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TECHNICAL MEMORANDUM (NASA) 89

PATH DISCREPANCIES BETWEEN GREAT CIRCLE AND RHUMB LINE

A simulation of a mathematical model to compute path discrepancies between great circle and rhumb line flight paths is presented. The model illustrates that the path errors depend on the latitude, the bearing and the trip length of the flight.

by

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I. INTRODUCTION

A mathematical model for a comparative analysis of great circle vs. rhumb line navigation in the continental United States has been developed at the Avionics Engineering Center, Ohio University. A FORTRAN simulation of the model has been implemented on the IBM 370 computer. The simulation predicts pertinent navigation information for the two flight paths. The basis for the project, which is a part of an M.S. thesis, is to provide a data base for computing discrepancies between the two flight paths. This document briefly describes the model and discusses the implications of the results obtained.

II. BACKGROUND INFORMATION

The standard en-route navigation system used in the United States is the VOR/DME. VHF Omnidirectional Range (VOR) is the basis for defining the airways and is therefore an integral part of air traffic control procedures. Two VOR stations are used to define a radial intersection, which is the basis of the established Victor airways. These Victor airways follow a flight path which maintains a path of constant heading (i.e. the course crosses the meridians at the same angle). Thus, a rhumb line course appears as a straight line on a Mercator projection.

Area/Random Navigation (RNAV) as defined in FAA Advisory Circular 90-45A (1) is

> '...a method of navigation that permits aircraft operations on any desired course within the coverage of station referenced navigation signals or within the limits of self-contained system capabilities.'

The principle advantage of RNAV is that it allows the navigator to fly a great circle course between two points. The great circle course is the shortest path between two points on the earth, and is formed by the intersection of the plane defined by the center of the earth and the points of origin and destination, projected on the surface of the earth. The flight path is projected as a curved line on a Mercator projection and hence changes heading constantly.

Loran-C, OMEGA, GPS, Doppler, and INS are considered RNAV systems. However, VOR in its basic form is not an RNAV system. A major concern is that the discrepancies between the two flight paths may cause navigation conflicts. This report attempts to quantize the discrepancies which depend on various factors.

III. PATH DISCREPANCIES

The magnitude of path discrepancies between the great circle and rhumb line courses between two points increases as

1) the latitude increases,
2) the difference in their latitudes decreases and
3) the difference in their longitudes increases.

When plotted on a Mercator projection map, the two flight paths start to diverge from the origin of the flight. The discrepancy is a maximum at the mid-point of the flight, and then the flight paths start to converge and meet at the destination point. The great circle path is always curved away from the equator and therefore intersects the meridians at higher latitudes than the rhumb line path. Also, the meridians converge as they depart from the equator and are closer together at higher latitudes. Thus, the deviation between the two paths increase at higher latitudes because the rhumb line path must curve rapidly to be able to fly a course of constant meridian crossing angle.

The bearing of the flight path, which is a simple function of the latitude and longitude, also has a considerable effect on the discrepancies between the two flight paths. By definition, all lines of longitude are also great circles. On a true north-south flight path the great circle and rhumb line courses are exactly the same. However, as the bearing becomes more easterly or westerly, errors between great circle and rhumb line paths start to increase. The maximum path errors due to bearing alone are on a true east-west course. It is important to note that on an east-west course at the equator the great circle and rhumb line flight paths are the same because the equator is also a great circle path.

Finally, another factor that has an effect on path discrepancy is the trip length, which is also a function of the latitude and longitude of the two points. As mentioned earlier, the great circle and rhumb line courses start to diverge from the point of origin of the flight and reach the point of maximum divergence half way into the flight. Therefore, as the trip length increases the paths simply diverge further till they reach the half-way point.

IV. THE MODEL

The mathematical model which was developed using the haversines formula was obtained from Bowditch (2). The equations developed for the model were designed for computer applications. The model is based upon a spherical approximation of the earth and adjusted using geodesic error equations for the North American datum based on the Clarke ellipsiod. The equations describing the model are given below.

ϕ_1 = Latitude of origin in degrees

λ_1 = Longitude of origin in degrees

ϕ_2 = Latitude of destination in degrees

λ_2 = Longitude of destination in degrees

$$\bar{\phi} = \frac{(\phi_1 + \phi_2)}{2}$$ --- mid-latitude point

$$Dlo = \lambda_2 - \lambda_1$$ --- difference in longitude

Dlox = Interval of longitude measured from point of departure in degrees

A. RHUMB LINE EQUATIONS

Rhumb line bearing (relative from origin to destination)

$$\alpha = Tan^{-1}\left(\frac{\phi_2 - \phi_1}{Dlo \cdot Cos\bar{\phi}}\right)$$

Latitude of points on a rhumb line path

$$\phi_{(RL)} = \phi_1 + Dlox \cdot Cos\bar{\phi} \cdot Tan\alpha$$

Rhumb line distance

$$RLDIST = 60\sqrt{[(\phi_2 - \phi_1)^2 + (Dlo \cdot Cos\bar{\phi})^2]}$$

B. GREAT CIRCLE EQUATIONS

Initial course of the great circle path

$$C = Tan^{-1}\left(\frac{Sin(Dlo)}{Cos\phi_1 \cdot Tan\phi_2 - Sin\phi_1 \cdot Cos(Dlo)}\right)$$

Vertex

The vertex is the point of highest latitude on a great circle path

$$Lv = Cos^{-1}[Cos(\phi_1) \cdot Sin(C)]$$

Difference in longitude between vertex and point of origin

$$Dlov = Sin^{-1}\left[\frac{Cos(C)}{Sin(Lv)}\right]$$

Latitudes of points on great circle path

$$\phi_{(GC)} = Tan^{-1}[Tan(Lv) \cdot Cos(Dlox - Dlov)]$$

Great circle distance

$$GCDIST = 60 \cdot Cos^{-1}[Sin(\phi_1) \cdot Sin(\phi_2) + Cos(\phi_1) \cdot Cos(\phi_2) \cdot Cos(Dlo)]$$

C. GEODESIC ERROR EQUATIONS

The geodesic error equations between the model and Clarke ellipsoid

$$\text{ERROR(east)} = [9.12951 \cdot \text{Cos}(\overline{\phi}) - 2.92495 \cdot \text{Cos}(3\overline{\phi})] \cdot (\lambda_2 - \lambda_1) \cdot \frac{\pi}{180}$$

$$\text{ERROR(north)} = 0.37414 \cdot (\phi_2 - \phi_1) \cdot (\frac{\pi}{180}) - 8.88543 \cdot [\text{Sin}(2\phi_2) - \text{Sin}(2\phi_1)]$$

Similar equations were also used by Hogle, Markin and Toth in their report 'Evaluation of Various Navigation Concepts' (3).

V. RESULTS AND CONCLJSIONS

The equations detailed in the previous section were modeled in FORTRAN on the Ohio University IBM 370 computer. Figures 1 and 2 are simulations of great circle and rhumb line paths on a mercator projection. The results indicate that the discrepancy between the two flight paths can be significant. Figure 1 is an east-west flight path from Baltimore-Washington International to Los Angeles International airport. At the point of maximum deviation the path discrepancy is 126 nautical miles for a trip length of 2017 nmi. On the other hand, a north-south flight path from St. Paul to Houston the discrepancy at the point of maximum deviation is only 5.25 nmi for a trip length of 903 nmi. (Figure 2.)

Since shorter trip lengths are of major importance to the pilot an effort was made to quantify the path errors for flight paths of up to 500 nmi. Figures 3 through 6 illustrate how the factors mentioned earlier effect the path discrepancies. The flights were simulated at a constant bearing with the mid-point of the flight at the specified latitude. Comparisions of the plots indicate the compound effect bearing and latitude of the mid-point of a flight have on the path errors. A flight at a mid-point latitude of 35 degrees north and a relative bearing of 76 degrees has maximum path errors of 1.1 nmi for trip lengths of up to 300 nmi (Fig. 3). However, a flight at the same mid-point latitude, but a relative bearing of 39 degrees has maximum path errors of more than 6 nmi for trip lengths up to 450 nmi (Fig. 5).

It is of vital importance to note here that the errors mentioned in this report are solely due to the discrepancies in the great circle and rhumb line paths. The errors do not account for reciever computational errors or pilot errors. Another source of error may be discrepancies among the earth models in various RNAV recievers. Offsets due to using different navigation systems in the same airspace also may cause considerable disparity (4). It is recommended that any decision made regarding this subject must take into account all discrepancies mentioned above, and not just the goemetrical errors.

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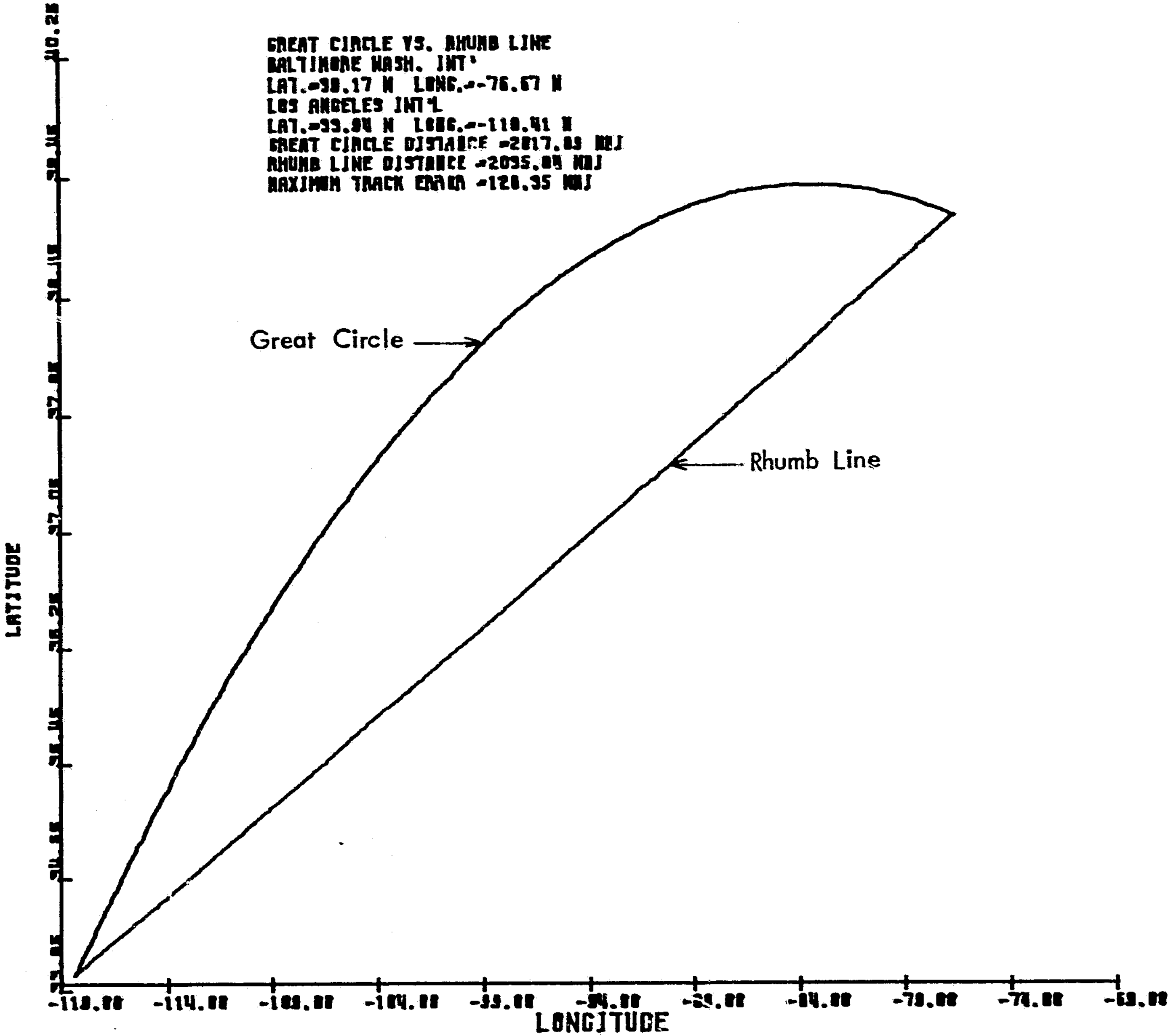

Figure 1. Great Circle vs. Rhumb line. Baltimore - Washington to Los Angeles.

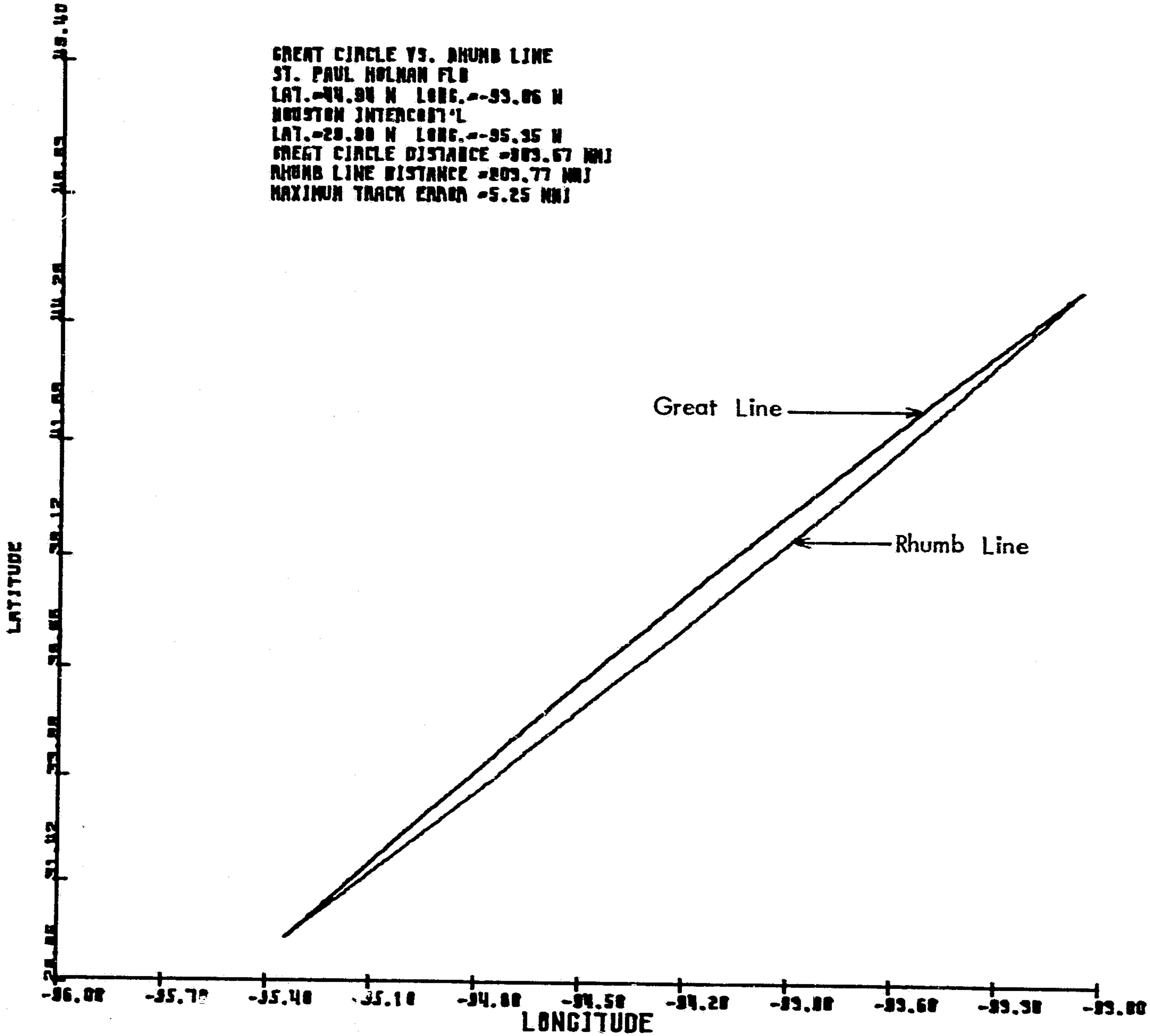

Figure 2. Great Circle vs. Rhumb line. St. Paul to Houston.

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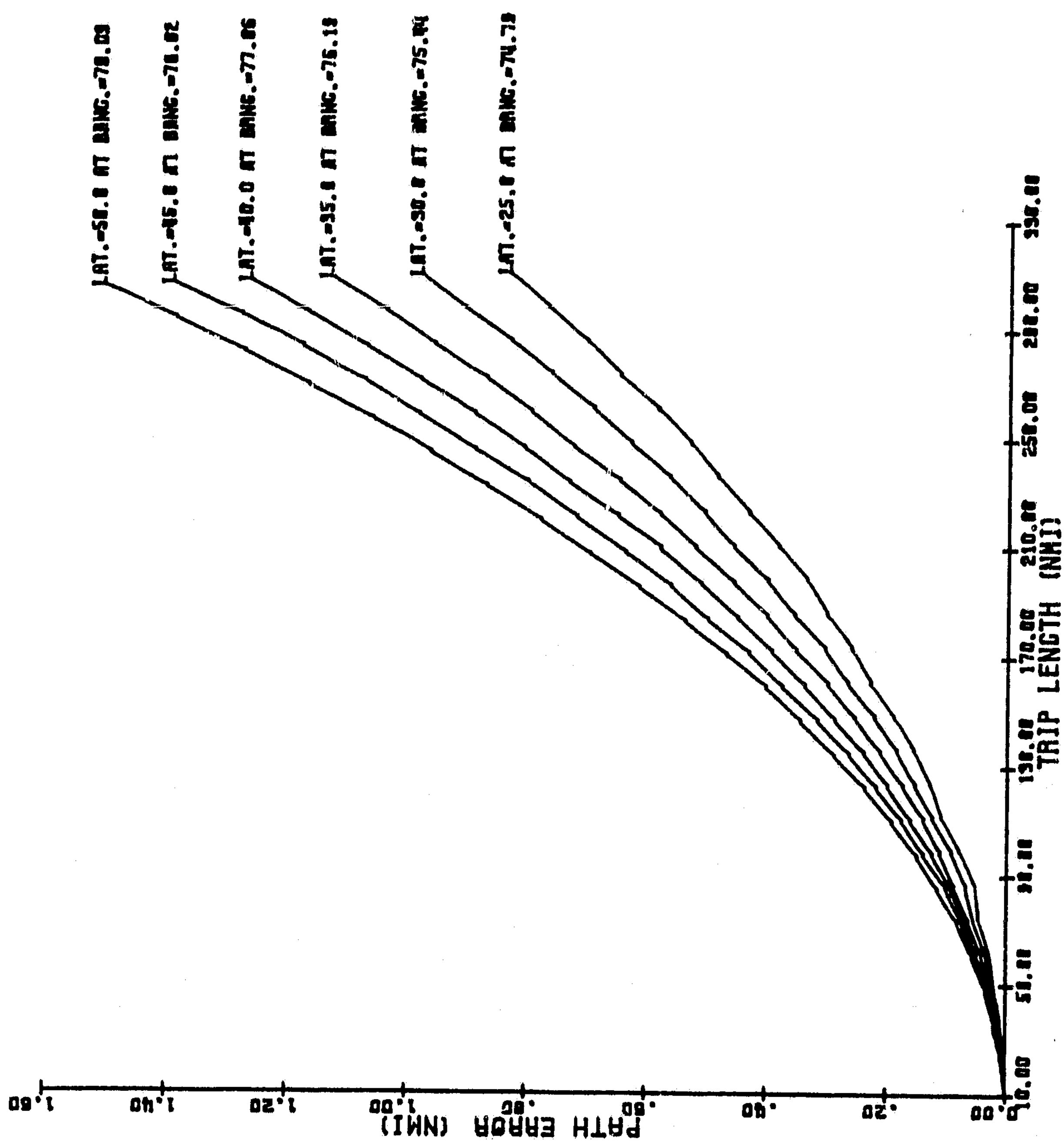

Figure 3. Path Errors vs. Trip Length at Average Bearing 76°.

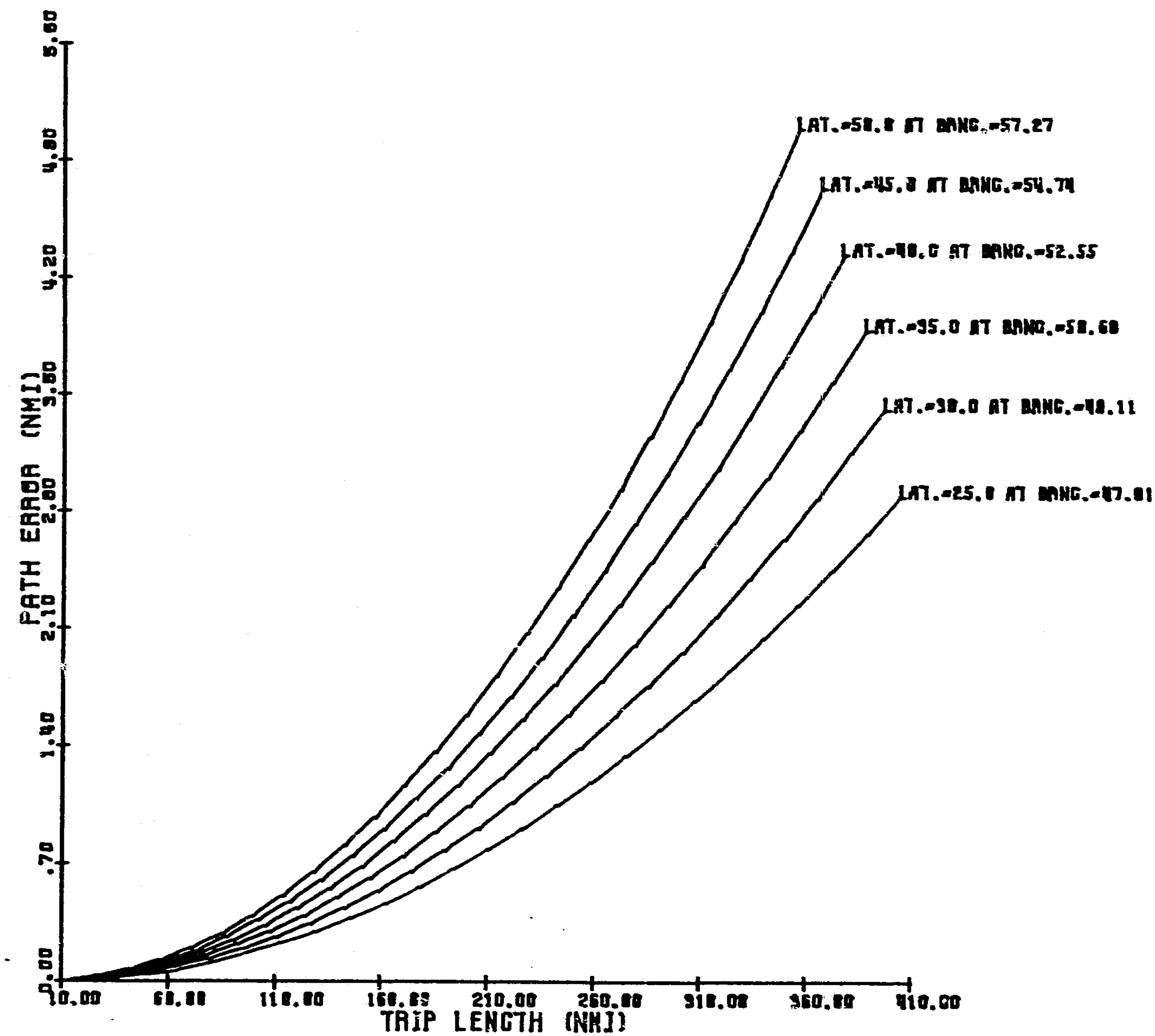

Figure 4. Path Errors vs. Trip Length at Average Bearing 52°.

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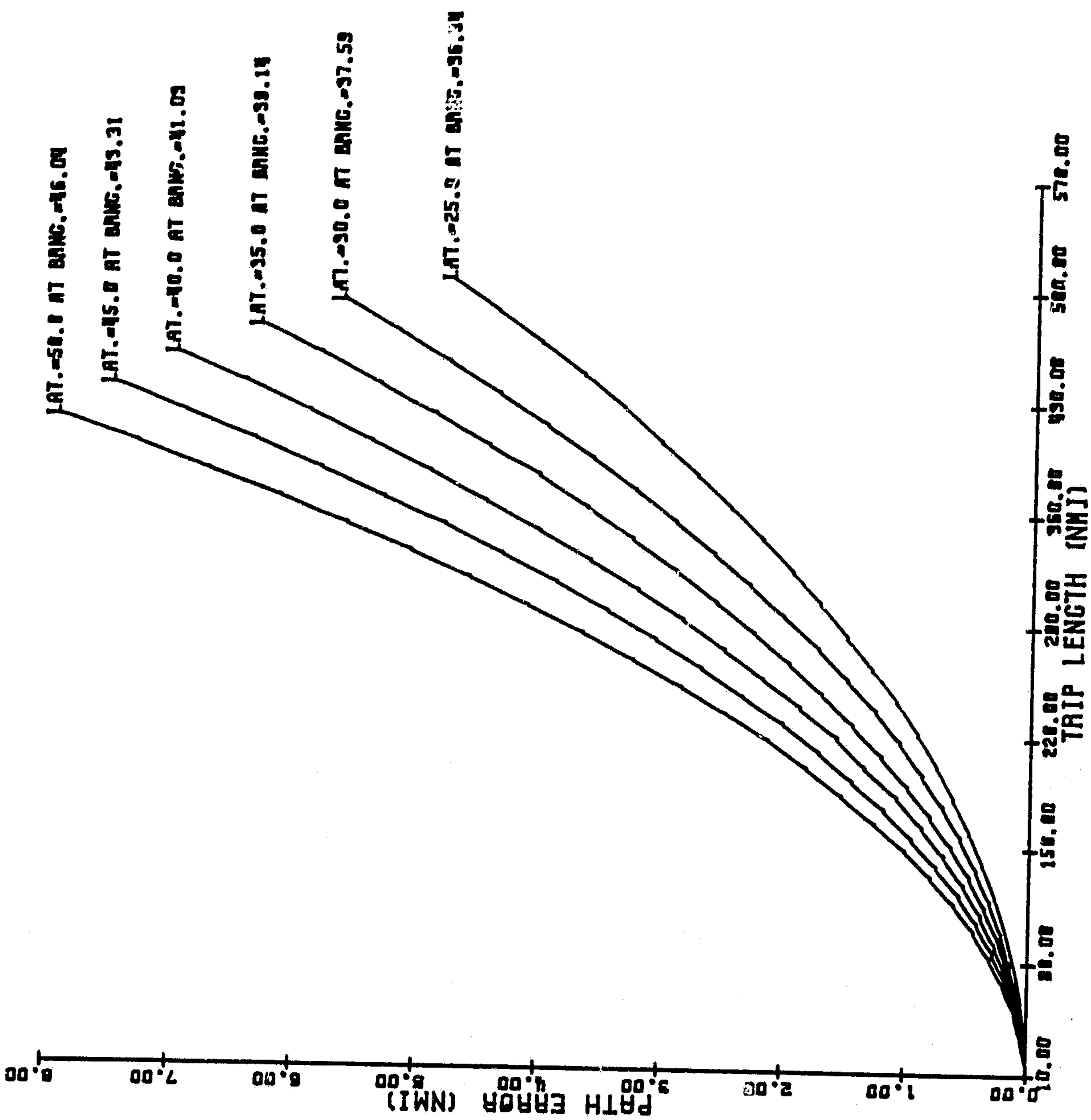

Figure 5. Path Errors vs. Trip Length at Average Bearing 40°.

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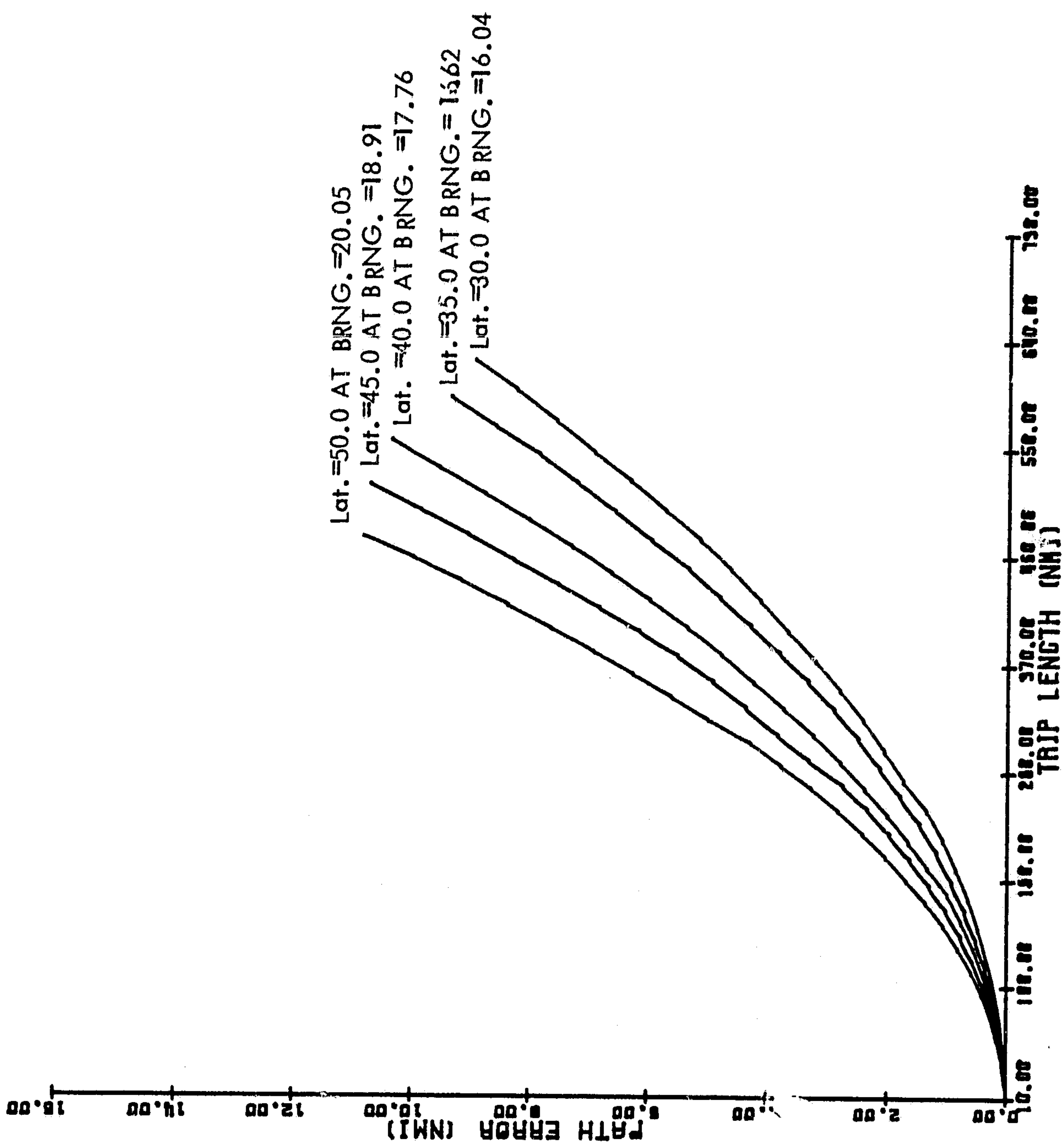

Figure 6. Path Errors vs. Trip Length at Average Bearing 18°.

VI. ACKNOWLEDGEMENTS

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[4] Ibid.