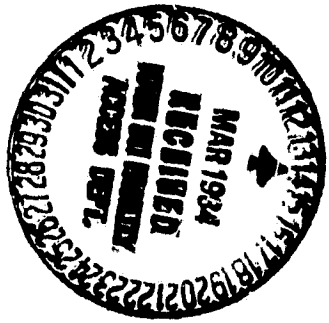


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A RENORMALIZATION GROUP MODEL FOR THE STICK-SLIP BEHAVIOR OF FAULTS

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Abstract

We treat a fault as an array of asperities with a prescribed statistical distribution of strengths. When an asperity fails the stress on the failed asperity is transferred to one or more adjacent asperities. For a linear array the stress is transferred to a single adjacent asperity and for a two-dimensional array to three adjacent asperities. Using a renormalization group (RG) method to extrapolate to an arbitrarily large scale we show that the solutions bifurcate at a critical applied stress. At stresses less than the critical stress virtually no asperities fail on a large scale and the fault is locked. At the critical stress the solution bifurcates and asperity failure cascades away from the nucleus of failure; we interpret this catastrophic failure as an earthquake and it corresponds to the transition from stick to slip behavior on the fault. Thus the stick-slip behavior of most faults can be attributed to the distribution of asperities on the fault. Our results explain why stick-slip behavior on faults is commonly observed rather than stable sliding, they explain why the observed level of seismicity on a locked fault is very small; and they explain why the stress on a fault is less than that predicted by a standard value of the coefficient of friction.

Introduction

This paper presents a simple friction model to provide an understanding of the stick-slip behavior of faults. A large fraction of active faults appear to behave in a stick-slip rather than a stable sliding manner. For a fault system such as the San Andreas, which has well over 1000 km of surface exposure, approximately 55 mm/yr of relative motion, and has been extensively studied (Turcotte, 1977), it is possible to differentiate between these two alternate behaviors. The San Andreas can be subdivided into three principal sections based on the type of behavior observed: northern and southern locked sections and a central creeping section. There is strong evidence that the northern and southern locked sections, the sites of the 1906 and 1857 earthquakes, behave in a stick-slip fashion. There has been no significant seismicity on these sections of the San Andreas in the period since adequate instrumentation has been available. In the central one, aseismic creep has been observed along with many small and moderate size earthquakes, suggesting a stable sliding type of behavior.

Since repetitive sequences of earthquakes are observed to occur on active fault systems it is appropriate to treat faults as approximately planar surfaces with a coefficient of friction. Using frequency magnitude and moment magnitude relationships Aki (1981) has shown that the fractal dimension of a fault is $D = 3b/c$, where b is the slope of the log frequency magnitude relation and c is the slope of the log moment magnitude relation. For $c = 1.5$ (Hanks and Kanamori, 1979) and $b = 1$, the fractal dimension is 2, the same as the topological dimension of a plane (Aki, 1981). For regions where the b value is near 1 the planar approximation should therefore be quite good. Since most earthquakes occur on preexisting faults it is not appropriate to model earthquakes on active faults as the fracture of pristine

rock. While surficial fault traces have been observed to go through previously unfractured rock, there is much evidence that earthquakes repeatedly occur on the same fault surface or zone of surfaces (Sieh, 1978).

We will therefore model a fault system which can contain fault trace offsets, multiple fault traces and varying amounts of curvature or roughness as an array of asperities with a statistical distribution of strengths. There is evidence that large asperities, or barriers, can control the propagation of an earthquake along a fault.

Several authors have previously modelled the stick-slip behavior of faults in terms of frictional effects. Weertmann (1979) modelled the instability in terms of a frictional stress on a fault that decreases with increasing slip velocity. Stuart and Mavko (1979) modelled the instability in terms of a strain-softening constitutive relation for the fault zone. In this paper we present an alternative hypothesis to explain the stick-slip behavior of faults.

In terms of mathematical modelling an earthquake is clearly a catastrophic change in the behavior of the system. Recently, renormalization group (RG) techniques have been successfully applied to models that exhibit catastrophic behavior of the type found in natural systems (Wilson and Kogut, 1974; Fisher, 1974). A classical example is that of a system undergoing a phase transition.

Renormalization group (RG) techniques have been used by Madden (1981) to relate the macroscopic electrical conductivity and fracture of rocks to the microcrack population, and by Allegre et al. (1982) to study the coalescence of fractures. Newman and Knopoff (1982, 1983) have also studied the coalescence of fractures and while they use the term renormalization in their

work no rescaling is done so their approach is substantially different from the usual RG methods.

Formulation of the Problem

In this paper we model a fault as an array of asperities with a statistical distribution of strengths. We will consider both linear and two-dimensional arrays of asperities. In order to illustrate the approach we will first consider a linear array as illustrated in Figure 1. This model should be appropriate for large scale asperities (barriers) on a long fault. The fault is broken into n elements of length δx and each element is assigned an asperity failure strength f ; the asperity will fail when the stress on the asperity reaches this value. The asperities have a distribution of strengths which will be specified by a statistical distribution function. When a stress σ is applied to the fault all asperities with a failure strength $f < \sigma$ will fail.

We will divide the linear array of n asperities into $n/2$ cells, each containing two asperities as illustrated in Figure 1. When one asperity in a cell fails we assume that the stress on that asperity is transferred to the other asperity in the cell. This is an essential feature of our model and is equivalent to the transfer of stress to adjacent regions when a crack is introduced into an elastic solid. The cell size is a measure of the distance over which the stress is redistributed after an asperity failure. The reason we assume that there are two asperities in a cell is that the stress in the failed region is applied over a length which is of the order of the length of the failed region.

A cell may contain two broken, two unbroken, or a broken and an unbroken asperity. If a cell contains a broken and an unbroken asperity the strength

σ_f of the unbroken asperity must be greater than 2σ if it is to survive the transfer of stress σ from the broken asperity. We assume that a cell fails only when both asperities in the cell fail. The basis of the RG approach is that after the first renormalization, each first order cell is now treated as a second order ($r = 2$) asperity and pairs of second order asperities form second order cells. This is illustrated in Figure 1. The statistics of failure of the second order asperities and cells is the same as the first order asperities and cells. The process is repeated by iteration to infinite order. It should be emphasized that the same statistical distribution of asperity strengths is applied to the higher order cells even though asperities are destroyed at each of the lower levels of calculation. This is an essential feature of the RG method.

The process of stress transfer and induced failure tends to increase the lengths of segments of broken asperities. As the applied stress is increased, a value is reached at which failure of an infinite length of asperities will occur and the behavior changes catastrophically from stick to slip. The stress at which this change occurs is equivalent to the temperature at which a phase transition occurs. The statistical distribution of energies in a solid, liquid, or gas is equivalent to the statistical distribution of asperity strengths in our model. The utilization of the RG method allows us to study the development of failed segments as the characteristic lengths of the failed segments increase with increased applied stress.

We first consider the distribution of asperity strengths. Clearly a wide variety of strengths on a wide variety of scales must exist on any real fault. For example, fault bends and offsets correspond to strong asperities. However, data on actual distributions of asperity strengths are

not available. It is possible that studies of the type given in this paper may allow asperity distributions to be inferred from such seismic observations as the dependence of earthquake frequency on magnitude. In the absence of applicable data we assume a quadratic Weibull distribution for the probability P_a that the failure strength σ_f of an asperity is less than the stress σ

$$P_a = 1 - e^{-(ax)^2} \quad (1)$$

where

$$x = \sigma/\sigma_0$$

and σ_0 is a reference asperity strength. Weibull distributions are often used to represent a statistical distribution of failure strengths (Harlow and Phoenix, 1982). It should be emphasized that our approach can be applied to any continuous distribution of asperity strengths. The probability

$$P_1 = 1 - e^{-x^2} \quad (2)$$

that $\sigma_f \leq \sigma$ is shown in Figure 2a as a function of σ/σ_0 . Ten percent of the asperities have failed when $\sigma/\sigma_0 = 0.32$, fifty percent of the asperities have failed when $\sigma/\sigma_0 = 0.83$, and ninety per cent of the asperities have failed when $\sigma/\sigma_0 = 1.52$. Since the relation between P_1 and σ is invertible, P_1 can be used as a measure of the applied stress. The probability that failure will occur at the applied stress σ/σ_0 is given by dP_1/dx and is shown in Figure 2b. The probability that $\sigma_f = \sigma$ is zero at zero stress and increases to a maximum at $\sigma = 0.71 \sigma_0$. The mean strength of an asperity is $\bar{\sigma} = (\sqrt{\pi}/2)\sigma_0 = 0.8862\sigma_0$.

An essential feature of our model is the transfer of stress from a failed asperity to its nearest neighbors. Without this transfer of stress

the behavior of the system is simple and uninteresting. Since strong asperities will not break until large stresses are applied they can block the propagation of broken segments and there is no change from stick to slip behavior on the fault. It is the transfer of stress from broken asperities onto the remaining unbroken asperities that leads to catastrophic behavior at an applied stress that is less than the average strength of the asperities. It is clear that stress transfer to the adjacent unbroken sections of a fault will occur on a real fault.

In order to quantify the failure of asperities due to the transfer of stress we introduce the conditional probability $P_{a,b}$ that failure will occur when a stress $(a-b)\sigma$ is transferred to an unbroken asperity supporting a stress $b\sigma$, so that the final stress on the asperity is $a\sigma$. This conditional probability is given by

$$P_{a,b} = \frac{P_a - P_b}{1 - P_b}, \quad (3)$$

with

$$P_a = 1 - (1 - P_1)^a \quad (4)$$

for the probability function given in (1).

In principle this problem could be solved without the use of the RG technique. However, the range of scales that could be studied is quite limited even with the largest computers available. Although we will utilize the renormalization group method in the standard manner, it should be recognized that the approach is semi-empirical and has been principally justified by its success in solving a variety of fundamental unsolved problems in physics. These problems fall in a broad class in which a

continuous system on the microscopic scale exhibits catastrophic behavior on the macroscopic scale. We argue that the stick-slip behavior of faults falls in this general classification of physical problems.

We will first illustrate the application of the renormalization group technique to the failure of a linear array of asperities using a basic cell composed of two "asperities" and the probability distribution given in (1). Note that the $(r+1)$ th order "asperities" which result from r iterations of the RG transformation contain 2^r actual first order asperities. The first three renormalizations are illustrated in Figure 1. For a cell containing two asperities which are either broken or unbroken, four states are possible: 1) [bb], 2) [bu], 3) [ub], and 4) [uu], where b represents a broken asperity and u represents an unbroken asperity. Note that states 2 and 3 are equivalent and can be combined into a single state with a multiplicity of 2. The probabilities for each of these states neglecting any interactions between asperities is given in Table 1a.

Next it is necessary to consider the influence of a broken asperity on an adjacent unbroken asperity. We use the conditional probability $P_{2,1}$ that an unbroken asperity already supporting a stress σ will fail when an additional stress σ is transferred to it from an adjacent broken asperity. Including the effects of such induced failures leads to the probabilities given in Table 1b for each of the cell states. Since the conditional probability from (3) is given by

$$P_{2,1} = \frac{P_2 - P_1}{1 - P_1}, \quad (5)$$

the probabilities with stress interactions in Table 1b can be expressed in terms of P_1 and P_2 .

We must prescribe a condition for determining whether a r th order cell is broken or unbroken. We assume that a r th order cell is broken only if both "asperities" in the cell are broken. Under this condition the probability that a first order cell is broken $P_1^{(2)}$ is given by

$$P_1^{(2)} = 2P_1P_2 - P_1^2, \quad (6)$$

and substitution of P_2 from (4) gives

$$P_1^{(2)} = 2P_1[1 - (1 - P_1)^4] - P_1^2. \quad (7)$$

For higher order cells (7) is used as an iteration equation to determine $P_1^{(r+1)}$ from $P_1^{(r)}$, where r is the order of the cell being considered. Implicit in the RG method is the assumption that the probability distribution applicable to a first order, $r+1$, cell is also applicable to higher order cells. This assumption is an essential feature of the method and is clearly an approximation. The general form of (7) is

$$P_1^{(r+1)} = 2 P_1^{(r)} [1 - (1 - P_1^{(r)})^4] - (P_1^{(r)})^2. \quad (8)$$

The dependence of $P_1^{(r+1)}$ on $P_1^{(r)}$ is given in Figure 3. The points 0 and 1 are stable fixed points of the system. The straight line corresponding to $P_1^{(r+1)} = P_1^{(r)}$ is also included in Figure 3. The iterative relation crosses this straight line at $P_1^{(r)} = P^* = 0.2063$. We will show that P^* is an unstable fixed point that separates the region of stick behavior from the region of slip behavior.

The RG iteration can be performed graphically using Figure 3. For example, we take $P_1 = 0.6$ and from (8) find $P_1^{(2)} = 0.8093$. This cell behavior at order 1 now becomes the asperity behavior at order 2. To do this graphically a horizontal line is extended to the line $P_1^{(r+1)} = P_1^{(r)}$

to reflect the total cell behavior at order 1 into the asperity behavior at order 2. Thus the probability of cell failure at order 2 is $P_1^{(3)} = 0.9615$. This procedure is repeated to give $P_1^{(4)} = 0.9985$, etc., and the probability of failure rapidly approaches unity as the order is increased. On the other hand, if we take $P_1 = 0.1$ we find $P_1^{(2)} = 0.05478$, $P_1^{(3)} = 0.02184$, $P_1^{(4)} = 0.00322$, etc., and the probability of failure decreases towards zero as the order is increased. If $P_1 > P^*$ failure occurs for infinite length scales and slip behavior results. If $P_1 < P^*$ the behavior is stable and failure occurs only on the smallest scales. Bifurcation of the solution occurs at $P_1 = P^* = 0.2063$ and the critical stress leading to failure is $\sigma^* = 0.4807 \sigma_0$ from (1). The dependence of $P_1(r)$ on r for several values of P_1 is given in Figure 4. The bifurcation of the solution at $P_1 = P^* = 0.2063$ is clearly illustrated. Note that the value of the critical stress is considerably less than the value of the mean strength of an asperity $\sigma = 0.8862 \sigma_0$.

The stable behavior of the system at $\sigma < \sigma^*$ can be characterized by a correlation length L which measures the maximum length over which failure occurs for $P_1 < P^*$. The rapid increase of L as the threshold is approached from below is described by a power law

$$L \sim (P^* - P_1)^{-\nu}, \quad (9a)$$

or equivalently

$$L \sim (\sigma^* - \sigma)^{-\nu}, \quad (9b)$$

where ν is the correlation length exponent (Wilson and Kogut, 1974).

According to this result the magnitude of precursory seismicity would be expected to increase as the critical stress on the fault is approached. The

onset of catastrophic behavior at $\sigma = \sigma^*$ corresponds to the divergence of the correlation length L .

The correlation length exponent ν is easily obtained from the RG transformation (8). Given the dependence of $P_1(r+1)$ on $P_1(r)$, the slope of the curve in Figure 3 at $P_1 = P^*$ is given by

$$\Lambda \equiv \left. \frac{dP_1(r+1)}{dP_1(r)} \right|_{P_1 = P^*} \quad (10)$$

As long as $(P^* - P_1(r)) \ll 1$, a linear approximation to (8) is valid, and

$$\Lambda = \frac{P^* - P_1(r+1)}{P^* - P_1(r)} \quad (11)$$

It then follows that (Wilson and Kogut, 1974)

$$\Lambda = b^{1/\nu} \quad (12)$$

where b is the linear rescaling factor. For the $b = 2$ RG transformation that led to (8) we obtain $\Lambda = 1.6189$, so that $\nu = 1.4388$.

So far we have considered only a linear array of asperities. We will next consider a two-dimensional array of asperities distributed uniformly on a planar fault as illustrated in Figure 5. We will divide the two-dimensional array of n asperities into $n/4$ cells each containing four asperities. The failure of individual asperities will be treated in the same way as in the linear case and (1) is assumed to be applicable. When one or more asperities in a cell fail we assume that the stress on those asperities is transferred equally to the remaining asperities in the cell. That is, if one asperity fails the stress on the three remaining asperities is $4\sigma/3$. We choose four asperities in a cell so that the stress in the failed region is

applied over a length which is of the order of the length of the failed region. We again assume that the cell fails when all asperities in the cell fail.

A second order cell is composed of four first order cells or second order asperities and, therefore, sixteen primary asperities as illustrated in Figure 5. The statistics of failure of the second order asperities and cells is the same as the first order asperities and cells. Again, the process is repeated by iteration to infinite order. The RG transformation thus constructed corresponds to a linear rescaling factor $b = 2$ on a two-dimensional array. This case is considerably more complex than the linear array example considered above. Following the same procedure illustrated in Table 1 and using the definition of the conditional probability we find that the probability that a cell fails is given by

$$\begin{aligned}
 P_1^{(2)} = & P_1^4 + 4P_1^3(1 - P_1)P_{4,1} + 6P_1^2(1 - P_1)^2[P_{2,1}^2 \\
 & + 2P_{2,1}(1 - P_{2,1})P_{4,2}] + 4P_1(1 - P_1)^3\{P_{4/3,1}^3 \\
 & + 3P_{4/3,1}^2(1 - P_{4/3,1})P_{4,4/3} + 3P_{4/3,1}(1 - P_{4/3,1})^2[P_{2,4/3}^2 \\
 & + 2P_{2,4/3}(1 - P_{2,4/3})P_{4,2}]\} \quad , \quad (13)
 \end{aligned}$$

and introducing (3) we obtain

$$\begin{aligned}
 P_1^{(2)} = & P_1^4 + 4P_1^3(P_4 - P_1) + 6P_1^2(P_2 - P_1)^2 + 12P_1^2(P_2 - P_1)(P_4 - P_2) \\
 & + 4P_1(P_{4/3} - P_1)^3 + 12P_1(P_{4/3} - P_1)^2(P_4 - P_{4/3}) \\
 & + 12P_1(P_{4/3} - P_1)(P_2 - P_{4/3})^2 \\
 & + 24P_1(P_{4/3} - P_1)(P_2 - P_{4/3})(P_4 - P_2) \quad . \quad (14)
 \end{aligned}$$

The dependence of $P_1^{(r+1)}$ on $P_1^{(r)}$ shown in Figure 6 follows from introducing P_a from (4) and using (14) as an iterative relation.

The general behavior of this two-dimensional case is the same as that of the linear example considered above. Again, an S-shaped curve is generated. The points 0 and 1 are stable fixed points. The crossing at $P^* = 0.1707$ separates stick from slip behavior. From (1) the bifurcation of the solution occurs at $\sigma^* = 0.4327 \sigma_0$. This is just about one-half the mean strength of the asperities $\bar{\sigma} = 0.8862 \sigma_0$. We also find that $\Lambda = 2.357$; the correlation length exponent $\nu = 0.8084$ follows from (12) with $b = 2$. The quantitative differences between these results and those for the linear array, as listed in Table 2, illustrate the effect of the physical dimensionality on the critical behavior of the system. Note the decrease in both the critical probability P^* and the correlation length exponent ν with increasing D . Simpler percolation models exhibit the same trend when D is increased from two to three (Stauffer, 1979).

Conclusions

We have shown that a statistical distribution of asperity strengths leads to stick-slip behavior of a fault. The transfer of stress from failed sections of the fault to adjacent locked sections is an essential feature of our model. We have used the RG approach to obtain the behavior of the model as a function of applied stress. The main result is the existence of a finite critical stress below which the fault breaks are always bounded in their growth.

We also find that the value of the critical stress is considerably smaller than the mean strength of the asperities. This may explain the low stress levels associated with displacements on the San Andreas fault. Laboratory studies of friction generally result in a coefficient of friction near 0.6 (Byerlee, 1978). However, the low measured heat flow adjacent to

the San Andreas fault is strong evidence that the equivalent coefficient of friction on the fault is about a factor of two less than the laboratory value (Turcotte et al., 1980). We find that the critical stress on a fault is about a factor of two less than the mean strength of asperities. A similar result has been found for fibrous materials (Harlow and Phoenix, 1982).

Clearly the model considered in this paper is based on a number of simplifications. These include:

1) The assumption that the applied stress σ on the fault is a constant is not a good approximation. The stress on an actual fault will have spacial variations. Stress concentrations are expected to occur at the edges of locked sections. These concentrations would be expected to initiate catastrophic failures on a fault. However, significant levels of stress will have accumulated on all sections of the fault and our model shows how a failure, once nucleated, can spread due to the transfer of stress.

2) The form of the asperity strength distribution given in (1) is arbitrary. However, the RG method given here is applicable to any continuous distribution of asperity strengths.

3) Failure on actual faults does not extend to infinity. There is ample observational evidence that strong asperities (barriers) can block the propagation of a zone of failure. This limiting behavior can be included in our analysis by utilizing a more complex relationship for the distribution of asperity strengths.

4) The assignment of the same scale δx to all asperities is a poor approximation. Clearly large asperities may have larger physical dimensions than smaller asperities. On the smallest scale asperities may have atomic dimensions whereas major barriers such as bends in a fault may have

dimensions of tens of kilometers. However, this additional complication would not be expected to affect the qualitative predicted behavior.

We would argue that our results make physical sense. Once a broken patch on a fault starts to grow the transfer of stress is sufficient to break adjacent asperities as long as the distribution of strong asperities is sufficiently small. Obviously real fault breaks have finite lengths. We argue that a fault break is terminated when a very strong asperity is reached. This is confirmed by the observation that long fault breaks are often terminated by bends or offsets in the fault trace. The scale upon which large asperities are distributed will determine the length of fault breaks and the behavior of the fault.

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[bb]	[ub]	[uu]
P_1^2	$2P_1(1 - P_1)$	$(1 - P_1)^2$

a) Probabilities without stress interactions

[bb]	[ub]	[uu]
$P_1^2 + 2P_1(1 - P_1)P_{2,1}$	$2P_1(1 - P_1)(1 - P_{2,1})$	$(1 - P_1)^2$
$2P_1P_2 - P_1^2$	$2P_1(1 - P_2)$	$(1 - P_1)^2$

b) Probabilities with stress interactions

[b ₂]	[u ₂]
$2P_1P_2 - P_1^2$	$1 + P_1^2 - 2P_1P_2$

c) Probability of failure applied to the next order cell

Table 1. Renormalization group applied to a basic cell composed of two asperities. The probability of the failure of an individual asperity is P_1 and the probability of an induced failure is P_2 .

D	P*	σ^*/σ_0	ν
1	0.2063	0.4807	1.4388
2	0.1707	0.4327	0.8084

Table 2. Renormalization group results for the critical probability P^* , the critical stress σ^*/σ_0 , and the correlation length exponent ν in one and two dimensions.

Figure Captions

- Figure 1. Illustration of the statistical distribution of asperity strengths σ_f for a linear array; σ_0 is a reference asperity strength. An asperity is assigned to each unit length δx . Also shown are the cell sizes for orders $r = 1$ to 4.
- Figure 2. (a) Dependence of the probability P_1 that failure of an asperity will have occurred on the normalized stress σ/σ_0 . (b) Dependence of the probability that failure will occur at the normalized stress σ/σ_0 . This is the change in the probability of failure δP_1 when there is a change in the normalized stress $\delta(\sigma/\sigma_0)$.
- Figure 3. Dependence of the probability of failure for the $r+1$ cell $P_1(r+1)$ on the probability of failure of the r cell $P_1(r)$ for cells containing two asperities with a quadratic Weibull distribution of strengths. The procedure described in the text for determining the probability of cell failure for successive iterations is illustrated for $P_1 = 0.6, 0.1$. The critical probability of failure P^* gives the bifurcation point for catastrophic failure of the system. If $P_1 < P^*$ the solution iterates to $P_1^\infty = 0$ and no failure occurs. If $P_1 > P^*$ the solution iterates to $P_1^\infty = 1$ and the system has failed.
- Figure 4. Dependence of the probability of failure $P_1(r)$ on the order r for several values of P_1 . The bifurcation of the solution at $P_1 = P^* = 0.2063$ is clearly illustrated.
- Figure 5. Illustration of the two-dimensional array of asperities with four asperities per cell. Second (2), third (3), and fourth (4) order cells are also shown.

Figure 6. Dependence of the probability of failure for the $r+1$ cell $P_1(r+1)$ on the probability of failure of the r cell $P_1(r)$ for cells containing four asperities with a quadratic Weibull distribution of strengths. The critical probability of failure P^* gives the bifurcation point for the system. If $P_1 < P^*$ the solution iterates to $P_1^\infty = 0$ and no failure occurs. If $P_1 > P^*$ the solution iterates to $P_1^\infty = 1$ and the system has failed.

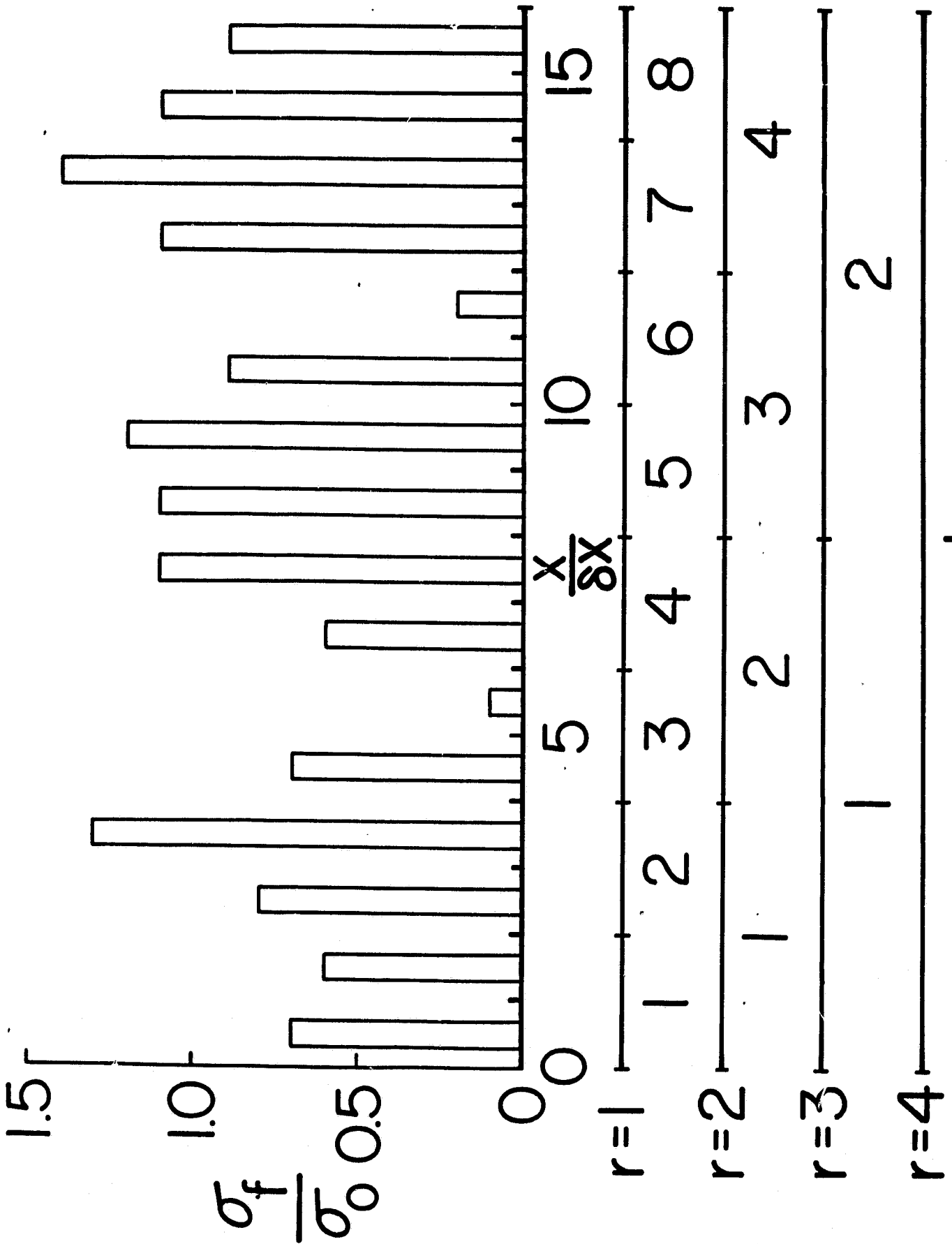
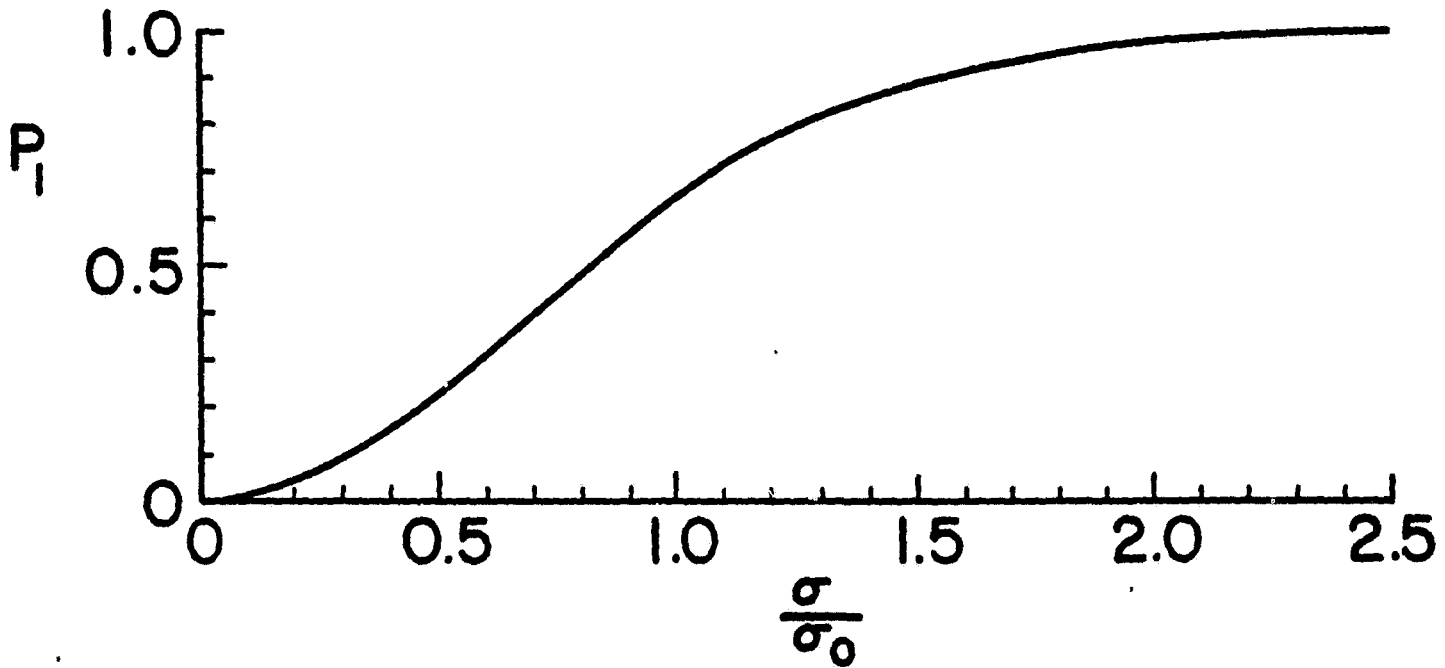


Figure 1

ORIGINAL PAGE IS
OF POOR QUALITY

(a)



(b)

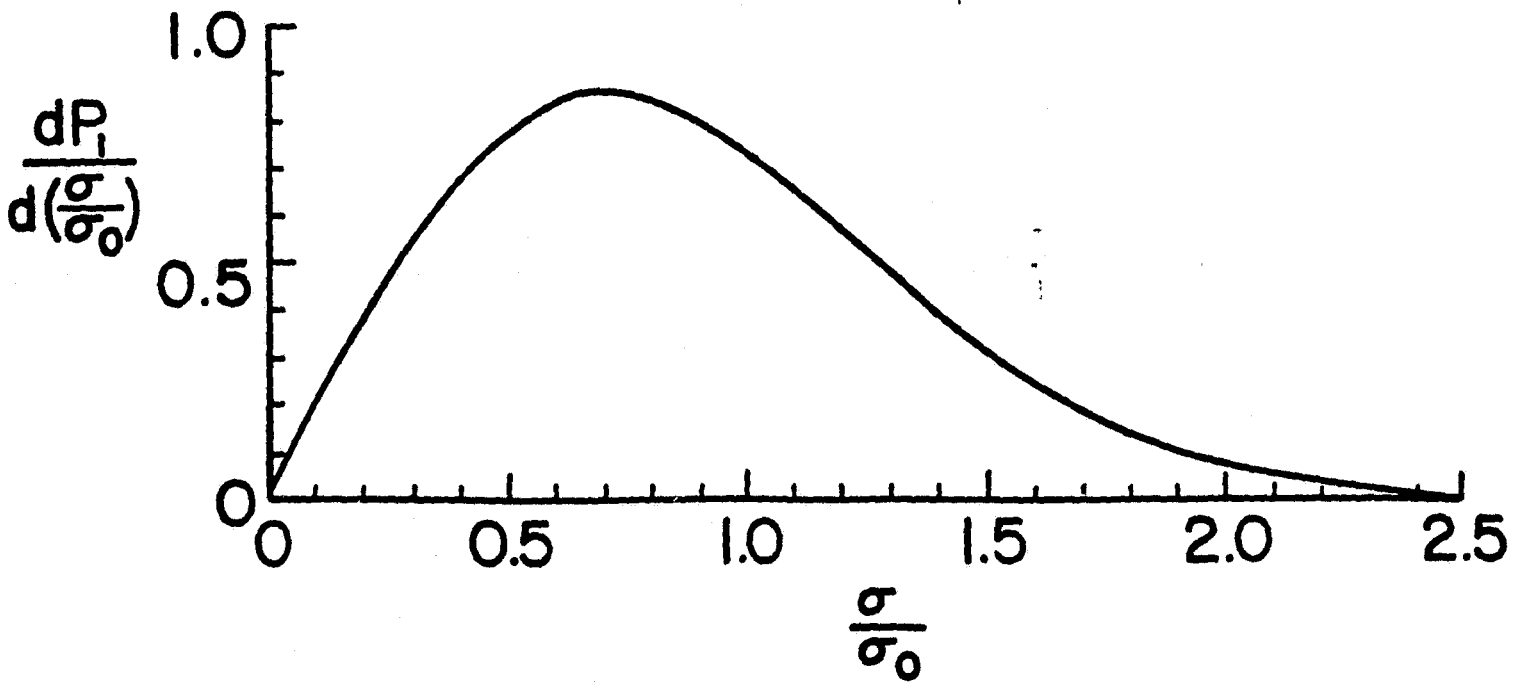


Figure 2

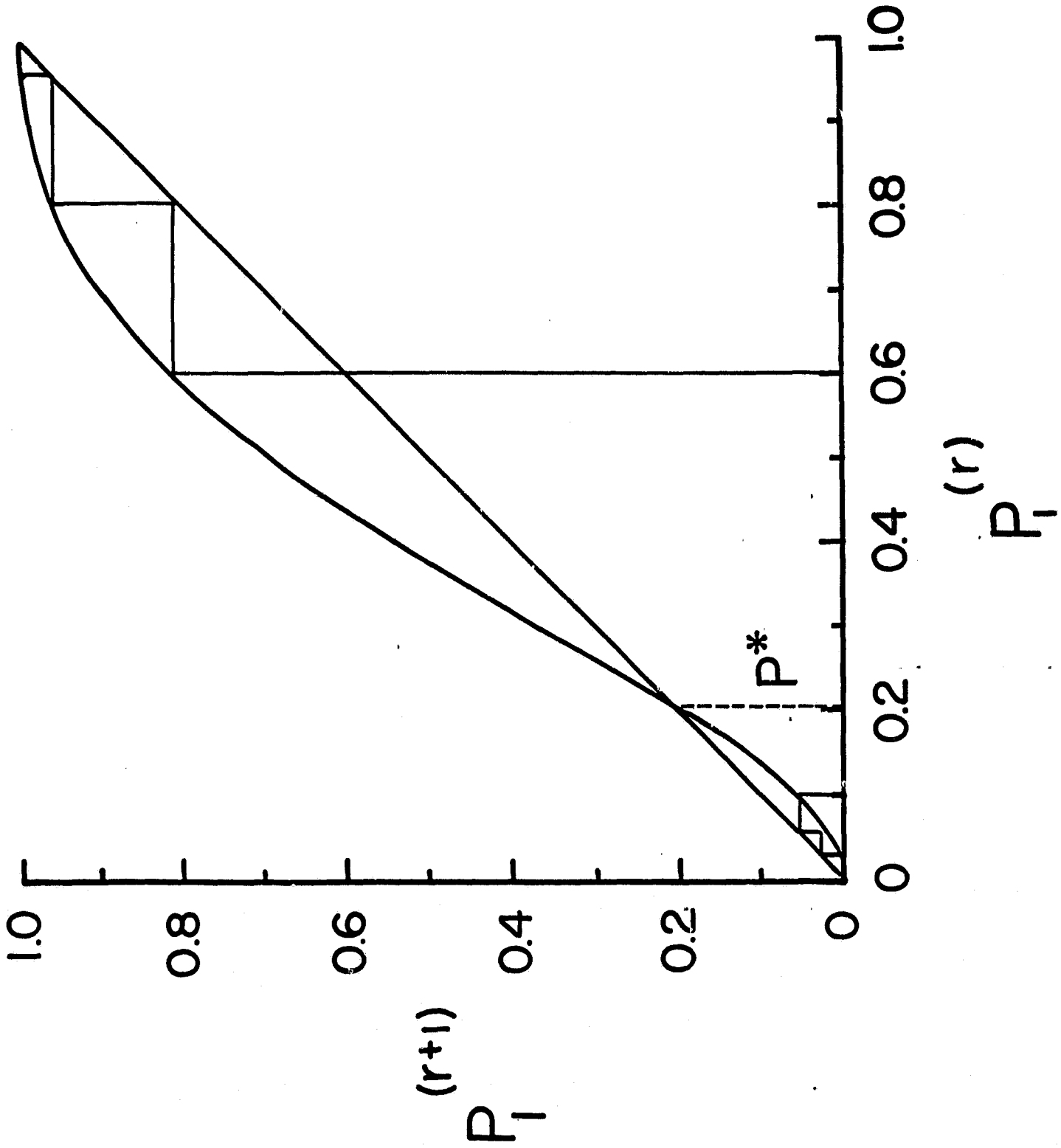


Figure 3

OF 100% QUALITY

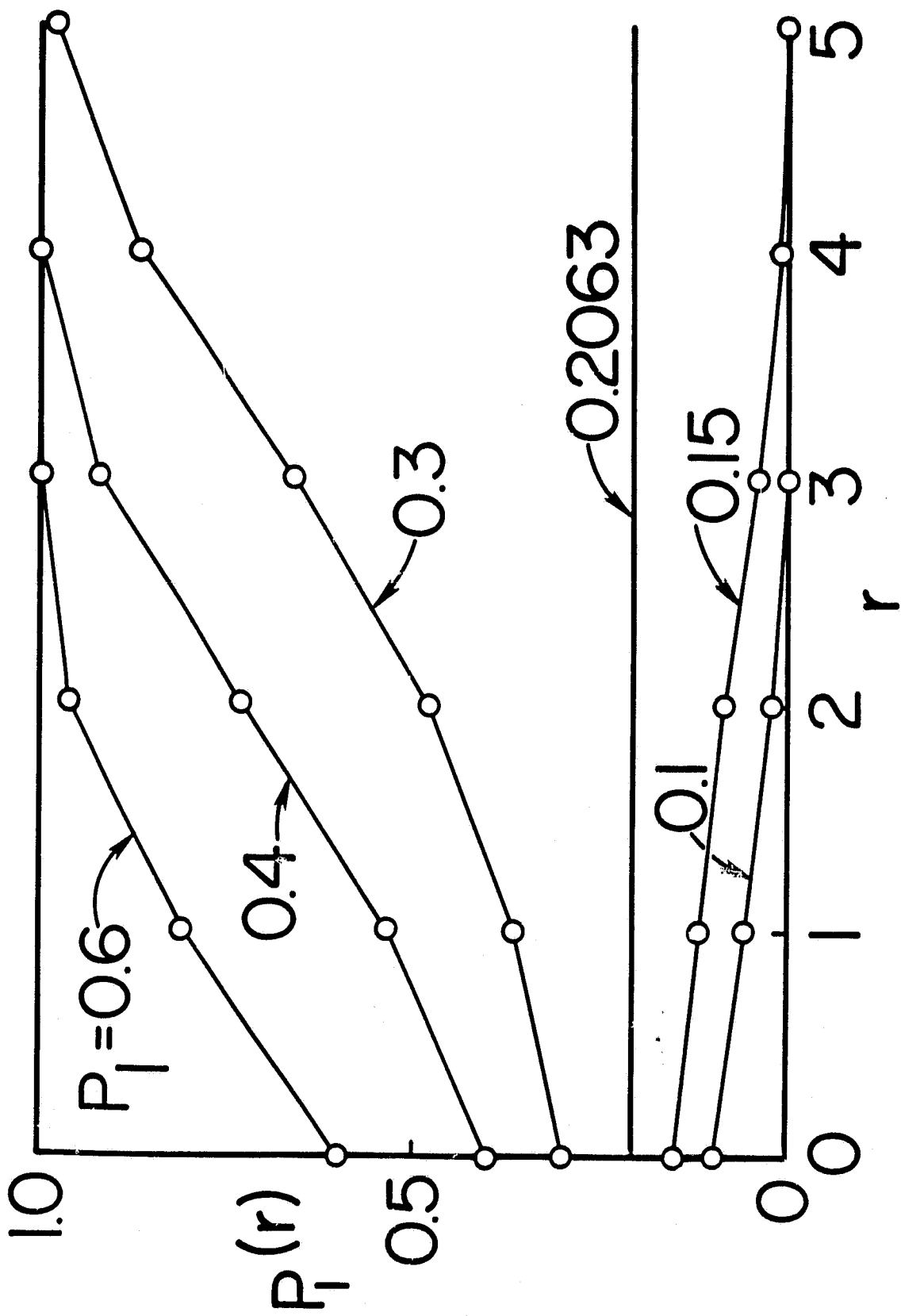


Figure 4

ORIGINAL PAGE IS
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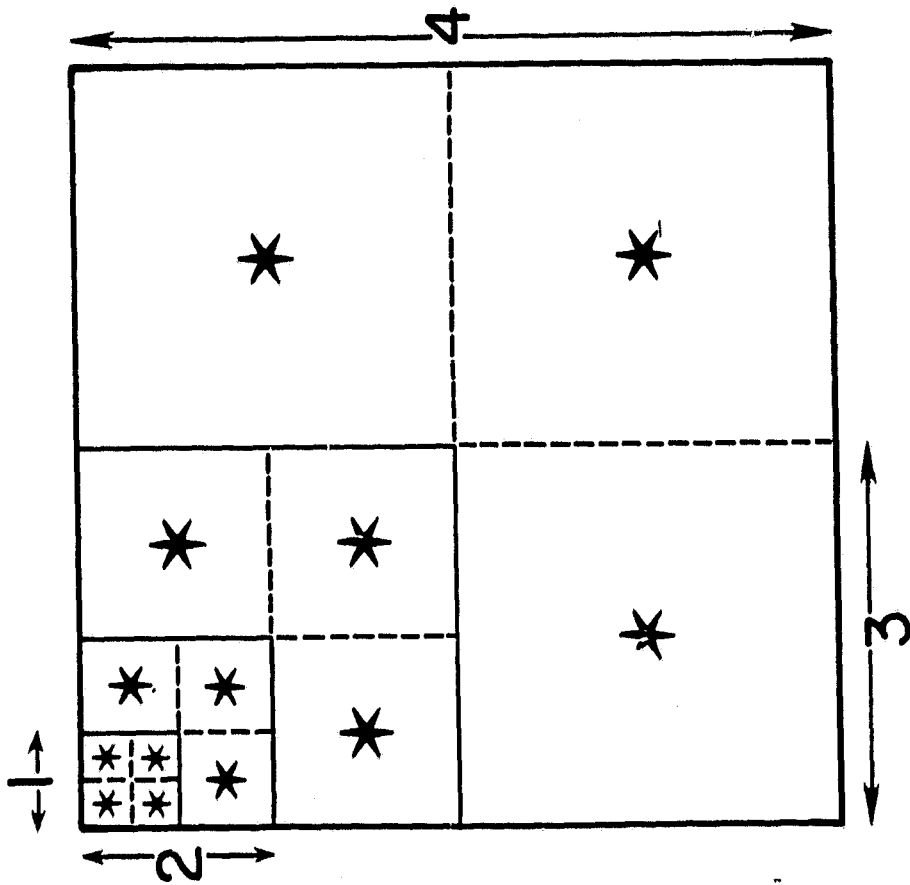


Figure 5

ORIGINAL WORK IS
OF POOR QUALITY

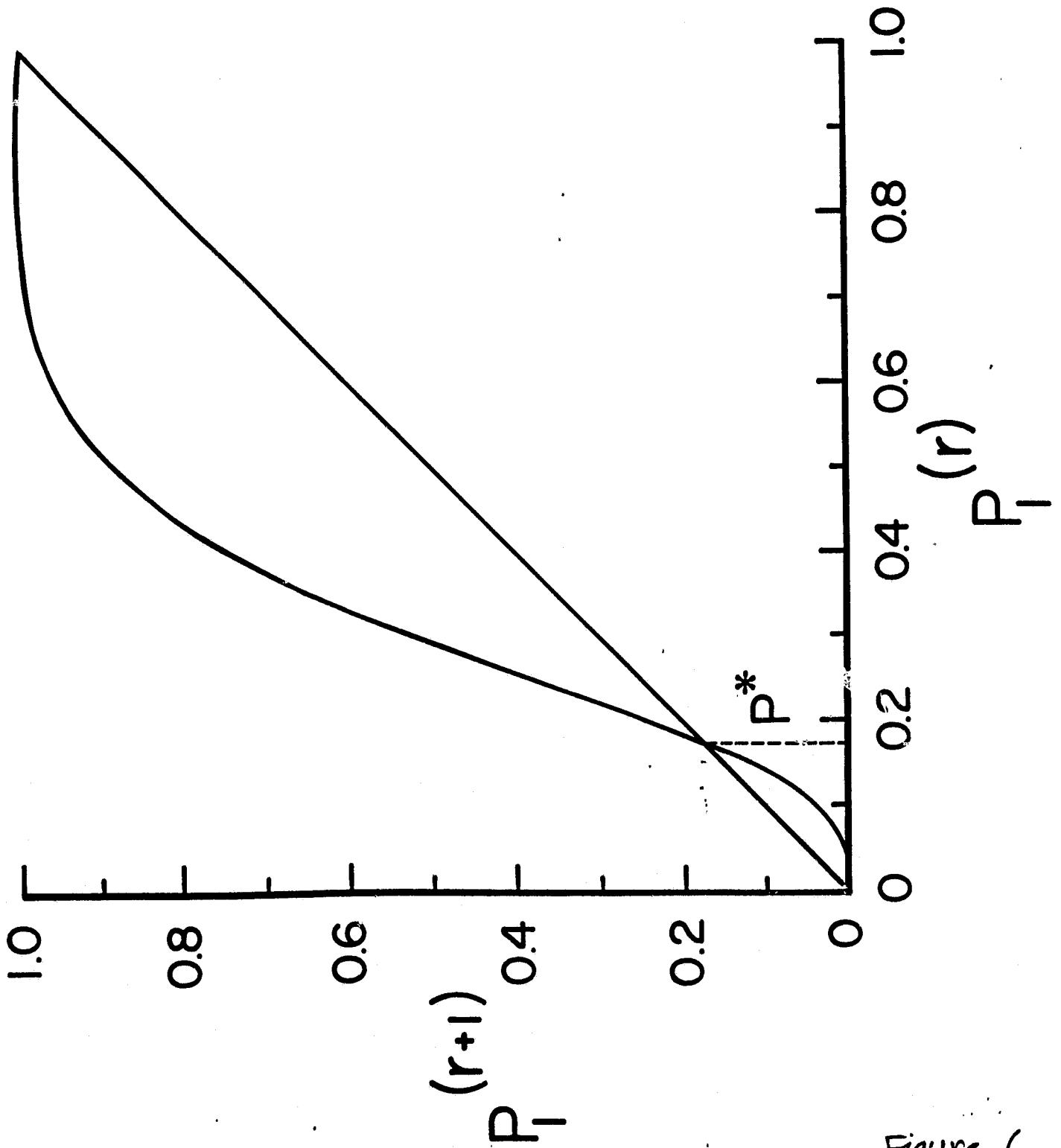


Figure 6