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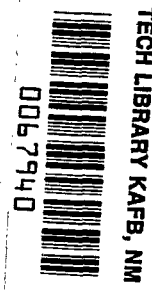
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Simplified Combustion Noise Theory Yielding a Prediction of Fluctuating Pressure Level

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NASA

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Summary

The first-order equations for the conservation of mass and momentum in differential form are combined for an ideal gas to yield a single second-order partial differential equation in one dimension and time. Small-perturbation analysis is applied. A Fourier transformation is performed that results in a second-order, constant-coefficient, nonhomogeneous equation. The driving function is taken to be the source of combustion noise.

A simplified model describing the energy addition via the combustion process gives the required source information for substitution in the driving function. This enables the particular integral solution of the nonhomogeneous equation to be found. This solution multiplied by the acoustic pressure efficiency predicts the acoustic pressure spectrum measured in turbine engine combustors. The prediction was compared with the overall sound pressure levels measured in a CF6-50 turbofan engine combustor and found to be in excellent agreement.

Introduction

The reduction of aircraft noise through the use of high-bypass-ratio turbofan engines has revealed another noise source for concern: core engine noise. Core engine noise is defined as the noise produced by the gas generator-turbine–tailpipe combination used to provide shaft power to drive the fan in a turbofan engine. Reference 1 summarizes various combustion noise prediction methods for aircraft engines available before 1974. The methods include both engine and combustor rig prediction.

Since the publication of reference 1 a data bank from various combustion noise experiments has been created by NASA (refs. 2 to 9). Additionally, a number of engine tests have been performed (refs. 10 to 19) to determine the effect of core engine noise on the far-field observer. The acoustic power determined from the data in references 2 to 9 has been correlated with engine operating parameters in reference 20. A number of theoretical investigations are reported in the literature: reference 21 sets forth several possible combustion noise source terms to be evaluated; reference 22 gives a correlation of data, based on theory, for the acoustic power in a combustor with engine operating parameters; reference 23 gives scaling laws for combustion noise radiating from open flames that are based on a theory evolved by using the chemical reactions; reference 24 calculates the 1/3-octave spectrum for a propane-air open flame; and reference 25 points out the need to know the heat release distribution in the combustor before the dynamic design can be performed. References 21 to 24 have focused attention on combustion noise theory and have presented a funda-mental, though at times complex, picture of possible combustion noise mechanisms.

In an effort to understand the dominant combustion noise mechanism and to provide a theoretically based prediction that allows insight into the noise-generating parameters in ducted combustors, a simple combustion noise model is postulated in this report. The classical small-perturbation approach is used to determine the acoustic source pressure spectrum and the parameters affecting the sound pressure. An acoustic pressure efficiency is introduced in the theoretical expression for the acoustic pressure to account for the inefficiency of converting turbulent energy to acoustic energy by the combustion process. The symbols used in this report are defined in appendix A.

The Governing Differential Equations and Assumptions

The general laws of conservation of mass and momentum and the assumptions used to limit the analysis to a nonviscous, adiabatic, perfect gas flowing through a constant-area duct are given here. The second-order partial differential equation governing the noise generated in a one-dimensional duct by the combustion process can be derived from the laws of conservation of mass and momentum. These laws given in equation form, taken from reference 26, are

Continuity:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho V}{\partial x} \tag{1}$$

Motion:

$$\frac{\partial \rho V}{\partial t} = -\frac{\partial \rho V^2}{\partial x} - \frac{\partial P}{\partial x} \tag{2}$$

For this analysis the following assumptions have been made:

(1) Turbulent shearing stress is negligible (i.e., the fluid is inviscid).

(2) Body forces due to gravity are negligible (i.e., $\rho g \equiv 0$).

(3) Heat transfer across boundaries is negligible (i.e., $(\nabla Q \equiv 0)$).

(4) Changes in the flow area are negligible.

(5) The fluid acts as a single-component gas (i.e., the effect of fuel mass in the mixture is negligible).

(6) The fuel contributes energy to the air and otherwise has a negligible effect on fluctuating pressure.

(7) The gas follows the perfect-gas law.

(8) Turbulence is dominant over acoustic fluctuating quantities in the combustor.

(9) Energy addition begins at the flame front. The rate of energy addition decreases exponentially with axial distance.

(10) The Mach number in the combustor is much less than unity.

(11) The combustor is located in an infinite pipe; hence, no acoustic reflections are present in the combustor or pipe.

Source Model for Combustion Noise

The model for combustion noise considers a one-dimensional duct flow (fig. 1). The flow is composed of a mixture of fuel and air. Turbulence is generated upstream of the flame front by turbulators. The flame front is stabilized aerodynamically at a point just downstream of the turbulators but does not touch the turbulators. The source region of combustion-related noise starts at the flame front. To simplify the calculations, the flame front is considered to be in a plane perpendicular to the flow direction. The regions upstream and downstream of the source region have constant though different properties and are infinite in length.

The objective of the present work is to describe the pressure fluctuations at the combustor that drive the acoustic system upstream and downstream of the combustor.
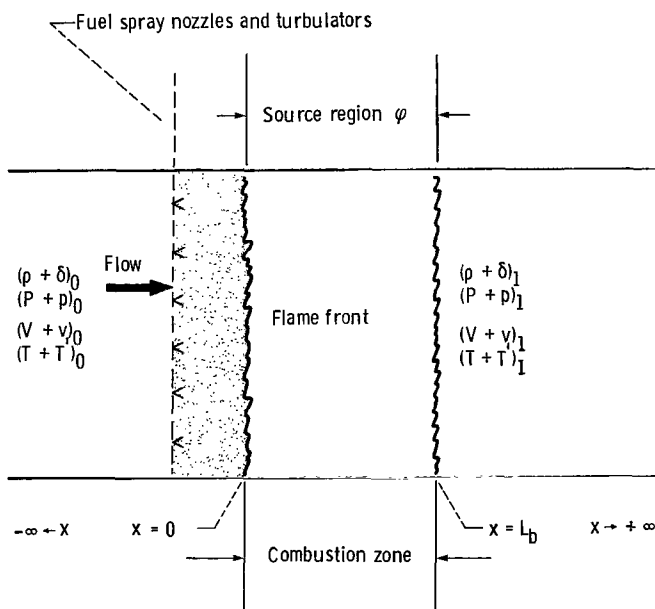

Figure 1. – Combustion noise source model.

Acoustic Source and Wave Equations

The acoustic wave equation describing the sound propagation in a duct is derived in appendix B by using the conservation equations (eqs. (1) and (2)) and assumptions 1 to 7. The resulting wave equation (eq. (B11)) may be applied to the duct either upstream or downstream of the combustion zone since the properties in either direction from the source are constant though different from each other. The wave equation (eq. (B11)), simplified by assuming constant mean properties in the region external to the combustor, is written here as

$$\frac{1}{C_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2}{\partial t^2}\left(\frac{p}{C_0^2} - \delta\right) + 2\frac{\partial^2 \rho V v}{\partial x^2} + \frac{\partial^2 \delta V^2}{\partial x^2} \tag{3}$$

In the derivation given in appendix B it has been assumed that the turbulence both upstream and downstream of the combustor is smaller than the acoustic fluctuations. It has also been assumed, for purposes of calculating the acoustic source in the combustor, that the turbulence generated in the combustor is much larger than any acoustic waves that pass through the combustion region.

The acoustic pressure is usually given in terms of finite-bandwidth, mean-square pressure fluctuations as a function of frequency. This is accomplished by taking the Fourier transform of equation (3), which removes time as a variable and thus simplifies the method of solution. The Fourier-transformed equation (appendix B, eq. (B19)) in terms of wave number is

$$\frac{\partial^2 p_\omega}{\partial x^2} + k^2 p_\omega = - \sum_{n=1}^{3} \varphi_{n,\omega}(x,\omega) \tag{4}$$

where $\varphi_{1,\omega}$, $\varphi_{2,\omega}$, and $\varphi_{3,\omega}$ are shown in appendix B (eqs. (B13b), (B14b), and (B15b)) to be the entropy, cold-flow, and direct-combustion noise sources.

Solution of Nonhomogenous Wave Equation

By using the methods given in reference 27, the complete solution of the homogeneous part (i.e., with $\varphi \equiv 0$) of equation (4) is given in appendix B as

$$p_{\omega,0} = C_1 e^{ikx} + C_2 e^{-ikx} \tag{5}$$

The acoustic pressure generated by the combustor installed in an infinite pipe can be calculated with the help of Green's function (ref. 28) as given in appendix B. For the left-running wave the acoustic pressure from appendix B (eq. (B22)) is

$$p_{\omega,L} = \frac{e^{ikx}}{2ik} \int_0^{L_b} \varphi_\omega(\xi) e^{-ik\xi} d\xi \qquad (6)$$

where

$$\varphi_\omega = -(\varphi_{1,\omega} + \varphi_{2,\omega} + \varphi_{3,\omega}) \qquad (7)$$

and

$$\varphi_{1,\omega} \equiv -\omega^2 \left(\frac{p_\omega}{C_0^2} - \delta_\omega \right) = \frac{-\omega^2 \rho S'}{C_P} \qquad (8)$$

$$\varphi_{2,\omega} \equiv 2 \frac{\dot{W}}{A} \frac{\partial^2 v_\omega}{\partial x^2} \qquad (9)$$

$$\varphi_{3,\omega} \equiv \frac{\dot{W}}{A} \frac{\partial^2}{\partial x^2} \left(\frac{\delta_\omega}{\rho} V \right) \qquad (10)$$

A similar result is given in appendix B for the right-running wave.

Physics of Noise Generation Process

Examination of the three acoustic source terms, $\varphi_{1,\omega}$, $\varphi_{2,\omega}$, and $\varphi_{3,\omega}$ given by equations (8), (9), and (10), respectively, shows that each is a function of frequency. The first acoustic source term $\varphi_{1,\omega}$ is a function of the turbulent fluctuating pressure and density. For a calorically and thermally perfect gas with sonic velocity C_0, $\varphi_{1,\omega}$ can be expressed in terms of the fluctuating entropy (eq. (8)). The conclusion is drawn then that the first source term $\varphi_{1,\omega}$ is a result of unsteady burning that may be the result of nonuniform fuel droplet size or mixing. In this work the entropy fluctuations are neglected.

The second acoustic source term $\varphi_{2,\omega}$ (eq. (9)) is given by the second derivative of the fluctuating velocity with respect to axial distance x. (The second derivative with respect to x is hereinafter called the Laplacian.) Turbulence, generated by turbulators in the combustor, is used to promote the mixing of the fuel and air prior to burning. The product of the mass flow rate per unit cross-sectional area and the Laplacian of the turbulent velocity describes a noise that is present even if the flame is off. Hence, $\varphi_{2,\omega}$ appears to be a cold-flow noise source.

The third acoustic source term $\varphi_{3,\omega}$ (eq. (10)) is due to the Laplacian of the product of the turbulent density and the mean velocity. It has been assumed in this work that the turbulent density fluctuations in the flame zone are constant throughout the zone. With this assumption the source term becomes a function of the Laplacian of the mean velocity, the result of the heat energy added by the combustion process. This source is investigated in detail in order to find the scaling coefficient or correlating parameters applicable to combustion noise.

An additional source results from $\varphi_{3,\omega}$. If the entropy fluctuations resulting from unsteady combustion produce fluctuating densities and these fluctuating densities pass through the combustion zone (i.e., the region having a nonzero Laplacian of the mean velocity), another noise, also termed entropy noise, is generated. For purposes of the present investigation this entropy noise is also neglected.

Acoustic Source Term Evaluation

Neglecting the temperature fluctuations $\varphi_{1,\omega}$ and the cold-flow noise $\varphi_{2,\omega}$ generated by the turbulators leaves the direct-combustion noise source $\varphi_{3,\omega}$ to be evaluated. Equation (6) calculates the acoustic pressure due to the acoustic source terms.

To evaluate $p_{\omega,L}$ (eq. (6)), the source term $\varphi_{3,\omega}$ must be given as a function of distance x. It is shown in appendix B that for Mach numbers much less than unity the velocity is directly proportional to the heat energy input to the air (appendix B, eq. (B36)).

By assuming that the fuel droplet mass decreases exponentially with axial distance, the velocity distribution due to the energy released to the air can be determined as a function of x. The second derivative with respect to x is then substituted into equation (10). The Fourier-transformed source term from appendix B (eq. (B43)) is

$$\varphi_{3,\omega} = -\frac{\dot{W}}{A} \frac{\delta_\omega}{\rho} \frac{V_0 \eta_c H_v f_0 \lambda^2}{C_P T_{T,0}} e^{-\lambda x} \qquad (11)$$

Inserting equation (11) into equation (6), assuming that the fluctuating density is constant with distance through the combustion zone, and integrating yield the acoustic pressure in polar form as (see derivation in appendix B, eq. (B46))

$$p_{\omega,L} = \frac{\eta_c}{2} \frac{\delta_\omega}{\rho} \frac{\dot{W}}{A} \frac{V_0 H_v f_0}{C_P T_{T,0}} \frac{\lambda}{k} \frac{e^{i[kx - (\pi/2) + \tan^{-1}(-k/\lambda)]}}{\sqrt{1 + (k/\lambda)^2}} \qquad (12)$$

where $0 < x < L_b$. Equation (12), with dimensions of pressure per unit angular frequency, is used for calculating the overall sound pressure level and the narrowband acoustic spectrum.

In deriving equation (12) the conversion of turbulent energy to acoustic energy was assumed to be complete. This, in reality, cannot be accomplished as evidenced by the fact that turbulence does exist downstream of a combustor. To account for this, an acoustic pressure efficiency η_p is introduced and defined as the ratio of the experimentally measured acoustic pressure generated by the combustor p'_ω to the acoustic pressure that would be

generated if all of the turbulent energy had been converted to an acoustic pressure p_ω. From appendix B (eq. (B45a)) η_p is given by

$$\eta_p \equiv \frac{p'_\omega}{p_\omega} \tag{13}$$

Thus the acoustic pressure efficiency can now be determined for any specific combustor. The value of η_p will probably depend on the combustor type and configuration. Therefore it is necessary that η_p be determined for a number of combustors so that the proper value can be selected when making predictions of combustor pressure fluctuations.

Overall Sound Pressure Level

An expression for the overall sound pressure level can be obtained by multiplying equation (12) by the acoustic pressure efficiency η_p, squaring, and then integrating over the frequency range of interest. The resulting equation for p'_ω is

$$\left(\frac{p'_\omega}{p_{ref}}\right)^2 = \left(\frac{\mathcal{F}\eta_p\eta_c}{2p_{ref}}\frac{\delta_\omega}{\rho}\frac{\dot{W}}{A}\frac{V_0 H_w f_0}{C_P T_{T,0}}\right)^2 \tag{14}$$

where the term $\mathcal{F}$ is a function containing all of the frequency dependence and is defined as

$$\mathcal{F}^2 \equiv \int_{f_L}^{f_U} \left(\frac{\lambda}{k}\right)^2 \frac{1}{[1+(k/\lambda)^2]} df$$

$$= \left(\frac{\lambda C}{2\pi}\right)^2 \left[\frac{1}{f} + \frac{2\pi}{\lambda C}\tan^{-1}\left(\frac{2\pi f}{\lambda C}\right)\right] \Bigg|_{f_U}^{f_L} \tag{15}$$

Equation (15) indicates that $\mathcal{F}^2$ is inversely proportional to the frequency. Therefore the magnitude of $\mathcal{F}$ will be determined by the lower limit on frequency, providing that the upper limit is much greater than the lower limit. In determining the OASPL the combustion noise frequency limits are of the order of 50 to 2000 Hz. For a fuel mass decay constant λ, defined as $2\pi/L_b$, and reasonable sonic velocities, equation (15) can be approximated by using the lower frequency limit by

$$\mathcal{F}^2 = \frac{C^2}{L_b^2 f_L}, \text{sec}^{-1} \tag{16}$$

The overall sound pressure level (OASPL) equation can now be written for the combustor from equations (14)

and (16) as

$$OASPL = 20\log_{10}\frac{\eta'_p\eta_c C}{2p_{ref}L_b\sqrt{f_L}}\frac{\delta_\omega}{\rho}\frac{\dot{W}}{A}\frac{V_0 H_w f_0}{C_P T_{T,0}} \tag{17}$$

The acoustic pressure efficiency η'_p can be determined from the measured overall sound pressure level and equation (17). The equation for experimentally determining the value of η'_p is

$$\eta'_p = \frac{2p_{ref}L_b\sqrt{f_L}}{\eta_c C}\frac{\rho}{\delta_\omega}\frac{A}{\dot{W}}\frac{C_P T_{T,0}}{V_0 H_w f_0} \times 10^{OASPL/20} \tag{18}$$

The downstream temperature is used in calculating the sonic velocity C because the measurements in combustors are in the hot region. Acoustic pressure waves moving upstream from the combustor would require that the inlet temperature be known when calculating their acoustic pressure.

The fluctuating pressure in the combustor of the CF6-50 turbofan engine has been measured (refs. 7 and 8). The measured overall fluctuating pressure (assumed to be acoustic) in the combustor at 3.8 percent of design thrust has been used in conjunction with equation (18) to calculate the acoustic pressure efficiency, $\eta'_p = 0.030$, for the CF6-50 combustor.

From this acoustic pressure efficiency the overall sound pressure level in the combustor was predicted over the engine thrust range by using equation (17). The results are shown in figure 2. The theory, given by the solid line, agrees well with the measured fluctuating pressure, given by the symbols. Thus the theory can be used to predict, at a very minimum, the trends in the overall acoustic pressure.

In light of this discussion it can be concluded that the theory predicts the trends in the combustor overall
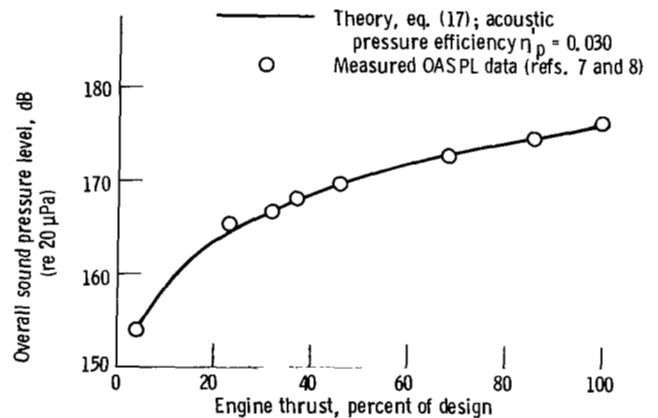

Figure 2. – Combustor pressure fluctuations as a function of CF6-50 turbofan engine thrust—experimental data and theory.

acoustic pressure with engine operating conditions and, for CF6-50 combustors and similar geometries, an acoustic pressure efficiency of 0.030 can be used in equation (17) to predict the combustor overall acoustic pressure. The acoustic pressure efficiency has also been determined for a YF102 engine combustor. This combustor, unlike the CF6-50, is a reverse-flow combustor and is much smaller than the CF6-50. Its acoustic pressure efficiency is 0.016 at 30 percent of design speed. Using the acoustic pressure efficiency of 0.016 yields only a 0.2-dB error in predicting the OASPL at the 95-percent speed.

Comparing the acoustic pressure efficiencies shows that either the CF6-50 combustor converts the turbulence to acoustic pressure more efficiently than the YF102 combustor or that the turbulence levels in the combustors are not equal.

Narrow-Band Sound Pressure Level Spectrum

The theoretical sound pressure spectrum is given by equation (12). Discarding the phase information contained in the exponential term and applying equation (13) to account for the efficiency of conversion of turbulence to acoustic pressure, equation (12) is written as

$$\left[\frac{2}{\eta_p \eta_c} \frac{\rho}{\delta_\omega} \frac{AL_b}{\dot{W}} \frac{C_P T_{T,0} p_\omega}{V_0 H_u f_0 C} \right] = \frac{1}{f} \frac{1}{\sqrt{1 + (fL_b/C)^2}} \qquad (19)$$

The right side of equation (19) represents the spectral shape of the acoustic pressure given by equation (12).

The sound pressure level spectrum is given by

$$\text{SPL} = 20 \log_{10} \frac{\eta_c \eta_p}{2p_{\text{ref}}} \frac{\delta_\omega}{\rho} \frac{\dot{W}}{AL_b} \frac{V_0 H_u f_0}{C_P T_{T,0}} C \frac{\Delta f}{f\sqrt{1 + (fL_b/C)^2}} \qquad (20)$$

This equation states that the acoustic pressure level is inversely proportional to the square of the frequency. The contribution to the spectrum of the terms contained under the radical is small for $f < C$.

The predicted spectral shape is compared with the measured narrow-band spectra for the CF6-50 engine (ref. 7) in figure 3. The measured fluctuating pressure level in the combustor of the CF6-50 turbofan engine operating at 3.8 percent of design thrust is shown in figure 3(a). Also shown in figure 3(a) is a plot of equation (20) with the level matched to the measured spectrum at a frequency of 2000 Hz. The acoustic pressure efficiency is determined from equation (20) and is given by

$$\eta_p = \frac{2}{\eta_c} \frac{\rho}{\delta_\omega} \frac{AL_b}{\dot{W}} \left(\frac{C_P T_{T,0} p_{\text{ref}}}{V_0 H_u f_0 C} \right) 10^{\text{SPL}(f)/20}$$

$$\times \frac{f}{\Delta f} \sqrt{1 + \left(\frac{fL_b}{C} \right)^2} \qquad (21)$$

At 2000 Hz the measured SPL is equal to 110 dB and the narrow-band spectral acoustic pressure efficiency, computed by using equation (21), is

$$\eta_p = 4.5 \times 10^{-2}$$

when δ_ω/ρ is assumed to be 0.3.

The trends in the spectral shape of both theory and experiment agree very well at the 3.8-percent-thrust point given in figure 3(a). The comparison at 99.8-percent thrust shown in figure 3(b), however, is not as good. The theory given for the 99.8-percent-thrust point uses the acoustic pressure efficiency obtained from the 3.8-percent-thrust point, namely $\eta_p = 4.5 \times 10^{-2}$. Examining figure 3(b) shows that the general trend is correct but that large oscillations in the spectrum are not
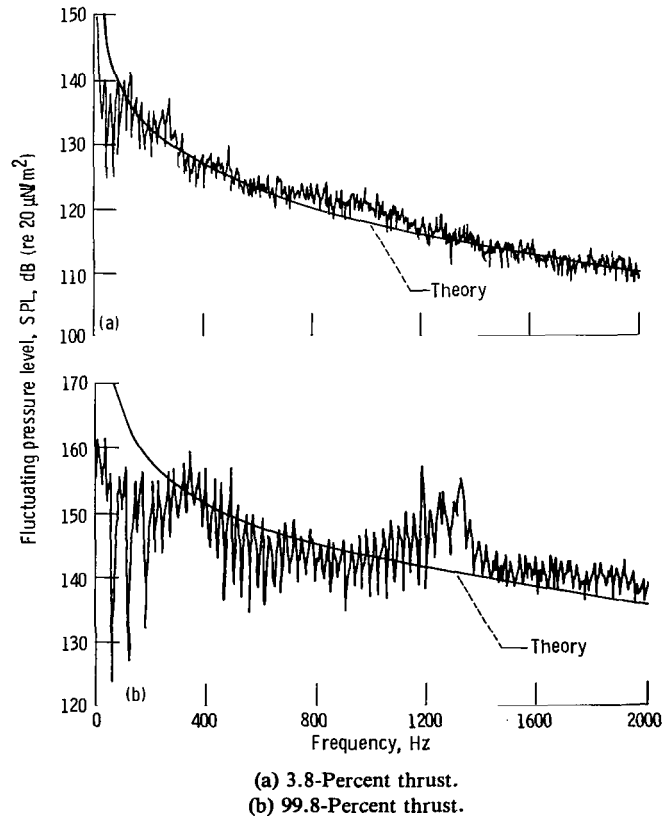

(a) 3.8-Percent thrust.
(b) 99.8-Percent thrust.

Figure 3. – Comparison of theory to measured sound pressure level spectrum—CF6-50 turbofan engine combustor. Acoustic pressure efficiency, η_p, 4.5×10^{-2}; dimensionless fluctuating density, δ_ω/ρ, 0.3.

predicted by equation (20). Acoustic reflections from the turbine may account for these oscillations.

From this discussion it is concluded that the spectrum predicted by equation (20) agrees substantially with the experimental measurements made in an operating engine. It can also be concluded that the simplified combustion noise theory given herein substantially predicts the trends in the acoustic pressures generated by the combustion process of turbine engine combustors. However, additional information on the turbulence in the combustor is required to evaluate its contribution to the combustion noise spectrum.

1/3-Octave-Band Sound Pressure Spectrum

The 1/3-octave-band spectrum of the combustor noise can be calculated for a simplified spectrum by neglecting the term under the radical contained in equation (20) and integrating over the bandwidth:

$$\Delta SPL = 10 \ log_{10} \ \frac{0.23}{f_c} \tag{22}$$

This equation has been plotted in figure 4 with its level matched, as for the narrow-band spectrum, at the 2000-Hz band for the CF6-50 operating at the 3.8-percent-thrust point. The 1/3-octave-band data given by the symbols match the theoretical trends for frequencies above 100 Hz. At frequencies less than 100 Hz the data are significantly overpredicted by the theory.

As has been discussed, the input to the theory considered the turbulence spectrum to be constant. The eddy size in the combustor must be limited to the height of the combustor. If the velocity is of the order of 18 m/sec (60 ft/sec) and the annular combustor is approximately 0.15 m (0.5 ft) high, the frequency of the eddy passing a point must be no less than 120 Hz. This may
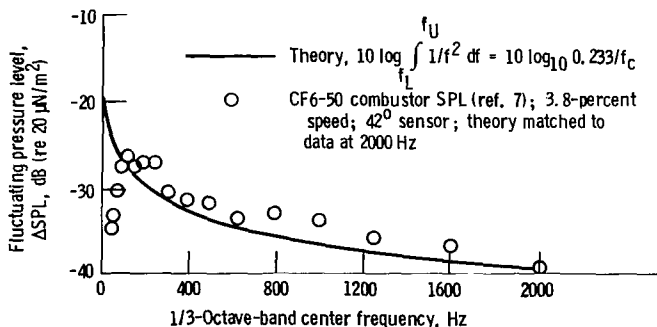
account for the discrepancy between the low-frequency theory and the data. This situation illustrates the need for turbulence measurements in combustors and the resulting spectral information.

Concluding Remarks

The theory derived herein can be applied, in modified form, to the noise generated by the turbine at low frequencies. This is apparent by inspection of the third source term. This source term indicates that any fluctuating quantity experiencing a mean flow acceleration (combustor) or deceleration (turbine) will produce a low-frequency sound (eq. (6)). For the turbine case the source term $\varphi_{3,\omega}$ becomes a function of the product of the fluctuating density and the Laplacian of the mean velocity through the turbine rotor blade.

The acoustic pressure generated by the combustor can be expressed in several different forms. For instance, the acoustic pressure can be said to be directly proportional to the square of the volumetric heat release rate, the turbulence density fluctuations, and the mean inlet velocity and inversely proportional to the entrance enthalpy and frequency. Because other variations in the governing equations are easily derived, it is understandable that the literature contains so many expressions for the acoustic pressure generated by the combustion process. The conclusions drawn herein are the result of the expression for the acoustic pressure generated by the interaction of the turbulence with the Laplacian of the mean energy created in the combustor. The existence of cold-flow noise and time-varying combustion has been noted, but they have not been evaluated.

Conclusions

By using the theory developed herein, expressions for the acoustic pressure generated by the combustion process have been derived. It has been shown that the overall acoustic pressure in large turbofan engine combustors can be predicted over the range of engine operating speeds, providing that the acoustic pressure efficiency, defined herein, can be determined. The following conclusions are based on the theory:

1. The major source of combustor pressure fluctuations is the interaction of the turbulence and the mean internal energy (i.e., temperature) additions in the combustor.

2. For a typical large turbofan engine the ratio of the measured overall acoustic pressure in the combustor to the theoretical acoustic pressure (obtained by assuming that all of the turbulent energy is converted to acoustic pressure) is a constant with a magnitude of the order of 0.030. For smaller reverse-flow combustors the acoustic

Figure 4. – 1/3-Octave-band fluctuating pressure level spectrum shape for CF6-50 turbofan engine combustor. Thrust, 3.8 percent of design.

pressure efficiency has been found to be of the order of 0.020.

3. The turbulence-flame-generated noise is directly proportional to the square of the sonic velocity, the mass flow rate per unit cross-sectional area, the ratio of heat energy added per unit mass of air to the inlet enthalpy, the inlet velocity, the combustion efficiency, and the turbulence intensity. It is inversely proportional to the square of the frequency and the burning length of the combustor.

4. Because combustor size limits turbulence scales, larger scale low-frequency turbulence is suppressed.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, November 1, 1983

Appendix A
Symbols

A cross-sectional flow area, m^2

C sonic velocity, m/sec; constant of integration

C_P specific heat at constant pressure, J/kg K

C_0 flow or sonic velocity used in deriving generalized acoustic wave equation, m/sec

C_1, C_2 constants used in complementary function for solving differential equations

f frequency, Hz; function

f_L lower limit on frequency, Hz

f_U upper limit on frequency, Hz

f_0 ratio of mass of fuel to mass of air

Δf frequency bandwidth

g acceleration of gravity, m/sec^2

H_T total enthalpy, equals heat energy released by combustion, J/kg (cal/kg)

H_v heating value of fuel, J/kg (cal/kg)

i $\sqrt{-1}$

J mechanical equivalent of heat, J/kg m

k acoustic wave number, ω/C_0, 1/m

L_b burning length (i.e., axial distance measured from flame to point in combustor where chemical reaction ends), m

M Mach number, V/C

SPL   sound pressure level, dB (re 20 μPa)

OASPL   overall sound pressure level, dB (re 20 μPa)

P instantaneous or mean static pressure, Pa

p time-fluctuating part of instantaneous pressure, Pa

p_{ref} reference pressure used in calculating sound pressure level, 20 μPa

p_ω Fourier-transformed fluctuating pressure, Pa sec

$p_{\omega,L}$ acoustic pressure for left-running wave (i.e., $x<0$), Pa sec

$p_{\omega,0}$ acoustic pressure outside source region (i.e., $x<0$ and $x>L_b$), Pa sec

p'_ω acoustic pressure generated in combustor, Pa sec

Q heat energy, J/kg (cal/kg)

R universal gas constant for air, m/K

S' fluctuating component of entropy, J/kg K

T mean temperature, K

T' instantaneous or fluctuating temperature, K

t time, sec

U instantaneous or mean internal energy, J/kg

V mean velocity, m/sec

v instantaneous or fluctuating velocity, m/sec

$\dot{W}$ mean mass flow rate, kg/sec

x axial distance measured from flame front, positive in direction of flow, m

ΔSPL   sound pressure level difference

δ time-fluctuating component of density, kg/m^3

η_c combustion efficiency

η_p acoustic pressure efficiency

θ phase angle, deg

λ fuel mass decay constant, assigned a value of $2\pi/L_b$

ξ axial distance in combustor measured from flame front location, m

ρ instantaneous or mean density of fluid, kg/m^3

φ_ω Fourier-transformed source terms (eq. (4))

ω angular frequency, $2\pi f$, rad/sec

Subscripts:

c 1/3-octave center frequency

L left running

n acoustic source numbers

R right running

T total conditions

ω Fourier-transformed quantity; spectrum

0 upstream axial location

1 axial location downstream of combustion zone; first source term

2 second source term

3 third source term

Appendix B
Derivation of Governing Differential Equation Neglecting Heat Transfer, Body Forces, and Shearing Stresses

Under the assumption of negligible shearing stress, heat transfer, and body forces the time-dependent conservation equations in one dimension can be written as

Continuity:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho V}{\partial x} \tag{B1}$$

Motion:

$$\frac{\partial \rho V}{\partial t} = -\frac{\partial \rho V^2}{\partial x} - \frac{\partial P}{\partial x} \tag{B2}$$

Taking the $\partial/\partial t$ of equation (B1) gives

$$\frac{\partial^2 \rho}{\partial t^2} = -\frac{\partial^2 \rho V}{\partial t \partial x} \tag{B3}$$

Taking $\partial/\partial x$ of equation (B2) gives

$$\frac{\partial^2 \rho V}{\partial x \partial t} = -\frac{\partial^2 \rho V^2}{\partial x^2} - \frac{\partial^2 P}{\partial x^2} \tag{B4}$$

Adding equations (B3) and (B4) gives

$$\frac{\partial^2 \rho}{\partial t^2} - \frac{\partial^2 \rho V^2}{\partial x^2} - \frac{\partial^2 P}{\partial x^2} = 0 \tag{B5}$$

Rearranging terms and adding

$$\frac{1}{C_0^2} \frac{\partial^2 P}{\partial t^2}$$

to both sides of equation (B5) give

$$\frac{1}{C_0^2} \frac{\partial^2 P}{\partial t^2} - \frac{\partial^2 P}{\partial x^2} = -\frac{\partial^2 \rho}{\partial t^2} + \frac{1}{C_0^2} \frac{\partial^2 P}{\partial t^2} + \frac{\partial^2 \rho V^2}{\partial x^2} \tag{B6}$$

where C_0 can be defined generally, at this point in the derivation, as the velocity of propagation of either a sonic wave or a disturbance such as turbulence. The parameters in equation (B6) are instantaneous values of P, ρ, and V. They are assumed to be the sum of a mean value and a fluctuating component. In equation form they are defined as

$$P \equiv P(x) + p(x,t) \tag{B7}$$

$$\rho \equiv \rho(x) + \delta(x,t) \tag{B8}$$

$$V \equiv V(x) + v(x,t) \tag{B9}$$

Substituting equations (B7), (B8), and (B9) into equation (B6), dropping the terms containing time derivatives of mean quantities, and expanding the products yield an equation in both mean and fluctuating quantities. Equating the zero-order terms (mean quantities) yields

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 \rho V^2}{\partial x^2} = 0 \tag{B10}$$

Equating the first-order terms yields

$$\frac{1}{C_0^2} \frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \frac{\partial^2}{\partial t^2}\left(\frac{p}{C_0^2} - \delta\right) + 2\frac{\partial^2 \rho V v}{\partial x^2} + \frac{\partial^2 \delta V^2}{\partial x^2} \tag{B11}$$

For steady flow, continuity gives a relationship between the mean values of ρ and V as

$$\rho V = \frac{\dot{W}}{A} \tag{B12}$$

The fluctuating quantities are assumed to be the result of the turbulence generated by the turbulators just upstream of the flame front.

Ribner (ref. 29) in discussing equation (B11) follows Lighthill's interpretation of the source terms on the right side of equation (B11). Lighthill (ref. 30), neglecting the first term on the right, uses the turbulent stresses to formulate his jet noise theory. The source terms on the right side of equation (B11) are, for simplicity, defined as follows:

$$\varphi_1 \equiv \frac{\partial^2}{\partial t^2}\left(\frac{p}{C_0^2} - \delta\right) \tag{B13a}$$

or for C_0 defined as the acoustic velocity

$$\varphi_1 \equiv \frac{\partial^2}{\partial t^2}\left(\frac{\rho S'}{C_P}\right) \qquad\qquad \text{(B13b)}$$

as the fluctuating entropy source

$$\varphi_2 \equiv 2\frac{\partial^2 \rho V v}{\partial x^2} \qquad\qquad \text{(B14a)}$$

or by using equation (B12) as

$$\varphi_2 \equiv 2\frac{\dot{W}}{A}\frac{\partial^2 v}{\partial x^2} \qquad\qquad \text{(B14b)}$$

$$\varphi_3 \equiv \frac{\partial^2 \delta V^2}{\partial x^2} \qquad\qquad \text{(B15a)}$$

or again by using equation (B12) as

$$\varphi_3 \equiv \frac{\dot{W}}{A}\frac{\partial^2}{\partial x^2}\left(\frac{\delta}{\rho}V\right) \qquad\qquad \text{(B15b)}$$

The last term φ_3 indicates that the second derivative of the product of the fluctuating density and mean velocity with respect to axial distance represents a source in the combustor. For simplicity, the second derivative with respect to distance is hereinafter called the Laplacian of the quantity. This source term acts as a forcing function for the system described by the left side of equation (B11). It will be shown that the particular integral solution of equation (B11) with source φ_3 as its forcing function yields an expression for the fluctuating pressure that agrees with measured values in a combustor and thus indicates that the source φ_3 is dominant over the sources φ_1 and φ_2.

The second term φ_2 is considered to be the cold-flow noise source term. This noise is created by the turbulators generating a fluctuating velocity as the flow passes through them. The turbulence thus produced causes mixing of the fuel with the air as the mixture moves toward the flame front. Since the product of the mean density and velocity is constant and equal to the mass flow rate per unit cross-sectional area, the source term can be written as in equation (B14), leaving the Laplacian of the fluctuating velocity as the measure of the source strength. The relative magnitude of this source term can be determined from measurements of the fluctuating pressures in the combustor during flameout or cold-flow tests.

The first source term φ_1 can be written in terms of the fluctuating entropy by assuming that the gas is both calorically and thermally perfect and that the perturbations in pressure p and density δ are propagated at the speed of sound. The result of these assumptions is equation (B13b). Although the mean density and specific heat are not functions of time, it is likely that uneven combustion will occur and give a fluctuating heat release or, equivalently, a fluctuating entropy. This could be due to nonuniformity of fuel droplet size or the time-varying mixing process in the combustor. The first source term thus accounts for the noise source associated with the fluctuating burning process as controlled by the mixing process in the combustor. Although the fluctuating heat release is recognized as a potential noise source, it is not thought to be very important (ref. 5).

Entropy noise also results when a fluctuating quantity passes through a gradient in a mean quantity and may, if desired, be included in φ_3 by using the perfect-gas relationship

$$\frac{\delta}{\rho} = \frac{p}{P} - \frac{T'}{T} \qquad\qquad \text{(B16)}$$

giving

$$\varphi_3 = \frac{\dot{W}}{A}\frac{\partial^2}{\partial x^2}\left[\left(\frac{p}{P} - \frac{T'}{T}\right)V\right] \qquad\qquad \text{(B17)}$$

Equation (B11) can now be written as

$$\frac{1}{C_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = \sum_{n=1}^{3}\varphi_n(x,t) \qquad\qquad \text{(B18)}$$

Taking the Fourier transform of equation (B18) converts the equation from the time domain to the frequency domain. In terms of frequency then

$$\frac{\partial^2 p_\omega}{\partial x^2} + k^2 p_\omega = -\sum_{n=1}^{3}\varphi_{n,\omega}(x,\omega) \qquad\qquad \text{(B19)}$$

where $k \equiv \omega/C_0$. The solution to the homogeneous part of equation (B19), that is,

$$\frac{\partial^2 p_\omega}{\partial x^2} + k^2 p_\omega \equiv 0 \qquad\qquad \text{(B20)}$$

is

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$$p_{\omega,0} = C_1 e^{ikx} + C_2 e^{-ikx} \tag{B21}$$

Since the combustor is assumed to be located in an infinite pipe, no reflected waves are present and the solution for the acoustic pressure in the infinite pipe radiated from the source region can be found by using Green's function (ref. 28). For the left-running wave

$$p_{\omega,L} = \frac{e^{ikx}}{2ik} \int_0^{L_b} \varphi_\omega(\xi) e^{-ik\xi} d\xi \qquad x < 0 \tag{B22}$$

where the integration is performed over the source region (burning length). For the right-running wave

$$p_{\omega,R} = \frac{e^{-ikx}}{2ik} \int_0^{L_b} \varphi_\omega(\xi) e^{ik\xi} d\xi \qquad x > L_b \tag{B23}$$

where

$$\varphi_\omega \equiv -(\varphi_{1,\omega} + \varphi_{2,\omega} + \varphi_{3,\omega}) \tag{B24}$$

In this report, entropy and cold-flow noise ($\varphi_{1,\omega}$ and $\varphi_{2,\omega}$) are neglected. Furthermore, the temperature fluctuations given in equation (B17) are considered as entropy fluctuations and are negligible as compared with the pressure fluctuations. These assumptions leave the direct-combustion noise source term $\varphi_{3,\omega}$ in the acoustic pressure equations (eqs. (B22) and (B23)). To perform the integration required in equations (B22) and (B23), the dependence of the term $\partial^2/\partial x^2 \, (\delta/\rho \ V)$ on axial position must be known or assumed. As discussed later, δ/ρ is assumed to be constant. To get the velocity-distance relationship, we first obtain the relationship between the velocity and internal energy of the gas at low Mach numbers. It is assumed that the chemical reaction rate is much greater than the rate of mixing of the fuel with the air in the combustor. This assumption appears to be well justified because chemical reaction times for well-mixed reactants are of the order of microseconds and the mixing of fuel with the surrounding air appears to be of the order of milliseconds. Hence, the combustion process is rate controlled by the mixing process (ref. 31). The mixing process is represented by an exponential decay of the fuel droplet diameter with axial distance x. This assumption permits the calculation of the required velocity distribution with distance for use in equations (B22) and (B23).

The derivation using the mean-valued parameters is as follows:

From steady-state continuity

$$\rho \frac{dV}{dx} + V \frac{d\rho}{dx} = 0 \tag{B25}$$

From steady-state motion

$$\frac{dP}{dx} + \rho V \frac{dV}{dx} = 0 \tag{B26}$$

From the perfect-gas law

$$\rho = \frac{P}{RT} \tag{B27}$$

Substituting equation (B27) into equation (B25) gives

$$\frac{P}{RT} \frac{dV}{dx} + \frac{V}{R} \frac{d}{dx}\left(\frac{P}{T}\right) = 0 \tag{B28}$$

or expanding the second term gives

$$\frac{P}{RT} \frac{dV}{dx} + \frac{V}{RT^2}\left(T \frac{dP}{dx} - P \frac{dT}{dx}\right) = 0 \tag{B29}$$

or

$$P \frac{dV}{dx} + V\left(\frac{dP}{dx} - \frac{P}{T} \frac{dT}{dx}\right) = 0 \tag{B30}$$

Next, substituting equation (B27) into equation (B26) and rearranging give

$$\frac{dP}{dx} = -\frac{PV}{RT} \frac{dV}{dx} \tag{B31}$$

Substituting equation (B31) into equation (B30) and rearranging yield

$$P \frac{dV}{dx} = -V\left(-\frac{PV}{RT} \frac{dV}{dx} - \frac{P}{T} \frac{dT}{dx}\right) \tag{B32}$$

or

$$\left(1 - \frac{V^2}{RT}\right) \frac{dV}{dx} = \frac{V}{T} \frac{dT}{dx} \tag{B33}$$

$$(1 - M^2) \frac{1}{V} \frac{dV}{dx} = \frac{1}{T} \frac{dT}{dx} \tag{B34}$$

Because the Mach number in a combustor is usually of the order of 0.05, equation (B34) can be simplified as follows under the assumption that the Mach number is much less than unity:

$$\frac{1}{V}\frac{dV}{dx} = \frac{1}{T}\frac{dT}{dx} \tag{B35}$$

or

$$V = V_0 \frac{T_T}{T_{T,0}} \tag{B36}$$

At this point it is necessary to know the axial temperature distribution in the combustor. The heat energy released by the fuel droplet with x then is given by

$$Q(x) = H_T(x) = C_P[T_T(x) - T_{T,0}] \tag{B37}$$

and if we assume an exponential decay of the fuel droplet diameter,

$$H_T(x) = \eta_c H_v f_0 (1 - e^{-\lambda x}) \tag{B38}$$

The temperature ratio is given by assuming constant specific heats as

$$\frac{T_T(x)}{T_{T,0}} = \left[\frac{\eta_c H_v f_0}{C_P T_{T,0}}(1 - e^{-\lambda x}) + 1\right] \tag{B39}$$

where for convenience of computation, λ is defined as the fuel decay constant and is set equal to $2\pi/L_b$. This yields a vanishing exponential at $x = L_b$.

Substituting equation (B39) into equation (B36) gives the required velocity distribution as

$$V(x) = V_0\left[\frac{\eta_c H_v f_0}{C_P T_{T,0}}(1 - e^{-\lambda x}) + 1\right] \tag{B40}$$

To this point in the derivation little has been discussed about the nondimensional fluctuating density term δ/ρ in the acoustic source term φ_3 (eq. (B15)). Indeed, little is known about this term. However, some arguments can be made concerning its magnitude throughout the combustion zone. First, it is assumed that the fluctuating density is caused by the turbulators just upstream of the flame front. Next, because of the exponential heat addition assumption, it is possible to state that most of the effect of the Laplacian of the mean velocity takes place over a short distance downstream of the flame front. This assumption is further justified in reference 31 by gas samples taken along the combustor axial

centerline, which indicate that the bulk of the reaction takes place near the flame front. Reference 31 also shows that the turbulence decays over a large distance as compared with the maximum heat release region at the flame front. Thus, it is reasonable to expect that the fluctuating density can be approximated by assuming that it is constant near the flame front. Of course, this is desirable because it simplifies the solution of the problem.

This reasoning then, to some extent, justifies the assumption of constant fluctuating density with axial distance near the flame front. This allows the source φ_3 to be written from equation (B15b) as

$$\varphi_3 \equiv \frac{\dot{W}}{A}\frac{\delta}{\rho}\frac{d^2V}{dx^2} \tag{B41}$$

Taking the second derivative with respect to x of the expression for $V(x)$ (eq. (B40)) gives

$$\frac{d^2V}{dx^2} = -\frac{V_0 \eta_c H_v f_0 \lambda^2}{C_P T_{T,0}}e^{-\lambda x} \tag{B42}$$

Substituting this into the equation for φ_3 (eq. (B41)) gives the direct-combustion noise source term as

$$\varphi_{3,\omega} = -\frac{\dot{W}}{A}\frac{\delta_\omega}{\rho}\frac{V_0 \eta_c H_v f_0 \lambda^2}{C_P T_{T,0}}e^{-\lambda x} \tag{B43}$$

Assuming that the mass flow rate per unit area is constant and noting that the nondimensionalized density is assumed to be constant in the source region, the acoustic pressure can be calculated from equation (B22) as follows for the left-running wave:

$$p_{\omega,L} = \frac{e^{ikx}}{2ik}\frac{\dot{W}}{A}\frac{\delta_\omega}{\rho}\frac{V_0 \eta_c H_v f_0 \lambda^2}{C_P T_{T,0}}\int_0^{L_b} e^{-(\lambda+ik)\xi}d\xi \tag{B44}$$

Performing the integration and substituting the upper limit at $\xi = L_b$ yield zero since $e^{-\lambda L_b}$ is approximately zero. Substituting the lower limit $\xi = 0$ yields

$$p'_{\omega,L} = \frac{\eta_p \eta_c}{2}\frac{\delta_\omega}{\rho}\frac{\dot{W}}{A}\frac{V_0 H_v f_0}{C_P T_{T,0}}\frac{\lambda^2 e^{ikx}}{ik(\lambda+ik)} \tag{B45}$$

where η_p is defined as the acoustic pressure efficiency for the conversion of turbulence to acoustic pressure.

$$\eta_p \equiv \frac{p'_\omega}{p_\omega} \tag{B45a}$$

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The term η_p has been incorporated into equation (B45) to account for the fact that only a fraction of the turbulent density fluctuation is converted to acoustic pressure.

Equation (B45) converted to polar form becomes

$$p'_{\omega,L} = \frac{\eta_p \eta_c}{2} \frac{\delta_\omega}{\rho} \frac{\dot{W}}{A} \frac{V_0 H_\nu f_0}{C_P T_{T,0}} \frac{\lambda}{k} \frac{e^{j[kx - \pi/2 + \tan^{-1}(-k/\lambda)]}}{\sqrt{1 + (k/\lambda)^2}} \quad (B46)$$

The sound pressure level is

$$\mathrm{SPL} = 20 \log_{10} \frac{\eta_p \eta_c}{2} \frac{\delta_\omega}{\rho} \frac{\dot{W}}{A} \frac{V_0 H_\nu f_0}{C_P T_{T,0}} \frac{\lambda}{k} \frac{\Delta f}{\sqrt{1 + (k/\lambda)^2}}$$

$$- 20 \log_{10} p_{\mathrm{ref}} \quad (B47)$$

The rate of decay of fuel droplets as they pass through the combustion zone is, by assumption, proportional to the exponential decay factor λ, defined by equation (B38). The value of λ must be such that the exponential function

$$f(x) = e^{-\lambda(x/L_b)}$$

is approximately zero so that equation (B38) will yield negligible heat addition at the end of the combustion zone, $x = L_b$. If the value of the exponent is 2π, the $f(x)$ becomes 0.0019, and this indicates that practically no heat energy is released past the end of the combustion zone. For $\lambda = 2\pi/L_b$, the frequency parameter k/λ equals $f L_b/C_0$. This can be used to simplify the SPL calculation (eq. (B47)). For source $\varphi_{3,\omega}$ it can be shown by solving equation (B23) for the acoustic pressure in the right-running wave that the sound pressure level of the right-running wave equals that of the left-running wave given by equation (B47). However, it should be noted that the phase angle for the left-running wave is given by

$$\theta_L = kx - \frac{\pi}{2} - \tan^{-1}\left(\frac{k}{\lambda}\right)$$

and the right-running wave phase angle is

$$\theta_R = -kx - \frac{\pi}{2} + \tan^{-1}\left(\frac{k}{\lambda}\right)$$

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15. Supplementary Notes

16. Abstract

The first-order equations for the conservation of mass and momentum in differential form are combined for an ideal gas to yield a single second-order partial differential equation in one dimension and time. Small-perturbation analysis is applied. A Fourier transformation is performed that results in a second-order, constant-coefficient, nonhomogeneous equation. The driving function is taken to be the source of combustion noise. A simplified model describing the energy addition via the combustion process gives the required source information for substitution in the driving function. This enables the particular integral solution of the nonhomogeneous equation to be found. This solution multiplied by the acoustic pressure efficiency predicts the acoustic pressure spectrum measured in turbine engine combustors. The prediction was compared with the overall sound pressure levels measured in a CF6-50 turbofan engine combustor and found to be in excellent agreement.

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