) and the structure of the functions f and g , we write

$$\left. \begin{aligned} \frac{\partial f}{\partial \xi} \Big|_{\xi=0} &= -S(\sigma_m) , & \frac{\partial^2 f}{\partial \xi^2} \Big|_{\xi=0} &= -S'(\sigma_m) \\ g \Big|_{\xi=0} &= -D(\sigma_m) , & \frac{\partial g}{\partial \xi} \Big|_{\xi=0} &= -D'(\sigma_m) , & \frac{\partial^2 g}{\partial \xi^2} \Big|_{\xi=0} &= -D''(\sigma_m) \end{aligned} \right\} \quad (4.12)$$

By using (4.2) and (4.12) in (4.11) (with $S(\sigma_{cr}) > 0$), we arrive at the simple form

$$\mu = \frac{-\varepsilon^2 (\kappa S)^{1/2}}{4} \frac{d}{d\sigma_m} \frac{D'}{S} \Big|_{\sigma_m = \sigma_{cr}} \quad (4.13)$$

(The simplicity of this result suggests that it might be possible to derive it by a less formal method. This is indeed the case: We have verified the result by a physical approach familiar to workers in the field of nonlinear mechanics.) Equation (4.13) reveals that the sign of μ , and thus the stability of the bifurcation solution, is independent of the scalar and inertial properties of the aircraft. Rather, stability depends only on whether the aerodynamic property (D'/S) is increasing or decreasing on crossing the dynamic stability boundary $\sigma_m = \sigma_{cr}$. The two possibilities are well illustrated in the form of bifurcation diagrams as shown in Figs. 1a and 1b.

In a bifurcation diagram, the abscissa represents the parameter that is being varied, in our case the mean angle of attack σ_m . The ordinate is a parameter characteristic of the bifurcation solution alone. In our case it is ε , a measure of the amplitude of the periodic bifurcation solution. Stable solutions are indicated by solid lines, unstable solutions by dashed lines. Thus, over the range of mean angle of attack $\sigma_m < \sigma_{cr}$ where the steady-state motion is stable, ε is zero, and the

stable steady motion is represented along the abscissa by a solid line. The steady motion becomes unstable for all values of $\sigma_m > \sigma_{cr}$ as the dashed line along the abscissa indicates. Periodic solutions bifurcate from $\sigma_m = \sigma_{cr}$ either supercritically or subcritically.

When $(d/d\sigma_m)(D'/S)|_{\sigma_m=\sigma_{cr}} > 0$ (implying $\sigma_{m2} > 0$), the bifurcation is called supercritical and its characteristic form is shown in Fig. 1a. Stable periodic solutions (solid curves in Fig. 1a) exist for values of $\sigma_m - \sigma_{cr} > 0$. The amplitude of the periodic solution at a given value of $\sigma_m - \sigma_{cr}$ is proportional to ϵ , hence is vanishingly small when $\sigma_m - \sigma_{cr}$ is small, varying essentially as $(\sigma_m - \sigma_{cr})^{1/2}$.

When $(d/d\sigma_m)(D'/S)|_{\sigma_m=\sigma_{cr}} < 0$ (implying $\sigma_{m2} < 0$), the bifurcation is called subcritical and its characteristic form is shown in Fig. 1b. Periodic solutions exist for values of $\sigma_m - \sigma_{cr} < 0$, but they are unstable (dashed curve in Fig. 1b). Whether stable periodic solutions do or do not exist for $\sigma_m > \sigma_{cr}$ depends predominantly on the behavior of the damping-in-pitch derivative $D(\sigma_m)$ for $\sigma_m > \sigma_{cr}$. If no such stable periodic solutions exist for $\sigma_m > \sigma_{cr}$, then when the mean angle of attack σ_m is increased beyond σ_{cr} the aircraft may undergo an aperiodic motion whose departure from the steady motion at $\sigma_m = \sigma_{cr}$ is potentially large.

In the more likely event that stable periodic solutions do exist for $\sigma_m > \sigma_{cr}$ (an example is given later), their amplitudes must be finite, and not infinitesimally small, even for small positive values of $\sigma_m - \sigma_{cr}$. It is likely that this branch of stable periodic solutions will join that of the unstable branch in the way illustrated in Fig. 1b.

In this event, the form of the bifurcation curve for values of $\sigma_m < \sigma_{cr}$ helps explain the situation mentioned earlier, where it was noted that the steady-state motion could be stable to sufficiently small disturbances but become unstable to larger disturbances. Thus, Fig. 1b suggests that for $\sigma_m < \sigma_{cr}$, so long as disturbances are of small enough amplitude to lie below those of the unstable branch of periodic solutions (curve OB in Fig. 1b), they will die out with time and the steady motion will remain stable. However, disturbances with amplitudes sufficiently larger than those of the unstable branch may actually grow up to the ultimate motion as $\tau \rightarrow \infty$, which will be that of the stable branch of periodic solutions (curve BA in Fig. 1b). Finally, we note that if the motion does attain the stable branch of periodic solutions (say, for $\sigma_m < \sigma_{cr}$) then hysteresis effects will manifest themselves with further changes in σ_m . When σ_m is increased beyond σ_{cr} , the motion will continue to be periodic with finite amplitude (point A in Fig. 1b). If σ_m is now decreased below σ_{cr} , the periodic motion will persist, even at values of σ_m where previously there had been steady motion when σ_m was being increased. Not until σ_m is diminished beyond a certain point (point B in Fig. 1b) will the motion return to the steady-state condition (point C in Fig. 1b) that had been experienced when σ_m was increasing.

To further explore the implications of (4.13), we invoke some approximate relationships between the damping-in-pitch derivative $D(\sigma_m)$ and the stiffness derivative $S(\sigma_m)$. Tobak and Schiff have argued (see, in particular, [15]) that, to good accuracy, a linear relationship should

exist between D and S at any value of σ_m ; i.e., $D(\sigma_m) \approx a - bS(\sigma_m)$, with $b > 0$. In addition, we require that D vanish at $\sigma_m = \sigma_{cr}$.

The two requirements yield the form

$$D(\sigma_m) = b[S(\sigma_{cr}) - S(\sigma_m)] , \quad b > 0 \quad (4.14)$$

(The validity of the approximate relation (4.14) has been verified in the present study (via comparison with exact results in [7]) for use with oscillating flat-plate airfoils in supersonic/hypersonic flow and in [12] for use with oscillating flaps in transonic flow.) Replacing D in (4.13) by (4.14) casts the criterion solely in terms of S :

$$\mu = \frac{\epsilon^2 (\kappa S)^{1/2} b}{4} \frac{d^2}{d\sigma_m^2} [\ln S(\sigma_m)] \Big|_{\sigma_m = \sigma_{cr}} \quad (4.15)$$

Thus, near $\sigma_m = \sigma_{cr}$, if the stiffness derivative $S(\sigma_m)$ increases with σ_m slower/faster than exponential, the periodic bifurcation solution is stable (supercritical)/unstable (subcritical). Cast in terms of D instead of S , the criterion states that decreasing D with respect to σ_m slower/faster than exponential results in stable (supercritical)/unstable (subcritical) periodic bifurcation solutions.

Examples of $S(\sigma_m)$ and $D(\sigma_m)$ that obey (4.14) and exhibit the various possibilities are as follows.

Example 1:

$$\left. \begin{aligned} S_1(\sigma_m) &= S_0 \exp[k(\sigma_m - \sigma_{cr}) + m(\sigma_m - \sigma_{cr})^2] \\ D_1(\sigma_m) &= bS_0 \{1 - \exp[k(\sigma_m - \sigma_{cr}) + m(\sigma_m - \sigma_{cr})^2]\} \end{aligned} \right\} \quad (4.16)$$

Example 2:

$$\left. \begin{aligned} S_2(\sigma_m) &= S_0 \exp[k(\sigma_m - \sigma_{cr}) + m(\sigma_m - \sigma_{cr})^2 - n(\sigma_m - \sigma_{cr})^4] \\ D_2(\sigma_m) &= bS_0 \{1 - \exp[k(\sigma_m - \sigma_{cr}) + m(\sigma_m - \sigma_{cr})^2 - n(\sigma_m - \sigma_{cr})^4]\} \end{aligned} \right\} \quad (4.17)$$

where S_0 , k , and n are positive constants. According to (4.15), $m > 0$ corresponds to unstable periodic bifurcation solutions (subcritical bifurcation), and $m < 0$ corresponds to stable periodic bifurcation solutions (supercritical bifurcation). In the case of subcritical bifurcation ($m > 0$) in Example 1, the damping-in-pitch derivative $D_1(\sigma_m)$ continues to decrease to very large negative values as $(\sigma_m - \sigma_{cr})$ increases, which makes it very unlikely that the system (2.16) will have a stable periodic motion as a solution for $\sigma_m > \sigma_{cr}$. On the other hand, in Example 2 the damping-in-pitch derivative $D_2(\sigma_m)$ becomes positive for sufficiently large $|\sigma_m - \sigma_{cr}|$ so that a stable periodic motion may be a possible solution for $\sigma_m > \sigma_{cr}$, $m > 0$. Indeed, S_2 and D_2 in Example 2 fulfill all of the conditions set by a theorem of Filippov [16], the satisfaction of which guarantees the existence of a stable periodic motion of the system (2.16) for $\sigma_m > \sigma_{cr}$. As we have noted, resulting as it does from a subcritical bifurcation, the solution will exhibit hysteresis effects with variations in σ_m below σ_{cr} .

5. SUPERSONIC/HYPERSONIC FLAT-PLATE AIRFOILS. To illustrate the application of bifurcation theory in a concrete case, we consider a flat-plate airfoil in supersonic/hypersonic flow. The stiffness derivative $S(\sigma_m)$ and the damping-in-pitch derivative $D(\sigma_m)$ are known as

analytic functions of σ_m up to the angle-for-shock detachment [6-8]. They depend on the flight Mach number M_∞ , the ratio of specific heats γ (here taken to be that of air, $\gamma = 1.4$), and the (dimensionless) distance h of the center of gravity from the leading edge, here taken as a fraction of the chord length ℓ . Results are presented in Fig. 2 of $\log S(\sigma_m)$ versus σ_m (with $M_\infty = 2.0$, $h = 0$) and in Table 1 of the index μ for various combinations of M_∞ and h . It is shown in Fig. 2 that $S(\sigma_m)$ increases faster than exponential near $\sigma_m = \sigma_{cr} = 15.75^\circ$, and in Table 1 that μ is always positive. We conclude that whenever the flat-plate airfoil becomes dynamically unstable [$D(\sigma_{cr}) \equiv 0$], the ensuing bifurcation always will be subcritical.

Accordingly, there are two possibilities. One, a subcritical bifurcation curve such as that sketched in Fig. 1b exists, in which case the airfoil motion will find stability at values of $\sigma_m > \sigma_{cr}$ in a periodic oscillation of finite amplitude. At values of $\sigma_m < \sigma_{cr}$, the steady-state motion will be stable for small disturbances, but for larger disturbances the airfoil motion will again seek the stable periodic motion. Exchanges between these two stable modes at $\sigma_m < \sigma_{cr}$ will be accompanied by hysteresis effects. Two, alternatively, a bifurcation curve such as that sketched in Fig. 1b does not exist, in which case a potentially large aperiodic departure from the steady-state motion may occur as σ_m exceeds σ_{cr} . In either case, on exceeding σ_{cr} the loss of stability of the steady-state motion must entail a discrete change to a new stable condition.

6. CONCLUDING REMARKS. Bifurcation theory has been used to analyze the nonlinear dynamic stability characteristics of an aircraft subject to single-degree-of-freedom pitching-motion perturbations about a large mean angle of attack. Setting up the equations to which the bifurcation theory was applied required, first, determining conditions under which the inertial equations of motion and the gas-dynamic equations governing the flow could be decoupled and, second, showing how the required aerodynamic responses to finite-amplitude oscillations could be obtained from the responses to infinitesimal-amplitude oscillations.

Results of the bifurcation theory analysis revealed that when the mean angle of attack is increased past the critical point where the aerodynamic damping vanishes, new solutions describing finite-amplitude periodic motions bifurcate from the previously stable steady motion. The sign of a simple criterion, cast in terms of aerodynamic properties, determines whether the bifurcating solutions are stable (supercritical) or unstable (subcritical). For flat-plate airfoils flying at supersonic/hypersonic speed, the bifurcation is subcritical, implying either that exchanges of stability between steady and periodic motions will be accompanied by hysteresis phenomena, or that a potentially dangerous aperiodic motion may develop. In either case the loss of stability of the steady-state motion must be accompanied by a discrete change to a new stable state.

Acknowledgments

The authors thank Gary T. Chapman, Ames Research Center, for many stimulating discussions during the course of this research and Henry J. Van Roesell for helping with the computations. W. H. H. is grateful for the financial support provided by NASA Contract NAGW-130.

REFERENCES

1. K. J. Orlik-Rückemann, Dynamic Stability Testing of Aircraft — Needs Versus Capabilities, Progress in Aerospace Sciences, Vol. 16, No. 4, Pergamon Press, New York, pp. 431-447 (1975).
2. G. D. Padfield, Nonlinear Oscillations at High Incidence, AGARD CP-235, Dynamic Stability Parameters, Paper No. 31, May 1978.
3. G. Iooss and D. D. Joseph, Elementary Stability and Bifurcation Theory, Springer-Verlag, New York, 1980.
4. R. K. Mehra and J. V. Carroll, Bifurcation Analysis of Aircraft High Angle-of-Attack Flight Dynamics, New Approaches to Nonlinear Problems in Dynamics, Ed. P. J. Holmes, SIAM, Philadelphia, pp. 127-146 (1980).
5. P. Guicheteau, Bifurcation Theory Applied to the Study of Control Losses on Combat Aircraft, La Recherche Aérospatiale, pp. 1-14 (1982-3).
6. W. H. Hui, Stability of Oscillating Wedges and Caret Wings in Hypersonic and Supersonic Flows, AIAA J., Vol. 7, No. 8, pp. 1524-1530 (Aug. 1969).
7. W. H. Hui, Supersonic/Hypersonic Flow Past an Oscillating Flat-Plate at High Angles of Attack, ZAMP, Vol. 29, Fasc. 3, pp. 414-427 (1978).

8. W. H. Hui, An Analytic Theory of Supersonic/Hypersonic Stability at High Angles of Attack, AGARD CP-235, Dynamic Stability Parameters, Paper No. 22, May 1978.
9. W. H. Hui and M. Tobak, Unsteady Newton-Busemann Flow Theory. Part I: Airfoils, AIAA J., Vol. 19, No. 3, pp. 311-318 (Mar. 1981).
10. W. H. Hui, Large-Amplitude Slow Oscillation of Wedges in Inviscid Hypersonic and Supersonic Flows, AIAA J., Vol. 8, No. 8, pp. 1530-1532 (Aug. 1970).
11. W. H. Hui and M. Tobak, Bifurcation Analysis of Aircraft Pitching Motions About Large Mean Angles of Attack. To appear in J. Guidance, Control, and Dynamics.
12. W. J. Chyu and L. B. Schiff, Nonlinear Aerodynamic Modeling of Flap Oscillations in Transonic Flow: A Numerical Validation, AIAA J., Vol. 21, No. 1, pp. 106-113 (Jan. 1983).
13. M. Tobak and L. B. Schiff, The Role of Time-History Effects in the Formulation of the Aerodynamics of Aircraft Dynamics, AGARD CP-235, Dynamic Stability Parameters, Paper No. 26, May 1978.
14. E. Hopf, Abzweigung einer periodischen Lösung von einer Stationären Lösung eines Differentialsystems, Berichten der Mathematisch-Physischen Klasse der Sächsischen Akademie der Wissenschaften zu Leipzig, Vol. XCIV, pp. 1-22 (1942).
15. M. Tobak and L. B. Schiff, On the Formulation of the Aerodynamic Characteristics in Aircraft Dynamics, NASA TR R-456, Jan. 1976.
16. A. F. Filippov, A Sufficient Condition for the Existence of a Stable Limit Cycle for an Equation of the Second Order, Mat. Sbornik, Vol. 30, pp. 171-180 (1952).

Figure Captions

Fig. 1 Typical forms of bifurcation diagrams near the dynamic stability boundary σ_{cr} where $D(\sigma_{cr}) = 0$.

(a) Supercritical, $(d/d\sigma_m)[D'(\sigma_m)/S(\sigma_m)]_{\sigma_m=\sigma_{cr}} > 0$.

(b) Subcritical, $(d/d\sigma_m)[D'(\sigma_m)/S(\sigma_m)]_{\sigma_m=\sigma_{cr}} < 0$.

Fig. 2 Stiffness derivative $S(\sigma_m)$ versus mean angle of attack σ_m for a flat-plate airfoil in a supersonic free stream: $M_\infty = 2.0$, $h = 0$, $\gamma = 1.4$, $\kappa = 1$.

Table 1 Values of stability criterion $\mu(M_\infty, h)$ for flat-plate airfoil:
 $\kappa = \epsilon^2 = 1$, $\gamma = 1.4$; $\mu > 0$ subcritical bifurcation, $\mu < 0$ supercritical
 bifurcation

M_∞	h					M_∞	h				
	0	0.1	0.2	0.3	0.4		0	0.1	0.2	0.3	0.4
1.5	36.5	19.0	--	--	--	2	39.5	25.8	17.2	13.0	14.7
1.6	39.6	23.4	13.6	8.8	10.7	3	57.2	38.6	26.8	20.9	23.3
1.7	40.0	24.8	15.5	11.0	12.8	4	84.7	58.4	41.4	32.8	36.4
1.8	39.7	25.3	16.3	12.0	13.7	5	107.3	74.8	53.7	42.9	47.5
1.9	39.4	25.5	16.8	12.6	14.3	6	123.6	86.7	62.6	50.3	55.6
2.0	39.5	25.8	17.2	13.0	14.7						

M_∞	h											
	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.34	0.35	
1.6	10.6	10.1	9.7	9.3	9.0	8.8	8.6	8.5	8.4	8.4	8.5	
1.7	12.6	12.2	11.8	11.5	11.2	11.0	10.8	10.7	10.6	10.6	10.7	
1.8	13.6	13.2	12.8	12.5	12.2	12.0	11.8	11.7	11.6	11.7	11.7	
1.9	14.1	13.7	13.4	13.0	12.8	12.5	12.4	12.3	12.2	12.2	12.3	
2.0	14.5	14.1	13.8	13.5	13.2	13.0	12.8	12.7	12.6	12.7	12.7	

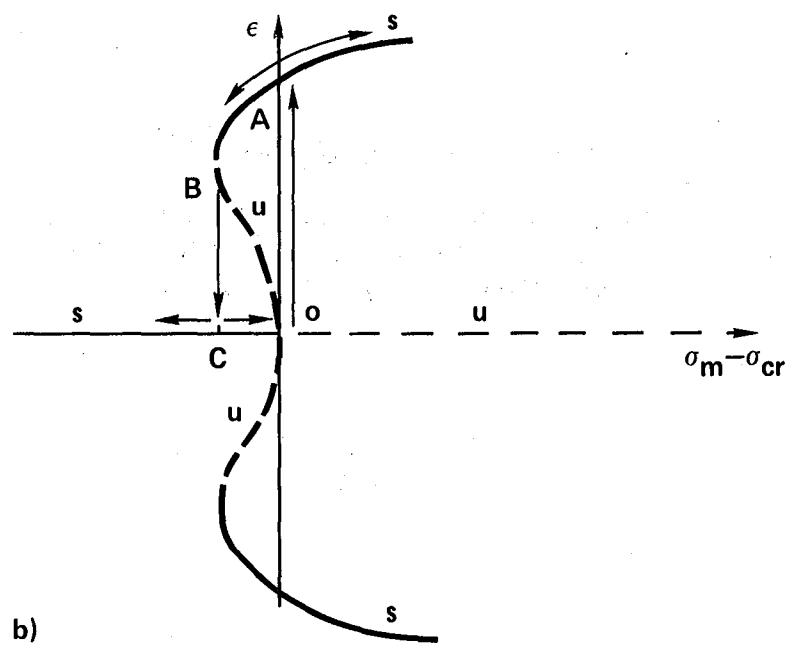
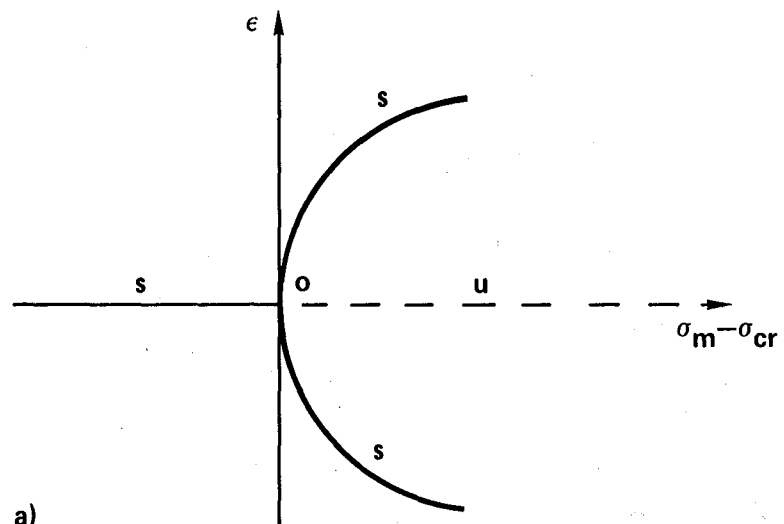


Fig. 1

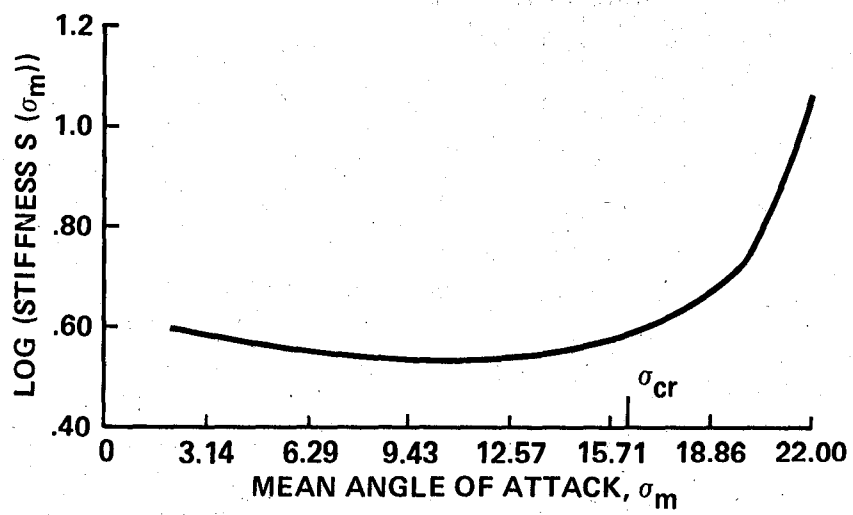


Fig. 2

1. Report No. NASA TM-85881		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle BIFURCATION ANALYSIS OF AIRCRAFT PITCHING MOTIONS NEAR THE STABILITY BOUNDARY				5. Report Date January 1984	
				6. Performing Organization Code	
7. Author(s) W. H. Hui (Univ. of Waterloo, Waterloo, Ontario, Canada) and Murray Tobak				8. Performing Organization Report No. A-9604	
				10. Work Unit No. T-4204	
9. Performing Organization Name and Address Ames Research Center Moffett Field, CA 94035				11. Contract or Grant No.	
				13. Type of Report and Period Covered Technical Memorandum	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546				14. Sponsoring Agency Code	
15. Supplementary Notes Originally presented as a lecture to the Berkeley-Ames Conference on Nonlinear Problems in Control and Fluid Dynamics, Univ. of California, Berkeley, Calif., May 31-June 10, 1983. Point of Contact: Murray Tobak, Ames Research Center, MS 234-1, Moffett Field, CA 94035 (415) 965-5398 or FTS 448-5398					
16. Abstract Bifurcation theory is used to analyze the nonlinear dynamic stability characteristics of an aircraft subject to single-degree-of-freedom pitching-motion perturbations about a large mean angle of attack. The requisite aerodynamic information in the equations of motion can be represented in a form equivalent to the response to finite-amplitude pitching oscillations about the mean angle of attack. It is shown how this information can be deduced from the case of infinitesimal-amplitude oscillations. The bifurcation theory analysis reveals that when the mean angle of attack is increased beyond a critical value at which the aerodynamic damping vanishes, new solutions representing finite-amplitude periodic motions bifurcate from the previously stable steady motion. The sign of a simple criterion, cast in terms of aerodynamic properties, determines whether the bifurcating solutions are stable (supercritical) or unstable (subcritical). For flat-plate airfoils flying at supersonic/hypersonic speed, the bifurcation is subcritical, implying either that exchanges of stability between steady and periodic motion are accompanied by hysteresis phenomena, or that potentially large aperiodic departures from steady motion may develop.					
17. Key Words (Suggested by Author(s)) Nonlinear aerodynamics Nonsteady aerodynamics Large angles of attack			18. Distribution Statement Unlimited Subject Category - 02		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 33	22. Price* A03

1. The first part of the document discusses the importance of maintaining accurate records of all transactions and activities. It emphasizes that proper record-keeping is essential for transparency and accountability, particularly in financial matters. This section also touches upon the legal implications of failing to maintain such records, which can lead to severe consequences for individuals and organizations alike.

2. The second part of the document delves into the specific requirements for record-keeping, including the types of documents that must be retained and the duration for which they should be kept. It provides a detailed overview of the various categories of records, such as financial statements, contracts, and correspondence, and outlines the best practices for organizing and storing these documents to ensure they are easily accessible when needed.

3. The third part of the document addresses the challenges associated with record-keeping, particularly in the context of digital information. It discusses the risks of data loss, corruption, and unauthorized access, and offers strategies to mitigate these risks. This includes the use of secure storage solutions, regular backups, and access controls to protect sensitive information.

4. The fourth part of the document provides a comprehensive guide to the legal and regulatory requirements governing record-keeping. It covers the various laws and regulations that apply to different industries and sectors, and explains how these requirements may vary over time. This section is particularly useful for organizations that operate in highly regulated environments, where compliance with record-keeping standards is a critical component of their operations.

5. The fifth and final part of the document offers practical advice and tips for implementing an effective record-keeping system. It discusses the importance of developing clear policies and procedures, training staff on the correct use of the system, and regularly reviewing and updating the system to reflect changes in requirements and technology. This section concludes with a summary of the key points discussed throughout the document, reinforcing the message that proper record-keeping is a fundamental aspect of sound business and legal practice.



