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A Mathematical Model for the Doubly-Fed Wound Rotor Generator

Part II

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Work performed for
U.S. DEPARTMENT OF ENERGY
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A MATHEMATICAL MODEL FOR THE DOUBLY-FED WOULD ROTOR GENERATOR – PART II

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Abstract – This paper is a mathematical analysis of a doubly-fed wound rotor generator. It is a continuation of a previous paper, which applied the constraints of constant stator voltage and frequency to the circuit equations and obtained expressions for the currents and voltages in the machine. In this paper the previously derived variables are redefined as direct and quadrature components. In addition, the apparent (complex) power for both the rotor and the stator are derived in terms of these redefined components.

INTRODUCTION

In the previous paper [1], the doubly-fed wound rotor generator was analyzed. The machine included a three-phase winding on both the rotor and the stator. The general circuit equations were reduced to two simplified equations, one for the rotor voltage, another for the stator voltage.

Applying the constraints of constant stator voltage and frequency to these two circuit equations, it was possible to obtain the necessary rotor excitation. The current and voltage expressions, which resulted, were used to derive expressions for rotor and stator real powers; and also, to show the relationships between shaft, rotor, and stator powers. Those results will be used in this paper. However, the variables are redefined into direct and quadrature components. The expressions for apparent power on both the rotor and the stator are derived using these redefined components.

SYMBOLS

a	ratio of stator number of turns to rotor number of turns
e_R, e_S	induced voltage, rotor, and stator, respectively
e_{SD}, e_{SQ}	induced stator voltage, direct, and quadrature components, respectively
i_R, i_S	instantaneous rotor and stator current, respectively
i_{RO}	rotor current required to generate open-circuit stator voltage
$i_{R\varepsilon}$	rotor current required to compensate for stator impedance drop
i_{RD}, i_{RQ}	rotor current, direct and quadrature components, respectively
K	coupling coefficient
L_R, L_S	self inductance, rotor, and stator, respectively
n_R, n_S	number of turns, rotor, and stator, respectively
R_R, R_S	resistance of winding, rotor, and stator, respectively
S	slip
t	time
v_R, v_S	terminal voltage, rotor, and stator, respectively
$v_{RO}, v_{R\varepsilon}$	rotor voltage required to establish i_{RO} and $i_{R\varepsilon}$, respectively
$\overline{x}$	the bar above a variable indicates it is a phasor
$\theta_R, \theta_S, \theta_L$	phase angle shift, rotor, stator, and load, respectively
ω, ω_S	angular frequency, mechanical and synchronous electrical, respectively

PREVIOUSLY DERIVED VARIABLES

Before proceding with any new derivations, some of the results of the previous paper [1] will be restated. To make the development of the model easier to follow, some terms will be made more compact.

From the previous paper, the voltage and current at the stator terminals is,

$$\overline{v}_S = V_S e^{j(\omega_S t - \theta_R + \pi/2)} \tag{1-1}$$

and

$$\overline{i}_S = -I_S e^{j(\omega_S t - \theta_R - \theta_L + \pi/2)} \tag{1-2}$$

The induced stator voltage was given as,

$$\frac{3}{2} aKL_R \frac{d}{dt}\left[e^{j(p\omega t/2)}\overline{i}_R\right] \tag{1-3}$$

This variable was not derived explicitly. But if the expression for $\overline{i}_R$ is substituted into (1-3), the induced stator voltage becomes,

$$\overline{e}_S = V_S e^{j(\omega_S t - \theta_R + \pi/2)} + |Z_S| I_S e^{j(\omega_S t - \theta_R - \theta_L + \theta_S + \pi/2)} \tag{1-4}$$

It should be noted that the derivative of the product in (1-3) produces the sum of two terms. The induced stator voltage is, more precisely,

$$\bar{e}_S = (1 - S)\bar{e}_S + S\bar{e}_S \tag{1-5}$$

Later in this paper, when the active and reactive powers are derived, this separation of terms will have more meaning.

The rotor variables that were previously derived are as follows:

Rotor current,

$$\bar{i}_R = \bar{i}_{R0} + \bar{i}_{R\epsilon}$$

$$= \frac{2}{3}\frac{1}{aKX_R}\left[V_S e^{j(S\omega_S t-\theta_R)} + |Z_S| I_3 e^{j(S\omega_S t+\theta_S-\theta_R-\theta_L)}\right] \tag{1-6}$$

Rotor voltage,

$$\bar{v}_R = \bar{v}_{R0} + \bar{v}_{R\epsilon}$$

$$= \frac{2}{3}\frac{1}{aKX_R}|Z_R|\left[V_S e^{j(S\omega_S t)} + |Z_S| I_S e^{j(S\omega_S t+\theta_S-\theta_L)}\right] + \frac{3}{2}\frac{K}{a} SX_S I_S e^{j(S\omega_S t-\theta_R-\theta_L)} \tag{1-7}$$

The voltage induced on the rotor is given by,

$$\frac{3}{2}\frac{K}{a} L_S \frac{d}{dt}\left[e^{-j(\rho\omega t/2)}\bar{i}_S\right] \tag{1-8}$$

If the expression for $\bar{i}_S$ is substituted into (1-8), the result is the induced rotor voltage, $\bar{e}_R$.

$$\bar{e}_R = \frac{3}{2}\frac{K}{a} SX_S I_S e^{j(S\omega_S t-\theta_R-\theta_L)} \tag{1-9}$$

In order to make the expressions for $\bar{v}_R$ and $\bar{i}_R$ more compact, the following definitions will be made:

$$I_{R0} = \frac{2}{3}\frac{1}{aKX_R} V_S \tag{1-10}$$

$$I_{R\epsilon} = \frac{2}{3}\frac{1}{aKX_R} |Z_S| I_S \tag{1-11}$$

$$E_R = \frac{3}{2}\frac{K}{a} SX_S I_S \tag{1-12}$$

When these definitions are substituted into equations (1-6) and (1-7), the results are,

$$\bar{i}_R = I_{R0} e^{j(S\omega_S t-\theta_R)} + I_{R\epsilon} e^{j(S\omega_S t+\theta_S-\theta_R-\theta_L)} \tag{1-13}$$

$$\bar{v}_R = |Z_R|\left[I_{R0} e^{j(S\omega_S t)} + I_{R\epsilon} e^{j(S\omega_S t+\theta_S-\theta_L)}\right] + E_R e^{j(S\omega_S t-\theta_R-\theta_L)} \tag{1-14}$$

ROTOR PHASOR DIAGRAM

Equations (1-13) and (1-14) describe the voltage and current as seen at the rotor terminals of the machine. Since these expressions are in phasor form, a phasor diagram can be constructed from them directly. This diagram is a visual aid to what happens on the rotor as slip and stator current varies. it will also be used to explain the basis for the direct and quadrature currents that will be derived later.

The phasor diagram (Fig. 1) shows the relative phase angles between the rotor variables. The reference phasor is $\bar{v}_{R0}$, since it was assumed to have zero phase angle. In order to draw the other variables, three assumptions must be made. First, the value of $|Z_R|$ is arbitrary, but the relationship $V_{R0} = |Z_R| I_{R0}$ must be maintained. Second, by manipulation of equations (1-10) and (1-11) it can be shown that the relationship $I_{R\epsilon} = (|Z_S|/|Z_L|)I_{R0}$ must hold. Finally, the variable, E_R can be scaled using equation (1-12), and assuming that $X_S \cong |Z_S|$.

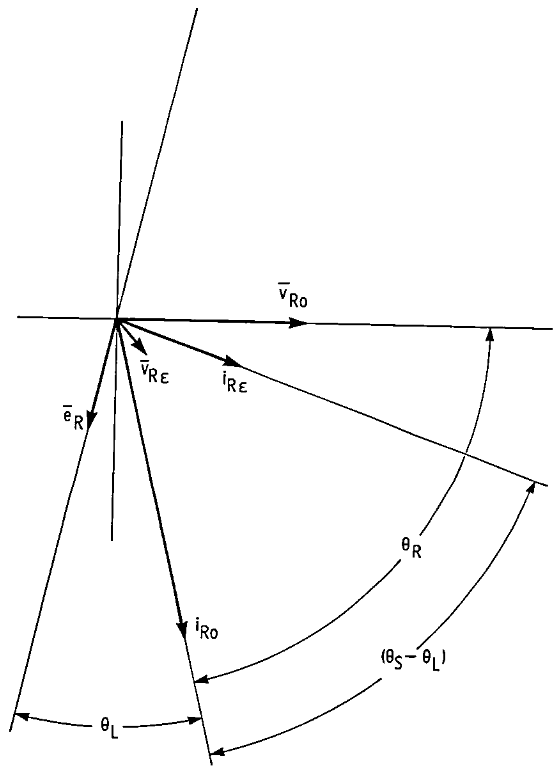

Figure 1. - Rotor phasor diagram. (Shown for positive slip.)

DIRECT AND QUADRATURE ROTOR CURRENTS

The induced stator voltage consists of two components, one that is in phase with the stator current, another leading the current by 90°. This can be shown by modifying equation (1-4). In order to make a comparison between voltage and current, the voltage must be placed in the same reference frame as the current. This is done by factoring equation (1-4) into the following form,

$$\bar{e}_S = \left[V_S e^{j\theta_L} + |Z_S| I_S e^{j\theta_S}\right] e^{j(\omega_S t-\theta_R-\theta_L+\pi/2)} \tag{2-1}$$

Comparing this equation to (1-2), it can be seen that both phasors $\bar{e}_S$ and $\bar{i}_S$ have the same rotational reference; namely,

$$e^{j(\omega_S t-\theta_R-\theta_L+\pi/2)}$$

The terms within the brackets of equation (2-1) are the phasor components within the reference frame.

As they are written, the two components in equation (2.1) are not orthogonal. They can be made so by using the identity,

$$e^{j\theta} = \cos\theta + (\sin\theta)e^{j(\pi/2)} \qquad (2\text{-}2)$$

and combining like components.

Equation (2-1) becomes

$$\bar{e}_S = \left[(V_S \cos\theta_L + R_S I_S) + (V_S \sin\theta_L + X_S I_S)e^{j(\pi/2)}\right] \times e^{j(\omega_S t-\theta_R-\theta_L+\pi/2)} \qquad (2\text{-}3)$$

The first component is in phase with the stator current. It will be called the direct component, that is,

$$\bar{e}_{SD} = (V_S \cos\theta_L + R_S I_S)e^{j(\omega_S t-\theta_R-\theta_L+\pi/2)} = E_{SD}e^{j(\omega_S t-\theta_R-\theta_L+\pi/2)} \qquad (2\text{-}4)$$

The second component leads the current by 90°. It will be called the quadrature component, that is,

$$\bar{e}_{SQ} = (V_S \sin\theta_L + X_S I_S)e^{j(\pi/2)}e^{j(\omega_S t-\theta_R-\theta_L+\pi/2)} = E_{SQ}e^{j(\omega_S t-\theta_R-\theta_L+\pi)} \qquad (2\text{-}5)$$

The components $\bar{e}_{SD}$ and $\bar{e}_{SQ}$ will be shown to have a relationship to the rotor current.

The expression for rotor current, equation (1-6) can be rewritten in the following form

$$\bar{i}_R = \frac{2}{3}\frac{1}{aKX_R}\left[V_S e^{j\theta_L} + |Z_S| I_S e^{j\theta_S}\right]e^{j(S\omega_S t-\theta_R-\theta_L)} \qquad (2\text{-}6)$$

Using equality (2-2), $\bar{i}_R$ can be separated into orthogonal components.

$$\bar{i}_R = \frac{2}{3}\frac{1}{aKX_R}\left[(V_S \cos\theta_L + |Z_S| I_S \cos\theta_S) + (V_S \sin\theta_L + |Z_S| I_S \sin\theta_S)e^{j(\pi/2)}\right]e^{j(S\omega_S t-\theta_R-\theta_L)} \qquad (2\text{-}7)$$

If the appropriate substitutions are made, using equations (1-10) and (1-11), the rotor current becomes,

$$\bar{i}_R = \left[(I_{RO} \cos\theta_L + I_{R\epsilon} \cos\theta_S) + (I_{RO} \sin\theta_L + I_{R\epsilon} \sin\theta_S)e^{j(\pi/2)}\right]e^{j(S\omega_S t-\theta_R-\theta_L)} \qquad (2\text{-}8)$$

The rotor current can now be expressed in orthogonal components. Equation (2-8) becomes

$$\bar{i}_R = \left[I_{RD} + I_{RQ}e^{j(\pi/2)}\right]e^{j(S\omega_S t-\theta_R-\theta_L)} \qquad (2\text{-}9)$$

where

$$I_{RD} = I_{RO} \cos\theta_L + I_{R\epsilon} \cos\theta_S \qquad (2\text{-}10)$$

and

$$I_{RQ} = I_{RO} \sin\theta_L + I_{R\epsilon} \sin\theta_S \qquad (2.11)$$

When equations (1-10) and (1-11) are substaituted into equations (2-10) and (2-11), the rotor components and stator componments are related as follows.

The relationship between direct components is,

$$E_{SD} = \frac{3}{2} aKX_R I_{RD} \qquad (2\text{-}12)$$

The relationship between quadrature components is,

$$E_{SQ} = \frac{3}{2} aKX_R I_{RQ} \qquad (2\text{-}13)$$

ROTOR VOLT-AMPERES (APPARENT POWER)

In order to properly size the power supply feeding the rotor, the volt-ampere requirements of that circuit must be known. This can now be done by making use of the expressions for $\bar{i}_R$ and $\bar{v}_R$.

The rms value squared of rotor current, I_R^2, can be obtained by inspection of equation (2-9).

$$I_R^2 = \frac{1}{2}\left(I_{RD}^2 + I_{RQ}^2\right) \qquad (3\text{-}1)$$

To obtain a corresponding expression for rotor voltage, equation (1-14) can be expressed in terms of direct and quadrature rotor currents. The result is,

$$\bar{v}_R = \left\{|Z_R|\left[I_{RD} + I_{RQ}e^{j(\pi/2)}\right] + E_R e^{-j\theta_R}\right\}e^{j(S\omega_S t-\theta_L)} \qquad (3\text{-}2)$$

In order to obtain the rms value by inspection, the terms within the braces of equation (3-2) must be orthogonal. This can be done by using the identity,

$$e^{-j\theta_R} = \cos\theta_R + (\sin\theta_R)e^{-j(\pi/2)} \qquad (3\text{-}3)$$

Equation (3-2) becomes

$$\bar{V}_R = \left[\left(|Z_R|I_{RD} + E_R \cos r_R\right) + \left(|Z_R|I_{RQ} + E_R \sin \theta_R\right)e^{j(\pi/2)}\right]e^{j(S\omega_S t - \theta_L)} \qquad (3\text{-}4)$$

Since the phasors of equation (3-4) are now separated by 90°, the rms value squared of rotor voltage, V_R^2, can be written as,

$$V_R^2 = \frac{1}{2}\left(|Z_R|I_{RD} + E_R \cos \theta_R\right)^2 + \frac{1}{2}\left(|Z_R|I_{RQ} + E_R \sin \theta_R\right)^2 \qquad (3\text{-}5)$$

The rotor volt-amperes can be found by taking the product of equations (3-1) and (3-5), $V_R^2 I_R^2$; the result is

$$V_R^2 I_R^2 = \frac{1}{4}\left[R_R\left(I_{RD}^2 + I_{RQ}^2\right) + \frac{L_S}{a^2 L_R} SE_{SD}I_S\right]^2 + \frac{1}{4}\left[(1 + K)SX_R\left(I_{RD}^2 + I_{RQ}^2\right) + \frac{L_S}{a^2 L_R} SE_{SQ}I_S\right]^2 \qquad (3\text{-}6)$$

Inspection of equation (3-6) gives the rotor real power, P_R; and reactive power, Q_R.

The second term within each bracket of equation (3-6) is the real and reactive powers transferred between rotor and stator. To show that these same terms appear in stator power expressions, equation (2-3) can be written as follows,

$$\bar{e}_S = \left[E_{SD} + E_{SQ}e^{j(\pi/2)}\right]e^{j(\omega_S t - \theta_R - \theta_L + \pi/2)} \qquad (3\text{-}7)$$

The rms value squared of $\bar{e}_S$, from equation (3-7), is

$$E_S^2 = \frac{1}{2}\left(E_{SD}^2 + E_{SQ}^2\right) \qquad (3\text{-}8)$$

However, referring back to equation (1-5), it can be seen that the stator induced voltage expands into two components; one related to mechanical speed, another, to slip speed. It can be shown that an equivalent form of equation (3-8) is the following,

$$E_S^2 = \frac{1}{2}\left[(1 - S)E_{SD} + SE_{SD}\right]^2 + \frac{1}{2}\left[(1 - S)E_{SQ} + SE_{SQ}\right]^2 \qquad (3\text{-}9)$$

When this equation is multiplied by I_S^2, the result is the apparent (complex) power for the stator circuit; namely

$$E_S^2 I_S^2 = \frac{1}{4}\left[(1 - S)E_{SD}I_S + SE_{SD}I_S\right]^2 + \frac{1}{4}\left[(1 - S)E_{SQ}I_S + SE_{SQ}I_S\right]^2 \qquad (3\text{-}10)$$

When equation (3-10) is compared with equation (3-6), the terms $SE_{SD}I_S$ and $SE_{SQ}I_S$ are the real and reactive powers, respctively, that are transferred between rotor and stator. Equation (3-10) gives the stator real power, P_S; and the reactive power, Q_S.

$$P_S = E_{SD}I_S \qquad (3\text{-}11)$$

$$Q_S = E_{SQ}I_S \qquad (3\text{-}12)$$

SUMMARY

The rotor currents and induced stator voltages are redefined into direct and quadrature components. The quantative relationship between direct rotor current and direct stator voltage is shown; also, a similar relationship is given for quadrature rotor current and quadrature stator voltage. These components are used to derive expressions for the apparent (complex) power on both the rotor and stator.

This paper, along with the first paper, provide a complete description of rotor and stator variables, as well as the real and reactive power flowing in the doubly-fed generator.

REFERENCE

1. F. J. Brady, "A Mathematical Model for the Doubly Fed Wound Rotor Generator," NASA-TM-83454, 1983. IEEE Power Engineering Society, 1983 SM479-3.

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